Cylindrical magnetic domains

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This review is devoted to a systematic exposition of theoretical and experimental results related to the physics of cylindrical magnetic domains (bubbles). The stability problem is discussed for an individual cylindrical domain and for a lattice of such domains, with allowance for the effect of the coercivity on the stability and dynamics of cylindrical domains. A detailed discussion is given of the laws of motion of cylindrical magnetic domains under the influence of inhomogeneities of magnetic field, temperature, and plate thickness. Phase transitions in a lattice of domains are considered, and the singularities of the magnetization and magnetic susceptibility in such phase transitions are determined. A theory is given for waves propagated in a lattice of domains. Detailed consideration is given to the effect of external factors and of the parameters of the magnetic material on the dynamical properties of a lattice of domains.

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CONTENTS

INTRODUCTION

Investigation of the magnetic phenomena and processes that lead to magnetization of matter has its roots in extreme antiquity.[1)] Modern scientific descriptions of the physics of magnetism are based on ideas that originated after the creation of quantum mechanics. The first quantitative theory of magnetism, based on the concept of the existence of a "molecular field," was constructed by Weiss in 1907.[3] The nature of this field was revealed only in 1927–1928, after the fundamental work of Heisenberg,[4] Dorfman,[5] and Frankel.[6] As a result of these investigations it became clear that the interaction of atomic magnetic moments with Weiss's molecular field is an approximate description (quite good) of the more complicated atomic exchange interaction, which is of electrostatic nature and is due to the quantum character of the motion of electrons in a solid.

In addition to exchange interaction, the magnetic anisotropy energy exerts a significant influence on the magnetic properties of materials; this is energy that depends on the orientation of the magnetic moments of the atoms with respect to the crystallographic directions. This energy was first studied by Akulov.[7] Investigation of the interaction of variations of the magnetic moments of the atoms with variations of the lattice had its beginning with works of Akulov[8] and Heisenberg.[9]

It is well known[1] that magnetically ordered bodies of finite dimensions possess, as a rule, a domain structure. Within each of the domains, the magnetization is constant both in magnitude and in direction. On transition from one domain to another, the direction of the magnetization changes abruptly. The idea of a domain structure was introduced by Weiss[3] to explain the processes of magnetization and demagnetization of ferromagnets. An important step in the study of domains was taken by Bloch[10] and Néel,[11] who found the law of varia-

[1)] Interesting and complete historical sketches on magnetism may be found in the monographs of Vonsovskiĭ[1] and of Mattis.[2]

tion of the magnetization within a domain wall, determined the energy stored in the boundaries between the domains, and showed that the change of magnetization direction occurs over distances that are small in comparison with the dimensions of the domains but large in comparison with the interatomic distances.

In the fundamental work of Landau and Lifshitz[12] it was shown that the formation of a domain structure in a crystal corresponds to a decrease of the total energy of the specimen by virtue of a decrease of the energy of magnetic dipole interaction. The domain dimensions are determined by competition between the energy of magnetic dipole interaction and the energy necessary for domain boundaries. Their work gave a powerful push to investigations of domain structure. It soon became clear[13,14] that domain structure (especially at the surface of the body) is very complicated, and that its specific form is strongly influenced not only by the orientation of the surface with respect to the crystallographic axes, but also by the purity of the surface, surface deformations, etc. Important results relating to the magnetization curve were obtained in papers of Akulov,[15] Kondorsky,[16] Bitter,[17] v. Hámos and Thiessen,[18] Kersten,[19] Bozorth,[2b] and others.

In[15] it was shown that the magnetization process is due, in weak fields, to displacement of the domain boundaries, such that the component of the magnetic moment in the direction of the magnetic field increases; and in strong fields, to processes of rotation of the magnetic moment into the direction of the external magnetic field.

In[15,16,17] the use of powders to fix the positions of domain boundaries was proposed, and the method of investigation of domain structures by means of powder patterns (the Akulov–Bitter method) was developed.

In[16,17] a theory was given for interaction of domain walls with crystal defects, and the nature of the coercive force was thereby explained. Under the influence of an external magnetic field, there occurs a simplification of the domain structure by virtue of the natural diminution of those domains in which the magnetization is directed opposite to the external magnetic field. It was established that simplification of the domain structure is accomplished also by virtue of a decrease of the thickness of the walls, which possess considerable magnetic anisotropy. During the last decade, very diversified methods have been applied to the study of domain structures (optical, electron-diffraction, neutron-diffraction, etc.). An important contribution to the development of these trends has been made by Soviet scientists S. V. Vonsovskiĭ, L. V. Kirenskiĭ, R. V. Telesin, Ya. S. Shur, and others.

The first systematic investigations of magnetic domains in thin films, located in an external magnetic field, were made by Kooy and Enz[20] and by Kaczér and Gemperle.[21] In these researches it was shown that under the influence of an external magnetic field, the usual stripe domain structure becomes unstable, and stripes in which the magnetization is directed opposite to the field break up into "drops" of circular shape. In thin films, these "drops" correspond to right circular cylinders extending through the whole film; the magnetization within a cylinder is directed opposite to the external field.

Notable features of these domains, which are called in our literature cylindrical magnetic domains (CMD) and in the foreign literature "bubbles," are their high mobility and their small size (down to microns).

Bobeck[22] called attention to the fact that these properties of CMD can be used for transmission and recording of information in computers. His work[22] fulfilled the role of an extraordinary trigger for a boom in investigations of CMD, both of fundamental and of applied character (but more of the latter). At present the total number of articles of CMD is about a thousand; they are dispersed among journals of various kinds, including very specialized ones. At the same time, a whole series of questions on the physics of CMD is of general interest. Such questions include primarily the problem of the stability of an individual CMD, the dependence of the dimensions of CMD on the magnetic field, the dynamics of an individual CMD, the problem of interaction of CMD with each other and formation of plane lattices of CMD, waves in CMD lattices, and a number of other problems.

The experimental and theoretical results so far obtained are in fair agreement and encourage confidence that understanding has been achieved of the fundamental laws of formation, existence, dynamics, and collective properties of CMD.

The present paper is a review of the foundations of the theory and of the most important experiments on CMD. The review is divided into two parts; in one of these (the first), the results of investigations of the static and dynamic properties of isolated CMD are presented; in the other, the properties of CMD lattices are discussed. Finally, the first section of the review serves to remind the reader of the fundamental concepts of the physics of magnetically ordered crystals; it may be omitted by those for whom employment in the physics of magnetism is a profession.

1. ENERGY OF A FERROMAGNET. DOMAIN BOUNDARIES

For description of the domain structure of ferromagnets and ferrites, the concepts of the phenomenological theory of magnetism are sufficient. According to these concepts (see[11,23–35]), the state of a ferromagnet is described by specifying the local magnetic-moment density (the local magnetization) $\mathbf{M}(\mathbf{r})$. The free-energy density can be represented as a series in powers of the magnetization and its derivatives. The ground state of a ferromagnet corresponds to a magnetization uniform over the whole body. Therefore if we are interested in the energy of states close to the ground state, it is appropriate to suppose that these states correspond to small nonuniformities of the magnetization. On the strength of these considerations, we shall represent the free-energy density of a magnet, w, in the form

$$w = AM^{-2}\left(\frac{\partial M_i}{\partial x_i}\right)^2 + K_1 \sin^2\theta - MH_0 + \frac{H_m^2}{8\pi}. \quad (1.1)$$

In this formula, the first term is the energy of exchange origin due to inhomogeneities of magnetization; A is an exchange constant ($A>0$), equal in order of magnitude to $A \approx \varkappa a^2 (T_C/\mu_B M) M^2$, where $\varkappa$ is Boltzmann's constant, T_C is the Curie temperature, μ_B is the Bohr magneton, and a is the lattice constant.

The second term in formula (1.1) is the magnetic-anisotropy energy; K_1 is the first magnetic-anisotropy constant ($K_1 > 0$), and θ is the angle between the axis of easy magnetization and the magnetization direction.

The third term is the Zeeman energy, and the last term is the energy of the field produced by the magnetic moments of the atoms (the energy of magnetic dipole interaction). The field H_m is determined by the equations of magnetostatics and the boundary conditions on the surface of the body. The ground state is the state that corresponds to the minimum of the total energy of the body

$$W = \int_V w\, dV. \quad (1.2)$$

This minimum must be sought under the supplementary condition $\mathbf{M}^2 = \text{const}$. This supplementary condition reflects the fact that the magnetization of a ferromagnet is of spin character. The atomic spins change in magnitude only upon change of the electronic shell, which requires energies considerably exceeding the thermal energy at temperatures below the Curie temperature.

We consider first the simplest case, in which the external magnetic field is absent and the field $\mathbf{H}_m$ also vanishes (we shall discuss later the conditions under which $\mathbf{H}_m = 0$). Then in the expression (1.1) there remain only the first two terms. Supposing for simplicity that the changes of the magnetization direction occur in some fixed plane (for example the plane zoy), so that $M_x = 0$, $M_y = M\sin\theta$, $M_z = M\cos\theta$, and that the angle θ depends only on the coordinate x, we rewrite expression (1.1) in the form

$$w = A\dot{\theta}^2 + K_1 \sin^2\theta, \quad \dot{\theta} \equiv \frac{d\theta}{dx}. \quad (1.3)$$

On varying this expression with respect to the angle θ, we find

$$z_B^2 \ddot{\theta} - \sin\theta \cdot \cos\theta = 0, \quad (1.4)$$

where

$$z_B = \sqrt{\frac{A}{K_1}}.$$

We see that this equation has three uniform solutions:

$$\theta_1 = 0, \quad \theta_2 = \pi, \quad \theta_3 = \frac{\pi}{2}. \quad (1.5)$$

The first two of these correspond to two equal minima of the magnetic anisotropy energy, in accordance with the fact that the axis of easy magnetization does not distinguish directions as "up" or "down," while the third solution corresponds to a maximum of the anisotropy energy.

Besides the uniform solutions (1.5), a nonuniform solution of Eq. (1.4) also exists. The first integral corresponding to it has the form[2)]

$$z_B^2 \dot{\theta}^2 - \sin^2\theta = \text{const}. \quad (1.6)$$

Following Bloch, we shall treat the nonuniform magnetization distribution as a transitional distribution between two uniform solutions with $\theta_1 = 0$ and $\theta_2 = \pi$, which occupy the regions in the body with $x = +\infty$ and $x = -\infty$ respectively. In order that the condition (1.6) may describe this transition layer, it is necessary to set the constant of integration equal to zero. On integrating the first-order equation thus obtained, we find[10]

$$\operatorname{tg}\frac{\theta}{2} = \exp\left(-\frac{x}{z_B}\right). \quad (1.7)$$

The quantity z_B obviously has the meaning of thickness of the transition layer; or, as it is usually stated,

$$z_B = \sqrt{\frac{A}{K_1}} \quad (1.8)$$

is the thickness of the Bloch boundary between domains (see Fig. 1). This thickness is directly proportional to the square root of the ratio of the exchange energy to the anisotropy energy. On using the estimate for A and noting that for most films $K_1 = \beta M^2$, where $\beta \approx 10$–100, we obtain for z_B the following estimate: $z_B \approx \sqrt{T_C/\beta\mu_B M a} \approx (10 \text{ to } 100)a$; that is, the thickness of a Bloch wall amounts to ten to a hundred interatomic layers.

We shall now discuss the conditions under which a Bloch wall occurs. By noting that $M_x = 0$, $M_y = M\sin\theta$, $M_z = M\cos\theta$ and that $\theta = \theta(x)$, one can easily show that $\operatorname{div}\mathbf{M} = 0$. Therefore to find the field $\mathbf{H}_m$ one must start from the equations $\operatorname{div}\mathbf{H}_m = 0$, $\operatorname{rot}\mathbf{H}_m = 0$. In conjunction with zero boundary conditions, which are realized for example in a plate with surfaces parallel to the plane zoy, this gives $\mathbf{H}_m = 0$.

In the general case, the magnetization distribution in a specimen is determined from the equations[11]

$$AM^{-2}\Delta\mathbf{M} + \mathbf{H}_i + K_1 M^{-2}\,\mathbf{n}(\mathbf{nM}) + \lambda\mathbf{M} = 0, \quad (1.9)$$
$$\operatorname{rot}\mathbf{H}_m = 0, \quad \operatorname{div}\mathbf{H}_m = -4\pi \operatorname{div}\mathbf{M},$$

where $\mathbf{H}_i = \mathbf{H}_0 + \mathbf{H}_m$, and the corresponding boundary conditions[3)]

$$\mathbf{H}_{m,t}^{(i)} = \mathbf{H}_{m,t}^{(e)}, \quad (\mathbf{H}_m + 4\pi\mathbf{M})_n^{(i)} = \mathbf{H}_n^{(e)}, \quad \frac{\partial\mathbf{M}}{\partial x_n} = 0, \quad (1.9')$$

the indices n and t denote components normal and tangential to the interface, the upper indices i and e denote

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[2)] This integral becomes especially lucid if we consider w as the Lagrangian function $w = L = T - U$ with kinetic energy $T = A\dot{\theta}^2$ and potential energy $L = -K\sin^2\theta$.

