The light ray (contribution to the theory of the light field)

G. V. Rozenberg

Institute of Atmospheric Physics, USSR Academy of Sciences
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The electrodynamic aspects of the concept of the light ray, of the principal concepts and laws of photometry, polarimetry, and ray optics as a unified approximation of actual optics are explained. The apparatus-related origin of these concepts is demonstrated and the structure of the general theory of the light field is explained, including algebraic optics and the theory of radiative transfer. The principal premise of the "photometric" or "ray" approximation is the concept of the wavelet or the wave packet, the energy and the dynamic parameters of which are defined essentially in frequency-momentum rather than coordinate-time representation. The ray (photometric) approximation operates exclusively with observable quantities that are connected with the finite character of the dimensions and characteristic times of the square-law receivers used in optics. The generalization of photometric, polarimetric, and ray concepts to include a radiation field of arbitrary structure follows from the fact that the action of such a field on an *optical* receiver is equivalent to the action on this receiver of a beam of incoherent wavelets, the region of coherence of which is determined by the parameters of the receiver. This makes it possible to regard the field as an aggregate of light rays, each of which is described in a photometric approximation generalized with allowance for the polarization effects (Stokes parameters), which leads to formulation of the principal laws of photometry and polarimetry, and also to the photometric formulation of the conservation laws, and makes it possible to establish a direct relation between photopolarimetry and general theory of coherence. The limited nature of the region of coherence of the wavelet train leads to the problem of its transformation in time and in space, the description of which is effected by methods of algebraic optics by introducing the operators of differential and local transformations of the ray, and this leads directly to formulation of the radiative-transfer equation and to a delineation of the limits of its applicability. It is shown that the Stokes parameters are insufficient for a complete physical and mathematical description of the light ray, and it is necessary to introduce the three-dimensional distribution function of the wavelet trains over the polarization states; the problem of spin spectroscopy, i.e., of spatial selection of incoherent trains in accordance with the state of their polarization, is discussed.

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CONTENTS

What is the light ray and what is the scope of ray optics? What is the meaning of "measurement of light"? Whenceforth stem the concepts and laws of photometry? What does polarimetry deal with? What is the electrodynamic significance and what are the limits of applicability of ray and photopolarimetric concepts?

These long-standing problems of optics, which have troubled even its founders, and have at times received answers nowhere close to reality, have entirely new light cast on them by modern statistical optics. The exposition and discussion of the corresponding already sufficiently well formulated and verified concepts is the subject of the present review, which is based on an earlier publication,[1] but has now been substantially reviewed and expanded. It is in a certain sense also a continuation of the review[2] and, just as the latter, consists to a large degree of hitherto unpublished results by the author, with no cited literature. As to the published references, they do not claim to be complete and are aimed only to guide a reader with further interest in the subject.

1. SCOPE OF RAYS OPTICS

A. Photometry, ray optics, and Fourier transforms of the radiation field

Although system of concepts, quantities, and relations, which are used in optics to describe the energetics of the light field (brightness, intensity of illumination, spherical illumination, Stokes parameters, etc.) and which can be unified under the headings "photometry," "polarimetry," and "ray optics," is related to the system employed for the same purposes in electrodynamics (energy density, Poynting vector, coherence functions, etc.) there is at the same time a great difference between them. This difference, the elucidation of which is equivalent to the establishment of the position that photometry occupies in the system of electrodynamics, is on very firm ground, having taken root, as it turns out, in the means of realizing the measurement process. To explain this circumstance, let us turn first to certain general considerations.

Photometry in the broad sense of this word is customarily defined as the branch of optics devoted to the energetics of the radiation.[3] The photometry concepts, which are basically intuitive, are definitely of empirical origin. Formulated during the early Renaissance (see, e.g., Dante[4]), they were fused in the middle of the eighteenth century into an orderly system of views, which have made the names Bouguer[5] and Lambert[6] immortal. Since that time, the concepts have remained practically unchanged to this very day and can be found in all physics textbooks as an independent branch of optics, not connected in any way with its principal content, and having therefore an applied significance.

A clear example of this fact is the theory of radiative transfer,[7–14] which is a direct development of the photometric ideas and which has long turned into an independent well developed section of mathematical and applied (but not at all theoretical!) physics, perfectly independent of the theory of radiation, and existing to the very latest time without any electrodynamic foundation.

This raises inevitably the following questions: What is the reason for isolating this branch of optics? How did the rather primitive concepts of which it is made up withstand the surge of modern science? Why have they not been swept away either by the century of development of classical electrodynamics, or by such "explosions" as the appearance of quantum electrodynamics and statistical optics? It should be remembered that we are not dealing here with some details, but with fundamental "truths" that every schoolboy must know.

The source of this isolation and confinement of photometry was precisely the failure of all the rather numerous attempts to fit it within the framework of the electrodynamic theory of light. An exception is the trivial case of a plane monochromatic wave, for which it became possible to set in correspondence the illumination intensity produced by it with the Poynting vector, which then led to the quadratic dependence of the photometric quantities on the intensity of the light wave electric field. For a long time, the more general case of an arbitrary radiation field has not been subjected to a photometric treatment at all.

It is important to note here that actually, as we shall show, photometric (and polarimetric) concepts apply not at all to a plane monochromatic wave but, and furthermore exclusively, to the light ray, which is a stochastic mixture of wave packets or *wavelets* that are not coherent with one another. Without additional treatment these concepts can certainly not be extended to include a field of arbitrary structure. This is evident, say, from the fact that the concepts of brightness and polarization have no physical meaning whatever if we are dealing with a superposition of at least partially coherent light beams that differ in direction, for example in the case of standing waves in thin-layer systems,[14,15] or in the case of a light field inside small scattering particles.[16,17]

Actually, a generalization of photometry to include a light field of complex structure is possible only in an approach in which the field is treated as an aggregate of incoherent light rays. The understanding of this circumstance, although as yet not explicit, turned out to be the cornerstone for the development of photometric theory of the light field. This theory is the main accomplishment of the photometry of the twentieth century, including both its earlier forms (see, e.g.,[18]) and the general theory of radiative transfer in all its modifications,[7–14,21] and especially its electrodynamic foundation (see, e.g.,[2,22–36]).

The same circumstance was the reason for the phenomenological character of this theory, since it was impossible to discern its connection with electrodynamics without a more profound analysis of the concepts of coherence, i.e., before the modern development of statistical optics.[37–42] This is also why most physicists invariably prefer to this day to use the undefined concept "intensity" of light rather than the standardized photometric quantities (see, e.g.,[8,9,17,18,38–45]). For the same reason, in the analysis of polarization phenomena physicists usually choose (see, e.g.,[46,47]) to discuss the behavior of the field vector, instead of using the Stokes parameters, which are photometric in nature. Yet without a special analysis of the coherence problems such a treatment is frequently incorrect and, at any rate, inadequately represents the gist of the phenomena, since the very concept of polarization of light is inseparately linked to the concept of the ray and has an essentially photometric character (see below).

Attention should also be called to the fact that photometry came into being and developed until recently in parallel with geometrical optics, but independently of it. The point is that these sciences deal with two essentially different aspects of *one and the same* concept, namely the concept of the light ray as a stream of photons. Whereas geometrical optics deals with the influence of the medium on the trajectory of the photon jet, the subject of photometry is the dynamics of the photons, and consequently the entire assembly of the pertinent conservation laws. On the one hand, this calls for extending the scope of photometry to include polarimetry, and on the other hand it becomes necessary to consider ray optics as a synthesis of geometrical optics and photometry in its extended meaning.[2,12,21–23,48–56]

The unity of the photopolarimetric and geometrical aspects becomes most clearly manifest if we turn directly to Maxwell's equations and use the so called geometrical-optics approximation in its vector form. It is known that this procedure leads in the best manner to the notion of a monochromatic plane electromagnetic wave and its transformation in a quasi-inhomogeneous medium, which properly speaking is usually in fact the basis for introducing concepts that pertain to light wavelets and are subsequently transferred, with the necessary emendations, to the treatment of the light ray. The eikonal equation serves in this case as the condition that the system of zeroth-order linear vector equations have a solution, and these equations themselves, in conjunction with the condition that the first-order approximation equations have a solution, describe the behavior of the field vectors.[55,56]

This leads to a conclusion of fundamental character.

To the extent that the prototype of the train of light wavelet is the plane electromagnetic wave, and it is precisely its characteristics that serve as the basis for introducing photometric and polarimetric concepts, these concepts are by the same token meaningless in the space-time representation, but pertain to individual components of the Fourier transform of the radiation field, i.e., they are defined in its momentum-frequency representation. The latter is a direct consequence of the treatment of the light ray as a stream of photons, and, on the other hand, of the known impossibility of determining the wave function of a photon in the coordinate-time representation.

As we shall see, it is precisely this circumstance, when added to the inevitable temporal, spatial, frequency, and angular filtration of the Fourier components of the radiation field by the optical measuring devices, which is the reason for the aforementioned difference between the optical and electrodynamic descriptions of the radiation field in terms of energy. In other words, it turns out that the space-time representation, which is so convenient, for example, in radiophysics, is organically foreign to the optical manner of thinking.

In this lies, in particular, the internal contradiction of the suggestion made by Soleillet, [48] used, for example, in[44] and subsequently developed in detail by Fedorov, [21,53,54] that the light field be covariantly described with the aid of a tensor $E_i E_k^*(i, k = x, y, z)$, defined in three-dimensional coordinate space, if it is applied to a plane wave or a wavelet train (as is done in[21,44,53,54] in contrast to[48], where such a description is consistently applied to a radiation field of arbitrary structure).

In the case of a wavelet train it seems logically consistent to use a system of coordinates that takes into account *ab initio* the transversality of the radiation field, and is rigidly connected with the direction of propagation of the light. This leads inevitably to the quantum-mechanical density matrix $J_{ik} = (cn/4\pi)E_i E_k (i, k = 1, 2)$, where the quantities E_i are specified in a plane perpendicular to the direction of the ray, or to the four-dimensional Stokes vector parameter made up of its components, $S_i = (cn/4\pi)\mathbf{E}\sigma^i\mathbf{E}^* (i = 1, 2, 3, 4; \sigma^i$ is the i-th Pauli matrix, see below), since the latter are explicitly defined precisely in the frequency-momentum representation (see, e.g., [2,22,37,57–59]). This question will be discussed in detail in Secs. C and E of Ch. 2.

On the other hand, the description of wavelets with the aid of an aggregate of parameters S_i (or J_{ik}), that are functions of the frequency and of the direction l of the propagation of the wavelets, i.e., specified at a point (more accurately, as we shall show, in a certain small vicinity of a point) in frequency-momentum space, makes it possible to formulate the *subject* of ray optics as an *investigation of transformation operators* for the *parameters* S_i (or J_{ik}) that characterize the ray when the point $(\omega, \mathbf{l})$ corresponding to this ray is displaced or mapped through the action of the medium on the light ray (see, incidentally, Sec. C of Ch. 1).

This statement of the problem, first succinctly formulated in[2,22,23], leads directly to the formation of specific concepts and premises of algebraic optics, which in final analysis are also of photometric nature and occupy an important place among the concepts and interests of modern ray optics[2,12,13,22,23,40,48–52,56,60 *et al.*]; for details, see Ch. 3.

