Gravitational experiments in space

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The latest achievements in the experimental verification of Einstein's theory are reviewed. The classical relativistic experiments, together with the possibility of testing the existence of non-Einstein effects, are discussed. Some programs for relativistic experiments with space probes are examined.

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CONTENTS

1. INTRODUCTION

The development of interplanetary space flights and the most recent astrophysical discoveries (quasars, pulsars, relict black-body radiation, black holes)[1–4] have changed attitudes to one of the most fundamental and most complicated physical theories of the 20th Century—Einstein's general theory of relativity.[5] This theory, which was formulated in detail as long ago as 1916, remained for a long time "pure science," but during the last decade it has attracted the serious attention of experimentalists as well. Consider the search for deposits of raw materials by means of satellites, the study of meteorology, and space navigation—this is just a selection from the list of practical problems for which precise knowledge of gravitational effects has become necessary.

In 1971, Thorne and Will predicted that the seventies would be the decade of verification of general relativity. Indeed, the decade opened with Weber's gravitational wave experiments, which attracted the attention of the whole world with his sensational announcement (although Weber's results were not subsequently confirmed).[7] Somewhat earlier, in 1967, Dicke and Goldenberg has discovered oblateness of the Sun, which made it necessary to reconsider all possible variants of gravitational theories, recalculate the relativistic effects, and test experimentally the hypotheses providing the foundation of general relativity.[9–18] Measurements made later by Hill's group[19,20] did not confirm Dicke and Goldenberg's results. As Chapman showed,[21] the oblateness effect could be mimicked by an inaccurate method of evaluation of the measurements that does not take into account solar faculae (it is true that Dicke is not in complete agreement with this[22]). The solar oblateness J_2 (quadrupole moment), if it exists, must lead to an additional precession of the orbit of a spacecraft and, for strongly elongated orbits, can in principle be measured. The Helios probes were planned with such measurements in mind. Measurements of J_2 will also be included in the European–American experiment planned for the beginning of the eighties with two solar probes out of the plane of the ecliptic.[23]

At the present time, the available experimental data indicate that Einstein's theory is confirmed to within 1–3%, and the Sun does not appear to be oblate. This is deduced from analysis of the following groups of experiments considered together:

1) experiments to test the axioms of general relativity (tests of the equivalence principle,[10,17,24–26,97] measurement of the red shift of spectral lines in the field of the Earth and the Sun,[27–30] improvement in the value of the gravitational constant[18,31,32,96]);

2) relativistic experiments using space probes and planets to measure the time delay of a reflected radio signal that passes near the Sun[33–36]; radio interferometric and optical measurements of the deflection of an electromagnetic signal in the field of the Sun[3,37–41]; data on the precession of planetary orbits[42–44]; experiments to measure the oblateness of the Sun and their evaluation[45–47];

3) the laser ranging experiments to the Moon, which demonstrated the absence of the non-Einstein Nordtvedt effect[48,49]; geodynamical and geophysical measurements[18,103];

4) astrophysical measurements that yield information about the evolution of the Universe and individual regions of it, thus making it possible to test different cosmological models (in particular, by means of the microwave background radiation and the cosmological red shift)[1–3]; investigation of models of stellar evolution and the behavior of interstellar matter near objects having a relativistic gravitational field: black holes and pulsars.[1–4,78]

In the post-Newtonian region, Einstein's theory is the only theory of gravitation that is not contradicted by any of these groups of experiments, whereas the non-Einstein theories may agree with some experiments but usually contradict others. Theories with parameters that are not fixed and contain additional gravitational fields (scalar–tensor, vector–tensor, tensor–tensor) agree with all experiments only under the condition that the contribution of the additional fields does not exceed

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the limits of the experimental errors.

Experimental tests of general relativity are very difficult because, as a rule, the deviations from Newtonian theory are small and there are various nongravitational sources of error present. But in some cases the role of the nongravitational forces may be small, which makes it possible to increase the accuracy of the experiments. This is the case if one considers the motion of planets or drag-free satellites,[10,51–53] and also the behavior of binary star systems in which one of the components is a black hole or a pulsar.[54–56] Therefore, the discussions in the present paper will concentrate on gravitational experiments in space, made using either cosmic bodies or space probes.

At the end of the sixties and the beginning of the seventies, several programs were proposed for testing general relativity by means of space probes.[1,10,23,57,58,83] In 1974, Pickering[60] made the suggestion that in the seventies and eighties of this century there should, by analogy with the International Geophysical Year, be an International Solar System Decade (ISSD), during which space probes would be used to investigate the planets and the interplanetary medium and also test the general theory of relativity. Then, at the scientific level, we would be able to construct more accurate models of astrophysical and geophysical phenomena and, at the technical level, be able to exploit better the configuration of gravitational fields for space flights. The ISSD has been initiated by the Viking program.

It so happens that the positions of the celestial bodies favor testing of general relativity in the seventies and eighties: 1979 is the most convenient year in the century for investigating Uranus, 1981 for launching a spaceship to Jupiter, 1981 and 1983 for studying Mars, and in 1982 an encounter with Encke's comet is planned. In addition, the significant progress made in experimental technology during the last 15 years makes it possible, in principle, to improve the accuracy of the relativistic experiments during the coming decade from the current 1–3% to 0.3%. However, the cost of these experiments is, as a rule, exceptionally high, so that a systematic investigation of the solar system at the level of relativistic effects cannot be fully carried out.

From the scientific point of view, the necessary prerequisites for the success of space experiments have been provided. During the sixties, a group of American physicists at the California Institute of Technology and a number of scientists in other countries have made a systematic theoretical analysis of the basic propositions and experimental consequences of the general theory of relativity and also other conceivable theories of gravitation that differ from general relativity in certain axioms (generally, in the field equations). This has led to the formulation of the currently most general approximate description of gravitational effects, making it possible to compare different theories of gravitation in the so-called parametrized post-Newtonian approximation (PPN approximation).[6,9,11] Thorne and his collaborators have prepared a catalog of viable and nonviable theories.[10,16] Theories are said to be viable if they are complete (in that they contain a set of physical laws sufficient to construct realistic models), are internally closed (i.e., predict uniquely the result of measurements), are relativistic, and have the correct Newtonian limit.

Although all viable theories have the same Newtonian limit in weak fields and may give predictions close to those of general relativity, in other situations (for example, in the problem of collapse) the non-Einstein theories may lead to very different conclusions. Details about Thorne's catalog and other catalogs, and also additional information about non-Einstein theories of gravitation can also be found in the review.[12] From the point of view of the PPN approximation, the "true" theory of gravitation must be selected on the basis of a more accurate (than hitherto) measurement of the two main parameters γ and β in the expansion of the metric in the PPN approximation. These parameters occur in the relativistic effects of light deflection in the field of a gravitating body, the time delay of a radio signal passing near the Sun, the precession of Mercury's perihelion, precession of a gyroscope axis, the Thirring–Lense effect, and some others. The values of the parameters γ and β in different theories of gravitation differ little, and to measure the difference reliably an accuracy $\sim 0.03\%$ is needed. In addition, it is necessary to measure accurately the oblateness of the Sun, for which modern measurements give an error of the order of magnitude of J_2 itself or even more.[8,19–22] An important role in the justification of the postulates of the PPN approximation itself will be played by terrestrial experiments to test at a higher level of accuracy the equivalence principle, the anisotropy of masses in the Universe, and variations in the gravitational constant, among others.[18,61] At the present time, it is not doubted that Einstein's is the "true" theory, although some years ago experimentalists considered that the Brans–Dicke theory could compete with general relativity.[62]

It must however be pointed out that in all theories of gravitation the correct interpretation of the experimental results requires a painstaking analysis of the procedure of the measurements since the theory of gravitation describes not only the behavior of the investigated test bodies but simultaneously the behavior of the frames of reference by means of which the experimental characteristics of the test bodies are established. The real bodies that form a frame of reference, like real test bodies, are not pointlike. During an experiment, they are subject to different nongravitational influences, to say nothing of the fact that rods and clocks themselves have definite dimensions and parameters only by virtue of the existence of nongravitational forces that keep the electrons in atoms in their orbits. The ambiguity in the determination of the mass of a real body due to the contributions of internal energy and the possibility of describing the mutual behavior of rods and clocks, on the one hand, and test bodies, on the other, differently lie at the basis of the different variants of theories of gravitation.[17,18] In Sec. 2 of the present paper, we consider the physical realization of the basic concepts of Einstein's theory. We shall find that the best plane to apply the test general relativity must be the behavior of massive cosmic objects and electromagnetic signals. We shall discuss various ways of realizing geodesic motion in setting up relativistic experiments. We shall briefly consider programs aimed at experimental verification of general relativity by means of space probes. We list the modifications in the axioms of gravitation theory which lead to non-Einstein theories and effects. In Sec. 3 we discuss the present state of experiments to test general relativity in space in the framework of the PPN approximation. Conclusions are drawn in the fourth section.

