Cosmic objects and elementary particles

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Usp. Fiz. Nauk **121**, 319–329 (February 1977)

A study is made of the connection between the parameters of elementary particles (mass and "size") and the characteristics of stars–red dwarfs, stars of the main sequence, white dwarfs, and pulsars. An elementary theory of the emission of black holes, in the framework of which the main features of this process are deduced, is presented in the paper. An empirical numerical sequence that relates the nucleon mass and the universal constants $G, \hbar$, and c to the masses of various cosmic objects is given. There are five tables and one illustration.

PACS numbers: 95.30.Cq, 97.20.–w, 97.60.Lf, 98.80.–k

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INTRODUCTION

Pythagoras was one of the first to be captured by the charm of mathematics as the fundamental harmony of nature. The list of great natural scientists who have attempted to penetrate to the essence of things on the basis of numerical regularities ends with Eddington and Dirac.

Games with numbers are very attractive; one must however recognize clearly that as yet no physical content has been found in the numerical sequences and they are probably merely a game of the imagination. Therefore, numerical regularities form the subject of only the final sections of this paper and are not the main content. Many monographs and papers have been written about stars, which *are* the main content of this paper. However, almost all material about stars is intended mainly for astronomers.[1] In my opinion, it is also desirable to "look" at stars as simple physical objects that demonstrate a transparent and intimate connection between the macroscopic world and microscopic particles.

Of course, to achieve maximal perspicuity we cannot avoid simplifications. But sensible idealization is not too high a price to pay for clarity, simplicity, and conciseness.

MAIN OBSERVATIONAL DATA

Let us first list the facts that are to be explained in the first place. There are three.

1. Stars exist for a long time:

$$t_0 \sim 10^{10} \text{ years} \sim 10^{17} \text{ sec.}$$

2. Stars emit, i.e., they give off a lot of energy.

3. The stars are grouped into definite families in the Hertzsprung–Russell diagram. Essentially, this diagram represents the luminosity L of the stars[2] as a function of the temperature of their emitting region. Tradiationally, stars are divided into spectral classes that correspond to the values of their temperature T. Table I gives the temperatures corresponding to the spectral classes.

Figure 1 is a typical Hertzsprung–Russell diagram: it contains the stars nearer than 5 pc from the Sun.

It follows from Fig. 1 that the majority of the stars are grouped near a curve that passes from the upper

[1] One of the few exceptions is the book by Zel'dovich and Novikov, (Teoriya Tyagoteniya i Evolyutsiya Zvezd (Theory of Gravitation and Evolution of Stars), Nauka, Moscow, 1971.

[2] The luminosity is the power of the emission of the stars.

TABLE I.

Spectral classes	Temperature, deg
B0	25 000
A0	11 000
F0	7 600
G0	6 000
K0	5 100
M0	3 600

left-hand corner to the lower right-hand corner. These are the stars of the main sequence. In the lower left-hand corner there is a group of stars known as white dwarfs, which do not belong to the main sequence.

Our Sun is a typical star of the main sequence. Table II gives its main parameters.

We are interested in the "average" star, and we have therefore rounded off all the characteristics of the Sun (in the last column of Table II). In Table III we give the characteristics of a typical white dwarf.

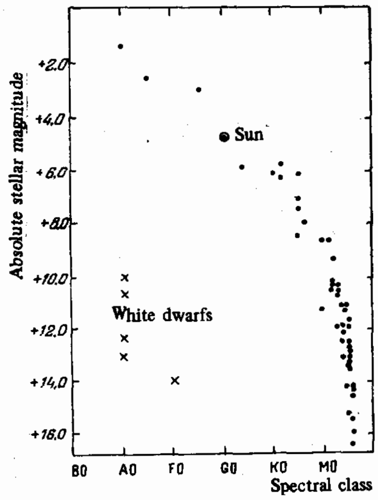

FIG. 1. Luminosity as a function of the temperature. The luminosity is plotted along the ordinate on logarithmic scale (absolute stellar magnitudes). The spectral classes are plotted along the abscissa. For the correspondence between the spectral classes and the temperature see Table I.