This reveals one more important circumstance. The described approach makes it possible to separate distinctly the characteristics of the radiation field (before and after it is acted upon by the medium) from the characteristics of the medium itself, the optical properties of which are exhaustively described by the operator of its action on the ray, independent of the state of the latter.[2,22,48]

An example of such an operator description may be the Fresnel coefficients, which make up in their totality the operators of reflection and refraction of light by an interface, whereas an example of the alternative traditional description is the aggregate of the formulas that are derived from them, which characterize the *result* of the action of these operators, i.e., say, the state of polarization of the reflected or refracted light at some state of the wavelets that illuminate the surface.[16,46,47,61] Other examples may be the matrices of light scattering by small particles[2,17,18,22,23,48,49] or dispersion matrices[2,12,62–64] (see also below), which also characterize the properties of the medium itself, and not the light field transformed by the medium as, for example, the degree of polarization or the degree of coherence of the scattered radiation; see, e.g.,[44,47].

The introduction of the operator of the action of the medium on the light ray explains immediately also the formulation of the so called inverse problems, intended for optical investigation of the properties of matter.[65,66] Their task is to interpret the experimentally determined operator of the action of matter on the light ray.

Thus, the geometrical-optics approximation leads, on the one hand, to the laws by which the medium (course of refraction, reflection, scattering, etc.) transforms the position of the point representing a plane monochromatic wave in frequency-momentum space and, simultaneously, to the laws governing the transformation of the vectors of the electric and magnetic field intensities of this wave, accompanying the displacement of the point representing the wave. The next step should consist of a transition to a train of light wavelets (i.e., to a certain finite vicinity of the point in the frequency-momentum space) and to the photometric parameters that describe this train.

B. Photometry and observability

This, however, raises a number of problems. First, how can we change from the relations connecting the field vectors of a plane wave to the photometric parameters and the laws of their transformations for a wavelet? Second, what set of these parameters is needed for an exhaustive description of the light wavelet? Third, how can one introduce photometric concepts in the case of an arbitrary space-time structure of the radiation field when it is necessary to regard the field as a superposition, generally speaking, of partial coherent plane waves having all possible directions and frequencies? Fourth, how can these concepts and quantities be generalized to include the light ray? Finally, what are the conditions and limits of applicability of these concepts, which are essentially foreign to the spirit of electrodynamics?

To answer these questions we must return to the empirical origin of photometry and to call attention to the fact that it deals exclusively with *observable* quantities. In other words, the origins of photometry cannot be revealed by considering only the electromagnetic field itself, as can be easily verified, say, from[37–42] or[53,54], and also from the treatment of photometric problems in[25,33,36] and especially in[67,68]. The nature of the photometric quantities and concepts is in principle inseparable from the specifics of the process of optical measurements[2,22,23,52] and is brought about by the *quadratic character* of the radiation receivers, and by the *finite character of their dimensions and of their time constant*,[30] and also by the fact that the receiver *filters out* the Fourier components of the radiation field (see below).

From the point of view of quantum electrodynamics, this means that the entire system of photometric concepts and quantities is generated by the operator of the action of the radiation field on the measuring device (see, e.g.,[57]). Moreover, the inseparable link between photometry and the concept of the light ray, and the treatment of the latter as a jet of photons, is a direct consequence of the properties of this operator.[30]

At first glance, the finite character of the dimensions and of the time constant of the measuring instrument, which brings about averaging of the values of the measured quantity both in space (over scales greatly exceeding the wavelength) and in time (the duration of which is much longer than the period of the optical oscillations), and also the associated cutoff of the high-frequency part and of the temporal and spatial spectra of the variability of the radiation field, are due exclusively to technical causes. Actually, however, it is dictated by considerations of fundamental nature. To illustrate the importance of this operation of space-time spreading, it suffices to recall the impossibility, in accordance with the uncertainty principle, of constructing an instantaneous operator for the flux density of the photons at a point (see, e.g.,[57,69]). Such a possibility appears only after sufficient averaging in space and in time or, equivalently, over a certain vicinity in the space of the frequencies and wave vectors, i.e., in other words, only for a train of wavelets or a ray.

This circumstance, which is fundamental for the description of the light field, was first indicated by Max Planck,[70] who noted that when optical quantities are used the time differential must be much larger than the period of the oscillations, and the space differential must greatly exceed the wavelength of the light.

Thus, the description of the light field with the aid of the electric and magnetic field intensity vectors as functions of the time and of the coordinates is certainly incompatible with the photometric conception. The latter calls for formulation of all of optics in terms of observable quantities defined in frequency-momentum space as applied to the specifics of the light ray as an aggregate of incoherent wavelets, and of the operators of the transformation of these quantities in the course of the transformation of the ray under the influence of the medium.[2,22,23,30] A planned or, more frequently, fortuitous realization of such a program comprises an appreciable part of the subject of modern statistical optics; see, e.g.,[2,12–14,17–19,21–42,48–56,65,71].

Norbert Wiener was apparently the first to call attention to the possibility of representing the polarization characteristics of a light wave in the form of observable quantities. Using correlation functions for different components of the electric vector of a plane wave, Wiener[52,59] arrived at a spectral matrix that accounts for the entire aggregate of the quantities observable in this case, and then used it to make up a system of four parameters identical with those introduced by Stokes[72] back in 1852 on a radically different basis, and almost forgotten for lack of any connection with the views of that time.

The same parameters were subsequently introduced independently by myself[2,22,23] and later by Fano[52] in a different manner—directly from an analysis of the action of a light ray on a measuring instrument equipped with a variable polarization device. The analysis was based on considerations identical with the ideas of Max Born (see, e.g.,[73]) that constancy of the measured parameters when the measurement method is varied is the criterion for the objectivity of the measurement result.

In such a treatment, the Stokes parameters, which pertain in explicit form (as an exhaustive aggregate of observables) not to a plane wave and not to the radiation field at all, but concretely to the light ray, i.e., to a small vicinity of a point in the frequency-momentum representation, have acquired a clearly pronounced photometric meaning not only by unifying photometry with polarimetry, but also by connecting them rigidly with the ray concept.[2,22,23]

A direct consequence of this fact was the already mentioned possibility of a consistent algebraic interpretation of the action of a medium on a light beam as an operation of a linear transformation of the Stokes parameters, which made it possible, in turn, to introduce the transformation of the state of the polarization of the light under the influence of the medium (in particular, to introduce the law of conservation of the angular momentum) into the theory of radiative transfer in scattering media—see Ch. 3 for details. It is important to note once more that the latter turns out to be possible not only because the transfer equation is formulated in principle within the framework of photometric concepts and is essentially based on the assumption that the light field is split up in the scattering medium into light wavelets that are not coherent with one another—see Ch. 2.

C. Coherence and light wavelet

The transition to observable quantities has inevitably brought to the forefront the problem of coherence. In general outline, the connection between coherence and observability was clear already to Michelson.[74] On the other hand, a quantitative investigation of this connection has become in recent years the subject of a persistent study in statistical optics; see, e.g.,[37–42,75–77].

The usual treatment of this question, however, is somewhat one-sided—attention is focused on investigations of the statistical properties of the radiation field itself. The coherence phenomena are dealt with here as a manifestation of the correlation between the fluctuations of the field at different space-time points, while the concept of coherence is directly connected with the measure of the compatibility of the phases of the components of the Fourier expansion of the space-time variation of the radiation field in plane monochromatic waves; see, e.g.,[75].

There is, however, another no less important aspect, which has remained so far in the shadow. The point is that the very concept of coherence stems from the quadratic character of the light receivers and from the fact that both their dimensions and their time constant are finite. Without resorting to the measurement process, i.e., without going over to observable quantities, there is no need whatever for the coherence concept. This photometric character of the coherence concept is emphasized by the fact that its measure—the degree of coherence—is introduced via the same correlation functions[37–42] as the photometric quantities[30] (see below).

Inasmuch as we shall henceforth deal with the splitting up of the radiation field into an aggregate of incoherent (i.e., statistically independent) wavelets, it is necessary to bear in mind also the following considerations.

The concept of a wavelet, being a modification of the concept of a plane wave, can pertain only to radiation that propagates in a quasi-homogeneous medium—this concept becomes meaningless in substantially inhomogeneous media. Subject to this limitation, the wavelet can be regarded as a formation whose Fourier-expansion components fill tightly a narrow spectral interval of frequencies $\omega = \bar{\omega} - \Omega$ ($\bar{\omega}$ is the average radiation frequency; $|\Omega| \leqslant \Omega_0 \ll \bar{\omega}$), of inseparably connected wave numbers $k = (\omega/c)m$ (c is the speed of light, $m = n - i\varkappa$ is the complex refractive index of the medium), and, generally speaking, complex wave normals $\vec{\mathfrak{l}} = \mathbf{l} + \rho$ ($\mathbf{l}$ is the wave normal in the direction of the beam axis, $\mathbf{l}^2 \approx \vec{\mathfrak{l}}^2 = 1$, $\mathbf{l}\rho = 0$, $|\rho|/|\mathbf{l}| = |\rho| \leqslant \rho_0 \ll 1$) with a smoothly varying spectral density $\mathbf{E}_0 \mathcal{G}(\Omega, \rho)$, which vanishes outside the interval $|\Omega| \leqslant \Omega_0$, $|\rho| \leqslant \rho_0$. In other words, the electric and magnetic field intensities in the light beam, regarded as functions of the coordinate and the time t, with allowance for the connection between E and H for a plane wave[16,55,56,75,135] are written in the form

$$\begin{aligned} \mathscr{E}(t, \mathbf{r}) &= \bar{\mathbf{E}}_0 \exp\left[i\bar{\omega}\left(t - m\frac{\mathbf{l}\mathbf{r}}{c}\right)\right] g(t, \mathbf{r}), \\ \mathscr{H}(t, \mathbf{r}) &= m[\mathbf{l}\mathbf{E}_0] \exp\left[i\bar{\omega}\left(t - m\frac{\mathbf{l}\mathbf{r}}{c}\right)\right] g(t, \mathbf{r}), \end{aligned} \tag{1}$$

where $\mathbf{E}_0$ is generally speaking a complex phasor, and the function

$$g(t, \mathbf{r}) = \int\limits_{-\Omega_0}^{+\Omega_0} \int\limits_{|\rho|=\rho} \mathcal{G}(\Omega, \rho) \exp\left[i\Omega\left(t - m\frac{\mathbf{l}\mathbf{r}}{c}\right) - i\frac{\omega}{c} m\rho\mathbf{r}\right] d\Omega\, d\rho, \tag{2}$$

which describes the space-time structure of the beam, depends relatively little on t and $\mathbf{r}$.

