Stimulated effects upon "jarring" of an electron in an external electromagnetic field

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Various elementary processes that involve imparting a large momentum to an electron, and which occur in an external laser field, stimulate effects of absorption and emission of quanta of this field. As a rule, the first stage of this process (e.g., perhaps the scattering of the electron, its motion in an inhomogeneous medium, emission or absorption of a light quantum by the electron, the Compton effect, radioactive decay, the photoelectric effect, etc.) occurs in times τ much shorter than the period $2\pi/\omega$ of the low-frequency motion of the electron caused by the external field. In this case, the second stage (stimulated emission-absorption) does not depend on the physical nature of the first stage, and it is universal for all processes. The problem of stimulated effects of the stated type can be solved for once and for all as a problem of the "jarring" of the electron in the presence of the external field. In non-relativistic fields ($v/c \ll 1$), the probability of the stimulated processes is determined by the parameter $N \sim |(v/c)c\delta\mathbf{p}/\hbar\omega|$, which denotes the rms number of quanta emitted or abosrbed. In the limiting cases $N \gg 1$ and $N \lesssim 1$, the problem of "jarring" is solved classically or quantum-mechanically, respectively. In the Compton effect, where $\delta\mathbf{p} = \hbar(\mathbf{k}_1 - \mathbf{k}_2)$, or in "jarring" due to emission (or absorption) of a hard photon, when $\delta p = \hbar\mathbf{k}$, the parameter N does not contain the Planck quantum. Hence one can explain the effect of satellites appearing in the emission spectrum from a purely classical standpoint for any N. In addition to the Compton effect, the article treats also stimulated processes in β-decay and in the photoelectric effect occurring in an external laser field.

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CONTENTS

1. INTRODUCTION

As laser technology has progressed, problems are being posed in a number of studies of the effect of strong electromagnetic fields on various elementary atomic processes.[1–5]

In spite of the seeming diversity of the studied problems, they contain much in common. This generality is manifested in the fact that problems that are quite different in their formulation, such as bremsstrahlung in the presence of an ac field[1,4] and modulation of an electron beam as it passes through a dielectric plate in the presence of a laser field,[6] contain answers that differ from one another actually only by change of nomenclature. As we see it, this coincidence has deep physical causes, and it involves the fact that an essential feature of the studied phenomena proves to be a change in the motion of an electron in times much shorter than the period of the low-frequency movements in the external field. This common feature permits us to describe phenomena of this kind from a unified standpoint as being the "jarring" of the electron in the presence of the electromagnetic field.

We shall show with a number of concrete examples that the studied phenomena can be pictured as occurring in two stages. A certain change in the motion of the charged particle occurs in the first stage (collision with another particle or with a wall, absorption or scattering of an x-ray quantum by it, creation by β-decay, or the photoelectric effect, etc.). All of the physical specifics of the problem is manifested in this first stage. The amplitude of the probability a^{I} for the first stage is calculated for each concrete process by its own rules. However, since we assume that this stage is executed in a time τ much shorter than the period $2\pi/\omega$ of the low-frequency motions, the moderately strong field cannot substantially affect the motion of the charged particle within this time. This means that the amplitude a^{I} can be calculated in the same way as in the absence of the external field. The second stage consists in stimulated absorption or emission of a certain number of photons of the external field. Although this second stage would not arise by itself in the absence of the first stage, the amplitude a^{II} that corresponds to it does not depend on the physical nature of the first stage. In this sense, the second stage is universal for all processes of the studied type. In line with what we have said, the amplitude of the probability of any process involving absorption or emission of n quanta of the external field is written as the product $A_n = a_n^{\rm I} a_n^{\rm II}$.

We shall treat moderately strong fields that are attainable at the present level of development of laser technology, in which the ac velocity $\bar{v} = eE/m\omega$ acquired by the electron is much smaller than the velocity of light c. We note that even in these (nonrelativistic) fields, the contribution of stimulated multiphoton processes to the studied processes can become decisive.

In Chap. 2, we calculate the amplitude of the probability $a_n^{\rm II}$ of the second stage, and as we have mentioned, we don't need to concretize the process corresponding to the amplitude $a_n^{\rm I}$ in order to solve this problem. In Chap. 3, we give a brief review of certain effects that had been treated previously by various authors for physically differing processes. We wish to show that they all are of one common nature, and they result from the jarring of the electron in the external field.

In Chap. 4, we treat the purely classical effect of appearance of satellites in the spectrum of scattered hard quanta in the Compton effect involving an electron lying in an external laser field. Finally, we study in Chap. 5 the spectrum of photoelectrons and β-particles that arise in the presence of a laser field.

