Ion-beam plasma and the propagation of intense compensated ion beams

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Studies of the properties of plasma produced during the neutralization of the space charge of an intense ion beam are reviewed. Compensation of an ion beam by charges produced as a result of the ionization of a gas by the beam or by charges introduced from outside is considered. Particular attention is devoted to collective phenomena in ion-beam plasmas and, in particular, to nonlinear effects restricting the amplitude of the excited oscillations. It is shown that the propagation of compensated ion beams is influenced not only by dynamic decompensation but also by coulomb ion-electron scattering and by collective oscillations. All these processes must be taken into account when questions connected with the production of "ultradense" compensated beams are considered.

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CONTENTS

1. INTRODUCTION

The propagation of intense ion beams is accompanied by a spreading due to the mutual repulsion of the ions. This effect can be reduced by compensating (neutralizing) the beam space charge by charges of another sign, and the net effect of this is that the resulting system is essentially a specific "ion-beam" plasma. In view of the extensive applications of ion beams in science and technology,[1–5] and the very extensive range of new investigations of physical phenomena in compensated ion beams, it is now possible and useful to review and summarize the current state of development of this branch of plasma physics. This is the aim of the present review in which we will consider the following problems.

a) Compensation of previously formed and accelerated extensive ion beams, which leads to the establishment of a residual radial electric field. In particular, attention is drawn to the inadequacy of previous descriptions of the compensation process in a stable ion beam, namely, the suggestion that the minimum residual field is connected with the Coulomb interaction between the beam ions, on the one hand, and the neutralizing electrons, on the other.

b) Collective processes in unstable ion-beam plasma, are given considerable space in this review. In addition to studies of the linear stage of development of the different branches of oscillations, there has been considerable experimental work in recent years on the corresponding nonlinear effects.

c) Certain topical problems in the propagation of compensated ion beams, which have previously been discussed in relation to single-component beams[6] or were ignored altogether (influence of collective processes and Coulomb collisions on beam transport).

2. COMPENSATION OF THE SPACE CHARGE OF ION BEAMS

A. Ion-beam compensation methods and ion-beam plasma

The space charge of an ion beam can be compensated as follows: 1) by charges produced during the ionization of atoms in a low-pressure gas by the ion beam (self-compensation or gas compensation) and 2) by charges introduced into the beam from outside, i.e., by superimposing the ion beam on a preformed accelerated beam of particles of opposite sign ("forced" compensation or production of synthesized beams and synthesized plasma).

The gas compensation process is essentially different for positive and negative ion beams.[7–9] Thus, when a positive ion beam traverses the residual gas, or the gas specially introduced into the ion guide to achieve compensation, the number of electrons produced per unit volume per unit time is $\nu_e = n_b v_b n_a \sigma_e$. In this expression, n_b and v_b are, respectively, the concentration and velocity of beam ions, n_a is the concentration of the gas atoms, and σ_e is the electron production cross section. The number of slow ions produced at the same time is $\nu_{pi} = n_b v_b n_a \sigma_{pi}$, where σ_{pi} is the corresponding ion production cross section. When the gas pressure is relatively low [so that the concentration of atoms is much less than the quantity given by (1.5) below], the concentration of slow ions is low and the resulting ion-beam plasma can be described by a model consisting of

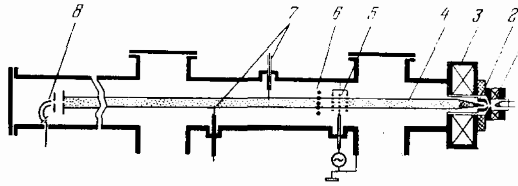

FIG. 1. Apparatus for investigating the compensation of ion beams and collective oscillations in ion-beam plasma: 1—duoplasmatron, 2—extractor, 3—magnetic lens, 4—ion beam, 5—modulator, 6—neutralizer, 7—probes, 8—ion energy analyzer.

a "cold" ion beam, the residual potential well of which is filled by an electron gas with a Maxwell velocity distribution corresponding to temperature T_e and concentration satisfying the quasineutrality condition

$$n_e \approx n_b. \tag{1.1}$$

The further condition for the plasma state, which must be satisfied in addition to (1.1), is the inequality

$$d_e = \sqrt{\frac{T_e}{4\pi n_e e^2}} \ll R, \tag{1.2}$$

where d_e is the Debye length and R is the beam radius. Since the beam current is given by $I_b = en_b v_b \pi R^2$, we have from (1.2) the following equivalent condition:

$$\frac{I_b}{v_b} \gg \frac{T_e}{e}, \tag{1.3}$$

which shows that the ion-beam plasma is produced only when the beam intensity is such that the radial potential drop prior to compensation, $\Delta\varphi_b = I_b/v_b$, substantially exceeds the ratio T_e/e. As the gas pressure increases, condition (1.1) is replaced by

$$n_e \approx n_b + n_{pi}, \tag{1.4}$$

and the system takes the form of an ion beam passing through the two-component plasma produced by it and consisting of an electron gas and slow ions with concentration n_{pi} comparable with the concentration of the other charged particles. These slow ions can only formally be said to have the "temperature" T_{pi}. This type of plasma is formed, for example, when an ion beam passes through a charge-transfer chamber: beams with currents of 1–10 A produce at entry into the chamber a plasma with $n_{pi} \approx 5 \times 10^{10} - 5 \times 10^{11}$ cm^{-3} and $T_e \approx 1$ eV, but the concentration decreases with increasing distance from the ion source, and the electron temperature substantially increases for reasons that are not as yet clear.[10,11]

It is important to note the following point. If $f(v_e)$ and $f(v_{pi})$ are the velocity distribution functions for electrons and ions when they are formed, the corresponding mean velocities are given by

$$\bar{v}_e = \int_0^\infty f(v_e)\, v_e\, dv_e \Big/ \int_0^\infty f(v_e)\, dv_e,$$

$$\bar{v}_{pi} = \int_0^\infty f(v_{pi})\, v_{pi}\, dv_{pi} \Big/ \int_0^\infty f(v_{pi})\, dv_{pi}.$$

Since $\bar{v}_{pi} < \bar{v}_e$ and $\nu_{pi} \geq \nu_e$, the most that the gas compensation of freely propagating positive ion beams can achieve is to reduce the electric field producing the spreading of these beams. A change in the sign of the field is not possible. However, in the case of compensation of a negative ion beam, when the neutralizing particles are slow positive ions, an increase in the concentration of atoms beyond the value defined above, i.e.,

$$n_{a0} = \frac{2\bar{v}_{pi}}{Rv_b\sigma_{pi}} \tag{1.5}$$

leads both to a change in the sign of the potential and in the corresponding radial electric field, so that "gas focusing" of this beam is produced.[9] Unfortunately, this focusing occurs for $n_a > n_{a0}$ when the mean free path for the detachment of an electron from a negative ion satisfies the condition

$$\lambda_{\bar{1},0} < \frac{Rv_b\sigma_{pi}}{2\bar{v}_{pi}\sigma_{\bar{1},0}}, \tag{1.6}$$

and an appreciable proportion of the ions lose their charge by collisions with atoms. For the ion-beams plasma produced by compensation of a negative ion beam, the quasineutrality conditions corresponding to (1.1) and (1.4) assume the form

$$n_{pi} \approx n_b \tag{1.7}$$

and

$$n_{pi} \approx n_b + n_e. \tag{1.8}$$

Another method of compensation—forced compensation[12,13]—is based on the use of some known form of emission of charged particles by solids or the emission of particles by plasma, for example, the cold plasma produced as a result of the ionization of cesium atoms on a hot surface.[14] The forced compensation system in which the synthesized beam consists of interacting ion and electron beams traveling in the same direction has been widely used. When the concentrations and velocities of the components of the ion-electron beam are equal, there is not only a compensation of the space charge but also a current compensation, which is frequently necessary. To ensure that the ion and electron velocities are equal, the corresponding accelerating potential differences must be related by $\varphi_b/\varphi_e = M/m$. The necessary superposition of the ion and electron beams is achieved by placing a neutralizer in the form of a thermionic source of electrons[13,15–17] in the path of the ions (Fig. 1), by passing the electron beam through the ion source,[18,19] by rotating the particles in magnetic fields until the beams are coincident,[20] or by producing electron and ion beams traveling in opposite directions.[21]

The ion-ion beam[22] produced by superimposing positive and negative ion beams of equal density, velocity, and energy is a particular form of synthesized plasma. By passing positive ions through a charge-transfer chamber, it is also possible to produce an analogous ion-ion beam, but the currents associated with the two components are then low.[23] The ion-ion beam has been

TABLE I. Parameters of some compensated ion beams.

	Ref. 29	Ref. 30	Ref. 31	Ref. 32
Beam energy, keV	40	600	25	35
Beam current, mA	50(H_2^+)	100(H_2^+)	600(H_1^+)	15(He^+)
Beam density, mA/cm^2	4	5	2	500
Distance from point of measurement, cm	300	200	350	50
Beam ion concentration, cm^{-3}	$1.2\cdot10^8$	$0.4\cdot10^8$	$0.6\cdot10^8$	2.10^{10}
Debye length, cm	$\sim10^{-1}$	$\sim10^{-1}$	$\sim10^{-1}$	$\sim10^{-2}$
Electron Langmuir frequency, sec^{-1}	10^8	$6\cdot10^7$	$7\cdot10^7$	10^9
Ion Langmuir frequency, sec^{-1}	$1.7\cdot10^6$	10^6	$1.6\cdot10^6$	$1.2\cdot10^7$

used as a model for verifying the theory of nonlinear interaction of charged-particle beams,[24] and successes in the development of high-intensity sources of negative ions[25–27] suggest that it may well be possible to use such beams for the injection of fast particles into magnetic traps.[28]

Table I shows the parameters of a number of extended ion beams propagating at relatively low gas pressure in the absence of external fields. The beams are prefocused by a magnetic lens. Inspection of the table will show that, because of incomplete compensation, the beam current densities are relatively low and decrease with beam length. The reasons for the incomplete compensation are discussed in the succeeding sections.

B. Violation of compensation through escape of neutralizing particles along the beam

The escape of electrons along the beam, which is one of the reasons for the violation of gas compensation, is facilitated by the electric field in the region of primary beam shaping and acceleration. Measures used to prevent this phenomenon include accelerating-decelerating ion-beam shaping systems and focusing magnetic lenses or transverse magnetic fields.[29,33]

C. Dynamic decompensation of ion beams

The phenomenon of dynamic decompensation,[7,34–36] i.e., the appearance of an alternating potential producing the spreading of the ion beam, is connected with pulsations in the ion current, and is due to the fact that the rate of increase of this current exceeds the rate of accumulation of neutralizing electrons. The electron accumulation time up to the point where (1.1) is satisfied, and hence the positive ion beam compensation time,[7,30,31] are given by

$$\nu_e\tau_c \approx n_b, \quad \tau_c \approx \frac{1}{v_b n_a \sigma_e}. \tag{1.9}$$

Dynamic decompensation sets in when the ion-current pulsation frequency is sufficiently high: $f \gtrsim 1/\tau_c$. At very high frequencies, when the number of electrons accumulated during one period of the oscillations is small, the alternating component of the potential drop is given by

$$\Delta\tilde{\varphi} = \Delta\varphi_b\cdot\mu, \tag{1.10}$$

where $\mu = \Delta I_b/I_b$ is the relative magnitude of the current pulsations.