In rough outline, the statistical structure of such a formation is characterized by a coherence time T and by a coherence area s, which are defined by

$$T \approx \frac{1}{2\Omega_0}, \quad s \approx \frac{4\pi^2}{(k\rho_0)^2} = \frac{\lambda^2}{\vartheta^2}, \tag{3}$$

where $\lambda = \lambda_0/n$ is the wavelength in the medium and $\vartheta \approx \rho_0$ is apex angle of the light beam.[41,75]

It is appropriate to recall here that, as first shown by Gabor (see, e.g.,[37,39]) Maxwell's equations operate not with real field-intensity vectors, but with their complex-conjugates, for which the Fourier-transform components exist only at $\omega \geqslant 0$ and take the form (1), where $g(t, r) \equiv 1$. As explained in[16], the complex character of the vector phasor $\mathbf{E}_0 = E_0(\mathbf{f} + i\mathbf{g})$, where the real vectors $\mathbf{f}$ and $\mathbf{g}$ are connected by the relation $\mathbf{f}\cdot\mathbf{g} = 0$ and $\mathbf{f}^2 + \mathbf{g}^2 = 1$, means that the vector $\mathbf{E}_0$ describes an ellipse with semiaxes $E_0\mathbf{f}$ and $E_0\mathbf{g}$, while the complexity of the wave normal $\vec{\mathfrak{l}}$ corresponds to inhomogeneity of the plane wave.

The expansion of a monochromatic field into a spatial spectrum is compatible with Maxwell's equations only when it is limited by plane waves of only the given frequency (or, equivalently, with a given wave vector satisfying the dispersion relation). Therefore, generally speaking, such an expansion is possible only if one resorts only to the spectrum of inhomogeneous plane waves having the same wave number $k = m\omega/c$[78] (the Weyl expansion). This means[16] that the aggregate of the wave normals spans generally speaking the entire domain of complex vectors satisfying the unitarity condition quadratically $\vec{\mathfrak{l}}^2 = 1$, which can be rewritten in the form

$$\mathbf{a}^2 - \mathbf{b}^2 = 1, \quad \mathbf{ab} = 0, \quad \mathbf{a} = \mathrm{Re}\,\vec{\mathfrak{l}}, \quad \mathbf{b} = \mathrm{Im}\,\vec{\mathfrak{l}}. \tag{4}$$

It can be verified, however, that the concept of the light ray presupposes essentially that the Fourier components of the functions $\mathscr{E}(t, r)$ and $\mathscr{H}(t, r)$ do not include plane waves with strongly pronounced inhomogeneity. More accurately speaking, quasi-homogeneity of the system is not sufficient for the existence of a light wave. It is necessary also that the damping of each of the electromagnetic-field Fourier components making up the ray remain unobservable at least within the limits of the coherence area during the coherence time T; this can be regarded as a refinement of the concept of the space-time quasi-stationarity of the field (see, e.g.,[79]).

In the case of an inhomogeneous plane monochromatic wave of frequency ω_r propagating in the $\vec{\mathfrak{l}}_r$ direction, the radiation-power flux density, i.e., the real part of the complex Poynting vector

$$\mathbf{L} = \mathrm{Re}\,\frac{c}{4\pi}[\mathbf{E} \times \mathbf{H}^*], \tag{5}$$

takes the form[16,53,56]

$$\mathbf{L} = \frac{c}{4\pi} e^{-2k_0\mathbf{K}\mathbf{r}} \{(\mathbf{E}\mathbf{E}^*)\mathbf{N} + i[\mathbf{K}[\mathbf{E} \times \mathbf{E}^*]]\}, \tag{6}$$

where the wave normal is

$$\mathbf{N}=\mathrm{Re}\,\frac{\mathbf{k}}{k_0}=n\mathbf{a}+\varkappa\mathbf{b}, \tag{7}$$

the damping vector is

$$\mathbf{K}=-\mathrm{Im}\,\frac{\mathbf{k}}{k_0}=\varkappa\mathbf{a}-n\mathbf{b} \tag{8}$$

and $\mathbf{k}=k\vec{\mathfrak{l}}=k_0 m\vec{\mathfrak{l}}$ is the wave vector.

Accordingly, from the estimate of the damping along the wavelet axis (1) and across this axis, the requirement that all the Fourier components of the wavelet plane be quasi-homogeneous can be written in the form[56]

$$\left.\begin{array}{l} 2\varkappa\omega T \ll 1\left(\text{ or } \varkappa \ll \frac{\Omega_0}{\omega}\right), \quad |\mathbf{j}| \ll \frac{l}{4\pi k_0 \sqrt{s}} \approx \frac{\rho_0}{4\pi} \ll 1, \\ |\mathbf{l}| \approx 1+\frac{|\mathbf{j}|^2}{2}, \quad \rho \approx \rho^*, \quad \mathfrak{l}=\mathbf{l}+i\mathbf{j}, \quad \mathbf{lj}=0, \\ \overline{\mathbf{N}} \approx n\mathbf{l}, \quad \mathbf{K} \approx \varkappa\mathbf{l}-n\mathbf{j}. \end{array}\right\} \tag{9}$$

In other words, one can refer to the wavelet train (i.e., also to the beam) only in sufficiently weakly absorbing media, and the restrictions on the allowed inhomogeneity of the Fourier components of the field are more stringent the smaller the angle aperture of the wavelet train, i.e., the larger its coherence area.

This requirement, to which no due attention has been paid so far, imposes most important restrictions on the structure of the electromagnetic fields in which light wavelet trains can be separated, and by the same token, for which a photometric description of the field can be introduced. In fields that do not satisfy this requirement (say in the case of total internal reflection in a region behind a reflecting interface or in the case of diffraction inside small bodies), the results of the energy and correlation measurements cannot be expressed in photometric terms. In particular, it is impossible to use for them the radiative-transfer equation.

Under the restrictions (9), as can be easily verified, we have

$$[\mathbf{E}\times\mathbf{E}^*]=-2i\,(\mathbf{EE}^*)[\mathbf{f}\times\mathbf{g}]\approx i(\mathbf{EE}^*)\,q\mathbf{l}, \tag{10}$$

where

$$q=2\,|\mathbf{f}|\,|\mathbf{g}|, \tag{11}$$

and from (6) with allowance for (7) and (8)[56,107] we have

$$\mathbf{L}=\frac{cn}{4\pi}\,e^{-2k_0\mathbf{Kr}}\,(\mathbf{EE}^*)\,(1+q[\mathbf{l}\times\mathbf{j}]). \tag{12}$$

We now write down equations for the densities, averaged over space and time, of the principal dynamic quantities that characterize the field of the wavelet train under the natural (owing to the smallness of Ω and ρ_0) assumption that there is no frequency or spatial dispersion.

1) The average density of the electric energy of the radiation

$$W_E=\frac{\mathrm{Re}\,\varepsilon}{8\pi}\,(\vec{\mathscr{E}}\vec{\mathscr{E}}^*)=\frac{\mathrm{Re}\,\varepsilon}{8\pi}\,(\mathbf{E}_0\mathbf{E}_0^*)\,e^{-2k_0\mathbf{Kr}}\,\mathscr{G}\,(\Omega_0,\,\rho_0). \tag{13}$$

2) The average density of the magnetic energy of the radiation

$$W_M=\frac{1}{8\pi}\,(\vec{\mathscr{H}}\vec{\mathscr{H}}^*)=\frac{mm^*}{8\pi}\,[\mathbf{l}\times\mathbf{E}_0][\mathbf{l}^*\times\mathbf{E}_0^*]\,e^{-2k_0\mathbf{Kr}}\,\mathscr{G}\,(\Omega_0,\,\rho_0). \tag{14}$$

3) The average density of the radiation power flux[16,56,107]

$$\mathbf{L}=\mathrm{Re}\,\frac{c}{4\pi}\,[\vec{\mathscr{E}}\times\vec{\mathscr{H}}^*]=\frac{cn}{4\pi}\,(\mathbf{E}_0\mathbf{E}_0^*)\,e^{-2k_0\mathbf{Kr}}(1+q\,[\mathbf{l}\times\mathbf{j}])\,\mathscr{G}\,(\Omega_0,\,\rho_0). \tag{15}$$

4) The average density of the radiation momentum after Abraham[80-82]

$$\mathbf{p}=\mathrm{Re}\,\frac{1}{4\pi c}\,[\vec{\mathscr{E}}\times\vec{\mathscr{H}}^*]=\frac{n}{4\pi c}\,(\mathbf{E}_0\mathbf{E}_0^*)\,e^{-2k_0\mathbf{Kr}}(1+q\,[\mathbf{l}\times\mathbf{j}])\,\mathscr{G}\,(\Omega_0,\,\rho_0). \tag{16}$$

5) The average density of the spin angular momentum of the radiation[2,56,57,83] with allowance for (10) and (11)

$$\mathbf{M}=\frac{ip}{k_0}\,\frac{[\mathbf{E}\times\mathbf{E}_0^*]}{(\mathbf{E}\,\mathbf{E}^*)}=\frac{in}{4\pi\omega}\,e^{-2k_0\mathbf{Kr}}\,[\mathbf{E}_0\times\mathbf{E}_0^*]\,\mathscr{G}\,(\Omega_0,\,\rho_0) \approx\frac{n}{4\pi\omega}\,e^{-2k_0\mathbf{Kr}}\,q\mathbf{l}\,\mathscr{G}\,(\Omega_0,\,\rho_0)\,, \tag{17}$$

where according to (2) we have

$$\mathscr{G}\,(\Omega_0,\,\rho_0)=\langle g\,(t,\,\mathbf{r})\,g^*\,(t,\,\mathscr{G})\rangle_{T,s} \tag{18}$$

and $\langle\ \rangle_{T,s}$ denotes averaging over the time and over the coherence area.

We note also[56] that the momentum $\mathbf{p}$ and the angular momentum $\mathbf{M}$ are transported in the direction of the power flux, i.e., in the direction of $\mathbf{l}+q[\mathbf{l}\times\mathbf{j}]$, with a group velocity

$$U=\frac{c}{N+\omega\,(dN/d\omega)}\approx\frac{c}{n+\omega\,(\partial n/\partial\omega)}, \tag{19}$$

the direction of $\mathbf{p}$ coinciding with the direction $\mathbf{L}$ of the power transport, and differ in general from either the direction of the wave normal $\mathbf{N}$ or the direction of $\mathbf{M}$.

Relations (13)–(19) constitute the aggregate of the dynamic characteristics of the wavelet train as a unit, and by the same token constitute the energy basis for our representations of the light ray.

The concept of the wavelet train is fundamental in the system of ray optics. To estimate the extent to which it connects the modern concepts with the opinions of the past centuries and ensures continuity of the photometric and geometrical-optics concepts, we shall review cursorily the history of this concept.

In the antiquity, the concept of light-bearing rays emitted by the eye, stemming from the inevitable radiance required to see the celestial bodies, has enabled Euclid and Ptolemy to construct a working theory of the reflection and the refraction of light.[84] The subsequent development of geometrical optics (for example, by Al-Hazen) reversed the ray path and filled the rays with a certain material substance, but did not touch upon their external appearance, to which Descartes, Newton, and Huygens added the property of independence of one another, i. e., noncoherence. The next step was made by Leonard Euler, [85] who transformed the ray into a wave channel. In essence, this concept was preserved, at any rate in computational optics, to our very day, and underwent only slight refinements.

Principal among them was the idea of motion of energy along the wave channel, i. e., the replacement of the disembodied ray by a *light tube*, thus providing a connection between geometrical-optics constructions with photometry, albeit only from the viewpoint of the physics of the middle of the last century. In particular, this has led to a photometric formulation of the law of energy conservation in the form of the so called Straubel invariant (see, e. g., [20,86]).