2. EINSTEIN'S THEORY AS A THEORY OF COSMIC SPACE

Whenever the experimental verification of any physical theory is under consideration, it is necessary to decide how the experiment should be arranged in order to obtain the result predicted by the theory. In each particular case, it is necessary to stipulate how and with what we identify the events that we observe. This sometimes leads to a fairly complicated procedure for evaluating the measurements, and it may occupy months or even years. A negative experimental result cannot be taken as an argument against a theory (or a positive result as an argument in its favor) until it has been established that the identification of the theoretical concepts with real physical objects and processes has been made correctly. In what follows, we shall refer to this identification as the physical realization of theoretical concepts. The point is that, in the structure of its axioms, every physical theory contains implicitly or at least embryonically the properties which the instruments and measurement procedures used to verify it must have.[63,64] Usually, the experimentalists find these properties empirically, "by feel," but a study of the conditions under which one can, with a given accuracy, realize the axioms of the theory enables one to predetermine the possible field of applicability and the most adequate methods of verification of it. In the case of Einstein's theory, we shall see below that under real conditions the best test bodies are massive ones such as planets, widely separated from one another, or drag-free space probes with communication by means of electromagnetic signals. Therefore, Einstein's theory must describe the behavior of massive bodies in space, and in this sense general relativity can be regarded as a theory of cosmic space.

The verification of any particular theory of gravitation must take into account the following aspects:

1) determination of the type of inertial (distinguished) motions by means of the equations of motion;

2) determination of the conditions of realization of such motions by test bodies and the choice of the test body;

3) the derivation of equations for the relative motion of the test bodies and measurement of the parameters of the relative motion;

4) the relating of the results to the properties of the field source through the field equations.

In the general theory of relativity, inertial motions are defined as motions along the so-called geodesics of Riemannian spacetime. The geodesics are extremals of the action integral of the system, which has the form of a four-dimensional interval (path length) in a curved spacetime with metric dependent on the point under consideration. The extremals are sometimes called straightest or shortest lines. But since the metric in the Riemannian space changes from point to point, they are, as a rule, neither straight nor the shortest. Within the solar system, the geodesics obtained in Einstein's theory almost coincide with the lines along which the total energy of a body moving in the Newtonian gravitational field of the Sun and the planets in flat spacetime is constant. Therefore, in the nonrelativistic region the effects that permit one to distinguish the gravitational theories of Newton and Einstein are very small and require a careful conception of the experimental scheme. For example, the relativistic precession of the orbit of Mercury (which is the largest of the precessions of the planetary orbits in the solar system) is two orders of magnitude smaller than the Newtonian precession.[1,3] The relativistic time delay of a radio signal that passes near the solar disk is comparable with the delay in the solar corona. The relativistic precession of the axis of a gyroscope on a satellite circling the Earth due to the Thirring–Lense effect is comparable with the precession of the axis due to the deformation of the gyroscope itself as a consequence of its motion,[65] and so forth. Therefore, let us consider in more detail the conditions which ensure the possible application of Einstein's theory to real bodies, i.e., the problem of a test body and the realization of geodesic motion.

Inertial motion along a geodesic is realized by a Lorentzian frame of reference in which a free mass that is at rest at the origin at a certain time remains at rest for all time. Under real conditions, geodesic motion can be realized only approximately since any body is subject to various nongravitational influences as well as the gravitational force. The extent to which a motion is geodesic can be estimated as follows. All nongravitational influences (except electromagnetic and thermal) take the form of surface forces, in contrast to the body forces of gravitation. Therefore, a body for which the ratio of the surface to the body forces is minimal will have a motion that is most nearly geodesic. Obviously, under otherwise equal conditions, such body must have the shape of a massive sphere. If such a sphere at one astronomical unit from the Sun, where the main nongravitational influence is pressure of the solar wind, is to move along a geodesic with an accuracy corresponding to a level of compensation 10^{-8} cm/sec^2 of nongravitational influences, it must have mass $\sim 10^9$ g with radius ~ 3 m and mean density ~ 10 g/cm^3. Test bodies of such a mass already have an appreciable self-gravitational field. Therefore, if we wish to observe several test bodies at once, we must keep them far from one another. In other words, it is not fortuitous that the behavior of cosmic objects is the field of application of Einstein's theory of gravitation. For the Earth, the compensation of nongravitation influences is $\sim 10^{-14}$ cm/sec$^2 \approx 2 \cdot 10^{-14}$ $g_\odot$, whereas a qualitative estimate under the same conditions for nuclei, treated classically, gives only 10^{-6} cm/sec^2 despite their small size and huge density.[1)] Therefore, a small size of an investigated object does not by itself mean it can be regarded as a test body. The criterion must be the extent to which its motion is geodesic. To obtain geodesic motion to the same accuracy for a small body as for a large, it is necessary to have a matter density exceeding the nuclear, or shield the test body from nongravitational influences. As an example of the rapid deterioration in the accuracy of general-relativistic experiments in the case of small bodies we can mention the experiment to test the equivalence principle on electrons falling freely in a tube. This experiment was made by Witteborn and Fairbank (Phys. Rev. Lett. **19**, 1049 (1967)) and gave an accuracy of 10% instead of the 10^{-12} obtained in experiments with a torsion balance.[66]

Space-probe tests of general relativity require special measures to guarantee geodesic motion. The most effective for this purpose are drag-free space probes and satellites.[10,51–53,58,67,68,73] In contrast to all other space instruments, they are equipped with a device that enables them to move along geodesic paths without appreciable deviations. Such a probe automatically "locks onto" a geodesic line. Geodesics are universal in the following sense. The trajectory of a test body, i.e., motion under the influence of gravitational forces alone, is determined solely by the initial conditions, i.e., by the coordinate and momentum of the body at the initial time, but not by its mass or shape. Thus, we can make a chart of the geodesics of the solar system or some other part of space in which we are interested. It would be similar to a chart of ocean flows, and a journey on a drag-free spaceship resembles a journey on a raft through the oceans. Energy would be required only to go over from one inertial trajectory to another, which could be achieved either by means of motors (as usual) or by means of a "solar sail,"[75] which catches, not gusts of air, but fluxes of solar photons. Thus, the configuration of gravitational fields in space could be used for "economic" space navigation.

The first example of the use of the gravitational field configuration of the Earth based on the idea of geodesic motion was the U.S. Navy navigation satellite Triad 1 (the first in the series of Transit satellites).[67] Its general form is shown in Fig. 1. A French satellite of similar type, Castor, was launched in May, 1975.[68] Triad 1 was launched on September 2, 1972 into a polar orbit around the Earth. It is a development of a model of an artificial planet proposed by Schwarzschild.[17] This model was specially conceived for the realization and

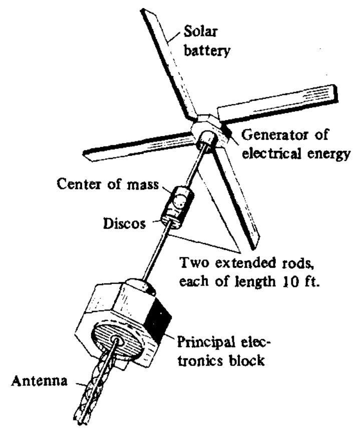

FIG. 1. Triad 1.

testing of Einstein's conjecture of the geodesic motion of bodies in a gravitational field. Schwarzschild proposed that the artificial planet should be surrounded by a rigid shell to shield it from external nongravitational forces. The shell has a set of gas motors which correct the position of the shell with respect to the artificial satellite to ensure that the centers of mass of the planet and the shell coincide (Fig. 2).

The "heart" of Triad 1 is the accelerometer Discos, which is placed at the center of mass of the satellite. It is constructed in accordance with the principle of Schwarzschild's artificial planet and makes it possible to compensate nongravitational forces to the level $5 \cdot 10^{-9}$ cm/sec.[2,73] The test showed that Triad 1 departs from a geodesic by ~ 200 m/month, which, in principle, makes it possible to predict its motion over a long period and test the influence of general relativity effects on the orbits of satellites on a firm basis.[51,73]

Another way of realizing geodesic motion is to minimize the ratio of the surface to body forces by the choice of the shape and material for the satellite. A sphere made of a nonmagnetic material with large specific weight will behave on an orbit in the neighborhood of the Earth in approximately the same way as a drag-free satellite. The degree of compensation of nongravitational forces will be the same for all spheres satisfying the relation $\rho R = \text{const}$, where ρ is the matter density of the sphere and R is the radius.

