# Charmonium and quantum chromodynamics

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The properties of levels of charmonium—the bound system consisting of the charmed quark $\bar{c}$ and antiquark $\bar{c}$—are considered. A brief review is given of the experimental data on the different levels of charmonium, and the classification of the states and their decays are discussed. Of the latter, radiative transitions between levels and the annihilation of levels of charmonium to give photons (or lepton pairs), and also light hadrons ($\pi$, $\eta$ and $K$ mesons), are paid the most attention. Such decays have fundamental significance, inasmuch as they are connected in the most direct manner with the properties of quarks and their interactions. The theoretical foundation of the review is quantum chromodynamics—the theory of the interaction of colored quarks and gluons. The review contains the results of calculations performed in the framework of quantum chromodynamics and pertaining to the annihilation decays of charmonium levels and also to other phenomena: photoproduction of charmed particles, leptonic decays of charmed particles, and nonleptonic decays of strange particles.

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## CONTENTS



## INTRODUCTION

This review is devoted to the theoretical interpretation of the properties of charmonium—a system of narrow hadron resonances with masses in the 3–4 GeV range. We shall discuss the classification of the levels of charmonium and their electromagnetic and strong decays. Lying at the base of the whole treatment is the hypothesis that charmonium consists of a charmed quark $c$ and a charmed antiquark $\bar{c}$, and the strong interactions of these quarks with each other and with other, lighter quarks are realized through exchange of gluons. According to the theoretical hypothesis, gluons are electrically neutral vector particles with zero mechanical mass. Both the quarks and the gluons possess specific charges (sources of the strong interaction), which have been given the name of color charges. The quarks exist in three color varieties and the gluons in eight.

The theory of the interaction of colored quarks and colored gluons—quantum chromodynamics—is still not completely worked out, and by no means all physicists working in the field of the theory of elementary particles regard it as a real candidate for the role of the final theory of the strong interactions. However, quantum chromodynamics, being, like quantum electrodynamics, a renormalizable theory, already explains at the present time a whole series of properties, both of charmonium and of ordinary hadrons. These properties pertain principally to short distances, less than or of the order of $10^{-14}$ cm. Today, the principal unsolved problem of quantum chromodynamics is the problem of the confinement of colored quarks and gluons from colorless hadrons. This problem (the trapping or confinement problem) is a large-distance (of the order of $10^{-13}$ cm) problem.

Further experimental and theoretical investigation of charmonium may lead to quantitative verification of certain predictions of quantum chromodynamics, and thereby to progress in the creation of a theory of the strong interaction.

The review is constructed as follows. In Chap. 1 the principal experimental data pertaining to charmonium are given, the concept of charm is briefly explained, and an introduction to quantum chromodynamics is given. In particular, it is explained how, in quantum chromodynamics, the strong interaction becomes weaker at short distances (in the literature, this property has been named asymptotic freedom). Chapters 2–4 are devoted to a description of the consequences of the nonrelativistic model of charmonium that treats the $c$ and $\bar{c}$ quarks in charmonium as heavy nonrelativistic particles situated in a potential with infinitely high walls. In Chap. 2 the widths of the annihilation of charmonium to give photons and ordinary hadrons are calculated, and in Chap. 3 the radiative transitions between the levels of charmonium are calculated. Whereas in Chaps. 2–3 the charmonium levels lying below 4 GeV are considered,

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these levels being described as the levels of an atom-like system, in Chap. 4 we consider the charmonium levels lying above 4 GeV. The latter are interpreted as molecular charmonium, consisting of two charmed hadrons, e.g., a $D$ and a $\bar{D}$ meson, each of which consists of a heavy and a light quark. In Chap. 5 the annihilation of charmonium is treated outside the framework of the nonrelativistic model, using such general properties of the theory as asymptotic freedom, unitarity and analyticity. The sum rules obtained here give a number of clear predictions for the widths of the charmonium levels and make it possible to determine the mass of the deep-virtual $c$-quark, which turns out to be equal to 1.25 GeV.

In Chap. 6 we briefly discuss the results of calculations performed within the framework of quantum chromodynamics but pertaining to other phenomena: the photoproduction of charmed particles, leptonic decays of charmed mesons, and nonleptonic decays of strange particles.

## 1. CHARMONIUM AND GLUONS

### a) Principal experimental facts

The discovery of charmonium was announced in November 1974 by two independent groups: MIT-BNL, led by Samuel Ting (see Ref. 1a), and SLAC-LBL, led by Burton Richter (see Ref. 1b). Both groups observed the same new particle, which was designated by letter $J$ by the first group, and by $\psi$ by the second. This discovery, which brought Ting and Richter the 1976 Nobel prize in physics, induced a chain reaction of brilliant experimental discoveries and very interesting theoretical studies.[1)]

Almost immediately after the discovery of $J/\psi$ it was realized[3] that this particle is just one of the levels (the most noticeable) of the system called charmonium. According to the theoretical hypothesis, charmonium is a bound system consisting of the so-called charmed $c$ quark and its antiquark $\bar{c}$. Theorists had suspected the existence of charmed quarks since 1964. The possibility that they exist was first discussed by Hara[4] and Bjorken and Glashow[5a] (cf. also Ref. 5b), who were attempting to construct a symmetric picture of four quarks $(u, d, s, c)$ and four leptons $(\nu_e, e, \nu_\mu, \mu)$. The need for a fourth quark became especially pressing after Glashow, Iliopoulos and Maiani[6] had shown that certain serious difficulties in the theory of the weak interactions of kaons could be solved with its help. The spin of the $c$-quark, like that of the other quarks, is equal to 1/2, and the charge is fractional: $Q_c = 2/3$ (the charges of the other quarks are respectively $Q_u = 2/3$, $Q_d = Q_s = -1/3$); the $c$ quark mass is large, of the order of 2 GeV.

The $J/\psi$ particle is the ground $^3S_1$ state of charmonium. (We use the usual spectroscopic notation $^{2S+1}L_J$, where $J$ is the total angular momentum of the system, composed of the orbital angular momentum $L$ and spin $S$.) The parity of this state is negative ($P = (-1)^{L+1} = -1$), and the charge-conjugation parity is also negative ($C = (-1)^{L+S} = -1$). We see that $J/\psi$ has the same quantum numbers as the photon: $J^{PC} = 1^{--}$; however, this particle is very massive: $M_{J/\psi} = 3095 \pm 4$ MeV. It is much more massive than all the other mesons that were known before the discovery of $J/\psi$.

However, the most striking characteristic of the $J/\psi$-meson is not so much its large mass as its small width. Its decay to hadrons is only an order of magnitude more intense than its decay to a lepton pair $e^+e^-$ or $\mu^+\mu^-$:

$$\Gamma_{\text{tot}}^{J/\psi} = 69 \pm 7\,\text{keV}, \Gamma(J/\psi \to e^+e^-) = 5.0 \pm 0.4\ \text{keV}.$$

The decay to a lepton pair occurs as a result of the electromagnetic interaction (Fig. 1). There exist no selection rules forbidding decays of the type "$J/\psi \to$ hadrons" by way of the strong interaction. However, the normal widths of the strong decays of heavy mesons are at least three orders of magnitude greater than the value of the $J/\psi$ width. Thus, in $J/\psi$ decays a very distinctive, very weak form of the strong interaction is manifested. There are serious theoretical reasons to suppose that further study of this form of the strong interaction, in combination with the other forms that manifest themselves in other properties of charmonium, may lead in the final analysis of the construction of a complete theoretical scheme of the strong interaction.

The next level of charmonium, $\psi'$, was discovered at SLAC[7] ten days after the discovery of $J/\psi$. Like $J/\psi$, this level appears as a very narrow resonance in the $e^+e^-$-annihilation cross-section. The mass of $\psi'$ is $3684 \pm 5$ MeV, its width is $\Gamma_{\text{tot}} = 228 \pm 56$ keV, $\Gamma(\psi' \to e^+e^-) = 2.1 \pm 0.3$ keV, and its quantum numbers are $J^{PC} = 1^{--}$. In the framework of the charmonium model, $\psi'$ is the $2\,^3S_1$ state, where the 2 signifies that this is the first radial excitation of the $^3S_1$ state. The observed decays of $\psi'$ can be divided into four basic classes:

a) the decays $\psi' \to e^+e^-$ and $\psi' \to \mu^+\mu^-$, with relative probability $B \sim 1\%$ for each of these decays;

b) strong decays into ordinary hadrons (principally to $\pi$- and $K$-mesons),[8] with $B \sim 10\%$;

c) strong decays with the production of $J/\psi$ in the final state:

$$B(\psi' \to J/\psi\pi\pi) \approx 49\% \text{ (see[9])},$$
$$B(\psi' \to J/\psi\eta) \approx 4\% \text{ (see[10])};$$

d) electromagnetic cascade transitions with emission of two photons, or of one photon and hadrons. (Some of these transitions also lead to $J/\psi$ in the final state.) These decays form a fraction $B \sim 30\%$.

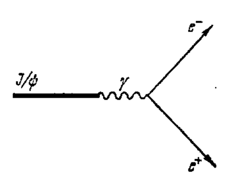



FIG. 1. Decay of a $J/\psi$-meson to an electron-positron pair.

[1)]For an early review of these investigations, see Ref. 2a.

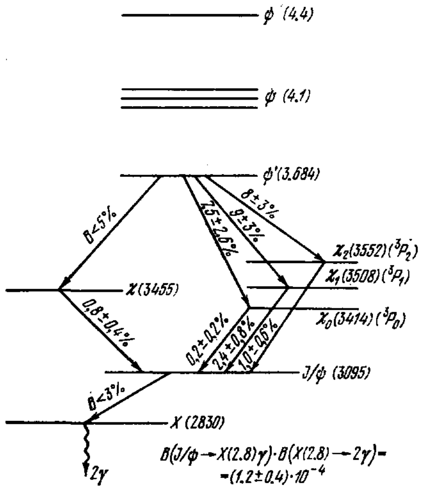



FIG. 2. Family of levels of charmonium and radiative transitions between them. [For the decays $\psi' \to \chi\gamma$ the relative probabilities $B(\psi' \to \chi\gamma)$ are given. For the decays $\chi \to J/\psi\gamma$ the products $B(\psi' \to \chi\gamma)B(\chi \to J/\psi\gamma)$ are given.]

The most interesting of the decays of $\psi'$ are the electromagnetic transitions (d), which were first observed at DESY.[11]

These transitions revealed the existence of a group of charmonium levels called $\chi$ particles. The properties of some of the $\chi$ particles are now very reliably known, but a whole series of experimental problems, concerning not only the identification of all these levels but even the question of the existence of some of them, still remain unsolved. It was predicted theoretically that three triplet $P$ levels should exist between $J/\psi$ and $\psi'$: $^3P_0$, $^3P_1$ and $^3P_2$, with quantum numbers $J^{PC}=0^{++}$, $1^{++}$ and $2^{++}$. These states have positive $C$-parity, and, consequently, the decays $\psi' \to {}^3P_J + \gamma$ and ${}^3P_J \to J/\psi + \gamma$, which are very similar to radiative transitions in ordinary atoms, are possible. The most likely correspondence between the quantum numbers and the $\chi$ levels is as follows[12]:

$$
\begin{gathered}
0^{++} \leftrightarrow \chi_0\,(3415),\\
M = 3414 \pm 4\ \text{MeV},\\
1^{++} \leftrightarrow \chi_1\,(3500),\\
M = 3508 \pm 4\ \text{MeV},\\
2^{++} \leftrightarrow \chi_2\,(3550),\\
M = 3552 \pm 6\ \text{MeV}.
\end{gathered}
$$

The experimental numbers for the relative probabilities of the radiative transitions of the charmonium levels are indicated in Fig. 2. Two more groups of particles are represented in the same figure. First, represented in the left-hand side of the scheme are the two levels $X(2830)$ and $\chi(3455)$, which are candidates to be $1\,{}^1S_0$ (also called $\eta_c$) and $2\,{}^1S_0$ ($\eta_c'$), respectively, of para-charmonium, with the quantum numbers $J^{PC}=0^{-+}$. Secondly, in the upper part of the scheme the two structures $\psi(4100)$ and $\psi(4400)$, with the quantum numbers of the photon, $J^{PC}=1^{--}$, are indicated. The rise in the $e^+e^-$-annihilation cross section at energy 4.1 GeV is not described by a Breit-Wigner curve. It is very plausible that the corresponding structure is a manifestiation of three superimposed resonances. The electron widths of each of them are about 0.5 keV.[13] The maximum cross-section is found at energy 4.028 GeV.

We shall first make some remarks about the candidates for para-charmonium with $J^{PC}=0^{-+}$. The state $X(2830)$ was observed at DESY,[14] but its existence is not yet confirmed by the SLAC-LBL group. It was expected theoretically that the mass difference $M_{J/\psi} - M_{\eta_c}$ should be several times smaller than 250 MeV. If, nevertheless, $X(2830)$ is indeed the $\eta_c$ meson, then, from the theoretical estimates, the relative probability of the decay $J/\psi \to X(2830) + \gamma$ should be much greater than the upper bound obtained at SLAC.[8] As regards $\chi(3455)$, three or four events, corresponding to the cascade

$$
\psi' \to \chi(3445) + \gamma,\quad \chi(3445) \to J/\psi + \gamma,
$$

have been observed at SLAC,[15] and, possibly, one event at DESY. The problem with the interpretation of $\chi(3455)$ as $\eta_c'$ is that the experimentally observed relative probability of the decay $\chi(3455) \to J/\psi + \gamma$ is two orders of magnitude greater than the theoretical estimates. The problem of para-charmonium is treated in more detail in Chap. 3.

We turn now to the states $\psi(4.1)$ and $\psi(4.4)$. Although the electron widths of these levels are of the same order as that of $\psi'$:

$$
\Gamma\,(\psi\,(4.1) \to e^+e^-) \approx 2\ \text{keV},\ \Gamma\,(\psi\,(4.4) \to e^+e^-) = 0.44 \pm 0.14\ \text{keV}\ (\text{see}^{[14]}),
$$

their total widths are three orders of magnitude greater than those of $J/\psi$ and $\psi'$:

$$
\Gamma_{\text{tot}}(\psi\,(4.1)) \sim 150\ \text{MeV},\ \Gamma_{\text{tot}}\,(\psi\,(4.4)) = 33 \pm 10\ \text{MeV}.
$$

As already mentioned, it is highly probable that $\psi(4.1)$ is a superposition of several resonances. The theoretical interpretation of $\psi(4.1)$ and $\psi(4.4)$ is a long way from complete certainty, but the large total widths of these resonances are in agreement with the theoretical expectations. The principal difference between $\psi(4.1)$ and $\psi(4.4)$, on the one hand, and $J/\psi$ and $\psi'$, on the other, is that new decay channels are opened for the former, namely, channels for decays to pairs of charmed mesons.

The existence of charmed mesons was already predicted in the first paper by Bjorken and Glashow.[5] A charmed meson is a bound state of the charmed quark $c$ with one of the light antiquarks. For the pseudoscalar ($J^P=0^-$) and vector ($J^P=1^-$) mesons the notation in Table I is used. $D$-mesons were discovered at SLAC in 1976 from their creation in $e^+e^-$-annihilation and their

TABLE I.

| Quark Composition | $c\bar{u}$ | $c\bar{d}$ | $c\bar{s}$ | $\bar{c}u$ | $\bar{c}d$ | $\bar{c}s$ |
|---|---|---|---|---|---|---|
| Pseudoscalars | $D^0$ | $D^+$ | $F^+$ | $\bar{D}^0$ | $D^-$ | $F^-$ |
| Vectors | $D^{*0}$ | $D^{*+}$ | $F^{*+}$ | $\bar{D}^{*0}$ | $D^{*-}$ | $F^{*-}$ |

subsequent decays to $K^{\mp}\pi^{\pm}$, $K^{\mp}\pi^{\pm}\pi^{\pm}\pi^{\mp}$ and $K^{\mp}\pi^{\pm}\pi^{\pm}$. The $D$-meson masses were found to be equal to[16,17] $M_{D^0} = 1865 \pm 15$ MeV and $M_{D^+} = 1876 \pm 15$ MeV. Thus, $\psi(4.1)$ and $\psi(4.4)$ are above the threshold for production of a $D\overline{D}$ pair. In the decays of $\psi(4.1)$ and $\psi(4.4)$ to $D\overline{D}$ the $c$-quarks are conserved and, therefore, these decays are not suppressed, unlike the decays of $J/\psi$ and $\psi'$, which are below the $D\overline{D}$-production threshold. In the latter case the quark pair $c\overline{c}$ must be annihilated in the decay process. There are also experimental proofs of the existence of vector $D^*$ mesons. The masses of these particles amount to about 2.01 GeV, and they decay through the strong interaction to $\pi D$ and through the electromagnetic interaction to $\gamma D$.