It is easily seen that the train considered above is none other than the wave model of a light pulse[75,135] moving along an elementary light tube. An aggregate of the incoherent wavelet trains with a common propagation direction, i. e., an aggregate of pulses moving along one and the same light tube, makes up the "*light ray.*" In turn, an aggregate of rays of nearly equal direction, i. e., a sheaf of rays or light tubes, makes up a "*light beam.*"

However, the concept of the light ray, i. e., of a light tube along which energy flows, is rigidly connected with the space-time representation, whereas the parameters characterizing the state of the wavelet train are defined, as we have seen, in the frequency-momentum representation. Therefore the possibility of combining these concepts, of "transfering" the parameters of the wavelet train from one representation to the other, is not a trivial one. It is ensured exclusively by the finite character of the coherence region of the wavelet train and is restricted by the requirements of the uncertainty principle. Within the framework of these restrictions, the parameters of the wavelet train (including the photometric parameters, say the Stokes parameters) referred in frequency-momentum space no longer to the point $(\omega, \mathbf{l})$ but to a finite vicinity around it, can be regarded as parametric functions of the coordinates and of the time.

It is precisely this duality of the concept of the wavelet train and its parameters, with an obvious physical basis, which makes it possible also to identify the wavelet train with the light tube and to include photometry in the system of ray optics.

At the same time, this raises the question of tracing and studying the processes of the space-time transformation of the wavelet train, as a unit, by the medium. In other words, the scope of ray optics can be treated as the biography of the train (with the entire aggregate of the structure and dynamic parameters that characterize it) as an object of various physical actions exerted by the medium or by bodies immersed in the medium (including optical devices).

We shall return to this group of questions in Ch. 3.

2. OPTICAL MEASUREMENTS AND PHOTOPOLARIMETRY OF THE LIGHT FIELD

We turn now to a photometric description of a light field with arbitrary space-time structure, or equivalently, to the question of the electrodynamic interpretation of the photometric concepts and the limits of their applicability.

The idea of constructing a consistent and closed theory of the light field was first advanced by Gershun. [87] The concept developed by him, which played an important role in illumination engineering, was arrived at phenomenologically, without any connection with electrodynamics, as a classical theory of a vector field of energy parameters of the radiation. It turned out that the latter are the optical equivalents of the volume energy density and the Poynting vector, and a method was indicated of deriving them from photometric quantities.

Radical progress in this direction is connected with the advent of statistical optics, [37–42] one of the main objects of which is precisely the theory of the field of the energy parameters (correlation functions) of the radiation, the latter being constructed directly on the basis of Maxwell's equations.

Such an approach to the light-field theory, however, as well as Gershun's approach, is connected in essence with renouncing the universally accepted and well verified photometric concepts, and calls for the development of special methods to surmount this barrier. It is easy to note that an appreciable part of the efforts aimed at an electrodynamic justification of the radiation-transport equation[23–36] is aimed precisely at developing procedures that connect the formalism of the correlation functions with the formalism of ray optics (primarily the choice of the averaging region).

At the same time it turns out that there is another possible purely photometric approach to the construction of a consistent general theory of the light field within the framework of statistical electrodynamics.

A. Photometric role of the light receiver

In spite of the great variety of the existing or possible light receivers, they can be divided into two fundamentally different classes, which give rise respectively to different photometric concepts and quantities. The first includes instruments that can be called *flux meters*, i. e., instruments that accumulate and convert the flux of some dynamic quantity (power, momentum, angular momentum) delivered to the receiver by electromagnetic radiation. If the reference is to energy flux meters, then they imitate to one degree or another a black body in which the energy of trapped radiation is transformed into another directly discernible form of energy (heat, electricity, etc). Frequently the role of such a black or "grey" body is played by the coating or volume of the working body of the receiver. On the other hand, if we are dealing with momentum flux (e. g., [88]) or angular momentum flux (e. g., [89,90]), then the receiving element is an absorbing or reflecting surface of a vane of one form or another.

In either case, a flux meter is characterized by the existence of a receiving window or a receiving surface (which we shall assume, without loss of generality, to be flat) with area $\Sigma \gg \lambda^2$, with simultaneous angular filtration of the radiation-field Fourier components that reach the receiver. The measured quantity in this case is the flux of the power (momentum, angular momentum) of the radiation passing through the entrance opening (or the receiving surface) and subjected to averaging over the area Σ and over the time constant $\tau \gg 1/\omega$ of the receiver, with subsequent normalization to a unit surface and a unit time.

In the case of radiation-power flux, this quantity is, generally speaking, by no means the Poynting vector, but the *energy illumination* F of the receiving surface, defined by the relation

$$F = \langle \tilde{\mathbf{L}} \times \vec{\mathfrak{f}} \rangle_{\Sigma,\tau}, \qquad (20)$$

where $\vec{\mathfrak{f}}$ is the inward normal to the receiving surface and $\tilde{\mathbf{L}}$ is the Poynting vector for the field *filtered* by the radiation receiver:

$$\tilde{\mathbf{L}} = \frac{c}{4\pi} e^{-2k_0\mathbf{K}\mathbf{r}} \operatorname{Re} \int\int\int\int f(\mathbf{l}) f(\mathbf{l}') [\mathbf{E}(\omega, \mathbf{l}) \mathbf{H}^*(\omega', \mathbf{l}')] \times e^{i(\omega-\omega^{*\prime})t} e^{-ik_0(\mathbf{N}-\mathbf{N}^{*\prime})\mathbf{r}}\, d\omega\, d\omega'\, dO\, dO', \qquad (21)$$

where dO is the solid-angle element corresponding to the direction of the vector $\mathbf{N}$, while $f(\mathbf{l})$ is the filtering factor and depends on the construction of the instrument.

Thus, if we have two identical light sources, then we can see that half-way between them we have $\mathbf{L}=0$. A similar situation occurs inside a non-absorbing layer of a cloud or in the interior of a snow cover, where the brightness of the light field is identical in all directions (see, e.g., [12,14,91–93]). At the same time, for an instrument with an angle filter

$$f(\mathbf{l}) = \begin{cases} 1 & \text{if } \mathbf{l}\vec{\mathfrak{f}} \geqslant 0, \\ 0 & \text{if } \mathbf{l}\vec{\mathfrak{f}} < 0, \end{cases} \qquad (22)$$

under the aforementioned situations we certainly have $\tilde{\mathbf{L}} \neq 0$. Thus, for a flux meter with a filter (22), oriented towards one of two receivers, the second, so to speak, ceases to exist. In exactly the same way, in a field of diffuse radiation inside a scattering medium, filtration of the type (22) excludes, for a vertically oriented receiver, the action of all, say, rising or descending Fourier components.

It is precisely this circumstance which prevents us in the general case from comparing the energy illumination F with the power flux density of the light field. In the example with the two sources, the latter is equal to the *difference* between the illuminations of the area on the opposite sides, and under the conditions of a diffuse light field the establishment of a corresponding connection calls for a more detailed analysis.[20]

We encounter a similar situation in all types of calorimeters, thermocouples, bolometers, and receivers based on the use of thermostriction, pyroelectric, and pyromagnetic phenomena, effects of light pressure, etc.

The action of recivers of the other radically different type, namely *photoelectric* receivers, is based on the principle of counting (in one way or another) the electrons that change their state under the influence of light, via some type of photoeffect, within a certain time $\tau \gg 1/\omega$ and in a certain volume $v \gg \lambda^3$. These include all the receivers in which one uses photoemission, photoconductivity, photoluminescence, photochemical reactions, etc.

In contrast to flux meters, where a phenomenological analysis is sufficient, the understanding of the properties of photoelectric receivers calls already for a detailed quantum-mechanical analysis of the photoelectric process. Such an analysis, however, leads[39,41,71] to the conclusion that, regardless of the individual characteristics of the process, the probability of registering photoelectric action of the radiation in a volume v with coordinations $\mathbf{r}$ within a time τ is equal to

$$P(\mathbf{r})\, v\tau = v\tau \int_0^\infty \alpha(\omega)\, \vec{\mathscr{E}}(\omega, \mathbf{r})\, \vec{\mathscr{E}}^*(\omega, \mathbf{r})\, d\omega, \qquad (23)$$

where $\vec{\mathscr{E}}(\omega, \mathbf{r})$ is the spectral density of the temporal Fourier expansion of the complex intensity vector $\vec{\varepsilon}(\mathbf{r}, t)$ of the electric field of the radiation at the point $\mathbf{r}$. What is essentially assumed here is a weak spectral dependence, on a scale of τ^{-1}, of both the field itself and of the volume quantum efficiency $\alpha(\omega)$ of the receiver, i.e.,

$$\frac{1}{\tau}\frac{d \ln \mathscr{E}(\omega, \mathbf{r})}{d\omega} \ll 1, \quad \frac{1}{\tau}\frac{d \ln \alpha(\omega)}{d\omega} \ll 1. \qquad (24)$$

Inasmuch as to be able to count photoelectrons any photoelectric receiver must have a finite volume $v \gg \lambda^3$, the object of the measurement is the value of $P(\mathbf{r}, t)$ averaged over the volume v and over the time:

$$\langle P \rangle_{\tau, v} = \frac{1}{v} e^{-2k_0\mathbf{K}\mathbf{r}} \int_v \int_0^\infty \alpha(\omega) \oint\oint f(\mathbf{l}) f(\mathbf{l}')\, \mathbf{E}(\omega, \mathbf{l})\, \mathbf{E}^*(\omega, \mathbf{l}') \times \exp(-ik_0(\mathbf{N}-\mathbf{N}')\mathbf{r})\, dO\, dO'\, d\omega\, d\mathbf{r}, \qquad (25)$$

where account is taken of the already stipulated requirement that the change of the factor $\exp(-2k_0\mathbf{K}\cdot\mathbf{r})$ be not discernible within the limits of the measurement volume v.

B. Splitting of the radiation field by a receiver, and the principles of photometry

We consider first the simplest case of a wavelet train (1) incident on a flux meter. The receiver, as already mentioned, averages the quantities $\tilde{\mathbf{L}} \cdot \vec{\mathfrak{f}}$ both over the area of the input window, and over a time period equal to its time constant $\tau \gg 1/\omega$. Taking into account the smallness of Ω_0/ω and ρ_0, the practical constancy of $\exp(-2k_0\mathbf{K}\cdot\mathbf{r})$ within the limits of the entrance window of the receiver, as well as the quasi-homogeneity of all the Fourier components, we obtain in accordance with (7), (11), and (15)

$$F = \frac{cn}{4\pi} e^{-2k_0\mathbf{K}\mathbf{r}} (\mathbf{E}\mathbf{E}_0^*)(\mathbf{l}\vec{\mathfrak{f}})\, \mathscr{G}(\Omega_0, \rho_0), \qquad (26)$$

where $\mathscr{G}(\Omega, \rho)$ is defined by (18), except that the averaging regions T and s are replaced by τ and Σ.

