Vacuum polarization in strong fields and pion condensation

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INTRODUCTION

A vacuum alteration analogous to an ordinary phase transition can take place in a sufficiently strong field. This transition occurs when the energy of an individual particle or a pair vanishes in an external field and spontaneous particle production becomes possible. To this end, the particle energy gain in the field must compensate for the rest energy of the particle. This is why this process begins with particles of smallest mass. Since it will be shown that the mechanism of the phenomenon differs substantially in bosons and in fermions, we shall hereafter be particularly interested in the fermions and bosons that have the smallest known mass, i.e., electrons and pions.

Such an alteration of the vacuum takes place near nuclei with charges $Z > Z_c = 170$. The new ground state of the vacuum corresponds to a nonzero charge. This charge is equal to the number of electronic states that drop below the value $-mc^2$ as the charge of the nucleus is increased. These phenomena were investigated in[1–3] and expounded in detail in the review[4]. The present article deals with the mechanism of vacuum alteration near a nucleus with $Z > Z_c$, and offers a consistent physical interpretation of the phenomenon. The alteration of the vacuum should be accompanied by the emission of positrons, and these should be observable in collisions between two uranium nuclei, when a system having a field larger than critical is produced for some time by the mutual approach of the nuclei. It will be shown that allowance for the electron-positron interaction leads to the appearance of bound states of electron-positron pairs and influences the energy distribution of the positrons emitted when the nuclei come close together.[5] In the case of supercharged nuclei with $Ze^2 \gg 1$ (whose possible existence is discussed below), the electrons produced as a result of the alteration of the vacuum ("electron condensation") screen the nuclear charge and decrease substantially its Coulomb energy.[6] Electron condensation, as will be shown, influences the interaction of the charged particles at ultrashort distances.

No less interesting are the consequences of the alteration of a boson vacuum in an external field.[7] Particular interest attaches to the alteration of a pion field in a sufficiently dense nucleonic medium. In this case the role of the potential we-1 is played by the effective field acting on the pion and due to the nucleons of the medium. At a sufficiently high nucleon density, the pion energy vanishes and a phase transition takes place—"pion condensation." This produces an additional pion field ("pion condensate"). The most important physical consequence of this phase transition is the possibility, in principle, of the existence of superdense nuclei in which the energy gained in the phase transition offsets the energy loss due to compression.

The pion instability of vacuum in strong field and the ensuing possible existence of superdense nuclei was theoretically deduced in[7].

The uncertainty in the estimate of the critical density does not exclude the possibility that the phase transition had already taken place in ordinary nuclei. This possibility was first discussed in[8], where a method was developed for determining the spectrum of an excitation having pion quantum numbers. This method makes use of the result of the Landau theory of the Fermi liquid and the theory of finite Fermi systems.[9] The presence of π condensate in a nucleus would manifest itself in a periodic nucleon spin-density structure, having a wave vector $k_0 \approx p_F$ and capable of influencing the scattering of nucleons and electrons by nuclei.[10]

Regardless of whether the phase transition has taken place or not, the proximity of the nuclei to condensation manifests itself in a large number of experimental facts, namely, in all phenomena in which a substantial role is played by processes with exchange of one pion excitation. Proximity to condensation makes the pion degree of freedom "soft," thereby leading to enhancement of the matrix elements having pion symmetry. Among the phenomena strongly influenced by the decrease of the pion energy in nuclear matter are: shifts of the levels $0^-, 1^+, 2^-, \ldots$ relative to their positions in the shell mode, enhancement of $M1$ transitions with change of orbital momentum by two units (l-forbidden transitions), and enhancement of Gamow–Teller β transitions. The softening of the pion degree of freedom must also be taken

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into account in calculations of the suppression of the spin parts of the magnetic moments in the nucleus. Proximity to condensation exerts a particularly strong influence on the intensity of the l-forbidden transitions—the intensity of these transitions is in some cases several dozen times larger than the calculated values obtained without allowance for the proximity to condensation.[11] The theoretically predicted decrease of the energy of the pion in the nucleus, manifests itself directly in the spectral data of the π-atom (see Sec. 3 of Chap. III).

Sawyer[12] and Scalapino[13] have proposed a π-condensation model in a neutron star, with the π^--meson condensate assumed in the form of a traveling wave, thereby greatly simplifying the calculations. Exclusion of the π^+ mesons corresponds to a description of the pion field with the aid of a Schrödinger equation instead of a Klein–Gordon–Fock (KGF) equation. A consistent solution of the problem of nucleons interacting with a traveling-wave-type pion field was given in[14]. The pion-field Lagrangian constructed there corresponds to a description of the pions with the aid of the KGF equation, i.e., takes the positive and negative pions automatically into account. Analogous results were obtained in[15] with the aid of the Hamiltonian formalism, and in[16] by a variational method.

The traveling-wave method proposed in[12,13] and developed in[15,16] turned out to be essential to further work aimed at finding the energy of a strongly developed condensate in a more realistic model (see[17,18]). These results make it possible to estimate the energy of the nucleus at a density greatly exceeding that of ordinary nuclei, and are used to verify the possible existence of superdense nuclei[19] (see Sec. 4 of Chap. III).

Pion condensation leads to a number of interesting phenomena associated with the structure of neutron stars. We start with an explanation of the physical gist of π condensation in a neutron medium.

Sawyer and Scalapino[12,13] interpreted condensation as the result of instability of the neutron matter to the reaction $n \to p + \pi^-$. Yet, as shown in[14], neutron matter is stable with respect to this reaction, since the chemical potentials of the neutron, proton, and π^--meson satisfy the inequality

$$\mu_n < \mu_p + \mu_{\pi^-}.$$

For a correct physical interpretation of the condensation it is necessary to use the language of pion and nucleon excitations (quasiparticles), as is customary in the theory of phase transitions, rather than the language of bare particles. It then becomes clear that the instability observed in[12,13] is a manifestation of an instability investigated in a more realistic model,[6,20] and consists in the following: At a neutron density noticeably lower than the nuclear density $n_0 (n_c^+ \sim 0.4 n_0)$, a new excitation mode appears in the medium, with the quantum numbers of the π^+ meson and with negative energy $(\omega_s^+ < 0)$. The quasiparticles corresponding to this excitation mode (π_s^+ mesons) can be interpreted as bound states of a proton and a neutron hole (just as excitation of zero sound is interpreted as a bound state of a particle and a hole[9]). At a neutron density $n > n_c^+$, condensation of the π_s^+ mesons begins. With further increase of the density, the energy ω_s^+ of the π_s^+ mesons decreases (its absolute value increases) and at a certain density $n = n_c^{\pm} \approx n_0$ the sum of the energies of the π^- and π_s^+ particles becomes equal to zero:

$$\omega^- + \omega_s^+ = 0,$$

i.e., the system becomes unstable to the production of $\pi^-\pi_s^+$ pairs. It is just this instability which appears in the model of[12,13]. This instability leads to a strong softening of the equation of state of the star and can reverse the sign of the compressibility of the neutron matter.

As a result, a noticeable part of the neutron star should go over in a short time into a state with a density corresponding to a new phase

$$n = n_m = (3 - 6)\, n_0.$$

This transition should be accompanied by a release of an energy comparable with the gravitational energy of the star.

To understand all these phenomena, it is useful to trace the mechanism of π condensation by first using simple examples of condensation in an external scalar or electrical field, and only then proceed to the most interesting case, π condensation in a nucleon medium.

Pion condensation in an external field, besides being of methodological interest, is of physical interest in itself, because of the possible existence of supercharged nuclei in which the energy gain from π condensation in the electric field of the nucleus is partially offset by the energy loss due to the Coulomb repulsion of the protons (see Sec. 5 of Chap. III).

I. FERMIONS IN STRONG FIELDS

1. Alteration of electron-positron vacuum in the field of a nucleus with a large charge

Let us ascertain how the electron-positron vacuum is altered in the field of a nucleus with a large charge Z, when the energy level of the K electron drops to a value $-mc^2$.

It is known that the Dirac equation in the field of a pointlike nucleus becomes meaningless at $Z > Z_c = 137$. In fact, the ground-state energy is given by $(\hbar = m = c = 1)$

$$\omega_0 = \sqrt{1 - (Ze^2)^2}$$

and becomes imaginary at $Z > 137$. Allowance for the finite dimensions of the nucleus eliminates this difficulty. At $Z \approx 170$, however, the energy of the lowest state reaches a value $\omega_0 = -1$ and the pair energy becomes equal to zero, i.e., the vacuum becomes unstable to the production of electron-positron pairs. Thus, at $Z = Z_c$ the Dirac equation loses the meaning of an equation for one particle. If the K shell is not filled, two pairs can be produced; if the K shell has one electron, then according to the Pauli principle only one pair can be produced; finally, if the shell is filled the vacuum remains stable notwithstanding the appearance of a level $\omega_0 = -1$.

As shown in[4], at $Z > Z_c$ the vacuum is altered—the ground state corresponds to a state with charge $-2e$. At $Z - Z_c \ll Z_c$ this charge is distributed in space with a density close to the charge distribution in the K shell for $Z = Z_c - 0$, i.e., the charge is localized near the nucleus. The transition to this state is the result of production of one or two electron-positron pairs, with the positrons going off to infinity and the electrons distributed near the nucleus to form a new vacuum state.

The foregoing picture of the restructuring of the electron-positron vacuum near $Z = Z_c$ was obtained without allowance for the electron-positron interaction. Yet at $Z \approx Z_c$ the problem has degeneracy. In fact, in the case of an unfilled shell, if the interaction is not taken into account, the following three states have the same energy: 1) bare nucleus, 2) nucleus with one pair, 3) nucleus with two pairs. For the case of one electron on the K shell, two states have the same energy: 1) one electron, 2) one electron + a pair. The filled-shell state is not degenerate. Allowance for the interaction lifts the degeneracy and influences strongly the system level positions at $Z \approx Z_c$.[5]

The physical meaning of the results is the following: As shown in[21], at $Z > Z_c$ the positron acquires a long-lived quasistationary state described by a wave function close to the ψ function of the K electron. The interaction mixes the aforementioned degenerate states, with the pair corresponding to an electron on the K shell and a positron in the quasistationary state.

The onset of a positron quasistationary state is very natural. If we write down the particle equation of motion in the form of an equivalent Schrödinger equation, then the effective potential in this equation is[3]

$$U = -\frac{1}{2}V^2 + \omega V,$$

where V is the usual potential and ω is the particle energy. For Bose particles this is an exact expression (see Sec. 1 of Chap. II), and for Fermi particles it includes also small spin corrections. Thus, at any sign of the potential V (i.e., at any sign of the particle charge) the effective potential U is negative at large V, meaning attraction.

For a positron of energy $\omega \approx 1$, the effective potential is the same as for an electron with energy $\omega \approx -1$—an attraction region near the nucleus and a potential barrier outside this region. At $Z = Z_c$ the quasistationary level of the positron has an energy $\omega = 1$, so that allowance for the attraction to the electron located on the K shell suffices to obtain a bound electron-positron pair state. Indeed, allowance for the interaction leads to pair bound states in the interval $\Delta Z = Z - Z_c \sim 1$, and these states go over into a quasistationary state with further increase. If the K shell is filled, then at $Z > Z_c$ the electrons go over, as it were, into a negative continuum—a vacuum K shell is produced.

The state with one electron in the K shell goes over at $Z > Z_c$ into a state with one electron in the vacuum. In the language of the "new" vacuum this is the ground state (two electrons in the vacuum) plus a wave packet that describes a hole in the new occupation. The state with unfilled shell corresponds to two holes in the new occupation.

Since the width of the quasistationary state is small in comparison with the interaction energy, all the states with different numbers of electrons in the vacuum shell can be regarded as stationary, and any of them can be called the vacuum state.

None the less, the "old" vacuum has a physical advantage over the "new" one, its somewhat higher energy notwithstanding. A hole in the "new" vacuum does not always denote the presence of a positron. Thus, for example, a state without a charge on the vacuum K shell, which corresponds to two holes in the "new" vacuum, does of course not mean that two physical positrons are present. A physical positron is a positively charged particle produced in the "old" uncharged vacuum. Two positrons should interact with each other, whereas two such holes in the "new" vaccum do not interact.