[3)] The Lagrangian multiplier λ takes account of the condition $M^2 = \text{const}$.

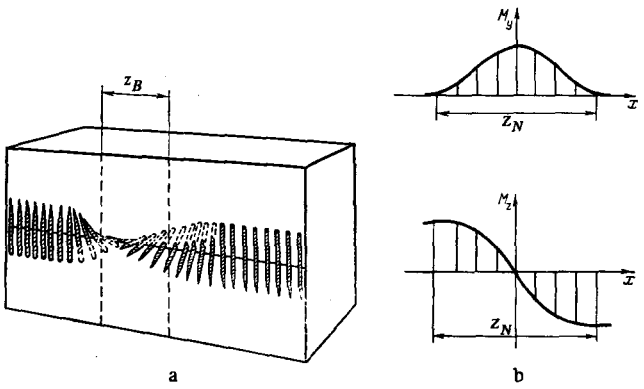

FIG. 1. a) Bloch wall; b) Néel wall.

the fields inside and outside the body.

The system of equations (1.9) is called the "micromagnetic" equations, and in the general case their solution is a very complicated problem. Therefore one usually does not solve the system of equations (1.9) but, with the aid of Bloch walls, selects various domain structures and, following[12], uses the parameters of these structures as variational parameters.

We shall present one other solution of the system (1.9) in an infinite body; like the distribution (1.7), it describes the transitional layer between regions with $\theta_1 = 0$ and $\theta_2 = \pi$. As before, we set

$$M_z = M\cos\theta, \quad M_y = M\sin\theta, \tag{1.10}$$

but now

$$\theta = \theta(y).$$

With this choice of the dependence of the magnetization on the coordinates, the equations of magnetostatics take the form

$$\frac{dH_{m,y}}{dy} = -4\pi M\frac{d}{dy}\sin\theta, \quad \frac{dH_{m,x}}{dy} = 0, \quad \frac{dH_{m,z}}{dy} = 0. \tag{1.11}$$

Hence

$$H_{m,x} = H_{m,z} = 0, \quad H_{m,y} = -4\pi M\sin\theta(y). \tag{1.11'}$$

The choice of the solution (1.11') for the magnetic field corresponds to the boundary condition that the magnetic field vanishes far from the domain boundary. On substituting (1.11') in (1.1), we get

$$w = A\left(\frac{d\theta}{dy}\right)^2 + (K_1 + 2\pi M^2)\sin^2\theta. \tag{1.12}$$

This expression differs from (1.3) by replacement of x by y and of K_1 by $K_1 + 2\pi M^2$. Therefore

$$\operatorname{tg}\frac{\theta}{2} = \exp\left(-\frac{y}{z_N}\right), \tag{1.13}$$

where

$$z_N = \sqrt{\frac{A}{K_1} + 2\pi M^2}. \tag{1.14}$$

This distribution of the magnetization in the transitional layer between domains was found by Néel, and the "wall" corresponding to it is called a Néel wall. The quantity z_N has the meaning of width of the Néel transitional layer (see Fig. 2b). In contrast to the Bloch wall, the Néel wall has on it local magnetic charges, whose density is $\rho_m = \operatorname{div}\mathbf{M} = M(\sin^2\theta)/z_N$. It is easy to see that $\rho_m = 0$ for $y \to \pm\infty$. Therefore the formation of Néel walls entails the occurrence also of a magnetic field H_m. The formation of Néel walls turns out to be preferable in thin films. The distribution (1.13) found by Néel plays a fundamental role also in the theory of CMD. Knowing the distributions (1.7) and (1.13) and using (1.3) and (1.12), one easily finds the energy σ per unit surface of a domain boundary:

$$\sigma = 4\begin{cases}\sqrt{AK_1} & \text{for a Bloch boundary,}\\ \sqrt{A(K_1 + 2\pi M^2)} & \text{for a Néel boundary.}\end{cases} \tag{1.15}$$

The larger value of the surface energy for a Néel boundary is due to the presence in it of a magnetic field H_m.

We shall state briefly the results of this section. The thickness of a domain boundary is a macroscopically small quantity. The magnetization distribution in a wall is of exponential character. These facts enable us to treat the transitional layer as infinitely thin, with a definite surface energy.

2. DOMAIN STRUCTURE AND ITS RESPONSE TO AN EXTERNAL MAGNETIC FIELD

As has already been mentioned in the introduction, the domain structure of real specimens may be very complicated. The variety of forms of domain structure and its "easy vulnerability" to external influences are

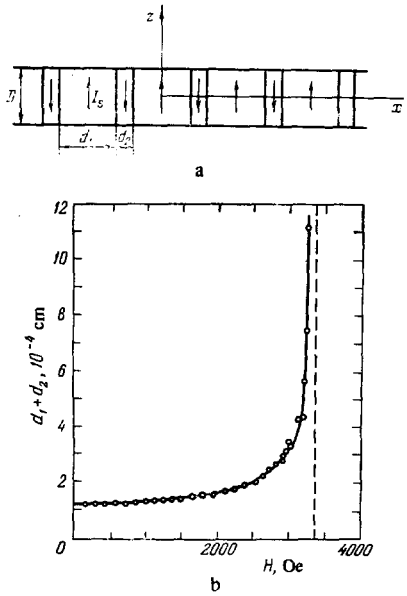

FIG. 2. a) Plane-parallel domain structure in a plate (d_1 is the dimension of a domain in which the magnetization is directed along the field, d_2 of one in which the magnetization is directed opposite to the field); b) theoretical curve and experimental points determining the dependence of the period of the domain structure on the magnetic field.

due to the fact that it is formed because of weak magnetic dipole interactions. Nevertheless, a number of general properties of domain structure can be analyzed quite simply. We shall first consider, for the example of a plane-parallel plate, the problem of the mean dimensions of the domains. We shall suppose that the plate has a rather large magnetic anisotropy energy, $K_1 > 4\pi M^2$, and an axis of easy magnetization (the z axis) perpendicular to the plate surface. Let there be N domains per unit length along the x axis, so that the distance between them is d; then if d is much smaller than the plate thickness h, the magnetic energy can be expressed in the form

$$\mathcal{E} = \sigma h L_y \frac{L_x}{d} + \xi M^2 d^2 L_y \frac{L_x}{d}. \qquad (2.1)$$

In this expression, $\sigma L_y h$ is the energy associated with a single domain boundary, L_x/d is the number of domain boundaries in the specimen, $\xi M^2 d^2 L_y \approx \int (H_m^2/8\pi)\, dV$ is the energy of magnetic dipole interaction associated with a single domain (the field $H_m \approx M$ is concentrated in a surface region with dimensions d^2 in the plane zOx and extending along the domain walls, that is over a distance L_y), and ξ is a numerical factor of order of magnitude unity. The minimum of the energy (2.1) occurs for domain dimensions of the order

$$d \approx \sqrt{\frac{\sigma}{M^2} h}. \qquad (2.2)$$

The conclusion that the domain dimensions increase with the linear dimensions of the specimen is in good agreement with experimental data. The relation (2.2), however, becomes incorrect both for specimens of sufficiently small dimensions and for bulk specimens. If the linear dimensions of the body are less than a characteristic length $l = \sigma/4\pi M^2$ for the magnetic energy, then formation of domain walls in it becomes so unfavorable that domains do not occur. Specimens of these dimensions are magnetized uniformly. In specimens of large dimensions, there begins a branching of the domain boundaries at the surface, and the dependence $d \sim h^{1/2}$ changes to the slower dependence $d \sim h^{1/3}$.[13]

Under the influence of an external magnetic field there is a change not only of the dimensions of those domains in which the magnetization is directed opposite to the field, but also of the period of the domain structure. The change of both these quantities was investigated, both experimentally and theoretically, in[20,21]. The results are shown in Figs. 2a and 2b. At fields $H_0 \approx 4\pi M$, when the width of a domain magnetized opposite to the field becomes sufficiently small (of order l), instability of the stripe domain occurs,[26] and the long stripe domain breaks up into separate cylindrical domains of circular cross section. Because of the magnetic dipole repulsive force, they separate from each other and distribute themselves more or less uniformly over the whole surface of the plate. This phenomenon was first observed in[20]. The dimensions of the domains thus formed are tens of microns. A hexagonal lattice of cylindrical domains was discovered in[21]. The properties of CMD show up most clearly in thin magnetic plates of thickness $h \approx l$. The dimensions of CMD in such plates are of the same order as the thickness of the plate.

3. ENERGY AND EQUILIBRIUM DIMENSIONS OF A CYLINDRICAL MAGNETIC DOMAIN

In this section we shall consider the static properties of an individual CMD; that is, we shall suppose that the distance between the domains is much larger than the radius of a domain and that their interaction may be neglected.

In a monocrystalline, magnetically uniaxial plate (film) of thickness h, with the plane of the surface perpendicular to the axis of easy magnetization (AEM), along which is directed an external magnetic field of intensity H_0, coinciding in direction with the z axis, let there be an isolated CMD (Fig. 3). We shall find the change of energy of the plate caused by formation of the domain[27,28]. What energies must be taken into account can be easily understood by starting from the following qualitative considerations. The presence of domain walls leads to a positive energy E_W due to them. The value of E_W is proportional to the area of the domain boundaries, so that the domain will strive to decrease its dimensions in order to decrease E_W as much as possible. But decrease of the dimensions of the CMD leads to increase of the energy E_M of magnetostatic dipole interaction, which is unfavorable. In the absence of an external magnetic field, the necessity for decrease of E_M will lead to a spreading and distortion of the CMD. In order that this may not occur, there must be an external magnetic field directed opposite to the magnetization of the CMD. A gain in the energy of magnetization in this field obviously occurs on compression of the CMD. Thus the equilibrium dimensions of the CMD are determined by competition between the energy of magnetostatic interaction, on the one hand, and the energy of the domain walls and the energy of magnetization in the external magnetic field, on the other.

For the further discussion, we make the following simplifying assumptions:

a) The ferromagnetic plate is infinite in the plane perpendicular to the z axis. This permits us to suppose that all positions of the CMD in the plane of the plate are equivalent. In other words, when the CMD is displaced in the plane of the plate, its energy does not change (translational invariance).

b) The thickness of the domain boundary is much smaller than the domain dimensions, and its energy σ per unit area is independent of the curvature of the domain wall, of the coordinates, and in a monocrystalline

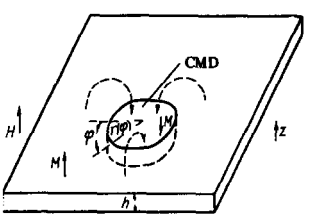

FIG. 3. Cylindrical magnetic domain.

TABLE I. Films of rare-earth garnets for CMD devices.

Film material	Substrate material	Orientation in direction	Method of preparation	CMD diameter, μm	Characteristic length l, μm	Saturation magnetization, G	Anisotropy field, Oe
$Tb_{2.4}Er_{0.6}Fe_5O_{12}$	$Sm_3Ga_5O_{12}$	(100)	Chemical deposition	8	1.0	220	4000
$Y_3Ga_xFe_{5-x}O_{12}$, $0.4 \leqslant x \leqslant 1.4$	$Gd_3Ga_5O_{12}$	(110)	Chemical deposition	5–10	5	500	500
$Eu_2ErGa_{0.7}Fe_{4.3}O_{12}$	$Gd_3Ga_5O_{12}$	(111)	Chemical deposition	5–10	5	500	500
	$Gd_3Ga_5O_{12}$	(110)	Epitaxy from liquid phase	6–17	0.7	173	4000
$EuEr_2Ga_{0.7}F3_{4.3}O_{12}$	$Gd_3Ga_5O_{12}$	(111)	Epitaxy from liquid phase	...	2.2	120	6500
$Gd_{3-x}Tb_xFe_5O_{12}$	Nd_3GaO_{12}	(111)	Epitaxy from liquid phase	...	...	...	...
$Y_{3-x}Gd_xFe_5O_{12}$	$Gd_3Ga_5O_{12}$	(111)	Epitaxy from liquid phase	...	...	...	...

plate also of the orientation of its individual sections with respect to the crystallographic axes.

c) The domain has the form of a right cylinder with axis perpendicular to the plate surface (the curvature of the walls along the axis vanishes).

d) The magnetization **M** at all points of the plate is directed strictly along the z (or $-z$) axis.