The weighting factor $\mathscr{G}(\Omega_0, \rho_0)$ is determined essentially by the relation between the parameters of the instrument (τ, Σ) and the coherence region of the wavelet train (T, s). If $1/\omega \ll \tau \gtrsim T$ and $\lambda^2 \ll \Sigma \lesssim s$, then we have on the beam axis $\mathcal{G}(\Omega_0, \rho_0) = 1$. This means that the beam is received by the instrument as an integral fully coherent formation (i.e., a plane monochromatic wave) with an effective amplitude

$$\mathbf{E}_{\text{eff}} = E_0 e^{-k_0 \mathbf{K r}} \int_{-\Omega_0}^{+\Omega_0} \int_{|\rho| \leq \rho_0} \mathcal{G}(\Omega, \rho)\, d\Omega\, d\rho. \tag{27}$$

To the contrary, if $\tau \gg T$ and $\Sigma \gg s$, then the instrument, so to speak, splits the wavelet train into an aggregate of partial trains that are not coherent with one another, for each of which the coherence area and time are equal to Σ and τ respectively, and each of which acts on the instrument independently in accordance with (27), but with changed integration regions.

We now place our receiver in an arbitrary radiation field, which we assume to be quasi-stationary in time and in space. The averaging over the time again leads to a splitting of the field (in the sense of its action on the receiver) into an aggregate of incoherent components with coherence time τ each. Turning further to expansion of the monochromatic component into a spatial spectrum of plane waves, we must take into account that the entrance window of the receiver acts here not only as a spatial but also as an angle filter of the type (22), and eliminates all the Fourier components for which $\mathbf{l} \cdot \vec{\mathfrak{f}} < 0$. This circumstance, as we have seen, no longer enables us to identify the energy illumination intensity with the Poynting vector, and makes it necessary to resort to its truncated analog $\tilde{\mathbf{L}}$ (21). The original technique of Fourier transformation, filtration of Fourier components, and subsequent averaging of the projection (20) of the analog $\tilde{\mathbf{L}}$ (21) of the Poynting vector for the residual formation over the area Σ of the receiver and over the time interval τ, again lead to a splitting of the field into aggreate incoherent wavelet trains with different directions, so that the coherence time and coherence area of each of them are equal to

$$T = \tau, \quad s = \Sigma \cdot \cos\theta \qquad (\cos\theta = \mathbf{l}\vec{\mathfrak{j}}), \tag{28}$$

each of the wavelet trains separated in this manner acting on the receivers independently.

We have in fact assumed here that the field does not contain any Fourier components with strongly pronounced inhomogeneity, i.e., in other words, we have assumed a statistical spatial quasi-homogeneity and a statistical temporal quasi-stationarity of the radiation field, which limit the region of the photometric treatment of the results of optical measurements.

Thus, as a consequence of the finite dimensions of the entrance window and of the time constant of the instruments of the flux-meter type, the average energy illumination intensity measured by them in the direction $\mathbf{l} = \vec{\mathfrak{f}}$ can be expressed in the form

$$F(\mathbf{l}) = \int F(\omega, \mathbf{l})\, d\omega, \tag{29}$$

where its spectral density is

$$F(\omega, \mathbf{l}) = \int I(\omega, \mathbf{l}) \cos\theta\, dO \tag{30}$$

dO is the element of the solid angle in the direction of $\mathbf{l}$ and, in accordance with (26),

$$I(\omega, \mathbf{l}) = \frac{cn}{4\pi} e^{-2k_0 \mathbf{K r}}\, \mathbf{E}(\omega, \mathbf{l})\, \mathbf{E}^*(\omega, \mathbf{l}) \tag{31}$$

has the obvious photometric meaning of the spectral density of the brightness of the wavelet train propagating in the direction $\mathbf{l}$.

These arguments are valid if the angular dimensions of the luminous object exceed the angular dimensions ΔO of the wavelet train, as determined by the parameters of the receiver. This situation obtains, say, in the interior of a scattering medium. In the opposite case, when the angular dimensions of the source ΔO_s (say a distant star) is smaller than ΔO, the illumination intensity F does not depend on $\Delta\theta$, and we have in place of (30)

$$F(\omega, \mathbf{l}) = I(\omega, \mathbf{l}) \cos\theta \cdot \Delta O_s, \tag{32}$$

i.e.,

$$I(\omega, \mathbf{l}) = \frac{cn}{4\pi} e^{-2k_0 \mathbf{K r}} \frac{(\mathbf{E}\mathbf{E}^*)}{\Delta O_s}, \tag{33}$$

and for a small source $\Delta O_s \ll 1$ we have $(\mathbf{E} \cdot \mathbf{E}^*) \sim \Delta O_s$, from which follows invariance of the brightness of a light beam propagating from a small source in a nonabsorbing medium. Obviously, (31) coincides with (33) if we assume for an extended source $\Delta O_s = 1$.

Thus, as already noted above on the basis of general considerations, the principal photometric characteristic of the light field—the spectral density of its brightness—is determined exclusively for a train with a given direction $\mathbf{l}$, i.e., for the vicinity of a certain point $(\omega, \mathbf{l})$ in frequency-momentum space. The specifics of receivers of the flux-meter type leads in this case to the definition of one more photometric quantity, namely the energy illumination intensity in the direction of $\mathbf{l}$, and establishes their connection with the spectral densities of the brightness in different directions.

By way of example, let us consider the classical experiment with Young's two-slit interferometer, the slits of which are seen by the receiver as shifted by a small angle ϑ relative to each other. If the linear dimensions of the entrance window of the receiver are much smaller than the scale of the measured interference pattern, then ϑ is smaller than the angle ΔO subtended by the wavelet-train receiver. In other words, for such a receiver, the two slits are indistinguishable and are perceived by the receiver as a single small combined source, for which we have in accordance with (32) and (33), with allowance for the fact that $\mathbf{K} = 0$,

$$F = \frac{cn}{4\pi} (\mathbf{E}_1 + \mathbf{E}_2,\ \mathbf{E}_1^* + \mathbf{E}_2^*) \cos\theta, \tag{34}$$

where $\mathbf{E}_1$ and $\mathbf{E}_2$ are the phasors of the field produced in the vicinity of the receiver by the radiation from the slits 1 and 2 of the interferometer. Therefore, as it moves along the interference pattern, the receiver reproduces its structure.

On the other hand, if the linear dimensions of the receiver exceed the scale of the interference pattern, then the angular dimensions of the incoherent trains, into which the receiver splits the light field, is smaller than ϑ, although it is much larger than the angular dimensions of each of the slits taken individually. Therefore the receiver will see the radiation of the two slits as incoherent, i.e., according to (30)–(33)

$$F' = \frac{cn}{4\pi}(\mathbf{E}_1\mathbf{E}_1^* + \mathbf{E}_2\mathbf{E}_2^*)\cos\theta, \tag{35}$$

which can be treated in another language as a result of averaging of the interference pattern over the receiver area: $F' = \langle F \rangle$.

We return now to the photoelectric receiver and place it in an arbitrary light field, without resorting to an angle filter, (i.e., $f(\mathbf{l}) \equiv 1$). If we assume that the measurement volume is a parallelepiped with sides X, Y, and Z, then we get from (25)

$$\langle P \rangle_{\tau, v} = e^{-2k_0 \mathrm{Kr}} \int\int\int \alpha(\omega)\, \mathbf{E}(\omega, \mathbf{l})\, \mathbf{E}^*(\omega, \mathbf{l}')\, \mathrm{sinc}\,(k_0 \Delta N_x X) \times \mathrm{sinc}\,(k_0 \Delta N_y Y) \times \mathrm{sinc}\,(k_0 \Delta N_z Z)\, d\omega\, dO\, dO', \tag{36}$$

where $\mathrm{sinc}\, z = \sin z / z$, in accordance with (7) with allowance for the smallness of $\varkappa$ and Im 1

$$\Delta N_x = n\,(\mathbf{l}'_x - \mathbf{l}_x) \tag{37}$$

and analogously for ΔN_y and ΔN_z, whence, as a result of the condition $X, Y, Z \gg 1/k$,

$$\langle P \rangle_{\tau, v} = e^{-2k_0 \mathrm{Kr}} \int\int \alpha(\omega)\, \mathbf{E}(\omega, \mathbf{l})\, \mathbf{E}^*(\omega, \mathbf{l})\, d\omega\, dO, \tag{38}$$

or, introducing a quantity that has the obvious photometric meaning of the spectral density of spherical illumination intensity[20]

$$\Phi(\omega) = \oint I(\omega, \mathbf{l})\, dO, \tag{39}$$

we obtain

$$\langle P \rangle_{\tau, v} = \int_{\omega} A(\omega)\, \Phi(\omega)\, d\omega, \tag{40}$$

where

$$A = \frac{4\pi}{cn}\alpha(\omega). \tag{41}$$

For a nonselective receiver ($\partial \alpha / \partial \omega = 0$) we have

$$\langle P \rangle_{\tau, v} = A\Phi, \tag{42}$$

where

$$\Phi = \int \Phi(\omega)\, d\omega = \int I(\mathbf{l})\, dO \tag{43}$$

is the total spherical illumination of the volume v and

$$I(\mathbf{l}) = \int I(\omega, \mathbf{l})\, d\omega \tag{44}$$

is the total brightness of the light beam.

The quantity

$$\Phi = \frac{cn}{4\pi} \langle \vec{\mathscr{E}}(t, \mathbf{r})\, \vec{\mathscr{E}}^*(t, \mathbf{r}) \rangle_{\tau, v} \tag{45}$$

differs in accordance with (13) and (14) from the average spatial energy density of the light field $\langle W \rangle = \langle W_E + W_M \rangle = 2\langle W_E \rangle$ by a factor c/n:

$$\Phi = \frac{c}{n}\langle W \rangle_{\tau, \Sigma}, \tag{46}$$

or in analogy with (21) and (25) for an arbitrary light field

$$\langle W_E \rangle_{\tau, \Sigma} = \frac{\mathrm{Re}\,\varepsilon}{8\pi} e^{-2k_0 \mathrm{Kr}} \int\int (\mathbf{E}_0 \mathbf{E}_0^*)\, dO\, d\omega. \tag{47}$$

The need for introducing such a scalar characteristic of the light field as Φ in order to complete its photometric description was first pointed out by Gershun,[20] who indicated also a method for its direct measurement, based on relation (43).

Thus, in the case of receivers of the photoelectric type, the fact that their time constants and their geometric dimensions are finite leads likewise to a splitting of the radiation field into an aggregate of independent (in the sense of their action on the receiver) wavelet trains of different frequency and different directions, with a coherence time τ and with a coherence area Σ determined by the dimensions of the receiver. In other words, the possibility of splitting up a light field into an aggregate of incoherent wavelet trains is ensured by the very method of the field measurement with the aid of a quadratic receiver with finite τ and finite dimensions, and the structure parameters of the trains are determined entirely by the parameters of the receiver. This is a fact that must be borne in mind when many problems of instrumental optics are solved, including the action of an ordinary lens.

It is obvious without further argument that the foregoing reasoning pertains to the entire assembly of the dynamic parameters of the radiation (13)–(17), which after averaging over τ and Σ are formed additively from the corresponding parameters of the individual wavelet trains.

It should be added that when light is scattered by a medium we are interested again in forms quadratic in the intensity of the electric field of the scattered radiation, averaged over a certain "effective" volume element,[23,30,31,36] as can be easily verified by a suitable analysis of the corresponding arguments, as given for example in[38,44,45]. It is precisely this circumstance which enables us to treat the light field in a scattering medium likewise as an aggregate of incoherent wavelet trains, and consequently to use the phenomenological ideas of the radiation-transport theory—see below.