2. STIMULATED EMISSION OR ABSORPTION UPON "JARRING" OF AN ELECTRON

A. Classical treatment

Let us first treat the classical problem of the jarring of a nonrelativistic electron that is moving in an ac electric field: $(1/c)\dot{\mathbf{A}}(t)$. For the sake of argument we shall assume that the field $\mathbf{A}(t)$ is turned off adiabatically as $t \to \pm\infty$.

The momentum of the electron in the field is

$$\mathbf{p}(t) = \mathbf{p}_- - \frac{e}{c}\mathbf{A}(t) \text{ where } t < t_0,$$

$$\mathbf{p}(t) = \mathbf{p}_+ - \frac{e}{c}\mathbf{A}(t) \text{ where } t > t_0;$$

Here t_0 is the instant of time at which any process happens to the electron that leads to a fast (as compared with the period of the electric field) change in its motion (jarring), $p_\pm$ is the value of the momentum of the electron as $t \to \pm\infty$. The resultant change in the momentum of the electron is

$$\frac{p_+^2 - p_-^2}{2m} = \delta\varepsilon + \frac{e}{mc}\delta\mathbf{p}\mathbf{A}(t_0) \equiv \delta\varepsilon + \Delta, \tag{1}$$

where

$$\delta\mathbf{p} = \mathbf{p}(t_0+0) - \mathbf{p}(t_0-0) = \mathbf{p}_+ - \mathbf{p}_-, \quad \delta\varepsilon = \frac{p^2(t_0+0) - p^2(t_0-0)}{2m}$$

are the momentum and the energy imparted to the electron in the fast stage of the process. Their values coincide with the corresponding values in the absence of the electric field.

The quantity Δ in Eq. (1) is the field contribution to the energy imparted to the electron. Since this contribution depends on the instant t_0 of jarring, the electrons in the final state will have an energy spread. In order to calculate the averages, we must average over the phase of the field $\phi_0 = \omega t_0$ at which the jarring happened, while assuming all phases to be equally probable. Since $A \sim \cos\phi_0$, we get $\langle\Delta\rangle = 0$ for the mean energy, and the following for the scatter in energy:

$$\Delta_0 \equiv \sqrt{\langle\Delta^2\rangle} = \frac{|e\mathbf{E}_0\delta\mathbf{p}|}{\sqrt{2}\, m\omega}; \tag{2}$$

Here $\mathbf{E}_0$ is the amplitude of the electric field having the vector potential $\mathbf{A}$. We can also easily derive the energy distribution of the electrons in the final state:

$$W(\Delta)\,d\Delta \equiv \frac{d\varphi}{2\pi} = (\Delta_m^2 - \Delta^2)^{-1/2}\frac{d\Delta}{2\pi}, \tag{3}$$

Here Δ_m is the maximum value of the energy imparted to the field or taken from it, which is connected with the rms energy scatter by the relationship $\Delta_m = 2\sqrt{\Delta_0}$.

In order that the presented classical calculation be applicable, the characteristic number of quanta of the electric field emitted or absorbed in the jarring process

$$N \equiv \frac{\Delta_m}{\hbar\omega} = \frac{|e\mathbf{E}_0\delta\mathbf{p}|}{\hbar m\omega^2} \tag{4}$$

must be much larger than unity. But this is just the case where the stimulated effects are large. The introduced parameter N is of the order of magnitude of the ratio of the amplitude of the oscillations of the electron in the external field $a_0 = eE_0/m\omega^2$ to the de Broglie wavelength of the electron as calculated from the transferred momentum, $\lambda\!\!\!^{-}_e = \hbar/\delta p$.

In line with what we have said above, the derived probability distribution for the second stage is universal, and it is given in the classical limit by Eq. (3). As for the first stage, it can be (and most often is) doubly of quantum type. We note that the total probability of the process does not depend on the existence of the field, since

$$\int d\Delta W(\Delta) = 1.$$

B. Quantum treatment

A quantum treatment of the second stage of the process is needed in the case in which $N \lesssim 1$. As before, just as in the classical variant, the electron can be jarred at an arbitrary instant of time, which corresponds to an arbitrary phase of the field. However, now we must sum the amplitudes that describe the different pathways of evolution of the electron, and this leads to quantum interference effects.