Thus, after the vacuum has acquired additional states coming from the discrete spectrum, just as the single-particle analysis with the aid of the Dirac equation is invalidated, the hole interpretation of the positron becomes modified.

The state with two electrons on the vacuum K shell and a positron in a quasistationary state does not reduce to the state of one electron on the K shell.

The change of the interpretation of the positron as a hole in the vacuum occupation applies to only one state in the continuum of the vacuum states, and is therefore practically immaterial. The only danger is that according to the hole interpretation the positron quasistationary state corresponds to the same distribution over the negative states of the continuous spectrum as for the elec- of the vacuum K shell, and it may seem on the face of it that the appearance of a positron is equivalent to the vanishing of an electron. This feeling contradicts the physical picture of a positron as an independent particle that can be in its own quasistationary state regardless of the charge of the vacuum K shell.

The existence of a quasistationary state of a positron at any charge of the vacuum K shell is verified by a Gedanken experiment of positron scattering by a nucleus. The existence of the positron quasistationary state is determined only by the depth of the effective potential hole and manifests itself in the form of a pole in the scattering amplitude at any occupation of the vacuum K shell.

These statements will made particularly clear later, in the analysis of Fig. 2, which shows how the electron levels of the vacuum shell appear. Simultaneously with the appearance of these levels, positron quasistationary states come into being and exist independently of the occupation of the vacuum shell.

That the primitive hole interpretation of the positron is incorrect can be seen also from the following reasoning. Consider the state "electron on a vacuum K shell and a positron in a quasistationary state." When account is taken of the interaction, the electron moves in a field having a charge somewhat larger than that of the nucleus, owing to the influence of the positron charge. On the other hand, because of the influence of the electron, the positron is acted upon by a field having an effective charge smaller than that of the nucleus. This shifts the position of the maximum of the distribution over the continuum functions of the electron and of the holes corresponding to the quasistationary state of the positron. As a result, the positron state corresponds to holes in another vacuum that has a nuclear charge different from that of the electron vacuum. The maximum in the distribution of the holes corresponding to the positron state is shifted relative to the electron maximum by an energy $\Delta\omega$ much larger than the width of the distribution (at $\Delta\omega \gg \gamma$).

Thus, to prevent errors, it is necessary to use the language of the old vacuum. In this language a state with a charge can have two variants: 1) one electron on the vacuum K shell, 2) two electrons on the K shell plus a positron.

The state due to the unfilled shell is realized in three ways: 1) no electrons in the vacuum, 2) one electron and one positron), 3) two electrons and two positrons. Two states in the first case and three states in the second have nearly equal energies if the wave function of each of the positrons makes up a packet corresponding to the quasistationary state.

Thus, as $Z > Z_c$ the problem becomes degenerate. To lift the degeneracy it is necessary to solve the problem of the electron-positron field with interaction taken into account.

Allowance for the interaction alters little the coordinate dependence of the wave function that describes the positrons in the cases listed above, but alters substantially the distribution in the eigenfunctions in the wave packet. This physical picture will serve as the basis for the method developed in the next section for taking the electron-positron interaction into account.

In the case of an unfilled K shell, three levels appear with a spacing of the order of e^2 and independent of $Z - Z_c$ (in first order in e^2). These levels describe a system of 0, 1, or 2 pairs. In the case of a K shell with one electron, two levels appear with a spacing of the same order. Accordingly, when two heavy nuclei come close together, positrons are emitted with an energy spectrum that has several maxima corresponding to transitions between the indicated split states.

2. Distribution of the vacuum charge near supercharged nuclei

We see that at $Z > Z_c$ the electron-positron vacuum is so altered that the ground level of the system corresponds to a state with charge $-2e$.

In the case $Z \gg Z_c$ the ground state of the vacuum corresponds to a large number of electrons whose charge cancels almost the entire charge of the nucleus. In so far as such a charged vacuum is present in the ground state of the system, it is natural call this phenomenon electron condensation, to correspond with the analogous phenomenon for Bose particles.

The distribution of the vacuum charge around a nucleus with a charge $Z \gg Z_c$ can be easily determined, since in this case the solution of the Dirac equation can be obtained in the quasiclassical approximation, and the electron density is calculated by the Thomas–Fermi method.[6]

It has been shown that the electron density in a potential well with depth $V(r) \gg 1$ is given by

$$n(r) = \frac{V^3(r)}{3\pi^2}. \tag{1.1}$$

At sufficiently large Z ($Ze^3 \gg 1$—"supercharged nucleus") the vacuum electrons land inside the nucleus in such a way that the cancel completely the charge of the protons inside the nucleus, leaving an uncompensated charge only in a narrow layer near the surface of the nucleus.

As we shall see, electron condensation plays an essential role in the investigation of the possibility of formation of a charged π condensate and in the calculation of the energy of supercharged nuclei.

Allowance for the electron condensation is essential in the investigation of the interaction between charged particles at ultrasmall distances, and may perhaps explain the nature of the electrodynamic divergences or help eliminate them.

3. Dielectric constant of vacuum in strong nonuniform fields

The polarization of vacuum in strong field was already investigated long ago.[22] In a strong electric field, allowance for the perturbation in the motion of the electrons and positrons of the vacuum yields, besides the usual expression for the energy density $E^2/8\pi$, an additional term (at $eE \gg 1$)

$$W' = -\frac{e^2E^2}{24\pi^2}\ln eE.$$

The polarization vector is

$$\mathcal{P} = \frac{\partial W}{\partial E} = -\frac{e^2E}{12\pi^2}\ln eE - \frac{e^2}{24\pi^2}E.$$

The last term can be omitted if $\ln eE \gg 1$, and consequently the dielectric constant is

$$\varepsilon = 1 - \frac{e^2}{3\pi}\ln eE. \tag{1.2}$$

This expression was obtained under the assumption that the field varies slowly in space, namely, it changes little over the Compton wavelength of the electron or, in our units, $E'/E \ll 1$.

We shall show that the expression (1.2) is valid in sufficiently strong fields even if E varies very strongly from point to point. The criterion that we shall derive is

$$\left(\frac{E'}{E}\right)^2 \ll eE. \quad (1.3)$$

The point that in the earlier derivation the authors obtained a single expression for arbitrary field, while for their expression to be valid in weak fields it is actually necessary to satisfy the condition $E'/E \ll 1$. This is a very frequently encountered case, when the restrictions imposed by the method of obtaining the result become applicable to result itself. Assume that some physical result has been obtained theoretically for parameter values $\xi < \xi_1$. If the characteristic values of ξ over which the investigated quantity changes are $\xi \sim \xi_2 \gg \xi_1$, then the result will be valid also for values of ξ much larger than those assumed in the derivation. In more formal language: the result obtained at $\xi < \xi_1$ can be analytically continued into the region of large ξ up to values that are determined by the nearest singular point of the function under consideration. We proceed now to determine the region of validity of (1.2). Let the external field applied to the vacuum be determined by charges with density $\rho_0(r) = e_0 n_0(r)$, where e_0 is the bare charge. Then the potential is determined by the Poisson formula

$$\Delta V = -4\pi e_0^2 (n_0 + n_1),$$

where $n_1(r)$ is the additional particle density resulting from the polarization of the vacuum in the field. We have defined the potential V as the electric potential multiplied by e_0. We express $n_1(r)$ in the form

$$n_1(r) = \int \Pi(r, r') V(r') dr'. \quad (1.4)$$

In weak fields $\Pi(r, r') = \Pi^0(r-r')$. In addition, since a constant increment to the field cannot change the observed quantity, we have

$$\int \Pi^0(\mathbf{r}-\mathbf{r}') dr' = 0.$$

This condition is the simplest consequence of gauge inveriance, i.e., the invariance of physical quantities under a gauge transformation of the four-dimensional vector potential

$$A'_\nu = A_\nu + \frac{\partial f}{\partial x_\nu}.$$

We consider first $n_1(r)$ in weak fields. Expanding V in a series about the point r, we obtain

$$n_1(r) = \frac{1}{6} \int \Pi^0(\rho) \rho^2 d\rho \Delta V.$$

We have used the fact that Π^0 depends only on the absolute value of the vector ρ. Substituting in the Poisson equation, we obtain

$$\Delta V = -\frac{4\pi e_0^2 n_0(r)}{\varepsilon_0},$$

where ε_0 is the dielectric constant of the vacuum in weak fields:

$$\varepsilon_0 = 1 + \frac{4\pi}{6} e_0^2 \int \Pi^0(\rho) \rho^2 d\rho.$$

Since the electric fields are so defined that the dielectric constant of vacuum in weak fields is equal to 1, we must introduce an observable (in weak and slowly varying fields) electron charge

$$e^2 = \frac{e_0^2}{1 + (4\pi/6) e_0^2 \int \Pi^0(\rho) \rho^2 d\rho}. \quad (1.5)$$

The simplest dimensional analysis of the quantities in (1.4) shows that $\Pi^0(\rho)$ has the dimensionality $1/L^5$. Since the Compton length $1/m$ can not enter in the problem at $\rho \ll 1$, it follows that

$$\Pi(\rho) = \frac{A}{\rho^5}. \quad (1.6)$$

At $\rho \gg 1$ the value of $\Pi^0(\rho)$ must decrease even more rapidly (calculation yields $\Pi^0(\rho) \sim e^{-2\rho}$).

Substitution in (1.4) yields an integral that diverges at the lower limit

$$n_1(r) = \frac{4\pi A}{6} \int_{r_0}^{1} \frac{d\rho}{\rho}.$$

We have set the lower limit of the integration at r_0, which is the minimal distance at which the simple expression (1.6) is still valid. After introducing the observable charge in place of the bare e_0, the quantity r_0 drops out of the final expression. Using for A the numerical value obtained by calculation, we obtain from (1.5)

$$e^2 = \frac{e_0^2}{1 + (e_0^2/3\pi) \ln(1/r_0^2)}. \quad (1.7)$$

This is the well-known formula for the charge renormalization.

Let us return to the case of strong fields. Formula (1.4) can be interpreted in the following manner: At the point r' the field produces a virtual pair that contributes, as it moves in the field, to the charge density at the point r. It is clear that if the distance $R = |\mathbf{r} - \mathbf{r}'|$ is small compared with the curvature radius R_c of the particle trajectories in the field, then the particle motion can be regarded as free. Consequently at $\rho \ll R_c$ we have $\Pi(r, r') = \Pi^0(r-r')$. In the opposite case $\rho \gg R_c$ the particles produced at the point r' will not reach the point r at all, but will be turned away by the field, so that at $\rho \gg R_c$ we have $\Pi(r, r') = 0$. It remains to estimate R_c. This quantity is determined by the condition that the change of the momentum in the field be of the same order as the momentum itself:

$$\frac{\Delta p}{p} = \frac{eER_c}{p} \sim 1.$$

Momenta that are significant at $|\mathbf{r} - \mathbf{r}'| = R_c$ are of the order of $p \sim 1/R_c$. As a result we get

$$eER_c^2 \sim 1.$$

Since R_c is the only length that characterizes the motion of the particles in strong fields, it follows that formula (1.2) is valid when the fields change little over this length:

$$\left(\frac{E'}{E}\right)^2 R_c^2 < 1, \quad \left(\frac{E'}{E}\right)^2 < eE.$$

The relations obtained by us make it possible not only to estimate the region of applicability of expression (1.2), but also to derive this expression.[23]

Introducing the observable charge, we get

$$\Delta V = 4\pi e^2 (n_0(\mathbf{r}) + \tilde{n}_1(\mathbf{r})),$$

where $\tilde{n}_1$ is given by

$$\tilde{n}_1(\mathbf{r}) = \int \Pi(\boldsymbol{\rho}) V(\mathbf{r}+\boldsymbol{\rho})\, d\boldsymbol{\rho} - \int \Pi^0(\rho)\rho^2\, d\rho \Delta V.$$

Using the "locality" property of $\Pi(r, r')$:

$$\Pi(\mathbf{r}, \mathbf{r}') = \begin{cases} \Pi^0(\mathbf{r}-\mathbf{r}'), & |\mathbf{r}-\mathbf{r}'| \ll R_c, \\ 0, & |\mathbf{r}-\mathbf{r}'| \gg R_c, \end{cases}$$

we readily obtain

$$\tilde{n}_1(r) = \frac{4\pi A}{6} \int_{R_c}^{1} \frac{d\rho}{\rho} \Delta V,$$

which leads directly, after introducing the numerical value used above, to the expression

$$\Delta V = \frac{4\pi e^2 n_0(r)}{\varepsilon(E)},$$

with a dielectric constant that coincides with (1.2).