Later we shall discuss the consequences of abandoning some of these assumptions. Here, however, anticipating a bit, we may say that the consequences of a theory based on these assumptions agree well with experimental data. In particular, it is clear that fulfillment of assumption d) requires materials with a sufficiently large anisotropy constant. Table I shows values of the anisotropy field and of the magnetization of various materials. From this table it is evident that there are a number of ferrites and ferromagnets in which the anisotropy constant is sufficiently large. If the diameter of the CMD is denoted by d, then obviously

$$E_W = \pi\, dh\sigma = (2\pi M)^2\, dhl,$$
$$E_H = 2MH_0 \cdot \frac{1}{4}\pi d^2 h = \frac{1}{2}(2\pi M)^2 d^2 h \frac{H_0}{4\pi M}. \qquad (3.1)$$

The calculation of the energy of magnetic dipole interaction is a somewhat more complicated problem than the calculation of the surface energy of the CMD and of the energy of magnetization in the external field $\mathbf{H}_0$. This is due to the fact that the energy E_M is not expressed directly in terms of the shape of the CMD and the magnetization but is determined by the field $\mathbf{H}_m$, to find which it is necessary to solve the appropriate magnetostatic problem. Without going into the solution of this problem which is carried out in standard fashion,[14] we shall give the final expression for the energy E_M[27]:

$$E_M = -(2\pi M)^2 h^3 I(x), \qquad (3.2)$$

where $x = d/h$,

$$I(x) = \frac{1}{2}x^2 - \frac{2}{3\pi}x^3 + \frac{1}{3}x^3 \int_0^\infty J_1^2(y)\, y^{-2} \exp\left(-\frac{2y}{x}\right) dy \qquad (3.3)$$

and $J_1(y)$ is the first-order Bessel function. Thus formation of the CMD entails the energy

$$E = E_W + E_H + E_M. \qquad (3.4)$$

To the minimum of the energy (3.4) corresponds a domain diameter determined by the relation

$$\frac{l}{h} + \frac{xH}{4\pi M} - \frac{dI}{dx} = 0. \qquad (3.5)$$

It was first obtained by Thiele.[27,28] Equation (3.5) determines the dependence of the CMD diameter on the value of the magnetic field, the plate parameters, and the characteristics of the material. The function $d = d(H)$ is shown in Fig. 4. Since the function $I(x)$ contains no parameters and $H \approx 4\pi M$, it follows from (3.5) that $x \approx (l/h)\xi$ or $d \approx \xi l$ (ξ is a numerical parameter of order of magnitude 1 to 10); that is, the dimensions of equilibrium CMD are of the same order as l (or an order or two larger than l). Equation (3.5) is a necessary but by no means a sufficient condition for a minimum of the CMD energy. In order to clarify the sufficient conditions for a minimum of the CMD energy, it is necessary to consider the stability of the CMD with respect to small deformations of its generators. Here one must

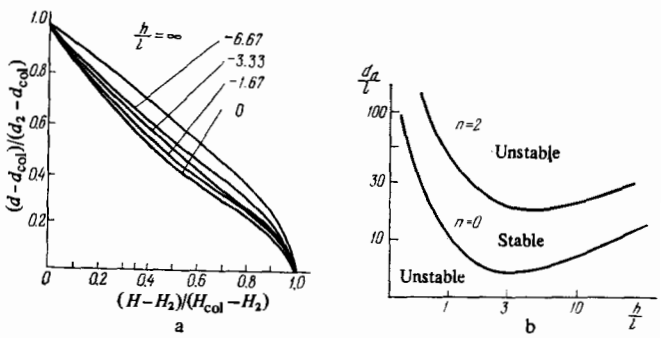

FIG. 4. a) Dependence of CMD diameter on magnetic field; b) dependence of diameters for collapse and for elliptic instability on the ratio h/l.

expect at least two variants of the onset of instability of the CMD. The first of these should bound the magnetic-field range for existence of a CMD from above. Specifically, in sufficiently strong fields the plate should be magnetized uniformly; that is, CMD must disappear. The fields at which collapse of the CMD occurs have been named collapse fields, H_{col}. The second variant of instability is related to fields that bound the range of existence of CMD from below. Specifically, in sufficiently weak fields a stripe domain structure is stable. Therefore there should be a field H_2 at which the CMD undergoes deformations that tend to convert it from a circle to a figure elongated along some direction, for example an ellipse. A detailed analysis carried out by Thiele[27] showed that, in fact, the collapse field and the field at which the circular cross section of the CMD becomes unstable with respect to elliptic deformations are the fields that bound the magnetic-field range in which a CMD is stable. Figure 4 shows the variation of the collapse diameter $d_{col} = d(H_{col})$ and of the elliptic-instability diameter $d_2 = d(H_2)$ on the ratio l/h, and Fig. 5 shows the variation of H_{col} and of H_2 with l/h.

An important quantity in the theory of CMD is the field H_c at which the energy of the CMD vanishes. Figure 5 shows a graph of the variation of H_c with l/h.

In the above treatment, it was assumed that the thickness z_B of the domain wall is small in comparison with the dimensions of the CMD. We shall consider the conditions under which this approximation is possible. First of all, it is evident from Fig. 4 that d_{col} as a function of the thickness has a minimum. This minimum value of d_{col}, which we shall denote by $d_{col,min}$, can be calculated and is about $4l$; it occurs in plates of thickness $h = 3.3l$. Thus for validity of the theory developed above, it is necessary that over a wide range of specimen thicknesses $d_{col,min} \gg z_B$. If we use the fact that $z_B = \sqrt{A/K_1}$, this condition takes the form

$$\frac{d_{col,min}}{z_B} = \frac{4\sigma}{4\pi M^2 z_B} \equiv 8q \gg 1, \quad q = \frac{K_1}{2\pi M^2} \equiv \frac{H_A}{4\pi M}. \qquad (3.6)$$

Hence it is evident that the above approximation is valid if q is at least greater than unity; for the majority of materials in which CMD are observed, this is satisfied (see, for example, Table I). Values of H_{col} and d_{col} are given in Table II.

Tu and Lin[29,30] give results of a numerical calculation of the energy of a CMD with allowance for the finite thickness of the domain wall. They show that the thickness of the domain wall may exert a definite influence

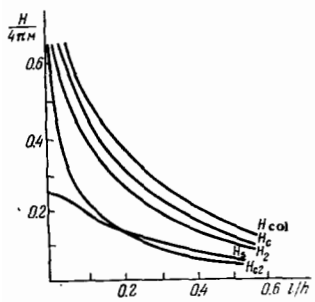

FIG. 5. Variation of critical fields with the ratio h/l.

TABLE II.

Compound	h, μm	$4\pi M$, G	l, μm	H_{col}, Oe	d_{col}, μm
$Y_{2.4}Eu_{0.6}Fe_{3.93}Ga_{1.07}O_{12}$	7.4	210	0.6	120	5
$Y_{2.5}Eu_{0.5}Fe_{4.01}Ga_{0.99}O_{12}$	9.7	290	0.6	180	5
	19	175	0.43	130	6
	7.5	156	0.63	88	6
$Y_{2.6}Eu_{0.4}Fe_{3.83}Ga_{1.17}O_{12}$	6.4	355	0.78	175	3.5

on the conditions for stability of a CMD. Thus with increase of the domain-wall thickness the range of stability of the CMD may broaden. This is due to the fact that by considering the wall infinitely thin we actually overestimate the magnetostatic energy, and the total energy of a specimen containing a CMD is found to be somewhat higher than in the case of a CMD with a finite domain-wall thickness. Investigations of the actual change of the field interval in which CMD are stable were not carried out in[29,30], although it is possible that in some cases these changes may prove important. This is due to the fact that the total energy E of a CMD is considerably smaller than the separate energy contributions and may prove sensitive to the small changes of these contributions that result from allowance for the finiteness of the domain-wall thickness.

Thiele[31] investigated the effect of a small anisotropy of the energy of domain walls on the static properties of CMD; it was shown that the presence of this anisotropy practically always leads to the result that the CMD is not circular in cross section, but elliptic. This ellipticity is least pronounced near the collapse of the CMD; it has practically no effect either on the collapse field or on the diameter of the collapsing domain, and it becomes increasingly noticeable with increasing distance from the collapse field. No conclusion can be drawn, within the framework of the approximations,[28] regarding the effect of this ellipticity on the elliptic-instability field. But experimental data on plates of $TmFeO_3$ (see[28]), in which the ratio of the anisotropic part of the domain-wall energy to the isotropic amounts to 3%, show that the decrease of the ratio of the collapse field to the elliptic-instability field caused by anisotropy of the domain-wall energy does not exceed 1%.

We note finally that allowance for the interaction of the field of the CMD with the plate boundaries may exert an appreciable influence on the stability range of the CMD.[32] This influence can be estimated most simply for a disk on whose center there is a CMD. The corresponding calculations[32] show that the range of existence of CMD broadens and that elliptic instability may not show up down to field $H = 0$. Thus the magnetic-field range for existence of CMD is bounded and determined by the exchange length and the plate thickness. In strong fields the domains collapse; in weak fields they change to stripes.

4. EFFECTS OF DEFECTS ON THE STABILITY OF CYLINDRICAL MAGNETIC DOMAINS

Defects (dislocations, magnetic and nonmagnetic inclusions) that are present in materials with CMD may

have an effect on the dynamics of the CMD, and also on their static properties. The problem of the interaction of the domain wall of a CMD with defects is very important and was already being considered in one of Thiele's first papers.[31] By the force of interaction of a domain boundary with defects we may understand the field that must be applied in order to tear it away from the defect. In general, this quantity will be a function of the point; but if, for simplicity, we suppose that the crystal contains defects of a single kind distributed uniformly, then the field mentioned can be identified with the coercive force H_{coer}, which is characteristic of the whole specimen. A treatment of the stability conditions of a CMD in this case can be carried out by a method similar to that used in treatment of the conditions for growth of nuclei of reverse magnetization.[29]

First of all, we note that defects exert on domain walls an influence analogous to the action of frictional forces, since either increase or decrease of the domain dimensions has a braking effect. Therefore a CMD will be unstable if the force $-dE/dr$ produced by change of the domain dimensions is larger in absolute value than the force of "friction," which according to the assumptions made above can be represented in the form $2\pi rhMH_{coer}$. Contrariwise, stability of the CMD requires[33]

$$\left|\frac{dE}{dr}\right| \leqslant 2\pi rhMH_{coer};$$

here E is determined according to (3.4). Using this, we find that the range of stable CMD dimensions is included between the curves $d^{\pm}(H)$, which can be constructed by numerical solution of the equations

$$\frac{l}{h}+\frac{d}{h}\frac{H\pm H_{coer}}{4\pi M}-\frac{dI}{dx}=0. \tag{4.1}$$

Then $d^{\pm}(H)$ are the solutions of the equation with $H+H_{coer}$ and $H-H_{coer}$, respectively.

Figure 6 shows both curves $d^{\pm}(H)/d$ as they depend on $H/4\pi M$.

In the range $d_{col}<d<d_2$, where the diameters of equilibrium CMD were concentrated in the case $H_{coer}=0$, for every fixed field there will now be contained a certain quantity of stable CMD, differing in diameter. Thus even for uniform H_{coer}, a spread in diameter of stable CMD will be observed at fixed H. But in specimens used practically, this spread may turn out to be small, since one usually tries to make H_{coer} as small as possible.

More interesting is another fact that emerges from consideration of Fig. 6. Thus, for example, when $H_{coer}=0$ the section of the $d(H)$ curve with $d<d_{col}$ (dotted) corresponded to absolute instability of CMD. Now, there occurs here also a whole range of values of diameters of stable CMD, stabilized solely by the presence of H_{coer}. The same is true of the section of the curve above $d=d_2$.

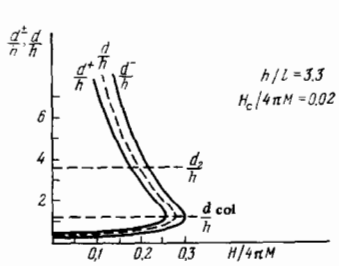

FIG. 6. Variation of CMD diameter with magnetic field, with allowance for the coercive force.

Thus for very small H_{coer}, in principle CMD of two significantly different dimensions should be observed at each fixed H. Such a situation was in fact observed in[34].

In closing this section, we note that the concepts described enable us to suggest a very simple method of determination of H_{coer}, by visual observation of the magnitude of the spread of CMD diameter.