It is possible that the structure in the $e^+e^-$ annihilation in the 4-GeV region is due to $P$-wave resonances in the $D\overline{D}$, $D^*\overline{D}^*$ and $D^*\overline{D}$-$D\overline{D}^*$ systems (Voloshin and Okun'[18]). We call such objects charmonium molecules and consider them later. In particular, there are reasons to suppose that the peak at 4.028 GeV is the $D^*\overline{D}^*$ molecule.[19]

According to the theoretical scheme, charm is conserved in the strong and electromagnetic interactions; therefore, the decays of the $D$ and $F$ mesons should occur via the weak interaction. Since the weak interactions do not conserve parity, parity-nonconservation effects should be manifested in the decays. A weighty proof of parity violation in $D$-meson decays was obtained at SLAC[20] with the aid of a Dalitz plot for the decays $D^{\pm} \to K^{\mp}\pi^{\pm}\pi^{\pm}$.

Semileptonic decays of $D$ mesons have been observed at DESY.[21] Thus, on this question the agreement between theory and experiments is billiant. We shall not discuss the weak interactions further, since the principal subject of this review is the strong and electromagnetic properties of charmonium.

To conclude this section we list the principal facts proving the existence of the new quantum number—"charm":

1) the narrow resonances $J/\psi$ and $\psi'$;

2) the broad peaks in the 4 GeV region;

3) the intermediate $\chi$ levels;

4) $D$ mesons, decaying with nonconservation of parity;

5) excited states—$D^*$ mesons.

The behavior of the ratio

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

is also in agreement with the idea of "charm"; near 4 GeV this ratio undergoes an appreciable "jump."

A detailed discussion of the experimental data and also an exhaustive list of references are given in the reviews.[22,23]

Below we attempt to describe the positions of the levels of charmonium and the widths of their electromagnetic and hadronic decays. The basis of our discussion is the theory of strongly interacting quarks and gluons—so-called quantum chromodynamics (QCD).

### b) Quantum chromodynamics (QCD)

The term "chromodynamics," coined by Gell-Mann, refers to the fundamental property of quarks that we are now about to discuss, namely, color. The concept of "color" arose almost as long ago as the concept of "charm." It was first introduced into the theory[24–27] in order to resolve the well-known paradox of the parastatistics of quarks: the three identical fermions ($s$-quarks) forming the $\Omega^-$ hyperon are in the same state.

According to the color hypothesis, each quark exists in three varieties, all the properties of which are completely identical, except for one. The varieties differ only in the value of a certain new quantum number, which Gell-Mann later called color. It is convenient to introduce the colors red, blue and green. In this terminology all the existing hadrons can be called white—colorless. The baryons consists of quarks of three different colors; e.g., $\Omega^- = \varepsilon_{\alpha\beta\gamma} s^\alpha s^\beta s^\gamma/\sqrt{6}$, where $\varepsilon_{\alpha\beta\gamma}$ is a completely antisymmetric tensor, and $\alpha, \beta, \gamma = 1, 2, 3$ are the color indices. Thus, the Pauli principle is restored. The mesons are white states of a quark and an antiquark; e.g., $\pi^+ = \overline{d}_\alpha u^\alpha/\sqrt{3} = (\overline{d}_1 u^1 + \overline{d}_2 u^2 + \overline{d}_3 u^3)/\sqrt{3}$.

Such a structure of the hadronic states implies the existence of a new symmetry group $SU(3)'$ (where the prime distinguishes the color group from the usual $SU(3)$ flavor group). Unlike flavor $SU(3)$, the group $SU(3)'$ is not an approximate but an exact symmetry group. The quarks form an $SU(3)'$ triplet, while the hadrons are singlets. By analogy with quantum electrodynamics it is assumed that the forces acting between the quarks are due to the exchange of massless vector particles, called gluons. With respect to the quark-flavor group the gluons are singlets, and, in particular, are electrically neutral.

In QCD the interaction is determined by the color $SU(3)'$-charge, in exactly the same way as it is determined by the electric charge in QED. The distinguishing feature of QCD is that the gluons themselves possess color charge, while the photon is electrically neutral. This means that the gluons are directly coupled with each other, and, thus, the equations of the gluon fields are nonlinear. A theory of such a type was first proposed by Yang and Mills in 1954.[28]

We proceed now to describe the Lagrangian of the theory. It has the following form:

$$L = \sum_q \overline{q}\,(i\gamma_\mu D_\mu - m_q)q - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}; \tag{1.1}$$

here $q$ denotes the quark field and the sum is taken over all flavors: $q = u, d, s, c, \ldots$. As regards the color degrees of freedom, these are implied in this notation; e.g., $\overline{q}q = \overline{q}_\alpha q^\alpha$ ($\alpha = 1, 2, 3$). $D_\mu$ denotes a covariant derivative:

$$D_\mu q = \left(\partial_\mu - \frac{i}{2} g \lambda^a b^a_\mu\right) q,$$

where $g$ is the universal quark-gluon coupling constant

$(g^2/4\pi \equiv \alpha_s)$, $b^a_\mu$ is the gluon field $(a = 1, 2, \ldots, 8)$, and the $\lambda^a$ are the usual Gell-Mann $SU(3)$ matrices:

$$\mathrm{Sp}\,(\lambda^a\lambda^b) = 2\delta^{ab},\quad [\lambda^a, \lambda^b] = 2if^{abc}\lambda^c;$$

here the $f^{abc}$ are the structure constants of the group $SU(3)$: they are completely antisymmetric in all three indices and satisfy the relation

$$f^{abc}f^{abd} = 3\delta^{cd}.$$

The stress tensor $G^a_{\mu\nu}$ of the gluon field is defined as follows:

$$G^a_{\mu\nu} = \partial_\mu b^a_\nu - \partial_\nu b^a_\mu + gf^{abc}b^b_\mu b^c_\nu.$$

The Lagrangian (1.1) is invariant under a gauge transformation of the form

$$q \to Sq,\quad \lambda^a b^a_\mu \to S^{-1}\lambda^a b^a_\mu S + \frac{2i}{g} S^{-1}\partial_\mu S,$$

where $S$ is an arbitrary unitary $(SS^+ = 1)$ and unimodular $(\det S = 1)$ matrix, depending on the space-time coordinates.

The gauge invariance implies that the vector field $b^a_\mu$ contains an unobservable part and to quantize the theory it is necessary to eliminate the unphysical components or else fix them. An example of this elimination is given by the Coulomb gauge. In this gauge the condition of three-dimensional transversality is imposed on the field $b^a_\mu$:

$$\partial_m b^a_m = 0 \qquad (m = 1, 2, 3).$$

Thus, there remain only two spatial components $(b^a_m)_\perp$, corresponding to two polarization states of the gluon. As regards the time component $b^a_0$, the equation for it does not contain any time derivatives and $b^a_0$ can be expressed in terms of $(b^a_m)_\perp$, the corresponding conjugate canonical momenta $(\pi^a_m)_\perp$ and the quark fields. After this quantization is obvious.[2)]

The so-called axial gauge gives another example of a description free from unphysical degrees of freedom. This gauge is defined by the condition $b^a_3 = 0$. However, in calculations it is convenient to use an explicitly Lorentz-covariant description. For this it is necessary to introduce unphysical fields.[30] The covariant gauge is fixed by adding a gauge-noninvariant term $(1/2\xi)(\partial_\mu b^a_\mu)^2$ to the Lagrangian. In the case of QED such a modification of the Lagrangian does not spoil the theory, since the contributions of the unphysical longitudinal and time-like photons cancel each other. As was pointed out by Feynman,[30] in the case of a Yang-Mills field the situation is different and the introduction of such a term into the Lagrangian leads to violation of unitarity. Therefore, in order to eliminate, in its turn, the unphysical contribution violating the unitarity, the auxiliary fields

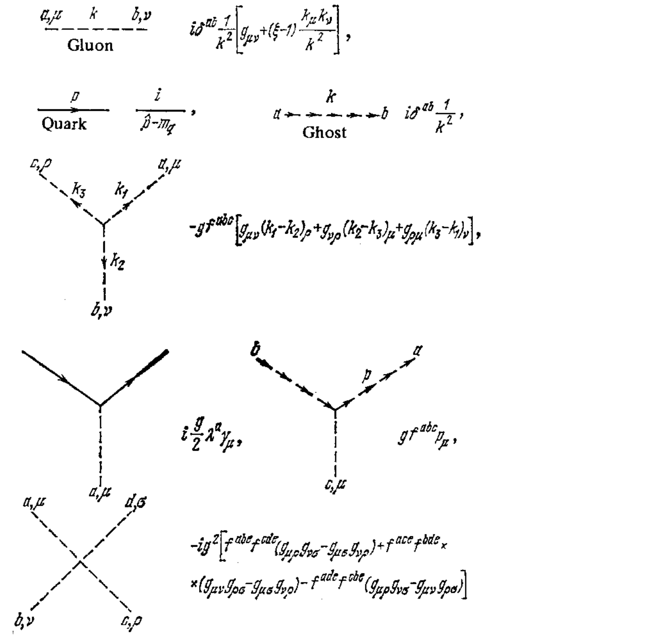


FIG. 3. Feynman rules for quantum chromodynamics.

of Faddeev, Popov and de Witt[31] are introduced. With the aid of continuous integrals it was shown[31,32] that addition of the term

$$\Delta L = -\frac{1}{2\xi}(\partial_\mu b^a_\mu)^2 + \partial_\mu\varphi^{*a}(\partial_\mu\varphi^a + gf^{abc}b^b_\mu\varphi^c) \qquad (1.2)$$

to the Lagrangian does not change the physical sector of the theory. In the quantization of the fields $\varphi$ anticommutators should be used, so that in the calculation of a diagram each closed loop with a $\varphi$-field gives a factor $(-1)$. The Feynman rules corresponding to $L + \Delta L$ (cf. (1.1) and (1.2)) are given in Fig. 3. It can be seen from Fig. 3 that the frequently used Landau gauge corresponds to the limit $\xi = 0$.

### c) Asymptotic freedom in QCD

In this section we try to explain how the change from one vector field (the case of QED) to a multiplet of vector fields (QCD) fundamentally alters the behavior of the interaction at short distances.[33,34]

We first recall the situation in QED. The problem of the interaction of two charges at short distances can be formulated as the problem of the relation between the bare and observable charges of a particle. In fact, the short-distance interaction studied in processes with large momentum transfers is determined by the bare charge, while the observed charge determines the coefficient in the Coulomb law at large distances.

The formulation given makes it possible to give a qualitative answer to the question of the influence of vacuum fluctuations. It is clear that the creation of a virtual electron-positron pair leads to a decrease of the initial charge, inasmuch as the latter attracts toward itself that component of the pair with charge opposite to its own. This screening effect was studied by Landau and Pomeranchuk[35] and led to the formulation of the famous

[2)]It should be noted in this connection, however, that for Yang–Mills fields of sufficiently high intensity even the Coulomb gauge does not fix the field uniquely (Gribov[29]). This effect, however, is unimportant in the framework of perturbation theory.


a b

FIG. 4. Electromagnetic interaction of two heavy particles (a) in lowest order in the coupling constant and (b) with allowance for the screening of the initial charges by a virtual electron-positron pair. [In the figure the dashed-dotted line denotes a "Coulomb" photon.]

zero-charge problem: any finite bare charge is screened to zero (see the excellent review by Berestetskiĭ[36]).

For what follows it is expedient to elucidate the screening of the charge in QED in the language of Feynman graphs. We shall consider the electromagnetic interaction of two heavy (test) charges. In lowest order this is described by the diagram of Fig. 4a:

$$M^{(0)}=\frac{4\pi\alpha}{q^2}\Gamma_\mu^{(1)}\Gamma_\mu^{(2)},$$

where $q$ is the momentum of the virtual photon, and $\Gamma_\mu^{(1,2)}$ are the electromagnetic vertices. From the conservation of the electromagnetic current follows the relation $q_0\Gamma_0=q_3\Gamma_3$ (we have chosen the $z$ axis in the direction of the three-dimensional momentum of the photon), which can be used to eliminate $\Gamma_3$:

$$M^{(0)}=\frac{4\pi\alpha}{q^2}\left[\Gamma_0^{(1)}\Gamma_0^{(2)}\left(1-\frac{q_0^2}{q_3^2}\right)-(\Gamma_1^{(1)}\Gamma_1^{(2)}+\Gamma_2^{(1)}\Gamma_2^{(2)})\right]$$
$$=-4\pi\alpha\left[\frac{1}{q_3^2}\Gamma_0^{(1)}\Gamma_0^{(2)}+\frac{1}{q^2}(\Gamma_1^{(1)}\Gamma_1^{(2)}+\Gamma_2^{(1)}\Gamma_2^{(2)})\right].$$

The first term is the Fourier transform of the Coulomb interaction. It may be said of this term that it has arisen from the exchange of a Coulomb quantum, if we remember, however, that a real "Coulomb" particle does not exist. This can be seen, in particular, from the fact that the imaginary part of the propagator $1/q_3^2$, unlike that of the term $(\Gamma_1^{(1)}\Gamma_1^{(2)}+\Gamma_2^{(1)}\Gamma_2^{(2)})/q^2$, is equal to zero. The latter term describes the exchange of a photon with transverse polarization and, in coordinate space, corresponds to the retarded interaction. In the case under consideration—that of heavy charges, it is obvious that the Coulomb part of the interaction is dominant.

The diagram of Fig. 4b describes the correction to the Coulomb interaction that arises from a virtual electron-positron pair. For the sum of the diagrams of Fig. 4 in the limit $|\mathbf{q}|^2\gg m_e^2$ we have the expression (in the center-of-mass frame)

$$M^{(0)}+M^{(1)}\approx M^{(0)}\left(1-\frac{\alpha_0}{3\pi}\ln\frac{\Lambda^2}{\mathbf{q}^2}\right)$$

where $\alpha_0$ is the bare constant corresponding to the cutoff parameter $\Lambda$. The negative sign of the correction is a consequence of unitarity and analyticity, since the absorptive part of the logarithmic correction, associated with the intermediate state $e^+e^-$, is positive. This statement about the sign corresponds precisely to the well-known theorem from quantum mechanics that the second-order correction always decreases the energy of the ground state.

From what has been said it is clear that the statement about the screening of the interaction is extremely general. A specific property of QCD, leading to a change in the sign, is the existence of vacuum fluctuations of a new type, not determined by the imaginary part of the amplitude (i.e., by real intermediate states). To elucidate where such contributions come from, we recall that in QCD the interaction is determined by the color, which plays a role analogous to that of the electric charge in QCD. Since the gluons form a color multiplet, i.e., they have nonzero color charge, they interact with each other. This means that a gluon, like a quark, is a source of the gluon field. Therefore, polarization of the gluon vacuum arises, described by the diagram of Fig. 5.

In these diagrams, following Khriplovich,[37] we have distinguished explicitly the contributions of the transversely-polarized gluons and of the Coulomb quanta. The appearance of both the "natural" vertex of the interaction of a Coulomb quantum with two transverse quanta, and a vertex at which two Coulomb quanta and one transverse quantum meet, is a "hallmark" of the nonabelian character of the theory.

The fact that the contribution of the diagram of Fig. 5a leads to screening can be seen from the same arguments as in QED. However, these arguments do not determine the sign of the diagram of Fig. 5b, since the imaginary part of this diagram is equal to zero. In fact, the Coulomb field does not correspond to the propagation of a physical particle (its propagator $1/\mathbf{q}^2$ has no imaginary part), and, consequently, the diagram of Fig. 5b does not have a section through physical states.

The fact that the diagrams of Figs. 5a and 5b have opposite signs can be seen from the following arguments. We cut the upper line, corresponding to a transverse gluon, in the loop in each graph of Fig. 5. The inner parts of these graphs are conveniently drawn in the form given in Fig. 6. The graph in Fig. 6a contains the exchange of a transverse gluon, while the graph in Fig. 6b describes the exchange of a Coulomb-like gluon. The answers for these graphs can be guessed by making use of the analogy with electrodynamics.

Namely, the exchange of a Coulomb quantum leads to the Coulomb law and the exchange of a transverse quantum leads to the Biot-Savart law for the interaction of currents, and two like charges repel while two parallel currents attract each other. Thus, we arrive at the conclusion that the diagrams of Fig. 6 give contributions of opposite signs; consequently, the diagram of Fig. 5a corresponds to screening while the diagram of Fig. 5b leads to antiscreening. The explicit answer is

a b

FIG. 5. Screening of the interaction of two color charges in quantum chromodynamics. [In the figure the dotted lines denote "Coulomb" gluons and the dashed line a transverse gluon.]