4. Interaction of point charges at short distances

Expression (1.2) for the dielectric constant makes it possible to determine directly the deviations from Coulomb's law.

Consider a nucleus of arbitrarily small radius, having at infinity a charge $Z (Z > 1,\ Ze^2 \ll 1)$. The charge inside the small radius will be larger than Z, since the charge is screened in a dielectric medium.

In the absence of external charges we have

$$\operatorname{div} D = 0,$$

where D is the induction, hence

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \varepsilon \frac{dV}{dr}\right) = 0.$$

Introducing the charge $Q(r)$ inside a sphere of radius r, multiplied by e, we obtain $dV/dr = Q/r^2$ and $\varepsilon Q = \text{const}$. Since $Q \to Ze^2$ at $r \gtrsim 1$, we get (see[23])

$$Q(r) = \frac{Ze^2}{1 - (e^2/3\pi)\ln[Q(r)/r^2]}. \tag{1.8}$$

Our expression for the dielectric constant can be used when $R_c^2 \ll r^2$, which corresponds to the condition $Q \gg 1$. To have an interpolation formula suitable at $Q \lesssim 1$ we replace Q under the logarithm sign by $Q+1$. At $Q \ll 1$ we obtain

$$Q(r) = Ze^2\left(1 + \frac{e^2}{3\pi}\ln\frac{1}{r^2}\right).$$

This expression coincides with the formula obtained in quantum electrodynamics for the corrections to Coulomb's law. At very short distances, when $Q \gg 1$, it is necessary to take into account in the Poisson equation for the potential V, the role of the electrons that condense in the vacuum near the positive charge. We have seen that this condensation sets in at $Ze^2 > 1$, and in our case, when $Ze^2 < 1$ at infinity, the condensation will take place near the external charge in regions where $Q > 1$. As a result, the equation for the distorted Coulomb potential, with (1.1) taken into account, takes the form

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \varepsilon \frac{dV}{dr}\right) = -4\pi e^2 \frac{V^3}{3\pi^2}. \tag{1.9}$$

The appearance of condensed charges is not taken into account in the electrodynamic calculations and may lead to substantial changes of the interaction at ultrashort distances.

II. BOSONS IN STRONG FIELDS

1. Instability of the boson vacuum in external fields

We have seen that the restructuring of the fermion vacuum in strong fields is restricted by the Pauli principle.

A much more substantial alteration of the vacuum takes takes place in the case of Bose particles, when there is no Pauli exclusion and many particles can be produced in the same state. The alteration of the vacuum is limited in this case only by the interaction between the particles. Once a sufficient number of particles is accumulated in a "dangerous" state, further particle production becomes energywise unprofitable because of the repulsion between the particles. We note that in the case of attraction between the Bose particles, the vacuum will be unstable even without an external field. Indeed, at a sufficiently large particle density, the energy loss to particle production (mc^2) is offset by the gain due to the attraction, and the system energy decreases with further particle production.

We consider first the case of a scalar external field. The boson energy U in an external scalar field is determined by solving the equation

$$\Delta\varphi + (\omega^2 - 1 + U)\varphi = 0; \tag{2.1}$$

we use here the units $\hbar = m = c = 1$.

The vacuum instability manifests itself in simplest form in the case of a scalar field in the form of a broad square well. The influence of the external field in this case reduces to replacement of the particle mass ($c = 1$) by an effective mass

$$\tilde{m}^2 = 1 - U_0,$$

where U_0 is the depth of the well.

When the effective mass vanishes, instability sets in. When the depth of the well is increased further, the problem becomes meaningless, because the lowest boson energy

$$\omega_{\min} = \sqrt{\tilde{m}^2}$$

becomes imaginary. The boson field will increase until the repulsion between the particles makes further increase of the field energywise unprofitable. An analogous instability sets in also in an electric field. In this case the Klein–Gordon–Fock equation takes the form

$$\Delta\varphi + [(\omega - V)^2 - 1]\,\varphi = 0. \tag{2.2}$$

We rewrite this equation in the Schrödinger form

$$\Delta\varphi + 2(E - U)\,\varphi = 0,$$

where the energy E is equal to

$$E = \frac{\omega^2 - 1}{2},$$

and the effective potential U is connected with the electric potential V by the relation

$$U = -\frac{1}{2}V^2 + \omega V. \tag{2.3}$$

The first term corresponds to attraction at any sign of the particle charge. This explains the surprising fact that in a deep potential well, when the potential tends rapidly enough to zero at infinity, a bound state is produced not only for a particle for which the potential V corresponds to attraction, but also to a particle of opposite charge, for which the potential $(-V)$ corresponds to repulsion. In a broad rectangular well, the lowest energy is determined, accurate to terms $\sim 1/R^2$, by the relation

$$(\omega + V_0)^2 = 1,$$

where V_0 is the depth of the well. It follows from this relation that the boson energy vanishes at $V_0 = 0$, and reaches a value -1 at $V_0 = 2$. At the latter value of V_0 the antiparticle energy is equal to 1. Consequently, instability with respect to production of single particles sets in at $V_0 = 1$, and pair production becomes possible at $V_0 = 2$. Of course, production of single pairs is possible only if the boson charge can change. We consider by way of example the case of pions in a well that is produced by protons, with the chemical potentials of the neutrons and protons identical (this corresponds to equilibrium relative to β decay). Then at $V_0 = 1$ the pion energy vanishes and instability sets in relative to the reaction

$$n \to p + \pi^-.$$

In this case the alteration of the vacuum consists of pion accumulation. The equality of the chemical potentials, $\mu_n = \mu_p$, will be restored on account of the β decay

$$p \to n + e^+ + \nu,$$

if the electrons can leave the system. Such a case might be realized in supercharged nuclei if they exist (see Sec. 5 of Chap. III).

Greatest interest attaches to restructuring of the pion field in a nucleon medium.

We regard the nucleon medium as the source of the field acting on the pions. The pion energy as a function of the momentum k can be obtained from the known relation ($\hbar = c = m_\pi = 1$)

$$\omega^2 = 1 + k^2 - 4\pi n F(k), \tag{2.4}$$

where n is the nucleon density and $F(k)$ is the amplitude for the scattering of a pion by a nucleon through zero angle. The first two terms yield the energy of the free pion, and the third term constitutes the effective field that acts on the pions in the nucleonic medium. For simplicity, we omit the isotopic indices. The sign of the scattering amplitude F corresponds to attraction for both π^+ and π^- mesons ($F > 0$), and therefore at sufficient density n the frequency can vanish, meaning instability of the pion field. However, $F(k)$ is small at small k and instability sets in at $k = k_0$, which corresponds to the maximum value of $F(k)$. The instability condition is $\omega^2 = 0$ or

$$1 + k_0^2 = 4\pi n F(k_0).$$

When the condition $\omega^2 = 0$ is satisfied for any one of the three pion types, a pion field of the corresponding type will accumulate at the corresponding level ($k = k_0$). The relation (2.4) does not take into account the possible excitation of the nucleonic medium by the moving pion—the nucleons are regarded as an external field (the "gas" approximation).

2. Motion of pions in a nucleon medium. Use of the methods of the Fermi-liquid theory.

In the preceding section we regarded the effect of the nucleon medium on the pion motion as the action of a certain effective field (formula (2.4)).

This approach gives only the qualitative picture. For more exact calculations we must take into account the possibility of virtual excitations of the nucleon medium by the moving pion.

To this end we write down the pion energy as a function of the momentum in the form

$$\omega^2 = 1 + k^2 + \Pi(k, \omega), \tag{2.4'}$$

where $\Pi(k, \omega)$ (the "polarization operator") is determined both by terms of the type (2.4) and by terms that take the possibility of excitation of the nucleon medium into account. We proceed to discuss the method of finding the polarization operator.

In the case of an electromagnetic field, the analogous quantity $\Pi^{(\gamma)}(k)$ is directly connected with the dielectric constant $\varepsilon(k, \omega)$, inasmuch as in this case

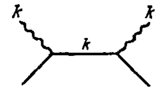
$$\omega^2 = \frac{k^2}{\varepsilon(k, \omega)} = k^2\left[1 + \frac{1}{k^2}\Pi^{(\gamma)}(k, \omega)\right].$$

This analogy is frequently used to obtain, in the case of pions, a formula similar to the Lorenz-Lorentz formula.[24] It must be assumed here that the amplitude of the virtual πN scattering (i.e., off the mass shell) is δ-like and does not differ from the real amplitude. These assumptions certainly are not satisfied in nuclear matter with nuclear density. Yet, as we shall see, there exists a consistent method of determining the polarization operator, free of these restrictions. Of course, the exact calculation of the polarization operator in a medium of strongly interacting particles is an unsolvable problem. It is easy, however, to separate the slowly varying quantities, which can be regarded as constants and determined from experiment, and express them in terms of other quantities that vary significantly in the region of interest to us, in analogy with the procedure used in Fermi-liquid theory.[9] This method is based on the fact that all the virtual processes that determine $\Pi(k, \omega)$ can be divided into two classes: those occurring at distances smaller than or of the order of $1/m_N$, and those occurring at distances on the order of unity in pion units. Processes of the former type, in a medium with a density that is low in comparison with $m_N^3 \sim 300$, proceed just as in vacuum, whereas processes of the latter type are appreciably distorted by the medium. Thus, for example, the local pion-nucleon interaction vertex, as can be verified by estimating the graphs that enter in it, is determined by the small distances $r_0 \sim 1/m_p$ or $r_0 \sim 1/m_N$, and consequently the πN-interaction constant in a medium of nuclear density differs little from the interaction in vacuum.

Let us make a few remarks concerning the graphic calculation method. Graphs or diagrams constitute primarily a convenient method of illustrating the occurring processes. They can be given the meaning of quantitative relations by assuming that each graph describes a definite transition amplitude. Then, according to the superposition principle, the total transition amplitude is the sum of all the possible physically different amplitudes and, in addition, any amplitude can be represented as a sum over all the intermediate states of the products of the amplitudes of the transition from the initial state to an intermediate state and from the intermediate to the final state, integrated over all the intermediate instants of time. If we introduce time-independent amplitudes, then this statement corresponds to the known quantum-mechanical formula

$$A_{01} = \sum \frac{B_{0i}C_{i1}}{E_0 - E_i}. \qquad (2.5)$$

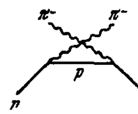
Any process, no matter how complicated, is determined by consecutive use of several simple amplitudes, which can be obtained once and for all by comparing the corresponding element of the graph with perturbation theory. Thus, the graphic method in the form in which we shall use it constitutes a simple utilization of the formulas of ordinary quantum mechanics and calls for no additional knowledge. Thus, for example, the pole part of the forward scattering amplitude of a π^+-meson by an immobile neutron can be written in the form

$A^{+\,n}_{\mathscr{P}} =$  ;

in the intermediate state there is a proton with momentum k. According to (3.1), this amplitude is equal to

$$A^{+\,n}_{\mathscr{P}} = \frac{|\Gamma|^2}{\omega + m_N - E(k)},$$

where Γ is the amplitude for the absorption of the pion by a nucleon, ω is the pion energy, and $E(k)$ is the nucleon energy. In the case of pole scattering of a π^--meson by a neutron, the only possible diagram is

$A^{-\,n}_{\mathscr{P}} =$  ,

which corresponds to the fact that the final meson is emitted first, after which the initial meson is absorbed. The amplitude in this case is

$$A^{-\,n}_{\mathscr{P}} = \frac{|\Gamma|^2}{\omega + m_N - [2\omega + E(k)]} = \frac{|\Gamma|^2}{-\omega + m_N - E(k)}. \qquad (2.6)$$

More complicated diagrams will be explained as they appear.