For example, let an electron of momentum $\mathbf{p}_0$ lie within the field $\mathbf{A}(t)$ at some instant of time t_0. The state of electron subsequently evolves. The field is adiabatically turned off as $t \to \infty$. Consequently the electron energy $p_+^2/2m$ does not have a definite value, but is characterized by the probability amplitude $a^{\rm II}$. Upon using the well-known expression for the wave function of the electron in the field $A(t)$,

$$\psi_p = C \exp\left[\frac{i}{\hbar}\mathbf{pr} - \frac{i}{2m}\int dt\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2\right], \qquad (5)$$

we get the amplitude of the above-described process

$$a^{II} \sim \exp\left[-i\frac{p_0^2}{2m}t_0 + \frac{i}{2m}\int_{t_0}^{\infty} dt\left(\mathbf{p}_+ - \frac{e}{c}\mathbf{A}\right)^2\right]. \qquad (6)$$

Upon integrating it over the instant of jarring, t_0, we find

$$\langle a^{II}\rangle \sim \int dt_0 \exp\left[-i\frac{p_0^2}{2m}t_0 + \frac{i}{2m}\int_{t_0}^{\infty} dt\left(\mathbf{p}_+ - \frac{e}{c}\mathbf{A}\right)^2\right]. \qquad (7)$$

In a field of the form $\mathbf{A} = (c\mathbf{E}_0/\omega)e^{-\Gamma t}\cos\omega t$, with $\Gamma \to +0$, if we assume for simplicity that $p_+ \gg (e/c)A$, we have

$$\langle a^{II}\rangle = \sum_{n=-\infty}^{\infty} a_n^{II}\delta\left(\frac{p_+^2 - p_0^2}{2m} + n\hbar\omega\right),$$
$$|a_n^{II}| = J_n^2(N), \quad N = \frac{|e\mathbf{E}_0\mathbf{p}_+|}{\hbar m\omega^2}, \qquad (8)$$

where J_n is a Bessel function. Thus the energy spectrum of the electron as $t \to \infty$ amounts to a set of satellites that differ from $p_0/2m$ by a multiple of the quanta $\hbar\omega$ that are absorbed or emitted in the external field. The appearance of these new channels is characterized by the weight $J_n^2(N)$, and it is determined by the value of N.

Owing to the properties of the Bessel functions $J_n(N)$, the only amplitudes a_n^{II} that are important at large values of the argument N are those whose index $|n| \lesssim N$. This corresponds to the classical result of the preceding section, and it actually means that the effective broadening of the energy spectrum of the electron at infinity is of the order of $N\hbar\omega \equiv \Delta_m$.

In a number of cases, e.g., β-decay or the photoelectric effect with absorption of a hard quantum, the emerging electron can possess a relativistic velocity. In order to study stimulated emission or absorption under such conditions, let us treat the problem of incidence of a relativistic electron having the 4-momentum p_0 into the field of a plane electromagnetic wave $A^\mu = A^\mu(\phi)$, with $\phi = k_0 x$ (we are using 4-dimensional notation, and $k_0 x = \mathbf{k}_0 \cdot \mathbf{r} - \omega t$).

If the field $\mathbf{A}(t)$ is of moderate intensity, i.e., $\bar{v}/c \ll 1$ (we recall that we mean by $\bar{v}$ the nonrelativistic increment of the generally relativistic velocity of the electron that it acquires in the external field), then Volkov's[7] solution for the wave function of the electron is considerably simplified:

$$\psi_p = Cu_p \exp\left[-ipx - \frac{ie}{c(k_0 p)}\int^{k_0 x} d\varphi\left(pA - \frac{e}{2c}A^2\right)\right], \qquad (9)$$

All of the spinor expressions here are represented by the single coefficient u_p, which is the bispinor amplitude of the free plane wave. Yet the spin properties of the electron with respect to its motion in the electromagnetic field are not manifested in any way. This simplification does not require other spinor transformations in calculating the cross-sections, apart from those that are also needed in the absence of the external field.

Therefore, even in the case of relativistic jarring, the amplitude $\langle a^{II}\rangle$ in the studied fields is written by analogy with (8) as

$$\langle a^{II}\rangle = \sum_n a_n^{II}(N)\,\delta(p_+ - p_0 - nk_0). \qquad (10)$$

The functions a_n^{II} and the parameter N in (10) have the same meanings as before. For example, for the field

$$A = e^{-\Gamma t}(a_1\cos\varphi + a_2\sin\varphi), \quad a_1 a_2 = a_1 k_0 = a_2 k_0 = 0,$$

their form as $\Gamma \to +0$ is as follows:

$$|a_n^{II}|^2 = J_n^2(N), \quad N = \frac{|e|}{c(k_0 p_+)}\sqrt{(a_1 p_+)^2 + (a_2 p_+)^2}.$$

Since each of the channels of index n corresponds to its own law of conservation of 4-momentum $p_+ = p_0 + nk_0$, the probability of the fast stage $|a_n^I|^2$ also proves to depend on n. Yet if we can neglect this dependence for all $|n| \lesssim N$, as often happens, then the following sum rule holds for the total contribution of all channels:

$$\sum_n |A_n|^2 \approx |a^I|^2 \sum_n |a_n^{II}|^2 = |a^I|^2. \qquad (11)$$

C. Applicability of the "jarring" concept

In order that the "jarring" approximation be applicable to the problem of stimulated emission or absorption, the first stage of the process must occur in times τ that are short in comparison with the period of the low-frequency motion in the external field: $\omega\tau \ll 1$. We note that this inequality is easily satisfied. Moreover, in the case of reversed inequality in which the "jarring" approximation does not hold, the effects of stimulated emission or absorption will be extremely small. Therefore the "jarring" approximation is actually always applicable for studying stimulated effects of the stated type.