5. DYNAMICS OF AN ISOLATED CYLINDRICAL DOMAIN

In the preceding section, the equilibrium dimensions of an isolated domain were considered, and problems related to the stability of a domain were discussed. But it is clear that the shape of a domain and the stability-field bounds may depend on its velocity of motion. This is due to the fact that a moving CMD possesses kinetic energy, caused by the appearance of an additional magnetic field. The kinetic energy has the form

$$E_{kin}=\frac{1}{2}h\oint m_{ik}v_iv_k\,dl, \tag{5.1}$$

where $\mathbf{v}$ is the velocity of motion of the domain, and where m_{ik} is the tensor density of effective mass per unit surface of the domain boundary[35]; the integration in (5.1) extends over the generators of the domain.[36,37] If the domain boundary is homogeneous (there are no magnetization gradients connected with the azimuthal angle), then the effective-mass tensor has only one nonvanishing component, describing motion along the normal to the surface.[36] Therefore[36,37]

$$E_{kin}=\frac{1}{2}hm\oint(\mathbf{v},\mathbf{n})^2\,dl, \tag{5.2}$$

where $\mathbf{n}(l)$ is the unit normal to the CMD surface at the point l (it is assumed that there is no nonuniformity of the velocity $\mathbf{v}$ along z). We shall further suppose that the velocity of motion of the CMD is small enough so that the magnetic fields that arise because of the motion may be treated as small corrections. As a dimensionless small parameter characterizing these corrections we choose the ratio

$$b=\frac{mv^2}{16\pi hM^2}. \tag{5.3}$$

Allowance for motion of the domain leads not only to a change of the domain dimensions but also to a change of its shape, so that the minimum of the Lagrangian function of the CMD, $L=E_{kin}-E_{pot}$,[38] corresponds to a generator described by the formula[37,39]

$$r(\varphi)=r_0+\frac{1}{2}bh\left[S_0\left(\frac{d}{h}\right)-\frac{l}{h}\right]^{-1}-2bd\left[\frac{l}{h}-S_2\left(\frac{d}{h}\right)\right]^{-1}\cos\varphi. \tag{5.4}$$

In this formula $r(\varphi)$ is the distance in the plane xOy

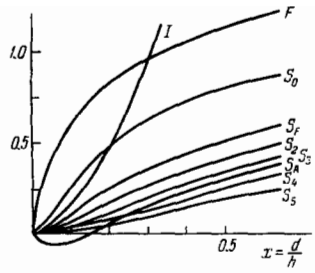

FIG. 7. Graphs of the functions S_n, S_A, S_F, I, and F.

from the CMD axis to its generators, φ is the azimuthal angle measured from the direction of motion of the domain, and $S_n(x)$ are special functions encountered in the theory of CMD. Their graphs are shown in Fig. 7. The function S_0 determines the stability of the CMD with respect to collapse, and the function S_2 with respect to elliptic instability; specifically, for stability of the domain the following conditions must be satisfied:

$$S_0\left(\frac{d(H)}{h}\right)-\frac{l}{h}>0,\qquad S_2\left(\frac{d(H)}{h}\right)-\frac{l}{h}<0. \tag{5.5}$$

From formula (5.4) we see that the domain elongates across the direction of motion, and that the role of the corrections due to the motion is especially important near the stability bounds; that is, near the collapse field H_{col} and the elliptic-instability field H_2.

It is not difficult to determine the effect of the domain motion on the collapse field H_{col} and the critical domain dimension r_{col}. Appropriate calculations show that the collapse field increases under the influence of the motion, so that

$$H_{col}(v)=H_{col}(0)+\frac{\pi m l^2}{2r_{col}}, \tag{5.6}$$

and that the dimensions of collapsing domains decrease:

$$r_{col}(v)=r_{col}(0)-\frac{1}{2}\frac{hb}{S_0'(d_{col}/h)}. \tag{5.7}$$

Analogous results can be obtained also for the elliptic-instability field H_2.

We shall now discuss free oscillations of domain walls.[36,40,41] The simplest oscillations are oscillations of the walls that are uniform through the thickness of the plate. To describe them, it is sufficient to consider only small deformations $\Delta(\varphi)$ of the CMD generators,

$$r(\varphi)=r_0+\Delta(\varphi)=r_0+\sum_{n=-\infty}^{\infty}\Delta_n e^{in\varphi}. \tag{5.8}$$

The expansions of the potential and kinetic energies in powers of Δ_n have the form

$$E_{pot}=2(2\pi M)^2h^2\sum_{n=-\infty}^{\infty}(n^2-1)\left[\frac{l}{h}-S_n\left(\frac{d}{h}\right)\right]\frac{h}{d}\left|\frac{\Delta_n}{h}\right|^2, \tag{5.9}$$

$$E_{kin}=\frac{\pi}{2}m\,dh\sum_{n=-\infty}^{\infty}|\dot\Delta_n|^2, \tag{5.10}$$

where

$$S_n(x)=\frac{x^2}{n^2-1}\int_0^\infty[J_1^2(y)-J_n^2(y)](1-e^{-2y/x})\,dy$$

and where J_n is the n-th order Bessel function. Thus the total energy of a domain with oscillating walls is[40]

$$\Delta E=\frac{1}{2}\pi m\,dh\sum_{n=-\infty}^{\infty}[|\dot\Delta_n|^2+\omega_n^2|\Delta_n|^2], \tag{5.11}$$

where

$$\omega_n^2=(4M)^2\frac{\pi(n^2-1)h}{md^2}\left[\frac{l}{h}-S_n\left(\frac{d}{h}\right)\right]. \tag{5.11'}$$

From these formulas it is evident that the energy of a domain with oscillating walls is the sum of the energies of oscillators with frequencies ω_n and that the functions S_n have a simple meaning: they determine the elastic constants of the domain walls with respect to deformations $\Delta r_n=\Delta_n e^{in\varphi}+\Delta_n^* e^{-in\varphi}$. The frequency of the first harmonic is $\omega_1=0$; this is a consequence of the translational invariance of the domain energy. At large values of the number n, the frequency of the oscillations increases almost according to the linear law $\omega_n\sim n$. We note that the frequency ω_0 vanishes at $H=H_{col}$ and frequency ω_2 at $H=H_2$. The vanishing of the frequencies is due to the fact that the fields H_{col} and H_2 are bounds to the stability of the CMD. The dependence of the frequencies ω_n on the external magnetic field is shown in Fig. 8 for a plate with thickness $h=3l$ and with $K_1=8\pi M^2$. The oscillations under consideration may be interpreted as waves propagated along the contour of the CMD. This is evident from the formula that determines the dependence of a point on the contour upon the coordinate φ and the time t:

$$r(\varphi,t)=r_0+\sum_{n=-\infty}^{\infty}[\Delta_n e^{-i(\omega_n t-n\varphi)}+\Delta_n^* e^{i(\omega_n t-n\varphi)}]. \tag{5.12}$$

Above, we have considered the motion of a CMD with constant velocity and the free oscillations of an isolated CMD. This leaves still to be clarified two questions that are important for CMD dynamics: what is the nature of the frictional force and of the external forces that act on a CMD? We shall first consider the question of the external forces. In order to determine the force it is necessary, as is well known, to differentiate the energy E of the CMD with respect to the coordinate of the "center of gravity": that is, with respect to the coordinate $\mathbf{R}$ of the center of the CMD:

$$\mathbf{F}=-\frac{\partial E}{\partial \mathbf{R}}=-\nabla E. \tag{5.13}$$

The energy of the CMD was found earlier, in Sec. 1. Within the framework of the assumptions adopted there, the equilibrium position of the CMD is neutral with re-

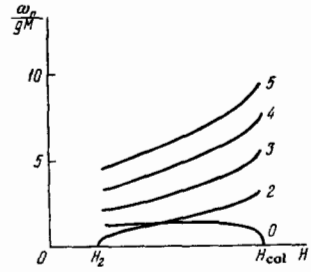

FIG. 8. Dependence of frequencies ω_n ($n=0,2,3,4,5$) on magnetic field.

spect to translation of the domain in an arbitrary direction in the plane of the plate. The picture changes in the presence of a gradient of the external magnetic field H or of the temperature T. If these gradients are small enough so that a characteristic distance of change of H and T is considerably larger than the CMD radius, then it may be supposed that the energy of the CMD is, as before, determined by formula (3.4); but the parameters h, M, H, and σ are to be interpreted as their local values, that is the values at the point where the CMD is located. Therefore

$$\nabla E = \frac{\partial E}{\partial h}\nabla h + \frac{\partial E}{\partial H}\nabla H + \frac{\partial E}{\partial M}\nabla M + \frac{\partial E}{\partial \sigma}\nabla \sigma + \frac{\partial E}{\partial r}\nabla r. \qquad (5.14)$$

Since the radius of the domain is determined by the condition $(\partial E/\partial r) = 0$, we have for the force acting on the domain[42]

$$\mathbf{F} = -\frac{\partial E}{\partial h}\nabla h - \frac{\partial E}{\partial M}\nabla M - \frac{\partial E}{\partial H}\nabla H - \frac{\partial E}{\partial \sigma}\nabla \sigma \qquad (5.15)$$

(we recall that h is the plate thickness, H is the external magnetic field, M is the magnetization, and σ is the surface energy density of the domain wall). This formula determines the force acting on the CMD.

We shall discuss in greater detail the action of the gradient of the external magnetic field. Let $\nabla h = \nabla M = \nabla \sigma = 0$. Then

$$\mathbf{F} = -\frac{\partial E}{\partial H}\nabla H. \qquad (5.16)$$

In this formula, the derivative of the energy with respect to magnetic field is taken at fixed values of the remaining CMD parameters (h, M, σ, r); that is, only the Zeeman part of the total CMD energy is subject to differentiation:

$$\frac{\partial E}{\partial H} = \frac{\partial E_H}{\partial H} = 2\pi r^2 h M$$

and

$$\mathbf{F} = -2\pi r^2 h M \nabla H. \qquad (5.17)$$

Thus we see that the force $\mathbf{F}$ is directed opposite to ∇H and that under the action of this force, the CMD is displaced into a weak-field region; the value of the magnetic field $H(R)$ plays for a CMD the same role as does the potential energy $U(R)$ for a material point.

If there is a temperature gradient in the body, then it produces gradients of M, σ, and h, and in this case

$$\mathbf{F} = -\left(\frac{\partial E}{\partial h}\frac{\partial h}{\partial T} + \frac{\partial E}{\partial M}\frac{\partial M}{\partial T} + \frac{\partial E}{\partial \sigma}\frac{\partial \sigma}{\partial T}\right)\nabla T. \qquad (5.18)$$

The terms that enter in this formula may have different signs, and the resultant sign depends on the properties of the specific materials. Thus, for example, in the orthoferrite $Sm_{0.55}Tb_{0.45}FeO_3$[42] the resultant sign of the terms in parentheses is positive, and the domains move opposite to the temperature gradient, i.e. from hot to cold parts of the body. In the garnet $Gd_{1.3}Tb_{0.7}Fe_5O_{12}$ the resultant sign of the terms is negative, and the force $\mathbf{F}$ is directed along the temperature gradient; that is, under the action of this force the domains move from cold to hot parts. The CMD property of moving from hot sections to cold or vice versa can be used to control the motion of CMD by means of a laser beam.

We pass now to consideration of the frictional force that acts on CMD. It can be exhibited in the form of a sum of two forces: a force of "static friction," which under certain assumptions can be identified with the coercive force, and a force of viscous friction, proportional to the velocity of the CMD.

The force of "static friction" can be represented in the form

$$\mathbf{f}^n_{fr} = -\pi\, dh F_{coer}\frac{\mathbf{v}}{v}. \qquad (5.19)$$

If for simplicity we suppose that the defects in the plate are distributed uniformly and that the character of f^s_{fr} is dependent on the surmounting of barriers in the direction normal to the CMD surface, then $F_{coer} = (4/\pi)MH_{coer}$. As regards the force proportional to the velocity, in writing down an expression for it we must remember that we are dealing with a frictional force in a magnetically polarized medium, characterized by magnetization $\mathbf{M}$. Therefore in obtaining a formula for such a force, we shall turn to the general relations of the thermodynamics of irreversible processes.