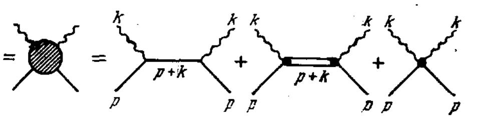
We proceed now to consider the method of separating the essential diagrams and to the calculation of the polarization operator.

The increment contributed by the medium to the square of the pion energy is expressed in the gas approximation in terms of the zero-angle scattering amplitude in the energy normalization (Formula (2.41)). Since the polarization operator is in fact this increment, we have in the gas approximation

$$\Pi(k, \omega) = -4\pi n F = nA(k, \omega),$$

where $A = -4\pi F$ is the scattering amplitude in the energy normalization. The normalization of the amplitude A is determined by the fact that in the Born approximation A becomes the volume integral of the energy of the perturbation due to one nucleon. To get rid of the gas approximation, it is necessary to introduce in place of the total density of the nucleons the Fermi distribution density $n(p)$ for the neutrons and protons and to take into account in the calculation of A the Pauli principle and the interaction between the nucleons in the intermediate states. As a result, the amplitude A itself turns out to depend on the distribution $n(p)$.

Before we proceed to the calculation of $\Pi(k, \omega)$, let us ascertain which processes determine the (π, N) scattering amplitude in vacuum. It is known that the (π, N) scattering at low pion energies $\omega \sim 1$ is described with good accuracy by the following processes:

$A =$  (2.7)

The first of the graphs corresponds to one nucleon in the intermediate state (the "pole" term of the scattering). The second diagram corresponds to a transition to the N^*_{33} resonance (the resonant part of the scattering). We shall show that both terms describe P scattering. The last of the terms in (2.7) is S scattering.

Besides the contribution from the remote resonances via the S channel, the S scattering contains, in particular, a term corresponding to N^*-resonance exchange in the u channel. Since this term is due to intermediate states that have large 4-momenta, it can be regarded as local and assumed to be independent of the pion momentum. The S scattering can then be represented by a point. For the same reason we use also points to represent the vertices $(N\pi N)$ and $(N\pi N^*)$, which can be easily seen to contain 4-momenta $m_N c$ in the intermediate states. However, the contribution to the P scattering is not restricted to these processes, and furthermore the S scattering is substantially changed on going off the mass shell. We shall determine below the additional contribution made to the scattering amplitude by these factors from the experimental data on πN scattering, using the low-energy theorems of current algebra, which make it possible to determine the changes in occurring in the amplitude on going off the mass shell.

The $(N\pi N)$ vertex is written in the form (see, e.g., [25])

$$\Gamma(N\pi N) = f\bar{\psi}\gamma_\nu\gamma_5\tau_\alpha\psi\partial_\nu\varphi_\alpha, \tag{2.8}$$

where ψ is the wave function of the nucleon, γ_ν are Dirac matrices, τ_α are the nucleon isospin matrices, and φ_α are the components of the pion field. The fields of the π^+, π^-, and π^0 mesons are connected with φ_α by the relation

$$\varphi^{\pm} = \frac{\varphi_1 \pm i\varphi_2}{\sqrt{2}}, \quad \varphi^0 = \varphi_3.$$

The constant f in (2.8) equals $g/2m_N$, where g is a dimensionless interaction constant; $g^2/4\pi \approx 14.6$ (in pion units, $m_N = 6.7$ and $f = 1.0$).

For nonrelativistic nucleons, expression (2.8) simplifies to

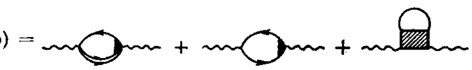
$$\Gamma(N_\pi N) \approx f\psi^+\sigma_\alpha\tau_\beta\psi\nabla_\alpha\varphi_\beta; \tag{2.9}$$

σ_α is the nucleon spin matrix.

As follows from (2.9), the vertex is proportional to the pion momentum, and the first term of (2.7) describes P scattering. Since the spin of the N^* isobar is 3/2 (the resonance $N^*_{33}(1232)$), the second term also corresponds to P scattering and its vertex is also proportional to the wave vector of the pion; the proportionality coefficient can be obtained quite accurately from the cross section for the scattering of pions with energy close to resonance.

Accordingly, for the third term of (2.7), which determines the πN-scattering amplitude, the polarization operator at pion 4-momenta $\omega \sim 1$, and $k \sim 1$, as will be shown, is determined in a medium by the same πN-scattering mechanisms. The pole or resonant interaction of the pion with the nucleons of the medium can be described in two ways: 1) scattering of a pion with a transition of the nucleon either into a state lying above the Fermi boundary or into an isobar; 2) the production of a nucleon or an isobar and the appearance of a hole in the nucleon Fermi filling. The second approach is for many reasons more convenient than the first and is in fact the one used in the many-body problem and in the Fermi-liquid theory whose results we shall use.

Thus, the polarization operator is represented by a sum of three diagrams

$\Pi(k, \omega) =$ 

$$= \Pi_R + \Pi_{\mathcal{P}} + \Pi_S. \tag{2.10}$$

Lines with arrows directed to the left and to the right represent holes and particles, respectively. The shaded triangles represent vertices that take into account the NN and NN^* correlations in nuclear matter. Expressions connecting these vertices with the constants of the NN and NN^* interactions will be given later on. The first term, designated Π_R, corresponds to production of a nucleon hole in the Fermi filling and the isobar N^*_{33} (1232) (resonant term"). The "pole" term $\Pi_{\mathcal{P}}$ corresponds to excitation of a particle-hole type in the medium. The third term takes into account the S scattering. All the remaining diagrams that have no parts connected by a particle and hole or by a hole and isobar are determined by the large 4-momenta of the intermediate states ($\sim m_N$) and either make a small contribution, or else differ little from the corresponding vacuum graphs (which have already been included in the observed pion mass or, finally, are contained in the effective mass m^* of the nucleon, which will be used below ($m^* \approx 0.9 \times m_N$).

In other words, these graphs are characterized by spatial dimensions $\sim 1/m_N$ and are not greatly distorted in nuclear matter, where the distance between particles is of the order of m_π^{-1}. These graphs depend little on the 4-momenta of the input ends, since we are interested in 4-momenta $\sim m_\pi$. They can therefore be replaced by constants, which should be obtained from experiment.

As is well known, the same idea is used in Fermi-liquid theory to introduce the constants that determine the interaction near the Fermi surface, and also to introduce the effective mass and the effective local "charge" of quasiparticles in an external field.[9]

By way of illustration we estimate now the pion-mass error resulting from the fact that the incoming pion ends in Π are taken not on the mass shell, but at $k^2 - m_\pi^2$

$=\Pi\sim m_\pi^2$. Since the vacuum part of the polarization operator changes significantly at momenta on the order of m_N or on the order of the mass of the corresponding resonance, it follows that

$$\delta m_\pi^2 \sim \frac{\delta \Pi_{\mathrm{vac}}}{\delta k^2}(k^2 - m_\pi^2) \sim \left(\frac{m_\pi}{m_N}\right)^2 m_\pi^2.$$

We see that this error is small.

Thus, the use of the methods of the many-body problem makes it possible to separate and calculate diagrams that vary strongly in the range of variables of interest to us, and replace the remaining diagrams by constants obtained from experiment.

Analysis of the diagrams (2.10) shows that all but the pole graph are determined by high energies in the intermediate states. The contribution of these diagrams (which we shall call local) to the polarization operator can therefore be written in the form (2.6):

$$\Pi_{\mathrm{loc}}(k,\omega) = n\tilde{A}(k,\omega),$$

where $\tilde{A}$ is the amplitude of the forward πN scattering in the energy normalization after subtracting the pole term of the amplitude. Allowance for interaction (say, NN^*) in the nuclear matter reduces to multiplying the amplitude by a factor that varies in the range $\Gamma\approx 0.8-1.2$, depending on the assumptions made concerning the character of the NN^* interaction. In estimates we can put $\Gamma=1$.

The amplitude $\tilde{A}$ enters in the problem at $\omega^2\neq 1+k^2$, i.e., off the pion mass shell.

Let us demonstrate the determination of the amplitude $\tilde{A}(k,\omega)$. We denote the incoming and outgoing 4-momenta of the pion and the nucleon by q, q' and p, p' and introduce the standard symbols

$$s=(p+q)^2=(p'+q')^2,$$
$$t=(q-q')^2,\quad u=(p-q')^2.$$

Let the nucleon be on the mass shell $p^2=p'^2=m^2$. The scattering amplitude can be regarded as a function of the variables t, $\nu=(s-u)/4m=\omega+(t/4m)$ and $v=(q^2+q'^2)/2$, where ω is the pion in the l.s. On the mass shell i.e., at $q^2=q'^2=1$ $(v=1)$, the scattering amplitude was obtained from an analysis of the experimental data at $k\sim 1$ and $\omega\sim 1$ with the aid of the dispersion relations.[26] Since the amplitude $\tilde{A}$ does not contain terms that have singularities near the mass shell, it can be expanded in powers of $(v-1)$. Confining ourselves to the amplitude increment linear in $(v-1)$, we get

$$\tilde{A}=\tilde{A}_{\text{mass shell}}+\alpha(v-1).$$

To determine α we must use the "consistency condition" derived in current algebra (see the review[27]). According to this condition $\tilde{A}$ should vanish at $q^2=1$, $q'\to 0$, i.e., at $t=1$, $\nu=0$, $v=\frac{1}{2}$. It is this condition that determines the constant α.

The result is the following expression for the zero-angle scattering amplitude ($t=0$):

$$\tilde{A}^+ = 0.7-0.8k^2-(0.4+0.2\omega^2)\,\omega^2,$$
$$\omega^{-1}\tilde{A}^- = -1.5+0.2\omega^2, \tag{2.11}$$

where $\tilde{A}^+$ and $\tilde{A}^-$ are the isotopically symmetrical and isotopically antisymmetrical parts of the scattering amplitude ($\tilde{A}^+$ and $\tilde{A}^-$ are respectively the half-sum and half-difference of the amplitudes of π^- and π^+ scattering by a proton or π^+ and π^- scattering by a neutron).

The scattering amplitudes, and accordingly the polarization operators, for the π^+ and π^--mesons are interconnected by the isotopic invariance condition together with the crossing-symmetry requirement. Crossing symmetry means that any transition amplitude (and, in particular, any polarization operator) should remain unchanged if we change from the particle to the antiparticle and simultaneously reverse the signs of the energy of the momentum (absorption of a particle with 4-momentum k is equivalent to production of an antiparticle with momentum $-k$).

The following relations are obtained:

$$\Pi^{(\pi^+,n)}(\omega,\mathbf{k})=\Pi^{(\pi^-,n)}(-\omega,-\mathbf{k})=\Pi^{(\pi^-,n)}(-\omega,\mathbf{k}),$$
$$\Pi^{(\pi^+,n)}(\omega,\mathbf{k})=\Pi^{(\pi^-,p)}(\omega,\mathbf{k}).$$

With the aid of (2.6) we obtain

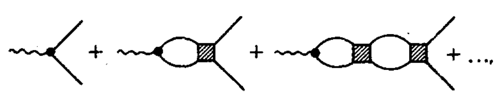
$$\Pi^+_{\mathrm{loc}}=\frac{\Pi^{(\pi^+,n)}+\Pi^{(\pi^-,n)}}{2}=(n_n+n_p)\,\tilde{A}^+,$$
$$\Pi^-_{\mathrm{loc}}=\frac{\Pi^{(\pi^+,n)}-\Pi^{(\pi^-,n)}}{2}=(n_n-n_p)\,\tilde{A}^-. \tag{2.12}$$

Let us show now how to determine the polarization-operator pole part that corresponds to the second graph of (2.10). It is easy to obtain for the scattering of a π^+ meson in a neutron medium, using (2.5) and (2.9),

$$\Pi_{\mathscr{P}}^{(\pi^+,n)}(\omega,\mathbf{k})=2f^2k^2\int\frac{n^{(n)}(\mathbf{p})\,d^3p}{\omega-E^{(p)}(\mathbf{p}+\mathbf{k})+E^{(n)}(\mathbf{p})}\,\frac{2}{(2\pi)^3}\,\Gamma_{\mathscr{P}}(\omega,\mathbf{k}), \tag{2.13}$$

where Γ $(\omega,\mathbf{k})$ is a vertex defined by the sum of graphs

$\Gamma_{\mathscr{P}}(\omega,\mathbf{k})=$



where the shaded rectangle denotes effective interaction in the nucleon medium.