Dynamic decompensation is the leading effect in many systems. It is frequently complicated by the fact that the current-density pulsations on the boundary of the plasma produced by the ion source (ion-current pulsations in the beam) give rise to a periodic variation in the shape of this plasma boundary and a corresponding change in the properties of the primary ion-optical system. Studies of the transport of beams in a transverse magnetic field[7,34–36] have shown that the reduction or the increase in the current density, which occur under these conditions in certain definite parts of the beam, are accompanied either by the escape of excess electrons or the appearance of an excess positive charge, and the net effect of this is an enhancement of the influence of oscillations in the ion current (the enhancement factor is $K = \Delta j/j\mu$). Reduction in the amplitude of the oscillations in the ion sources and in the factor K through the optimization of the ion-optical system, which includes the plasma boundary, has led, for example, to a substantial increase in the efficiency of electromagnetic separators.

Pulsed operation with beam life $\tau < \tau_c$ has been used, for example, at relatively high gas pressures to simulate the forced-compensation process under ultrahigh vacuum conditions.[37]

D. Self-decompensation of positive ion beams by Coulomb collisions between ions and electrons

The ion-beam modulation depth can be reduced by a suitable choice and improvement of the ion source. The question therefore arises as to what is the ultimate degree of compensation that can be achieved for such stable beams. This question has been considered[8,38] in terms of the balance equation for the compensating electrons

$$\nu_e\beta\pi R^2 = 2\pi R\,\frac{n_e v_e}{4}\exp\left(-\frac{e\Delta\varphi_c}{T_e}\right); \tag{1.11}$$

where β is the relative number of electrons produced by ionization, which do not have sufficient energy to leave the beam immediately and are captured by the residual potential well due to the beam, and $\Delta\varphi_c$ is the well depth. It follows from (1.11) that the formula

$$\Delta\varphi_c = \frac{T_e}{e}\ln\eta, \tag{1.12}$$

which is frequently used in the literature is, in fact, incorrect for the following reasons.

1) The mechanism assumed for maintaining the stationary state, i.e., energy transfer between electrons in the well and escape of these electrons through the Maxwell distribution "tail," on which the derivation of (1.12) is based, is meaningless in the absence of a source of additional energy transmitted to the electron gas. The correct solution of the problem must be based on the utilization of both the particle balance equation and the energy balance equation.[39] An attempt to in-

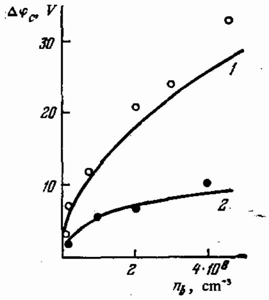

FIG. 2. Radial potential difference in compensated ion beam of 20 mA and 23 keV as a function of concentration of H_2^+ ions. Curve 1—gas pressure $p=1\times10^{-5}$ Torr, 2—$p=1\times10^{-4}$ Torr.

troduce the latter was made in[40], but the particle balance equation used there was incorrectly formulated and this led to the incorrect expression given by (1.12).

2) Equation (1.12) is in conflict with experimental data, since it does not predict the observed dependence of $\Delta\varphi_c$ on the beam density (Fig. 2). In contrast to the prediction given by (1.12), the potential difference $\Delta\varphi_c$ varies along the beam whereas the corresponding change in T_e is quite small.

The additional source of energy mentioned above is the ion beam itself. The mechanism responsible for maintaining a stationary state for a finite $\Delta\varphi_c$ is based on Coulomb collisions between beam ions and electrons captured by the well as a result of which the latter receive additional energy and therefore escape from the beam. The minimum energy that must be introduced per unit time into a compensated beam of radius R and length L in order to ensure that all the captured electrons may leave the well is

$$\mathscr{E}=L\int_0^R 2\pi\xi\,d\xi\int_0^{e\varphi(\xi)} f(\varepsilon)\,(e\varphi-\varepsilon)\,d\varepsilon. \tag{1.13}$$

The energy distribution of the resulting electrons will be taken to be $f(\varepsilon)\sim 1/(\varepsilon+\varepsilon_i)^2$, where $\int_0^\infty f(\varepsilon)\,d\varepsilon=\nu_e$. This distribution is obtained from the classical Thomson ionization theory ($\varepsilon_i=e\varphi_i$ is the ionization energy of the atom), and the shape of the potential well will be approximated by the function

$$\varphi(\xi)=\Delta\varphi_c\left(1-\frac{\xi^2}{R^2}\right). \tag{1.14}$$

Using the well-known expression for the energy loss experienced by an ion beam passing through an electron gas,[41] we obtain the following expression for the energy transferred to this gas per unit time:

$$Q_{\rm Coul}=\frac{\alpha n_b n_e e^4\pi R^2 L}{m v_b}, \tag{1.15}$$

where $\alpha=4\pi ln(m^{3/2}v_b^3/1.78\pi^{1/2}n_e^{1/2}e^3)$.

The energy transferred to the electron gas produced during ionization by fast electrons (not captured by the potential well) can be neglected in comparison with $Q_{\rm Coul}$, which is transferred directly by the beam ions. Thus, the ratio of the current of these fast electrons in a beam of length L to the ion current is less than $n_b v_b n_a \sigma_i L/n_b v_b$, and the latter is much less than unity under typical experimental conditions. The required quantity $\Delta\varphi_c$ can be obtained from $\mathscr{E}=Q_{\rm Coul}$ and the quasineutrality condition (1.4) in which the concentration of slow ions averaged over the cross section is given by

$$n_{pi}=\frac{n_b v_b n_a \sigma_{pi} R}{2\bar v_{pi}}, \tag{1.16}$$

which is obtained from the corresponding balance equation. The mean velocity $\bar v_{pi}$ of slow ions depends both on their initial energy and on the field in the beam. The cross section σ_{pi} includes the charge-transfer and ionization cross sections. In the case of good compensation, $\Delta\varphi_c<\varphi_i$, the depth of the potential well is given by

$$\Delta\varphi_c\approx\sqrt{3\alpha}\,e\sqrt{\frac{M_b}{m}}\sqrt{\frac{\varphi_i}{\varphi_b}}\left(\frac{1}{n_a\sigma_e}+\frac{v_b\sigma_{pi}R}{2\bar v_{pi}\sigma_e}\right)\sqrt{n_b}. \tag{1.17}$$

By measuring the radial potential drop in a focused ion beam passing through a gas, it has been possible to compare calculated functions $\Delta\varphi_c=\Phi(R, n_a, n_b)$ with experimental data and to show that the agreement between them is satisfactory[42] (Fig. 2). Thus, the self-decompensation of a positive ion beam by Coulomb collisions with the compensating electrons has been experimentally confirmed.

In a negative-ion beam with an overcompensated space charge (when $n_a>n_{a0}$), the analogous Coulomb collisions will also facilitate the escape of electrons, but this has the opposite effect in that it tends to compress the beam through the appearance of excess positive space charge. The potential drop in an overcompensated beam of negative ions can be estimated from (1.17) by replacing the sum in parentheses by the quantity $\sqrt{1-(n_{a0}/n_a)}/\sqrt{n_a}\sqrt{\sigma_e+\sigma_{T,0}}$; the observed potential drop when a beam of H^- ions is passed through krypton is found to be of the order of a few volts.

There is one further decompensation mechanism, namely, the escape of compensating electrons by recombination with ions. This mechanism does not require additional energy. The corresponding electron lifetime in the potential well of the beam is $\tau_r=(\alpha_r n_{pi}+\alpha_r' n_b)^{-1}$, where α_r is the recombination coefficient. The ratio of this lifetime to the lifetime determined by energy transfer through Coulomb collisions is

$$\theta=\frac{\tau_r}{\tau_{\rm Coul}}=\frac{\alpha n_b e^3}{(\alpha_r n_{pi}+\alpha_r' n_b)\,m v_b\,\Delta\varphi_c}. \tag{1.18}$$

Numerical estimates show that $\theta\gg1$ under typical experimental conditions, even when dissociative recombination is possible. This process is therefore of minor importance in comparison with decompensation due to Coulomb collisions.

E. Decompensation of a synthesized ion-electron beam

The radial electric field should be zero in the ideal model of synthesized plasma consisting of cold ion and electron beams with equal concentrations and velocities over the entire cross section. Since, however, the electron temperature T_e is finite, it follows that, even when the total number of positive ions per unit beam length is equal to the corresponding number of electrons, some of the latter are located outside the ion core of the syn-

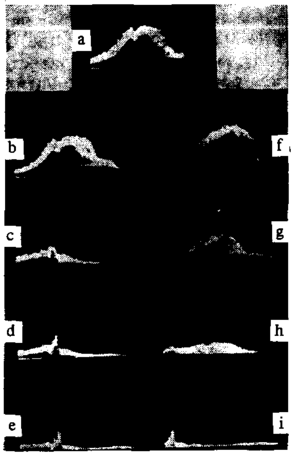

FIG. 3. Effect of ion-beam modulation on the spectrum of excited electron oscillations. Spectrum: a—modulation amplitude $\varphi_m=0$, b—$\varphi_m=14$ V, modulation frequency $f=27$ MHz, c—$\varphi_m=34$ V, d—$\varphi_m=50$ V, e—$\varphi_m=70$ V, f—$\varphi_m=20$ V, $f=20$ MHz, g—$\varphi_m=50$ V, h—$\varphi_m=100$ V, i—$\varphi_m=150$ V.

thesized beam and, therefore, a radial field will appear. The radial potential drop in this beam is given by[15,44]

$$\Delta\varphi_{i,e}=\sqrt{\frac{2I_bT_e}{v_be}}, \qquad (1.19)$$

where I_b is the current of the ion component, which is equal to the electron current, and v_b is the electron and ion velocity. The formula given by (1.19) is in agreement with the numerical solution[45] of the self-consistent field equations for a model with finite phase volumes of the ion and electron beams. This solution shows that the configuration of the phase volume changes during the propagation of the beams. The most important is the change in the phase volume of the electron beam, the equivalent increase in the electron temperature, and the escape of these electrons beyond the limits of the ion core.

Experiments with an ion-electron beam consisting of 20–30-keV helium ions and electrons of the same velocity show that the function $\Delta\varphi_{i,e}=f(I_b)$, which follows from (1.19), is in qualitative agreement with experimental data if T_e is taken to be equal not to the temperature of the neutralizer but to the directly measured actual temperature of the electron gas.[15] This is also in agreement with the conclusions in[45].