One can easily estimate the times τ in each concrete process. Thus, for example, when the hard quantum $\hbar\Omega$ is emitted, absorbed, or scattered by an electron, the inequality $\omega\tau \ll 1$ amounts to the following condition for the frequencies: $\Omega \gg \omega$. In collision processes involving atoms, the quantity τ is determined by the time of flight of the electron through the characteristic atomic dimensions. For example, in the case of electron scattering and in the photoelectric effect, we have, respectively:

$$E_{el} \gg \frac{\omega}{\Omega_{at}}\hbar\omega, \quad \Omega \gg \frac{\omega}{\Omega_{at}}\omega,$$

where the Ω_{at} are the characteristic atomic frequencies.

The inequality $\omega\tau \ll 1$ is a necessary but generally not a sufficient condition for application of the "jarring" concept. Evidently, an intense enough field can substantially affect the collision processes, even in the short time τ. Hence we must also specify a limitation on its intensity. The amplitude of the vector potential $\mathbf{A}(t)$ determines the size of the parameter N, and by using the latter, we can therefore formulate this limitation.

In order to seek a sufficient condition, we must require that the phase shift caused by the field $\Phi \sim \int dt \mathbf{p} \cdot \mathbf{A}$ within the times τ is not significant. Without going into the details of calculating $|a^{II}|^2$, this directly allows us to write the criterion

$$(N+1)\,\omega\tau \ll 1, \tag{12}$$

such that the "jarring" approximation is known to be applicable when it is satisfied.

Yet actually, we can relax the condition (12) to a considerable extent when $N \gg 1$. There are two different kinds of jarring: alternation $\delta\mathbf{p}$ of the momentum of the electron in a short time, or alteration $\delta\mathbf{A}$ of the field in such a time (e.g., when the electron passes through the phase boundary of two media). In both cases, the time-dependence of the integrand in Φ is: $f((t-t_0)/\omega)\sin\omega t$. The function f varies sharply in the time interval from t_0 to $t_0+\tau$, and the field is adiabatically turned off as $t \to \pm\infty$. The condition $\omega\tau \ll 1$ allows us to express the phase that we are interested in:

$$\Phi = N\omega \int_{-\infty}^{+\infty} dt\, f\left(\frac{t-t_0}{\tau}\right) \sin\omega t$$

in terms of the moments of the derivative f' of the function (if they exist):

$$\Phi \approx N\left(1-\frac{b}{2}\omega^2\tau^2\right)\cos\omega t_0 - Na\omega\tau\sin\omega t_0,$$

$$a\omega\tau = \int dx\,(x-x_0)\,f'\left(\frac{x-x_0}{\omega\tau}\right),$$

$$b\omega^2\tau^2 = \int dx\,(x-x_0)^2\,f'\left(\frac{x-x_0}{\omega\tau}\right).$$

Consequently, owing to the finite duration of the first stage, the following new argument appears in the corresponding expansion of $\exp(i\Phi)$ in the Bessel functions $J_n(\mathcal{N})$:

$$\mathcal{N} \approx N\left(1+\frac{a^2-b}{2}\omega^2\tau^2\right),$$

By hypothesis, this must be close to the quantity N. Thus, when $N \gg 1$, the sufficient condition (12) is replaced by the weaker inequality

$$N\omega^2\tau^2 \ll 1. \tag{13}$$

3. SOME EFFECTS THAT CAN BE DESCRIBED AS THE "JARRING" OF AN ELECTRON

Among these effects, we first recall the well-studied stimulated bremsstrahlung effect[1] (SBE—emission or absorption of several quanta of an external laser field induced by scattering of an electron). The stimulated processes become important when

$$N = \frac{|e\mathbf{E}_0(\mathbf{p}_+ - \mathbf{p}_-)|}{\hbar m\omega^2} \gtrsim 1.$$

In the studied nonrelativistic fields, this requires fast electrons, while the scattering event itself can be considered to be a "jarring" as compared with the effect of the external field. Interestingly, the electron does not change in energy here in the first stage of the process, and the entire jarring consists only in changing the direction of the momentum.

Detailed study of the stimulated Bremsstrahlung effect has arisen recently from the growing interest in the problem of heating of a plasma by laser fields.[8–10] Naturally, a compact expression for the probability of the second stage of the SBE, $|a_n^{II}|^2 = J_n^2(N)$, can be obtained only in the very simple case of the Born approximation (in which the energy of the electron is much larger than the ionization potential of the atom).