MAIN SEQUENCE STARS

On the basis of simple physical arguments, one can attempt to interpret the main parameters (M, R, ρ) of the main sequence stars, the causes of their evolution, and the characteristics of their final stages.

Let us take a simple and obvious model of a star in the form of a gigantic gaseous sphere in its own gravitational field. It follows from fact 1 (long existence of the star) that the star is in a *quasiequilibrium state*. Otherwise there would be a collapse, which would last $t_{\rm coll} \sim 10^3$ sec.[3] Equilibrium is possible *if there is finite number of simple configurations*. It is important that although this number is not large it is greater than 1 and is determined by the number of stable elementary particles. To ensure stable equilibrium there must exist forces that equalize the force of gravity. These forces may have a purely kinetic origin—pressure of heated plasma—and a more complicated origin due to the fact that elementary particles—the main building bricks of stars—have definite sizes. In other words, "nature abhors a vacuum." A star is a sphere consisting of particles pressed close to each other. This limiting configuration is due to gravitational contraction, and the properties of the various equilibrium configurations are due to the characteristics (primarily the "sizes" of the microscopic particles).[4]

We consider first a body of such small mass M that the gravitational forces are insufficient to break up the atomic shells. The conditions of existence of such a body are

$$\frac{GM}{R} m_p < \varepsilon_b, \tag{1}$$

$$\rho \sim 1 \text{ g-cm}^{-3}; \tag{2}$$

$\varepsilon_b \sim 10^{-11}$ erg is the binding energy of electrons in light atoms, m_p is the proton mass. In Eq. (2) it is necessary to average ρ over the complete volume, i.e., use $\bar{\rho}$. In what follows, we shall always assume such averaging over the volume. Since

$$M \sim \rho R^3, \tag{3}$$

we have

$$M^{(1)}_{\rm min} < 10^{30} - 10^{31} \text{ g}. \tag{4}$$

Therefore, the emission of bodies that satisfy the condition (4) is very low (in particular, planets are included among these bodies). Such bodies will consist mainly of nonionized atoms. The bodies can emit (i.e., be stars) if $M > M^{(1)}_{\rm min}$. In this case, the equilibrium must be maintained either by the kinetic pressure or by the interaction of elementary particles with one another. Let us consider first the first case. Here there can be two possibilities: the first corresponds to relatively weak emission of stars, so weak that gravitational energy alone is sufficient for the emission during a length of time close to the cosmological t_0. In the second this energy is insufficient, and a new source of energy is needed.

Let us consider first the first possibility. From very general considerations one can show that the luminosity

TABLE II. Parameters of the Sun.

Mass M, g	$2 \cdot 10^{33}$–10^{33}
Radius R, cm	$7 \cdot 10^{10}$–10^{11}
Luminosity L, erg/sec	$4 \cdot 10^{33}$–10^{34}
Density ρ, g/cm^3	1.4–1
Temperature of photosphere T, deg	6000–10^4

[3] $t_{\rm coll} \sim (R/c)\sqrt{R/R_g}$, where $R_g = 2GM/c^2$ is the gravitational radius; $G = 10^{-7}$ cm$^3 \cdot$ sec^{-2} g^{-1} is the gravitational constant.

[4] We do not consider the possibility of equilibrium due to centrifugal forces.

TABLE III. White dwarf.

Mass M, g	10^{33}
Radius R, cm	10^{8}–10^{9}
Luminosity L, erg/sec	10^{30}
Density ρ, g/cm^3	10^{7}
Temperature T, deg	10^{1}

of a star satisfies $L \propto M^3$.[5)] Therefore, the condition for gravitational energy to be sufficient to maintain the emission during the time t_0 is

$$\frac{G[M_{\min}^{(2)}]^2}{R} = 10^{51}\left(\frac{M_{\min}^{(2)}}{M_\odot}\right)^3; \tag{5}$$

where R is measured in centimeters and $M_{\min}^{(2)}$ in grams.