A satellite of this type was launched in France on February 4, 1975 (Starlette).[69] It is made of U^{238} and covered with 60 laser reflectors. Its weight is 47 kg and its diameter 25 cm. Observations of its orbit make it possible to determine the shape of the Earth with an accuracy ~ 0.2 m. The geodynamic satellite LAGEOS

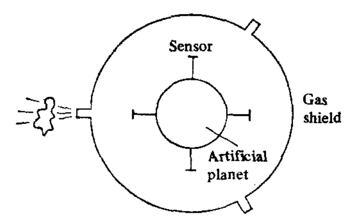

FIG. 2. Schematic diagram of Schwarzschild's artifical planet.

[1)]It is possible that "solar separation" (the blowing of lighter constituents to the periphery by solar light) is the cause of the survival of heavy nuclei in cosmic rays and the differences in the chemical composition of the planets of the solar system.

TABLE I.

Year	Type of spacecraft	Celestial body
1977	Mariner	Jupiter, Saturn
1978	Pioneer–Helios	Venus
1979	Mariner	Jupiter, Uranus
1980	Mariner–Helios C	Comet Encke, Sun
1980	Artificial satellite of the Moon	
1982	Pioneer with separable atmospheric probe	Jupiter
1982	Two space probes in orbit around Jupiter	
1983	Mariner–Helios	Asteroid Geograf
1984	Viking 3	Mars

(weight 411 kg, diameter 60 cm), launched on May 4, 1976 in the United States,[70] belongs to the same class of satellites. It is fitted with 426 corner reflectors and ensures an accuracy down to 2 cm in the location of terrestrial objects. This will make it possible to detect quickly deformations of the Earth's crust, to carry out geodynamic and relativistic measurements, and also predict earthquakes.

The accurately known, stable, and orientation-independent geometry of the satellite LAGEOS and its motion in a stable orbit make this satellite a fundamental global standard for determining the position of different points of the Earth's surface. It is intended that it should be used for 50 years, although it is estimated that it will stay in its orbit for nine million years. A tablet with a message to distant descendants is fixed to its frame. On it are engraved the positions of the terrestrial continents 200 million years ago, their present position, and their predicted position in ten million years.

A third method of realizing geodesic motion was proposed by Bertotti and Colombo[71] in 1972, and it is based on the use of two identical satellites that have the same surface but different density and, therefore, different masses (double-probe method). In principle, observation of the relative motion of these satellites enables one to determine a certain ideal point, whose motion is a geodesic. It is possible that a variant of this idea will be used in a planned experiment with two probes in an out-of-the-ecliptic orbit; these probes will first be sent to Jupiter, and then, using its gravitational field, returned to the Sun.[23,59,72] Besides studying the interplanetary medium and the polar regions of the Sun, these probes will make a number of high-precision experiments to test general relativity at the 0.1–0.01% level. The launch is planned for 1983.

A fourth way of reducing the influence of nongravitational forces is to use in relativistic experiments probes that become satellites of other planets or landers on their surfaces.[10,57,73,74] In such a case, the relativistic effects can be calculated from the position of the center of mass of the planet, which is a drag-free body. This approach was used to measure the delay of a radio signal to Mariner 9, which was launched in 1971 and became a satellite of Mars,[12,79] and also in December 1976 with the Viking lander.[36]

Experiments to test general relativity can also be made without special measures with standard radio apparatus. But in this case, to separate reliably the relativistic effects one must know accurately the characteristics of nongravitational forces (which, like the solar wind, may be very irregular), and also celestial-mechanical parameters (masses and shapes of planets, their mutual distances, etc.). In addition, it is necessary to use a long averaging time. The accuracy in the determination of the distance depends on the stability of the frequency standard (of atomic clocks) and may reach $\Delta r/r \sim 10^{-13}$.[66] On the other hand, the value of $\Delta(GM)$, where G is the Newtonian gravitational constant and M is the mass of a planet, is known as yet with accuracy 10^{-10}–10^{-12} cm/c^2, which (along with other factors) restricts the accuracy in the determination of relativistic parameters.

Experiments to test general relativity by means of standard radio apparatus have been made using the Helios probes launched into the region of the Sun on December 10, 1974 (Helios A) and January 15, 1976 (Helios B); Helios A approached to 0.3 a.u. of the Sun (46.3 million km), while Helios B approaches even closer (43.4 million km). As yet, no other space probe has approached so close to the Sun. The Helios probes are equipped with special screens to shield them from the solar radiation.

The program of investigations carried out by the Helios probes includes a measurement of the Sun's magnetic field, the solar wind, the intensity of cosmic rays, and also exact measurements of the orbital elements in order to improve knowledge of the parameters of gravitational theory (in particular, a measurement of J_2).[81] The probes Explorer 47, Explorer 50, Pioneer, Pioneer 10, and Pioneer 11 are equipped with similar radio apparatus.[82]

Some planned programs that include gravitational investigations are listed below in Table I.[59] It goes without saying that changes may be made in the table.[2)]

Experiments have also been proposed to test general relativity with artificial satellites of the Earth.[1,10,13,51,76,83] But in this case, the errors introduced by the nonsphericity of the Earth, the inhomogeneity of its mass distribution, and the influence of the Moon are so large that even drag-free satellites may be inadequate instruments for the investigations.[1,73] However, a group of physicists at Stanford have for many years been preparing an experiment with gyroscopes in a polar orbit around the Earth, which will probably be carried out in the coming years.[10,66,84] The results of investigations already carried out with space probes are discussed in the following section. They reveal agreement with general relativity to accuracy ~ 2%.

Thus, we see that the technology of space experiments currently makes it possible to realize geodesic motion

[2)]The first of the programs listed in the table has been carried out and given the name Voyager.

with sufficient accuracy to detect deviations from Newton's theory. The question therefore arises: Must these deviations agree with general relativity or are other variants of gravitational theory admissible? How "stable" is Einstein's theory to generalizations? To answer these questions, we must consider how the axioms and the equations of gravitational theory are related to the procedure of measurements made in relativistic experiments.

Let us consider Einstein's axioms. Like the axioms of Euclid's geometry, which reflect the properties of movable instruments (compass and ruler), the axioms of general relativity reflect the properties of the instruments and the measurement procedures used to test it. The postulate of the existence of a metric $g_{\mu\nu}$ with Lorentz signature and the equivalence principle indicate that curved Riemannian space must be regarded as a set of ordinary flat spaces associated with each point of the Riemannian space (namely, tangent to it) and rotated somewhat with respect to one another.[77,85] The amount of the rotation can vary in time and in space. It characterizes the curvature of spacetime in the following sense.

We introduce a frame (of four basis vectors) in one of the local flat Lorentz spaces and displace it round a closed contour formed of segments of geodesics. Since motion backwards in time is not possible in the real world, we shall regard this operation as a simultaneous transport of vectors (as certain rigid rods) and "twins" of them along opposite halves of the contour. When the vectors and their twins meet, they are rotated relative to one another through an angle proportional to the curvature of the space surrounded by the contour. If a standard vector is used rather than a "twin," we arrive at the conclusion that the frame vectors at different points of the curved space do not have a definite relative orientation. The attempt to measure the rotation of a frame at the point x with respect to some standard frame at the point y would not give a unique result since the result would depend on the path along which the investigated frame is moved until it coincides with the standard. Even a vector at rest at some spatial point changes its orientation with the course of time.

All that we have said above amounts to the following. Since parallel transport of vectors along geodesics preserves the length $v^2 = g_{\mu\nu}v^\mu v^\nu$ of vectors but not their orientation, a contravariant vector in Riemannian space should have as its physical characteristic, not its components (as in flat space), but $|v| \exp(iL_{\alpha\beta}\omega^{\alpha\beta}(x))$, where $|v| = \sqrt{g_{\mu\nu}v^\mu v^\nu}$, $L_{\alpha\beta}$ is the matrix of Lorentz rotation, $\omega^{\alpha\beta}(x)$ are the parameters of the Lorentz rotation, which vary from point to point, and $\mu, v, \alpha, \beta = 0, 1, 2, 3$. In this case, $|v| \exp(-iL_{\alpha\beta}\omega^{\alpha\beta}(x))$ is to be taken for the covariant components of vectors. The quantities $\omega^{\alpha\beta}(x)$ will then reflect the properties of the gravitational field. A similar situation occurs in the theory of gauge fields and, in particular, in the theory of the electromagnetic field, where the role of $|v|$ is played by $|\psi|$—the modulus of the wave function of a charged particle.[63,64,86]

The formulation of the theory of gravitation in the form of a gauge theory using the language of local symmetries makes it possible to answer a question once posed by Fock[93]: Why do we need general covariance of the theory and why would it not be better to replace it by invariance under some fairly large finite-parameter Lie group acting in flat space? This question did not arise by chance. Usually, the choice of a system of concepts reflects in some way or another the properties of the instruments and measurement procedures used in the experiment. As a rule, the measurement procedure consists of comparing the studied object and a standard. It is assumed that there exist classes of mutually identical objects, classes of mutually identical frames of reference, and classes of identical situations. These conditions ensure the reproducibility of the results and, therefore, their experimental verifiability, which is an important property of every scientific theory. But the equality relation has group structure. Therefore, irrespective of the method by means of which one establishes in practice which objects or frames of reference are identical to one another, one can make the assertion that every comparison (or measurement) procedure necessarily presupposes the existence of some symmetry group. This group determines the relativity principles of the theory. The invariants of this group become the characteristics used in a description of the properties of investigated objects. In the absence of symmetry, we have no language in which we could speak of measurements. Energy, momentum, angular momentum, mass, length, and spin are invariants of the symmetry groups of flat spacetime. In the general case, Riemannian space has no symmetry at all. What does it mean to measure in such a space, and in what terms must one formulate the results of experiments?