Sums of this type are expressed in[9] in terms of the nucleon-nucleon interaction constants and universal functions of k and ω.

Expression (2.13) becomes exact if the intermediate state is taken to mean not the state of the free nucleon and free hole, but the state of the corresponding quasiparticle and quasihole. By the same token, account is taken of all the graphs that distort the motion of the nucleon in the medium. The changeover to quasiparticles complicates the $E(p)$ dependence, but for energies not very far from the Fermi energies, the excitations can be characterized by two terms—the Fermi and the quasiparticle.

In a medium with $N\approx Z$ these quantities are known quite well from nuclear experiments ($m^*\approx 0.9m$, $\varepsilon_F=45$ Mev).

We present by way of illustration the pole part of the polarization operator for the case $N=Z$ (the polarization operator for an arbitrary ratio Z/N was obtained in[20]):

$$\Pi^{+}_{\mathcal{P}} = -2f^2k^2 \frac{m^* p_F}{\pi^2} \frac{\Phi(\mathbf{k}, \omega)}{1+g^- \Phi(\mathbf{k}, \omega)}. \tag{2.14}$$

where $g^- = g^{nn} - g^{np}$, g^{nn}, g^{np} are constants that characterize the spin-spin interaction in the nucleon medium. The constants g^{nn} and g^{np} were obtained with the aid of the theory of finite Fermi systems from nuclear experimental data ($g^{nn} \approx 1.5$, $g^{np} \approx -0.2$, $g^- \approx 1.7$).

The function $\Phi(k, \omega)$ is quite complicated in form (see[8]). We present its value only for $k \ll 2p_F$:

$$\Phi(k, \omega) \underset{k \ll 2p_F}{=} 1 - \frac{\omega}{2kv_F} \ln \frac{\omega + kv_F}{\omega - kv_F}. \tag{2.14'}$$

3. Separation of pion degrees of freedom. Outline of consistent theory of nucleon matter

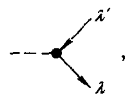
It was assumed in the theory of finite Fermi systems[9] that after the particle-hole exchange graphs are separated the remaining terms of the NN interaction have are δ-like and can be characterized by several parameters. This assumption is valid for phenomena in which the important role is played by sufficiently small momentum transfers. At momentum transfers ~ 1 in pion units it is necessary to separate, besides the particle-hole exchange, also the one-pion exchange. The remaining part of the interaction is then characterized, as can be readily verified, by 4-momentum on the order of m_N, and can be replaced by a δ-like interaction at momentum transfers on the order of unity. As a result, as shown in[20], the constant g^-, which characterizes the spin-isospin interaction of two nucleon quasiparticles in the nucleus, is replaced by a function $g_t^-(k, \omega)$ of the 4-momentum (ω, k) transferred via the particle-hole channel. We have

$$g_t^-(k) = (g^{nn} - g^{np})_t = g^- + 2\frac{dn}{d\varepsilon_F} \frac{f^2k^2}{\omega^2 - (1 + k^2 + \Pi')}. \tag{2.15}$$

Since the considered part of the interaction does not contain, by definition, any particle-hole graphs, these graphs must be separated from the pion operator. The denominator contains therefore the quantity $\Pi' = \Pi - \Pi_{\mathcal{P}}$.

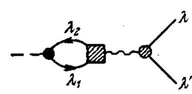
To avoid misunderstanding we note that expression (2.14) for the polarization operator contains the quantity g^- and not g_t, since the polarization operator, by definition, does not contain one-pion graphs.

The one-pion exchange graphs exert a decisive influence on the matrix elements of the effective field having the quantum numbers of the pion. As shown in the theory of finite Fermi systems, the exact matrix element of the single-particle transition under the influence of the field V_0 in the nucleus reduces to a matrix element of the effective field V produced in the nucleus under the influence of the field V_0. If the external field V_0 has the quantum numbers of the pion, then the field V contains a pole corresponding to the pion propagator. Near the critical point, when the pion energy vanishes, the effective-field matrix element should become infinite. If the matrix element of the external field is represented in the form

$V^0_{\lambda\lambda'} =$ 

,

then the matrix element of the effective field is given by the diagram

$V_{\lambda\lambda'} = V^0_{\lambda\lambda'} +$ 

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where the wavy line represents the pion propagator in the medium.

Thus, as a result of the substantial distortion of the pion propagator in the nuclear matter, the nucleon propagation does not reduce to pair interaction, as is customary assumed in the calculations of the theory of nuclear matter.

A correct theory of nuclear matter should be constructed in accordance with the following scheme: The one-pion exchange graph is subtracted from the pair interaction of the nucleons in vacuum. The remaining part of the interaction is included in the Hamiltonian as a paired NN interaction. Besides this interaction, πN interaction with the vacuum constant f is added to the Hamiltonian of the system (see the reasoning in Sec. 2 of Chap. II). In addition, one adds the pion-field Hamiltonian, which contains the vacuum $\pi\pi$ interaction. Of course, such a problem involving interacting nucleon and pion fields cannot be solved exactly. By assuming that the local quantities in the medium are equal to their vacuum values, we can greatly simplify the problem and develop a consistent theory that is suitable up to sufficiently high densities ($n < (m_N/m_\pi)^3 n_0$).

All the NN-interaction graphs except the one-pion exchange graph are assumed to be δ-like and reduce to constants that can be obtained from the vacuum interaction after subtracting from it the one-pion exchange graph. A one-pion exchange graph but with the pion propagator distortion in the medium taken into account is then added to the thus obtained δ-like interaction. The first task is to express the spin-spin NN interaction in nuclear matter in terms of its vacuum value and in terms of the interaction corresponding to exchange of one distorted pion.

To ensure that we are dealing here not with small corrections but with a significant modification of the theory of nuclear matter, we present for the pion energy the expression that follows from the formulas for the polarization operator at small k in nuclear matter

$$\omega^2 = 1.3 + \alpha k^2, \quad a = 1 - 0.4\frac{n}{n_0}.$$

This corresponds to a propagator $D = 1/[\omega^2 - (1.3 + \alpha k^2)]$. Exchange of such a "pion" over distance $r \gg 1$ leads to a nucleon-nucleon interaction that differs strongly from that in vacuum. By considering elastic scattering of

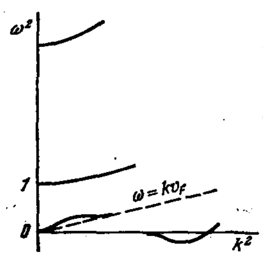
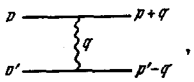


FIG. 1.

two nucleons



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corresponding to exchange of one "pion" with $q = (0, k \ll 1)$, and changing over to the coordinate representation, we readily obtain

$$V(r_1 - r_2) = \frac{1}{a} f^2 (\nabla_1 \sigma_1)(\nabla_2 \sigma_2) \frac{\exp(-\varkappa |r_1 - r_2|)}{|r_1 - r_2|}.$$

This expression differs from the vacuum value by a factor $\varkappa \approx \sqrt{1.3/a} = 1.5$ in the argument of the exponential, and by multiplication by the factor $1/a \approx 1.7$.

4. Spectra of pion excitations

The pion-excitation spectrum is determined from relation (2.4′), which can be rewritten in the form

$$\omega^2 = 1 + k^2 + \Pi_{\mathrm{loc}}(k, \omega) + \Pi_{\mathscr{P}}(k, \omega); \qquad (2.4'')$$

We have left out here the isotopic symbols. The quantity Π_{loc} is defined by relations (2.11) and (2.12), while $\Pi_{\mathscr{P}}$ is given by (2.14) and (2.14′). As seen from (2.14) and (2.14′), relation (2.4″) is a transcendental equation for $\omega(k)$.

The theory involves three types of field: pion field, particle-hole field, and isobar-hole field. For each pion charge there should therefore exist, in general, three branches of the solutions of (2.4″). These branches can be classified in accord with the excitations into which they go over when the πN interaction is turned on.

As should be the case of equations that describe relativistic particles, extra branches of the spectrum arise in the solution of (2.4″) and these should be interpreted as solutions for the antiparticle taken with a minus sign. The criterion for the selection of physical solutions with the quantum numbers of the π^+ mesons is the condition[8,20]

$$2\omega^+ - \left(\frac{\partial \Pi^+}{\partial \omega}\right)_{\omega=\omega^+} > 0.$$

A similar condition exists for the π^--mesons.

Figure 1 shows the excitation spectra obtained for the case $N = Z$ by numerically solving (2.4″). In this case all three pion types (π^+, π^-, π^0) have identical spectra (isotopically symmetrical medium).

There are three spectrum branches. The upper one can be called resonant. It should be interpreted as a bound state of an isobar and a nucleon hole. At $k = 0$ the excitation energy goes over into the difference between the isobar and nucleon masses.

The excitation energy of the middle (pion) branch goes over into the energy of the free pions when the πN interaction is turned on ($\omega^2 \to 1 + k^2$).

The lower branch can be called the spin-isospin-sound branch. As $f \to 0$ it goes over into a spin-isospin-sound excitation of the nucleon medium, i.e., into an excitation with symmetry $\sim \sigma_\alpha \tau_\beta$, where σ and τ are the spin and isospin matrices acting on the nucleons (these excitations were considered in[9]).

To clarify things we recall that Fermi systems are subject to collective excitations called zero sound, which can be interpreted as bound states of a particle and a hole. These excitations can be of four types: 1) scalar type—ordinary zero sound; 2) spin type—spin-density waves; 3) isotopic type, corresponding to isospin waves; 4) finally, spin-isospin waves, with the quantum numbers of the pion (0^-, $T = 1$). At $n > n_c$ a region with $\omega^2 < 0$ appears in the spin-isospin branch, meaning instability for π_s^0-meson and $\pi_s^+\pi_s^-$-pair production. The symbol s labels the type of branch.

The picture of the spectra in a neutron medium is somewhat more complicated.

At a density n_c^+ much less than the nuclear density $n_0 \times (n_c^+ \approx 0.4n)$, there is no spin-sound branch. Next, at a density $n = n_c^\pm (n_c^\pm \approx n_0)$ there appears in the spectra of the π^- and π_s^+-mesons a point with $d\omega/dk = -\infty$, corresponding to vanishing of the pair energy:

$$\omega_s^+ + \omega^- = 0.$$

The π^0-meson spectrum of each of the three branches is of the same form as in a medium with $N = Z$. The spectra of the π^+ and π^- excitations for the neutron medium are shown in Fig. 2. The resonance spectrum is left out for simplicity.

5. Pion condensation

Let us trace the restructuring of the vacuum after the instability sets in. For this analysis, it is immaterial to us in which field the instability has set in. It is only important that the frequency of some degree of freedom passes through zero. Inasmuch as the "condensation" consists in the fact that the field φ_k corresponding to this degree of freedom is strong, we can neglect the influence of the fields corresponding to all other degrees

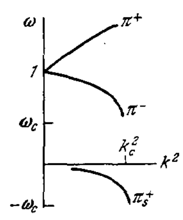


FIG. 2.

of freedom. Then the energy of the condensate can be written in the form

$$H=\int dr\left(\frac{\dot{\varphi}_k^2+\omega^2\varphi_k^2}{2}+\frac{\lambda\varphi_k^4}{4}\right). \tag{2.16}$$

At $\omega^2=1+k^2$ and $\lambda=0$, Eq. (2.16) goes over into the known expression for the energy of a free pion field. We have introduced phenomenologically the effective repulsion between the pions in the nucleon medium ($\mathscr{H}'=\lambda\varphi^4/4$, $\lambda>0$).