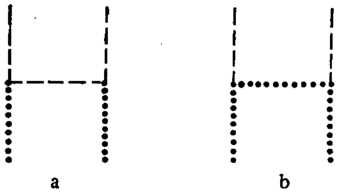


FIG. 6. Interaction associated with the exchange of (a) a transverse gluon and (b) a "Coulomb" gluon.

$$M = M^{(0)}\left(1 - \frac{\alpha_s^{(0)}}{4\pi}\ln\frac{\Lambda^2}{\mathbf{q}^2} + \frac{12\alpha_s^{(0)}}{4\pi}\ln\frac{\Lambda^2}{\mathbf{q}^2}\right), \tag{1.3}$$

where $\alpha_s^{(0)}$ is the bare strong-interaction constant. The second term in the right-hand side pertains to the diagram of Fig. 5a, and the third to the diagram of Fig. 5b.[37] Thus, the antiscreening contribution is twelve times greater than the screening contribution.

A "quark-antiquark" pair of each flavor adds the usual screening term

$$-\frac{2}{3}\frac{\alpha_s^{(0)}}{4\pi}\ln\frac{\Lambda^2}{\mathbf{q}^2}, \quad |\mathbf{q}| \gg \text{quark masses} \tag{1.4}$$

to the amplitude (1.3). The effecting coupling constant $\alpha_s(q^2)$ is defined in terms of the scattering amplitude for two heavy objects:

$$M(\mathbf{q}) = \frac{4\pi\alpha_s(\mathbf{q}^2)}{\mathbf{q}^2}\Gamma_\mu^{(1)}\Gamma_\mu^{(2)}; \tag{1.5}$$

here $\Gamma_\mu^{(1,2)}$ are the vertices describing the interaction of the color with the gluon field, and $\mathbf{q}$ is the momentum transfer. (N.B. In our discussion of the effective charge in QED we used the same notation $\Gamma_\mu^{(1,2)}$ for the electromagnetic vertices, which, of course, do not coincide with the vertices in QCD. In particular, the latter contain color indices, which are not written out explicitly in the expression (1.5). It is understood that they are incorporated in $\Gamma_\mu^{(1,2)}$.)

The resulting expression for the effective coupling constant in the logarithmic approximation has the form

$$\alpha_s(\mathbf{q}^2) = \alpha_s^{(0)}\left[1 + \left(11 - \frac{2}{3}N\right)\frac{\alpha_s^{(0)}}{4\pi}\ln\frac{\Lambda^2}{\mathbf{q}^2}\right], \tag{1.6}$$

where $N$ is the number of "operative" quark flavors. (The precise significance of the work "operative" will become clear somewhat later.)

When higher orders are taken into account a series in $[\alpha_s^{(0)}\ln(\Lambda^2/\mathbf{q}^2)]^n$ arises; the standard, and simplest, way of summing this series is given by the renormalization group.[38–40] It follows from the renormalizability of QCD that the quantity $d(\alpha_s(\mathbf{q}^2))/d\ln\mathbf{q}^2$, when expressed in terms of $\alpha_s(\mathbf{q}^2)$, does not contain the cutoff parameter $\Lambda$. Then it follows simply from dimensional considerations that $d\alpha_s(\mathbf{q}^2)/d\ln\mathbf{q}^2$ is a function of only the one argument $\alpha_s(\mathbf{q}^2)$. Thus, e.g., differentiating (1.6), we obtain

$$\frac{d\alpha_s(\mathbf{q}^2)}{d\ln\mathbf{q}^2} = -\left(11 - \frac{2}{3}N\right)\frac{\alpha_s^2(\mathbf{q}^2)}{4\pi}, \tag{1.7}$$

and direct integration gives

$$\alpha_s(\mathbf{q}^2) = \frac{\alpha_s(\mathbf{q}_0^2)}{1 + [11 - (2/3)N][\alpha_s(\mathbf{q}_0^2)/4\pi]\ln(\mathbf{q}^2/\mathbf{q}_0^2)} \to 0 \quad \text{as} \quad \mathbf{q}^2 \to \infty. \tag{1.8}$$

This is the celebrated asymptotic-freedom formula.[33,34]

We shall say a few words about effects arising from the quark masses. It is convenient to introduce, temporarily, the following terminology. We shall call quarks with mass $m^2 \gg \mathbf{q}^2$, $\mathbf{q}_0^2$ "heavy," and quarks with mass $m^2 \ll \mathbf{q}^2$, $\mathbf{q}_0^2$ "light." The contribution of the "heavy" quarks in formula (1.8) is negligibly small: it is suppressed by the power factor $\sim\mathbf{q}^2/m^2$. This fact seems almost obvious; a detailed proof can be found in Ref. 41. Thus, the parameter $N$ in (1.8) "counts" only the "light" quarks. Taking the mass of "light" quarks into account also leads to small power corrections $\sim m^2/\mathbf{q}^2$. Quarks with intermediate mass $m^2 \sim \mathbf{q}^2$ or $\sim\mathbf{q}_0^2$ give (nonlogarithmic) corrections of the order of $\alpha_s(\mathbf{q}^2 = m^2)$.

In conclusion we note that as $\mathbf{q}^2 \to \infty$ in the standard model with four quark flavors and three colors,

$$\alpha_s(\mathbf{q}^2) \sim \frac{1}{(25/12\pi)\ln(\mathbf{q}^2/\mathbf{q}_0^2)}.$$

### d) Confinement of quarks and gluons

The objects we are discussing—quarks and gluons—have never been observed in the free form. This fact may appear surprising, since everything suggests that quarks, at least deep inside hadrons, are light[42,43a]:

$$m_u + m_d \approx 10 \text{ MeV}, \quad m_s \approx 150 \text{ MeV}.$$

One of the most convincing arguments in favor of small mechanical (bare) masses for the quarks is the success of predictions based on chiral symmetry and, in particular, of the Weinberg sum rules,[43b] in the derivation of which the quark masses are neglected in comparison with the hadron masses. The description of deep-inelastic $eN$ and $\nu N$ scattering with momentum transfers $Q^2 \gtrsim 1$ GeV$^2$ using massless quark-partons is also in good agreement with the experimental data. Moreover, to realize the exact color gauge symmetry $SU(3)'$ the gluons must also be assumed to be massless. Thus, the reasons for the absence of quarks and gluons in experiment should be dynamical rather than kinematic, i.e., we must seek forces that keep colored objects constantly inside the hadrons.

In the literature there are a number of models (mainly of a descriptive character) of such quark-confinement mechanisms. A quantitative theory of confinement has not yet been constructed. Evidently, the formation of the hadrons and the confinement of quarks occurs at large distances. If we extend the effective-charge formula (1.8) describing the asymptotic freedom of quantum chromodynamics at short distances into the region of large distances, it will give an increase of the charge with increasing distance, and, finally, the charge will become not small but of order unity. In this region perturbation theory is no longer applicable (and neither is the effective-charge formula (1.8) itself). On the other hand, insofar as the value of the constant at short distances is known at present ($\alpha(m_\psi) = 0.2$), the distances at which perturbation theory is "spoiled" correspond exactly to the expected radius of confinement of the quarks. Although quantitative calculations in this region are absent, the universal hope is that the same theory—quantum chromodynamics—that describes so successfully the properties of hadrons at short distances will at large distances furnish forces confining all colored objects (quarks, gluons, diquarks, etc.).

The language most adequate to our physical intuition—that of a potential and forces acting between the quarks, pertains, essentially, only to the nonrelativistic potential model. Of the hadrons known at the present time it is evidently only for the levels of charmonium that these intuitive ideas can be tested as a dynamical model. It is not ruled out, therefore, that precisely the study of the properties of charmonium will enable us to lay a path from the intuitive expectations to a quantitative theory of quark confinement.

**e) Nonrelativistic potential model of charmonium**

So far as we can judge at present, the mechanism of quark confinement is due to the interaction of quarks at distances that are in any case greater than $(1\ \mathrm{GeV})^{-1}$ (at least, we know that, for momentum transfers $Q^2 \gtrsim 1$ $\mathrm{GeV}^2$ in, for example, inelastic $eN$-scattering, the quarks in the nucleon can be regarded as almost free). It is highly plausible that the range of the interaction confining the quarks amounts to $(0.5\ \mathrm{GeV})^{-1}$, or even $(2m_\pi)^{-1}$. We shall consider a meson, consisting of a quark and an antiquark. Since, roughly speaking, the radius $R$ of the system coincides with the confinement radius, we can estimate the characteristic momenta of the quark and antiquark from the uncertainty relation: $p \sim 1/R$. If the masses of the quarks constituting given hadrons are of the order of $1/R$, then only in a very crude approximation can the hadrons be treated nonrelativistically; this situation is realized in ordinary particles. However, if the quarks are sufficiently massive, a nonrelativistic picture becomes adequate for a dynamical description of the properties of a hadron. Inasmuch as the charmonium levels $(J/\psi, \psi', \chi)$ are extremely massive on the scale of the quantity $R^{-1}$, it may be thought that the latter possibility is realized in charmonium and that charmonium can be treated as a nonrelativistic system with a certain degree of accuracy. An additional argument in favor of the applicability of a nonrelativistic treatment is the experimental fact of the existence of an entire spectrum of charmonium states, the mass differences between which (200–600 MeV) are small compared with their masses (3–4 GeV).

One way or another, it makes direct sense to attempt to describe the observed properties of the levels of charmonium by means of a nonrelativistic potential approach.[3,43c] In the construction of such a nonrelativistic model an important role is played by the choice of the interaction potential binding the quarks. It is known that at short distances the interaction has almost the Coulomb form $V(r) \sim 1/r$, while at large distances, in order to ensure confinement, the potential should not tend to zero but should continue to increase. The rate of growth of the potential is not known—it is not even known whether this is a power-law growth and whether it continues to infinity. Most of the calculations in the literature[44–48] have been performed for a potential that grows linearly at large distances. Namely, the potential is chosen in the form

$$V(r) = -\frac{\tilde{\alpha}}{r} + gr + V_0, \tag{1.9}$$

where $\tilde{\alpha}$, $g$ and $V_0$ are adjustable parameters, for which the values given by different authors are close to the following:

$$\tilde{\alpha} = 0{,}27, \quad g = 0{,}25\ \mathrm{GeV}^2, \quad V_0 = -0{,}76\ \mathrm{GeV}.$$

(Fitting the quark mass for this model gives $m = 1.65$ GeV.) The graph of the potential (1.9) is drawn schematically in Fig. 7. Since the potential profile resembles a funnel, below we shall call it simply that. A linear growth of the potential with distance at large $r$ is predicted by, e.g., the string model. It is easy, however, to imagine other types of potential that ensure confinement, e.g., a potential with a steep wall at $r = R_0$, or a harmonic-oscillator potential. The latter potential differs pleasantly from the first two in that, if we neglect the Coulomb interaction of the quarks at short distances, all the wavefunctions and matrix elements can be calculated explicitly and have a simple form.

Below, therefore, we shall frequently invoke oscillator wavefunctions to estimate the order of magnitude of matrix elements. The results agree qualitatively, and in many cases also quantitatively, with those calculated in the "funnel" model by numerical solution of the Schrödinger equation.

The principal properties of the three-dimensional harmonic oscillator are described in the book of problems by Flugge.[49] We write the oscillator potential in the form

$$V(r) = \frac{\tilde{m}\omega_0^2 r^2}{2},$$

where $\tilde{m} = m/2$ is the reduced mass of the system ($m$ is the mass of the $c$ quark). Both $\tilde{m}$ and $\omega_0$ must be treated as adjustable parameters. For example, the energy parameter $\omega_0$ can be chosen in order to reproduce the mass difference between the 2S and 1S states (the $\psi'$ and $J/\psi$ mesons); then $\omega_0 \simeq 300$ MeV. Analysis of the decay $J/\psi \to e^+e^-$ gives $\tilde{m}\omega_0 \simeq 0.35\ \mathrm{GeV}^2$, and, consequently, $\tilde{m} = 1.17$ GeV, or $m \simeq 2.3$ GeV. (In the case of the funnel, $m = 1.65$ GeV.)

In the following two sections we proceed to a systematic description of the nonrelativistic model of charmonium. As will be seen from the following, a developed qualitative picture of a "world of hidden charm" arises. The principal properties of the whole spectrum of levels

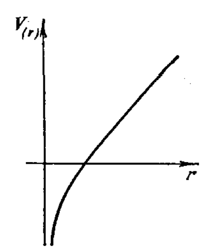



FIG. 7. Hypothetical "funnel"-type potential of the interaction of the $c$- and $\bar{c}$-quarks.

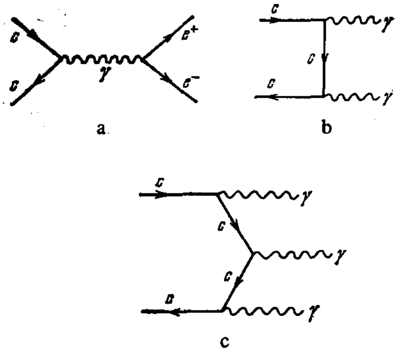


FIG. 8. Processes responsible for the annihilation of charmonium (a) to an electron-positron pair, (b) to two photons, and (c) to three photons.

become comprehensible. It may be thought that the potential approach, being the first, rather crude, but necessary step, will play a role in the construction of the future theory of confinement.

## 2. CHARMONIUM ANNIHILATION IN QUANTUM CHROMODYNAMICS. THE NONRELATIVISTIC APPROACH

As already mentioned above, charmonium is the most nonrelativistic of all the quark systems known at the present time. It is natural, therefore, that the first calculations of the probabilities of the decays of charmonium were performed in the framework of a potential model. In other words, the simple nonrelativistic problem of a bound state of two objects—a quark $c$ and an antiquark $\bar{c}$, with interaction characterized by some potential, e.g., an oscillator or a funnel, was solved. It is clear that any calculation of such a kind cannot pretend to high accuracy and is essentially approximate.

In this section the widths of the electromagnetic and hadronic annihilations of charmonium levels are calculated. In the potential model the widths are expressed in terms of $R(r\to 0)$, where $R(r)$ is the radial part of the wavefunction of $c$ and $\bar{c}$, and $r$ is the relative distance. The procedure reduces essentially to the following. First the amplitude is found for a transition of a pair of free quarks at rest, e.g., $c\bar{c}\to e^+e^-$ (Fig. 8a), and this amplitude is then converted to a probability by multiplying by $|R(0)|^2$ for $S$-wave decays, by $|R'(0)|^2$ for $P$-wave decays, and so on.

The electromagnetic-annihilation widths are calculated most reliably. We shall consider, e.g., the decay $J/\psi\to e^+e^-$, described by the diagram of Fig. 8a.

The matrix element for the transformation of a $J/\psi$ meson to a virtual photon has the form $Q_c e\langle 0|j_i|\psi_j\rangle$, where $Q_c$ is the charge of the $c$ quark. If we introduce the spin function of the triplet state of $c$ and $\bar{c}$: $\chi=(1/\sqrt{2})\bar{c}\sigma c$, where $c$ is the nonrelativistic spinor describing the $c$ quark, the electromagnetic current has the form $\mathbf{j}=-\sqrt{2}\,\chi$, and the vector wavefunction of the $J/\psi$ meson has the form $\psi=\chi\psi_S(p)$. The subscript $S$ denotes the fact that $c$ and $\bar{c}$ are in an $S$ wave in the $J/\psi$ meson, and $\mathbf{p}$ is the relative momentum of the $c$ and $\bar{c}$ quarks. As a result we obtain

$$Q_c e\langle 0|j_i|\psi_j\rangle=\sqrt{2}\,\delta_{ij}Q_c e\int\psi_S(\mathbf{p})\frac{d\mathbf{p}}{(2\pi)^3}=\delta_{ij}Q_c e\psi_S(r=0)\sqrt{2}\sqrt{3}, \tag{2.1}$$

where $\psi_S(0)=R_S(0)/\sqrt{4\pi}$ and the factor $\sqrt{3}$ arises from taking the color into account. The corresponding expression for the probability of the decay $J/\psi\to e^+e^-$ has the form

$$\Gamma(1^3S_1\to e^+e^-)=\frac{4\alpha^2Q_c^2}{M^2}|R_S(0)|^2; \tag{2.2}$$

here $R_S$ is the radial part of the $S$-wave $\psi$ function, normalized by the condition $\int R_S^2(r)r^2\,dr=1$, and $M$ is the meson mass.