Going outside the framework of the Born approximation[11–14] considerably complicates the final expression for the SBE cross-sections, but it remains as before in essence. The results of the Born approximation[1] are reproduced in an elegant method that used a transformation to an oscillating coordinate system bound to the electron.[15–16]

There is an entire set of phenomena in which the "jarring" of the electron arises from its emission or absorption of a photon. If the energy of the emitted (absorbed) quantum $(\mathbf{k}, \Omega)$ is large, so that the frequency $\Omega \gg \omega$, then the parameter

$$N = \frac{|e\mathbf{E}_0\mathbf{k}|}{m\omega^2}, \tag{14}$$

that determines the role of the stimulated processes can become large enough, even in nonrelativistic fields. The condition $\Omega \gg \omega$ means that the first stage (emission or absorption of the high-frequency quantum) amounts to the "jarring" of the electron in the external field.

Attention is called to the fact that Planck's constant $\hbar$ did not enter into the expression for the parameter in (14), and N is simply the ratio of the amplitude of oscillation of the electron in the field $a_0 = eE_0/m\omega^2$ to the wavelength of the emitted (or absorbed) photon, $N \sim a_0/\lambda$. Under the condition $\Omega \gg \omega$, the amplitude a_0 can become of the order of λ, even in a nonrelativistic external field, when $eE_0/m\omega c \ll 1$. This requires a field intensity such that $\bar{v}/c$ becomes of the order of ω/Ω. Evidently, in this case the motion of the electron in the external field can substantially change the pattern of the emission (or absorption).

The absence of $\hbar$ in Eq. (14) leads to the idea that the studied effect allows a classical interpretation, even in the case when N is not large. We shall show how one can find the probability of the second stage $|a_n^{II}|^2$ by starting with a purely classical treatment.

An electron moving along an arbitrary trajectory $\mathbf{r}(t)$ emits a spectrum of frequencies in the range $-\infty < \Omega < +\infty$. The Fourier component of the vector potential of the emission field is found from the following expression[17]:

$$\mathbf{A}_\Omega = e\frac{e^{ikR_0}}{cR_0}\int_{-\infty}^{\infty} dt\, \mathbf{v}(t)\, e^{i\Omega t - i\mathbf{k}\mathbf{r}(t)}. \tag{15}$$

R_0 is the distance from the charge to the point of observation at the instant t of time. Now let us assume that the overall trajectory $\mathbf{r}(t)$ differs from some assigned trajectory $\mathbf{r}_0(t)$ by an ac additive term:

$$\mathbf{r}(t) = \mathbf{r}_0(t) + \mathbf{a}_0 \sin\omega t.$$

The physical reasons for the appearance of this additive term can differ, e.g., an external laser or magnetic field, a spatial periodicity of the properties of the medium, etc. We shall assume henceforth that the following inequality holds: $v_0 = |\dot{r}_0| \gg a_0\omega$. If the emission field is determined by the vector potential A_{Ω_0}, then Eq. (15) implies for a trajectory $r(t)$ of the above-cited type that

$$A_\Omega = \sum_{n=-\infty}^{\infty} J_n(N) A_{\Omega_0+n\omega}, \quad N = |ka_0|. \tag{16}$$

Thus an entire set of frequencies $\Omega_0 + n\omega$ appears in the emission spectrum instead of the single frequency Ω_0, with the relative contributions $J_n^2(N)$ to the intensity.

The very simple examples of jarring of an electron owing to emission include bremsstrahlung at a nucleus in the presence of an external laser field. Among the less common jarring effects, we must list transition radiation and Vavilov–Cerenkov radiation occurring on a background of slow variations of trajectory in an external field. In transition radiation, the role of jarring is played by the change in the electric field as the electron crosses the phase boundary of the media.

The modulation of an electron beam while passing through a plate along which an electromagnetic wave is being propagated[6] is of the same nature as the effects of stimulated transition emission or absorption. Two jarring events occur as it passes through the plate: in entering and in emerging from it. The scatter in the final energy of the electrons gives rise to their spatial grouping, while the current density becomes modulated at the frequency of the field and its harmonics.

A number of studies[5,20] have investigated the effect of external fields on the Vavilov–Cerenkov effect. This phenomenon can also be studied as resulting from jarring of the electrons of the medium by the passage of the charged particle. Since this jarring stimulates the emission of satellites, the phase conditions are altered, and the Cerenkov cone is split into a system of cones, each of which corresponds to emission of a Cerenkov quantum and is accompanied by absorption (stimulated emission) of several quanta of the external field.

