Scientific session of the Division of General Physics and Astronomy, USSR Academy of Sciences (March 30–31, 1977)

Usp. Fiz. Nauk **123**, 683–689 (December 1977)

A scientific session of the Division of General Physics and Astronomy of the USSR Academy of Sciences was held on March 30 and 31, 1977, in the conference hall of the Lebedev Physics Institute.

On March 30, a joint session of the Division of General Physics and Astronomy and of the Division of Nuclear Physics was held and was devoted to the memory of Academician V. I. Veksler (on his seventieth birthday). The following papers were delivered:

1. *I. M. Frank.* Preamble.

2. *A. M. Baldin.* The JINR proton synchrotron and its development.

3. *Ya. B. Feǐnberg.* Collective methods of acceleration.

4. *E. I. Tamm.* Investigations of electromagnetic interaction at the Lebedev Institute.

The following papers were delivered at the session of March 31:

1. *M. I. Kaganov.* Singularities that the local geometry of the Fermi surface induces in the absorption and velocity of sound in metals.

2. *V. D. Natsik.* Conduction electrons and mobility of dislocations in normal metals and in superconductors.

A brief summary of two papers follows.

M. I. Kaganov. *Singularities that the local geometry of the Fermi surfaces induces in the absorption and in the velocity of sound in metals.* 1. The electrons in a metal are known to play a substantial role in sound absorption.[1] We call attention to the fact that the electronic part of the sound absorption coefficient Γ_e must be quite sensitive to changes in the topology of the Fermi surface—to a phase transition of order $2\frac{1}{2}$.[2] In fact, Γ_e can be expressed in the form[3]:

$$\Gamma_e = \frac{\omega}{\rho S}\frac{2}{(2\pi\hbar)^3}\oint\frac{dS}{v^2}|\Lambda|^2\varphi(ql,\ \mathbf{vn}),\quad \varphi(ql,\ \mathbf{vn}) = \frac{ql}{(ql)^2[\mathbf{vn}-(s/v)]^2+1},\quad \mathbf{v}=\frac{\mathbf{q}}{q};\tag{1}$$

[1])See the article by V. S. Berezinskiǐ and G. T. Zatsepin, "Possible experiments with very high energy neutrinos: The DUMAND project" (Usp. Fiz. Nauk **122**, 3 (1977); Sov. Phys. Usp. **20**, 361 (1977)).

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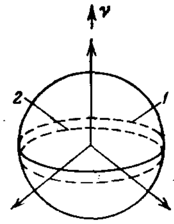

FIG. 1. 1—"strip," 2—equator.

ω, s, $\mathbf{q}$ are the frequency, velocity, and wave vector of the sound, $q=\omega/s$, ρ is the density of the metal, Λ is the corresponding component of the deformation potential, $\Lambda \sim \hbar^2/a^2 m$, a is the distance between the atoms, m is the electron mass, l is the electron mean free path, v is the electron velocity, and $\mathbf{n}=\mathbf{v}/v$. The integration is carried out over the Fermi surface $\varepsilon(\mathbf{p})=\varepsilon_F$, and the electron gas is assumed to be degenerate to the limit ($T \ll \varepsilon_F$).

It is the presence of v^2 in the denominator of the integrand in (1) which makes Γ_e particularly sensitive to phase transitions of order $2\frac{1}{2}$. We call attention to the fact that if the dispersion law is quadratic and isotropic ($\varepsilon = p^2/2m^*$), then Γ_e does not depend directly on p_F ($dS = p_F^2\,dO$, $v^2 = p_F^2/m)^{*2}$, nor therefore on the number of electrons. This means that the contribution of small cavities of the Fermi surface to Γ_e is commensurate with that of large cavities, since there are no grounds for assuming that Λ and l depend substantially on the dimensions of the cavity. The singularities of Γ_e in a phase transition of order $2\frac{1}{2}$ were considered in[4] for the case of short-wave sound, when (1) yields

$$\Gamma_e \approx \frac{\omega}{\rho S}\frac{2}{(2\pi\hbar)^3}\oint |\Lambda|^2 \frac{dS}{v^2}\,\delta\left(\mathbf{vn}-\frac{s}{v}\right), \qquad ql \gg 1, \tag{2}$$

and the Fermi electrons that participate in the absorption of the sound have a velocity that agrees with the velocity of the sound wave

$$\varepsilon(\mathbf{p}) = \varepsilon_F, \quad \mathbf{vn} = \frac{s}{v_F}. \tag{3}$$