Einstein's answer[5] is: One must use small (compared with the characteristic dimensions of the gravitational field) rigid rods and clocks. The symmetry group that can be obtained using a rigid rod as measuring instrument is the group of motions of Euclidean space. Therefore, the invariants in general relativity must be the same quantities as in Newtonian mechanics (or in special relativity). But, in contrast to special relativity, the symmetry group giving these invariants is no longer a group of motions of spacetime as a whole, but only the parts of it where the gradients of the gravitational field are small and the equivalence principle is satisfied. In such regions, one can eliminate the gravitational field by a choice of the frame of reference. The following doubt therefore arises: Is such language capable of describing the properties of a *real* gravitational field that *cannot* be eliminated? It can be shown that it is capable of doing so because the same group of transformations of local rigid rods and clocks can be regarded as holonomy group of curved Riemannian spacetime, i.e., the group of transformations (rotations) of vectors at a given point after their transport around closed contours in spacetime. The components of the curvature tensor occur among the generators of the algebra of the holonomy group. It is this circumstance that makes it possible to interpret the results of measurements of precessions in the gravitational field as a measurement of the curvature of spacetime. The holonomy groups at different points of Riemannian space are isomorphic to one another. Therefore, the results of measurements are reproducible.

Thus, in an ordinary arrangement of relativistic experiments in which we observe the behavior of test bodies by means of ordinary classical instruments, we are dealing with the so-called frame (tetrad) or gauge formulation of gravitational theory. As invariants, we use the invariants of the symmetry groups of the four-dimensional local flat spaces, and there is no need to change the symmetry group since the chosen group corresponds to the chosen means of measurement. The general covariance of the theory determines the form of the interaction Lagrangian and the field equations, which are the Einstein choices.

Indeed, if gravitation theory is regarded as a Lagrangian gauge theory, one can show[86,87] that the two basic postulates:

1) gravitation is described by a symmetric tensor field $g_{\mu\nu}$ of second rank and

2) the theory is invariant under arbitrary continuous transformations of the corrdinates $x^{\mu'}=f^{\mu}(x)$ (sometimes this requirement of general covariance of the theory is identified with the equivalence principle[3]) have the following consequences: 1) field equations for $g_{\mu\nu}$ not containing higher derivatives are Einstein's equations[3)]; 2) the inclusion of interaction with matter in the Lagrangian *independently of the concrete form of the field and matter Lagrangians* leads to the covariant conservation law $T^{\mu\nu}_{;\nu}=0$, where $T^{\mu\nu}$ is the energy–momentum tensor of all nongravitational fields.

The uniqueness of Einstein's equations as the field equations for a symmetric second-rank tensor was also demonstrated in[88], which used, rather than generally covariant transformations, identities relating the initially unknown field equations and the additional spin conditions of special form that must follow from the required field equations. In the language of perturbation theory (in the lowest orders) the uniqueness of Einstein's equations was demonstrated in[89,92].

From the field equations, as consequences, one can obtain equations of motion in the form of geodesic equations by choosing $T^{\mu\nu}$ in the form of the energy–momentum tensor for pressure-free dust or the hydrodynamic energy–momentum tensor.[3,93,94] In the general case, a direct connection between the conservation law $T^{\mu\nu}_{;\nu}=0$ and the equations of geodesics is not known, but it is known that if this conservation law is postulated and one requires that it follow from the field equations for a symmetric second-rank tensor $g_{\mu\nu}$ irrespective of the concrete form of the free field Lagrangian, the group of transformations of $g_{\mu\nu}$ generating this conservation law is the generally covariant group, and a Lagrangian not containing higher derivatives is the Einstein Lagrangian, which again leads us to general relativity.[86,87] Thus, in a certain sense mathematics here does the thinking for us. *By choosing to describe the gravitational field by a symmetric second-rank tensor $g_{\mu\nu}$, the generally covarient group, and the covarient conservation law $T^{\mu\nu}_{;\nu}=0$, and rejecting higher derivatives, we uniquely obtain Einstein's theory.*

With what can one dispense in order to generalize Einstein's theory and to what can this lead?

1) Forgo the equivalence principle, at least for real massive bodies. One then has a Nordtevdt effect (nongeodesic motion for self-gravitating bodies): nonequality of the inertial and gravitational masses of a body, leading to the existence of anomalous accelerations in the self-frame of reference of a body and to differences in the accelerations of free fall in an external gravitational field for different bodies.

2) Forgo the weak equivalence principle, i.e., the universality of free fall in an external gravitational field for small test bodies with negligibly small self-gravitational field. This leads to a contradiction with Dicke–Eötvös experiments.[17]

3) Forgo Einstein's conjecture that the free-fall trajectories of test bodies coincide with geodesics of the local Lorentzian metric $g_{\mu\nu}$. Then the acceleration of free fall of photons in a gravitational field must differ from the acceleration of test bodies, which contradicts measurements of the red shift of spectral lines.[6,27–30]

4) Assume that Newton's gravitational constant is not a true constant, but only a scalar field and can depend on position in space as well as on time. Then the behavior of rods and clocks, on the one hand, and test bodies on the other, will be different. Clocks and rods will no longer measure ds^2. Monopole gravitational waves and a Nordtvedt effect are then possible. The rest masses of elementary particles and the sizes of atoms are not constant.[6,7,17,95,96]

5) Forgo Lorentz invariance. The gravitational constant becomes dependent on the direction in space (anisotropy of the Newtonian constant) and the velocities of propagation are different for gravitational and electromagnetic waves.[98]

6) Introduce a second metric field that controls the behavior of rods and clocks. Allow the possibility of vector gravitation. There is anisotropy of the gravitational constant and of the inert mass of bodies, and the possibility of detecting an ether.[9,10,14,99–104]

7) Forgo the Lagrangian formalism and the conservation laws. Anything goes.

The possibilities 1)–6) have been discussed and tested experimentally with measurements of genuinely relativistic effects such as the precession of planet and satellite orbits, deflection and delay of electromagnetic signals passing by the Sun, and so forth. The negative results of these experiments confirm the validity of the basic theoretical principles that form the foundation of the general theory of relativity. Calculations of the majority of modern gravitational experiments are made in the framework of the PPN approximation, which we shall consider in the next section.

[3)]This result was obtained by Einstein.

3. PPN APPROXIMATION. THE EXPERIMENTAL SITUATION CURRENTLY

Modern experiments to test the theory of gravitation are formulated in the so-called PPN approximation (parametrized post-Newtonian approximation). This approximation has been developed in different variants (Eddington, 1922; Robertson, 1962; Schiff, 1967; Baierlein, 1967; Nordtvedt, 1968)[11,105–108] which differ in the number of free parameters. Initially, they were intended to describe individual relativistic effects. The most general variant of the PPN approximation is Will's formulation,[95] which contains ten parameters. It is obtained by generalizing Chandrasekhar's equations for hydrodynamics in the post-Newtonian approximation and corresponds to a model of a planet as a perfectly fluid body. Individual PPN parameters do not have physical meaning since they depend on the choice of the coordinate system. Definite linear combinations of the parameters are measurable. One of these parameters (Σ) is always fixed, and in a standard gauge $\Sigma=0$, while two others, β and γ, take into account the basic relativistic effects. The seven additional parameters enable one to describe the possible deviations from the equivalence principle (or the hypothesis of the geodesic motion) for massive extended bodies, which under real conditions play the role of point test particles in general relativity. These parameters determine the expansion of the components of an arbitrary metric tensor in terms of integrals over the volume of massive extended field sources as follows[95] ($i, k=1, 2, 3$)

$$g_{00}=1-2U+2\beta U^2-4\Phi+\zeta\mathfrak{A}+\Sigma\mathfrak{B},$$
$$g_{0i}=\frac{7}{2}\Delta_1 V_i+\frac{1}{2}\Delta_2 W_i,\quad g_{ik}=-(1+2\gamma U)\,\delta_{ik},$$