Let us examine this problem in greater detail. The laws of conservation of 4-momentum in the process of Cerenkov emission with simultaneous emission of n quanta of the external field $(\mathbf{k}_0, \omega)$

$$p_1 = p_2 + \hbar k + n\hbar k_0, \quad n = 0, \pm 1, \pm 2, \ldots$$

(p_1 and p_2 are the d-momenta of the electron before and after emission) imply the following frequency-angular relationship for the satellites:

$$\cos\theta_n = \frac{v_{ph}}{v} + \frac{n\omega}{\Omega}\left(\frac{v_{ph}}{v} - \cos\alpha\right); \tag{17}$$

Here θ_n is the angle between the directions of emission and of the velocity of the particle, v_{ph} is the phase velocity of the light in the medium, and α is the angle between the direction of propagation of the laser wave and the velocity of the particle.

Equation (17) has a number of interesting consequences. Thus, when $\alpha = \theta_0$, when the laser field propagates along a generator of the fundamental Cerenkov cone, all the satellites of (17) coincide, since $\theta_n = \theta_0$ for any n. The satellite cones, e.g., those with positive n, can lie either inside or outside the Cerenkov cone, depending on the relationship between the quantities $\cos\alpha$ and v_{ph}/v, i.e., on the direction of propagation of the laser wave.

The effect of an ac external field on Vavilov–Cerenkov radiation can involve two mechanisms. The first amounts to a change in the law of motion of the charged particle when acted on by the ac field (a dc magnetic field that gives rise to cyclotron rotation of the charged particle also plays an analogous role). The second mechanism involves the direct effect of the laser field on the motion of the charges of the medium. The relative contribution of these two mechanisms can differ. In the case in which the Cerenkov radiation arises from the passage of a heavy charged particle (e.g., a proton), the second mechanism proves to predominate.

The first of these mechanisms is simple, since it is caused by a change in the motion of a single particle. The second one is considerably more complicated, since it involves the nonlinear characteristics of the medium. This problem lies outside the scope of this article, and we shall spend no more time on it.

The emission intensity in each of the Cerenkov cones in the first mechanism is proportional simply to $I_n \sim J_n^2(N)$, where $N \sim |\mathbf{k}\cdot\mathbf{a}|$, and $\mathbf{a}$ is the amplitude of the low-frequency motion of the fast particle in the external field. As we see from this formula, the emission pattern does not possess axial symmetry in any of the cones but the fundamental cone ($n = 0$). The inhomogeneity in the intensity distribution over the cross section of a cone becomes larger as its index number increases.

It is interesting to note that the effect of an external laser field of frequency $\omega < \omega_p$ (where ω_p is the plasma frequency) in semiconductors having a small effective mass m^* of free electrons increases greatly. The parameter N can increase thereby by two orders of magnitude. Therefore, intensities of laser radiation smaller by a factor of 10^4 will suffice for observing the same stimulated effects (we shall not treat here the problem of breakdown and absorption in semiconductors, which, of course, remain open).

4. THE EFFECT OF A LASER FIELD ON SCATTERING OF HARD RADIATION BY ELECTRONS

The Compton effect occurring in an external laser field is very convenient for illustrating all the features of the stimulated processes that we are discussing. Let us study the scattering of a high-frequency quantum $(\mathbf{k}_1, \Omega_1)$ by an electron lying in an ac field of frequency ω. It is evident from general considerations, just as in the emission case, that the parameter N that characterizes the contribution of the satellites to the Compton spectrum should not contain Planck's constant $\hbar$. Hence, even when $N \lesssim 1$, the effect of appearance of satellites is purely classical.

Hence the posed problem can be solved as the problem of the emission from an electron lying in the field of two electromagnetic waves of differing intensities. The classical solution neglects the Compton recoil, as is valid in the frequency range $\hbar\Omega \ll mc^2$. This allows us to assume that $\Omega_1 = \Omega_2 \equiv \Omega$. Moreover, the interaction of the electron with the laser field is accounted for in the dipole approximation.

Solution of the equation of motion

$$m\ddot{\mathbf{r}} = e\mathbf{E}_0 \sin \omega t + e\mathbf{E}_1 \exp (i\mathbf{k}_1\mathbf{r} - i\Omega t)$$

in the first order with respect to E_1 ($E_1 \ll E_0$, $\mathbf{r} = \mathbf{r}_0 + \mathbf{r}_1$, $r_1 \ll r_0$) gives

$$m\ddot{\mathbf{r}}_1 \approx e\mathbf{E}_1 \exp (i\mathbf{k}_1\mathbf{r}_1 - i\Omega t - i\rho \sin \omega t), \quad \rho = \frac{e\mathbf{E}_0\mathbf{k}_1}{m\omega^2}. \qquad (18)$$