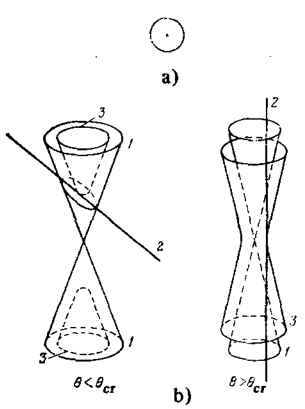

FIG. 2. a) Appearance of a new cavity (equal-energy surface $\varepsilon(\mathbf{p})=z_{\rm cr}=m^*s^2/2$; the surface degenerates into a point at $z=0$); b) Breaking of the neck (the equation of the coincal surface 1 is $(p_\perp^2/2m_\perp)-p_\parallel^2/2m_\parallel)=0$; 2—trace of plane, $s=(p_2\sin\theta/m_\perp)-(p_\parallel/m_\parallel)\times\cos\theta$; 3—critical surfaces, $(p_\perp^2/2m_\perp)-(p_\parallel^2/2m_\parallel)=z_{\rm cr}(\theta)$ are tangent to the plane 2).

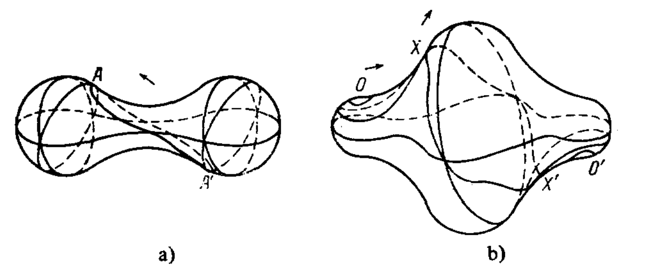

FIG. 3. Examples of changes of the "strip" topology. a) The strip $\boldsymbol{\nu}\mathbf{n}=0$ has two X-type points (A, A'); b) cavity containing points of type O and X (O and O', X and X'). The arrows mark the critical directions.

The electrons that interact activity with the sound wave are situated in the "strip" (3) passing over the Fermi surface. If the Fermi surface is a sphere, then the "strip" coincides with the "parallel" closest to the equator ($v_F \gg s$; Fig. 1). According to[4], when a Fermi-surface cavity is produced (or vanishes), Γ_e experiences a jump $\delta\Gamma\sim\Gamma_e$ at $z>z_{\rm cr}$, and when the neck is broken (is decreased) the character of the singularity depends substantially on the direction of propagation of the sound relative to the axis of the neck: Γ_e has a discontinuity $\delta\Gamma\sim\Gamma_e$ at $\theta<\theta_{\rm cr}=\tan^{-1}\sqrt{m_\perp/m_\parallel}$ and a logarithmic singularity $\delta\Gamma\sim\Gamma_e\ln|(z-z_{\rm cr})/z_{\rm cr}|$, at $\theta>\theta_{\rm cr}$, where $z=\varepsilon_F-\varepsilon_{\rm cr}$ and $\varepsilon_{\rm cr}$ is the energy at which the topology of the equal-energy surface changes. The thermodynamic characteristics do not depend on the character of the change of the topology and have singularities at $z=0$; when a new cavity with an effective mass m^* is produced, we have $z_{\rm cr}=(1/2)m^*s^2$, and when the nect is broken we have $z_{\rm cr}=z_{\rm cr}(\theta)$, where θ is the angle between $\boldsymbol{\nu}$ and the axis of the nect (the remaining symbols are explained in Fig. 2).