We shall denote the frictional force being sought by $\mathbf{f}_v$. Then the dissipative function $\dot{Q} = T\dot{s}$ is obviously

$$\dot{Q} = T\dot{s} = \mathbf{f}_v\mathbf{v}. \qquad (5.20)$$

If we choose as generalized fluxes the components of the vector velocity, which also describe the deviations from a state of thermodynamic equilibrium, then the components of the frictional force will be, except for a factor T^{-1}, the components of the corresponding generalized forces.[4)] Therefore

$$v_i = \Gamma_{ik} f_{v,k}, \qquad f_{v,k} = -\gamma_{kl} v_i \pi\, dh. \qquad (5.21)$$

The kinetic coefficients Γ and γ satisfy the Onsager symmetry relations $\Gamma_{ik}(\mathbf{B}) = \Gamma_{ki}(-\mathbf{B})$, $\gamma_{ik}(\mathbf{B}) = \gamma_{ki}(-\mathbf{B})$, where $\mathbf{B}$ is the magnetic induction. On separating the symmetric and antisymmetric parts of γ_{ik}, as in[43], we can represent $\gamma_{ik}(\mathbf{B})$ in the form

$$\gamma_{ik}(B) = \left(\gamma_0 \varepsilon_{ikl} B_l + \frac{1}{\eta}\delta_{ik}\right), \qquad (5.22)$$

where γ_0 and η are even functions of B. Therefore

$$\mathbf{f}_v = -\left(\gamma_0[\mathbf{v}\times\mathbf{B}] + \frac{1}{\eta}\mathbf{v}\right)\pi\, dh. \qquad (5.23)$$

On adding to this expression the term (5.19) that determines the force of "dry" friction, we get

$$\mathbf{F}_{fr} = -\pi\, dh\left(F_{coer}\frac{\mathbf{v}}{v} + \gamma_0[\mathbf{v}\times\mathbf{B}] + \frac{1}{\eta}\mathbf{v}\right). \qquad (5.24)$$

The value of $\mathbf{B}$ that enters in this formula has the

---

[4)] We recall that according to Onsager $\dot{s} = \dot{x}_i X_i$, where $X_i = -\partial s/\partial x_i$, and $\dot{x}_i = \gamma_{ik} X_k$; the kinetic coefficients γ satisfy the symmetry principle; s here is the entropy.

TABLE III. Mobilities of cylindrical domains.

Material	Domain mobility, cm sec^{-1} Oe^{-1}	Material	Domain mobility, cm sec^{-1} Oe^{-1}
$Sm_xTb_{1-x}FeO_3$, $x=0.55$	970	$EuFeO_3$	300
$DyFeO_3$	370	$HoFeO_3$	230

meaning of the mean magnetic induction in the region of the magnet in which the singularity in the magnetization distribution is located. In the case of domain walls, that is of a specimen with a domain structure, the mean value of the magnetic field H inside the specimen is either zero (for thick specimens) or proportional to M (for thin films). Therefore it may be supposed that $B \sim M$, and formula (5.24) may be put into the form[31,44-46]

$$\mathbf{F}_{fr} = -\pi d h \left(F_{coer} \frac{\mathbf{v}}{v} + \gamma_1 [\mathbf{v} \times \mathbf{M}] + \frac{1}{\eta} \mathbf{v} \right). \quad (5.25)$$

This formula determines the structure of the frictional force that acts on a singularity of the magnetization distribution that is being propagated with velocity $\mathbf{v}$ in a magnetically polarized medium.

The coefficients γ_1 and η in the treatment presented here are certain phenomenological parameters. Micromagnetic equations enable us to relate them to the number of so-called Bloch lines, the exchange constant, the anisotropy constant, and the relaxation constant in the equation of motion of the magnetic moment. We note finally that formula (5.25) describes the frictional force in the case of sufficiently small velocities $\mathbf{v}$. Domains in which the magnetization distribution in the walls is such that $\gamma_1 = 0$ are called simple or ordinary CMD. If $\gamma_1 \neq 0$, then domains with such a magnetization distribution in the wall are called hard CMD.[45]

By studying pulsations of CMD, one can determine the coefficient η experimentally from the mobility of the domain walls. Bobeck[46] proposed and used, for determination of η in ordinary CMD, an experiment on observation of collapse. The idea of this experiment is as follows. Let there be a CMD with equilibrium radius r_i corresponding to external field H_i. Then the field is suddenly increased to a value H_f that exceeds the collapse field. This field is kept constant for a time interval t, then suddenly decreased to its original value H_i. It is obvious that there is a time τ such that for $t > \tau$, the CMD succeeds in reaching the instability region (r becomes less than r_{col}), and the domain collapses. If $t < \tau$, then the domain does not succeed in

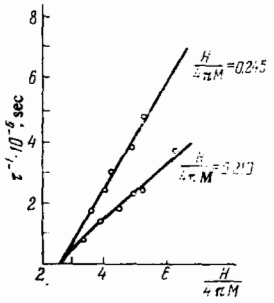

FIG. 9.

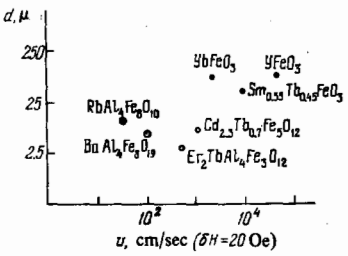

FIG. 10.

collapsing, and it enlarges to its original state r_i. The time τ is determined by the mobility of the CMD and by the values of the original field H_i and of the field H_f. By studying experimentally the variation of τ with the values of H_i and H_f, one can find the mobility μ (see Table III). Figure 9 shows the results of a study of collapse.[47] There are also other methods of determining the mobility of domain walls.[48,49] By equating the frictional force to the force (5.17) exerted by the magnetic-field gradient, we find the velocity of uniform motion of the CMD

$$v = \begin{cases} \frac{1}{2}\mu\left(\delta H - \frac{8}{\pi} H_{coer}\right), & \text{if} \quad \delta H > \frac{8}{\pi} H_{coer}, \\ 0, & \text{if} \quad \delta H < \frac{8}{\pi} H_{coer}, \end{cases} \quad (5.26)$$

where we have introduced the notation $\mu = \eta M$, $\delta H = (dH/dR)d$.

The value of the coercive force is determined by interaction of the domain wall with crystal defects. As regards the viscosity, it is determined by transfer of energy to spin waves and phonons. We shall give expressions for the mobility that are obtained from the equation of motion of the magnetic moment,

$$\dot{\mathbf{M}} = \gamma[\mathbf{M} \times \mathbf{H}_e] + \frac{\alpha}{M}[\dot{\mathbf{M}} \times \mathbf{M}] \quad (5.27)$$

where α is a relaxation constant. For a Bloch wall[5)]

$$\mu = \frac{\gamma}{\alpha}\sqrt{\frac{A}{K_1}}, \quad (5.28)$$

for a Néel wall[5)]

$$\mu = \frac{\gamma}{\alpha}\sqrt{\frac{A}{K_1} + 2\pi M^2}. \quad (5.29)$$

These two formulas can be combined into one by use of the expressions (1.8) and (1.14) for the thicknesses of Bloch and Néel domain walls:

$$\mu = \frac{\gamma z_0}{\alpha}, \quad z_0 = \{z_B, z_N\}. \quad (5.30)$$

It is evident from formula (5.30) that in order to increase the velocity of motion of a CMD (other things being equal),[47,50,51] it is necessary to decrease the relaxation constant α. Table III shows values of the mobility for a number of materials. Figure 10 shows velocities and, simultaneously, diameters of CMD in various materials for $\delta H = 20$ Oe.[67]

We shall now discuss the motion of a hard CMD in a

[5)]See Appendix.

nonuniform external magnetic field. The total force acting on a CMD has the form

$$\mathbf{F}_{\text{total}} = -\frac{\partial E}{\partial H}\nabla H - \pi dh\left(F_{\text{coer}}\frac{\mathbf{v}}{v} + \gamma_1[\mathbf{v}\times\mathbf{M}] + \frac{1}{\eta}\mathbf{v}\right). \quad (5.31)$$

The sum of the first two terms is analogous to the Lorentz force exerted on a charged particle by external electric and magnetic fields. Therefore the motion of a hard CMD should be reminiscent of the motion of a charged particle of external magnetic and electric fields with allowance for friction; that is, the velocity of the domain will have not only a longitudinal but also a transverse component with respect to ∇H. Actually these peculiarities of CMD motion were originally detected experimentally[45] in 1970, in observation of the motion of domains in a monocrystalline plate of $Gd_{2.3}Tb_{0.7}Fe_5O_{12}$, and not theoretically. Different domains behaved differently. One CMD moved to the left of the field gradient, others to the right. The transverse components of the velocity of motion of these domains reached very large values and exceeded the values of the longitudinal components. The largest value of the ratio of velocities was 10 : 1. These domains possessed still another peculiarity, that they were more stable with respect to collapse. Their diameter changed, from nucleation to collapse, by a factor 10. Subsequently, CMD with the same properties were detected in high-quality epitaxial films.

By a series of experiments, it was shown that this property of hard CMD was caused by the structure of their domain walls. Until the discovery of hard domains, it was assumed that the domain boundaries of CMD were simple Bloch boundaries; that is, that all the spins in them lay in planes parallel to the plane of the wall. But transillumination of thin cobalt plates with an electron microscope showed that in the domain boundaries there are transitional regions, separating two neighboring sections of Bloch type with opposite directions of rotation of the spins in the wall. These transitional regions have been called Bloch lines (or Néel segments). The kinetic coefficient γ_1 in formula (5.25) can be simply expressed in terms of the number n of Bloch lines,[44]

$$\gamma_1 = \frac{n+2}{\gamma r}, \quad (5.32)$$

where γ is the gyromagnetic ratio. As regards the ratio between the coefficient $1/\eta$ in the expression (5.25) for the frictional force and the relaxation constant in the equation of motion of the magnetization, it remains, except for corrections of order (z_0/r), the same for hard CMD as for ordinary CMD.[44]

In closing this section, we shall discuss the behavior of a CMD when the frictional force is small enough so that the free path length of a CMD, $\lambda = v\pi$ (π is the braking time of the CMD, determined by the relaxation constant α), is much larger than either the dimension of the CMD or the "Larmor radius" of the domain. Noting that the mass of the CMD is

$$\mathfrak{M} = 2\pi rhm = 2\pi^2 rh\gamma^{-2}\sqrt{\frac{K_1}{A}}, \quad (5.33)$$

and using the expression for the force that acts on the CMD, we find that the "Larmor radius" is

$$R_L = \frac{v\mathfrak{M}}{\pi dh\gamma_1 M} = \sqrt{\frac{K_1}{A}}\frac{\pi vr}{(n+2)\gamma M}, \quad (5.34)$$

and that the free path length is

$$\lambda = \frac{\pi v}{\alpha\gamma M}. \quad (5.35)$$

Thus if the condition

$$\lambda > R_L, \quad (5.36)$$

is satisfied, then in the absence of ∇H hard CMD revolves around a circle of radius R_L with frequency

$$\omega_L = \frac{v}{R_L} = \frac{\pi dh\gamma_1 M}{\mathfrak{M}}. \quad (5.37)$$

If a hard CMD moves in a film in the presence of ∇H, then its trajectory, like the trajectory of a charged particle, will be a trochoid. The drift velocity is determined by the formula

$$v_{\text{dr}} = \frac{(\partial E/\partial H)\,|\nabla H|}{\pi dh\gamma_1 M} = \frac{1}{2}\frac{\delta H}{\gamma_1 M}. \quad (5.38)$$

It is evident from formula (5.38) that the sign of the drift velocity depends on the sign of γ_1. We shall discuss also the behavior of a group of CMD moving in a plate of finite dimensions. If a constant flow of CMD is maintained as a result of generation of CMD at one of the ends and absorption at the other, then in the direction of the flow there occurs a magnetic-field gradient whose value is determined by the CMD parameters and by the value of δH. Measurement of this field can give important information about the characteristics of the CMD. This effect is analogous to the Hall effect in ordinary metals and in semiconductors, where there are two types of carrier.[6)]

Finally, we shall give estimates of the "Larmor radius" and of the free-path length.

Setting $r = 10^{-4}$ cm, $z_0 = 10^{-6}$ cm, $\gamma M = 10^9$ sec^{-1}, $\alpha \approx 10^{-3}$, $n \approx 1$, $\delta H = 10^{-4}$ Oe, and $v = 10^4$ cm/sec, we get $R_L \approx 10^{-3}$ cm, $\lambda = 10^{-3}$ cm, and $v_{\text{dr}} = 10^{-1}$ cm/sec.

We see from these estimates that the condition $R_L < \lambda$ can be satisfied by appropriate choice of the parameters of the ferromagnet.

Thus the dynamics of a normal CMD is analogous to the dynamics of an ordinary particle in a viscous medium, if the parameters of the CMD are appreciably smaller than the distances over which its energy changes appreciably. The dynamics of a hard CMD under similar conditions is reminiscent of the dynamics of a charged particle in crossed electric and magnetic fields.

6. ENERGY OF A LATTICE OF CYLINDRICAL MAGNETIC DOMAINS

In the preceding section, we considered the properties of an individual CMD. Quite often, however, what

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[6)] This fact can be used for construction of devices for the sorting of CMD with respect to sign and value of γ_1.