In the nonrelativistic approximation we neglect the difference between the mass $M$ of the level and the sum of the masses of the $c$ and $\bar{c}$ quarks. Therefore, in this approximation the same mass $M$—a certain average mass of the nonrelativistic charmonium—appears in the expressions for the widths of different levels of charmonium. This leads to uncertainty of the order of a factor of 2 in the theoretical predictions.

Two-photon and three-photon decays of levels of charmonium are described by the diagrams of Figs. 8b and 8c, respectively, and are calculated in an analogous way[3,44,50–54]:

$$\frac{\Gamma(J/\psi\to 3\gamma)}{\Gamma(J/\psi\to e^+e^-)}=\frac{\Gamma(\psi'\to 3\gamma)}{\Gamma(\psi'\to e^+e^-)}=\frac{4(\pi^2-9)}{3\pi}\alpha Q_c^4=5.32\cdot 10^{-4}, \tag{2.3}$$

$$\frac{\Gamma(\eta_c\to 2\gamma)}{\Gamma(J/\psi\to e^+e^-)}=3Q_c^2=4/3, \tag{2.4}$$

$$\frac{\Gamma(\chi^0\to 2\gamma)}{\Gamma(\eta_c\to 2\gamma)}=9\left(\frac{R'_{1P}(0)}{mR_{1S}(0)}\right)^2\approx 0.5, \tag{2.5}$$

$$\frac{\Gamma(\chi_2\to 2\gamma)}{\Gamma(\chi_0\to 2\gamma)}=\frac{4}{15}, \tag{2.6}$$

$$\frac{\Gamma(1^3D_1\to e^+e^-)}{\Gamma(J/\psi\to e^+e^-)}\approx 50\left(\frac{R''_{1D}(0)}{M^2R_{1S}(0)}\right)^2\approx 2.5\cdot 10^{-2}, \tag{2.7}$$

$$\frac{\Gamma(^1D_2\to 2\gamma)}{\Gamma(\eta_c\to 2\gamma)}=\left(\frac{R''_{1D}(0)}{m_c^2R_{1S}(0)}\right)^2\approx 10^{-2}. \tag{2.8}$$

The numerical estimates in the formulas (2.5), (2.7) and (2.8) containing ratios of wavefunctions are given for an oscillator potential, for which

$$\frac{R'_{1P}(0)}{R_{1S}(0)}=\frac{m_c\omega_0}{3},\quad \frac{R''_{1D}(0)}{R_{1S}(0)}=\frac{4}{15}(m_c\omega_0)^2. \tag{2.9}$$

The values of the parameters $m_c$ and $\omega_0$ were given in Chap. 1.

One should note the unexpectedly large ratio of the probabilities of the two-photon annihilations of the $\chi_0$ and $\eta_c$ mesons (cf. (2.5)), which is of order unity despite the fact that the former corresponds to annihilation of $c$ and $\bar{c}$ in a $P$ wave and the latter to annihilation in an $S$ wave.

Of the decays cited above, only for two are the widths known experimentally: these are $\Gamma(J/\psi\to e^+e^-)=5$ keV and $\Gamma(\psi'\to e^+e^-)=2$ keV. In the oscillator model their widths should be in the ratio 2 : 3. The suppression of the decay of $\psi'$ could have several causes: a large admixture of the $^3D_1$ state of the $c$ and $\bar{c}$ quarks in the wavefunction of the $\psi'$ meson, a large admixture of $D\bar{D}$ mesons in this wavefunction, and, finally, the chief one: the oscillator potential may be very unlike the true potential between the two quarks.

It is possible that the decay $X(2.83)\to 2\gamma$ that was observed at DESY is the decay $\eta_c\to 2\gamma$. However, in this case, only a lower bound for the relative width is known: $B(X(2.83)\to 2\gamma)>4\times 10^{-3}$. This quantity, as will be shown, is substantially higher than the theory predicts. As regards the remaining electromagnetic annihilation decays, these have not yet been observed experimentally.

We turn to the discussion of the hadronic widths of the charmonium levels. In the framework of QCD the annihilation of charmonium to give ordinary hadrons corresponds to the following picture. First, a $c\bar{c}$ pair, at short distances of the order of the Compton wavelength of the $c$ quarks, is transformed into gluons (Fig. 9), which are then transformed, at large distances of the order of the confinement radius, into observable particles—$\pi$ mesons, $K$ mesons, nucleons, etc. We do not know the mechanism of the transformation, and we can say practically nothing about the relative probability of a decay along any one or other exclusive channel. Nevertheless, the total probability can be found. As was noticed by Appelquist and Politzer,[3] the total width of the charmonium annihilation to ordinary hadrons should be approximately equal to the probability of the transition to gluons. This prescription will certainly not surprise the reader who is familiar with the parton model and has become accustomed to the fact that the probability of the transition of a parton to hadrons is assumed to be equal to unity. In the present case the gluons appear in the role of the partons.

Quantum chromodynamics has inherited this property of the parton model. Thus, e.g., in Chap. 5 we show that the cross-section

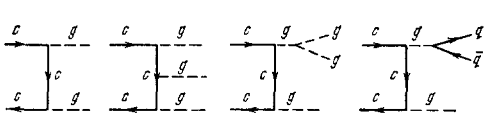
$$\sigma\,(e^+e^-\to\gamma^*\to \text{ordinary hadrons}) \tag{2.10}$$

at high energies $E=\sqrt{s}$ coincides with the corresponding quark cross section

$$\sigma\,(e^+e^-\to\bar{u}u+\bar{d}d+\bar{s}s)=\frac{4\pi\alpha^2}{S}\,(Q_u^2+Q_d^2+Q_s^2) \tag{2.11}$$

to within a small correction $\sim\alpha_s(s)$. The Appelquist-Politzer prescription, which states that the probability of, say, "$J/\psi\to$ hadrons" coincides with the probability of "$J/\psi\to 3$ gluons" to within $\alpha_s(4m_c^2)$, is the direct generalization. The difference reduces to the fact that gluons appear in place of the quarks, and the role of the "external source" is played by the $c\bar{c}$ pair in place of the virtual photon.

Thus, the problem consists in calculating the annihilation of the charmonium levels to the minimum kinematically allowed number of gluons. For the $\eta_c$, $\chi_0$ and



a b c d

FIG. 9. Diagrams describing the annihilation of the system $c\bar{c}$ (a) to two gluons, (b, c) to three gluons, and (d) to a pair of light quarks $q\bar{q}$ and a gluon. [Here and below a gluon is denoted by the letter $g$ and is depicted by a dashed line.]

$\chi_2$ mesons and the $^1D_2$ level, these are two-gluon decays, and for the $J/\psi$ and $\psi'$ mesons they are decays into three gluons (see Figs. 9a and 9b). The meson $\chi_1\equiv(^3P_1)$ is a special case, which we shall discuss a little later.

It is not difficult to calculate the gluon widths, since we already know the expressions for the widths of the two-photon and three-photon annihilations, and the corresponding diagrams are similar. To carry out the conversion it is necessary to replace $Q_ce$ at each vertex by $g\lambda^a/2$, where the $\lambda^a$ $(a=1,2,\ldots,8)$ are the Gell-Mann $SU(3)$ matrices and $g^2=4\pi\alpha_s$, just as $e^2=4\pi\alpha$. The replacement "2 photons $\to$ 2 gluons" corresponds to the factor[50,52]

$$\frac{9\alpha_s^2}{8\alpha^2}\approx 845, \tag{2.12}$$

and the replacement "3 photons $\to$ 3 gluons" to the factor[3,44]

$$\frac{135\alpha_s^3}{128\alpha^3}\approx 2.17\cdot 10^4. \tag{2.13}$$

The annihilation of $c\bar{c}$ to gluons occurs at distances of the order of the Compton wavelength of the $c$ quark, so that the effective quark-gluon constant appearing in the relations (2.12) and (2.13) corresponds to distances $\sim 1/m_c$, or, which is the same thing, to virtual momenta $p^2\simeq -m_c^2$. The numerical estimates in (2.12) and (2.13) are given for the value $\alpha_s=0.2$ obtained from analysis of the $J/\psi$ decays. In fact,[3,44]

$$\frac{\Gamma\,(J/\psi\to \text{hadrons})}{\Gamma\,(J/\psi\to e^+e^-)}=\frac{5}{18}\,\frac{(\pi^2-9)}{\pi}\,\frac{\alpha_s^3}{\alpha^2}. \tag{2.14}$$

(Here direct annihilation, not via a virtual photon, of $J/\psi$ to hadrons is meant.) In experiment this ratio is approximately equal to 10, whence follows the value $\alpha_s\simeq 0.2$.

For the widths of the annihilations of the $\eta_c$, $\chi_0$ and $\chi_2$ mesons and the $^1D_2$ level to hadrons we obtain the values 6 MeV, ~3 MeV, ~0.8 MeV and ~60 keV, respectively.

The $^1P_1$ level, having negative $C$-parity, does not decay to two gluons but, like $J/\psi$, decays to three gluons (see Fig. 9b). The probability of the decay $^1P_1\to 3g$ is easily calculated in the so-called logarithmic approximation. The point is that, for the $^1P_1$ state, as the binding energy of the nonrelativistic $c$ and $\bar{c}$ quarks tends to zero, the amplitude has an infrared divergence, so that

$$\frac{\Gamma\,(^1P_1\to 3g)}{\Gamma\,(\chi_0\to 2g)}\approx\frac{10}{27}\,\frac{\alpha_s}{\pi}\ln MR; \tag{2.15}$$

here $R$ is the confinement radius: $R\sim(300\ \text{MeV})^{-1}$. For $\ln MR\sim 2$ we obtain a value of ~5% for this ratio.

The probability of the decay $^1P_1\to 3g$, as in the case of $J/\psi$, is obtained by simple conversion of the QED formulas; namely, we use the result of Alekseev[55] for the $^1P_1$-level of positronium.

As regards the $^3P_1$-level, its decay to two gluons is impossible because of the well-known Landau-Pomeranchuk-Yang exclusion.[56,57] The three-gluon decay, however, does not reduce to the answer known in quantum electrodynamics, since diagrams that are specific for QCD (see Fig. 9c) give a contribution. These diagrams describe the transformation of the $^3P_1$ state into two gluons, of which one is real and the other virtual, and the latter is already transformed to $2g$ at short distances $\sim 1/m_c$. Inasmuch as the real and the virtual gluon are not identical to each other, the Landau-Pomeranchuk-Yang exclusion cannot be extended to this transition.

If we try to calculate the decay $^3P_1 \to 3g$ in the logarithmic approximation, it turns out that the term containing $\ln mR$ cancels in the sum of the diagrams of Fig. 9b and 9c. Such a term remains, however, in the diagram of Fig. 9d, describing the transition of the $^3P_1$ level to a gluon and a $q\bar{q}$ pair, where $q$ is a light quark ($u$, $d$ or $s$). Thus, the total hadronic width of the $^3P_1$ state is principally given not by the three-gluon annihilation but by the annihilation to $gq\bar{q}$, the $q\bar{q}$ pair being created at short distances $\sim 1/m_c$.

The ratio of the widths of $\chi_1 \to gq\bar{q}$ and $\chi_0 \to gg$ is equal to $(4\alpha_s/9\pi)\ln MR \sim 1/15$, so that the hadronic width of the $\chi_1$-meson should be smaller than that of $\chi_0$ and even $\chi_2$ ($\Gamma(\chi_1 \to \text{hadrons}) \sim 100$–$400$ keV). The ratio of the widths of $^1P_1 \to 3g$ and $^3P_1 \to gq\bar{q}$ is equal to 5/6.

The hadronic widths of the triplet $P$ levels have not yet been measured (the singlet $P$ level has not been observed at all), but we can judge their magnitude from the relative probabilities of hadronic annihilation and the radiative transitions $\chi_J \to J/\psi + \gamma$. As will be shown in the next section, these relative probabilities are in good agreement with the theory.

The widths of the annihilation to $e^+e^-$, $2\gamma$ and $2g$, calculated in this section in the framework of a potential model of charmonium, will be calculated again in Chap. 5 on the basis of dispersion sum rules. We shall see that the agreement between the results obtained by the two different methods is extremely good.

To conclude this section we note that for the $J/\psi$ meson, decays to a photon + hadrons should constitute an appreciable fraction of the decays. This fraction can be determined by comparing the diagram of the type in Fig. 10 with the diagram of Fig. 9b. One obtains[52,58,59]

$$\frac{\Gamma(J/\psi \to \gamma + \text{hadrons})}{\Gamma(J/\psi \to \text{hadrons})} = \frac{16}{5}\frac{\alpha}{\alpha_s} \approx 0.12. \tag{2.16}$$

Consequently, the photon-hadronic width should amount to about 5.5 keV for $J/\psi$ and about 2.3 keV for $\psi'$. It should be stressed that we are talking not of the internal bremsstrahlung but of hard structural radiation. The expected probability of emission of a photon increases practically linearly with increase of the photon energy.

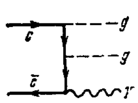



FIG. 10. Diagram describing the annihilation of the $c\bar{c}$ system to a photon and two gluons and responsible for decays of the type $J/\psi \to \gamma +$ hadrons.

TABLE II

| Level | $\Gamma(\psi' \to \chi_J+\gamma)$, keV | | $\Gamma(\chi_J \to J/\psi+\gamma)$, keV | | | |
|---|---|---|---|---|---|---|
| | Oscillator | Funnel | Upper bound | Oscillator | Funnel | Lower bound |
| $\chi_0$ (3.41) | 27 | 38 | 200 | 200 | 155 | 100 |
| $\chi_1$ (3.50) | 22 | 34 | 400 | 430 | 320 | 320 |
| $\chi_2$ (3.55) | 16 | 30 | 500 | 580 | 355 | 300 |

## 3. RADIATIVE TRANSITIONS IN CHARMONIUM

Whereas the annihilation of charmonium occurs at short distances, the radiative transitions between the levels of charmonium depend essentially on the behavior of the wavefunctions at large distances.

Radiative transitions in charmonium are very similar to radiative transitions in ordinary terms. Like the latter, they are divided into electric and magnetic transitions. The greatest widths are possessed by the electric transitions in which the $P$-levels of charmonium ($\chi_0$, $\chi_1$ and $\chi_2$) take part.

### a) Electric dipole transitions and the *P* levels of ortho-charmonium

Since the photon wavelength is large compared with the linear dimensions of charmonium, the usual dipole formulas (cf., e.g., Ref. 62) are valid for the widths of the radiative transitions $\psi' \to \chi_J + \gamma$ and $\chi_J \to J/\psi + \gamma$:

$$\Gamma(2^3S_1 \to 1^3P_J + \gamma) = (2J+1)\frac{4}{27}\alpha Q_c^2 \omega^3 |\langle r\rangle_2|^2, \tag{3.1}$$

$$\Gamma(1^3P_J \to 1^3S_1\gamma) = \frac{4}{9}\alpha Q_c^2 \omega^3 |\langle r\rangle_1|^2, \tag{3.2}$$

where

$$\langle r\rangle_a = \int_0^\infty R_{aS}(r) R_{1P}(r) r^3 \, dr \qquad (a = 1,\ 2);$$

here $J$ is the angular momentum of the $^3P_J$ level, $\omega$ is the transition frequency and $R(r)$ are the radial wavefunctions. Estimates of the $M2$-transitions and the relativistic corrections (some of these can be calculated at the present time) give us reason to think that the formulas for the widths are valid to within a factor 2. In the case of an oscillator potential, $|\langle r\rangle_1|^2 = 3/2\lambda$, where $|\langle r\rangle_2|^2 = 1/\lambda$ and $\lambda = m_c\omega_0/2 = 0.35\ \text{GeV}^2$. In the case of a potential of the funnel type the answer can only be obtained numerically. Table II gives the values of the widths, calculated for potentials of the oscillator and funnel types.[44–46,52,60]

Also given in the table are upper and lower bounds for the widths of the transitions $\chi \to J/\psi + \gamma$, which, as was noted by Jackson,[61] can be obtained from sum rules analogous to the well-known sum rules for atoms. These sum rules do not depend on the concrete form of the nonrelativistic potential in which the radiative transition occurs. They are a consequence of the commutation relations between the coordinate and momentum ($[r_k, p_l] = i\delta_{kl}$) and of the completeness of the set of wavefunctions describing the nonrelativistic system (see the book by Bethe and Salpeter[62]). The upper bounds are given by the Thomas-Reich-Kuhn sum rule, and the lower bounds by the Wigner sum rule:

$$\frac{4}{3}\frac{\alpha}{m_c}\omega_J^3 \geqslant \Gamma(\chi_J \to J/\psi+\gamma) \geqslant \frac{4}{9}\frac{\alpha}{m_c}Q_c^2\omega_J^3 + \frac{3}{2J+1}\left(\frac{\omega_J}{\omega_J'}\right)^2 \Gamma(\psi' \to \chi_J+\gamma); \quad (3.3)$$

here $m_c$ is the $c$-quark mass in the potential model. According to the best fits (for potentials of the funnel type), $m_c^{\text{fun}}=1.65$ GeV; $\omega_J$ is the energy of the photon in the transition $\chi_J \to J/\psi+\gamma$, and $\omega_J'$ is that in the transition $\psi' \to \chi_J+\gamma$. Unfortunately, in these relations spin effects have been taken into account only by substituting the observed transition energies $\omega_J$ and $\omega_J'$ into the sum rules in place of some spin-averaged energy $\omega_0$.

