M. A. Shifman. *Charmonium and asymptotic freedom.* We are witnessing the development of a new hadron theory. Dubbed quantum chromodynamics, this is a renormalizable theory of the Yang-Mills type, in which the interaction is realized by an octet of massless vector gluons connected with the color degrees of freedom of the quarks. A unique property of quantum chromodynamics is a logarithmic decrease of the coupling constant α_s over small distances[1]

$$\alpha_s(r) \sim \frac{1}{\ln(r_0/r)} \tag{1}$$

The unlimited growth of the constant at large distances gives grounds for hoping that all the objects with nonzero color charge (quarks, diquarks, gluons, etc.) have infinite mass, so that the physical sector of the theory covers only colorless states. It must be emphasized that despite the noticeable progress, color confinement has not yet been proved, and no quantitative approach to the problem of large distances has been developed in chromodynamics (as already mentioned in Polyakov's paper).

Whereas the theoreticians' attack on this flank of the theory is still tactical, strong positions have been won on another flank, in the region of short distances. The logarithmic exclusion of the interaction (1), dubbed asymptotic freedom, has explained qualitatively a large number of phenomena connected with deep-inelastic processes.[2] A large number of quantitative corollaries was obtained after the discovery of the Ψ family of particles. Being made up of heavy charmed quarks $c\bar{c}$, these particles (now usually designated by the term "charmonium") annihilate into ordinary hadrons over distances on the order of the Compton wavelength of the c-quark. At these distances, the effective coupling constant is already small,[3]

$$\alpha_s(m_c) \approx 0{,}2, \tag{2}$$

and the well-developed formalism of perturbation theory can already be used. For a long time, the rich possibilities connected with this circumstance could not be realized fully, since it was impossible to "get rid" of the effects of the relatively large distances inevitably encountered in charmonium, since the radius of the latter is quite large ~1–0.5 GeV. A solution of the problem was proposed in[4–6], where a new dispersion approach to charmonium was developed. Using the dispersion method, it is possible to calculate, solely from "first" (asymptotic freedom, unitarity, and analyticity), the leptonic widths of all the charmonium levels without exception. In conjunction with the Appelquist–Politzer prescription,[3] this predicts also the hadronic widths. None of the results contradict the presently available experimental data.

We shall illustrate the method using the calculation of the $\psi \to e^+e^-$ decay width as an example. We begin with an examination of the production of a pair of charmed particles in electron-positron annihilation at $s \sim 0$, where $\sqrt{s}$ is the total e^+e^- energy in the c.m.s. It is obvious that such a process cannot be real, in view of the energy-momentum conservation, but can only be virtual. According to the uncertainty principle, charmed particles are produced within a time $\tau \sim 1/2m_c$ and diverge to a distance $\lesssim c\tau$. We find ourselves thus in the region of asymptotic freedom. Bearing estimate (2) on mind, we can state that the contribution of the "charming" to the amplitude of the elastic e^+e^- scattering at s is given, at any rate accurate to 20%, by the bare quark loop (Fig. 1), while inclusion of one gluon (Fig. 2) improves the accuracy to ~4%. Diagrams 1 and 2 can be easily calculated.

On the other hand, the same amplitude can be expressed, by virtue of unitarity and analyticity, in terms of the dispersion integral of the cross section for the production of real mesons, both with latent charm of ψ, ψ', and of the pairs $D\bar{D}$, $F\bar{F}$, etc. We get thus a set of relations of the type

$$\int \frac{ds}{s^n}\,\sigma(e^+e^- \longrightarrow \text{charm}) = 4\pi^2 Q_c^2 \alpha^2 \frac{A_n}{(4m_c^2)^n} \qquad (n=1,\ 2,\ \ldots), \tag{3}$$