The other condition is associated with kinetic (sometimes called hydrostatic) equilibrium, which can be written in the form

$$\frac{GM}{R} = NkT, \tag{6}$$

where N is Avogadro's number and k is Boltzmann's constant.

It follows from the conditions (5) and (6) that $M_{\min}^{(2)} \sim 10^{31}$ g $\sim 10^{-2}$ $M_\odot$. Stars whose luminosities are maintained by gravitational energy can be red dwarfs in the right-hand lower corner in the Hertzsprung–Russell diagram. The parameters of white dwarfs (Table IV) follow from these estimates.

It is interesting to note that the values of $M_{\min}^{(1)}$ and $M_{\min}^{(2)}$ are approximately equal.

We now consider the second case, when the gravitational energy is insufficient to maintain radiation for the time t_0. (For example, for the Sun this energy would last for the time $\sim 10^7$ years.) It is then necessary to make an assumption about the existence of a new source of energy. It is well known that thermonuclear reactions provide such a source. Thermonuclear reactions proceed always between positively charged particles. Therefore, if the reactions are to take place copiously, one must have a high temperature T_t, sufficient for effective transmission through the Coulomb barrier. From experiments made in laboratories and theoretical estimates it follows that

$$T_t \gg (10^7)^0. \tag{7}$$

The emission of stars due to thermonuclear reactions is a consequence of the fact that the fusion process liberates an energy $\sim 10^{-3}$ Mc^2. From this one readily finds that the lifetime of the Sun due to thermonuclear reactions is $\sim t_0 \sim 10^{10}$ years. It follows from (2), (5), and (6) that the minimal mass of a star for which the thermonuclear reaction proceeds effectively is

$$M_{\min}^{(3)} \sim 10^{32} - 10^{33}\ \text{g} \tag{8}$$

and $M_{\min}^{(3)} \sim M_\odot$. Approximately the same value of $M_{\min}^{(3)}$ can be obtained from the conditions of total equilibrium: kinetic energy equal to the potential, which in its turn is equal to the radiation energy. Writing down these conditions per unit mass in the form of the chain of equations

$$\frac{GM}{R} = NkT = \frac{4\sigma}{\rho} T^4 \tag{9}$$

($\sigma = 5.7 \cdot 10^{-5}$ g $\cdot$ sec^{-3} $\cdot$ deg^{-4} is the Stefan–Boltzmann constant), we again obtain $M_{\min}^{(3)} \sim M_\odot$ for $\rho \sim 1$.

INEVITABLE DEATH OF STARS

The most characteristic feature of thermonuclear reactions is their one sidedness, in the sense that *light elements are always transformed into heavier elements during thermonuclear fusion.* There is an enrichment with heavy elements. A characteristic example is the so-called hydrogen cycle

$$\left.\begin{aligned} p + p &\to D^2 + e^+ + \nu + 1.4\ \text{MeV}, \\ D^2 + p &\to He^3 + \gamma + 5.5\ \text{MeV}, \\ He^3 + He^3 &\to He^4 + 2p + 12.9\ \text{MeV}. \end{aligned}\right\} \tag{10}$$

As a result of this cycle, four protons are transformed into the stable He^4 isotope. An analogous situation arises in other cycles. The enrichment of the star with heavy elements means that it gets older, and approaches its end. This can be readily understood from the well-known expression for the tunnel effect. The probability W of transmission through a potential barrier is

$$M \sim e^{-\alpha Z_1 Z_2}; \tag{11}$$

α is a constant and Z_1 and Z_2 are the charges of the colliding particles. The probability of fusion decreases exponentially with increase of the charges. On the other hand, the packing coefficient (the ratio of the mass defect to the mass number) decreases with increasing atomic number A (right up to $A \sim 40$). At $A \sim 50$ (iron) the fusion process becomes energetically disadvantageous. Therefore, there comes a time when the temperature T_t is insufficient for self-sustaining thermonuclear reactions. The star undergoes collapse, which will continue without limit unless new forces capable of overcoming gravitational attraction become operative.

Sometimes collapse is accompanied by a beautiful firework—the explosion of so-called supernovae. But since the subject of this paper is stationary states, we shall not dwell on this rare sight but turn to the final phases of stellar evolution.