Allowance for the effect of spin on the transition matrix elements can give two-figure percentage corrections (The fact that the oscillator widths for $\chi_1$ and $\chi_2$ in the table have turned out to be above the upper bound is connected with the inconsistent allowance for the spin effects. We draw attention specially to this discrepancy in order to stress the approximate character of the theoretical predictions cited.)

Experimentally, the widths of all the three transitions $\psi' \to \chi_J+\gamma$ are approximately $20\pm7$ keV (see Chap. 1), which agrees qualitatively with the predictions of the theory. We remark that for a different choice of the quantum numbers of the $\chi_J$ levels, because of the factors $(2J+1)$ and $(\omega_J')^3$, the widths of the transitions $\psi' \to \chi_J+\gamma$ would differ strongly from each other, and so the fact that theory and experiment are in agreement is not trivial.

For the transitions $\chi_J \to \psi\gamma$, only the relative widths $B(\chi_J \to \psi\gamma)$ have been measured experimentally; we can calculate these by making use of the theoretical values for the widths of the hadronic annihilations of the mesons. For the $\chi_0$, $\chi_1$ and $\chi_2$ mesons the latter amount to (3–4.5) MeV, (0.1–0.4) MeV and (1–2) MeV, respectively. (For $\chi_0$ and $\chi_2$ the lower values are closer to the results given by the potential calculation, and the higher values are closer to the dispersion results.) Comparing this with the bounds for $\Gamma(\chi_J \to \psi\gamma)$, we obtain

$$2\% \leqslant B(\chi_0 \to \psi\gamma) \leqslant 7\%, \quad 30\% \leqslant B(\chi_1 \to \psi\gamma) \leqslant 80\%, \quad 13\% \leqslant B(\chi_2 \to \psi\gamma) \leqslant 30\%.$$

We recall that in experiment the corresponding values lie in the following limits: (0–5)%, (13–53)%, (3–27)%, so that there is qualitative agreement between the theory and experiment.

If we assume that, in the transitions $\psi' \to \chi_J+\gamma$, the agreement is evidence that the calculations of the electric-dipole transitions are reliable, the data on the transitions $\chi_J \to \psi+\gamma$ can be used to check the extent to which the gluon calculations of the hadronic annihilation of the $P$ levels are correct. The fact that the ratio of the widths of the radiative transition and the hadronic annihilation is greatest for precisely the $\chi_1$ level, for which annihilation to two gluons is forbidden, is a serious qualitative confirmation of the correctness of these calculations.

### b) Para-charmonium

The situation with the levels of para-charmonium ($1\,^1S_0(\eta_c)$, $2\,^1S_0(\eta_c')$, $1\,^1P_1$ and $1\,^1D_2$) is much less certain.

We shall consider the difficulties that result from identifying $\eta_c$ with the particle $X(2.83)$ that was observed at DESY and $\eta_c'$ with the level $\chi(3.45)$. The width of the radiative $M1$-transition $J/\psi \to \eta_c\gamma$ is given by the formula

$$\Gamma(1^3S_1 \to 1^1S_0+\gamma) = \frac{16}{3}\mu^2\omega^3 I^2; \quad (3.4)$$

where

$$I=\int_0^\infty R_{^3S_1}(r)\,R_{^1S_0}(r)\,r^2\,dr, \quad \omega = M_{J/\psi}-M_{\eta_c},$$

and $\mu$ is the magnetic moment of the $c$ quark. If we neglect the spin forces, then $R_{^3S_1}=R_{^1S_0}$ and $I=1$. If, furthermore, we take the magnetic moment of the $c$-quark equal to its bare value:

$$\mu=\mu_0=\frac{Q_c\sqrt{\alpha}}{2m_c},$$

then

$$\Gamma(J/\psi \to \eta_c\gamma) \approx \frac{4}{3}\frac{Q_c^2\alpha\omega^3}{m_c^2} \approx 1.6\,(\text{keV})\left(\frac{\omega}{100\,(\text{MeV})}\right)^3.$$

This formula is also valid for the transition $\psi' \to \eta_c'+\gamma$. Thus, if $\eta_c \equiv X(2.83)$ and $\eta_c' \equiv \chi(3.45)$, then

$$\Gamma(J/\psi \to X(2.83)+\gamma) \approx 25\text{ keV}, \quad \Gamma(\psi' \to \chi(3.45)+\gamma) \approx 12\text{ keV}.$$

These widths have not been measured experimentally, but upper bounds have been established:

$$B(J/\psi \to X(2.83)+\gamma) < 3\%, \quad B(\psi' \to \chi(3.45)+\gamma) < 5\%.$$

The following products have also been measured (see Fig. 2):

$$B(J/\psi \to X(2.83)+\gamma)\,B(X(2.83) \to 2\gamma) = (1.2\pm0.5)\,10^{-4},$$
$$B(\psi' \to \chi(3.45)+\gamma)\,B(\chi(3.45) \to J/\psi+\gamma) = (8\pm4)\,10^{-3},$$

so that $B(X(2.83) \to 2\gamma) > (4\pm1.6)\times10^{-3}$, instead of $1.2\times10^{-3}$ from the gluon theory, and $B(\chi(3.45) \to J/\psi+\gamma) > (16\pm8)\%$. The latter number is in striking contradiction with the theoretical estimates of the width of the hadronic annihilation of the $\eta_c'$ meson. In fact, if $\Gamma(\psi' \to \eta_c'+\gamma) < 12$ keV, and $\Gamma(\eta_c' \to \text{hadrons}) = (1-4)$ MeV, then instead of 16% we ought to have a value less than 1%.

The estimates given above for the widths of the $M1$-transitions $J/\psi \to \eta_c\gamma$ and $\psi' \to \eta_c'\gamma$ can be reduced somewhat if we take into account that, because of spin effects $I<1$ (for the decays $\varphi \to \eta\gamma$, $\rho \to \pi\gamma$ and $K^* \to K\gamma$, $I \sim 0.7$–$0.8$), and $\mu \neq \mu_0$; however, as is easily seen, it is not possible to resolve all three contradictions between theory and experiment in this way. (If $\mu < \mu_0$, as follows, evidently, from the upper bound for the decay $\psi \to X(2.83)+\gamma$, the contradiction in the case of the decay $\psi' \to \eta_c'\gamma$ is only enhanced.) Thus, the hypothesis that $X(2.83)$ and $\chi(3.45)$ are respectively the $\eta_c$ and $\eta_c'$ mesons encounters serious difficulties.

We turn now to the levels ${}^1D_2$ and ${}^1P_1$ and, in particular, discuss whether the level $\chi(3.45)$ could be the ${}^1D_2$ level. (This hypothesis was put forward by Harari.[63]) Naive application of the Breit-Fermi equation does not rule out the possibility that this level could be so light, but the most plausible values for its mass lie 100–200 MeV higher (in this case we should expect that the mass of ${}^1P_1$ is about 3.15 MeV).

The probability of the transition $\psi' \to {}^1D_2\gamma$ depends sharply on the extent of the admixture of the ${}^3D_1$ state in the wavefunction of the $\psi'$ meson; with regard to this it is usually assumed that the meson corresponds principally to the $2\,{}^3S_1$ state. If this admixture is equal to zero, the width of the transition $\psi' \to {}^1D_2\gamma$ is negligible (~4 eV). The width of the transition ${}^1D_2 \to J/\psi + \gamma$ is not much greater (~15 eV) if $J/\psi$ does not contain any admixture of the $1\,{}^3D_1$ state. But if the wavefunctions of $\psi'$ and $\psi$ are superpositions

$$2^3S_1 + \varepsilon'(1^3D_1) \text{ and } 1^3S_1 + \varepsilon(1^3D_1),$$

the transitions are magnetic-dipole transitions and their widths amount to

$$\Gamma(\psi' \to {}^1D_2 + \gamma) \approx 20(\text{keV})(\varepsilon')^2,$$
$$\Gamma({}^1D_2 \to \psi + \gamma) \approx 40(\text{keV})(\varepsilon)^2.$$

In order that the ${}^1D_2$ level can play the role of $\chi(3.45)$ it is necessary that $(\varepsilon')^2 \lesssim 0.7$ and $\varepsilon^2 \simeq 0.5$. Such admixtures seem too large.

If the mass difference between ${}^1D_2$ and ${}^1P_1$ levels is sufficiently great, then, in the radiative transitions of the ${}^1D_2$ level, the $E1$ transition ${}^1D_2 \to {}^1P_1 + \gamma$, whose width in the oscillator model is equal to $12(\omega/100\text{ MeV})^3$ keV, should dominate. (We recall that the expected width of the hadronic annihilation of the ${}^1D_2$ level is approximately 60–100 keV.)

As regards the subsequent fate of the ${}^1P_1$ level, this depends strongly on its mass: the expected width of its hadronic annihilation is 100–400 keV, while the width of the $E1$ transition to the $\eta_c$ meson is $6(\omega/100\text{ MeV})^3$ keV.

Summarizing the content of this chapter, it may be said that at the present time we have a qualitative, and at many points quantitative, understanding of the spectroscopy of the levels of the ortho-charmonium and of the radiative transitions between them. As regards the levels of para-charmonium, here there are a whole series of experimental and theoretical unsolved questions, the elucidation of which should give, in the very near future, important information for our understanding of the interaction properties of charmed quarks and the structure of charmonium as a whole.

## 4. MOLECULAR CHARMONIUM

### a) Experimental data and interpretation

In this chapter we discuss the possible nature of the resonance structure in $e^+e^-$ annihilation at energies 4.1 and 4.4 GeV. As already mentioned in the Introduction, the resonance $\psi(4.4)$ is described by a Breit-Wigner curve with parameters $\Gamma_{\text{tot}} = 33 \pm 10$ MeV, $\Gamma(\psi(4.4) \to e^+e^-) = 440 \pm 140$ eV. The structure near 4.1 GeV looks like a superposition of two and three resonance peaks, the sharpest of which is located at energy $\sqrt{s} = 4028$ MeV. Of great interest are the decay properties of this latter resonance. We have in mind the decays to pairs of charmed mesons. It has been found that the magnitudes of the cross-sections $\sigma(D^0\overline{D}^0)$, $\sigma(D^0\overline{D}^{*0} + D^{*0}\overline{D}^0)$ and $\sigma(D^{*0}\overline{D^{*0}})$ at the 4.028 GeV peak are in the ratios $1:{\sim}8:{\sim}11$,[16,19] while the energy released for each of these channels amounts to ~300 MeV, ~160 MeV and ~18 MeV respectively ($M(D^0) \simeq 1865$ MeV and $M(D^{*0}) \simeq 2005$ MeV), which corresponds to the following values of the momentum $p$ in the center-of-mass frame (c.m.s.): 750 MeV, 550 MeV and 190 MeV. Since each of the pairs of charmed mesons is created in a $P$-wave (because of conservation of the spatial parity), the quantity $f^2 = \sigma/p^3$ serves as a true "measure of the interaction." From the numbers given above we find

$$f^2(D^0\overline{D^0}) : f^2(D^0\overline{D^{*0}}) + f^2(D^{*0}\overline{D^0}) : f^2(D^{*0}\overline{D^{*0}}) = 1 : {\sim}20 : {\sim}680. \quad (4.1)$$

The relations (4.1) become even more surprising if we take into account that, in the simple nonrelativistic model, one obtains[64] in place of the ratios (4.1)

$$1:4:7. \quad (4.2)$$

In fact, the process of production of charmed mesons is described by the diagram of Fig. 11.[3)] Corresponding to this diagram is an amplitude $A$ whose spin structure (in the nonrelativistic approximation both with respect to the $c$ quarks and also, which is important, with respect to the $q$ quarks) is as follows:

$$A \sim (j_l c^+ \sigma_l c)(p_k q^+ \sigma_k q). \quad (4.3)$$

Here $j_l$ is the electromagnetic electron current, $c$ and $q$ are nonrelativistic spinors, $\sigma_l$ are the Pauli matrices, $p_k$ is the momentum of either of the quarks in the c.m.s., and we have written out only the $P$-wave part of the amplitude, since it is precisely this which is responsible for the creation of pairs of charmed mesons. In the relation (4.3) it has been taken into account that in the nonrelativistic limit the quarks $c$ and $\bar{c}$ are created by a photon in a state with zero orbital angular momentum and total spin equal to unity, and that the spins of the $c$ quarks are not flipped in the process of formation of the $D$ meson.

Regrouping now the spinors in formula (4.3) by means of a Fierz transformation in such a way that combinations corresponding directly to the charmed mesons ($(c^+q) \sim D$, $(c^+\sigma_m q) \sim D_m^*$) are formed, we obtain

$$A \sim (\mathbf{jp})\overline{D}D + i\varepsilon_{lkm} j_l p_k(\overline{D}D_m^* - \overline{D}_m^* D) + j_l p_k\left(\overline{D}_l^* D_k^* + \overline{D}_k^* D_l^* - \tfrac{2}{3}\delta_{kl}\overline{D}_m^* D_m^*\right) - \tfrac{1}{3}(\mathbf{jp})(\overline{D}_m^* D_m^*). \quad (4.4)$$

---

3) An alternative mechanism, in which the photon creates a light-quark pair $q\bar{q}$ and a $c\bar{c}$-pair is picked up from the "vacuum," gives a negligibly small contribution because of the large mass of the $c$-quark.

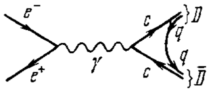


FIG. 11. Creation of $D\bar{D}$-molecule in $e^+e^-$ annihilation.

The first term describes the creation of $D\bar{D}$ pairs, the second the creation of $D^*\bar{D}$ and $D\bar{D}^*$, the third the formation of $D^*\bar{D}^*$ in the $^5P_1$ state, i.e., with total spin $S=2$, and, finally, the fourth the creation of $D^*\bar{D}^*$ in the $^1P_1$ state, i.e., with total spin $S=0$. The result (4.2) is now almost obvious. It is only necessary to square each of the matrix elements in (4.4) and sum over the spins of $D^*$. It is convenient to give the answer in the form in which it is obtained before summation of the $S=2$ and $S=0$ contributions in the $\bar{D}^*D^*$ channel:

$$f^2(D\bar{D}):\ [f^2(D\bar{D}^*)+f^2(D^*\bar{D})]:\ f^2(D^*\bar{D}^*,\ ^5P_1):\ f^2(D^*\bar{D}^*,\ ^1P_1) = 1:\ 4:\ \frac{20}{3}:\ \frac{1}{3}. \qquad (4.5)$$

Summarizing, we can state that the creation of $D\bar{D}^*$ and $\bar{D}D^*$ pairs at $\sqrt{s}=4.028$ GeV is enhanced approximately five times, and the creation of $D^*\bar{D}^*$ at least a hundred times, relative to the creation of $D\bar{D}$ pairs. A natural explanation of this fact is that the peak at $\sqrt{s}=4.028$ GeV is a $P$ wave resonance in the $D^*\bar{D}^*$ system, i.e., that it is constructed from a $D^*$ and a $\bar{D}^*$ meson.[19] The isotopic spin of this resonance is equal to zero, since it is produced by the $\bar{c}\gamma_\mu c$ component of the electromagnetic current (see Fig. 11). The existence of such states, which it is natural to call hadronic molecules inasmuch as they consist of hadronic "atoms"—mesons, was predicted earlier[18] on the basis of the arguments expounded below for the example of the $D\bar{D}$ system.