We shall not pause to consider the results reported in[46–48], in which a discussion is given of the existence of stationary solutions for a potential distribution along the ion-electron current, and draw attention only to the parameter $\chi=\bar{v}_{Te}/v_b$ used in these papers and equal to the ratio of the mean thermal velocity of electrons to the ion-beam velocity. Equation (1.19) is valid only when $\chi\ll 1$, i.e., when both electrons and ions move with a small velocity spread in the form of a beam. When $\chi \gtrsim 1$, the beam is more efficiently neutralized by the electron gas produced as a result of the reflection of some of the electrons by the electric field established along the beam.[49]

A transverse magnetic field acting on the ion-electron beam will give rise to charge separation, the appearance of an electric field, and violation of current compensation.[19]

3. COLLECTIVE PROCESSES IN ION-BEAM PLASMA

Theoretical papers on the interaction of charged particles with plasmas[50–56] predict, among other things, the instability of fast compensated ion beams with velocity exceeding the mean thermal velocity of electrons, i.e., they predict that the typical ion-beam plasma will be unstable. In this section, we shall confine our attention to these oscillations. They are interesting not only because the fields associated with the excitation of collective oscillations in compensated ion beams affect beam transport and focusing, but also in connection with a very wide range of problems in plasma physics.

A. Langmuir electron oscillations of ion-beam plasma

The dispersion relation describing the longitudinal electron resonance oscillations ($\omega_e/k\approx v_b$, $k=k_z$) in a homogeneous system consisting of plasma and a fast cold ion beam passing through it has the form

$$1-\frac{\omega_e^2}{\omega^2}-\frac{\alpha\omega_e^2}{(\omega-k_zv_b)^2}=0, \qquad (2.1)$$

where $\alpha=(\omega_b/\omega_e)^2\equiv n_b m/n_e M_b\ll 1$ and $\omega_b=\sqrt{4\pi n_b e^2/M_b}$ and $\omega_e=\sqrt{4\pi n_e e^2/m}$ are the Langmuir frequencies of the beam ions and plasma electrons. Solution of (2.1) will show that the maximum growth rate is

$$\gamma=3^{1/2}2^{-4/3}\omega_e\left(\frac{\omega_b}{\omega_e}\right)^{2/3}, \qquad (2.2)$$

and the oscillation frequency $\omega\approx\omega_e$. For ion-beam plasma formed in low-pressure gas, we can use the quasineutrality condition (1.1), and the expression given by (2.2) becomes somewhat simpler:

$$\gamma=3^{1/2}2^{-4/3}\omega_e\left(\frac{m}{M_b}\right)^{2/3}. \qquad (2.3)$$

The criterion for the validity of the above approximation based on a cold ion beam is obtained from the condition $|(\omega/k)-v_b|\gg v_{Tb}$:

$$\frac{v_b}{v_{Tb}}\gg 2^{4/3}\left(\frac{M_b}{m}\right)^{1/3}. \qquad (2.4)$$

In accordance with theory, ion-beam plasma produced by an ion beam passing through a gas is found experimentally[57–59] to support oscillations of frequency ω_e (in the oscillation spectrum shown in Fig. 3a, this frequency corresponds to the maximum amplitude) and phase velocity approaching the beam velocity. The amplitude of the oscillations grows exponentially in the direction of motion of the ions, and the growth rate calculated from (2.3) is somewhat different from that calculated from the experimental data, but is not too different. The uncertainty of the results obtained in the early studies[60,61] was probably connected with the small magnitude of the growth rate (2.3) and insufficient size of the beams investigated at the time, as well as the possible excitation of oscillations with similar frequencies by the accompanying electron beams. Thus, for example, it was subsequently shown[58] that, when the collector potential was such that the electron beam due to ion-electron emission by the collector traveled

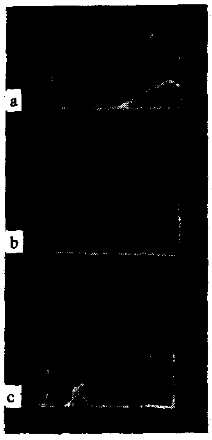

FIG. 4. Spectra of oscillations excited in ion-electron beam for different collector potentials. a—collector potential 0, b—40 V, c—160 V. Marker on the left corresponds to zero frequency and that on the right to 30 MHz.

against the ion beam, the resulting oscillations had lower frequency (less than ω_e) and larger amplitude. The amplitude increased in the direction opposite to that of motion of the ions. The difference between the frequencies of electron oscillations excited in this experiment by electron and ion beams was explained by the finite size of the system, i.e., the fact that, because of the high velocity of electrons $kR < 1$ in the first beam and $kR > 1$ in the second. Direct evidence for the effect of the finite radial size of real ion-beam plasma on the parameters of excited oscillations is provided by experiments with compensated negative ion beams.[62] The electron oscillations are excited when the concentration of atoms is increased along the path of this beam ($n_a > n_{a0}$), and the change in the sign of the potential in the beam leads to the accumulation of electrons. When $kR < 1$, the reduction in the beam radius produced by an iris diaphragm gives rise to a sharp reduction in the frequency and amplitude of the excited electron oscillations.

Spatial enhancement of oscillations in ion-beam plasma during the initial modulation of the ion beam was established in[63,64]. The superposition of a weak transverse magnetic field influencing the electrons leads to a reduction in this enhancement, so that the latter is, in fact, connected with the collective interaction of the ion beam with the electron background and not with purely kinematic phase bunching of ions during the modulation.

High-frequency oscillations are also excited in the synthesized ion-electron beam.[65] Figure 4 shows the spectra of oscillations obtained for different collector potentials. When the collector potential is close to the electron-emitter potential, and the electrons do not have directed velocities, oscillations are excited in accordance with (2.1) at the electron plasma frequency (Fig. 4a). When the collector is at a floating potential, and the ion and electron velocities v_{bi} and v_{be} are equal, there are no oscillations, as expected (Fig. 4b). Further increase in the potential with the electron velocity substantially exceeding the ion-beam velocity is accompanied by the excitation of oscillations with frequency less than the plasma frequency of the electron beam (Fig. 4c). This is in agreement with the prediction that, when $v_{be} \gg v_{bi}$, the frequency of the oscillations should approach the frequency of the Buneman oscillations $\omega = 2^{-4/3}\omega_{be}^{1/3}\omega_{bi}^{2/3}$, which is less than ω_{be}.

The excitation of oscillations in synthesized ion-electron plasma leads to the equalization of the electron and ion velocities[16,65] (equalization of current components).

The nonlinear effects restricting the amplitude of the oscillations are of the greatest interest. Whereas the main nonlinear effect restricting the wave amplitude during the passage of electron beams through plasma is the phase bunching of the electron beam and the capture of the resulting bunches by its field,[66–70] in the case of ion-beam plasma, this effect is replaced by the capture of the compensating electrons by the wave field.[59] The capture of cold plasma electrons should occur when the wave amplitude is

$$\tilde{\varphi}_{\max} = \frac{mv_b^2}{4e} = 0.5\,\frac{m}{M_b}\,\varphi_b, \tag{2.5}$$

and, in the ion-beam plasma with ion energy $e\varphi_b$ of the order of, say, some tens of keV, this amounts to only a few volts. Without considering the possibility of direct experimental detection of capture electrons in ion-beam plasma,[71] we note that the existence of this effect is confirmed by studies of the distribution of the fundamental harmonic of the potential $\tilde{\varphi}$ along the ion-beam plasma, measured by a probe insulated from the plasma.[59] As the potential φ_m which modulates the ion beam is increased (the method of modulation is shown in Fig. 1), the observed restriction on $\tilde{\varphi}$ sets in at a smaller distance from the modulator (Fig. 5). The measured maximum value of $\tilde{\varphi}$ is close to that calculated from (2.5).

To explain the relative importance of the capture of electrons by the wave, we must also consider the distribution of the fundamental harmonic of the beam current along the ion beam plasma, measured by a probe screened by a grounded grid. This distribution has a maximum corresponding to the phase focus, and the distance between this focus and the modulator decreases with increasing φ_m. The position S of the phase focus, established experimentally as a function of φ_m, is not in agreement with kinematic theory, but is satisfactorily described by the equation

$$\exp(\gamma_z S) - \exp(-\gamma_z S) = 2\gamma_z S_k, \tag{2.6}$$

which takes into account the interaction between the beam and plasma.[69,70] In this expression, γ_z is the spatial growth rate of the oscillations and $S_k = 3.68 v_b \varphi_b / \omega\varphi_m$

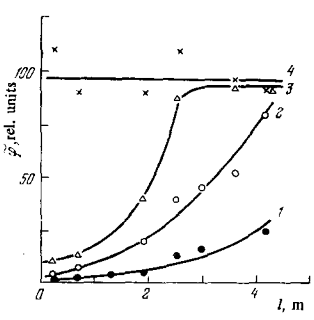

FIG. 5. Effect of the modulation amplitude on the distribution of the fundamental harmonic of the potential along the compensated ion beam. Curve 1—$\varphi_m = 18$, 2—$\varphi_m = 40$ V, 3—$\varphi_m = 120$ V, 4—$\varphi_m = 360$ V.

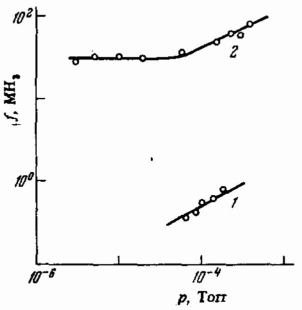

FIG. 6. Dependence of the frequency of excited oscillations in ion-beam plasma on hydrogen pressure. 1—Ion oscillations, 2—electron oscillations.

is the distance to the phase focus according to the kinematic theory. As a result of the beam-plasma interaction and the corresponding enhancement of oscillations, the inequality $S<S_k$ is satisfied. It is important to note that the distance between the modulator and the point of saturation of $\tilde{\varphi}$ is always less than the focal distance S. Thus, well before the onset of phenomena in ion dynamics that are characteristic for the phase focus, the wave amplitude becomes restricted by the capture of plasma electrons by the wave during the propagation of the compensated ion beam.

We note in conclusion that both the electron-beam modulation[72] and external modulation of the ion beam lead to the removal of instability at frequencies differing from the modulation frequency.[59] As the modulation frequency approaches ω_e, the oscillation cutoff occurs for smaller modulation amplitudes (Fig. 3).