where

$$U(x,t)=\int\frac{\rho(x',t)}{|x-x'|}dx',\quad \Phi(x,t)=\int\frac{\rho(x',t)\varphi(x',t)}{|x-x'|}dx',$$
$$\varphi=\beta_1 v^2+\beta_2 U+\frac{1}{2}\beta_3\Pi+\frac{3}{2}\beta_4 p\frac{1}{\rho},$$
$$\mathfrak{A}(x,t)=\int\frac{\rho(x',t)\,[(x_i-x_i')\,v_i(x')]^2}{|x-x'|^3}dx',$$
$$\mathfrak{B}(x,t)=\int\frac{\rho(x',t)\,\rho(x'',t)\,(x_i-x_i')\,(x_i'-x_i'')}{|x-x'|\cdot|x'-x''|^3}dx'\,dx'',$$
$$V_i(x,t)=\int\frac{\rho(x',t)\,v_i(x')}{|x-x'|}dx',$$
$$W_i(x,t)=\int\frac{\rho(x',t)\,v_k(x')\,(x_k-x_k')\,(x_i-x_i')}{|x-x'|^3}dx';$$

here, ρ is the density, p is the pressure, Π is the total nongravitational energy of the body, and v is the velocity of the fluid particles. It can be shown that the requirement of existence of integral conservation laws in any theory of gravitation treated in the PPN approximation reduces the number of essential expansion parameters to the two already mentioned β and γ, which determine the expansion of the metric in powers of v/c to fourth order, or in powers of r_g/r, where r_g is the gravitational radius, to second order. The theory then becomes invariant under post-Galileo coordinate transformations.[98] Different theories of gravitation are then basically compared through the values of the parameters β and γ corresponding to them, i.e., by their Newtonian limits. In the scalar–tensor theories of gravitation, in which, besides the ordinary gravitational field identified with the field of a symmetric second-rank tensor $g_{\mu\nu}$, a scalar gravitational field φ is introduced, there is an additional parameter ω, which determines the contribution of the scalar field to the gravitational potential. The number of PPN parameters is not thereby changed, but some of them (in particular, β and γ) become functions of ω. The field φ plays the role of a variable gravitational constant, and its derivatives in the Lagrangian make the theory invariant under conformal transformations of the metric. The parameter ω appears in the Lagrangian as a new coupling constant of the scalar gravitational field.[17]

Expansion of the metric in the parameters of the PPN approximation gives theories of gravitation a form convenient for comparison with experiment if these are experiments with slowly moving bodies in a weak gravitational field or, in other words, if the instruments in the experiments are ordinary Newtonian bodies.

The PPN approximation is based on the following hypotheses[6,9,16]:

1) On four-dimensional spacetime, which is a differentiable manifold, there exists a metric with signature 2, by means of which measurements of length and time are made in the usual manner:

$$ds^2=g_{\mu\nu}dx^\mu dx^\nu.$$

Gravitation, at least partly, is assumed to be related to this metric.

2) The interaction of matter and nongravitational fields with gravitation is described by the equation $\nabla_\mu T^\mu_\nu=0$, where ∇_μ is the covariant divergence with respect to the metric, and T^μ_ν is the energy–momentum tensor of all material and nongravitational fields.

Theories in which these hypotheses are satisfied are called metric theories.

In the general case, questions relating the existence of a Lagrangian or a Hamiltonian in the theory and also invariance properties, in particular general covariance of the theory, remain open. The PPN approximation does not prescribe any definite field equations. Therefore, one considers only the trajectories of particles or massive bodies in an external gravitational field. In this case, β and γ play the role of the free parameters of the theory.

Observations of the motion of material objects and electromagnetic waves in a real gravitational field make it possible to compare the theoretical trajectories with those observed experimentally, which leads to a determination of the parameters β and γ. On the other hand, the values of β and γ can be fixed if the metric is made to satisfy certain equations relating the gravitational field to its source. For example, Einstein's equations give $\beta=\gamma=1$. In Brans–Dicke theory, γ is somewhat less than unity for $\omega<\infty$, etc. In other words, near the Newtonian limit there exists the possibility for classifying different theories of gravitation according to the values of the parameters β and γ to which the corresponding field equations lead.

TABLE II.

Theory	γ	β
General relativity	1	1
Scalar–tensor theories	$\frac{1+\omega}{2+\omega}$	$1+\Lambda$
Brans–Dicke theory ($\Lambda=0$)	$\frac{1+\omega}{2+\omega}$	1

If no equations are introduced for the metric, then in the most general case in a centrally symmetric gravitational field produced by a body of mass M the four-dimensional interval in isotropic coordinates has the form

$$ds^2 = f(r)\,dt^2 - g(r)\,(dx^2 + dy^2 + dz^2). \qquad (1)$$

Coordinates are said to be isotropic if in them the spatial interval is proportional to the Euclidean expression. In Eq. (1), $r=\sqrt{x^2+y^2+z^2}$ is the radial distance from the attracting center, f and g are certain functions of r, which in the post-Newtonian approximation take the form

$$f = 1 - \frac{r_0}{r} + \frac{\beta r_0^2}{2r^2} + \ldots, \quad g = 1 + \frac{\gamma r_0}{r};$$

where $r_0 = 2GM/c^2$ is the gravitational radius of the field source (for the Sun, $r_0 = 2.9532$ km), $c=1$.

The field equations impose definite restrictions on the form of the functions f and g and the values of the parameters β and γ. In Einstein's theory,

$$f = \left(\frac{4r-r_0}{4r+r_0}\right)^2, \quad g = \left(1 + \frac{r_0}{4r}\right)^4,$$

i.e., $\beta=\gamma=1$.

In Table II we give the values of β and γ in different types of gravitational theory (Λ is the cosmological constant).

It can be seen from this table that in the limit $\omega\to\infty$ the scalar–tensor theories with and without cosmological term go over into general relativity.

The observable relativistic effects that contain a dependence on the parameters β or γ are shown in Table III.

Analytic expressions and the results of testing the main relativistic effects currently amenable to experimental measurements in space are as follows.

TABLE III.

Observable effect	Dependence on β, γ
Deflection of light rays and delay of ratio signals	$\frac{1}{2}(1+\gamma)$
Geodesic precession of gyroscope	$\frac{1}{3}(1+2\gamma)$
Thirring—Lense effect (drag of inertial systems by a rotating source)	$\frac{1}{2}(1+\gamma)$
Perihelion advance (secular)	$\frac{1}{3}[2(1+\gamma)-\beta]$

a) Deflection of light rays and microwaves

$$\delta\varphi = \left(\frac{1+\gamma}{2}\right)\frac{4GM}{c^2 d},$$

where G is the Newtonian gravitational constant, M is the mass of the field source, and d is the impact parameter. This expression was obtained for the first time by Einstein, who found that a ray of light passing near the edge of the solar disk must be deflected through 1.75". The effect was observed for the first time in 1919 and subsequently confirmed many times. However, even at the present time the accuracy of optical measurements remains low. The measured values of the deflection of light rays lie in the range 1.6"–2.2", which corresponds to $0.9<\gamma<1.3$ ([3,37,109]). During the expedition made in 1973 by Texas University and the Royal Greenwich Observatory, a deflection (1.66 ± 0.18)" was observed, which extrapolated to the solar limb gives $(0.95\pm0.11)L_E$, where $L_E = 1.75"$. Systematic errors in this experiment were not taken into account.[37]

The accuracy of measurements improves considerably on the transition from the optical to the radio range. Radio interferometers with very long base line (VLBI) have now made it possible to increase the accuracy in the measurement of the Einstein effect for radio waves to 2–3%.[38,39] Radio measurements were carried out on two groups of sources: the quasars 3C 273 and 3C 279, one of which passes behind the Sun every year on October 8, and the radio sources 0116+08, 0119+11, and 0111+02. An experiment to observe beats of radio signals from quasars was proposed by Shapiro and first realized in 1967 independently by Shapiro and Muhleman. With different antennas, they obtained the Einstein value of γ to within 12–15%. Gradually, the accuracy of the radio measurements has been improved, and the agreement with general relativity has been maintained. A review of the results of radio measurements of the deflection of microwaves by the gravitational field of the Sun made up to 1972 can be found in[10,40] and also in[3]. Evaluation of the measurements on quasars from observations of 1972 gave $\gamma = 0.98\pm0.06$ ([41], Shapiro's group). The more accurate data of measurements made in October 1973 on the quasars 3C 273 and 3C 279 agree to within 3% with Einstein's theory.[39]

The measurements made by Fomalont and Sramek in April–May 1974 on the radio sources 0116+08, 0119+11, and 0111+02 gave $\gamma = 1.030\pm0.22$. However, they have now improved the value of γ, having made additional measurements in March–April 1975, in which they obtained 1.014 ± 0.018.[38] This result agrees with general relativity but not with the Brans–Dicke theory if $\omega<23$. For $\omega>23$ the deviations from Einstein's theory are less than the errors of the experiment.

b) Time delay of radio signals

$$c\Delta t = r + \frac{1+\gamma}{2}\frac{4GM}{c^2}\ln\frac{r_0+r_1+r}{r_0+r_1-r} = r + \frac{1+\gamma}{2}\frac{4GM}{c^2}\ln\left(\operatorname{ctg}\frac{A}{2}\operatorname{ctg}\frac{B}{2}\right) \qquad (2)$$

(r is the Newtonian distance from the observer to the

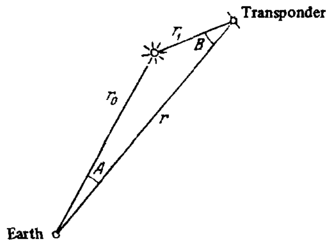

FIG. 3.

reflector, r_1 is the distance from the reflector to the Sun, r_0 is the distance from the observer to the Sun, and the angles A and B are indicated in Fig. 3).