## b) Dynamics of the $D\bar{D}$ system

A distinguishing feature of the charmed mesons is the fact that, besides the heavy $c$-quark, they also contain a light quark ($u$, $d$ or $s$) and can therefore emit and absorb ordinary mesons, e.g., $\rho$, $\omega$, $\varphi$, etc. (Fig. 12). Exchange of these mesons leads to strong interaction between $D$ and $\bar{D}$, with a range $r_0 \simeq m_\omega^{-1} \simeq m_\rho^{-1}$ that is large compared with the Compton wavelength $1/m_D$. The answer to the question as to whether or not levels can arise in the $D\bar{D}$ system depends on how strong this interaction is. We shall try to estimate it by confining ourselves to the $\rho$ and $\omega$ exchanges.[4)]

We write the potential acting between $D$ and $\bar{D}$ in the form

$$U^{(T)} = U_0 + \tau_1\tau_2 U_1 = U_0 + [2T(T+1) - 3]\, U_1, \qquad (4.6)$$

where $T$ is the total isospin of the $D\bar{D}$ system ($T=0, 1$). If $U_0$ ($U_1$) is due to $\omega$ ($\rho$) exchange, then, in the static limit,

$$U_0 = -\alpha_\omega \frac{1}{r} e^{-r/r_0}, \quad U_1 = \alpha_\rho \frac{1}{r} e^{-r/r_0},$$

[4)]Such an approach is analogous to treating the $N\bar{N}$ system by means of a one-meson exchange potential.[65]

where $\alpha_\omega = g^2_{\omega D\bar{D}}$ and $\alpha_\rho = g^2_{\rho D\bar{D}}$ are the coupling constants of $\omega$ and $\rho$ with $D\bar{D}$, normalized in the standard way. The resulting expression for the interaction potential in a channel with a particular isospin is:

$$U^{T=0(1)} = -\alpha^{0(1)}\frac{1}{r}e^{-r/r_0}, \qquad \alpha^{(0)} = \alpha_\omega + 3\alpha_\rho, \quad \alpha^{(1)} = \alpha_\omega - \alpha_\rho. \qquad (4.7)$$

The constants $\alpha_\omega$ and $\alpha_\rho$ are not known experimentally, but certain information can be obtained about them by invoking one or other of the models. Thus, e.g., we may expect that $\alpha^{(1)} \ll \alpha^{(0)}$. In fact, $\alpha_\omega = \alpha_\rho$ in both the vector-dominance model and the dual model. In the framework of the quark model, using the (not entirely unambiguous) data on the $\rho(\omega)NN$ constants, one obtains[18]

$$2.8 \leqslant \alpha^{(0)} \leqslant 4.4, \quad 0 \leqslant \alpha^{(1)} \leqslant 1.7. \qquad (4.8)$$

The criterion for the existence of a bound level with a given angular momentum $J_0$ in a given potential can be formulated in the language of Regge trajectories. The mass $M_0$ of the level is determined by the value $M=M_0$ at which the Regge trajectory $J(M)$ intersects the straight line $J=J_0$. The Regge trajectory for the Yukawa potential is characterized by the parameter $G=\alpha m_D r_0$. As shown by Lovelace and Masson,[66] a $1S$ level appears when $G \gtrsim 1.7$, while $1P$ has a resonance character (it lies above threshold) when $7.3 \lesssim G \lesssim 9.0$ and is stable when $G \gtrsim 9.0$. Starting from the estimate (4.8) and taking into account that $m_D r_0 \simeq 2.5$, we can postulate that one-meson exchange leads to the formation of at least isosinglet $S$ and $P$ levels of the molecular type. We recall that the 4.028 GeV peak is, apparently, precisely an isosinglet state.

In the positions and widths of the predicted levels there is considerable uncertainty. As regards the $P$-level, its mass should not be more than ~30 MeV above threshold, since is is just the height of the centrifugal barrier for $G \sim 8$.

For resonances of the molecular type, besides the "elastic" decays to $D\bar{D}$, transitions to the "atomic" levels of charmonium with emission of ordinary mesons are possible, e.g., decays to $\psi+2\pi$, $\eta_c+2\pi$, $\psi+\eta$, $\eta_c+\eta$, etc.

We note that the interaction between a charmed baryon $C$ and antibaryon $\bar{C}$ should be even stronger than in the $D\bar{D}$ system, since in the baryons there are more light quarks. Therefore, if molecules of the type $D\bar{D}$ exist, molecules of the type $C\bar{C}$ should certainly exist. It is also possible to imagine multihadronic molecules, e.g.,

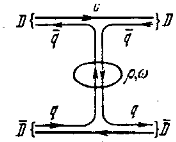


FIG. 12. Exchange of light quarks between a $D$ and a $\bar{D}$ meson. [The quark and antiquark that are exchanged form the mesons $\rho$ and $\omega$. This exchange leads to a potential of the Yukawa type between $D$ and $\bar{D}$.]

$DD\bar{D}\bar{D}$ or $C\bar{C}C\bar{C}$. In any case, the existence of a rich molecular spectroscopy of charmonium seems highly probable.

The estimates given above do not answer the question as to why the experimentally observed charmonium molecule arose in the system $D^*\bar{D}^*$ and not in $D\bar{D}$. The answer to this question depends on the spin forces, which we have not considered in view of our complete lack of knowledge concerning them. Further theoretical and experimental investigation of molecular charmonium will make it possible to establish their properties. Measurements of the angular correlations in the decays of molecular charmonium would be especially interesting in this respect.

### c) Angular correlations of the decay products at the 4.028-GeV peak

Investigation of the angular correlations would give a wealth of information about the internal structure of the 4.028-GeV charmonium molecule. Thus, the distribution of $D^*\bar{D}^*$ in the ${}^1P_1$ state is proportional to $(1-\cos^2\theta)$, where $\theta$ is the angle between the direction of emission of $D^*$ and the direction of the initial $e^+e^-$ beams, whereas the distribution in the ${}^5P_1$ state is proportional to $1-(1/7)\cos^2\theta$. Furthermore, for a ${}^5P_1$ $D^{*+}D^{*-}$ pair at rest (the velocity of the $D^{*\pm}$ mesons at the 4.028 GeV peak can be neglected, since it is $<0.1c$), the angular distribution of the $\pi$-mesons resulting from the decays $D^{*\pm}\to D^{\pm}\pi$ has the form $1-(21/47)\cos^2\theta$, while for the ${}^1P_1$ state the $\pi$ mesons are distributed isotropically.

The difference between the ${}^5P_1$ and ${}^1P_1$ states is manifested most clearly in the distribution with respect to the angle $\varphi$ between the planes of the decays $D^*\to D\pi$ and $\bar{D}^*\to\bar{D}\pi$. For the pure ${}^1P_1$ state this distribution is proportional to $\cos^2\varphi$, while for ${}^5P_1$ it is proportional to $1+(1/3)\cos^2\varphi$.

If it turns out that the 4.028 GeV molecule is, principally, the ${}^1P_1$ state of the $D^*\bar{D}^*$ pair, the enhancement of the $D^*\bar{D}^*$ channel relative to $D\bar{D}$ will amount not to $\sim 100$, as estimated above, but to approximately 2000, inasmuch as the "natural" ratio $f^2(D^*\bar{D}^*, {}^1P_1):f^2(D\bar{D})=1:3$; but if, on the other hand, it turns out that the angular correlations correspond to a ${}^5P_1$ structure for the 4.028 GeV molecule, then we should expect that, with high probability, there exists a ${}^1P_1$ $D^*\bar{D}^*$ molecule at a lower mass, most probably below the threshold $2M_{D^*}$. The width of the decay of such a molecule to $e^+e^-$ should be approximately twenty times smaller than that for the 4.028 GeV molecule $(f^2({}^1P_1):f^2({}^5P_1)=1:20)$. Its total width should also be smaller, since, being below the $D^*\bar{D}^*$ threshold, it cannot decay along this channel.

### d) Some conclusions

The idea of molecular charmonium, which was proposed before the actual discovery of the 4.028-GeV molecule, has turned out to be correct, at least in one case. We may expect that other molecules besides the one at 4.028 GeV will also exist. Unfortunately, the theoretical analysis of the spectroscopy and properties of the molecular states is on an almost purely qualitative level at the present time. Thus, e.g., it is rather obvious that $S$-wave isoscalar molecules should exist (inasmuch as a $P$-wave molecule—the 4.028-GeV peak—exists). However, estimates for the binding energy of $S$-wave molecules give a quantity of the order of 2 GeV, and so, of course, we must regard them not as dimeson structures but as truly four-quark structures.[67–70] There are no methods for analyzing four-quark systems; we do not even know a criterion to distinguish the molecular state $D\bar{D}$ from the "atomic" state $c\bar{c}$, since, in the case of an isoscalar molecule, their quantum numbers coincide. We can make a unique judgment about the molecular character of a state only if its quantum numbers cannot be realized in the system $c\bar{c}$, e.g., in the case of an isovector state. (We can judge $c$ quarks to be present from the small total width, the existence and width of transitions to atomic levels of charmonium, etc.) However, the existence of such states is highly problematical, since we have seen above that in an isovector state of a $D\bar{D}$ pair the interaction potential is small. (Even if it did exist, it would be manifested hardly at all in $e^+e^-$ annihilation.)

Another obvious question to which, apparently, an unambiguous answer cannot be given at present is the following: if a molecule $D^*\bar{D}^*$ exists, then do molecules $D\bar{D}^*$ and $D\bar{D}$ exist, and at what mass? In addition, in $e^+e^-$ annihilation there can be two $D^*\bar{D}^*$-resonances: ${}^5P_1$ and ${}^1P_1$, or two orthogonal combinations of them. If other molecular states besides that at 4.028 GeV are not discovered, this could mean that an important role in the formation of the 4.028 GeV molecule is played by the spin-spin forces acting between $D^*$ and $\bar{D}^*$. In this case it is almost obvious that the 4.028 GeV state should be the ${}^1P_1$ state. On the other hand, this statement could be checked using the angular correlations discussed above.

In any case, it may be stated that the study of such distinctive objects as hadronic molecules can yield much that is new. For example, the study of molecular charmonium is scarcely the only way of investigating the strong interaction between charmed hadrons.

## 5. DISPERSION THEORY OF CHARMONIUM

In this section we describe a new approach[71,72] to the problem of charmonium, based only on the asymptotic freedom of QCD and on dispersion relations. It will be seen from the following account that this approach works beautifully in precisely the case of hadrons consisting of heavy quarks, and makes it possible to obtain a whole spectrum of predictions for the decay constants of the charmonium levels.

### a) Sum rules

We shall consider the process of formation of a pair of charmed particles in the collision of an electron and a positron with a space-like total 4-momentum $q$: $q^2<0$. It is obvious that, because of energy-momentum conservation, such a process cannot be real but can only occur virtually. Here, by virtue of the uncertainty

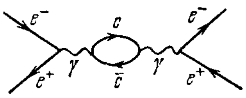



FIG. 13. Contribution of charmed quarks to the $e^+e^-$ elastic-scattering amplitude.

principle, the particles appear for a time less than or of the order of $\tau \sim (4m_D^2 - q^2)^{-1/2}$, where $m_D$ is the mass of the $D$ meson (the lightest of the charmed particles), and become separated by a distance not greater than $c\tau$. Even for $q^2 = 0$ this distance is considerably smaller than the confinement radius. If quantum chromodynamics is valid, at such short distances asymptotic freedom should obtain and we can assume the interaction of the quarks with the gluons to be rather weak. In zeroth order in this interaction the contribution of the "charm" to the $e^+e^-$ elastic-scattering amplitude is described by the bare $c$-quark loop (Fig. 13), which we can express purely formally in terms of the dispersion integral of the cross section for production of bare quarks. On the other hand, this same amplitude can be expressed in terms of the dispersion integral of the cross-section for creation of real charmed particles—both those with hidden charm ($J/\psi, \psi', \ldots$) and the pairs $D\bar{D}$, $F\bar{F}$, etc. As a result, to within terms $\sim \alpha_s$, we obtain the relation

$$\frac{s^2}{\pi}\int \frac{\sigma(s')\,ds'}{s'-s} = \frac{s^2}{\pi}\int\limits_{4m_c^2} \frac{\sigma_0(s')\,ds'}{s'-s}, \qquad (5.1)$$

which is the basis of the dispersion method of investigation of charmonium. The integral in the left-hand side of this relation contains the contributions of the $J/\psi$ and $\psi'$ poles and of the continuum of physical states starting at $(2m_D)^2$ (Fig. 14). The integral in the right-hand side is calculated trivially, since the diagram of Fig. 13 corresponds to

$$\sigma_0 = \frac{4\pi\alpha^2 Q_c^2}{s}\,\frac{3-v^2}{2}\,v, \qquad (5.2)$$

where $1 - v^2 = 4m^2/s$. It follows immediately from the relation (5.1), if we consider it for $s \to -\infty$, that $\sigma(s') \to \sigma_0(s')$ as $s' \to +\infty$. In fact, suppose that $\sigma(s') \to c/s'$ and $\sigma_0(s') \to c_0/s'$ as $s' \to \infty$ and that the constants $c$ and $c_0$ do not coincide. Then the difference between the left- and right-hand sides of (5.1) would tend asymptotically to $(c - c_0)(s/\pi)\ln(-s)$ as $s \to -\infty$. This contradicts the fact that the relation (5.1) for $s \to -\infty$ is fulfilled to within

$$\frac{s}{\pi^2}\ln(-s)\,\alpha_s(s) \sim s.$$

Thus, asymptotically, the physical cross-section for creation of charm tends to its bare quark value $4\pi\alpha^2 Q_c^2/s$. This fact was first noted by Appelquist and Georgi (see Refs. 73, 74), who also calculated the correction proportional to $\alpha_s$.

The experiments in a limited energy range, which have limited accuracy and cannot be interpreted completely unambiguously because of the contribution of the heavy lepton and, possibly, of new kinds of quark, do not contradict the possibility that $\sigma \simeq \sigma_0$ for $s \gtrsim (4\ \mathrm{GeV})^2$, and we shall rely on this equality in the following.

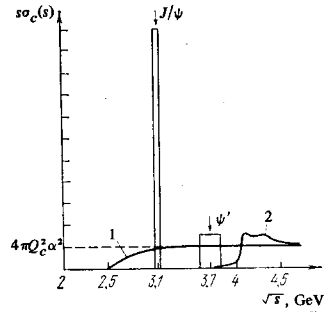



FIG. 14. Energy dependence of the quantity $S_c\sigma(s)$, where $\sigma(s)$ is the cross-section for $e^+e^-$ annihilation to hadrons with hidden and manifest charm. The contribution of the $J/\psi$- and $\psi'$-mesons is depicted conventionally by rectangles. The widths of the rectangles are 1000 times greater than the total widths of the corresponding mesons, and the height is reduced so that the areas of the rectangles correspond to the experimental values of the integrals $\int \sigma_c(s) s\, d\sqrt{s}$. The curve 1 corresponds to the cross-section for annihilation of $e^+e^-$ to a $c\bar{c}$ quark pair (cf. formula (5.2)); the curve 2 depicts schematically the cross-section for $e^+e^-$ annihilation to charmed hadrons in accordance with the existing data and theoretical expectations.

From the region $|s| \to \infty$ we turn now to $s = 0$. In this case too, arguments based on asymptotic freedom remain valid. The contribution of "charm" to the amplitude induced by the virtual photon (see Fig. 13) can be calculated even for small "virtuality" of the photon by series expansion in the small quark-gluon interaction constant $\alpha_s(4m_c^2)$. This simple fact, to which attention was first drawn by Shifman, Vaĭnshteĭn and Zakharov,[75] leads, as we show below, to far-reaching consequences.