A remark of fundamental character is in order here.

The splitting of the light field by the receiver into an aggregate of discrete wavelet trains that act on the receiver can be regarded as a realization of the sampling theorem, or more accurately of a rephrasing of this theorem as applied to space-time filtration of the radiation field by the receiver. We have already seen that the finite character of the space-time intervals separated by the receiver, a character which determines its frequency-contrast characteristics, leads to a high-frequency limit on the radiation-power spectrum sensed by the receiver. With the aid of the sampling theorem (see, e.g., [77]) it follows directly that the action of the entire radiation field on the receiver is equivalent to the aggregate of the actions of the radiation in certain selected points at selected instants of time.

It can be shown, however, on the basis of the same theorem, that the space-time restriction on the sensitivity of the receiver produces, in addition, equivalence of the field acting on the receiver to an aggregate of discrete incoherent (i.e., independently-acting) wavelet-train components, each of which has a frequency-angle structure in the form

$$\text{sinc}\,(\omega_n t - 2\pi n)\,\text{sinc}\,(k_x X - 2\pi p)\cdot\text{sinc}\,(k_y Y - 2\pi q)$$

and enters with a weight equal to the spectral density of the brightness at $\omega_n = 2\pi n/\tau$, $k_x(p) = 2\pi p/X$, $k_y(q) = 2\pi q/Y$.

The parameters n, p, and q are integers here, and it is assumed that a rectangular receiver window with sides X and Y is located in the vicinity of x and y (at a different window configuration, the actual expressions are different, but not the gist of the expansion; in particular, if the window of the receiver is circular, then the functions sinc is replaced by an equivalent expression made up of Bessel functions; see, e.g., [77]).

It is easily seen that the components of the expansion produced in this manner are none other than the light beams considered in photometry, the coherence time and coherence area of which are determined by the parameters of the instrument.

Thus, when speaking of a light field (including also an optical image), the concept of the light rays that make up the field is connected not with the field itself or with hypothetical diaphragms introduced into the field, but with the receiver of the light. In some respects such a representation comes close to the perspicacious but naive ideas of Euclid and Ptolemy, and this explains the viability of their theoretical constructions.

Obviously, equipping a receiver of any type with spectral or angular filters, as is the case in real measuring instruments, does not change anything in our reasoning, except for introducing weighting factors for $I(\omega, \mathbf{l})$.

We have seen that receivers of different types give rise in principle to different photometric quantities (F and Φ), i.e., they characterize the field from different points of view. Gershun[20] has shown long ago that only the aggregate of these characteristics ensures a complete energy-dependent description of the light field. However, on going to real instruments, it must be borne in mind that everything said above concerning flux meters pertains, generally speaking, only to an absolutely black body, and everything said concerning photoelectric receivers pertains to that element of the light-sensitive volume within which the brightness of the light beams can be regarded as invariant, and the averaging is carried out over the cross section of this volume perpendicular to the light beam.

A real light receiver is an aggregate of similar light-sensitive volume elements, interconnected by a certain system for gathering the information registered by them and for averaging this information over the entire volume of the receiver. This is the situation with a photocathode, a photographic plate, luminophors, as well as absorbing coatings such as in bolometers. Therefore the theory of the combined response of the receiver as a whole breaks up inevitably into two parts—optical, pertaining to the theory of light propagation in the light-sensitive body of the receiver, and informational — covering the physics of the processes that ensure the gathering and averaging of the information concerning the reaction (say, the heat rise or the photochemical yields) of each of the volume elements to this radiation. The singularities of both processes introduce significant changes in the photometric properties of real instruments, and in many respects blur the boundaries between instruments of different types. At the same time, they determine the individuality of each instrument and serve as a physical basis for the "corrections" peculiar to it, i.e., the deviations of the measured quantities from I, F, or Φ.

Thus, the purpose of the optical theory of a photoelectric receiver of light is obviously the establishment of a connection between the spherical illumination of each of the light-sensitive elements of the volume of the working body with the conditions of the illumination of the receiver as a whole. Assume that an analysis from the point of view of radiation transport theory shows that the spherical illumination Φ inside weakly absorbing scattering bodies illuminated from the outside, such as photographic emulsions, luminescent powders, magnesium oxide, opal glasses, leaves of vegetation, etc., can exceed significantly (by as much as a factor of four) the illumination F on their surface.[12,13,91,92] Even if the optical thickness of the layer is large enough, the absorption of the radiation in the layer is determined as a whole by a quantity that certainly differs (albeit not very strongly) from the illumination F of the layer.[13,14,91,92]

The optical regime in the layer of metallic black enamel covering the receiving surface of a bolometer can be considered from the same points of view.[93] The photometric properties of the layer as a whole depend significantly on its microstructure and resemble the characteristics of a black body only very roughly, even at very large thickness. In particular, the black coatings, just as coatings of weakly absorbing scattering media (for example, emulsions or luminophors) or dull-finished metallic surfaces, are by far not orthotropic, and one of the results of this fact is that the measured quantity differs from the illumination; the difference, incidentally, can be compensated for by an angle filter.

As to photocathodes having a thin-layer structure or placed in a layer-type interferometer,[16] as well as semitransparent metallic photocathodes,[16,94,95] the optical field in them does not admit of photometric treatment. However, such a treatment is possible for the layer as a whole, and its absorbing ability (and consequently also the response of the instrument) differs significantly from the absorbing ability of the aggregate of identical but separated elements.

The foregoing analysis, which culminates in the derivation of the main photometric relations, reveals clearly the electrodynamic meaning of the photometric concepts and quantities, and also the conditions of their applicability, and by the same token introduces photometry into the domain of electrodynamics or, more accurately speaking, statistical optics, eliminating its century-old isolation from electromagnetic theory of light.

C. Polarimetric generalization

Further generalization of photometric concepts is obtained from an analysis of the observability of the light field when the receiver is illuminated with polarization or interference apparatus of one type or another. The first step in this direction was the already mentioned introduction of the Stokes parameter into the system of photometry (i.e., ray optics)

$$S_i(\omega, \mathbf{l}) = \frac{cn}{4\pi} e^{-2k_0\varkappa r} \mathbf{E}(\omega, \mathbf{l})\, \sigma^i \mathbf{E}^*(\omega, \mathbf{l}), \tag{48}$$

where $i = 1, 2, 3, 4$;

$$\sigma^1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^4 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{49}$$

are Pauli spinor matrices.

As shown in[2,22,23,52,58,59] from different points of view, the aggregate of all these parameters, which make up in functional space the four-dimensional Stokes vector parameter $\vec{S}$, contains the complete photometric information on the wavelet train. The quantity $S_1(\omega, \mathbf{l}) \equiv I(\omega, \mathbf{l})$ has the meaning of its brightness, while the quantity $S_4(\omega, \mathbf{l}) = qS_1(\omega, \mathbf{l})$ (where q is the degree of ellipticity of the polarization of the radiation, defined by (11)), is connected by means of a relation that follows from (15) and (17), namely

$$S_4(\omega, \mathbf{l}) = \omega c M(\omega, \mathbf{l}) = \omega\left(n + \omega\frac{\partial n}{\partial\omega}\right) c\mathscr{M}(\omega, \mathbf{l}) \tag{50}$$

with the average density M of the spin angular momentum of the radiation in the light beam and the average density $\mathscr{M} = MU$ of its flux, where U is the group velocity defined by (19). We recall that the quantity $\mathscr{M}$, generally speaking, is measured by instruments of the flux-meter type.[89,90] The dynamic meaning of the remaining Stokes parameters calls for further investigation.

A generalization of the expressions for the components of the Stokes vector parameter to the case of an arbitrary representation, i.e., a representation of the field of the wavelet train as a superposition of two arbitrary, generally speaking alternately elliptically polarized components

$$\mathbf{E} = E_1\mathbf{e}_1 + E_2\mathbf{e}_2, \tag{51}$$

where the complex basis vectors satisfy the conditions[2,23,56]

$$\mathbf{e}_i\mathbf{l} = 0, \quad \mathbf{e}_i\mathbf{e}_j^* = \delta_{ij}, \quad \mathbf{e}_i = \mathbf{f}_i + i\mathbf{g}_i, \quad \mathbf{f}_i\mathbf{g}_i = 0, \tag{52}$$

has been introduced in[2,23]; see also[96].

As is evident from the foregoing analysis, all the dynamic characteristics of the radiation, including the Stokes parameters, are additive for incoherent wavelet trains of sufficiently close directions and close frequencies; i.e., for the components of the light ray that joins them. To describe the latter, i.e., the *result* of the stochastic superposition of wavelet trains that differ significantly only in their polarization states, it is necessary to introduce an additional statistical parameter that characterizes the *degree of homogeneity of the mixture.*[2,22] Such a parameter is the *degree of polarization coherence of the wavelet trains,*[2,22,37,38] defined in terms of the corresponding correlation functions for $E_i(t)$, and expressed in terms of the components of the Stokes vector parameter with the aid of the relation

$$r = \frac{\sqrt{S_2^2 + S_3^2 + S_4^2}}{S_1}. \tag{53}$$

Using this quantity, we can write down the Stokes vector parameter in the form

$$\vec{S} = I(1, r\mathbf{P}), \tag{54}$$

where $\mathbf{P}$ is a unit three-dimensional "polarization vector" in the corresponding functional space. This notation leads directly to the brilliant formalism developed by Poincaré[97] for the description of polarization effects (the "Poincaré sphere"); which is convenient for the solution of certain problems, but which has dropped out of the modern concepts.[2]

Any polarization device can also be characterized by a certain Stokes vector parameter $\vec{s} = A(1, \mathbf{P}_y)$, where $\mathbf{P}_y$ is the polarization vector of the fully polarized ($r = 1$) component separated by the device, and A is the energy "transparency" of the device to this component. Then (see, e.g.,[2]), the brightness I_H of the light beam with a vector parameter $\vec{S}$, observed through such a device, is equal to

$$I_H = \frac{1}{2}\vec{S}\vec{s} = \frac{A}{2} I(1, r\mathbf{P})(1, \mathbf{P}_y) = \frac{A}{2} I(1 + r\cos\theta), \tag{55}$$

where

$$\cos\theta = \mathbf{P}\mathbf{P}_y. \tag{56}$$

It is important to note that at $r \neq 1$, i.e., when we are dealing with a stochastic mixture of incoherent wavelet trains that are in different polarization states, the classic description of polarization phenomena with the aid of the polarization ellipse (see, e.g., [46,60,120]) loses all physical meaning. It remains valid only within the framework of the notion of the light wavelet train, and essentially has no direct bearing on the ideas of ray optics.