Measurement of the delay of a radio signal when it passes by the Sun is an alternative way of measuring $(1+\gamma)/2$. The effect is measured by reflecting a signal from a space probe or the surface of a planet (Fig. 3). The accuracy of the measurements is increased if the reflected signal is transmitted in a different range of frequencies. If the signal is reflected from the surface of a planet, the accuracy is about two orders of magnitude worse than when a space probe is used. For the measurements, it is necessary to know the distance to the probe when it passes behind the Sun with high accuracy. Therefore, in such experiments high stability frequency standards are used.[66] It can be seen from Eq. (2) that the closer a ray passes to the surface of the Sun, i.e., the smaller are the angles A and B, the larger is the relativistic effect, and an electromagnetic wave even has the possibility of going around the Sun. In practice, the time delay is equivalent to an additional distance ~ 60 km. However, these 60 km can be due to not only general relativity but also the delay of the radio signal in the solar corona. Eliminating the influence of the solar corona and increasing the accuracy of the measurement, one can attempt to establish a difference between theories of gravitation that give different values for $c\Delta t$.

The first attempts to measure the delay of a radio signal were undertaken by transmitting signals to Mercury, Venus, and Mars.[33] The time of flight of the reflected signal was measured, but the value $(1+\gamma)/2 > 0.99$ was obtained with a large error. The measurements were then repeated and gave $\gamma = 1.03 \pm 0.04$ (Shapiro and collaborators[34]). The errors in these experiments arose from inaccurate knowledge in the orbit of the planet, the extension and roughness of its surface, and inaccurate knowledge of the positon of the planet's center of mass. A better accuracy can be obtained by reflecting a signal from a space probe with an active transponder. Such an experiment was successfully made in 1967 and 1969 with Mariner 6 and Mariner 7. A coherent modulated signal was reflected. Using this method, Anderson and his collaborators obtained $(1+\gamma)/2 = 1.02 \pm 4\%$. The error of 4% was taken with a margin to allow for the large uncertain error introduced by absorption of radio waves in the solar corona. Radio waves with the frequency at which the link was made (S band, 2300 MHz) are strongly delayed by the solar corona.

According to the estimates of Anderson and Muhleman in 1970, the error due to the solar corona is equivalent to an error in the determination of the distance of about 90 km. To increase the accuracy of the measurements, the reflected signal was subsequently transmitted at frequency ~ 9600 MHz (X band), which is delayed less by the solar corona. This double link made it possible to reduce the error in the determination of the distance to the probe to < 1 m. However, there still remained systematic celestial-mechanical errors and departure of the probe orbit from a geodesic. Therefore, the total error remained at the level of few percent. An improved analysis of the data obtained by means of Mariner 6 and Mariner 7 leads at the current time to the value

$$(1+\gamma)/2 = 1.00 \pm 0.03\,.$$ [35]

In order to measure γ accurately, it is necessary to know about 20 different parameters: the masses of the planets and the distances to them, the parameters of the probe orbit, and the parameters of terrestrial tracking stations. None of these parameters are known exactly.[53,58,110] The evaluation of the results of the measurements is a very complicated statistical problem. Therefore, experiments using space probes behind the solar corona are carried out over a period of many years, which makes it possible to increase the accuracy of the measurements by averaging the errors. The gravitational field at the space probe itself is not taken into account. An advantage of the experiments with Mariner 6 and Mariner 7 was the possibility of independent analysis of data obtained from two probes.

The error due to the deviation of the probe from a geodesic can be reduced in different ways. For example, as we have already said in Sec. 2, one can make the probes drag free by compensating the nongravitational influences, or making it into the satellite of a planet and using in the calculations the parameters of the planet's orbit as a drag-free body. The second variant was used in experiments with Mariner 9, which was launched in 1971 and put into an orbit around Mars. The orbit of the probe was determined by the center of mass of the planet. The problems associated with the nongravitational forces that change the orbit of the probe were eliminated in this experiment and the main errors were due to the solar corona. The importance of this experiment is that its systematic errors differed from the systematic errors that arise with other methods of measurement. Therefore, the agreement between the new results and the previous measurements of γ give one greater confidence in all the general relativity results obtained by different methods. According to the preliminary estimates, the accuracy in the measurement of γ with Mariner 9 was better than 1%,[57] but a more accurate evaluation of the measurements gave the Einstein value of γ with an accuracy of 6%.

An experiment of the same type was made in December 1976 with the Viking 1 and Viking 2 orbiters and landers. According to the preliminary publications, the accuracy of the measurements was very high.[36]

c) Precession of the planetary perihelion

$$\delta\theta = \frac{2-\beta+2\gamma}{3}\cdot\frac{6\pi GM}{c^2 a(1-e^2)}$$

$$+ \text{precession due to solar oblateness } J_2$$

(a is the semimajor axis of the ellipse and e is the eccentricity).

An important means of investigating the properties of spacetime is through study of different types of precession, in the first place precession of the planetary orbits.

It is well known that general relativity predicts a precession of Mercury's perihelion of 43.03'' per century. However, this minute amount is obtained by subtracting various large contributions predicted by the Newtonian theory[1,3] from the large observed precession of the perihelion, which is 5600.73 ± 0.41''. The remaining 43'' is the part of the precession which cannot be explained by Newtonian theory. Fortunately, accurate data on the planetary orbits for several hundred years exist. These data for Mercury were analyzed by Clemence,[42] who found that $\delta\theta = 43.11 \pm 0.45''$. This result is currently the most accurate confirmation of general relativity. A combination of relativistic parameters is determined here to an accuracy of 1% ([31]):

$$\frac{2-\beta+2\gamma}{3} = 1.00 \pm 0.01.$$

It should also be noted that the precession of the perihelion is the only quantity sensitive to the parameter β in the second-order terms in g_{00}. In 1975, new estimates were published of the shift of Mercury's perihelion made *in a different way*.[43] They give a precession of 41''.9 ± 0''.5 per century.

The precessions of the orbits of the other planets and the asteroid Icarus also agree with the predictions of general relativity, although the accuracy in the determination of these effects is 1–1.5 orders of magnitude less good than in the case of Mercury.[3,111]

The advance of Mercury's perihelion has also been estimated by Shapiro's group, who analyzed data on the time delay of a radar echo from Mercury and Venus obtained during *five years of continuous radio observations* of these planets.[44] The data were obtained during a period 1966–1971. Evaluation of the data gave

$$\lambda_p = \frac{2-\beta+2\gamma}{3} = 1.005 \pm 0.007,$$

where 0.007 is the statistical standard error. Allowance for possible systematic errors increases the uncertainty of the result to 0.02. Combining this result with the determinations of γ by means of the time delay of radio signals,[34] Shapiro's group gave the following estimates for γ and β: $\gamma \approx 1.0 \pm 0.1$, $\beta = 1.1 \pm 0.2$.