Thus, we shall consider the relation (5.1) for $s \to 0$. Differentiating (5.1) $n-1$ times with respect to $s$ at $s = 0$, we obtain

$$\int \frac{\sigma(s')\,ds'}{(s')^n} = \int\limits_{4m_c^2}^{\infty} \frac{\sigma_0(s')\,ds'}{(s')^n}. \qquad (5.3)$$

It is clear that the integrals will be determined by smaller values of $s'$ the greater is $n$. But is is precisely at small values of $s'$ that the cross sections $\sigma$ and $\sigma_0$ differ most strongly (see Fig. 14), so that for sufficiently large $n$ the relation (5.3) should be strongly violated. Numerical calculations show that for, say, $n = 3, 4$, the relation (5.3) is still fulfilled to within a few percent, but, on the other hand, the contribution of the region $s > 4m_D^2$ is already small ($\lesssim 8\%$), so that the left-hand side of the relation (5.3) is essentially saturated by the contribution of the $J/\psi$ meson[5)]:

[5)]In principle, such a situation cannot be realized for ordinary hadrons. Indeed, if we were interested in the contribution of the light quarks $u$, $d$ and $s$ to the elastic $e^+e^-$-scattering amplitude, we would find that asymptotic freedom gives reliable predictions only for large negative values of $s$, say, $s = -3$ GeV$^2$. In this case the dispersion integral of the physical cross section in (5.1) is built up outside the resonance region at $s' \lesssim 3$ GeV$^2$, and, essentially, we obtain only information about the cross-section in the continuum region.

$$\sigma(e^+e^- \to J/\psi) = 12\pi^2\delta(s - M^2)\frac{\Gamma_{ee}}{M}, \quad (5.4)$$

where $M$ is the $J/\psi$ mass and $\Gamma_{e^+e^-}$ is the width of the decay $J/\psi \to e^+e^-$. Substituting (5.4) into (5.3) we can calculate the width $\Gamma_{ee}$ by expressing it in terms of the mass of the $J/\psi$ meson and the quark charge $Q_c$ [71]:

$$\Gamma(J/\psi \to e^+e^-) \approx \frac{Q_c^2\alpha^2 M}{3\pi}\frac{(64/315)^4}{(32/231)^3} \approx 5\,\text{keV}. \quad (5.5)$$

This relation is in excellent agreement with experiment. The theoretical predictions in the right-hand side of the formula (5.3) contain one unknown parameter, which we call, by convention, the $c$-quark mass. Of course, since the quarks are confined, the mass must be regarded only as a parameter appearing in the quark propagator. The value of the mass, determined from (5.3), is found to be equal to

$$m_c = 1.25\,\text{GeV}. \quad (5.6)$$

Inasmuch as the integral in the loop of Fig. 13 for $s = 0$ converges at values $p_c^2 \simeq -m_c^2$, where $p_c$ is the 4-momentum of the $c$ quark, the above value of 1.25 GeV is the mass term of the propagator of a deeply virtual $c$ quark with $p_c^2 \simeq -m_c^2$. For time-like $c$ quarks with $p_c^2 > 0$ the mass should be larger. It is clear that the concept of mass for a quark is to a certain degree arbitrary. Nevertheless, the value of $m_c$ found above is a key parameter for the physics of charmonium, determining the properties not only of $J/\psi$ but also of other levels of charmonium. Thus, the physical cross section for the creation of charm should be equal to the bare quark cross-section not only for $s \to \infty$ but also, in a certain averaged integral sense, for small values of $s$. This correspondence is sometimes called duality; one says that the corresponding cross sections are dual to each other. The integral of the bare quark cross-section in the range from $4m_c^2$ to $4m_D^2$ is dual (equal) to the integral of the $J/\psi$ and $\psi'$ resonances.

We turn now to another process, namely, the creation of charm in the collision of two photons. In this case too the integral relations (5.3) should hold, with the same value $m_c = 1.25$ GeV. Again it turns out that the integral of the cross-section for formation of a pair of bare quarks $c + \bar{c}$ from two photons (see Fig. 15), taken between the limits $4m_c^2$ and $4m_D^2$, is dual to the contribution of the resonances (with positive $C$-parity, this time) $\eta_c$, $\eta_c'$ ($J^P = 0^-$), $\chi_0$ ($J^P = 0^+$) and $\chi_2$ ($J^P = 2^+$), which can decay into two photons. Considering the cross sections $\sigma_\parallel$ and $\sigma_\perp$ for collisions of photons with parallel and perpendicular polarizations separately, it is not difficult to show that the contributions of the $0^-$, $0^+$ and $2^+$ mesons to $\sigma_\parallel$ and $\sigma_\perp$ are, respectively,

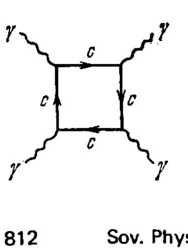



FIG. 15. Contribution of charmed quarks to photo-photon scattering.

$$(0,\ 16,\ 40\ \text{and}\ 16,\ 0,\ 40) \times \pi^2\frac{\Gamma_{\gamma\gamma}}{M}\delta(S - M^2). \quad (5.7)$$

If, in addition, we further assume that the ratio of the photon widths for $\chi_0$ and $\chi_2$ is equal to 15/4, as given by the nonrelativistic theory[50,52] (cf. formula (2.6)), we can find from the sum rules (5.3) for $\sigma_\parallel$ that $\Gamma(\chi_0 \to 2\gamma) = 5 \pm 0.5$ keV. By assigning masses to $\eta_c$ and $\eta_c'$ we can then find the photon widths of these mesons from the sum rules for $\sigma_\perp$. If we take $\eta_c \equiv X(2.83)$, then $\Gamma(\eta_c \to 2\gamma) \simeq 3.5$ keV; $\Gamma(\eta_c' \to 2\gamma)$ is approximately the same if $M_{\eta_c'} = 3.45$ GeV and somewhat larger if the mass of $\eta_c'$ is larger. This is very unlike the ratio of the electromagnetic decays of $J/\psi$ and $\psi'$: $\Gamma(\psi' \to e^+e^-)/\Gamma(J/\psi \to e^+e^-) \simeq 0.4$. At the same time, naive application of the nonrelativistic potential model of charmonium gives

$$\frac{\Gamma(\psi' \to e^+e^-)}{\Gamma(J/\psi \to e^+e^-)} \approx \frac{\Gamma(\eta_c' \to 2\gamma)}{\Gamma(\eta_c \to 2\gamma)},$$

since $|\psi(0)|^2$ should be the same in $S$-wave ortho- and para-charmonium. This contradiction indicates that either $\eta_c$ is substantially heavier than $X(2.83)$, e.g., $M_{\eta_c} \simeq 3$ GeV (then $\Gamma(\eta_c \to 2\gamma) \simeq 6$ keV and $\Gamma(\eta_c' \to 2\gamma) \ll \Gamma(\eta_c \to 2\gamma)$), or the hyperfine interaction and the $^3S_1 \to {}^3D_1$ mixing are very strong in charmonium, in contrast to what is usually assumed in the nonrelativistic model. In the latter case it is possible, e.g., that the 3.45 GeV level is the $^1D_2$ level and $\eta_c'$ lies higher.

Knowing the photon widths, by multiplying them by $(9/8)\alpha_s^2/\alpha^2 \simeq 850$ we can find the widths of the decays to two gluons, which, by assumption (see Chap. 2), are equal to the total hadronic widths. As a result we obtain

$$\begin{aligned}
&\Gamma(\chi_0 \to \text{hadrons}) \approx 3.9 - 4.6\ \text{MeV},\\
&\Gamma(\chi_2 \to \text{hadrons}) \approx 1.0 - 1.2\ \text{MeV},\\
&\Gamma(\eta_c \to \text{hadrons}) \approx 3\ \text{MeV, if}\ M_{\eta_c} = 2.83\ \text{GeV},\\
&\qquad\qquad\qquad\quad \approx 5\ \text{MeV, if}\ M_{\eta_c} = 3\ \text{GeV}.
\end{aligned}$$

These are fairly close to the numbers given by the potential model: 3 MeV, 0.8 MeV and 6 MeV, respectively.

Thus, we see that a dispersion analysis of charmonium, based on such general properties of quantum chromodynamics as the asymptotic freedom, analyticity and unitarity, confirms, in its general features, the correctness of the nonrelativistic model of charmonium based on potentials of the oscillator or funnel type that ensure the confinement of the quarks. The asymptotic freedom at short distances, the trapping of quarks at large distances, the existence of narrow levels of charmonium and the parton-like cross section for formation of charm above $2m_D$—all these are in good agreement with each other.

Above we applied the dispersion relation to the creation of charm by the electromagnetic current. In order to distinguish the contribution of mesons with given quantum numbers, and, particular, that of mesons corresponding to high orbital states such as $^1D_2$, it is convenient to make use of an artificial device[72]: we introduce fictitious currents with the quantum numbers that

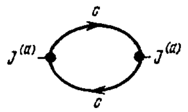



FIG. 16. Polarization of the vacuum $J^{(a)}$ by the external current .

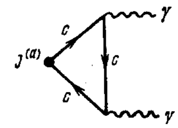



FIG. 17. Triangular diagram describing the transformation of the external current $J^{(a)}$ to two photons.

we need, e.g., $(c\bar{c})$ for the $\chi_0$ meson, $(\bar{c}\gamma_5 c)$ for $\eta_c$, $(\bar{c}\gamma_\mu\gamma_5 c)$ for $\chi_1$, $\bar{c}(p_\nu\gamma_\mu + \gamma_\nu p_\mu - (2/3)\eta_{\mu\nu}\hat{p})c$ for $\chi_2$, and $\bar{c}(p_\mu p_\nu - (1/3)\eta_{\mu\nu}p^2)\gamma_5 c$ for the ${}^1D_2$ level. Here $\eta_{\mu\nu} = \delta_{\mu\nu} - (q_\mu q_\nu/q^2)$, $q = k_c + k_{\bar{c}}$ and $p = k_c - k_{\bar{c}}$. We consider first the so-called biangular diagrams, describing the polarization of the $c\bar{c}$ vacuum by these fictitious currents (Fig. 16) and use again the duality of the integral of the fields in the quark loop between the limits $4m_c^2$ and $4m_D^2$ and the contribution of the corresponding resonances. In principle, by considering the integral moments with $n = 1, 2, 3, 4$ in (5.3), we can now find the masses of the resonances and the effective coupling constants $g$ of their interaction with the fictitious currents. These coupling constants are uniquely related to the quantity $\psi(0)$ for the $S$-wave states, to $\psi'(0)$ for the $P$-wave states, to $\psi''(0)$ for the $D$-wave states. etc. The results of the calculations are collected in Table III.

The predictions for $|R_S(0)|^2$, $|R_P'(0)|^2$ and $|R_D''(0)|^2$ obtained from the dispersion sum rules depend on the mass of the state and in this sense take into account the "hyperfine" splitting of the charmonium levels. By comparing these results with calculations in the potential models we can see clearly that the dispersion sum rules are in qualitative agreement with the potentials that correspond to confinement of the quarks (the "oscillator" and "funnel") and are in sharp disagreement with potentials of the Coulomb type.

We now consider the more complicated triangular diagrams (Fig. 17), which depict the creation of two photons by a fictitious current. Resonance contributions proportional to $g\sqrt{\Gamma_{\gamma\gamma}}$ are dual to these diagrams. Substituting the values of $g$ determined from the biangular diagrams, we can find $\Gamma_{\gamma\gamma}$ in this way. It is found that for $\eta_c$ and $\chi_0$ they are very close to the values that were obtained above by treating the collision of two photons. Thus, we see that the method of fictitious currents work well. We shall apply it now to the decays of mesons with $J = 2$. For ${}^3P_2$ we obtain $\Gamma(\chi_2 \to 2\gamma) \simeq 2 \pm 0.3$ keV, which is somewhat higher than the value obtained from the nonrelativistic calculation ($\sim 1.3$ keV). The width of the ${}^1D_2$ level, whose position is not yet known, depends on its mass: $\Gamma({}^1D_2 \to 2\gamma) = 115 \pm 25$ eV if $M_D = 3.45$ GeV. The corresponding hadronic width $\Gamma({}^1D_2 \to \text{hadrons})$ should then amount to approximately 100 keV.

**b) Gluon corrections to the sum rules**

Lying at the basis of the dispersion sum rules (5.3) is the assumption that the coupling constant $\alpha_s$ of the strong interaction of gluons with quarks is sufficiently small at short distances. It follows from a comparison of the hadronic width of $J/\psi$ with the gluon calculations that $\alpha_s$ in this case is approximately equal to 0.2. Below we give the results of a calculation of the corrections of first order in $\alpha_s$ to the dispersion sum rules and discuss the corrections of higher orders.

The correction of first order in $\alpha_s$ corresponds to diagrams with exchange of one virtual gluon (Fig. 18). Allowance for one-gluon exchange leads to the result that the quantity $\sigma_0(s)$, defined by the relation (5.2), appearing in the sum rule (5.3) and obtained in zeroth order in $\alpha_s$, should be replaced by the quantity

$$\sigma_1(s) = \frac{4\pi\alpha^2 Q_c^2}{s}\,\frac{3-v^2}{2}\,v\left\{1 + \frac{4}{3}\alpha_s\left[\frac{\pi}{2v} - \frac{3+v}{4}\left(\frac{\pi}{2} - \frac{3}{4\pi}\right)\right]\right\}. \quad (5.9)$$

The formula (5.9), obtained by Schwinger,[76] is an interpolation formula. It coincides with the exact formula when $v = 0$ and $v = 1$ and deviates from it by not more than 1% over the whole range of variation of $v$. In addition, we must take into account the correction of order $\alpha_s$ to the $c$-quark mass. The point is that the mass appearing in the definition of the velocity in the expression (5.9) ($v^2 = 1 - (4m^{*2}/s)$) is connected with the deep-virtual mass $m_c$ by the relation

$$\frac{m_c}{m_c^*} = 1 - \frac{2\alpha_s \ln 2}{\pi}, \quad (5.10)$$

if we use the Landau gauge for the gluon field. The fact that $m_c$ depends on the gauge should cause no surprise. In fact, only in the observable quantities does the dependence on the gauge drop out. The deep-virtual mass $m_c$ is defined in terms of the quark propagator off the mass shell, which is gauge-noninvariant. Nevertheless,

TABLE III.

| | $V(r) = \frac{m\omega r^2}{2}$ | $V(r) = -\frac{\tilde{\alpha}}{r} + gr$ | $V(r) = -\frac{\tilde{\alpha}}{r}$ | Dispersion sum rules |
|---|---|---|---|---|
| $\lvert R_S(0)\rvert^2$ | 0.5 GeV$^3$ | 0.5 GeV$^3$ | 0.1 GeV$^3$ | 0.5 GeV$^3$ for $J/\psi$<br>0.2 GeV$^3$ : $M_{\eta_c} = 2.85$ GeV } for $\eta_c$<br>0.45 GeV$^3$ : $M_{\eta_c} = 3$ GeV } for $\eta_c$ |
| $\lvert R_P'(0)\rvert^2$ | 0.12 GeV$^5$ | 0.09 GeV$^5$ | $1 \cdot 10^{-4}$ GeV$^5$ | 0.14–0.21 GeV$^5$ |
| $\lvert R_D''(0)\rvert^2$ | 0.066 GeV$^7$ | 0.07 GeV$^7$ | $1.5 \cdot 10^{-6}$ GeV$^7$ | 0.03 GeV$^7$ : $M = 3.45$ GeV<br>0.08 GeV$^7$ : $M = 3.7$–$3.8$ GeV |

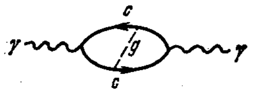



FIG. 18. Example of a diagram leading to corrections of order $\alpha_s$ to the polarization of the charmed vacuum.

it is reasonable to express the answer in terms of $m_c$, since then the gluon corrections are minimal and the final formulas do not in any case depend on the gauge. We introduce the dimensionless coefficients $A_n^{(1,0)}$ and $A_n$:

$$A_n^{(1,0)}(A_n)=\frac{(4m_c^2)^n}{4\pi Q_c^2\alpha^2}\int\frac{ds}{s^n}\sigma_{1,0}(\sigma), \tag{5.11}$$

here the superscripts on $A_n^{(1,0)}$ indicate the order of perturbation theory in $\alpha_s$. It is then easy to calculate the theoretical values of $A_n^{(1)}$ for the first five moments $n$:

$$A_1^{(1)}=\frac{4}{5}(1+0.\ 73\alpha_s),\quad A_2^{(1)}=\frac{12}{33}(1+0.71\alpha_s),$$
$$A_3^{(1)}=\frac{64}{315}(1+0.51\alpha_s),\quad A_4^{(1)}=\frac{32}{231}(1+0.22\alpha_s),$$
$$A_5^{(1)}=\frac{512}{5005}(1-0.14\alpha_s). \tag{5.12}$$

Inasmuch as all integrals over virtual loops converge for $p^2\simeq -m_c^2$, the constant $\alpha_s$ appearing in (5.12) corresponds to distances $\sim m_c^{-1}$. We shall use for it the value found from the width for $J/\psi\to$ hadrons (see Chap. 1):

$$\alpha_s(p^2=-m_c^2)=0.2. \tag{5.13}$$

As regards the $c$-quark mass determined with allowance for terms $\sim\alpha_s$, its value is practically unchanged from the estimate (5.6). (We recall that the parameter $m_c$ depends on the gauge. Our statement refers to the Landau gauge.)

In Table IV the theoretical values of $A_n^{(1)}$ and $A_n^{(0)}$ calculated for $\alpha_s=0.2$ and $m_c=1.25$ GeV are compared with the corresponding experimental numbers $A_n$. The contribution of the mesons $J/\psi$ and $\psi'$ and of the continuum to $A_n$ are shown separately; with a certain degree of arbitrariness, the latter is taken in the form $(4\pi\alpha^2Q_c^2/s)\times\theta(s-16\ \text{GeV}^2)$. Because of the small contribution of the continuum, the uncertainty associated with the fact that it is poorly known in the transitional region does not have a strong effect on the quantities $A_n$ for $n\geqslant 2$.