The interaction between the pions in the nucleon medium is the sum of their interaction in vacuum and the interaction due to exchange of excitations of the nucleon medium. The determination of this interaction is a complicated problem, but near the transition point, when the field φ_k is not very strong, the real $\pi\pi$ interaction takes the form assumed in (2.16) with a dimensionless constant $\lambda\sim 5-10$.

Near the instability point, the frequency of the considered degree of freedom can be written in the form

$$\omega^2=\alpha\,(n_c-n),\qquad \alpha>0. \tag{2.17}$$

The quantity α has a simple connection with the polarization operator. At $n>n_c$, when $\omega^2<0$, a static condensate field is produced, and its value can be obtained by minimizing (1.5) with respect to φ_k^2.

Using (2.17), we obtain

$$\langle\varphi_k^2\rangle=-\frac{\omega^2}{\lambda}=\frac{\alpha}{\lambda}\,(n-n_c). \tag{2.18}$$

The energy $\mathscr{E}_\pi$ of the condensate is obtained by substituting (2.17) in (2.16).

$$\mathscr{E}_\pi\underset{n>n_c}{=}-\frac{\omega^4}{4\lambda}=-\frac{\beta\,(n-n_c)^2}{2}. \tag{2.19}$$

In the case of a system of large size, $R\gg 1$, the frequency squared ω_k^2 is negative simultaneously for a large number of eigenvalues adjacent to the value of k_0 for which $|\omega_k|^2$ is maximal.

The minimum system energy, as follows from (2.19) corresponds to the state k_0($|\omega_{k_0}|^2$ is maximal). All remaining degrees of freedom will then have positive frequencies. In fact, the coefficient of φ_k^2 in the Hamiltonian consists, after making the substitution $\varphi\to\varphi_{k_0}+\varphi_k$, of two terms:

$$\omega_k'^2=\omega_k^2+3\lambda\,\langle\varphi_{k_0}^2\rangle.$$

It is easy to verify that the second positive term is larger than $|\omega_k|^2$, and consequently $\omega_k'^2>0$. Thus, the condensate stabilizes all the degrees of freedom. We note that this is precisely the scheme used to construct the Landau theory of second order phase transitions, in which the free energy was expanded in powers of the "order" parameter. Corresponding to the phase transition was the vanishing of the coefficient of the linear term. In our case the role of the "order" parameter is assumed by φ^2 and that of the free energy by H. Since the order parameter φ^2 increases from zero, we are dealing with a second-order phase transition.

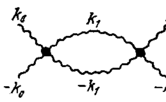
It is known that the theory of second-order phase transition becomes much more complicated when account is taken of the order-parameter fluctuations near the critical point. Analogously, in the case of our phase transition allowance for the pion-field fluctuations in the immediate vicinity of the transition point in the immediate vicinity of the phase transition distorts the simple results obtained above. A particularly important role is played by diagrams representing exchange of "soft" pions, i.e., π_s mesons.

As shown in[28], a long-range pion-pion interaction sets in near the phase-transition point. As a result, the effective 4-boson interaction constant Λ may reverse sign near the transition point at a density $n<n_c$. In this case, a first-order transition takes place. As we shall verify, this phase transition takes place with a small jump of the amplitude of the condensate field φ, and in practice differs little from the second-order phase transition considered above.

We confine ourselves below to symmetrical nuclear matter ($Z=N$).

Let us cite the results of[28]. First to be considered is the $\pi\pi$-interaction diagram corresponding to exchange of two "dangerous" pions:

$\Lambda_0=$



As shown in[28], Λ_0 has a pole near the critical point:

$$\Lambda_0=-\lambda\frac{\omega_0}{\omega_0},$$

where $\omega_1=0.05\lambda$ and $\omega_0=\omega(k_0)$ is determined by (2.4′) and passes through zero at the critical point. The diagrams Λ_0 must therefore be taken into account near the critical point in all orders of perturbation theory. Discarding the non-pole diagrams, we obtain for the effective interaction the expression

$$\Lambda=\lambda\frac{1-(\omega_1/\omega_0)}{1+(\omega_1/\omega_0)}.$$

Λ reverses sign at $\omega_0=\omega_1$ and with further increase of the density (decrease of ω_0) the system becomes unstable to a first-order phase transition. This does not change very significantly the results of the theory in which a second-order transition is assumed, since ω_1 is numerically small and leads to a small jump of the condensate field at the transition point. Even at a slight excess of density over the critical value the condensate energy takes the form (2.19), which corresponds to a second-order transition.

In nuclei (if a condensate exists) this phenomenon becomes blurred, in addition, by the fact that, since the system is finite, the value $k=k_0$ is reached with accuracy $\Delta k\sim 1/R$ and $\tilde{\omega}^2\sim 1/R^2\neq 0$ even at $\omega_0=0$.

The total energy density of nuclear matter can be expressed in the form

$$\mathscr{E}(n) = \mathscr{E}_N(n) + \mathscr{E}_\pi(n),$$

where $\mathscr{E}_N$ is the nucleon energy density. According to (2.19), a jump of the compressibility (a jump of d^2/dn^2) takes place at n. If this jump exceeds in absolute value the compressibility of the nuclear matter prior to condensation, then the compressibility becomes negative after the condensation and the system will be compressed until it goes over into a denser stable state.

At densities greatly exceeding the critical value, when the pion field becomes strong enough, the simple expression used in (2.16) for the effective $\pi\pi$ interaction is no longer valid. The criterion is the ratio φ/kv_F. With further increase of the pion field reaches a limiting value $\varphi \sim 1$ (in pion units) and the growth of the modulus of the condensate energy slows down.

The determination of the condensate energy at high densities is a very complicated problem that has been solved only for a condensate field assumed to have the form of a traveling wave.

The interaction of nucleons with a pion field in the form of a traveling wave was first considered in[12,13]. In this case the interaction mixes only two states: a neutron with momentum p and a proton with momentum $p-k$, where k is the wave vector of the π^--meson field. Therefore the determination of the nucleon energies in the pion field reduces to a solution of a quadratic equation. Knowing the nucleon energy we can find the condensation energy for an arbitrary amplitude of the pion field. However, a model of this type is quite far from the real conditions. Account must be taken first of the vacuum interaction of the pions and of the change of the πN interaction in the presence of a pion field, which follow from the Weinberg Lagrangian cited above (see[27]). It is necessary next to account for all the changes that occur in the $\pi\pi$ interaction in the nucleon medium, and finally, allowance must be made for the influence of the N^* resonance. We have seen that the allowance for these processes is a complicated problem even in the case of a weak pion field.

The calculation of the energy of a strongly developed condensate, with the N^* resonance and the nucleon correlation taken into account, was made possible by studies[17,18] in which the chiral symmetry approximation was used and N^* was described with the aid of the quark model. Analytic expressions are obtained only in the case of limiting fields.

III. PHYSICAL CONSEQUENCES OF PION CONDENSATION

1. Condensation in homogeneous nucleon matter and neutron stars

In a medium with $N \approx Z$, all three types of pions are under identical conditions (isotopically symmetrical medium), and the condensation sets in simultaneously for the π_s^+, π_s^- and π_s^0 mesons.

The picture of condensation in a neutron star is much more complicated. In this case the instability sets in originally for the π_s^+ mesons. When the density n_c^+ is reached and a spin-sound branch with energy $\omega_s' \lesssim -\varepsilon_F^{(n)}$ appears, the protons existing at $n < n_c^+$ go over into a bound state

$$p \to n + \pi_s^+.$$

The energy lost when a slow proton goes over into a neutron over the Fermi surface is offset by a negative energy ω_s^+ of large absolute value. The charge of the produced π_s^+ mesons is offset by the charge of the electrons present prior to the transition. With further increase of the neutron density as a result of the β process

$$n \to n + \pi_s^+ + e^- + \tilde{\nu} \tag{3.1}$$

the density of the π_s^+ mesons and the electron density, which is equal to it, will increase together with increasing $|\omega_s^+|$, inasmuch as at equilibrium, in accordance with (3.1), the Fermi boundary of the electrons should equal $|\omega_s^+|$.

It is easily seen that at densities close to n_c^+, the energy density of the π_s^+ condensate is equal to

$$\mathscr{E}_\pi = \omega_s^+ n_s^+ + \frac{|\omega_s^+|^4}{4\pi^2}, \tag{3.2}$$

and the density of the condensate is

$$n_e = n_s^+ = \frac{|\omega_s^+|^3}{3\pi^2}. \tag{3.3}$$

The second term in (3.2) is the kinetic energy of the electrons (at $\varepsilon_F^{(e)} \gg m_e c^2$).

Using (3.3), we obtain

$$\mathscr{E}_\pi = -\frac{|\omega_s^+|^4}{12\pi^2}. \tag{3.4}$$

The energy of the condensate assumes a finite value jumpwise. This jump, however, is offset by the change of the nucleon energy, so that the total energy of the system remains unchanged.

We see that near n_c^+ the condensate density and the condensate energy are limited not by the repulsion between the pions, but by the Pauli principle for the electrons. With further increase of the density, the increase of $|\omega_s^+|$ with density is slowed down by the influence of the repulsion between the pions. Furthermore, as we have seen, an instability for the production of the $\pi^-\pi_s^+$ pairs sets in (at $n > n_c^{(\pm)}$), and consequently a π_s^--meson field appears in the condensate in addition to the π_s^+.

Owing to the influence of the π_s^+ condensate, the energy of the condensate acquires a complicated dependence on the density. However, since the numerical factor in the denominator of (3.4) is large, the influence of the π_s^+ condensation is small, and formula (2.19) can be used at $n > n_c^{(\pm)}$.

It is easy to verify that at a density $n = n_c$ the com-

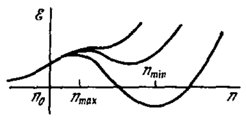


FIG. 3.

pressibility becomes negative (the compressibility is proportional to the second derivative of the energy density with respect to density). The condensate term (2.19) of the energy density makes a negative contribution to the compressibility, and this contribution is larger in absolute value (at a density n_0) then the contribution of the nucleon part of the energy density. Indeed, calculations of the neutron-matter energy density[29] without allowance for condensation yields for the second derivative of the nucleon energy density with respect to n (at $n \approx n_0$, in pion units):

$$\frac{d^2\mathscr{E}_N}{dn^2}\Big|_{n=n_0} \approx 0.2,$$

whereas for $d^2\mathscr{E}_\pi/dn^2$ we have

$$\frac{d^2\mathscr{E}_\pi}{dn^2} = -\beta \approx -1.$$

With further increase of the density, the repulsion between the nucleons at short distances assumes an ever increasing role, and in addition, when the pion field becomes strong enough, the growth of the condensate energy slows down, as a result of which the sign of the compressibility is restored. An approximate plot of $\mathscr{E}(n)$ is shown in Fig. 3.

Figure 4 shows the possible dependence of the pressure on the neutron density (the characteristic points are given for curve 2):

$$p = \mathscr{E}(n) - n\frac{\partial\mathscr{E}}{\partial n}.$$

The region between the points $n = n_c$ and $n = n_m$ is thermodynamically unstable. Therefore when the density at the center of an evolving star exceeds the critical value, the density distribution should change abruptly. Consider first the case of curve 2 (such a behavior was obtained in[30,31]). At the radius r_1, where the density is $n = n_1 < n_c$ and the pressure is $p = p_1$, a density jump should take place. The density in the inner part $n(r) > n_2$ is determined by the left branch of curve 2, whereas the density in the outer part $n(r) < n_1$ corresponds to the right-hand branch of this curve.

For curve 1 there is a point where $p = 0$. This point should lie on the surface of the star, where the pressure is equal to zero and the density distribution is determined by the left-hand branch of curve 1. In case 3, finally, an unperturbed density redistribution takes place, corresponding to a certain softening of the equation of state. All three cases lie in a reasonable region of not-too-well-known values of the NN- and πN-interaction constants for nuclear matter.