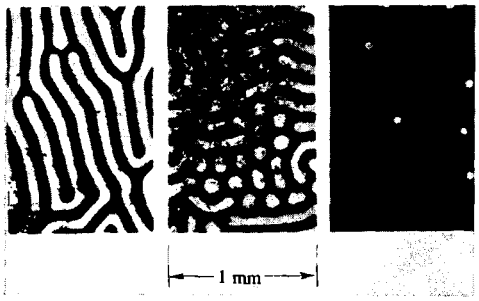

FIG. 11. Subdivision of a stripe structure into CMD on increase of the external magnetic field from 0 to 70 Oe.

appears is not an individual CMD but a system of cylindrical domains.[20,21] Thus, for example, the appearance of a system of CMD is observed experimentally[20,21] upon increase of the magnetic field applied normal to the surface of a specimen originally divided into plane parallel domains (Fig. 11). With increase of the magnetic field, domains in which the magnetization is oriented opposite to the field shrink (see Fig. 11) and ultimately break up into separate cylindrical domains, which then "repel" each other and fill the surface of the plate uniformly. Formation of a system of CMD may also have some other origin, for example as a result of artificial breakup of a stripe structure or of the operation of a source of nucleation of CMD, such as defects in the plate or of its edge.

The set of CMD, because of the dipole force of repulsion (Fig. 12), may form an ordered system, i.e., a two-dimensional lattice. Later we shall show that the forces that keep the individual CMD in this lattice are very small, and therefore small random effects can lead to a breakdown of the order, that is to the occurrence of a disordered structure.[7)]

A system of CMD, either ordered or disordered, has a number of new properties as compared with an individual CMD; for example, in such a system there can occur propagation of special types of spin waves, due to displacement of CMD from the equilibrium position or to oscillations of the boundaries of individual CMD. These waves show up in the magnetic susceptibility and affect the propagation of sound in the crystal and others of its properties.

We shall first study the static properties of a CMD lattice.

We shall consider a plate thickness h, infinite in the x and y directions, located in a magnetic field H directed along the axis of easiest magnetization (the z axis). Under these conditions, as has already been mentioned, it is possible for a domain structure to form that consists of a lattice of cylindrical magnetic domains of diameter d, magnetized opposite to the direction of the external magnetic field (Fig. 12). The rest of the plate is magnetized along the field. We choose an elementary rectangular cell with periods a and ap along the x and y axes respectively; here p is a numerical parameter that takes the values $p=\sqrt{3}$ for a hexagonal lattice and $p=1$ for a square lattice of cylindrical domains. The expression for the total energy density of the lattice can be put into the same form (3.4) as for an isolated domain. Thus we have for the energy of magnetization in the external magnetic field and for the energy of the domain walls, per unit volume of the specimen,

$$\mathcal{E}_H = -MH + (2\pi M)^2 \left(\frac{d}{a}\right)^2 \frac{H}{4\pi pM},$$
$$\mathcal{E}_W = 8\pi^2 M^2 \frac{ld}{a^2 p}. \tag{6.1}$$

The magnetostatic energy of a lattice of cylindrical magnetic domains was calculated in[52,53]:

$$\mathcal{E}_M = 2\pi M^2 - (2\pi M)^2 \frac{d^2}{a^2 p} + 4\pi^3 M^2 \left(\frac{d}{a}\right)^4 \frac{d}{pl} \sum_{k,m}{}' \Phi\left(\frac{\pi d}{a}\sqrt{k^2+p^2m^2}\right), \tag{6.2}$$

where

$$\Phi(x) = J_1^2(x)\, x^{-3}\left[1-\exp\left(\frac{-2\pi h}{d}\right)\right],$$

here the summation over k and m extends from $-\infty$ to ∞ under the condition that $k+m$ is an even number; J_1 is a Bessel function. $\Phi(0)$ is interpreted as

$$\Phi(0) = \lim_{k\to 0,\, m\to 0} \Phi\left(\frac{\pi d}{a}\sqrt{k^2+p^2m^2}\right).$$

In the practically most important case $a \gg d$ and $d \gtrsim h$, it is possible to obtain a simple expansion of the magnetostatic energy $\mathcal{E}_M$ in powers of the density of cylindrical domains.[54] In this case, the expression for $\mathcal{E}_M$ takes the form

$$\mathcal{E}_M = 2\pi M^2 - 8\pi^2 M^2 p^{-1} \left(\frac{h}{a}\right)^2 I\left(\frac{d}{h}\right) + (4\pi M)^2 p^{-1} R(p) \left(\frac{d}{a}\right)^4 \frac{h}{a}, \tag{6.3}$$

where $I(x)$ is defined by formula (3.3) and where

$$R(p) = \frac{\zeta(3)}{32} + \frac{\pi^3}{24p^3} + \frac{\pi^2}{2p^2} \sum_{n=1}^{\infty} \int_{np}^{\infty} \frac{\sqrt{t^2-p^2n^2}\, dt}{(-1)^n e^{\pi t} - 1}, \tag{6.4}$$
$$R(\sqrt{3}) \approx 0.1735.$$

By use of formulas (6.1) and (6.3), we can put the total energy density of the lattice into the following form:

$$\mathcal{E} = 2\pi M^2 - MH - 8\pi^2 M^2 p^{-1} \left(\frac{h}{a}\right)^2 V\left(\frac{d}{h}\right) + (4\pi M)^2 p^{-1} R(p) \left(\frac{d}{a}\right)^4 \frac{h}{a}, \tag{6.5}$$

where

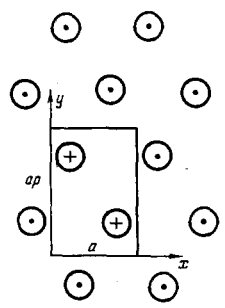

FIG. 12. Lattice of CMD. The coordinates of the CMD within an elementary cell are $(\frac{3}{4}a, \frac{1}{4}ap)$ and $(\frac{1}{4}a, \frac{3}{4}ap)$.

[7)] We remark that the high mobility of CMD, even in a lattice, makes a system of CMD a suitable model for the study of the properties of disordered systems.

$$V\left(\frac{d}{h}\right)=I\left(\frac{d}{h}\right)-\frac{l}{h}\frac{d}{h}-\frac{H}{8\pi M}\left(\frac{d}{h}\right)^2. \qquad (6.5')$$

The first term in the expression (6.5) corresponds to the energy density of a uniformly magnetized plate, the second to the interaction of the uniformly magnetized plate with the external magnetic field; the third term corresponds to the self-energy density of a cylindrical domain, while the last term describes the interaction between the domains in the dipole approximation. We note that, as is evident from formulas (6.5) and (6.5'), the energy per domain in a lattice of cylindrical domains is larger than the energy of an isolated domain of the same diameter.

7. EQUILIBRIUM PARAMETERS OF A LATTICE OF CYLINDRICAL MAGNETIC DOMAINS

The state of thermodynamic equilibrium of a lattice of cylindrical domains corresponds to the minimum of the total energy with respect to the variables, a, d, and p:

$$\frac{\partial \mathscr{E}}{\partial a}=0, \quad \frac{\partial \mathscr{E}}{\partial d}=0, \quad \frac{\partial \mathscr{E}}{\partial p}=0. \qquad (7.1)$$

The first two equations of the system (7.1) can be put into the following form by use of the expression (6.5) for the total energy of a lattice of cylindrical domains[54]:

$$\frac{l}{h}+\frac{H}{4\pi M}\frac{d}{h}-F\left(\frac{d}{h}\right)=-8R(p)\left(\frac{d}{a}\right)^3, \qquad \left(\frac{a}{d}\right)^3=\frac{5R(p)\,d}{V(d/h)\,h}, \qquad (7.2)$$

where

$$F(x)=\frac{dI}{dx}.$$

If we formally set $a=\infty$ in equation (7.2), it reduces to the known equation (3.5) obtained earlier for determination of the diameter of an isolated domain. Equations (7.2) can also be written in the following form:

$$\frac{l}{h}-\frac{1}{3}\frac{d}{h}\frac{H}{4\pi M}=S_A\left(\frac{d}{h}\right), \qquad \frac{a}{d}=\sqrt[3]{\frac{2R(p)}{S_E(d/h)-(l/h)}}. \qquad (7.3)$$

Figure 7 shows graphs of the functions S_A and S_F:

$$S_A=\frac{8}{3}\frac{I(x)}{x}-\frac{5}{3}F(x), \quad S_E=\frac{2}{x}I(x)-F(x).$$

By analyzing equations (7.3) one easily observes that the field H_c is the critical field for a lattice of magnetic domains, since on approach of H to H_c from below, the self-energy of the cylindrical magnetic domains approaches zero (this follows from the very definition of the field H_c), while the period a of the lattice meanwhile becomes infinite.[8)] Thus the field H_c is the field at which the lattice of cylindrical magnetic domains is transformed to one or several isolated cylindrical domains. At the point $H=H_c$, the energy of the lattice and of an isolated domain are equal. In fields $H>H_c$, the lattice becomes energetically disadvantageous.

We shall consider in greater detail the properties of a lattice of cylindrical domains near the critical point $H=H_c$.

[8)]At $H=H_c$ the condition $S_E(d/h)=l/h$ is satisfied.

On supposing that $H_c-H\ll 4\pi M$, we get the following approximate solutions of the system of equations (7.3):

$$x=x_c\left(1+\varkappa\frac{H_c-H}{4\pi M}\right), \qquad \frac{a}{d}=\sqrt[3]{\frac{10R(p)}{x_c}\frac{4\pi M}{H_c-H}}, \qquad (7.4)$$

where $\varkappa=[5S_F'(x_c)]^{-1}=x_c/5(S_0-S_E)$, $x_c=d_c/h\equiv d(H_c)/h$.

It follows from the expression (7.4) that the diameter of a domain decreases linearly with increase of the external magnetic field; the coefficient of proportionality $\varkappa$ in the state with a lattice of domains is five times smaller than for an isolated domain. The weaker dependence of the diameter of a cylindrical domain on the external magnetic field H when the domain belongs to a lattice is an expression of the fact that the magnetization of a plate with a lattice of cylindrical magnetic domains changes not only because of change of diameter of a domain, but also by decrease of the density of domains in the lattice. The variation of the period of the lattice with the external field H, as follows from formula (7.4), is determined by the power law $(H_c-H)^{-1/3}$.

By using formula (7.4), one easily calculates the dependence of the mean magnetization $\overline{M}$ on the value of the magnetic field:

$$\overline{M}=M\left[1-6\pi\chi_0\left(\frac{H_c-H}{4\pi}\right)^{2/3}\right], \qquad (7.5)$$

where

$$\chi_0=\frac{1}{6p}\left[\frac{x_c}{10R(p)}\right]^{2/3}.$$

We shall calculate also the static magnetic susceptibility near the critical point H_c:

$$\chi=\chi_0\left(\frac{4\pi M}{H_c-H}\right)^{1/3}. \qquad (7.6)$$

As is evident from the expression (7.6), the static magnetic susceptibility has a singularity at the critical point; it becomes infinite according to the law $(H_c-H)^{-1/3}$.

By using the relation (7.4), one can obtain an expression for the change of energy density due to formation of a lattice:

$$\Delta\mathscr{E}=-\frac{12\pi^2}{5}\frac{x_c}{10N(p)}\left(\frac{H_c-H}{4\pi M}\right)^{5/3}M^2, \qquad (7.7)$$

where

$$N(p)=p^{3/2}R(p).$$

The formulas obtained above contain the unknown parameter p. To find the value of this parameter, it is sufficient to investigate the minimum of the function $\Delta\mathscr{E}(p)$. The values of x_c and H_c are independent of p[54]; therefore the functional relation $\Delta\mathscr{E}(p)$ is completely determined by the form of the function $N(p)$, a graph of which is shown in Fig. 13. From Fig. 13 and formula (7.7) it follows that the function $\Delta\mathscr{E}(p)$ has two equal minima, at the points $p=\sqrt{3}$ and $p=3/\sqrt{3}$, and a maximum at the point $p=1$. Thus the minimum value of the energy density corresponds to a hexagonal lattice (the

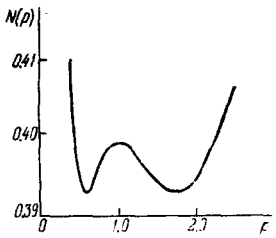

FIG. 13.

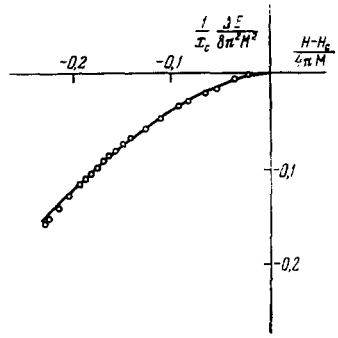

FIG. 14.

values $p=\sqrt{3}$ and $p=1/\sqrt{3}$ correspond to two equivalent hexagonal lattices, one turned through 90° with respect to the other). Obviously the values $p=1$, $p=3/\sqrt{3}$, and $p=\sqrt{3}$ comprise a complete set of solutions of the three equations of the system (7.1).