As can be seen from Table IV, the correction associated with $\alpha_s$ improves slightly the agreement between the theoretical and experimental results, which differ by 1% for $n=2$ and by 10% for $n=4$. (For $n=7$ the discrepancy amounts to $\sim$50%.) It should be emphasized that it is precisely this agreement of the first four moments with experiment in the case of the polarization of the charmed vacuum which is the basis of all our calculations and makes it possible to have confidence in their reliability in other cases too (when we turn to the $C$-even levels of charmonium). The second important argument in favor of the correctness of the approach to the annihilation of $C$-even levels described here is the agreement between the estimates obtained for the widths using the sum rules for scattering of light by light, on the one hand, and the estimates obtained by considering the biangular diagrams, on the other. This agreement is possible only when the masses of the resonances are close to the experimental values and is, therefore, not trivial.

The growing discrepancy between theory and experiment with increasing $n$ points to the fact that other corrections, both of higher order in $\alpha_s$ and of the power type $(1/Rm_c)^k$ ($R$ is the confinement radius), which we do not yet know how to calculate, begin to play an important role for large $n$.

If we try to estimate only the leading terms of order $\alpha_s^2$ and $\alpha_s^3$, we must iterate the contribution of the principal (for small $v$ and large $n$) Coulomb-like correction $2\pi\alpha_s/3v$ in the expression (5.8). The discrepancy between theory and experiment is then somewhat reduced but still remains considerable, and so it is apparent that the answer does not lie in the higher orders in $\alpha_s$. The question of the "Coulomb" corrections is intimately related to the question of how the large-distance behavior of the attractive potential between a quark and an antiquark affects the sum rules. It is clear that the behavior of the potential at large distances has a decisive effect on the properties of the high levels. For example, if we "cut off the tail" of the Coulomb potential at a value of $r$ such that $m^{-1}\ll r\ll(m\alpha)^{-1}$, all the levels disappear. However, modification of the potential at such values of $r$ affects the values of $A_n^{\text{theor}}$ only exponentially weakly, like $e^{-m_c r}$, since the quantity $A_n^{\text{theor}}$ is determined principally by the contribution from short distances: the deep-virtual quark is exponentially rarely at distances $r\gg 1/m_c$. It is precisely in the weak dependence of the values of $A_n^{\text{theor}}$ on the behavior of the interaction between the quarks at large distances that the strength of the dispersion approach lies.

In the discussion of corrections of higher orders in $\alpha_s$, those diagrams which have infrared divergences at small $s$ (Fig. 19) are usually considered to be the most dangerous. These diagrams, unlike the simple quark loop, contain intermediate states that are massless (or almost massless) in $s$, consisting of gluons $g$ or light quarks $q$, or both together. For these light particles at $s=0$ asymptotic freedom does not operate, since, unlike the heavy $c$ quarks, they are not deeply virtual in these conditions and their strong interaction at $s=0$ is truly strong. Such diagrams appear to be especially dangerous in the higher moments, with large values of $n$, in which their contribution to the dispersion integral is divided by $s^n$. The infrared divergences that arise here might outweigh a small factor of the type $\alpha_s^3$.

TABLE IV

| | Theory | | Experiment | | | |
|---|---|---|---|---|---|---|
| $n$ | $A_n^{(0)}$ | $A_n^{(1)}$ | $J/\psi$ | $\psi'$ | Continuum | $A_n$ (sum) |
| 1 | 0.800 | 0.917 | 0.420 | 0.103 | 0.391 | 0.914 |
| 2 | 0.343 | 0.391 | 0.274 | 0.048 | 0.077 | 0.398 |
| 3 | 0.203 | 0.224 | 0.178 | 0.023 | 0.021 | 0.222 |
| 4 | 0.139 | 0.144 | 0.116 | 0.011 | 0.005 | 0.132 |
| 5 | 0.102 | 0.099 | 0.076 | 0.004 | 0.002 | 0.082 |

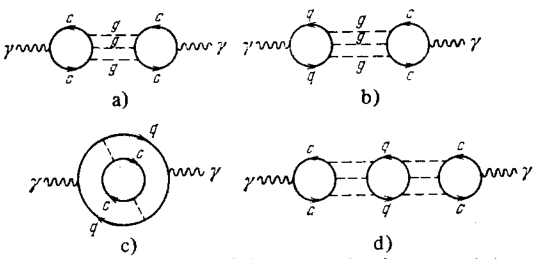



FIG. 19. Examples of diagrams leading to mixing of heavy and light quarks in high order in the coupling constant $\alpha_s$.

In fact, as we show now, these diagrams are not at all dangerous. We recall that $\psi$ and $\psi'$ are very bad at undergoing transitions to ordinary hadrons. Their hadronic decays are suppressed by 3–4 orders of magnitude compared with the decays of ordinary resonances. The same applies to the cross sections for their creation in collisions of ordinary hadrons. At the same time, their creation in colliding $e^+e^-$ beams is not suppressed at all: the electron widths of $J/\psi$ and $\psi'$ are of the same order as those of $\rho$, $\omega$ and $\varphi$. Therefore, in discussing the electron widths of $J/\psi$ and $\psi'$ we can disregard diagrams of the type in Figs. 19b and 19c, in which the photon interacts with a $q$ quark and not directly with a $c$ quark. The contribution of these diagrams in the physical region does not exceed a few percent of the contribution of the diagram of Fig. 13, and it is perfectly legitimate to neglect them in the physical region (i.e., for $s \geqslant m_\psi^2$). But this implies that we can treat the electromagnetic widths of charmonium consistently by simply "switching off" the electric charges of the light quarks.

However, not all the dangerous diagrams are eliminated from consideration in this way; some of them (of the type in Figs. 19a and 19d) remain. But these diagrams too do not present a serious threat. The point is that all the infrared divergences are connected not with the formation of charmed particles but with the formation of ordinary hadronic states, not containing even hidden charm: the physical cut in the dispersion integral starts at $(3m_\pi)^2$. Thus, these infrared parts of the diagram describe the contribution of virtual $c$-quarks in the creation of ordinary hadrons, in which we are not interested. The contribution of virtual light particles to the creation of charmed particles starts at $s = m_\psi^2$ and does not contain infrared divergences. Therefore, its order is determined by the order of perturbation theory in $\alpha_s$ and is not greater than $\alpha_s^3$.

## 6. OTHER PROCESSES WITH CHARMED PARTICLES

A heavy quark appears not only in charmonium but also in the composition of mesons $(D, F, \ldots)$ with manifest charm. Therefore, using quantum chromodynamics we can obtain definite predictions for the cross-sections for creation of these particles in various beams. An important point is that the same mass $m_c$ determines the scale of these cross sections, and, if the theory is correct, interesting relations between various quantities arise.

As an example we give the sum rule for the cross section $\sigma_c^\gamma$ for photoproduction of charmed particles (Shifman *et al.*[75]):

$$\int_{\text{threshold}}^{\infty} \frac{d\nu}{\nu^2}\sigma_c^\gamma = \frac{22\pi}{405}\alpha\alpha_s(m_c)\frac{m_N}{m_c^4}\rho\,(Q^2 = m_c^2), \tag{6.1}$$

where $\nu$ is the photon energy in the laboratory frame and $\rho(Q^2 = m_c^2)$ is the fraction of the nucleon momentum associated with the gluons. This fraction can be determined independently from the deep-inelastic electro-production of ordinary particles at $Q^2 = m_c^2$ and is approximately equal to $\frac{1}{2}$. The derivation of the sum rule (6.1) involves analyzing the diagram of Fig. 20 in the limit when the photon 4-momentum tends to zero.

Numerically, the right-hand side of the relation (6.1) amounts to approximately 20 nb/GeV, if for $m_c$ and $\alpha_s(m_c)$ we take the values determined in the theory of charmonium. The experimentally measured cross-section for photoproduction of $J/\psi$ gives an integral contribution of the order of 1 nb/GeV to the left-hand side of the equality. Thus, the cross-section for photoproduction of $D^+D^-$-meson pairs and other charmed particles should be more than an order of magnitude greater than the cross-section for photoproduction of the $J/\psi$ meson.

Analogous sum rules arise for the electro- and neutrino-production of charm.

Dispersion sum rules analgous to the charmonium rules can be obtained for the weak decays of the pseudoscalar charmed mesons ($D\to\mu\nu$ and $F\to\mu\nu$) and for the cross-sections for the diffractional production of vector mesons $D^*$ and $F^*$ in the collision of a neutrino with nucleons (Novikov *et al.*[71]). In particular, from these sum rules follow the inequalities

$$\begin{aligned}&\Gamma(F\to\mu\nu) < 5.3\cdot 10^{19}\ \text{sec}^{-1},\\ &\frac{g_{F^*}^2}{4\pi} > 3.6,\end{aligned} \tag{6.2}$$

where $g_{F^*}$ are the constants of the transformation of the current $\bar{c}\gamma_\mu s$ to $F^*$, normalized in the standard way. It should be emphasized, however, that in this case the expected power corrections to the bare quark loops should be large, since one of the quarks is light (see Fig. 21).

Virtual transitions to intermediate states with charmed quarks play an important role in the weak interactions of ordinary particles. In particular, in the standard four-quark scheme of Glashow, Iliopoulos and Maiani,[6] the mass difference of the $K_L$ and $K_S$ mesons is fairly well described by the quark diagram of Fig. 22. Allowance for gluon exchanges leaves the result practi-

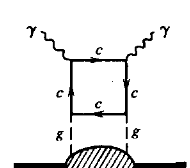


FIG. 20. Contribution of charmed quarks to the photon-nucleon Compton-scattering amplitude. [In the limit of zero photon frequency the corrections to this diagram are small and it determines the right-hand side of the relation (6.1).]

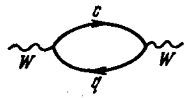



FIG. 21. Contribution of $c$-quarks to the polarization operator of the $W$-boson in zeroth order in the coupling constant of the strong interactions.

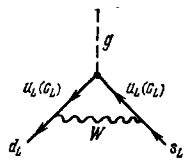



FIG. 23. Monopole and anopole transitions $s\to d+$ gluon. [The subscript $L$ denotes a left-handed spinor, e.g., $d_L\equiv(1+\gamma_5)d/2$.]

cally unchanged (cf. Vaĭnshteĭn *et al.*[77]). The answer here is proportional to $m_c^2$.

Quantum chromodynamics give the possibility of a theoretical explanation of the rule $\Delta T=\frac{1}{2}$ for the nonleptonic decays of strange particles. The dominant diagram is found to be that of Fig. 23, which gives the gluon monopole moment and anopole moment of the transition of a $d$ quark to an $s$ quark (Vaĭnshteĭn, Zakharov and Shifman[78]). Since the isospin of the gluons is equal to zero, this diagram obviously obeys the selection rule $\Delta T=\frac{1}{2}$. The dominant role of the diagram is connected with the fact that it leads to an effective four-fermion interaction with participation not only of left-handed quarks at the parity-nonconserving lower vertex in the diagram of Fig. 24a, but also of right-handed quarks (at the parity-conserving upper (strong) vertex). The corresponding term in the effective Hamiltonian of the weak nonleptonic interactions has the structure

$$H^{\text{eff}}(\Delta s=1)=\sqrt{2}\,G_F\sin\theta_C\cos\theta_C C\,[(\bar{s}_L u_R)(\bar{u}_R d_L)+(\bar{s}_L d_R)(\bar{d}_R d_L)+(\bar{s}_L s_R)(\bar{s}_R d_L)]+\dots\quad \psi_{L,R}\equiv\frac{1}{2}(1\pm\gamma_5)\psi, \tag{6.3}$$

where $G_F$ is the Fermi four-fermion constant and $\theta_C$ is the Cabibbo angle. The coefficient $C$ takes into account the effects of the strong interaction at short distances. It is practically independent of the choice of model for the weak interaction and is close to unity.[78] As a result the effective four-fermion interaction with $\Delta T=\frac{1}{2}$ converts left-handed quarks to right-handed quarks and gives matrix elements for the weak emission of $\pi$ mesons that are greater than the usual matrix elements arising from the left-handed currents by the factor $m_\pi/(m_u+m_d)$, where $m_\pi$ is the physical mass of the pion and $m_u$ and $m_d$ are the mechanical masses of the $u$- and $d$-quarks (an estimate of these was given in Sec. 4 of Chap. 1). Thus, a highly nontrivial connection arises between the exact character of the rule $\Delta T=\frac{1}{2}$ and the small mass of the light quarks. An investigation of the contribution of the Hamiltonian (6.3) to the amplitudes of the decays $K\to 2\pi$, $3\pi$, $\Lambda\to N\pi$, $\Omega\to\Xi\pi$, etc., gives numbers coinciding with the experimental values in sign and close to them in magnitude.[78]

We shall discuss briefly the possibility of applying the dispersion approach and asymptotic freedom to processes with $u$, $d$ and $s$ quarks and to hypothetical states of the charmonium type constructed from new, still heavier quarks.

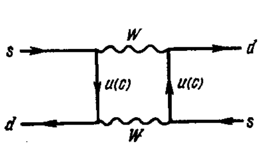



FIG. 22. Simplest quark diagram for the mass difference of the $K_L$ and $K_S$ mesons.

For the light quarks we can use asymptotic freedom only at large space-like external momenta. Therefore, simple dispersion sum rules exist only for the biangular diagrams and not for the triangular or quadrangular diagrams. The well-known Weinberg sum rules[43a] (see also Ref. 79) follow from consideration of the bihagular diagrams in the framework of quantum chromodynamics. However, because of the necessity of eliminating the continuum, the threshold for which is low in this case, the Weinberg sum rules impose considerably weaker restrictions on the properties of ordinary mesons than the $c$ quark sum rules do on the properties of the levels of charmonium.

Dispersion sum rules can turn out to be fruitful in the analysis of the levels of an "onium" (the term is due to Bjorken) consisting of quarks much heavier than the $c$ quarks, say, with mass of the order of 10 GeV. In this case the threshold for pair creation of new particles can also be appreciably greater than the energies of the "onium" levels and the sum rules can be saturated by the resonances. Without going into details, we note that the system of levels of the "onium" may be considerably richer than that of charmonium, if the potential between the heavy quarks is of the same type as in charmonium.

## CONCLUSION

The discovery and subsequent study of the properties of charmonium confirms the basic ideas of the quark model of the hadrons and the concepts concerning the properties of the interaction of quarks at short distances, built up in recent years. Charmonium is a unique system, both because of the richness of its levels and because of the large mass of the quark. The charmonium

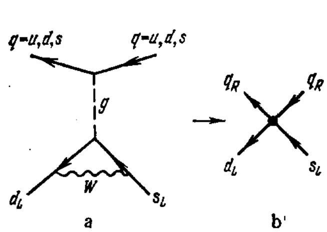



FIG. 24. Contribution, associated with the transition depicted in Fig. 23, to the effective Hamiltonian of weak nonleptonic interactions with change of strangeness. [Inasmuch as the lower vertex in the diagram (a) vanishes for real gluons, the diagram (a) reduces to the effective four-fermion point interaction represented in Fig. (b); $q_R$ denotes the right-handed component of the quark field: $q_R=(1+\gamma_5)q/2$.]

levels with mass less than 4 GeV are the most intuitive and convincing example of the quark spectroscopy of the hadrons. The study of the radiative transitions confirms the simple estimates of the atomic model of charmonium.

Owing to its large mass, the $c$ quark is an excellent test object for investigating the properties of strong interactions at short distances. In the framework of the dispersion sum rules, the properties of the wavefunction of the $c$ and $\bar{c}$ quarks for the lowest level of charmonium are practically uniquely determined by bare quark loops with zero external momenta. The success of the sum rules for the creation of charm in $e^+e^-$ collisions permits us to have confidence in the correctness of the predictions of quantum chromodynamics for a broad class of charm-production processes in the collisions of photons and leptons with hadrons.

Clearly, the picture discussed is still not a final picture, and further experimental data may turn out to be decisive. There can be little doubt that further study of the properties of charmonium will not only bring confirmation of the ideas that have accumulated but will also pose new questions. An excellent example of this kind is the discovery of the charmonium molecule with mass 4.028 GeV. Certain difficulties now arise in the interpretation of the properties of para-charmonium (the $\eta_c$ and $\eta_c'$ mesons). It is not ruled out that these difficulties are connected with fundamental problems of some kind.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