It is obvious that for all four Stokes parameters S_i it is possible to set up the photometric quantities F_i and Φ_i which are analogous to the spectral density of the illumination $F = F_1$ and the spherical illumination $\Phi = \Phi_1$. These quantities must be set up in order to consider the conservation laws for the radiation in the scattering medium, and in particular, when deriving the matrix equation of radiation transport. [2,12,22,36] The method and the physical meaning of such a generalization for S_4 are obvious, inasmuch as $\mathbf{F}_4$ and Φ_4 are vectors corresponding (apart from a multiplier) to the flux density of the spin angular momentum of the radiation through the surface ($\mathbf{F}_4$) and the total spin angular momentum of the radiation in the volume element (Φ_4). The situation is more complicated with the behavior of the quantities F_2, F_3, and Φ_2, Φ_3, the physical meaning of which is not quite as clear. Caution must be exercised in their definition, in view of the preliminary transformation of the reference plane, i.e., the representation (51), to which pertain the Stokes parameters for wavelet trains with different directions. [2,22,98] The photometric utilization of these quantities still awaits its investigation.

D. Photometric aspect of the convervation laws

The foregoing leads us to the problem of the conservation laws for the dynamic characteristics of a light field with arbitrary space-time structure. Referring the reader to an analysis of the contemporary status of this problem, for example in[60–62,99–101], we shall dwell here only on certain aspects that have a direct bearing on our topic, and confine ourselves accordingly to the case of quasi-homogeneous and quasi-stationary fields.

Turning to Maxwell's equations under the simplest assumptions of a medium that is isotropic, nonmagnetic, immobile, and having no space charge, and following with slight modification the universally known procedure (see, e.g., [101]), we obtain the conservation laws in the form

$$\begin{gathered}-\operatorname{div}\operatorname{Re}\frac{c}{4\pi}[\vec{\mathscr{E}}\times\vec{\mathscr{H}}^*]=\operatorname{Re}\left(\vec{\mathscr{E}}\frac{\partial\vec{D}^*}{\partial t}\right)+\frac{1}{2}\frac{\partial(\vec{\mathscr{H}}\vec{\mathscr{H}}^*)}{\partial t}+\operatorname{Re}(\mathbf{j}\vec{\mathscr{E}}^*),\\ i\operatorname{div}[\vec{\mathscr{E}}\times\vec{\mathscr{E}}^*]=\frac{2}{c}\operatorname{Im}\left(\vec{\mathscr{E}}\frac{\partial\vec{\mathscr{H}}^*}{\partial t}\right).\end{gathered}\tag{57}$$

Resorting again to Fourier expansion into a temporal spectrum and assuming, just as in[101] the existence of frequency dispersion, i.e., assuming for the electric induction $\vec{\mathscr{D}}$ and for the current j

$$\begin{gathered}\vec{\mathscr{D}}(\mathbf{r},t)=\int\varepsilon(\omega,\mathbf{r})\vec{\mathscr{E}}(\omega,\mathbf{r})e^{i\omega t}d\omega,\\ \mathbf{j}(\mathbf{r},t)=\int\sigma(\omega,\mathbf{r})\vec{\mathscr{E}}(\omega,\mathbf{r})e^{i\omega t}d\omega,\end{gathered}\tag{58}$$

where $\sigma(\omega, \mathbf{r})$ is the electric conductivity at the frequency ω, we obtain

$$\left.\begin{aligned}\operatorname{Re}\left(\vec{\mathscr{E}}\frac{\partial\vec{D}^*}{\partial t}\right)&=\frac{i}{2}\int\int_0^{+\infty}[\omega\varepsilon(\omega)-\omega'\varepsilon^*(\omega')]\vec{\mathscr{E}}(\omega)\vec{\mathscr{E}}^*(\omega')e^{i(\omega'-\omega)t}d\omega\,d\omega',\\ \operatorname{Re}(\mathbf{j}\vec{\mathscr{E}}^*)&=\frac{1}{2}\int_0^{+\infty}[\sigma(\omega)+\sigma^*(\omega')]\vec{\mathscr{E}}(\omega)\vec{\mathscr{E}}^*(\omega')e^{i(\omega'-\omega)t}d\omega\,d\omega',\\ \frac{\partial(\vec{\mathscr{H}}\vec{\mathscr{H}}^*)}{\partial t}&=\frac{i}{2}\int\int_0^{+\infty}(\omega-\omega')\vec{\mathscr{H}}(\omega)\vec{\mathscr{H}}^*(\omega')e^{i(\omega'-\omega)t}d\omega\,d\omega',\end{aligned}\right\}\tag{59}$$

$$2\operatorname{Im}\left(\vec{\mathscr{E}}\frac{\partial\vec{\mathscr{H}}^*}{\partial t}\right)=-2\operatorname{Re}\int\int_0^{\infty}\omega\vec{\mathscr{E}}(\omega)\vec{\mathscr{H}}^*(\omega')e^{i(\omega'-\omega)t}d\omega\,d\omega'.\tag{60}$$

Expanding further $\vec{\mathscr{E}}(\omega, \mathbf{r})$ and $\vec{\mathscr{H}}(\omega, \mathbf{r})$ in plane waves and averaging in time and in space, we obtain

$$\left\langle-\operatorname{div}\operatorname{Re}\frac{c}{4\pi}[\vec{\mathscr{E}}\times\vec{\mathscr{H}}]\right\rangle_{T,s}=\int_0^{+\infty}\oint[\sigma(\omega)+\omega\operatorname{Im}\varepsilon(\omega)]\mathbf{E}(\omega,\mathbf{l})\mathbf{E}^*(\omega,\mathbf{l})\,d\omega\,dO,\tag{61}$$

$$\langle-i\operatorname{div}[\vec{\mathscr{E}}\times\vec{\mathscr{E}}^*]\rangle_{T,s}=\frac{2}{c}\operatorname{Re}\int_0^{+\infty}\oint\omega\mathbf{E}(\omega,\mathbf{l})\mathbf{H}^*(\omega,\mathbf{l})\,d\omega\,dO,\tag{62}$$

or, taking into account the relation that is valid for a plane wave[16,56]

$$\mathbf{H}(\omega,\mathbf{l})=m(\omega)[\mathbf{l}\times\mathbf{E}(\omega,\mathbf{l})]=[\mathbf{N}(\omega,\mathbf{l})-i\mathbf{K}(\omega,\mathbf{l}),\mathbf{E}(\omega,\mathbf{l})],\tag{63}$$

and also (10)–(17) and (31), (39), we obtain after simple transformations the conservation laws for the photometric quantities:

$$\begin{aligned}\langle-\operatorname{div}\mathbf{L}\rangle_{T,s}&=-\int_0^{\infty}\oint\operatorname{div}\mathbf{L}(\omega,\mathbf{l})\,d\omega\,dO\\ &=\frac{4\pi}{c}\int_0^{+\infty}\oint[\sigma(\omega)+\omega\operatorname{Im}\varepsilon(\omega)]I(\omega,\mathbf{l})\,d\omega\,dO\\ &=\frac{4\pi}{c}\int_0^{+\infty}[\sigma(\omega)+\omega\operatorname{Im}\varepsilon(\omega)]\Phi(\omega)\,d\omega\\ &=\frac{4\pi}{n}\int_0^{+\infty}[\sigma(\omega)+\omega\operatorname{Im}\varepsilon(\omega)]W(\omega)\,d\omega,\end{aligned}\tag{64}$$

$$\langle-\operatorname{div}\mathbf{p}\rangle_{T,s}=\frac{1}{c^2}\langle-\operatorname{div}\mathbf{L}\rangle_{T,s}=-\int_0^{+\infty}\oint\operatorname{div}\mathbf{p}(\omega,\mathbf{l})\,d\omega\,dO,\tag{65}$$

$$\begin{aligned}\langle\operatorname{div}\mathbf{M}\rangle_{T,s}&=-\int_0^{+\infty}\oint\operatorname{div}\mathbf{M}(\omega,\mathbf{l})\,d\omega\,dO=2\int_0^{+\infty}\oint\omega\mathbf{K}(\omega,\mathbf{l})\mathbf{M}(\omega,\mathbf{l})\,d\omega\,dO\\ &\approx2\int_0^{+\infty}\omega\varkappa(\omega)\left[\oint\mathbf{M}(\omega,\mathbf{l})\,dO\right]d\omega=2\int_0^{+\infty}\omega\varkappa(\omega)\Phi_4(\omega)\,d\omega.\end{aligned}\tag{66}$$

In other words, when averaging over sufficiently large spatial and temporal regions, the dynamic characteristics of a quasi-homogeneous quasi-stationary radiation field are additive superpositions of analogous characteristics of the individual wavelet trains into which the field is split, and the conservation laws, considered "in the mean" for the corresponding region, do not contain "exchange" crossing terms—each of the trains experiences an independent action on the part of the medium. We emphasize that we are dealing here with another characteristic of the radiation field itself, and when the conservation laws are formulated one must bear in mind also the effects due to the action of the field on the medium (say, the action of the Abraham force)—see[81,82].

We note also that a relation of the type (64) (albeit in a somewhat more simplified form) was formulated first for the light field as self evident, from intuitive considerations, by Gershun,[20] and was obtained later in 1957 in[10,12,98] from the radiation-transfer equation (based, as already mentioned, likewise only on intuitive photometric considerations). Since that time it became the basis of the most widely used methods of measuring the absorptivity of scattering media (including natural waters) inasmuch as both $\langle \operatorname{div} L\rangle_{T,S}$ and $W(\omega)$ can be measured directly in a layer of a scattering medium.

It seems that introduction into the system of ray optics of the entire assembly of conservation laws for the dynamic parameters remains one of the most important problems of its further development, especially in connection with the study of the specifics of the matrices describing the action of a medium on a light ray (see below) at different types of action.

E. Photometry and general theory of coherence

It now becomes necessary to ascertain the connection between the photometric characteristics of the radiation field and its statistical structure, which is the subject of the general theory of coherence. The latter, as is well known (see, e. g., [37–42]) operates, generally speaking, with cross-relaxation functions (mutual-coherence functions) of the type

$$\Gamma_{\alpha\beta}(\theta, \rho) = \frac{cn}{4\pi}\langle \mathscr{E}_\alpha(t, \mathbf{r})\mathscr{E}^*_\beta(t+\theta, \mathbf{r}+\rho)\rangle, \qquad (67)$$

where $\mathscr{E}_\alpha$ is the α-th component of the field intensity ($\alpha, \beta = x, y, z$) and the averaging is over the time t and the coordinates r within the limits of quasi-stationarity and quasi-homogeneity of the radiation field.

Changing over to the Fourier expansion $\mathscr{E}_\alpha(t, \mathbf{r})$ in space and in time and averaging with respect to the parameters of the radiation receiver, we obtain

$$\Gamma_{\alpha\beta}(\theta, \rho) = \int_0^{+\infty} \oint m^{jk}_{\alpha\beta}(\mathbf{l}) J_{jk}(\omega, \mathbf{l})\, e^{-i\omega\theta + ik\rho}\, d\omega\, dO, \qquad (68)$$

where

$$m^{jk}_{\alpha\beta}(\mathbf{l}) = c_{\alpha j}(\mathbf{l})\, c^{*}_{\beta k}(\mathbf{l}),$$

$c_{\alpha j}(\mathbf{l}) = (\mathbf{e}_j)_\alpha$ are generally speaking the complex direction cosines of the j-th basis vector $\mathbf{e}_j$ for the direction $\mathbf{l}(j, k = 1, 2)$, defined in accordance with (51) and (52), while the functions

$$J_{jk}(\omega, \mathbf{l}) = \frac{cn}{4\pi} E_j(\omega, \mathbf{l}) E^*_k(\omega, \mathbf{l}) \qquad (69)$$

are the components of the quantum-mechanical density matrix for a wavelet train with frequency ω in the propagation direction l (see[2,22,23,57]).