In the case of long-wave sound ($ql \ll 1$) it follows from (1) that

$$\Gamma_e \approx \frac{\omega^2}{\rho s^2}\frac{2}{(2\pi\hbar)^3}\oint \frac{l|\Lambda|^2\,dS}{v^2}. \tag{4}$$

At low temperatures, the principal dissipation mechanism is scattering by impurities and $1/l = N_{\rm imp}\sigma$ ($N_{\rm imp}$ is the impurity concentration and σ is the cross section), i.e., l is practically independent of $\mathbf{p}$. It follows therefore from (4) that in a phase transition of order $2\frac{1}{2}$ Γ_e experiences a jump when a cavity is produced (or vanishes).

2. The Fermi surfaces of polyvalent metals are complicated and varied.[5,6] All, as a rule, contain lines of parabolic points—points where the sign of the principal curvature is reversed (Fig. 3). As the result, the strip (3) should undergo qualitative (topological) changes[2)] when the propagation direction is changed, namely: (a) breaking of the strip or merger or two strips into one, and (b) appearance or vanishing of the strip (see Figs. 3a and 3b). Those values of $\boldsymbol{\nu}$ at which the topology of the strips changes will be designated $\boldsymbol{\nu}_c$.

[2)]The text that follows is the content of article[7].

TABLE I.

Structure of "strips" at $\nu \approx \nu_c$		Im ω	Re ω
O-type points		$\delta\Gamma_e \sim \Gamma_e$	$\Gamma_e \ln|\boldsymbol{\nu}-\boldsymbol{\nu}_c|$
X-type points		$\Gamma_e \ln|\boldsymbol{\nu}-\boldsymbol{\nu}_c|$	$\delta \operatorname{Re}\omega \sim \Gamma_e$

At $\nu=\nu_c$ the "critical strip" either contains a self-intersection point (case (a), a type-X point), or degenerates into a point (case b), O-type points). The critical points (points of type X and O) are located near the lines of the parabolic points and if we neglect the speed of sound ($s/v \to 0$), there they are located on the line itself. The change of the topology of the strip produce a singularity (in terms of $|\nu-\nu_c|$) in $\Gamma_e=\operatorname{Im}\omega$ and, naturally, in $\operatorname{Re}\omega$ (see Table I). Each line of parabolic points corresponds to a cone of critical directions ν_c. The scale of the singularity is the same as in a phase transition of order $2\frac{1}{2}$: the magnitude of the jump and the coefficient of the logarithm is $\sim \omega\sqrt{m/M}$, where m is the electron mass and M is the ion mass. Since $\delta \operatorname{Im}\omega \sim q$ and $\operatorname{Re}\omega \sim q$, the singularities of $\operatorname{Im}\omega$ and $\operatorname{Re}\omega$ must be treated as singularities of the speed of sound $s=s(\nu)$. The inversion center which is mandatory for the Fermi surface causes each critical point to have an "antipode" with an antiparallel velocity. As the result, all the described singularities (in ν) should make up closely-lying pairs. We emphasize that at $T=0$ and $l=\infty$ the singularity (as $\omega \to 0$) is not spread out in any manner. Under real conditions, the spreading factor is the largest of the quantities $\hbar\omega/\varepsilon_F$, T/ε_F, $1/ql$.

3. The existence of singularities (in ν) in the structure of the strip (3) should lead to singularities of many characteristics of metals (the resistances of thin plates, the impedance under conditions of the anomalous skin effect, and others). Apparently one of the most sensitive effects may be the Pippard (geometric) resonance in sound absorption[8]: the topological change of the "strip" should be accompanied by a change of the frequency spectrum with increasing oscillation amplitude.

Since the parabolic points were points where the surface becomes flattened, it follows that the diameter joining two such points with antiparallel velocities should generate an enhanced Migdal-Kohn singularity[9] (see Fig. 3 and also[10]). All the singularities listed here are the consequences of singularities of the local geometry of the Fermi surface, which of necessity exist in practically all metals.

We take the opportunity to thank A. F. Andreev for a useful remark made at the session. It is reflected in the present exposition.

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