We note that the difference between the values of $\Delta\mathcal{E}$ for the hexagonal lattice and for the square amounts to $\approx 1\%$ of the whole value of $\Delta\mathcal{E}$. Therefore slight inhomogeneities of the thickness or of other parameters of the plate, and also inhomogeneities of the temperature of the plate or of the external magnetic field, may significantly distort the structure of the lattice and may lead to any degree of complication in the domain distribution on the surface of the specimen. In general, on the basis of the domain-distribution pattern on the surface of the plate one can make judgments about the distribution of inhomogeneities in the specimen or about the distribution of temperature gradients and of deformational stresses.

We shall now analyze the range of applicability of expressions (7.4)–(7.7). For this purpose it is necessary to estimate the next term in the expansion of the total energy $\mathcal{E}$ in the small quantity d/a and to compare it with the energy of dipole interaction. It is not difficult to obtain the value of the relative error, from which we get the formula

$$\varepsilon = 0.2\left(\frac{d}{a}\right)^2\left[1-\frac{4}{3}\left(\frac{h}{a}\right)^2\right]. \qquad (7.8)$$

As is seen from formula (7.8), the quantity d/a is actually an expansion parameter in the expression (6.5) only when $(d/h)\gtrsim 1$. In the contrary case, the natural expansion parameter will be the ratio h/a.

From Fig. 5 it is evident that the inequality $(d/h)\gtrsim 1$ is unconditionally satisfied in the most interesting case of thin films for which $(l/h)\approx 0.1$. The presence in the expression (7.8) of the small numerical parameter 0.2, which is determined by the geometry of the magnetization distribution in a plate, indicates that formulas (7.4)–(7.7) are valid, for sufficiently thin films, over the whole range of existence of a lattice.

TABLE IV. $l/h=0.25$, $H/4\pi M=0.12$.

	$\frac{d}{h}$	$\frac{d}{a}$
Theory	2.9	1.6
Numerical results	2.8	1.7

In the case of thick plates, however, $(l/h)\gtrsim 0.1$, the expressions (7.4)–(7.7) are valid only over the narrow field interval $0\leq (H_c-H)/4\pi M\leq l/h$. These conditions are easily derived by use of formulas (7.2) and (7.8). The structure of a lattice of cylindrical domains in fields $(H_c-H)/4\pi M > l/h$ was investigated in reference[55].

It is interesting to compare the relations determined by formulas (7.4)–(7.7) with the results of[53], in which numerical minimization of the total energy of the lattice was used to find d, a, and $\Delta\varepsilon$ as functions of the value of the external magnetic field for the parameter value $l/h=0.25$. Such a comparison is made in Table IV.

The curve in Fig. 14 shows the dependence of the energy density of a lattice on the value of the magnetic field, as found from formula (7.7). The points in the same figure represent the corresponding numerical values from[53].

The comparison made in Table IV and in Fig. 14 shows that the range of applicability of the theory we have developed encompasses practically the entire range of stability of a lattice of cylindrical magnetic domains in thin films. We mention also that in[56] a comparison was made between numerical results and experimental data on the lattice parameters, and good agreement was found between calculations and experiment (Table V).

In order to obtain a complete description of the properties of a lattice of cylindrical domains, it is necessary to investigate the question of the stability of the domain structure, both with respect to changes of the lattice parameters from their equilibrium values and with respect to small arbitrary changes of shape of the cylindrical magnetic domains. Such an investigation was made in[54]. There it was found that the upper bound for existence of a hexagonal lattice of CMD is determined by the onset of instability with respect to increase of the lattice parameter a, and that it coincides with the magnetic field H_c at which the energy necessary for formation of CMD vanishes. As for the lower bound, it is determined by the onset of elliptic instability of the CMD

TABLE V.

Material	Observed effect, cm		Calculated effect, cm	
$BaFe_{12}O_{19}$	$0.5-1.0\cdot10^{-4}$	$7.0\cdot10^{-4}$ cm	$1\cdot10^{-4}$	$6.7\cdot10^{-4}$
$TmFeO_3$	$1.3\cdot10^{-2}$	$4.3\cdot10^{-2}$ cm	$1.26\cdot10^{-2}$	$4.3\cdot10^{-2}$
GdIG	$4.5\cdot10^{-3}$	$6.0\cdot10^{-3}$ cm	$4.5\cdot10^{-3}$	$6.1\cdot10^{-3}$

of the lattice. The value of the elliptic-instability field in a lattice of CMD is lower than for an individual isolated CMD. This is due to the fact that in a lattice, neighboring domains produce a certain effective magnetic field that stabilizes individual domains and, in principle, makes possible the existence of a lattice of CMD even in zero field. Figure 14 [*sic*!] shows the fields H_c and H that bound the magnetic-field range in which a lattice of CMD is stable, as functions of the plate thickness.[54] It is evident that this range exceeds the range of stability of an isolated CMD.

8. LATTICE WITH FIXED DOMAIN DENSITY

As is seen from Fig. 5, in a thin film, depending on the value of the external magnetic field, the following types of domain structure may be realized. In the field range $H > H_{col}$, domains are absent. At fields $H_c \leq H \leq H_{col}$, metastable isolated cylindrical domains can exist; their destruction is prevented by the presence of an energy barrier. In the field interval $H_2 \leq H \leq H_c$, formation of an isolated domain becomes energetically advantageous, but the minimum of the total energy in this case corresponds to a lattice of cylindrical domains. This state is realized in the field interval $H_s < H < H_c$. At fields $H < H_s$, a stripe structure is energetically more advantageous.[56]

Nucleation or annihilation of a cylindrical domain usually involves the surmounting of an energy barrier; therefore in order to obtain thermodynamic equilibrium of a lattice of domains, it is necessary to take special measures.

The total number of domains in the specimen is determined by its previous history and does not change with time. Local ordering is established between the domains, so that one may speak of short-range order. As concerns ordering over the whole plate, it cannot be realized because of the presence of defects and inhomogeneities, so that the set of CMD is, as it were, either a polycrystal or a two-dimensional liquid, which is usually characterized by the CMD density ρ. The fields that bound the range of existence of such a lattice are the collapse field of an individual domain and the elliptic-instability field of an individual domain. The values of these fields naturally differ from the corresponding fields for an isolated CMD. Allowance for interaction of domains with each other leads to a stabilization effect; specifically, the interval between the collapse field and the elliptic-instability field in a disordered ensemble of domains with density ρ is larger than for an isolated CMD.[54] Near the collapse field, the magnetic susceptibility has a square-root singularity

$$\chi \sim (H_{col} - H)^{-1/2}.$$

9. HIGH-FREQUENCY PROPERTIES OF A LATTICE OF CYLINDRICAL MAGNETIC DOMAINS

We go on now to consideration of the high-frequency properties of a lattice of CMD.[57,61] In such a lattice, just as in an ordinary crystal lattice, there can be propagation of longitudinal and transverse oscillations caused by displacement of CMD from the equilibrium position. Besides these waves, in a lattice of magnetic domains waves can be excited that are due to change of shape of CMD.

In order to describe the high-frequency properties of a lattice of domains, we shall start from the following expression for the energy:

$$\mathcal{E} = \mathcal{E}_0 + \sum_i \varepsilon_i + \frac{1}{2}\sum_{i,k} V_{ik}, \tag{9.1}$$

where $\mathcal{E}_0$ is the energy of the uniformly magnetized plate, ε_i is the self-energy of the domain that is located at the ith site of the lattice, with coordinate $\mathbf{R}_i$, and V_{ik} is the interaction energy of the two domains with center coordinates $\mathbf{R}_i$ and $\mathbf{R}_k$ respectively. Since in thin films the lattice constant a is usually larger than either the CMD diameter or the plate thickness, the interaction energy of two CMD can be described as the energy of dipole–dipole interaction[9)]

$$V_{ik} = (\mathbf{m}_i\mathbf{m}_k R_{ik}^2 - 3(\mathbf{m}_i\mathbf{R}_{ik})(\mathbf{m}_k\mathbf{R}_{ik})]\, R_{ik}^{-5}. \tag{9.2}$$

We note that the vector $\mathbf{R}_{ik} = \mathbf{R}_i - \mathbf{R}_k$ lies in the plane of the plate; that is, $\mathbf{m}_i$ is orthogonal to $\mathbf{R}_{ik}$. Therefore the second term in (9.2) can be neglected in comparison with the first. By using the relation between the moments $\mathbf{m}_i$ and the CMD area S_i, we can put the interaction energy of the domains into the form

$$V_{ik} = (2hM)^2 S_i S_k R_{ik}^{-3}. \tag{9.3}$$

By using further the standard procedure for treatment of small oscillations, one can easily find the velocities c_l of longitudinal and c_t of transverse waves of CMD displacement:

$$c_{l,t}^2 = s_{l,t}^2 \left(\frac{d}{a}\right)^3 \frac{h}{m} M^2, \tag{9.4}$$

where $s_{l,t}^2$ are numerical parameters of order ten for longitudinal waves and of order unity for transverse. As in an ordinary lattice, the velocity of sound in a CMD lattice is inversely proportional to the square root of the mass; the factor $(d/a)^3$ reflects the fact that the interaction responsible for the waves in a CMD lattice is magnetic dipole interaction (a is the lattice parameter, d the CMD diameter). These waves, and also waves of pulsation of CMD, cause the appearance of poles in the tensor high-frequency magnetic susceptibility of a CMD lattice.[61]

10. CONTROL WITH CYLINDRICAL MAGNETIC DOMAINS, AND THEIR APPLICATIONS

The preceding sections were devoted to the physical properties of CMD. As was mentioned in the introduction, considerable work has now been done on the technical application of these domains and the use of CMD devices in computers. In this section we shall briefly describe specific methods of generation, control, and recording of CMD in use at the present time. A detailed elucidation of these questions can be found in the reviews[62,63] and in the monographs.[64,68]

At the basis of schemes for control of CMD lies the

[9)] If this condition is violated, then in order to calculate the interaction energy one must carry out numerical calculations.

idea of producing in the film a series of potential wells for domains, and of means of moving these potential wells or moving the domains from one potential well to another. As is clear from the preceding (see sections 3, 5, and 6), this can be achieved by change of an inhomogeneous external magnetic field, temperature, plate thickness, anisotropy, or other parameters of the magnetic film.

At present there are in fact exploitations of the use of each of these possibilities. Thus, for example, there are being developed systems for control by means of a laser beam, on the basis of local heating of the film; by change of the surface anisotropy by implantation of ions; and by production of magnetic inhomogeneities by means of magnetic attachments and current-carrying conductors.

Magnetic-attachment methods are at present the most widely used.

According to the configuration of the magnetic attachments, there are the various control schemes *T*, *Y*, *X*, and other types. Figure 15 shows an example of a control structure of the *T* type. The structure consists of *T*-shaped and strip elements; the material used for these is magnetically soft permalloy (~80% Ni and 20% Fe) with a low value of the coercive force. The thickness of these elements is about 5000 Å; the ratio of the length to the width of a strip is 5; and the distance between strips and their width is about half the diameter of a domain. The relatively large thickness is chosen for the purpose of preventing magnetization of the elements to saturation by the stray fields of the domains themselves. When the ratio of the sides of the elements is 5 : 1, there is a quite appreciable shape anisotropy the demagnetizing field along the strip is considerably smaller than across it), and as a result an external magnetic field parallel to the length of a strip magnetizes the element to saturation, whereas a perpendicular field of the same magnitude does not affect the magnetization. A field of about 10–20 Oe is sufficient to magnetize the element to saturation. Periodic motion of the field gradients along the elements of the structure,

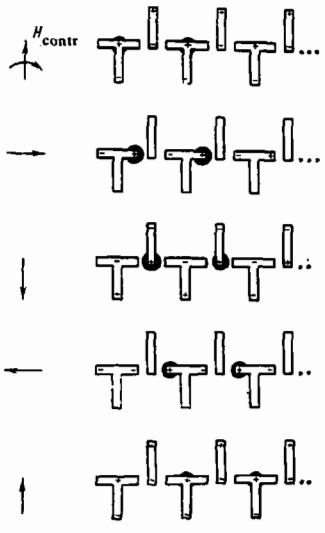

FIG. 15.

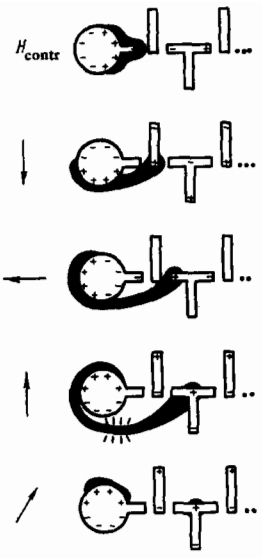

FIG. 16

which is necessary in order to move the domains over large distances, is effected by means of an external magnetic field that is rotated in the plane of the film. The action of the *T* structure shown in Fig. 15 may be regarded as the result of interaction of the domains, behaving as magnetic dipoles, with the magnetic poles induced in the permalloy elements by the rotating field. As is seen from the figure, during one period of variation of the rotating control field, the domains shift by one spatial period of the *T* structure. Change of sign of the rotating field leads to reversal of the direction of motion of the CMD. A rotating field can be produced by a pair of mutually perpendicular Helmholtz coils, fed with alternating currents 90° out of phase.