The density redistribution should occur within a short time, of the order of hydrodynamic times, with an energy release comparable with the gravitational energy of the star.

We shall discuss below the possibility of the existence of superdense neutron nuclei. If such nuclei exist, then neutron stars of arbitrary dimensions should exist, for in this case the equilibrium neutron state is attained because of the nuclear forces and not on account of the force of gravity, as in ordinary neutron stars.

The sharp change of the nucleon density along the radius of the star is accompanied by a sharp change of the π_s^+-meson energy, and consequently of the Fermi energy of the electrons. But a change in the end-point energy of the electrons means that a change takes place in the depth of the electric potential well $V(r)$ that retains the electrons.

At equilibrium we have

$$\varepsilon_F^{(e)}(r) + V(r) = \text{const},$$
$$\varepsilon_F^{(e)}(r) + \omega_s^+ = 0;$$

As a result, strong electric fields are produced, and can be obtained from the relation

$$\frac{dV}{dr} = \frac{d\omega_s^+}{dr}. \tag{3.5}$$

Thus, π condensation exerts a strong influence on the structure of the neutron stars.

2. Condensation in a finite system

Estimates of the critical density corresponding at $N = Z$ to the vanishing of the frequencies $\omega_s^{+,-,0}(k_0)$ gives a value $n_c \approx n_0$. The inaccuracy of this estimate is due to the inaccuracy of the constants of the NN and πN interactions in the medium, which were introduced in the theory. The uncertainty of the estimate n_c is such that it admits fully of the possible existence of a π condensate in ordinary nuclei. It is therefore of great interest to analyze the experimental data in which a π condensate might appear, and also experiments that make it possible to establish how close the nuclei are to condensation if the condensation has not yet set in, and by the same token refine the constants introduced into the theory. This refinement of the constants is particularly important in order to assess the possibility of the existence of superdense nuclei.

To this end it is necessary first of all to consider π condensation in a finite system. Such an analysis shows that in medium and heavy nuclei one obtains a conden-

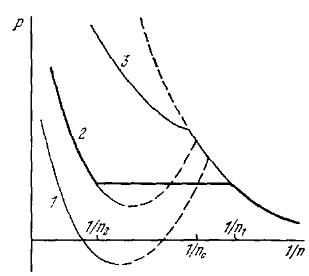


FIG. 4.

sate-energy density that differs from the case of an infinite system only in a thin layer $\delta \ll R$ near the boundary of the nucleus.[32] A periodic flat structure is realized of the condensate field

$$\varphi = a(r) \cos k_0 z, \tag{3.6}$$

with the amplitude $a(r)$ constant inside the volume and zero in the layer δ at the boundary of the nucleus. In the case of a deformed nucleus, the layers are oriented perpendicular to the major axis.

The additional surface energy connected with the π condensation is proportional not to the total surface of the nucleus, but to the smallest equatorial section. Consequently the condensation contributes to elongation of the nucleus and could lead in principle to the appearance of a second minimum on the plot of the nuclear energy against the deformation, i.e., to shape isomerism. When these results were derived, the nucleus was regarded as a sufficiently large system. In[33], the critical conditions for the condensation were obtained by methods of the theory of finite Fermi systems,[9] i.e., from the exact equation for a scattering amplitude having the quantum numbers of the pions in the particle-hole channel. Instead of finding the critical density, the critical value of the constant g^- at which the pion-excitation energy vanishes was determined. It was shown that for light nuclei the instability sets in first for the S states. For medium and heavy nuclei, the results hardly differ from those of the macroscopic approach.

The layered structure of the condensate field (1.16) leads as a result of the (πN) interaction, in second order in the field amplitude, to a layered structure of the density of the neutrons and protons with wave vector $2k_0$

$$n^{(n,p)} = n_0^{(n,p)} (1 + \xi^2 \cos 2k_0 z). \tag{3.7}$$

The layered structure (1.17) may cause a rotational spectrum to appear in nuclei that are spherical in the sense of the deformation parameter. In addition, a layered structure of the proton densities should influence the nuclear electric form factor that appears in electron scattering.

The strong decrease of the pion energy in the nucleus, predicted by the theory, manifests itself in a number of experimental facts. Thus, the spectral data of the π atom yield the "optical" potential of the pion in the nucleus (i.e., the effective potential well of the pion). It is clear that the optical potential is directly connected with the polarization operator $\Pi(k, \omega)$. Reasonable agreement is obtained between the theoretical optical potential and the experimental one.

The symmetry breaking due to the existence of the condensate leads to the appearance of low-lying Goldstone excitations. The condensate upsets the translational, rotational and (in the case of a traveling wave) the isotopic symmetries. Accordingly, three modes of Goldstone oscillations are possible. In an infinite system the frequencies of these modes should start from zero. In a finite system the minimal frequency contains in the denominator the system radius R raised to some power. According to[24], the lowest energy is possessed by the frequency corresponding to oscillations of the direction of the condensate structure relative to the direction of the elongation of the nucleus. The corresponding frequency is of the order of

$$\omega_{\rm rot}^2 \sim \frac{\beta\omega_0}{R(k_0R)^2},$$

where β is the deformation parameter. The translational and isotopic modes have large minimal frequencies, that are difficult to distinguish from other excitations with the same quantum numbers. Observation of a Goldstone oscillation among the nuclear excitations would be a decisive argument in favor of the existence of the condensate.

One more substantial difference between condensation in a finite system and an infinite medium is that the pion field in the ground state executes zero-point oscillations such that the average condensate field at each point is zero.[35] The mean square of the pion field is, of course different from zero and is determined by the formulas given above. Therefore the amplitude, linear in the field, for the scattering of any particles contains not the average field $\overline{\varphi}$ but the field matrix element between the ground and first excited states of the pion field, i.e., the quantity $\varphi_{01} e^{-i\omega_{01} t}$. The energy is $\omega_{01} \sim 5-10$ MeV and in the case of sufficiently large energy transfer the cross section contains the quantity $\varphi^2 = |\varphi_{01}|^2$.

To check on the employed expression for $\Pi(k, \omega)$ and to determine the constants more accurately it is essential to compare with experiment the energies of the levels that have pion symmetry. These states pertain to the levels $0^-, 1^+, 2^-, \ldots$ The energy shift of these levels compared with their shell-model values is determined to a large degree by the nucleon interaction on account of exchange of a "softened" pion. The agreement with experiment is satisfactory.[36]

A particularly strong influence is exerted by proximity to condensation on l-forbidden $M1$ transitions and on transitions with a change of the orbital angular momentum by two units.[37]

The intensity of such transitions contains a term due to one-pion exchange and having a pole at the critical point (i.e., at $\omega_s(k_0) = 0$). The intensities of these transitions are in some cases dozens of times larger than the value calculated without allowance for exchange of a "soft" pion. This fact attests to the proximity of the system to condensation, but leaves open the question whether a phase transition took place.

Significant information is obtained from an analysis of the influence of one-pion exchange on the magnetic moment (in this case the influence is not very strong) and on the probability of Gamow–Teller β transitions. It is of great interest to search for anomalies in the scattering of nucleons by nuclei, as well as an analysis of the nuclear magnetic form factor obtained in experiments on large-angle electron scattering. These experiments might reveal the spin structure of the nucleon density (in contrast to the electric form factor, which is determined by the charge-density structure).

Thus, analysis of the available experiments confirms the main conclusions of the theory and so far does not contradict the assumption that a condensate exists in nuclei.

It might be assumed that a more thorough analysis of the available facts as well as of the data obtained in scattering experiment can confirm or deny the existence of a condensate in nuclei, and at any rate will make it possible to refine the constants introduced into the theory to such an extent that the predictions concerning the possible existence of superdense nuclei can be made more definite.

We proceed to analyze possible experiments that assess the proximity of nuclei to condensation.

3. Experiments that establish the proximity of nuclei to condensation

If a condensate is present in the nucleus, i.e., the nucleon spin density has a periodic structure, then this structure can influence the angular dependence of the particle-scattering amplitude.

We consider first the simplest case, when the periodic structure of the spin density leads to a periodic structure of the nucleon density. As already mentioned, in the case of a traveling wave, the $\pi^+\pi^-$ fields do not lead to a periodic structure—this structure is due only to the π^0 condensate, which takes the form of a standing wave because the π^0 meson field is real. Thus, assume that in a coordinate frame fixed in the nuclear deformation direction, we have a periodic density structure

$$n(r) = n_0 (1 + \xi^2 \cos 2k_0 r).$$

If the nuclei are not polarized, then in experiments on elastic scattering, when the rotational levels are not excited, $n(r)$ in the problem is averaged over the directions of the vector k_0, i.e.,

$$\bar{n}(r) = n_0 \left(1 + \xi^2 \frac{\sin 2k_0 r}{2k_0 r}\right).$$

Thus, the electron elastic-scattering form factor must have an additional term compared with the form factor of the smooth density distribution. This additional term has a narrow maximum at a momentum transfer $q = 2k_0 \approx 2p_F \approx 3f^{-1}$. It is known that such an anomaly of the form factor is indeed observed at momenta q close to this value. It should be noted that a Thomas-Fermi calculation of the form factor shows that for a certain choice of the interaction constants the form-factor anomaly can be explained also without assuming the existence of a condensate. It is difficult to say which explanation is more convincing. A more convincing proof of the existing of a condensate in the nucleus might be provided by experiments on electron scattering by polarized nuclei, and this would lead to an appreciable increase of the anomalous scattering.

Experiments on nucleon scattering by an unpaired nucleon of an even-odd nucleon might yield information on the proximity of the nucleus to condensation, inasmuch as near the critical point the pion field produced by the odd nucleon is enhanced for wave vectors $k \approx k_0$. Indeed, the pion field produced by the nucleon is proportional to

$$\varphi \sim \frac{\psi^+ \sigma \tau_3 \psi}{\omega^2(k)}.$$

$\omega^2(k)$ has a minimum at $k = k_0$. Elastic scattering of nucleons by an odd nucleon should therefore have a maximum at a momentum transfer $q \approx k_0$.

The most convincing and seemingly easiest to perform is an experiment on single-nucleon capture of π^- mesons.[39] In the absence of a condensate, single-nucleon capture of a π^--meson from the shell of a π-atom should be very small, since the excess momentum should be transferred to the nucleus as a whole. In the presence of π condensate with amplitude $a^2 \approx 0.04$, the probability of single-nucleon capture, as shown in[39], increases by a factor of 100.

Observation of large single-nucleon capture would be an argument favoring the presence of a sufficiently well developed condensate in the nucleus.

It should be noted that the condensate periodic structure becomes smeared out near the condensation point by the radial zero-point Goldstone oscillations of the pion field, and therefore all the experiments connected with momentum nonconservation due to condensation can give a positive result only in the case of a well developed condensate, when the zero-point oscillations are insignificant.

The decisive experiment may turn out to be one on photoproduction of pions and a nucleus. If a condensate exists in the nucleus, then the photoproduction amplitude, as a function of the momentum transfer $q = k_\gamma - k_\pi$, should have a maximum at $q = k_0$, and this would correspond to photoproduction with a transition of the condensate field into an excited state (see Sec. 2 of Chap. III).

Significant information on the polarization operator of the pions in the nucleus can be gained from an analysis of the spectral data on the π-atom, which can yield information on the nuclear optical potential of the pions. As shown in[40], the nuclear optical potential of the pions is connected with the polarization operator $\Pi(k, \omega)$ by the relation (we confine ourselves for simplicity to the case $Z \approx N$)

$$V_{\mathrm{opt}} = -\left\{\frac{(\partial\Pi/\partial\omega^2) + (\partial\Pi/\partial k^2)}{2[1-(\partial\Pi/\partial\omega^2)]}\right\}_{\substack{k=0,\\ \omega=1}} \Delta. \qquad (3.8)$$

4. Possible existence of superdense and neutron nuclei and ways of their observation

Assume that the estimates given above are valid and that at $n > n_c$ the compressibility of the nuclear matter reverses sign. It does not necessarily follow, however, that superdense nuclei must exist (see[7, 19, 41]). For such nuclei to exist it is necessary to satisfy a number of conditions. First, the energy of such a nucleus should be less than the sum of the masses of the neutrons and

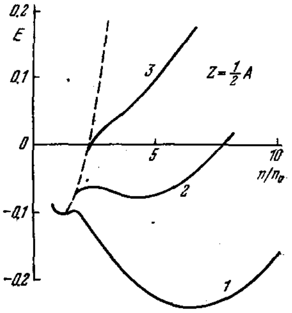


FIG. 5.

protons, otherwise it will break up into individual particles. In addition, it must be stable against fission.