One integrates this equation after expanding the last exponential in series in Bessel functions $I_n(\rho)$. It suffices to consider the coefficients $[1 + n(\omega/\Omega)]$ that arise in the corresponding terms of the expansion to be equal to unity, since the only substantial ones are those with $|n| \lesssim N$, and even with $N(\omega/\Omega) \sim \bar{v}/c \ll 1$. In this sense, we can say that the last exponential in (18) varies slowly as compared with the others, so that we get

$$\dot{\mathbf{r}}_1 \approx \frac{ie\mathbf{E}_1}{m\Omega} \exp (i\mathbf{k}_1\mathbf{r}_1 - i\Omega t - i\rho \sin \omega t). \qquad (19)$$

This gives an expression for the vector potential of the field of the high-frequency radiation from the electron:

$$\mathbf{A}_2 = \frac{e}{cR_0} \dot{\mathbf{r}}_1 \Big|_{t - \frac{R_0}{c} + \frac{\mathbf{r}\mathbf{n}_2}{c}} \approx \sum_{n=-\infty}^{\infty} \mathbf{A}_2(n)\, J_n(N), \qquad N = \frac{|e\mathbf{E}_0(\mathbf{k}_1 - \mathbf{k}_2)|}{m\omega^2}; \qquad (20)$$

Here R_0 is the radius vector drawn from the electron to the point of observation of the field, the velocity $\dot{\mathbf{r}}$ is substituted in with account taken of retardation, $\mathbf{n}_2 = c\mathbf{k}_2/\Omega$, and the quantity $\mathbf{A}_2(n)$ differs from the corresponding value in the absence of the low-frequency field by the substitution $\Omega \to \Omega + n\omega$.

If we calculate in the usual way[17] the energy flux of the radiation of each of the satellites in (20), we find the partial scattering cross sections with radiation of the frequency $\Omega + n\omega$:

$$\frac{d\sigma_n}{dO_2} = \frac{d\sigma_{\text{Thomps}}}{dO_2} J_n^2(N). \qquad (21)$$

Since the condition $\Omega \gg \omega$ is required for observing satellites in nonrelativistic fields, the first stage can be treated as a "jarring" with respect to the interaction of the electron with the low-frequency field. In order to get the required answer in this case to the problem treated in Sec. 2a, we must replace the notation $\delta p \to \hbar(\mathbf{k}_1 - \mathbf{k}_2)$, which reflects the conservation of momentum in the scattering process.

We note that the Compton frequency shift $\delta\Omega = \Omega_1 - \Omega_2$, which is obtained only in the quantum-mechanical approach, is comparable with the spacing between the satellites, even at energies $\hbar\Omega \sim \sqrt{\hbar\omega mc^2}$. Hence one must use the quantum treatment at high frequencies Ω and in studying satellites with large indices n. Naturally, this does not alter the essence of the effect, and in the region of the peak of (23), it leads only to a general shift in all the frequencies by the amount $\delta\Omega$.

The results of the quantum-mechanical calculation can be written most simply in the first order in $\hbar\Omega/mc^2$. The frequency of the scattered quantum $\Omega_2(n)$ is found from the laws of conservation of 4-momentum

$$p_1 + k_1 = p_2 + k_2 + nk_0, \qquad n = 0, \pm 1, \pm 2, \ldots$$

The corresponding scattering cross section

$$\frac{d\sigma_n}{dO_2} = \frac{d\sigma_{\text{Thomps}}}{dO_2} \left[\frac{\Omega_2(n)}{\Omega_1}\right]^2 J_n^2(N) \qquad (22)$$

in the peak region differs from (21) in the replacement of the Thompson by the Klein–Nishina cross section, in which the terms of the first order in $\hbar\Omega/mc^2$ have been kept. The expression for the parameter N in a linearly polarized field is the same as in (20), while in a circularly polarized field,

$$N = \frac{|e|}{m\omega} \sqrt{[\mathbf{a}_1(\mathbf{k}_1 - \mathbf{k}_2)]^2 + [\mathbf{a}_2(\mathbf{k}_1 - \mathbf{k}_2)]^2};$$

Here the $\mathbf{a}_{1,2}$ are the amplitudes of the vector potential of the two linearly polarized waves.

The effect of appearance of satellites in the spectrum of the scattered high-frequency wave is characterized by the size of the single parameter N, and is plays an appreciable role only under the condition $N \gtrsim 1$. If we even do not distinguish the partial cross section for different n, one can easily measure under the condition $N \gg 1$ the effective "broadening" of the spectrum of scattered frequencies. Its characteristic half-width is

$$\Delta\tilde{\Omega} \sim N\omega = \frac{|e\mathbf{E}_0(\mathbf{k}_1 - \mathbf{k}_2)|}{m\omega}. \qquad (23)$$

This recalls the broadening of a Compton line in scattering by a bound electron

$$\Delta\Omega_{\text{bound}} \sim \alpha Z\Omega \sin\frac{\theta}{2},$$

but it has a different nature (α is the fine-structure constant, Z is the effective charge, and θ is the scattering angle of the photon). We note that $\Delta\tilde{\Omega}$ can be comparble with $\Delta\Omega_{\text{bound}}$ even in relatively weak fields, when $E_0 \ll E_{\text{at}}$, if only $\omega \ll \Omega_{\text{at}}$ (for example, $\Delta\tilde{\Omega} \sim \Delta\Omega_{\text{bound}}$ in a field $E_0/E_{\text{at}} \sim \omega/\Omega_{\text{at}}$), where E_{at} is the characteristic atomic field.