An important aspect of the evaluation of the results of the observations by Shapiro's group was the assumption of a zero quadrupole moment J_2 of the Sun. If the quadrupole moment of the Sun is nonzero, the excellent agreement between the predictions of general relativity and the observations of the precession of Mercury's orbit is a *mere coincidence*. This possibility was pointed out by Dicke and Goldenberg, who in 1967 observed a difference between the polar and equatorial radii of the Sun at the level $\Delta r/r \sim 5.10^{-5}$.[8] If they were correct, the observed precession of 43'' would consist of two parts: a precession of a few seconds due to J_2, and a residual part, which would now have to be explained by a non-Einstein theory of gravitation. Such a theory could be Brans and Dicke's for $\omega = 6$ in conjunction with a model of the Sun with a rapidly rotating core.[10,17,45] However, both the model of an inhomogeneously rotating Sun and Dicke's theory for small ω ($4 < \omega < 7$), and the actual observation of solar oblateness were criticized by a number of people (see the reviews of Hill,[46] Chapman,[47] Roxburgh[10] (pp. 525–528), and also the book of Zel'dovich and Novikov.[2] New data on the measurement of J_2 do not confirm the result of Dicke and Goldenberg. The result obtained by Hill and his collaborators gives $J_2 = (0.10 \pm 0.43)\cdot 10^{-5}$.[19,20] New experiments measuring the deflection of microwaves in the field of the Sun and laser ranging to the Moon do not agree with the Brans–Dicke theory if $\omega < 29$.[38,48,49] An indirect estimate made by Dicke on the basis of the lunar lasing results gives $|J_2| \lesssim 0.6 \cdot 10^{-5}$.[48] The model of an inhomogeneously rotating Sun also encounters difficulties.[10,50] In addition, during the summer of 1976 a French orbiting solar observatory observed oscillations of the solar atmosphere with period 14 min. During this time the atmosphere is raised and lowered through 1300 km, which corresponds to $\sim 10^{-5}\ r_\odot$.[80]

Thus, at the present time there are no reasons to doubt the agreement between the predictions of general *relativity and the data of precession of the planetary* perhelia.

d) Precession of gyroscope axis

Relativistic precession of a gyroscope axis in a gravitational field can be produced by two causes: motion of the gyroscope in orbit (geodesic precession) and rotation of the field source (the Thirring–Lense effect). The expression for the total precession with allowance for possible deviations from Einstein's equations is[3]

$$\frac{d\mathbf{S}}{dt} = -(1+\gamma)(\mathbf{vS})\nabla\varphi - \gamma(\mathbf{v}\nabla\varphi)\mathbf{S} + \gamma(\mathbf{S}\nabla\varphi)\mathbf{v},$$

where $\mathbf{S}$ is the angular velocity vector of the gyroscope rotation, $\mathbf{v}$ is the velocity of the gyroscope in orbit, and $\varphi = -GM/r$ is the Newtonian potential.

Introducing a different conserved spin vector,

$$\mathbf{S} \equiv (1+\gamma\varphi)\mathbf{S} - \frac{1}{2}\mathbf{v}(\mathbf{vS}),$$

we cast the formula for the precession of the gyroscope axis into the simplified form

$$\frac{d\mathbf{S}}{dt} = [\mathbf{\Omega}\times\mathbf{S}],$$

where $\mathbf{\Omega} = -(\frac{1}{2}+\gamma)[[\mathbf{v}\times\nabla\varphi] + [\nabla\times\xi]]$, $\xi = (2G/r^3)(\mathbf{xJ})$, $\mathbf{J}$ is the angular momentum of the Earth.

Estimates show that for a gyroscope set up on board an artificial satellite moving in a polar orbit around the Earth the geodesic precession is $\sim 7''$ per year, while the Thirring–Lense effect is $\sim 0''.05$ per year. It can be seen that if the change in the magnetic moment is measured in an experiment, the effect is proportional to the rotational velocity of the gyroscope. Therefore, the conditions of observation are improved if the angular velocity of the gyroscope increases. However, at a high rotational velocity the gyroscope itself may be deformed, which introduces additional errors into the measurements. At the present time, a group of physicists at Stanford is preparing an experiment to carry a superconducting gyroscope on board an artificial satellite.[10,66,84] It is planned to carry out the experiment in 1979. The gyroscope will be a quartz sphere covered by a superconducting film of niobium suspended in a magnetic field. The error introduced by the deformation of the sphere in its rotation may be $\sim 0''.0254$ per year in this experiment and must be taken into account.[65] The planned accuracy of the experiment is $0''.001$ per year. Experiments with gyroscopes are interesting in that the measured effect introduces a new combination of components of the metric.

Cosmology, astrophysics, and graviational waves are not the subject of the present review, although extremely interesting results have been obtained in recent years in the first two fields. Our reason for not including them is the impossibility of separating in these cases the gravitational effects in a pure form in isolation from other interactions (see, for example, the discussion on the cosmological red shift in[112]). However, at the present level of knowledge, there are no observable effects that contradict Einstein's theory in cosmology, whereas there are contradictions with non-Einstein theories.[1–3,103]

A separate group of experiments aims to find deviations from general relativity by looking for effects that must be absent in general relativity but are predicted by non-Einstein metric theories of gravitation. The most promising are various variants of the Nordtvedt effect, i.e., the appearance of anomalous accelerations in the center of mass system. These accelerations arise from violation of the equivalence principle for massive bodies due to the nonequality of the gravitational and inertial masses. The non-Einstein effects also include changes (with different origins) in the gravitational constant, anisotropy of the velocity of light in a gravitational field, and nonequality of the propagation velocities of electromagnetic and gravitational signals.

In 1968, Nordtvedt[11] published a fundamental and general investigation of possible violations of the equivalence principle in metric theories of gravitation. The PPN approximation used by Nordtvedt for this purpose was subsequently perfected by Will[95] and others; the existence and behavior of conserved quantities in non-Einstein gravitational theories and the influence of boundary conditions on the parametric dependence of various effects were also considered. Theories in which conservation laws hold were called conservative theories. Forms of non-Einstein effects and their parametric dependence in different variants of gravitational theory are given in Table IV[98] (see also[16]).

It can be seen from Table IV that the requirement of existence of conservation laws (conservative theories) or asymptotic Lorentz invariance greatly reduces the number of independent parameters of the PPN approximation, or the number of possible observable effects. Therefore, the existence of such effects would indicate a violation of fundamental laws of nature or an inadequate realization of theoretical concepts in the experiment (for example, an inadequate choice of standards or test bodies).

The most serious experiment of this series was the six-year (from August 1969 to May 1975) investigation of the Moon's orbit by means of reflected laser signals.[48] A laser beam is reflected by corner reflectors set up on the Moon by American astronauts. Altogether, 1523 points were obtained. Evaluation of the measurements demonstrated the absence of the Nordtvedt effect (inequality of the inertial, M_i, and gravitational, M_g, masses) for the Moon. The Nordtvedt parameter was

TABLE IV.

Experimental test or observable effect	PPN theory of general form	Conservative theories	Asymptotically Lorentz invariant theories
Nordtvedt effect ($m_{pass} \neq m_i$):			
a) isotropic	$7\Delta_1 - 3\gamma - 4\beta$	$-(4\beta - \gamma - 3)$	$7\Delta_1 - 3\gamma - 4\beta$
b) anisotropic	$2\beta + 2\beta_2 - 3\gamma + \Delta_2 - 2$	0	$2\beta + 2\beta_2 - 3\gamma + \Delta_2 - 2$
Perturbations in terrestrial gravimetric experiments:			
a) variations of G due to:			
1) the Sun and planets	$3\beta + 2\gamma - 2\beta_2 - 2$	$4\beta - \gamma - 3$	$2\beta + 2\gamma - 2\beta_2 - 2$
2) motion through the "ether"	$4\beta_1 + 2\gamma + 1 - 7\Delta_1$	0	0
3) anisotropy due to motion through the ether;	$\Delta_2 + \xi - 1$	0	0
b) other effects due to:			
1) external field gradients	$7\Delta_1 + \Delta_2 - 4\gamma - 4$	0	0
2) internal structure of the Earth	$5\gamma - 4\beta_2 - \Delta_2$	$4\beta - \gamma - 3$	$5\gamma - 4\beta_2 - \Delta_2$

found to be $\eta = 0.00 \pm 0.03$, which corresponds to $M_i/M_g = 1 \pm 1.5 \cdot 10^{-11}$. To within the accuracy of the experiment, this result can also be reconciled with the Brans–Dicke theory if $\omega > 29$. But then, to within the limits of the experimental errors, this means that the Sun has no quadrupole moment J_2. From the point of view of the five-parameter PPN theory with conserved energy-momentum tensor, the interpretation of the results of the measurements[48] leads to the value $|\beta - 1| \lesssim 0.02$. If the PPN theory contains only two parameters, β and γ, then $|\beta - 1| \lesssim 0.01$. The general conclusion is that there is no Nordtvedt effect to within the error ± 30 cm.

Another group of experimentalists measured the time delay of laser signals reflected from the Moon during a four-year period (1970–1974).[49] Altogether 1389 measurements were made. The evaluation of the results gave $M_i/M_g = 1 \pm 7.10^{-12}$, $\eta = -0.001 \pm 0.015$, $\beta = 1.003 \pm 0.05$, $\gamma = 1.008 + 0.008$ (with correlation 0.6). The values $\beta = 1.03 \pm 0.04$ and $\gamma = 1.02 \pm 0.02$ are obtained with correlation 0.9.