B. Ion oscillations of ion-beam plasma

For phase velocities $v_{Ti} \ll \omega/k \ll v_{Te}$, where v_{Ti} and v_{Te} are the mean thermal velocities of ions and electrons, it is possible to obtain the following dispersion relation for the ion oscillations of nonisothermal plasma excited by a cold ion beam[55]:

$$1+\frac{1}{k^2d_e^2}-\frac{\alpha'\omega_{pi}^2}{(\omega-k_zv_b)^2}-\frac{\omega_{pi}^2}{\omega^2}=0, \tag{2.7}$$

where $\alpha'=(\omega_b/\omega_{pi})^2\equiv(n_bM_{pi}/n_{pi}M_b)$ and M_{pi} is the mass of slow ions in the ion-beam plasma. Solution of (2.7) will show that the maximum of the growth rate

$$\gamma=3^{1/2}2^{-4/3}\left(\frac{\omega_b}{\omega_{pi}}\right)^{2/3}\omega_k \tag{2.8}$$

is reached for

$$\omega_k^2=\frac{\omega_{pi}^2}{1+(k^2d_e^2)^{-1}}\approx(k_zv_b)^2. \tag{2.9}$$

The beam excites ion-acoustic (when $k^2d_e^2\ll 1$ and $n_{pi}=n_e$, the frequence is $\omega_k=kC_s$, where $C_s=\sqrt{T_e/M_{pi}}$ is the ion sound velocity) and ion Langmuir oscillations (when $k^2d_e^2\gg 1\omega_k\approx\omega_{pi}$). The latter are associated with the maximum growth rate. When $v_b\gg C_s$, it follows from (2.9) that

$$\left(\frac{k_\perp}{k_z}\right)^2=\frac{M_{pi}v_b^2(1+k_\perp^2d_e^2)}{T_e}\gg 1. \tag{2.10}$$

Thus, a fast ion beam should excite short-wave oscillations propagating at nearly right-angles to the ion beam ($k_\perp\gg k_z$).

The branch of ion-acoustic and ion-Langmuir oscillations excited in ion-beam plasma by a fast beam of positive ions was detected and investigated experimentally in[73,78]. The frequency f corresponding to the maximum amplitude in the observed spectrum of low-frequency oscillations, measured as a function of gas pressure, is of the form shown by curve 1 in Fig. 6.[73] This figure also shows the analogous dependence for the frequency of electron oscillations. Both a reduction and an increase in the gas pressure can be used to ensure that only electron oscillations are excited. In the former case, electrons are still accumulated in sufficient numbers in the potential well due to the ion beam whereas the ions are expelled from it so that their concentration is quite low. In the second case, when there is appreciable attenuation of oscillations due to collisions with atoms, the electron oscillations, which have a high growth rate, are damped out at higher frequencies. The growth rate calculated from (2.8) is close to the value calculated from the experimental data.

Equation (2.9) was verified by modulating the ion beam and measuring the phase of the oscillations as a function of the position of a mobile probe from the beam axis. This was used to determine k for each given modulation frequency ω.[75] The dispersion relations $\omega=f(k)$ (Fig. 7) obtained in this way showed, in accordance with (2.9) that $\omega=kC_s$ for low values of k, and, as the modulation frequency approached ω_{pi}, which corresponded to the maximum of the amplitude of spontaneously excited oscillations, the wavelength tended to zero. It was suggested that this could be used to determine the local concentration of slow ions. Having determined the frequency $\omega=\omega_{pi}=\sqrt{4\pi n_{pi}e^2/M_{pi}}$ of spontaneously excited oscillations with maximum amplitude for each value of the coordinate r, it is possible to calculate the concentration of slow ions and the dependence of this concentration on pressure for different cases (Fig. 8).

The dispersion relation for almost "transverse" ($k_\perp\gg k_z$) ion oscillations in ion-beam plasma in a longitudinal magnetic field, i.e.,

$$1+\frac{\omega_{pi}^2}{\omega_{Hi}^2-\omega^2}+\frac{\omega_b^2}{\omega_{Hi}^2-(\omega-k_zv_b)^2}=0 \tag{2.11}$$

shows that the corresponding growth rate is a maximum near $\omega=\sqrt{\omega_{pi}^2+\omega_{Hi}^2}$ and decreases with increasing magnetic field ($\omega_{Hi}=eH/M_{pi}c$). This suppression of ion

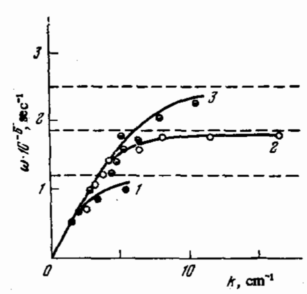

FIG. 7. Dispersive properties of ion oscillations for different gas pressures. Curve 1—$p=3\times10^{-5}$ Torr, 2—5×10^{-5} Torr, 3—8×10^{-5} Torr.

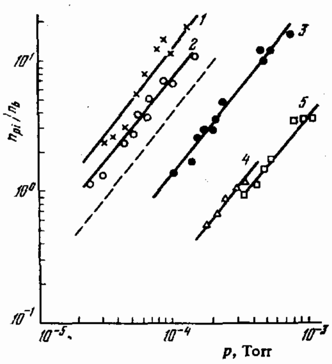

FIG. 8. Ratio of the concentration of slow ions to the concentration of beam ions (H_2^+) as a function of gas pressure. Curve 1—argon, 2—air (broken line calculated), 3—neon, 4—hydrogen, 5—helium.

oscillations by a magnetic field was established experimentally in[74].

The excitation of ion oscillations in plasma produced by a beam of negative ions has its own particular properties.[78] Three essentially different types of excitation of ion oscillations are possible in this plasma. At very low gas pressures, when the potential in the beam is negative, positive ions accumulate in the system whereas electrons are efficiently repelled in the direction of the chamber walls. When the electron concentration is reduced down to a value corresponding to $n_e/n_{pi} \ll (C_s/v_b)^2$, so that the second term in (2.7) is negligible, it is, in principle, possible to excite oscillations propagating along the beam even for $v_b \gg C_s$. At high pressures, when $n_e/n_{pi} > (C_s/v_b)^2$ but $d_e > R$, the excited ion oscillations are not as yet longitudinal in relation to the beam, but the effect of electrons on these oscillations can still be neglected. A standing wave is established in the radial direction in the system, and the radial mode of this wave is determined only by the initial conditions. Finally, when $n_e/n_{pi} \gg (C_s/v_b)^2$, $d_e \ll R$, and $\omega < \omega_{pi}$, progressive waves are excited in the system in the transverse direction, and the dispersion of these waves is in accordance with the theory of infinite plasma. When $\omega = \omega_{pi}$, there is a standing wave, the radial structure of which is determined by the ratio d_e/R in accordance with the theory of bounded plasma.

We must now consider nonlinear effects restricting the amplitude of ion oscillations in ion-beam plasma. The reaction of ion oscillations on the ions in the beam producing the oscillations leads to the bunching of these ions along the direction of propagation of the wave and to the rotation of the beam. The amplitude of the wave oscillates before it reaches its maximum value. This has been demonstrated by a computer experiment[79] (Fig. 9). The bunching effect in a fast ion beam in which longitudinal fields can be neglected and a monochromatic wave is initially excited is considered theoretically and established experimentally in[76]. If the beam ions are given an initial transverse velocity perturbation $v_x|_{z=0} = v_0 \sin kx \cdot \sin\omega t$, then, when the gas pressure is low and the ion-beam plasma consists largely of beam ions and electrons, the ion trajectories will cross at a certain point z_k. At high pressures, when there is a slow ion component and "transverse" ion oscillations are excited, the increase in the transverse velocity of the beam ions produces an earlier crossing of the trajectories of these ions (for $z_f < z_k$) and this is given by

$$\frac{kv_0}{\gamma_z v_b} \operatorname{sh}(\gamma_z z_f) \approx 1, \qquad (2.12)$$

where $\gamma_z = \omega_{bi}/v_b \sqrt{1-(\omega/\omega_{pi})^2}$.

The charge density in the bunches increases sharply at the point z_f and the crossing of the ion trajectory begins. The space-time structure of the modulated beam, which follows from the theory, and the focusing of the beam as a whole has been observed for both positive[76] and negative[77] ions. The space-time focusing of the ion beam, the formation of bunches, and the crossing of the trajectories form a mechanism for restricting the amplitude of the ion oscillations in the ion-beam plasma. If we suppose that the alternating potential associated with the excitation of ion oscillations increases exponentially right up to the focus, defined by (2.12), the maximum amplitude of this potential, which is related to the ion dynamics, is given by

$$\tilde{\varphi}_{max} = \frac{M_b \gamma^2}{4ek^2} \qquad (2.13)$$

and this is of the same order of magnitude as the experimental result. In contrast to the maximum amplitude of electron oscillations, given by (2.5), the maximum amplitude of ion oscillations depends on the growth rate or, more precisely, on γ/k.

C. Collective oscillations of synthesized ion-beam plasma

The solution of the dispersion relation for the oscillations of the system consisting of mutually penetrating cold beams of positive and negative ions of mass M, traveling at velocities $v_+ = v_b + \Delta v$ and $v_- = v_b - \Delta v$ with densities $n_+ = n_- = n_b$, shows that the enhancement of oscillations at given frequency is possible only when

$$0 < \frac{\omega \Delta v}{\omega_b v_b} < \sqrt{2}, \qquad (2.14)$$

where $\omega_b = \sqrt{4\pi n_b e^2/M}$. In addition to the critical value of the velocity difference $\Delta v_{crit} = \sqrt{2} v_b \omega_b/\omega$, there is an optimum value $\Delta v_{opt} = 0.5\sqrt{3} v_b \omega_b/\omega$, for which the spatial growth rate reaches the maximum value $\gamma_{z\,max} = 0.5\omega_b/$

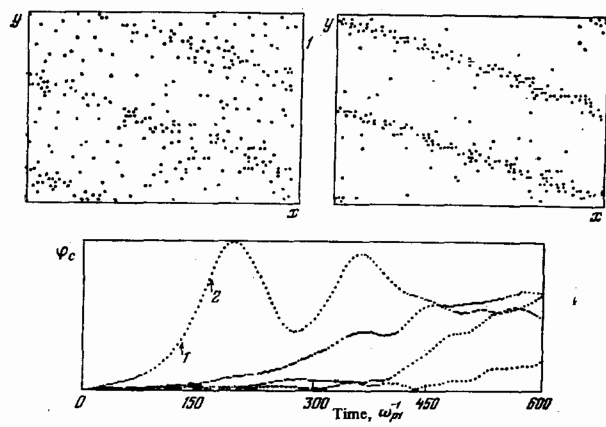

FIG. 9. Amplitude of oblique waves excited by the ion beam as a function of time, and the bunching of particles in space at times 1 and 2.

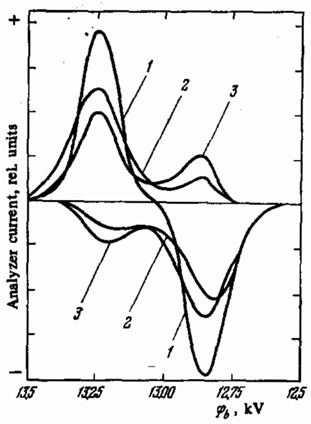

FIG. 10. Effect of modulation on the energy distribution function for interacting beams of positive and negative ions of hydrogen: $\tilde{v}/\Delta v=0$ (1), 0.15 (2), and 0.3 (3).

v_b. In accordance with theory, experimental investigations of the enhancement of oscillations in synthesized plasma consisting of modulated beams of positive and negative ions of hydrogen with energies of 10–15 keV, and currents of the order of a few milliamperes, have demonstrated the existence of the optimum and critical beam-velocity differences, and the fact that the product $\omega\Delta v_{opt}$ remains constant as the frequency is varied.[22] During this linear stage, the system behaves in the same way as the system consisting of beams of charges of the same sign, for example, electrons.