FINAL STAGES IN THE EVOLUTION OF STARS AND THE "SIZES" OF ELEMENTARY PARTICLES

It follows from the principle that "nature abhors a vacuum" that gravitational attraction will continue until the distances between the particles become equal to

TABLE IV.

M, g	10^{31}
R, cm	10^{10}
ρ, g/cm^3	1

[5)]See, for example, the monograph of I. S. Shklovskiĭ: Zvezdy (Stars), Nauka, Moscow, 1975, p. 117.

their "size." Since we have eliminated electromagnetic forces, all we have at our disposal for characteristic sizes are the Compton wavelengths of the corresponding particles:

$$\frac{\hbar}{m_e c},\ \frac{\hbar}{m_\pi c}\ \text{etc.}$$

A star consists of fermions. Therefore, when the particles have approached to distances equal to their Compton wavelength, the Pauli principle becomes effective. *It is this quantum mechanical principle which is the origin of the stability of very compact stars.*

Of course, the approach to definite boundaries corresponding to the sizes of the elementary particles is a necessary but not sufficient condition. It is obvious that if M increases unrestrictedly one can always find a value of the mass at which the influence of the identical nature of the fermions is insufficient.

The next problem is to determine the critical "equilibrium" values of the masses M of stars corresponding to the sequence of distances

$$\frac{\hbar}{m_e c},\ \frac{\hbar}{m_\pi c}\ \text{etc.}$$

WHITE DWARFS

We consider the situation when the particles have approached to the distance $r \sim \hbar/m_e c$.[6] This situation corresponds to the following parameters of the star. The volume v_d occupied by one particle is

$$v_d \sim \left(\frac{\hbar}{m_e c}\right)^3. \tag{12}$$

The electron density is

$$n_d \sim \left(\frac{m_e c}{\hbar}\right)^3. \tag{13}$$

The density is

$$\rho_d \sim (m_p + m_e)\left(\frac{m_e c}{\hbar}\right)^3 \sim m_p\left(\frac{m_e c}{\hbar}\right)^3 \sim 10^9\ \text{g-cm}^{-3}. \tag{14}$$

The only characteristic velocity v_e for the electrons is the velocity c. Therefore, in a first approximation $v_e \sim c$, and the total energy of an electron–proton pair for thermal equilibrium is[7]

$$E_{d\,\mathrm{kin}} \sim 2m_e c^2. \tag{15}$$

The condition of equilibrium of the star as a whole can be expressed in the form of equality of the potential and kinetic energy per proton–electron pair:

$$\frac{GM_d}{R_d} = m_e c^2. \tag{16}$$

Since $R_d = (3M_d/4\pi m_p n_d)^{1/3}$,

$$R_d = \left(\frac{3}{4\pi}\frac{M_d}{m_p}\right)^{1/3}\frac{\hbar}{m_e c}, \tag{17}$$

$$M_d = \left(\frac{3}{4\pi}\right)^{1/2}\left(\frac{\hbar c}{G}\right)^{3/2} m_p^{-2}. \tag{18}$$

Substituting numerical values of the constants, we obtain $M_d \sim M_\odot \sim 10^{33}$ g and $R_d \sim 10^8$ cm. As we see (from Table III), the values of M_d, R_d, ρ_d agree well with the parameters of white dwarfs. The white dwarf is a star in which the distances between the particles is $\sim \hbar/m_e c$. Note that these parameters correspond to the upper limit of the mass of white dwarfs (the momentum p can be smaller than $m_e c$).