It is interesting to compare the discussed effects with the analogous scattering by conduction electrons in semiconductors. Here the parameter N is fundamentally changed in two respects. Since we generally need frequencies Ω that exceed the plasma frequency, the electrons behave with respect to the high-frequency field practically as free electrons. If here $\omega < \omega_p$, then, first, the effective mass m^* of the electrons moving in the low-frequency field is considerably reduced, and hence, the parameter N is increased. Second, the expression for N should manifest the anisotropic nature of the effective mass. As we have noted, laser fields of intensities smaller by a factor of 10^4 suffice for observing the same effects, under the condition $m^* \sim m/100$.

5. THE PHOTOELECTRIC EFFECT AND β-DECAY IN AN EXTERNAL LASER FIELD

The problem of the photoelectric effect occurring in an external laser field amounts to the above-studied problem of occurrence of an electron in an ac field (see Sec. 2b), if the condition $\Omega \gg \omega^2/\Omega_{at}$ is satisfied. Thus, in a laser field of the following form ($\Gamma \to +0$):

$$\mathbf{A} = e^{-\Gamma t}(\mathbf{a}_1 \cos \omega t + \mathbf{a}_2 \sin \omega t), \quad a_1^2 = a_2^2 = a_0^2, \quad \mathbf{a}_1\mathbf{a}_2 = 0,$$

if we neglect the interaction of the photoelectron with the atomic core, and assume for simplicity that $p \gg (e/c)A$, we get the following expression for the partial cross sections of the photoelectric effect:

$$\frac{d\sigma_n}{dO_e} = \frac{d\sigma^0(\mathbf{p}_n)}{dO_e} J_n^2(N), \quad N = \frac{|e|}{mc\hbar\omega}\sqrt{(\mathbf{a}_1\mathbf{p}_n)^2 + (\mathbf{a}_2\mathbf{p}_n)^2}. \tag{24}$$

Here $d\sigma^0/dO_e$ is the cross section of the photoelectric effect in the absence of the field A, for which we have substituted the following value in Eq. (24):

$$\frac{p_n^2}{2m} = \hbar\Omega - |E_{\text{bound}}| - \frac{e^2 a_0^2}{2mc^2} - n\hbar\omega.$$

The satellites in the spectrum of the scattered radiation in the Compton effect correspond in the photoelectric effect to satellites in the energy spectrum of the photoelectrons. At the same frequencies Ω and ω, the parameter N for the photoelectric effect in the nonrelativistic region of quantum energies considerably exceeds the parameter that is characteristic of scattering. In fact, $\delta p_{\text{photo}} \sim \sqrt{m\hbar\Omega}$, while $\delta p_{\text{scatt}} \sim \hbar\Omega/c$, and therefore,

$$\frac{N_{\text{photo}}}{N_{\text{scat}}} \sim \frac{\delta p_{\text{photo}}}{\delta p_{\text{scat}}} \sim \sqrt{\frac{mc^2}{\hbar\Omega}} \gg 1.$$

All of the difficulties that arise in formulating the corresponding experiments involve the relatively small cross sections of the Compton effect and the photoelectric effect. Here it is far easier to achieve them in the photoelectric effect, since the methodology of recording and monochromatizing electrons is especially well developed (see, e.g., [18]). However, the stated difficulties involving the small cross sections disappear in other cases, e.g., in β-decay of nuclei or of free neutrons in an external laser field.

For a relativistic calculation of the processes of the photoelectric effect of β-decay occurring in a nonrelativistic laser field, the final state of the electron should be described by a wave function of the type of (9). In line with (24), this leads to the following expression for the partial cross sections:

$$d\sigma_n = d\sigma^0(p_n) J_n^2(N). \tag{25}$$

For example, this implies that the energy spectrum of particles having momenta $p \sim mc$ is effectively broadened by the amount $\Delta E \sim eE_0c/\omega$, which does not depend on the mass m, and is thus determined by the parameters of the laser field alone. For example, the limiting energy of the β-spectrum in the decay of the free neutron[19] $E_{\max} = 782\,470 \pm 50$ eV is shifted upward by this broadening. Thus, in the radiation field of a CO_2 laser, $E_{\max}$ is increased by ~ 50 eV, even at intensities $I \sim 10^6$ W/cm^2.

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