Collective processes in ion-ion beams are of interest not only in connection with the properties of the nonlinear stage of the interaction between beams of charges of different sign, but also in connection with the possibility of a separate experimental determination of the distribution functions for each of the beams in an overlapping velocity interval. This cannot be done, for example, in studies of the interaction between electron beams. The energy distribution functions for each of the interacting H^+ and H^- beams are shown in Fig. 10.[80] As the amplitude of the initial modulation is increased (Δv assumes optimum value), the distribution function becomes asymmetric, the mean energy of the fast beam is reduced, whereas the mean energy of the slow beam is increased, and an additional peak appears on the distribution function for each of the beams. Mathematical simulation has shown[24] that, during the instability development, phase grouping in the two beam results in the appearance of particle bunches in both beams, which move in opposite directions in the center-of-mass system. The density in these bunches continues to increase and, as the particle bunches of different sign pass through one another, this is accompanied by the appearance of a strong electric field. This reduces the velocity of the bunches down to zero, and gives them a directed velocity in the opposite direction. The bunches again penetrate one another but no longer retain their individuality, the electric field changes sign and is damped out, and a multivelocity particle distribution is produced. In contrast to the bunches of positive and negative ions, electron bunches formed during the interaction of electron beams, are retarded, and are dissipated without penetrating each other.[81] This description of the nonlinear stage of the interaction has been confirmed experimentally by direct measurements of alternating fields using an electron beam intersecting an ion-ion beam.[82] The evolution of a small initial sinusoidal perturbation eventually leads to the formation of a nonlinear wave in the form of sharp electric field pulses (Fig. 11). It is clear that the passage of bunches through one another (between $z=35$ cm and $z=75$ cm) is followed by the appearance of the maximum field which is subsequently damped out.[83]

D. Instability of ion-beam plasma in a magnetic field

In spatially inhomogeneous systems, and the ion-beam plasma is a special case of such systems, "hybrid" instabilities, due to both the presence of the beam and of the spatial inhomogeneity of the beam and plasma in the direction perpendicular to the external magnetic field, may be present in addition to the pure beam instabilities. One of the manifestations of this instability is the presence of long-wave perturbations resembling surface waves and localized near the boundary of the ion-beam plasma in a longitudinal magnetic field.[84–86] In the experiments described in[85,86], the interacting beams of argon or helium ions traveling in opposite directions with energies of 10 keV and beam currents up to 10 mA were allowed to pass through their own gas at pressures of 10^{-4}–10^{-5} Torr, so that the concentration of the resulting plasma was much greater than the concentration of the beam ions. Typical values of beam and plasma parameters were as follows: argon ion beam $n_b=10^7$–4×10^7 cm^{-3}, $\omega_b=0.7\times10^6$–1.4×10^6 sec^{-1}, $v_b=0.5\times10^7$–2×10^7 cm/sec, $n_{pi}=4\times10^8$–1×10^9 cm^{-3}, $\omega_{pi}=4\times10^6$–6.5×10^6 sec^{-1}, $\omega_{Hi}=10^6$–2.5×10^6 sec^{-1}, $T_e=8$–12 eV, $v_{T_e}=1.8\times10^8$ cm/sec, $C_s\approx5\times10^5$ sec^{-1}. This plasma was found to support oscillations with frequencies exceeding ion cyclotron frequencies by a substantial factor. The maximum of the oscillation amplitude was localized near the beam boundary in the region of maximum density gradient. In the azimuthal direction, the oscillations were nearly sinusoidal in form and propagated in the direction of the Larmor rotation of the ions. An increase in the beam current or gas pressure led to sudden transitions to higher azimuthal modes whereas, an increase in the magnetic field led to the excitation of lower modes. When one of the beams was removed, the amplitude was found to fall by two orders of magnitude, which was explained by a transition from absolute instability to drift instability. These experimental facts are in agreement with theory[84,85] in which the excitation of the oscillations is looked upon as a consequence of the drift-beam instability which is the analog of the drift-beam instability of electron beams in plasmas in a longitudinal magnetic field.[87–90]

Surface wave type oscillations[93] are also found to be present in a system consisting of adjacent ion beams

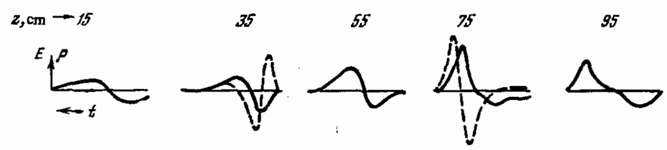

FIG. 11. Evolution of the field and space-charge density (broken curves) along a modulated synthesized ion-ion beam.

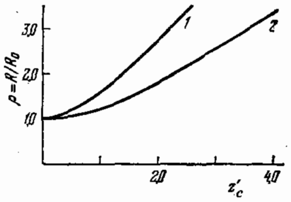

FIG. 12. Profile of a spreading compensated ion beam for different values of a. Curve 1—$a=1$, 2—$a=0$.

traveling in opposite directions, one of the beams being cylindrical and the other tubular. The maximum oscillation amplitude occurs in the region where the beams come into contact. The appearance of this instability is connected with the presence of a velocity gradient (discontinuity) in compensated ion beams on their contact boundary. [94,95]

When the ion beam passes through a gas in the direction of a magnetic field, the resulting plasma will also exhibit purely azimuthal oscillations with frequency equal to the ion cyclotron frequency of plasma ions and its harmonics. [91,96–98] There is evidence that the role of the ion beam in these experiments reduces only to the formation of the plasma, [97] and the associated instability is connected with the radial inhomogeneity of this plasma and the radial electric field. [92]

4. PROPAGATION AND FOCUSING OF COMPENSATED ION BEAMS

The questions considered in this section should be classified as belonging to plasma optics. [99] We note, however, that the compensated ion beams considered there differ from the beams which we shall discuss. Thus, in plasma optics, the compensated beams are usually regarded as given, collective processes are neglected, and the beams are regarded as fully compensated prior to the application of external fields. In the present review, on the other hand, we consider the process of compensation itself, the reasons for incomplete compensations, and the influence of incomplete compensation and of collective processes on beam transport.

A. Spreading of stable compensated beams of positive ions

Dynamic decompensation of excitation of collective oscillations may lead to a considerable spreading of a compensated ion beam. To determine the minimum spreading, we must assume that these effects have been removed and there is only self-decompensation of the beam due to Coulomb collisions between ions and the compensating electrons. If we start with the popular model in which the particle concentration is assumed constant across the beam, we can use the well-known equation of the beam envelope[100] for the compensated beam. This may be written in the form[101]

$$\frac{d^2R}{dz^2}-\frac{E^2}{R^3}-\frac{\Delta\varphi_c}{\varphi_b R}=0, \tag{3.1}$$

where E is the emittance and $\Delta\varphi_c$ the potential difference given by (1.17). In the case of relatively low gas pressures, when $(n_a\sigma_e)^{-1}\gg v_b\sigma_{pi}R/2v_{pi}\sigma_e$, integration of (3.1) leads to the following equation:

$$\left(\frac{d\rho}{dz'_c}\right)^2=\frac{C\rho^2-\rho-a}{\rho^2}; \tag{3.2}$$

where $\rho=R/R_0$ (R_0 is the initial radius of the beam), $z'_c=(z_c/R_0)\sqrt{2\Delta\varphi_{c0}/\varphi_b}$ is the reduced length of the compensated beam (the true length is z_c), $\Delta\varphi_{c0}=\Delta\varphi_c R/R_0=\sqrt{3\alpha e}\sqrt{M_b/m}(\varphi_i/\varphi_b)^{1/2}(n_a\sigma_e)^{-1/2}\sqrt{I_b/\pi R_0^2 e v_b}$, and $a=E^2\varphi_b/2R_0^2\Delta\varphi_{c0}$ is a parameter representing the thermal velocities of the ions. For an initially parallel beam, $C=1+a$. The required relationship between ρ and z'_c is obtained by integrating (3.2). The result is

$$z'_c=\frac{\sqrt{(1+a)\rho^2-\rho+a}}{1+a}+\frac{1}{2(1+a)\sqrt{1+a}}\ln\frac{2\sqrt{1+a}\sqrt{(1+a)\rho^2-\rho-a}+2(1+a)\rho-1}{1+2a}. \tag{3.3}$$

Figure 12 shows this function, i.e., the profile of the spreading compensated beam for two values of a. If we use the well-known expression $E=2^{3/2}R_{00}\sqrt{T_b/M_b}v_b$, where R_{00} is the beam radius in the region where the ions are removed from the plasma surface and T_b is the ion-beam temperature, then $a=2R_{00}^2T_b/R_0^2e\Delta\varphi_{c0}$.

The compensation effect can be characterized by the ratio of the compensated and uncompensated ion beam lengths (z_c and z), assuming that the beams have the same parameters and the same spreading (equal values of ρ). With this in mind, we recall that the spreading of the single-component uncompensated beam due to the intrinsic space charge, but without allowing for the thermal velocities, is given by

$$f(\rho)=2^{-3/4}\left(\frac{M_b}{e}\right)^{1/4}I_b^{1/2}\varphi_b^{-3/4}R_0^{-1}z, \tag{3.4}$$

where $f(x)$ is the well-known and tabulated function

$$f(x)=\frac{1}{2}\int_1^x\frac{dy}{\sqrt{\ln y}}. \tag{3.4'}$$

Comparison of (3.3) and (3.4) will show that

$$\frac{z_c}{z}=F(\rho)\sqrt{\frac{\Delta\varphi_b}{\Delta\varphi_{c0}}}, \tag{3.5}$$

where $F(\rho)$ is a function which is not very different from unity for very large values of ρ. It follows from (3.4) that, for given energy and length, the spreading of a single-component beam is unambiguously determined by the initial beam-current density. In contrast to this, the spreading of a stable uncompensated beam is determined not only by the initial current density, but also by the initial beam radius (it decreases with increasing initial radius or total beam current).

B. Optimum focusing of compensated positive ion beams

One of the most important problems in ion optics is the determination of the smallest beam cross section in a given plane z_c for optimum focusing of the beam by a lens in the $z=0$ plane. [6] When this problem is solved for a compensated beam, we can confine our attention to the stable ion beam, just as in the preceding section.

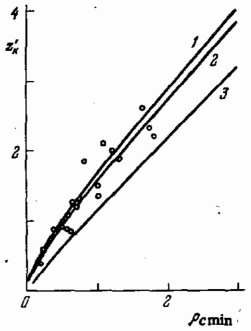

FIG. 13. Minimum radius of compensated ion beam in a given plane as a function of the reduced beam length for different values of emittance. Curve 1—$a=0$, 2—$a=0.04$, 3—$a=0.04$.