NEUTRON STARS

Suppose $r \sim \hbar/m_\pi c$; then the elementary volume is

$$v_N \sim \left(\frac{\hbar}{m_\pi c}\right)^3, \tag{19}$$

the concentration is

$$n_N \sim \left(\frac{m_\pi c}{\hbar}\right)^3, \tag{20}$$

and the density

$$\rho_N \sim m_p\left(\frac{m_\pi c}{\hbar}\right)^3 \sim 10^{15}\ \text{g-cm}^{-3}. \tag{21}$$

The characteristic nucleon momentum is $p_N \sim m_p c$. More precisely, this is an upper limit since in this case one can also form the characteristic momentum $m_\pi c$ from the corresponding parameters. We take the first value; then the condition of equilibrium becomes

$$\frac{GM_N}{R_N} = \frac{c^2}{2}. \tag{22}$$

Note that for a strongly compressed gas the reaction $e^- + p \to n + \nu$ is energetically advantageous. The electrons can "decrease" their size only by combining with protons. Then

$$R_N = \left(\frac{3}{4\pi}\frac{M_N}{m_p}\right)^{1/3}\frac{\hbar}{m_\pi c}, \tag{23}$$

$$M_N = \left(\frac{3}{32\pi}\right)^{1/2}\left(\frac{\hbar c}{G}\right)^{3/2} m_p^{-1/2} m_\pi^{-3/2}, \tag{24}$$

$$M_N \sim 3M_d \sim 3M_\odot \cdot R_N \sim 10^6\ \text{cm}.$$

These parameters correspond to the characteristics of neutron stars—pulsars. More precisely, since we have set $p_N \sim m_p c$, the mass value (24) is an upper bound for such objects. Precisely, $M_\odot \lesssim M_N \lesssim 3M_\odot$ [see Eq. (18)]. The mass (24) is the upper limit of the mass of a neutron star.

BLACK HOLES

If strong interactions are insufficient to compensate the gravitational forces, contraction will continue until a structureless object is formed, which absorbs all types of matter within it, has an essentially relativistic nature, and properties determined by the position of the observer. But, true to the general line of this paper,

[6] This is of course the condition of degeneracy of a relativistic electron gas.

[7] We retain here the numerical factor since, as we shall see later, the difference between the masses of other final states of stars is small. Therefore, this factor cannot be ignored.

we shall restrict ourselves to a classical analysis, defining a black hole (collapsar) as an object that absorbs any point particle moving within it in accordance with classical laws. Such a definition in the framework of the general theory of relativity is to a certain extent equivalent to assuming an infinitely distant (from the collapsar) observer. From these assumptions, there follows a value for the radius of the black hole, which was apparently obtained for the first time by Laplace as long ago as 1798:

$$R_g = \frac{2GM}{c^2}. \tag{25}$$

Since the relation (25) is actually equivalent to (22) and (23), we can conclude formally that neutron stars do not exist. In reality, the condition (22) is too crude. We have noted above that our value $M_N \sim 3M_\odot$ is only an upper limit for the mass of a neutron star [see Eqs. (18) and (24)].

BLACK HOLES EMIT

Until recently it was assumed that black holes are dead objects that absorb everything and emit nothing. However, in the last few years (1974–1975) it has been "unexpectedly" discovered (of course, theoretically) that black holes not only can but *must emit*. The hardest stumbling block was the idea that a black hole is an absolutely static object. In reality, as Hawking has shown in the framework of general relativity, black holes become absolutely static only as $t \to \infty$; if the time from the onset of collapse is finite, then cosmic objects tend to their gravitational limit, but do not reach it. In this sense, strictly speaking, a collapsar is a nonequilibrium object. Like every nonstatic object, a black hole must emit.[8)] Here we shall restrict ourselves to a simple estimate of the characteristics of black hole emission.

The physical meaning of this radiation is simple; the arguments, which are due to Laplace already, are based on a pointlike test body or, rather, one with size much less than R_g. However, it is obvious that elementary arguments are not valid if this condition is not satisfied. Suppose that the size of the test body exceeds R_g; then the "part" of the body outside the black hole cannot enter it. In particular, such a "body" can be radiation with wavelength $\lambda \gtrsim R_g$. In this case, a photon has a finite probability of being outside the black hole and can leave it without hindrance.