An advantage of control schemes of the type considered is the absence of electrical connections to the elements of the control structure, and the possibility of controlling the motion of CMD in many channels by means of a single rotating field.

Similar principles—interaction of domains with an inhomogeneous magnetic field—lie at the basis of the action of CMD generators. Such generators may be made in the form of a mosaic of permalloy elements, which are so located that under the action of a rotating field they cause extension of a nucleating domain. The extended nucleating domain is then split into two domains by transmission of a current pulse along a special conductor, the splitter. One of the newly produced domains, under the action of the rotating field, is guided into the propagation channel, while the other remains in the generator as a nucleating domain.

Another type of permalloy-element generator in widespread use produces one domain in one period of variation of the rotating field, without participation of current-carrying conductors. The scheme of such a generator is shown in Fig. 16. A nucleating domain is located under a permalloy element in the form of a disk. As is seen from the figure, under the action of the rotating control field the nucleating domain undergoes extension, and from it there splits off a new domain, which is guided into the propagation channel.

Also obvious are the problems of annihilation of cylindrical domains. The simplest method consists of raising the magnetic field H perpendicular to the film plane to a value equal to or greater than the collapse field. When it is necessary to annihilate a domain in a specified section of the film, increase of only the local magnetic field is required; this can be done by means of a plane current circuit.

The most widespread schemes for detection of CMD are based on the action of the stray magnetic field of a domain on the detector.[65] Such a detector may be a circuit of conducting film, in which an emf is induced because of the change of magnetic flux when a cylindrical domain passes under the circuit. The magnitude of the signal depends on the diameter of the domain and on its velocity of passage under the circuit. With a domain diameter of about 100 μm and with a velocity of motion corresponding to operating frequencies of about 1 MHz, the output signal may be as high as 100 μV. This signal can be increased if, before the detection, the area of the domain is increased by extension of it.

For practical application, however, there are other, more convenient methods of detection, using the Hall effect and the magnetoresistive effect, in which the signal originates not directly under the action of the magnetic field of the CMD but by modulation by this field of electrical power supplied by an external source.

Thin-film Hall microdetectors have an active area of the order of the domain area. The axial component of the magnetic field of a domain, acting on the detector, leads to the appearance of a Hall difference of potential in the direction perpendicular to the field and current directions. The detectors used are of silicon or of InSb, which because of the high mobility of the carriers has the best sensitivity. With a domain diameter of about 100 μm, the output signal of a silicon detector amounts to 0.5 mV for input voltage 9 V; in the case of an InSb detector, a signal of about 1 mV can be obtained with input voltage 0.3 V.

A Hall detector is a four-pole device, and this complicates its wiring diagram. A magnetoresistive permalloy detector, whose operation is based on the change of resistance under the influence of a magnetic field, is made in the form of a two-terminal network. The permalloy magnetoresistor is a rectangular film of thickness 200–300 Å, with size about equal to the domain size, through which is passed electric current from an external source (Fig. 17). By virtue of the magnetoresistance effect, when the film is magnetized perpendicular to the direction of the current it has a resistance a few percent lower than when it is magnetized parallel to the direction of the current. The crystallographic anisotropy, in combination with shape anisotropy, leads to the result that when no domain is near the detector, its magnetization is directed along its long side; that is, along the direction of the current. If a domain appears near the detector, then under the influence of the radial component of the stray field of the domain there occurs a rotation of the magnetization of the film toward the axis of hard magnetization; that

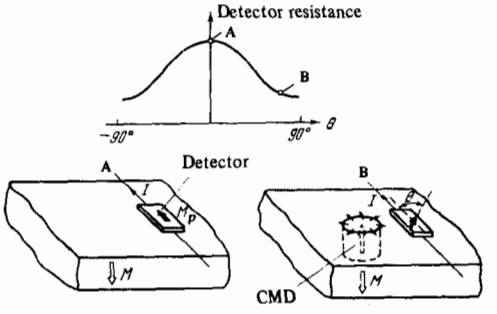

FIG. 17.

is, in the direction of increase of the angle θ (Fig. 17). The effective anisotropy field of the film for rotation of the magnetization through angle $\theta = 90°$ is made up of the magnetocrystalline anisotropy field and of the demagnetizing field (the latter is proportional to the thickness of the film) and, for a film of thickness 200–300 Å, has a value of the order of 10 Oe. The radial component of the magnetic field of a cylindrical domain is 0.1 to 0.3 of its magnetization $4\pi M$, and therefore the domain field leads to a significant change of the angle θ and consequently to an appreciable change of the resistance of the magnetoresistor. The largest change of the angle θ and the largest output signal of the detector are observed when the edge of the domain is under the center of the magnetoresistor. The output signal of a typical magnetoresistive detector, with domain diameter 100 μm, is about 2 mV when the input voltage to the detector is about 0.3 V.

Use of CMD *in computer technology*. From the data presented above, it follows that by use of CMD, memory devices (MD) can be constructed that differ favorably, with respect to a number of parameters, from the MD usually applied. With domain diameter about 5 μm (iron garnets), MD based on CMD guarantee a density of information storage of about $5 \cdot 10^5$ bit/cm^2; that is, higher than with other forms of magnetic MD.[66] Such storage densities have now already been attained. By choice of a material with domain diameter about 2 μm, it will be possible to obtain a storage density up to 10^7 bit/cm^2. With such a high storage density, production of control and readout circuits whose characteristic dimensions are of the order of magnitude of the domain dimensions requires the use of electron-beam lithography methods, since photolithography in this case no longer provides the necessary dimensions for the circuit elements. Devices with storage density about 10^7 bit/cm^2 will apparently be made within the next few years. MD and CMD are assembled from a set of ferrite plates or ferrite films on substrates of area 1–10 cm^2 and occupy a relatively small physical volume. There are now already MD with total capacity about 10^6 bits; it is quite realistic to expect realization in the near future of MD with capacity up to 10^9 bits in a physical volume of about 1000 cm^3 (a significant part of the volume is occupied by the magnets that produce the shift and control fields).

As always, a very important parameter of a memory device is the time for selection of information. It depends to an important degree on the mode of organiza-

tion of a CMD memory. The most advantageous organization is the following. The basic structural unit of the MD is a ferrite plate (film), on which are arranged the attendent circuits. Such a plate is called a chip. The information on the chip is kept in circular storage registers, i.e., closed channels of motion of CMD, with a definite digit capacity n. Usually $n = 10^2 - 10^4$ bits. Access to individual sites in writing and reading occurs within each register in accordance with the advance of domains along it. During a single cycle, equal to the period of rotation of the magnetic control field, the domains are shifted one site. In order to simplify the attendant circuits, the set of a large number of storage registers of a single chip is connected, by use of a coupling register, with a single general circuit for writing, erasing, and reading information. The storage registers are connected to the coupling register through switching devices. Such an organization of the memory insures cyclic access to information; the arrangement and circulation of domains in the shift registers is equivalent to separate writing paths on magnetic drums or disks. The selection time with this organization is half the time of a complete shift in the register; that is, $\tau = n/(2\nu)$, where ν is the timing frequency. This time corresponds to the rotation waiting time in the case of a disk or drum. With timing frequency about 1 MHz and with $n = 10^2 - 10^4$ bits, the selection time will be from 10 μsec to 1 msec. With MD organization with blockwise access, when a given register stores information of just one block, the time equivalent to the rotation waiting time is eliminated, and the selection time can be shortened to 1 – 0.1 μsec. Thus with respect to their capacity and fast action, MD based on cylindrical domains significantly surpass MD on magnetic disks and drums. The absence of mechanical motions in CMD devices (the only motion is that of the information itself with respect to a fixed informational medium) makes the operation of these devices very reliable. Furthermore, a CMD memory device compares favorably, with respect to low cost per bit (this cost corresponds in order of magnitude to the cost of a magnetic tape MD) and small energy consumption, with ferrite-core MD. By appropriate design, cylindrical domains can also be used to make memory devices with arbitrary (and also with associative) selection, and with parameters that satisfy the requirements for operational memory devices; but CMD memory devices will apparently be most widely used as external and buffer MD of large and medium capacity, with cyclic access. Such MD are capable of filling the large gap in speed of action that exists, in the traditional hierarchy of computer memory devices, between external and internal memory devices. There are predictions that within the next few years CMD memory devices will completely displace magnetic-disk memory devices.

CONCLUSION

Thus it is evident that cylindrical magnetic domains possess a number of interesting physical properties, which manifest themselves both in individual cylindrical domains and in an assemblage of CMD. The most interesting properties of those of hard CMD; the dynamics of these domains in slightly inhomogeneous external magnetic fields is as diversified as the dynamics of charged particles in external electric and magnetic fields.

CMD can arrange themselves in a lattice, in which unique waves exist, and which can simulate both ordered and disordered two-dimensional crystals.

All this variety of properties at present not only has scientific interest, but also is finding practical application in a whole series of memory and radiotechnical devices.

It must be mentioned that there are a number of current problems in the physics of CMD that have still not found a sufficiently complete solution. Among these problems are the dynamics of CMD at high velocities, investigation of the magnetization distribution in a CMD wall and the effect of motion on the wall structure, study of varieties of CMD, CMD in extremely anisotropic media, and the dynamical properties of lattices of hard CMD.

APPENDIX

Relation between the mobility and the relaxation constant in the equation of motion of the magnetic moment

We shall start from the following expression for the variation of the energy of a ferromagnet:

$$\delta E = -\int \mathbf{H}_e \delta \mathbf{M}\, dV, \tag{A.1}$$

where $\mathbf{H}_e = -\delta E/\delta \mathbf{M}$ is the effective magnetic field acting on the magnetization. Hence

$$\dot{E} = -\int \mathbf{H}_e \dot{\mathbf{M}}\, dV. \tag{A.2}$$

By using the equation

$$\dot{\mathbf{M}} = \gamma [\mathbf{M}, \mathbf{H}_e] - \alpha M^{-1} [\mathbf{M} \times \dot{\mathbf{M}}], \tag{A.3}$$

we get

$$\dot{E} = \frac{\alpha}{M} \int (\mathbf{H}_e [\mathbf{M} \times \dot{\mathbf{M}}])\, dV. \tag{A.4}$$

Allowing for the smallness of the damping, we have

$$\dot{E} = -\frac{\alpha}{\gamma M} \int \left(\frac{\partial \mathbf{M}}{\partial t}\right)^2 dV. \tag{A.5}$$

For a domain wall moving with velocity $\mathbf{v}$,

$$\frac{\partial \mathbf{M}}{\partial t} = \frac{\partial \mathbf{M}}{\partial x_i} \frac{\partial x_i}{\partial t} = (\mathbf{v}\nabla)\,\mathbf{M}. \tag{A.6}$$

Therefore

$$\dot{E} = -\frac{\alpha}{\gamma M} v_i v_k \int \frac{\partial M_i}{\partial x_i} \frac{\partial M_i}{\partial x_k}\, dV. \tag{A.7}$$

If the wall is plane, then

$$\dot{E} = -\frac{\alpha S}{\gamma M} v^2 \int \left(\frac{\partial \mathbf{M}}{\partial x}\right)^2 dx, \tag{A.8}$$

where S is the area of the domain wall, and where the x axis is chosen along the normal to the wall.

On the other hand, if a force of viscous friction $\mathbf{F} = -1/\eta \mathbf{v}$ acts on unit area, then the dissipation of energy during the motion of the domain wall is determined by the following expressions

$$\dot{E} = S\mathbf{F}\mathbf{v} = -Sv^2/\eta. \tag{A.9}$$

Therefore

$$\frac{1}{\eta} = \frac{\alpha}{\gamma M}\int\left(\frac{dM}{dx}\right)^2 dx = \frac{\alpha M}{\gamma}\int\left(\frac{d\theta}{dx}\right)^2 dx, \tag{A.10}$$

where θ is the angle that determines the magnetization distribution in a Bloch or Néel wall. This formula also determines the relation of the mobility coefficient η to the relaxation constant α and to the parameters of the magnetization distribution in the domain wall. By using the specific form of the dependence of θ on the coordinate x, we easily find

$$\frac{1}{\eta} = \frac{2\alpha M}{\gamma}\begin{cases}\sqrt{K_1 A^{-1}} & \text{for a Bloch wall,}\\ \sqrt{(K_1 + 2\pi M^2)A^{-1}} & \text{for a Néel wall}\end{cases} \tag{A.11}$$

and

$$\mu = \frac{\gamma}{\alpha}\begin{cases}\sqrt{A/K_1}, \\ \sqrt{A(K_1 + 2\pi M^2)^{-1}}.\end{cases} \tag{A.12}$$

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