For relatively low gas pressures, the problem reduces to the simultaneous solution of the equations[102]

$$\int_{\rho_m}^{1} \frac{\rho\, d\rho}{\sqrt{C\rho^2-\rho-a}} + \int_{\rho_m}^{\rho_c} \frac{\rho\, d\rho}{\sqrt{C\rho^2-\rho-a}} = \int_0^{z'_c} dz, \qquad (3.6)$$

$$\left.\frac{\partial \rho_c}{\partial f}\right|_{z_c} = 0; \qquad (3.7)$$

where f is the focal length of the lens, $\rho_m = (1+\sqrt{1+4aC})/2C$ is the beam radius at the crossover point ($\partial\rho/\partial z = 0$), obtained from (3.2) for given focal length f, and ρ_c is the beam radius in the plane z_c, the smallest value of which, $\rho_{c\,min}$, is to be determined (it corresponds to a certain optimum value of f). Simultaneous solution of (3.6) and (3.7) on a computer yields the required minimum radius $\rho_{c\,min}$ of the compensated beam as a function of z'_c (Fig. 13). The problem can be solved in a similar fashion in the case of a relatively high-pressure gas. Figure 13 also shows experimental data. These can be seen to be in satisfactory agreement with the above calculations of optimum focusing, which take into account self-decompensation of the ion beam due to Coulomb collisions between ions with the compensating electrons.

C. Ion-beam transport in the potential well due to the electron space charge

Because the ions are heavy, a magnetic field is not by itself an effective means of restricting the spreading of ion beams.[103] The more rational solution is to use the potential well formed by the electron space charge in the magnetic field. The potential well can, for example, be produced as follows.[104] The ion beam is allowed to pass through the magnetic field and, on reaching the collector, releases electrons from its surface. When a negative potential is imposed on the collector and on the end electrode facing it, the electrons injected in this way cannot escape along the magnetic field and leave the region occupied by the ion beam. Instead, they move across this field toward the positive electrode. The result of this is the inversion of the potential in the beam (Fig. 14). The radius of the ion beam propagating in the resulting potential well changes periodically from its initial value to some minimum value, and the ion-beam current density is a maximum in the corresponding beam cross sections.

We note, by the way, that, in the intense beams that are currently used, for example, for the injection of fast particles into thermonuclear installations, the compressing Lorentz force acting on the compensated space charge may turn out to be greater than the electrostatic force producing beam spreading. The self-compression effect has not as yet been investigated, but should occur when the compensated beam current is

$$I_b > \frac{\Delta\varphi c^2}{v_b}, \qquad (3.8)$$

where $\Delta\varphi$ is the potential drop in the residual potential well of the ion beam, the shape of which is described by (1.14).

We also note the possibility of focusing of ion beams passing through plasma carrying a current which, in turn, produces a sufficiently strong azimuthal magnetic field.[116]

D. Effect of collective processes on the propagation of compensated ion beams

Langmuir electron oscillations influence the propagation of compensated ion beams by acting on the ions either through the static field of the residual potential well, which increases when the electron gas is heated by these oscillations, or directly by its own high-frequency field. When oscillations with amplitude $\tilde{\varphi}$ are excited, the energy transferred to the electron gas can be estimated from the formula

$$Q_{coll} = \gamma \frac{k^2\tilde{\varphi}^2}{4\pi}, \qquad (3.9)$$

where γ is the growth rate under real conditions. This expression and the equation $\mathscr{E} = Q_{Coul}$ ($\mathscr{E}$ is the energy obtained from (1.13)) can be used to explain the parallel increase in the electron-gas temperature and the radial potential drop observed in a compensated ion beam when the modulating voltage is increased,[58,59] (Fig. 15) and certain other properties. The heating of the electron gas during the excitation of Langmuir electron oscillations and Coulomb collisions between ions and compensating electrons may thus lead to an appreciable spreading of the compensated beam. The depth of the residual potential well, calculated in Sec. 2, can thus be generalized by using the equation $\mathscr{E} = Q_{Coul} + Q_{coll}$.

The high-frequency fields associated with the collective oscillations are also found to lead to another undesirable effect, namely, an increase in the spread of longitudinal[59,105] and transverse velocities in the compensated ion beam. The maximum spread in the longitudinal velocities of the beam ions in the coordinate

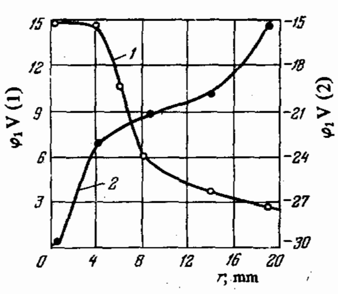

FIG. 14. Radial distribution of potential in the region of an ion beam during the free motion of an electron beam traveling in the opposite direction (1) and during the formation of a potential well due to the electron space charge (2).

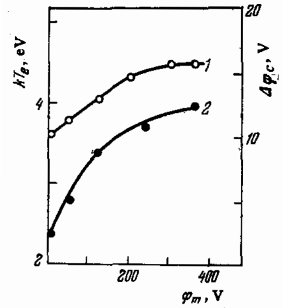

FIG. 15. Radial potential drop (curve 1) and electron gas temperature (2) as functions of the modulation amplitude of a compensated ion beam. Beam current 30 mA, energy 30 keV, gas pressure 9×10^{-6} Torr.

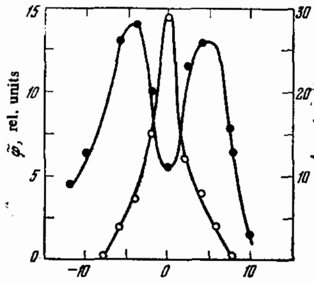

FIG. 17. Radial distributions of current density in a compensated ion beam, and amplitudes of the surface wave excited in the beam.

frame attached to the monochromatic wave is $\Delta v_b = 2\sqrt{e\tilde{\varphi}_{max}/M_b}$. In the laboratory frame, the energy spread is $\Delta\mathscr{E}_{lab} = 2e\varphi_b\sqrt{m/M_b}$, which agrees to within an order of magnitude with the experimental data (Fig. 16). Figure 16 also shows the energy distribution functions for the beam ions[59] obtained under different conditions with the aid of the Hughes-Rojansky analyzer placed at the end of a 500-cm interaction chamber. As the gas pressure is raised, and the growth rate increases appreciably due to the increase in the electron concentration, the energy spectrum is found to spread somewhat. External modulation of the beam, which leads to a function increase in the amplitude of the excited oscillations, produces an additional broadening of the energy spectrum and beam energy losses. It is important to note that these effects occur only when the modulation frequency is close to ω_e. The increase in the longitudinal velocity spread in the ion beam during the excitation of the electron oscillations should also be accompanied by an increase in the transverse velocity spread, i.e., an increase in the phase volume of the beam. The maximum transverse velocity spread does not exceed $\sqrt{2e\tilde{\varphi}_{max}/M_b}$ and, consequently, the maximum phase volume of the beam connected with the excitation of the electron oscillations is $V_{ph\,e} \approx (R/c)\sqrt{m/M_b}\,v_b$. An increase in the ion-beam emittance was observed in[106] during the propagation of a compensated ion beam in drift space.

The increase in the phase volume may also be a consequence of the excitation of "transverse" ion oscillations by a fast ion beam. This is accompanied by a spread in transverse velocities, which is of the order of the ion-sound velocity, and the corresponding phase volume is $V_{ph\,i} \approx (R/c)\sqrt{T_e/M_{pi}}$.

The above unfavorable influence of collective effects on the propagation of compensated ion beams does not exclude the possibility of using these effects to improve focusing, restrict the spreading of beams, and so on. Examples illustrating this possibility include the above influence of excited ion oscillations on beam focusing,[76,77] the use of negative ε, and the compression of a compensated ion beam by a surface wave produced in ion-beam plasma.[108] Let us consider the last effect in greater detail. A surface mode with frequency in the range $0<\omega<2^{-1/2}\omega_e$ can be excited[108,109] in addition to volume oscillations in ion-beam plasma. This is confirmed by studies of the dispersion relation and, in particular, by the observation of the oscillation cutoff at the frequency $2^{-1/2}\omega_e$ and of the radial distribution of the amplitude of the excited oscillations[108] (Fig. 17). In contrast to the volume wave field, characterized by the presence of a maximum on the axis of the system, the electric field due to the surface wave is a minimum on the axis and a maximum near the beam boundary. We may therefore expect that, in this case, electrons in the inner layers of the compensated ion beams will be compressed toward the axis by the Miller force and, as their space charge drags the beam ions, they produce an increase in the current density in the ion beam in the axial region. On the periphery of the beam, on the other hand, the change in the direction of the Miller force should produce the opposite effect, i.e., the motion of electrons and ions away from the axis. All this has, in fact, been observed[108] during the excitation of surface waves in compensated beams (Fig. 18).

5. CONCLUSIONS

The transport of intense ion beams used for the transfer of mass (atoms of different elements), energy, or momentum, is substantially influenced by processes in the ion-beam plasma produced during the compensation of the ion space charge. These processes include the following.

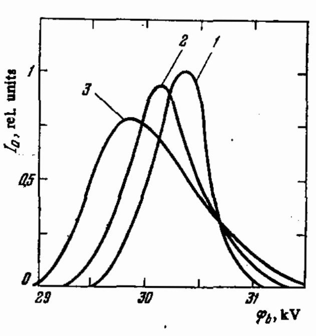

FIG. 16. Energy distribution function for ions in a compensated beam. 1—$p=1\times10^{-5}$ Torr $\varphi_m=0$, 2—$p=8\times10^{-5}$ Torr, $\varphi_m=0$, 3—$p=8\times10^{-5}$ Torr, $\varphi_m=150$ V.

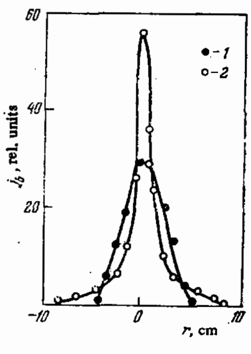

FIG. 18. Radial distribution of current density in a compensated ion beam prior to excitation of oscillations (1) and after the excitation of a surface wave (2).

a) Dynamic decompensation due to pulsations in the ion current. This is frequently the main reason for the spreading of a compensated ion beam. Further studies of the dynamic decompensation of high-current beams are desirable. Some of the features of this process have recently been established[110]: gas ionization by fast electrons captured by the potential well of a high-current beam cannot be neglected; the radial distribution of the time-average potential exhibits a plateau; and so on.

b) Self-decompensation of a stable positive-ion beam in which Coulomb collisions result in the transfer of energy to the compensating electrons and thus stimulate their escape from the beam. The radial potential drop $\Delta\varphi_c$ due to this leads to the spreading of the compensated beam. Rigorous theoretical analysis of the potential distribution (a two-step distribution under certain conditions) over the entire cross section of the ion guide containing a stable compensated ion beam would be of considerable interest.

c) Collective processes connected with the instability of ion-beam plasma and leading to beam spreading, due to the heating of the electron gas by the oscillation field, and to the increase in the phase volume of the ion beam. Studies of nonlinear effects, such as the capture by the wave field of compensating electrons which restrict the amplitude of the electron oscillations, and effects in the dynamics of the beam ions which restrict the amplitude of ion oscillations, have enabled calculations to be made of the spreading of beams and of the increase in their phase volume due to collective processes.