It is possible that a number of interesting conclusions about non-Einstein effects will be obtained from observations of the recently discovered pulsar PCR 1913 + 16, which belongs to a binary system.[54–56]

Should the gravitational and inertial masses not be equal, this could lead to anisotropy of the inertial properties of different bodies on the Earth due to inhomogeneity in the distribution of matter in surrounding space (for example, in the Galaxy). A search for such anisotropy, in connection with a test of Mach's principle and outside the PPN framework, was undertaken in 1960 by Hughes, Robinson, and Beltran-Lopez, and also Drever[113] (see also[17]). The experiments showed that there is no mass anisotropy $\Delta m/m$ to accuracy 10^{-22}–10^{-23}, although the expected effect was in the range $3 \cdot 10^{-10}$ to $2 \cdot 10^{-5}$.

Anisotropy of inertial properties is also predicted by certain non-Einstein theories, in particular the bimetric theories.[10,96,99,100] In such theories, the value of the Newtonian gravitational constant G measured locally in Cavendish type experiments[10] may depend on the direction. In addition, the velocity of light in them and the velocity of propagation of gravitation may be different. Such an anisotropy of G could be measured by a gravimeter on the Earth's surface through the 12-hour periodic variations produced by local gravitational accelerations.

However, the gravimetric data show that anisotropy of G at the 10^{-9} level is absent, which rules out theories of Whitehead's type, which predict an anisotropy effect 200 times greater than the experimental limit. The combination $(\Delta_2 + \zeta - 1)$ of PPN parameters is zero to accuracy 3%, and the velocity of light coincides with the velocity of propagation of gravitation to within 2%.[96] Motion relative to an ether or some privileged frame of reference is also not observed.[17,18,96,114] According to the estimates of Thorne, Will, Nordtvedt, and Ni, who have analyzed the complete set of astrophysical data as well as data obtained in relativistic experiments, the parameter combinations occurring in the non-Einstein effects are zero in the range from 10^{-5} to 0.4 depending on the experiment.[80]

Variations of the gravitational constant G can be manifested in the most varied geophysical, celestial-mechanical, and cosmological phenomena. They could cause drift of continents, changes in the orbits of celestial bodies, variations in the characteristics of the tides, and influence the evolution of stars and the Universe. Estimates made by many authors[15,17,18,97,115,116] who have analyzed gravimetric, geophysical, and celestial-mechanical data for possible variations of G give an upper limit on $\dot{G}/G$ of order 10^{-9}–10^{-10}. In 1975, Van Flandern[32] published an estimate for $\dot{G}/G$ based on analysis of lunar eclipses from 1955 to 1974. A feature of this experiment was the replacement of ephemeris time based on the motion of the Sun around the Earth by atomic time. The value of $\dot{G}/G$ was found to be $(-8 \pm 5) \cdot 10^{-11}$ year^{-1}, which is the same order of magnitude as the Hubble rate of expansion of the Universe: $(5.6 \pm 0.7) \cdot 10^{-11}$ year^{-1} ([117]). This value of $\dot{G}/G$ does not agree with the Brans–Dicke theory.

The weak equivalence principle, i.e., the existence of a universal set of trajectories for all laboratory bodies irrespective of their chemical composition and mass has now been established with accuracy 10^{-11} (in[24]) to 10^{-12} (in[26]). Varden and Everitt are preparing a satellite experiment,[10] in which it is intended to test the equivalence principle at the 10^{-17} level. Such experiments demonstrate the physical justification for the hypothesis that geodesics exist.

Experiments to measure the gravitational red shift, or rather red–blue shift, of spectral lines make is possible to establish that the universal trajectories of test bodies coincide with the geodesics of the metric of the local Lorentz frame of reference and, thus, make it possible to determine experimentally a class of inertial motions in Einstein's sense. The need for such a determination is due to the circumstance that the inertial property is not contained in moving bodies and cannot be extracted from them by simple observation. One and the same motion and one and the same frame of reference can be regarded as inertial or noninertial depending on the choice of the equations of motion of the theory which we choose to describe observable physical effects. The choice is connected to the choice of the means of measurement.

Thus, a laboratory system at rest on the surface of the Earth, i.e., inertial according to Newton, must be regarded in Einstein's theory as moving with respect to the local inertial system with a constant acceleration directed from the center of the Earth and the same for photons and massive bodies. This is confirmed by direct calculations[6,9] and by experiments to measure the red shift of spectral lines in the Earth's field by means of the Mössbauer effect (Pound and Rebka,[27] and also Pound and Snider,[28]). Thus, the red shift of spectral lines can serve as a measure of the difference between inertial frames of reference in the sense of Newton and Einstein.

Measuring the red–blue shift of spectral lines, we "translate" the description of gravitational field properties from the "language of forces" (Newton's theory) into Einstein's forceless geometrical language. The magnitude of the red (or blue) shift, i.e., the "conversion coefficient" is $\Delta\nu/\nu = gh/c^2$, where ν is the photon frequency, g is the acceleration of free fall of test bodies in the gravitational field, and c is the velocity of light. The accuracy of measurements of the gravitational red shift in the field of the Earth is $\sim 1\%$ in the experiments of[27] and[28]. The accuracy is somewhat worse, $\sim 5\%$, for measurement of the red shift of spectral lines in the field of the Sun.[29,30]

In 1976, a group of physicists at the Smithsonian Astrophysical Observatory performed an experiment to measure the red shift by means of an atomic clock in a satellite. On June 18, 1976, a gravitational probe was launched in the United States with a hydrogen frequency standard on board. The clock on the craft was synchronized with an analogous clock on the surface of the Earth ($\nu = 1.42 \cdot 10^9$ Hz). At a distance of 10 000 km from the surface of the Earth, the frequency of the clock on the satellite must increase and exceed the frequency of the terrestrial clock by about $\Delta\nu/\nu \approx 10^{-9}$. A fault which occurred when the probe was being separated from the final stage of the rocket had an adverse effect on the conditions of observations, but the organizers of the experiment hope that this will affect only the time required to evaluate the data. The expected accuracy[84] of the experiment is 0.01%.

The displacement of spectral lines in a gravitational field occupies a special position among other gravitational effects. Since it is not directly related to the field equations of gravitational theory but only to the choice of the class of inertial motions (i.e., to the equations of motion and the relativity principle), and also the energy–momentum conservation law, the "correct" red shift may survive alongside non-Einstein effects if general covariance or the equivalence principle are violated.

In nonmetric theories of gravitation, the weak equivalence principle is violated. But if the energy conservation law is satisfied, there is a simultaneous change in the dependence of the red shift on the gravitational potential.[118] Therefore, experiments to measure the red shift can also be regarded as a way of testing departure from nonmetrical behavior.[119] But from our point of view, this departure from metrical behavior is equivalent to an unusual choice of standards and types of inertial motions (or frames of reference) and in the usual scheme of relativistic experiments must be absent.

Thus, we see that modern experimental technology makes it possible to confirm general relativity to accuracy 1–2%. It is found that Einstein's theory is the only one among theories of gravitation that does not encounter internal difficulties at the theoretical level (in the sense of being logically closed) or at the experimental level from laboratory measurements to cosmic scales, including the evolution of the Universe.

4. CONCLUSIONS

Thus, in the last decade, the fundamental concepts and axioms of the general theory of relativity have been intensively analyzed from both the point of view of their physical realization and the preparation of an experimental basis for future practical applications.

As regards the theoretical side, we have seen the appearance of numerous non-Einstein theories of gravitation which question the most fundamental laws and principles of physics: the energy–momentum conservation law and the special theory of relativity, which denies an "ether." At the same time, in cosmology and astrophysics there is a tendency to use not only the linear approximation of general relativity and local results but also, invoking topology, to apply the full Einstein theory for large space-time regions (evolution of the Universe), strong gravitational fields (black holes), and to take into account other interactions.[1–4,86,120–124]

Experimentally, there are now a number of precise confirmations of Einstein's theory at the 1–3% level made by means of new technical means in observations of celestial bodies and space probes. To a high accuracy, there are no deviations from the predictions of general relativity. In addition, new programs are being prepared in which it is hoped to improve the accuracy of the measurements by 1–2 orders of magnitude.

In the present paper we have summarized various results, both purely theoretical and experimental, to show how Einstein's theory stands with regard to other theories of gravitation, on the one hand, and gravitational experiments, on the other. Since the theory of gravitation is simultaneously a theory of spacetime, which, we are inclined to believe, is given to us only in a single example, the question of measurement of its characteristics is far from trivial. This applies to the very concept of "measurement," i.e., comparison with a standard and, therefore, with a *different* spacetime as well as to the method of realization of the measurement process. In particular, it must be borne in mind that in modern experiments information about Einstein effects is obtained by means of the same instruments as in Newtonian mechanics. Therefore, in the present paper, besides giving information about the results of experiments, we have drawn attention to various problems associated with the realization of the theoretical concepts of general relativity in relativistic experiments. These problems are in fact related to those that arise when one prepares to confront any theory with experiment.[17,63,124–126] "We must understand that what we observe is not nature itself but nature manifested in a form corresponding to our way of asking questions."[127]

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