It is known that the lifetime of ordinary transuranium nuclei decreases sharply with increasing charge, owing to their instability against fission. The stability condition of ordinary nuclei is given by $Z^2/A < 50$. A similar inequality should be satisfied for anomalous nuclei. Furthermore, to be able to observe anomalous nuclei in cosmic rays, they must be stable relative to β decay. To formulate these conditions quantitatively, we must have an expression for the nuclear energy at both low nucleon densities $n \approx n_0$ and at densities for which stable anomalous nuclei are expected (calculation shows this density to be $(3 \text{ to } 6)n_0$). The energy of the nucleus consists of the pure nucleonic energy and of the energy gained when the condensate is formed. Since the pure nucleonic energy is minimal at the density $n = n_0$, it follows that at $n > n_0$ it is positive and increases with increasing n. The pion-condensation energy is negative and cancels the increase of the nucleon energy partially or fully.

Figure 5 shows the energy of the nucleus (reckoned from the sum of the rest energy of the nucleons) as a function of the density. The first minimum on the curves 1–3 corresponds to ordinary nuclei. The second minimum, if it exists at an energy less than zero, corresponds to anomalous nuclei. The total energy is the difference of two large quantities, the positive nucleon energy and the negative energy of the pion condensate. Therefore even a slight inaccuracy in the calculation of each of the term can lead to a large error in the total energy. As seen from Fig. 5. depending on the choice of the insufficiently well known nucleon-nucleon interaction parameters, the second minimum on the curve can either be absent or lie above zero, corresponding to stable superdense nuclei.

It should be noted that calculation of both the nucleon energy and of the energy of the condensate constitutes, at high densities, a complicated problem that has been solved only approximately. On top of the inaccuracy in the choice of the interaction parameters, there is also the inaccuracy of the theory itself. It is therefore impossible to make a definite conclusion that anomalous nuclei exist. It can only be stated that the existence of anomalous nuclei is likely enough to undertake most serious efforts to prove or refute this assumption.

We note that Lee[42] has proposed a mechanism for the production of superdense nuclei, based on the assumption that sufficiently dense nuclear matter is unstable relative to nucleon-pair production.

According to convincing estimates made in[19], this phenomenon, can occur, if at all, at densities hundreds of times higher than nuclear density. We present a simple argument indicating that a nucleon–antinucleon instability is impossible at densities comparable with nuclear ones.

For such an instability to set in it is necessary that the depth of the effective well for an individual nucleon become of the order of $m_N c^2 = 930$ MeV. Yet at nuclear density the depth of the well is only 50 MeV. If correct account is taken of repulsion at short distances, it should decrease with increasing density and should even reverse sign at a density on the order of nuclear.

Let us return to the curves of Fig. 5. One of these curves has a minimal energy lower than that of normal nuclei. If this case were to be realized in nature, then the ordinary nuclei would have to be unstable and should go over into superdense nuclei. It is possible that the time of this transition is so long that the number of the superdense nuclei produced from the ordinary ones during the lifetime of the universe is very small, and the superdense nuclei are a small admixture to the ordinary nuclei.

As already mentioned, even the case when the curve on Fig. 5 has a second minimum with an energy lower than zero, the question of the stability of the anomalous to fission and to β decay still remains open. To answer these questions it is necessary to know the energy of the nucleus not only at high density, but also at an arbitrary ratio of the nuclear charge Z and the number of neutrons N. A recent calculation of the energy as a function of the density n and of the ratio Z/N has shown that superdense nuclei with $Z \approx N$ should have a lower energy than nuclei with $Z \ll N$. Therefore nuclei with $Z \ll N$ should go over via a cascade of β decays into nuclei with $N \approx Z$. The energy of the β electrons at the start of the cascade is 100–200 MeV, corresponding to a lifetime 10^{-6}–10^{-8} sec. Under certain reasonable assumptions concerning the interaction constants, there should exist "neutron" nuclei that are stable to β decay and fission. Allowance for the screening of the Coulomb field of the nucleus by the vacuum electrons (see Sec. 5 of Chap. III) greatly extends the region of the stability of such nuclei.[43]

We make now a few remarks concerning the possible experiments on the observation of anomalous nuclei.

If superdense nuclei exist, it is not clear to which nuclei—normal or superdense—the larger binding energy corresponds. It is possible in principle that the larger binding energy is possessed by superdense nuclei. Interest attaches in this connection to the experimental limitation on spontaneous transitions of normal nuclei to the superdense state. We note that the searches of nuclei with anomalously high binding energy have so far yielded negative results.

It is of interest to search for stable or short-lived β-active anomalous nuclei with small dimensions ($A \sim 100$)

in the fission products of ordinary nuclei.

Superdense nuclei can possibly be produced in collisions of heavy ions with energies on the order of several hundred MeV per nucleon (the energy per nucleon should be noticeably higher than the Fermi energy $\varepsilon_F \approx 40$ MeV).[1]. The resultant shock wave can increase appreciably the nuclear matter. It is quite probable that the compressibility of the system becomes negative already at $n = n_c$. It suffices therefore to compress the system to a density $n = n_c$ to initiate the formation of the superdense phase. Regardless of whether stable superdense nuclei exist or not, pion condensation should greatly influence the dynamics of the collision and should manifest itself in the angular and energy distributions of the reaction products. This possibility was considered in[44]. A more detailed investigation of the influence of the phase transition on the dynamics of shock waves in nuclear matter was carried out by Galitskiĭ and Mishustin[45] and showed that the presence of a negative compressibility to the equation of state should lead to a splitting of the shock wave into two. In the first wave there is a jump of density from n_0 and n_c, while in the second there is a jump from n_c to the density $n_m (n_m \approx (3-6)n_0)$ of the superdense phase. This phenomenon can lead to the appearance of two (instead of one) maxima in the angular distribution of the emitted particles.

Experimental and theoretical study of the collision of heavy ions permits an approach to the solution of the problem existence of superdense nuclei.

Finally, one can hope to observe anomalous nuclei in cosmic rays, as noted already in the first paper on this subject.[7]

The possibility of observing in cosmic rays stable anomalous nuclei or their β-active fragments with anomalous Z/A, produced in interaction with the nuclei of the atmosphere, should be taken into account in the formulation and analysis of experiments. Thus, for example, a photoemulsion track attributed erroneously to the hitherto unobserved Dirac monopole should perhaps be interpreted as the track of an anomalous (neutron) nucleus. It is also of interest to search for superdense nuclei of cosmic origin, which have accumulated in the course of cosmological time in the surface layer of lunar soil and in meteorites.

5. Supercharged nuclei

A few remarks concerning the possible existence of supercharged nuclei. The idea of such nuclei in its initial form[8] was based on the following.

At $Ze^2/R > m_\pi c^2$, which corresponds to $Ze^3 \gtrsim 1$, it is possible for $\pi^+\pi^-$ condensation to occur in a supercharged nuclei, and the energy gain is larger than the nucleon Coulomb energy if Z is sufficiently larger than the critical value. As a result, such a nucleus may turn out to be unstable. In the field of such a nucleon, however, e^+e^- pairs are produced, the positrons move off to infinity, and the electrons are distributed inside and outside the nucleus, and screen the nuclear charge. The distribution of the vacuum electrons near a supercharged nucleus was obtained in[6]. It turned out that it precisely at $Ze^3 \sim 1$ that strong screening of the nuclear charge sets in, and at $Ze^3 \gg 1$ the proton charge is screened inside the nucleus in such a way that only the charge in a layer adjacent to the surface of the nucleus remain uncompensated. Thus, the Coulomb energy is strongly reduced by this screening. The electron kinetic energy, however, which is added to the system energy, makes such a nucleus unstable. A much larger energy gain is obtained when account is taken of one-pion condensation, wherein the proton charge is screened by π^--mesons. Even in this case, however, the system energy is still positive. The existence of supercharged nuclei is possible only if account is taken of the influence of the nucleon and if condensation is considered with a wave vector k_0 corresponding to the lowest pion energy $\tilde{\omega}^2(k_0)$.

If the critical density n is only slightly higher than n_0, then π^- condensation can occur also in stable nuclei.

In fact, consider for simplicity a nucleus with $N = Z$ the π^--meson energy in the nucleus is then determined from the equation

$$(1 + k^2 + \Pi(k, \tilde{\omega}) - \tilde{\omega}^2)\psi = 0,$$

where $k = (1/i)\nabla$, $\tilde{\omega} = \omega - V$.

Expanding $\Pi(k, \omega)$ in a series and confining ourselves to the first terms (this is permissible if $|V| < kV_F$), we get

$$\left[\left(1 - \frac{\partial\Pi}{\partial\omega^2}\right)\tilde{\omega}^2 - \tilde{\omega}^2(k^2)\right]\psi = 0, \tag{3.9}$$

where $\tilde{\omega}^2(k^2) = 1 + k^2 + \Pi(k, 0)$ has a minimum at $k = k_0$.

Multiplying (3.9) by ψ and integrating, we easily obtain

$$\omega^2 - 2\bar{V}\omega - \frac{\tilde{\omega}^2(k_0)}{1 - (\partial\Pi/\partial\omega^2)} + \overline{V^2} = 0. \tag{3.10}$$

The bar denotes averaging over ψ^2 (the integral of ψ^2 is normalized to unity). From (3.10) we have

$$\omega = \bar{V} + \sqrt{\bar{V}^2 - \overline{V^2} + \frac{\tilde{\omega}^2}{1 - (\partial\Pi/\partial\omega^2)}}.$$

It is easily seen that

$$\overline{V^2} - \bar{V}^2 \ll \bar{V}^2, \left(\bar{V} \approx -\frac{Ze^2}{R}{}^{2)}\right).$$

Neglecting this quantity and using by way of estimate

$$\tilde{\omega}^2 = \omega_0^2 + \varkappa \frac{\overline{(k^2 - k_0^2)^2}}{4k_0^2} \approx \omega_0^2 + \frac{C_1}{R^2},$$

where C_1 is a number of the order of unity, we have, using (2.17) for ω_0^2 (2.17)

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[1] Such an experiment was proposed by B. M. Pontecorvo in 1971 in a discussion of a paper by the author.[7]

[2] Assuming the amplitude ψ to be constant over the volume of the nucleus, we get $\overline{V^2} - \bar{V}^2 \approx 0.017\ \bar{V}^2$, $\bar{V} = -(6/5)Ze^2/R$.

$$\omega=\bar{V}+\sqrt{\frac{\alpha(n_c-n_0)+(C_1/R^2)}{1-(\partial\Pi/\partial\omega^2)}}.$$

The onset of π^- condensate begins at $\omega=0$. Therefore, at a sufficiently small excess of n_c over n_0 the π^- condensation can occur even in the region of stable nuclei if $|V|>\sqrt{C_1/1-(\partial\Pi/\partial\omega^2)}1/R$.

For nuclei with large change, instability to fission becomes significant. This instability can be eliminated if the Coulomb energy of the nucleus is greatly weakened. To this end, the π^--meson charge must be of the order of Z. Calculation shows that $Z_\pi\sim Z$ at $\Lambda\sim 1$ if $Ze^3\sim 1$.

Thus, an appreciable reduction of the Coulomb energy can make supercharged nuclei ($Ze^3\gtrsim 1$) stable. The question of the relation between the energy of these nuclei and the energy of superdense nuclei having the same charge remains open. We hope to deal with this question in the future.

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