The inequality $I_b/v_b \gg \Delta\varphi_c$, which resembles the condition for the plasma state of the beam (1.3), but does not include the undetermined quantity T_e, can be used to calculate the beam current for which a substantial compensation effect is possible. It can also be used as a definition of an *intense* ion beam.

If we suppose that the spreading of a compensated beam has, in some way, been prevented, the limitation of its current is connected with instability, for example, with the formation of a virtual anode for positive potential perturbations.[112] This type of instability of an extended compensated ion beam of finite cross section has not as yet been investigated, but the natural lower limit for the corresponding maximum current is the maximum current $I_{b\,\max} = v_b\varphi_b$ of the uncompensated beam. The production of extended, compensated, high-current ion beams, including beams with high specific power, and studies of the properties of such beams are of considerable interest.

The production of quasistationary high-current ion beams[2,4,99] and of high-current ion beams with high initial current density and specific power under short-pulse operation[113–115] are important steps in this direction.

[1]G. N. Flerov and V. S. Barashenkov, Usp. Fiz. Nauk 114, 351 (1974) [Sov. Phys. Usp. 17, 783 (1975)].

[2]N. N. Semashko, in: Plazmennye uskoriteli (Plasma Accelerators), Mashinostroenie, M., 1973; AE 38, 348 (1975).

[3]T. Takagi, Elektronikusu (in Japanese) 17, 618 (1972) (a Russian translation is given in VINITI No. Ts-9675, M., 1973).

[4]Proc. Second Symposium on Ion Sources and Formation of Ion Beams, Berkeley, USA, 1974.

[5]N. V. Pleshivtsev, Katodnoe raspylenie (Cathode Sputtering), Atomizdat, M., 1968.

[6]M. D. Gabovich, Usp. Fiz. Nauk 56, 215 (1955).

[7]M. V. Nezlin, Plasma Phys. 10, 337 (1968).

[8]J. Koch, R. Dawton, *et al.*, Electromagnetic Isotope Separators and Applications of Electromagnetically Enriched Isotopes, Interscience, New York, 1958.

[9]M. D. Gabovich, A. P. Naĭda, I. M. Protsenko, L. S. Simonenko, and I. A. Soloshenko, Zh. Tekh. Fiz. 44, 861 (1974). [Sov. Phys. Tech. Phys. 19, 546 (1974)].

[10]K. Berkner, W. R. Baker, W. S. Cooper, K. W. Ehlers, W. B. Kunkel, R. V. Pile, and J. W. Stearns, cited in[4], paper VI-12.

[11]V. M. Kulygin and A. A. Panasenkov, *ibid.*, paper P-11.

[12]A. Engel and R. N. Franklin, Proc. R. Soc. London 264, 335 (1961).

[13]Electrical Rocket Engines, ed. by Yu. A. Ryzhov (Russ. Transl., Mir, M., 1964).

[14]N. F. Balaev and R. N. Kuz'min, Zh. Tekh. Fiz. 40, 1537 (1970) [Sov. Phys. Tech. Phys. 15, 1187 (1971)].

[15]M. D. Gabovich, I. A. Soloshenko, and A. A. Ovcharenko, Ukr. Fiz. Zh. 16, 812 (1971).

[16]K. S. Golovanivskiĭ and A. I. Lushchik, Zh. Tekh. Fiz. 37, 2234 (1967) [Sov. Phys. Tech. Phys. 12, 1647 (1968)]; 39, 1446 (1969) [Sov. Phys. Tech. Phys. 14, 1085 (1970)]; 40, 1490, 1497 (1970) [Sov. Phys. Tech. Phys. 15, 1149, 1155 (1971)].

[17]M. N. Yakloveva, G. M. Mantrova, and T. A. Novskova, Zh. Tekh. Fiz. 42, 2472 (1972) [Sov. Phys. Tech. Phys. 17, 1928 (1973)].

[18]W. L. Stirling, Am. Rocket Soc. J. 32, 929 (1962).

[19]V. F. Virko, M. D. Gabovich, G. S. Kirichenko, and O. K. Nazarenko, Zh. Tekh. Fiz. 44, 2296 (1974) [Sov. Phys. Tech. Phys. 19, 1418 (1975)].

[20]B. I. Verkin, A. M. Markus, V. M. Suslo, and N. A. Tsurikov, in: Radiatsionnaya fizika nemetallicheskikh kristallov (Radiation Physics of Nonmetallic Crystals), Naukova Dumka, Kiev, 1967, p. 437.

[21]H. S. Maddix, P. Chorney, and E. F. Paik, Rev. Sci. Instrum. 40, 1471 (1969).

[22]M. D. Gabovich and A. P. Naĭda, Zh. Eksp. Teor. Fiz. 60, 965 (1971) [Sov. Phys. JETP 33, 517 (1971)].

[23]M. Baribaud, J. M. Dolique, J. Monte, and F. Zadworny, Rev. Sci. Instrum. 46, 768 (1975); Plasma Phys. 16, 865 (1974).

[24]V. S. Imshennik, O. V. Lokutsievskiĭ, L. G. Khazin, M. D. Gabovich, and A. P. Naĭda, Preprint IPM AN SSSR, Moscow, 1974.

[25]Yu. I. Belchenko, G. I. Dimov, and V. G. Dudnikov, Zh. Tekh. Fiz. 43, 1720 (1973) [Sov. Phys. Tech. Phys. 18, 1083 (1974)] Nucl. Fusion 14, 113 (1974).

[26]K. Prelec and T. Sluthers, in: Proc. Particle Acceleration Conference, BNL 19833, Washington, USA, 1975.

[27]M. A. Abroyan, V. P. Golubev, and Ch. V. Chemyakin, Istochniki otritsatel'nykh ionov (Sources of Negative Ions), NII EFA, L., 1974.

[28]M. D. Gabovich, E. A. Pashitskiĭ, and A. P. Naida, Ukr. Fiz. Zh. 18, 1748 (1973).

[29]G. G. Kelley, IEEE Trans. Nucl. Sci. NS-14, 29 (1967).

[30]R. Bernas, L. Kaluszyner, and J. Druaux, J. Phys. Radium 15, 273 (1954).

[31]A. A. Panasenkov and N. N. Semashko, Zh. Tekh. Fiz. 40, 2091, 2525 (1970) [Sov. Phys. Tech. Phys. 15, 1628, 1979 (1971)].

[32]B. E. Paton, O. K. Nazarenko, M. D. Gabovich, and I. A. Soloshenko, Avtomaticheskaya Svarka, No. 10, 1 (1973).

[33]R. Bernas and J. Surroy, C. R. Acad. Sci. 233, 1092 (1951).