Let us consider this possibility, making the natural assumption that the emission which leaves the black hole is in thermodynamic equilibrium. We have at our disposal one characteristic length, R_g; it is therefore natural to assume that the maximum of the emission is at the wavelength $\lambda \sim R_g$. Then

$$\frac{\hbar c}{R_g} \sim kT \tag{26}$$

and the energy density is

$$\varepsilon_b \sim \left(\frac{kT}{\hbar c}\right)^4 \sim \frac{\hbar c}{R_g^4}. \tag{27}$$

The luminosity of the black hole is $L_b = dE_b/dt$:

$$L_b = \frac{dE_b}{dt} \sim \varepsilon_b c R_g^2 \sim \frac{\hbar c^2}{R_g^2}. \tag{28}$$

The lifetime t_l of the black hole can be readily estimated from the relation

$$\frac{dE_b}{dt} t_l \sim M_b c^2; \tag{29}$$

$$t_l \sim \frac{G^2}{\hbar c^4} M_b^3; \tag{30}$$

M_b is the mass of the black hole. The relation (30) can be written in the form

$$t_l \sim 10^{17} \left[\frac{M_b(\mathrm{g})}{10^{15}}\right]^3 \text{ sec}. \tag{31}$$

Since 10^{17} sec $\sim t_0$ is the cosmological time, it follows from (31) that there now exist and can emit copiously only black holes with $M_b \sim 10^{15}$ g. If $M_b \ll 10^{15}$ g, such a black hole has already evaporated, and if $M_b \gg 10^{15}$ g such an object hardly emits at all (see Eq. (28)).

It is obvious that black holes with $M_b \sim 10^{15}$ g are not formed as a result of the collapse of stars; if such black holes do exist, they have a cosmological origin, i.e., were formed as a result of fluctuations during the early stages in the evolution of the universe.

Let us consider further the emission of particle–antiparticle pairs from the vacuum in the gravitational field. If pair emission is to be effective, it is sufficient if over the distance r (the "size" of the particle) the difference between the potentials of the gravitational field is in order of magnitude equal to the particle rest mass[9)]:

$$\frac{GM_b}{R_g^2} r \gtrsim c^2. \tag{32}$$

This condition is equivalent to the inequality $kT \gtrsim mc^2$. It follows from this inequality, as also from the condition (32), that emission of particle–antiparticle pairs with mass m is possible if the gravitational radius satisfies $R_g \lesssim h/mc$, i.e., does not exceed the Compton wavelength of the particle m. Therefore, for electron–positron pairs the mass M_b of a black hole at which intensive production begins is

$$M_b \sim \frac{\hbar c}{Gm_e}. \tag{33}$$

For nucleon–antinucleon pairs

$$M_b \sim \frac{\hbar c}{Gm_p}. \tag{34}$$

If the mass of a black hole reaches the critical value

8) This of course was long known, but it was assumed that the emission of black holes is negligibly small if $t \gg R_g/c$.

9) Because of the tunnel effect, this emission begins somewhat earlier.

TABLE V.

Class of objects	Diameters of objects, cm	Corresponding "sizes" of microscopic particles, cm
Red dwarfs	10^{10}	Atomic, 10^{-8}
White dwarfs	$10^{8}-10^{9}$	10^{-11}
Neutron stars	10^{6}	10^{-13}
Collapsars from stars	10^{5}	

given by (33) or (34), emission of pairs will predominate over radiation. If the emission of hadrons begins in accordance with (34), then (as is usually the case in processes with the participation of strong interactions) the process will proceed very strongly. The black hole "explodes" with the emission of hadrons.

WHAT HAPPENS IF THE PARAMETERS OF ELEMENTARY PARTICLES CHANGE

We have considered the connection between the parameters of stars and elementary particles. From the simple relations we have obtained, we can conclude that the existence of all four classes of stars is, to a certain extent, fortuitous and a reflection of the parameters of the elementary particles. Table V shows the correspondence between the sizes of the cosmic objects and the radius of the microscopic particle.

Suppose that the temperature needed for thermonuclear reactions increases by several orders of magnitude. For example, one could imagine that the elementary charge of the electron is 10 times greater than the actual charge. Then, accordingly, the mass needed for self-sustaining thermonuclear reactions must also rise by two orders of magnitude (corresponding to the increase in the Coulomb barrier). In this case, stars with $M \sim M_\odot$ could not exist at the present time; previously existing stars of this mass would already have collapsed with the formation of white dwarfs and pulsars.