where $Q_c = 2/3$ is the charge of the c-quark, $\alpha = 1/137$, A_n are dimensionless constants known in the forms of series in $\alpha_s(m_c)$, and the mass m_c of the c-quark is the only fit parameter of the theory. With increasing n, the accuracy of the calculation of the constants A_n by perturbation theory becomes worse, inasmuch as at sufficiently large n the relations (3) should be violated. Direct calculations, however, show that, say at $n=3$ or 4, the relation (3) are still accurate within several per cent, but on the other hand the contribution of the region $s > 4m_D^2$ is already small, less than several per cent. At these values of n, the left-hand side of (3) is saturated mainly by the contribution of the ψ meson. Eliminating m_c, we find[4]

$$\Gamma(\psi \longrightarrow e^+e^-) = \frac{\alpha^2}{\pi} \cdot \frac{2^{11} \cdot 11^3}{3^7 \cdot 5^4 \cdot 7}\, m_\psi = 5\ \text{keV}. \tag{4}$$

Let us recall that the experimental width of the ψ

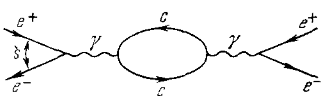

FIG. 1.

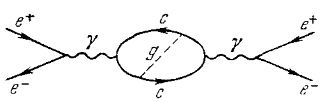

FIG. 2.

$\to e^+e^-$ decay is 5 ± 0.5 keV. It is easy also to determine the mass of the c-quark from (3).

Having obtained such splendid agreement in the e^+e^- channel and having determined m_c, it is natural to proceed to the next step—the calculation of the widths of charmonium in other channels. Analysis of the amplitude of scattering of light by light yielded the probabilities of the decays[4]

$$\chi_0 \to 2\gamma, \quad \chi_2 \to 2\gamma, \quad \eta_c \to 2\gamma, \quad \eta_c' \to 2\gamma. \tag{5}$$

The contributions of the other charmonium levels ($^1S_0$, $^1D_2$) could be separated by introducing auxiliary external currents with suitable quantum numbers.[5] For example, to find the $^1D_2$ widths it is necessary to consider the polarization of the vacuum by the current

$$j_{\alpha\beta} = i\left[k_\alpha k_\beta - \frac{1}{3}\left(g_{\alpha\beta} - \frac{q_\alpha q_\beta}{q^2}\right)k^2\right]\bar{c}\gamma_5 c, \tag{6}$$

where $k=(p_c-\bar{p}_c)/2$ and $q=p_c+\bar{p}_c$ in the annihilation channel. Thus method turned out to be useful also in another respect. The point is that after going through several of the simplest variants of external currents and calculating the corresponding polarization operators at $s\sim 0$, we found again the probabilities of the decays (5). The results obtained in this manner are in splendid agreement with the preceding estimates. Bearing in mind that the two calculation methods are utterly independent, we can state that the dispersion approach to charmonium is internally self-consistent.

In Refs. 4–6 are given also the charmonium hadronic widths obtained from the leptonic widths by the recalculation procedure of Appelquist and Politzer.

We see that charmonium is a splendid theoretical laboratory that allows us to investigate quantum chromodynamics even now, without awaiting the final solution of the large-distance problem. The "neat" predictions that follow from the dispersion approach are in good agreement with experiment. Moreover, they agree also qualitatively with the results of potential models in the case when the potential between the c and $\bar{c}$ quarks ensures confinement, and are in strong disagreement with the results for potentials that do not increase with distance. In this sense it can be stated that asymptotic freedom at short distances plus the dispersion relations require confinement of the quarks at large distances.

Many applications have not been touched upon in this paper for lack of time. We list only the most important ones. Restrictions on the constants of pure leptonic decays of the charmed mesons D, F, D^* and F^* were obtained in[4], where it was shown that an analysis of these restrictions and their comparison with experimental data, when obtained, can cast light on the mechanism of spontaneous breaking of chiral symmetry. A sum rule is derived in[8] for the cross section of the photoproduction of charmed particles. It is predicted that photoproduction of pairs of charmed mesons, not yet observed, should be more intense by approximately one order of magnitude than the photoproduction of ψ mesons. In[9] is considered the production of charmed particles in beams of electrons, muons, and neutrinos at moderate values of Q^2 corresponding to the existing experimental capabilities.

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