[34]A. V. Zharinov, Pis'ma Zh. Eksp. Teor. Fiz. 17, 508 (1973) [JETP Lett. 17, 366 (1973)].
[35]V. M. Raĭko, Zh. Tekh. Fiz. 33, 244 (1963) [Sov. Phys. Tech. Phys. 8, 175 (1963)].
[36]M. V. Nezlin and P. M. Morozov, in: Trudy 2-ĭ Mezhdunarodnoĭ konferentsii po mirnomu ispol'zovaniyu atomnoĭ énergii, Doklady sovetskikh uchenykh (Proc. Second Intern. Conf. on the Peaceful Applications of Atomic Energy, Soviet Papers), Glavatom, M., 1959, p. 117.
[37]T. Bolzinger, C. Manus, and G. Spiess, Plasma Phys. 11, 411 (1969).
[38]V. S. Anastasevich, Dokl. Akad. Nauk SSSR 105, 442 (1955); Zh. Tekh. Fiz. 26, 1487 (1956) [Sov. Phys. Tech. Phys. 1, 1448 (1956)].
[39]M. D. Gabovich, Ukr. Fiz. Zh. 19, 692 (1974); Zh. Tekh. Fiz. 44, 2425 (1974) [Sov. Phys. Tech. Phys. 19, 1502 (1975)].
[40]T. S. Green, Rep. Prog. Phys. 37, 1257 (1964).
[41]I. P. Shkarofsky *et al.*, The Particle Kinetics of Plasmas, Adv. Bk. Prog., Addison–Wesley 1966 (Russ. Transl., Atomizdat, M., 1969, p. 243).
[42]M. D. Gabovich, L. P. Katsubo, and I. A. Soloshenko, Fiz. Plazmy 1, 304 (1975) [Sov. J. Plasma Phys. 1, 162 (1975)].
[43]M. D. Gabovich, I. A. Soloshenko, and A. A. Ovcharenko, Ukr. Fiz. Zh. 16, 934 (1971).
[44]M. D. Gabovich, Fizika i tekhnika plazmennykh istochnikov ionov (Physics and Technology of Plasma Ion Sources), Atomizdat, M., 1972.
[45]M. D. Gabovich, V. S. Kuznetsov, I. A. Soloshenko, and G. I. Trubnikov, Zh. Tekh. Fiz. 43, 2178 (1973) [Sov. Phys. Tech. Phys. 18, 1372 (1942)].
[46]S. Ya. Lebedev, Yu. A. Stavisskiĭ, I. I. Bondarenko, S. A. Maev, I. P. Stakhanov, and É. A. Stumbur, Zh. Tekh. Fiz. 31, 1202 (1961) [Sov. Phys. Tech. Phys. 6, 878 (1962)].
[47]J. M. Dolique, Nucl. Fusion Suppl. 2, 767 (1962).
[48]H. Derefler, Phys. Fluids 7, 10 (1964).
[49]J. M. Sellenm, W. Bernstein, and R. F. Kemp, Rev. Sci. Instrum. 36, 316 (1965).
[50]Ya. B. Faĭnberg, Atom. Energ. 11, 313 (1961).
[51]A. A. Vedenov, E. P. Velikhov, and R. Z. Sagdeev, Usp. Fiz. Nauk 73, 701 (1961) [Sov. Phys. Usp. 4, 332 (1961)].
[52]B. B. Kadomtsev, v kn. Voprosy teorii plazmy (Problems in Plasma Theory), No. 4, Atomizdat, M., 1964, p. 188.
[53]M. V. Nezlin, Usp. Fiz. Nauk 102, 105 (1970) [Sov. Phys. Usp. 13, 608 (1971)].
[54]A. I. Akhiezer, I. A. Akhiezer, R. V. Polovin, A. G. Sitenko, and K. N. Stepanov, Élektrodinamika plazmy (Plasma Electrodynamics), Nauka, M., 1974.
[55]A. B. Mikhaĭlovskiĭ, Teoriya plazmennykh neustoĭchivosteĭ (Theory of Plasma Instabilities), Atomizdat, M., 1970.
[56]E. K. Zavoĭskiĭ and L. I. Rudakov, Fizika plazmy (Physics of Plasma), Znanie, M., 1967.
[57]W. Herrmann and T. Fessenden, Phys. Rev. Lett. 18, 535 (1967).
[58]M. D. Gabovich and I. A. Soloshenko, Zh. Tekh. Fiz. 41, 1627 (1971) [Sov. Phys. Tech. Phys. 16, 1281 (1972)]; *ibid.*, 43, 1656 (1973) [18, 1044 (1974)].
[59]M. D. Gabovich, I. A. Soloshenko, and L. S. Simonenko, Zh. Eksp. Teor. Fiz. 62, 1369 (1972) [Sov. Phys. JETP 35, 721 (1972)] Ukr. Fiz. Zh. 16, 1747 (1971); 17, 1362 (1972).
[60]A. M. Kudryavtsev and N. S. Buchel'nikova, in: Tezisy dokladov 1-ĭ Vsesoyuznoĭ konferentsii po nizkotemperaturnoĭ plazme (Abstracts of Papers read to the First All-Union Conf. on Low-Temperature Plasma), Naukova Dumka, Kiev, 1966, p. 61.
[61]E. Z. Tarumov, Yu. A. Bakshaev, *et al.*, Zh. Tekh. Fiz. 38, 163 (1968) [Sov. Phys. Tech. Phys. 13, 113 (1968)].
[62]L. S. Simonenko, I. A. Soloshenko, and N. V. Shkorina, Ukr. Fiz. Zh. 19, 1886 (1974).
[63]Z. S. Chernov, P. S. Voronov, N. A. Ovchinnikova, and G. A. Bernashevskiĭ, Zh. Eksp. Teor. Fiz. 57, 725 (1969) [Sov. Phys. JETP 30, 397 (1970)].
[64]Z. S. Chernov, P. S. Voronov, G. A. Bernashevskiĭ, and N. A. Ovchinnikova, Radiotekh. Elektron. 15, 2120 (1970).
[65]I. A. Soloshenko and L. S. Katsubo, Zh. Tekh. Fiz. 44, 2126 (1974) [Sov. Phys. Tech. Phys. 19, 1317 (1975)].
[66]B. B. Kadomtsev and O. P. Pogutse, Phys. Rev. Lett. 25, 1155 (1970).
[67]V. D. Shapiro and V. I. Shevchenko, Vzaimodeĭstvie voln konechnoĭ amplitudy s plazmoĭ (Interaction Between Finite Amplitude Waves With Plasma), Kharkov Physicotechnical Institute, 1972.
[68]B. B. Kadomtsev, in: Trudy konferentsii po teorii plazmy (Proc. of a Conference on Plasma Theory), Kiev, 1972, p. 271.
[69]M. D. Gabovich and V. P. Kovalenko, Zh. Eksp. Teor. Fiz. 57, 716 (1969) [Sov. Phys. JETP 30, 392 (1970)]; Dokl. Akad. Nauk SSSR 199, 799 (1971) [Sov. Phys. Dokl. 16, 640 (1972)].
[70]V. P. Kovalenko, Zh. Eksp. Teor. Fiz. 60, 2122 (1971) [Sov. Phys. JETP 33, 1142 (1971)].
[71]Z. S. Chernov, Radiotekh. Élektron. 17, 2527 (1972).
[72]A. K. Berezin, Ya. B. Faĭnberg, and I. A. Bez'yazychnyĭ, Pis'ma Zh. Eksp. Teor. Fiz. 7, 156 (1968) [JETP Lett. 7, 119 (1968)].
[73]M. D. Gabovich, I. A. Soloshenko, and A. A. Goncharov, Zh. Tekh. Fiz. 43, 2292 (1973) [Sov. Phys. Tech. Phys. 18, 1450 (1974)].
[74]M. D. Gabovich, A. A. Goncharov, V. Ya. Poritskiĭ, and I. M. Protsenko, Zh. Eksp. Teor. Fiz. 64, 1291 (1973) [Sov. Phys. JETP 37, 655 (1973)].
[75]M. D. Gabovich, L. P. Katsubo, and I. A. Soloshenko, Zh. Tekh. Fiz. 44, 2286 (1974) [Sov. Phys. Tech. Phys. 19, 1412 (1975)].
[76]L. P. Katsubo *et al.*, Zh. Eksp. Teor. Fiz. 67, 110 (1974) [Sov. Phys. JETP 40, 57 (1975)].
[77]L. S. Simonenko and I. A. Soloshenko, Fiz. Plazmy 1, 635 (1975) [Sov. J. Plasma Phys. 1, 350 (1975)].
[78]M. D. Gabovich, L. S. Simonenko, I. A. Soloshenko, and N. V. Shkorina, Zh. Eksp. Teor. Fiz. 67, 1710 (1974) [Sov. Phys. JETP 40, 851 (1975)].
[79]A. A. Ivanov, V. V. Parail, *et al.*, Proc. Fifth Conf. on Plasma Physics and Controlled Nuclear Fusion Research, Tokyo, 1974, Vol. 2, IAEA, 1975, p. 203.
[80]M. D. Gabovich and A. P. Naida, Zh. Eksp. Teor. Fiz. 62, 183 (1972) [Sov. Phys. JETP 35, 98 (1972)]; Pis'ma Zh. Eksp. Teor. Fiz. 14, 3 (1971) [JETP Lett. 14, 1 (1971)].
[81]I. M. Gel'fand, N. M. Zueva, V. S. Imshennik, O. V. Lokutsievskiĭ, V. S. Ryaben'kiĭ, and L. G. Khazin, Zh. Vych. Mat. Mat. Fiz. 7, 322 (1967).
[82]M. D. Gabovich, A. M. Gladkin, V. P. Kovalenko, Yu. N. Kozyrev, and A. P. Naida, Pis'ma Zh. Eksp. Teor. Fiz. 18, 343 (1973) [JETP Lett. 18, 202 (1973)].
[83]Yu. N. Kozyrev and A. P. Naĭda, Zh. Eksp. Teor. Fiz. 67, 2104 (1974) [Sov. Phys. JETP 40, 1044 (1975)].
[84]A. A. Goncharov and É. A. Pashitskiĭ, Zh. Tekh. Fiz. 42, 528 (1972) [Sov. Phys. Tech. Phys. 17, 418 (1972)].
[85]M. D. Gabovich, E. A. Pashitskiĭ, I. M. Protsenko, V. Ya. Poritskiĭ, and L. S. Simonenko, Zh. Eksp. Teor. Fiz. 62, 195 (1972) [Sov. Phys. JETP 35, 104 (1972)].
[86]M. D. Gabovichm, E. A. Pashitskiĭ, I. M. Pirotsenko, and L. S. Simonenko, in: Proc. Tenth Intern. Conf. on Phenomena in Ionized Gases, Oxford, 1971, p. 358.
[87]A. B. Mikhaĭlovskiĭ and E. Q. Pashitskiĭ, Zh. Eksp. Teor. Fiz. 48, 1787 (1965) [Sov. Phys. JETP 21, 1197 (1965)].
[88]A. B. Mikhaĭlovskiĭ, Zh. Eksp. Teor. Fiz. 48, 380 (1965) [Sov. Phys. JETP 21, 250 (1965)].
[89]L. S. Bogdankevich, E. A. Lovetskii, and A. A. Rukhadze, Nucl. Fusion 6, 176 (1966).
[90]M. V. Nezlin, M. I. Takatishvili, and A. S. Trubnikov, Zh. Eksp. Teor. Fiz. 55, 397 (1968) [Sov. Phys. JETP 28, 208 (1969)].
[91]W. Dommaschek, in: Plasma Physics and Contr. Nucl. Fu-

sion Research, Vol. 2, IAEA, Vienna, 1969, p. 765.
[92]T. A. Davydova, Ukr. Fiz. Zh. 15, 969 (1970).
[93]M. D. Gabovich, I. M. Protsenko, V. N. Tovmachenko, and V. M. Kolochko, in: Proc. Eigth Intern. Conf. on Phenomena in Ionized Gases, IAEA, Vienna, 1967, p. 366.
[94]I. S. Baĭkov and A. A. Rukhadze, Zh. Tekh. Fiz. 35, 1913 (1965) [Sov. Phys. Tech. Phys. 10, 1477 (1966)].
[95]É. A. Pashitskii, Zh. Tekh. Fiz. 38, 1020 (1968) [Sov. Phys. Tech. Phys. 13, 853 (1969)].
[96]M. Perulli, C. Etievant, and E. Lutaud, J. Phys. Radium 26, 493 (1965).
[97]M. D. Gabovich and I. A. Soloshenko, Zh. Tekh. Fiz. 40, 254 (1970) [Sov. Phys. Tech. Phys. 15, 184 (1970)].
[98]A. P. Goede, Ion Beam Interaction, Thesis, Amsterdam, 1975.
[99]A. I. Morozov and S. V. Lebedev, in: Voprosy teorii plazmy (Problems in Plasma Theory), Atomizdat, M., 1974.
[100]I. M. Kapchinskiĭ, Dinamika chastits v lineĭnykh rezonansnykh uskoritelyakh (Dynamics of Particles in Resonance Linear Accelerators), Atomizdat, M., 1974.
[101]M. D. Gabovich, Fiz. Plazmy 2, 163 (1976) [Sov. J. Plasma Phys. 2, 90 (1976)].
[102]M. D. Gabovich, Zh. Tekh. Fiz. 46, 1731 (1976) [Sov. Phys. Tech. Phys. 21, 1001 (1976)].
[103]L. Smith, V. Perkins, and A. Forrester, transl. in: Usp. Fiz. Nauk 35, 556 (1948).
[104]M. D. Gabovich, A. A. Goncharov, V. Ya. Pritskii, and I. M. Protsenko, in: Proc. Twelfth Intern. Conf. on Phenomena in Ionized Gases. Eindhoven. 1975. p. 287.
[105]M. Fukao, Y. Takeda, *et al.*, Jpn. J. Appl. Phys. 14, 1017 (1975).
[106]V. M. Kulygin and N. N. Semashko, in: Proc. Ninth Intern. Conf. on Ionized Phenomena in Gases, Bucharest, 1969, p. 564.
[107]V. B. Krasovitskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. 9, 679 (1969) [JETP Lett. 9, 422 (1969)].
[108]M. D. Gabovich, S. M. Levitskiĭ, and I. A. Soloshenko, Pis'ma Zh. Thech. Fiz. 1, 416 (1975) [Sov. Tech. Phys. Lett. 1, 195 (1975)].
[109]G. J. Brackenhoff, A. Baan, and T. Matiti, Plasma Phys. 15, 157 (1973).
[110]V. M. Kulygin, Author's Abstract for Candidate Thesis, IAE, M., 1975.
[111]C. Lejune, cited in[4], Paper No. I-1.
[112]Yu. S. Popov, Pis'ma Zh. Eksp. Teor. Fiz. 4, 352 (1966) [JETP Lett. 4, 238 (1966)].
[113]S. Humphries, cited in[4], Paper No. III-1.
[114]S. Humphries, J. J. Lee, and R. N. Sudan, Appl. Phys. Lett. 25, 20 (1974); J. Appl. Phys. 46, 187 (1975).
[115]J. M. Creedon, I. D. Smith, and D. S. Prono, Phys. Lett. 35, 91 (1975).
[116]M. D. Gabovich, A. A. Goncharov, and I. M. Protsenko, Ukr. Fiz. Zh. 21, 10 (1976).

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