Suppose further that the mass of an exchange particle is $m_\pi > m_p$. In this case, the proton momentum would be strictly equal to $m_p c$, and therefore the mass of a neutron star would always to be equal to the mass of the black hole; in other words, neutron stars could not exist at all.

Let us imagine a more general case: suppose that the masses of the elementary particles could be arranged in the series $m_1 < m_2 < \cdot < m_k$ and that the mass of the exchange particle for object i is $m_{i+1} > m_i$; then the existence of final evolution stages of stars other than collapsars would be impossible.

To illustrate this situation, let us consider the possible existence of stars consisting of quarks. In accordance with various models, $m_{qu}c^2 \sim 300$–500 MeV (m_{qu} is the quark mass), and the quark "size" is $\sim \hbar/10 m_{qu}c$. If we ignore the uncertainty in the concept of the mass and size of the quark and assume that the length given here is equal to the range of the interaction between quarks, then such a quark star cannot exist—collapse must necessarily occur.

An analogous situation would arise if the mass of fundamental particles increased significantly. For example, if the masses of the proton and pion increased by 10 times, then the upper limit of the mass of such a "neutron star" would be reduced by 100 times (see Eq. (24)), and therefore there would again exist just a single final form of evolution of the stars—the collapsar.

The existence of stars of the main sequence in our epoch, white dwarfs, and neutron stars is fortuitous in the sense that it is due to the numerical values of the parameters e and m_π, which, according to modern ideas, are in no way related to the other fundamental constants G, $\hbar$, c, m_p.

"MAGIC" NUMBERS

It follows from various arguments that the mass of a star which exists for $t_0 \sim 10^{17}$ sec must be of order 10^{32}–10^{33} g.

The masses of white dwarfs are near the limit of this interval: the masses of stars of the main sequence are near $M_\odot$ because of equilibrium considerations independently of the need for effective thermonuclear reactions. The mass of the equilibrium state of white dwarfs and neutron stars is also near $M_\odot$. All these almost equal values have been obtained by independent arguments. Is this fortuitous? So far, this question is rhetorical. To emphasize these "coincidences," I give a remarkable sequence of numbers, which offer no physical interpretation.

Let us find the mass m_M of a black hole at which its gravitational radius is equal to its Compton wavelength:

$$\frac{G m_M}{c^2} \sim \frac{\hbar}{m_M c}, \tag{35}$$

$$m_M = \sqrt{\frac{\hbar c}{G}} = M_0. \tag{36}$$

It is well known that this value of the mass possibly plays an important role in the nature of fundamental interactions. We form the quantity

$$M_1 = \frac{m_M^2}{m_p} \sim 10^{15}\ \text{g}, \tag{37}$$

This mass $M_1 \sim 10^{15}$ g has a distinguished significance in the emission of black holes. The gravitational radius of such a "particle" is equal to the nucleon radius. Only this "particle" can now exist in the form of a black hole and emit. Beginning with the mass value M_1, the emission of nucleon–antinucleon pairs begins to predominate.

An even more important role is played by the mass

$$M_2 = \frac{m_M^3}{m_p^2}. \tag{38}$$

This is the mass of a standard star; $M_2 \sim M_\odot$; as we have seen, the value of M_2 is essentially distinguished in stellar objects [cf. (34) with (18) and (24)]. We form the final term of this series:

$$M_3 = \frac{m_M^4}{m_p^3}, \tag{39}$$

$M_3 \sim 10^{54}$ g is approximately (in order of magnitude) the mass of the visible part of the universe with radius $\sim 10^{28}$ cm.[10)]

Unfortunately, there is no basis for commenting on the physical meaning of the series m_M^n/m_p^{n-1} at the present time.

[10)]It has been established that in the universe there are approximately 10^{10} galaxies, in each of which there are on the average 10^{10}–10^{11} stars.

Translated by Julian B. Barbour