The density matrix J is connected with the Stokes parameters S_i by the relations[57]

$$S_i(\omega, \mathbf{l}) = \operatorname{Sp}(J\sigma^i), \quad J(\omega, \mathbf{l}) = \frac{1}{2} S_i(\omega, \mathbf{l})\, \sigma^i, \qquad (70)$$

in which σ^i are Pauli matrices (see (49)). Accordingly, we obtain from (69)

$$\Gamma_{\alpha\beta}(\theta, \rho) = \frac{1}{2}\int_0^{+\infty} \oint m^{jk}_{\alpha\beta}(\mathbf{l})\, \sigma^i_{jk} S_i(\omega, \mathbf{l})\, e^{-i\omega\theta + ik\rho}\, d\omega\, dO \qquad (71)$$

and, in particular,

$$\Gamma(\theta, \rho) = \int_0^{+\infty} \oint I(\omega, \mathbf{l})\, e^{-i\omega\theta + ik\rho}\, d\omega\, dO, \qquad (72)$$

where

$$\Gamma(\theta, \rho) = \frac{cn}{4\pi}\langle \vec{\mathscr{E}}(\mathbf{r}, t)\vec{\mathscr{E}}^*(\mathbf{r}+\rho, t+\theta)\rangle. \qquad (73)$$

Thus, the main photometric quantities $I(\omega, \mathbf{l})$, $J_{jk}(\omega, \mathbf{l})$, $S_i(\omega, \mathbf{l})$ turn out to be the coefficients of the space-time Fourier expansions of the corresponding correlation functions of the electric field strength of the radiation.

If we put in (72) $\rho = 0$ and integrate in the right-hand side over the directions, then, taking (39) into account, we have

$$\Gamma(\theta, \rho = 0) = \int_0^{+\infty} \Phi(\omega)\, e^{-i\omega\theta}\, d\omega, \qquad (74)$$

i. e., the spectral density of the spherical illumination has themeaning of the spectral density of the expansion of $\Gamma(\theta = 0, \rho)$ into a temporal spectrum. In exactly the same way, after integrating the right-hand side of (72) with respect to ω at $\theta = 0$ we obtain, taking (44) into account,

$$\Gamma(\theta = 0, \rho) = \oint I(\mathbf{l})\, e^{ik\rho} dO, \qquad (75)$$

i. e., the brightness of the ray has the meaning of the spectral density of the expansion of $\Gamma(\theta = 0, \rho)$ into a spatial spectrum.

As applied to the wavelet train, the last relation was first obtained and analyzed by Dolin[25] and has led it to the first electrodynamic formulation of the brightness concept, which coincides with (31) at $K = 0$. The general case of a diffuse light field was not considered by L. S. Dolin.

A later attempt to obtain the photometric quantities and relations from the general theory of radiation-field coherence was undertaken in[67,68], again not for the general case but as applied to the radiation field of a certain object of finite area. In essence the authors have shown that to construct the photometric quantities it is necessary to carry out spatial averaging (they do not consider temporal averaging) of correlation functions that are quadratic in the electric field intensity. However, by disengaging this point of view from the measurement process and from the receiver properties, and by connecting it entirely with the properties of the luminous object, they have deprived it of generality (by eliminating, in particular, the possibility of analyzing a diffuse optical field) and encountered a number of difficulties which could not be consistently resolved.

At the same time, they have hit upon ways of introducing, within the framework of electrodynamics, such properties as luminosity or brightness of the object, although their analysis calls for some review in light of the conception developed here—cf. (31)–(33).

It appears also that through a misunderstanding the authors of[68] regard as a branch of photometry Lambert's law, i.e., the orthotropy of diffuse radiation or of diffusely reflecting objects. Actually, the problem of the angular distribution of the intrinsic or reflected radiation belongs entirely to the field of radiative-transfer theory; see, e.g., [7–14,91–93,102], and it has been established both theoretically and experimentally that, generally speaking, Lambert's law cannot hold, and is only a rough albeit convenient approximation.

Returning to the connection between the quantities $\Gamma(\theta, \rho)$ and $I(\omega, \mathbf{l})$ let us consider by way of example an interferometer of the Michelson type, which produces a path difference $c\theta$ between two components of one and the same light beam with $I(\omega, \mathbf{l}) = I_0(\omega)\, \delta(\mathbf{l} - \mathbf{l}_0)$. The brightness of the interference picture at the exit of the interferometer is determined in this case by a relation similar to (55) (see, e.g., [37])

$$I_s = \int_0^{+\infty} I_s(\omega)\, d\omega = (A+B) \int_0^{+\infty} I_0(\omega)\, d\omega + 2\sqrt{AB}\, \mathrm{Re}\, \Gamma(\vartheta, \rho = 0), \qquad (76)$$

where A and B are the effective transparencies of the corresponding channels of the interferometer, and account is taken of relation (44). Substituting here the value of Γ from (72), we obtain the results in photometric form

$$I_s = \int_0^{+\infty} [(A+B) + 2\sqrt{AB} \cos(\omega\theta)]\, I_0(\omega)\, d\omega. \qquad (77)$$

In the general case, the treatment of the interference experiments leads to the need for using cross-correlation functions of the type (67), and cannot always be reduced to photometric form. We encounter a situation of this type, for example, when considering interference of waves scattered by individual elements (say by particles) of a stochastically inhomogeneous medium, when allowance for the multiple interference phenomena creates the basis for holographic study of similar media, and in the electrodynamic derivation and explanation of the limits of applicability of the radiative-transfer equation (see, e.g., [36]).

There are also many phenomena that can be understood only by invoking statistical moments of higher order; see[37,41,76]. We shall discuss one of them in the next section.

Most generalizations of this type, however, pertain already to statistical optics itself and have no connection with the principles of photometry as a dynamic ray-optics aspect operating with the laws of conversion and methods of measurement of the Stokes parameters of light rays in those cases when the concept of light rays is admissible. Once these parameters are introduced into the photometry system and the electrodynamic origin of the concepts and relations of the system are explained, the formulation of photometry as an independent branch of statistical optics should be regarded as completed.

In the general structure of statistical electrodynamics, photometry stands out clearly as some limiting case. The region of its applicability is distinctly outlined by the possibility of using ray concepts, and the subsequent analysis is necessary here only from the point of view of introducing into photometric practice the entire assortment of the dynamic characteristics of the wavelet train, including the conservation laws, and also to spell out concretely individual details concerning the connection between the photometric quantities and the more general correlation characteristics of the radiation field.

Perhaps the most important problem of this type remains the comprehensive analysis of the conditions for reconciling the time constant and the area of the receiver (i.e., the averaging scale) with the statistical structure of the radiation field (i.e., the space-time scales of the interference picture), and the refinement of the limits of applicability of the photometric approach as a function of the character of this reconciliation.

Problems of this kind arise, for example, in the observation of light scattered by a colloidal system. It has already been mentioned that in the case of scattered light at $\Sigma \gg s$ and $\tau \gg T$ the receiver destroys, as it were, the interference pattern by splitting the field into an aggregate of incoherent wavelet trains and receiving them separately, corresponding to incoherence of the individual scattering acts. At small scattering angles, however, the characteristic scales s and T for the scattered light increase to such an extent, that the situation changes and leads to the formation of a so-called "grainy structure" of the scattered-light field and of the holograms of the scattering medium, and also to fluctuations in their form and brightness, [12] such as were observed, for example, in[103] under the conditions $\Sigma \approx s$ and $\tau \approx T$. Similar phenomena are observed also at larger scattering angles under conditions when τ and Σ are suitably decreased, and this has become an effective method of investigating Brownian motion of small particles suspended in a medium—see, e.g., [104–107].

This leads directly to the need of studying fluctuations of photometric quantities, i.e., phenomena of "interference of brightnesses"[37] or in other words to incorporation of the fourth statistical moments relative to the radiation-field intensities into the system of ray optics.

In this connection we emphasize once more that in the case of scattering medium the very applicability of the ray treatment, and consequently also of the radiative-transfer equation, is ensured (to the extent that they are attainable) by a suitable choice of the parameters of the volume element to which the operation of quadratic averaging pertains, and of the time of this averaging. [30,36] It is important to bear in mind that this averaging should take into account the entire aggregate of the cooperative dispersion phenomena. These phenomena can greatly influence the effective three-dimensional phenomenogical parameters of the scattering medium, with which the photometric theory, and in particular, radiative-transfer theory, operates. [2,12,18,30,36]

All other problems concerning the correlation properties of the radiation field, including their transmission through the medium, as well as the treatment of results of measurements made outside the framework of the applicability of the photometry, are already the subject of statistical optics proper, just as diffraction phenomena are beyond the scope of geometrical optics and pertain entirely to the domain of wave theory.

Thus, in the act of measurement with the aid of an optical receiver, the radiation field serves as an aggregate of incoherent light rays, and this leads to the main concepts and laws of photometry, and subsequently also to their polarimetric generalization. The conservation laws also acquire in this case a "ray" formulation that reflects the independence of the action of the medium on each of the wavelet trains.

The resultant system of concepts and representations of ray optics (brightness, illumination, spherical illumination, Stokes parameters, etc.) are related in a definite manner with the concepts of energy density and Poynting vector, which are used in electrodynamics, and also with the traditional formalism of general coherence theory. The fundamental difference between the corresponding quantities and concepts lies in the fact that they are defined in different representations, namely the electrodynamic quantities are defined in a space-time representation and the optical quantities in a frequency-momentum representation.

We must now consider the processes of transformation of light rays under the action of the medium, and the methods of describing these processes within the framework of the ray-optics approximation.

3. LIGHT RAY AND ITS TRANSFORMATION

A. Light ray and selection of incoherent wavelet trains

We have established that the main photometric quantities, namely the aggregate of the four Stokes parameters (48), including the brightness (31), pertain by definition to a certain wavelet train of form (1). The measurement apparatus required to determine these quantities must therefore be provided with an angle filter $f(\mathbf{l})$ and a frequency filter $f(\omega)$, which ensure separation of sufficiently small intervals of direction ($\Delta\mathbf{l}$) and frequency ($\Delta\omega$), accessible to the Fourier components of the radiation field reaching the receiver and acting on it. In other words, it is necessary to limit in frequency-momentum space the region of the radiation sensed by the receiver to a small vicinity of a certain point (ω, $\mathbf{l}$). According to (3), the time T and the area s of the coherence of the formation separated by the filters are equal to $T=1/\Delta\omega$ and $s=\lambda^2/(\Delta\mathbf{l})^2$.

Assume now that the time constant of the investigated receiver is $\tau \gg T$ and its geometric area is $\Sigma \gg s$. Then, as we have seen, the radiation incident on the receiver will be split into an aggregate of incoherent wavelet trains, independently acting on the receiver, $T'=\tau$ and $s'=\Sigma$, which the receiver will sense as a stochastic formation for which, owing to the assumed smallness of $\Delta\mathbf{l}$ and $\Delta\omega$, the Stokes parameters of individual wavelet trains are additive.[2,22,23,37,38,40]

This allows us to extend the photometric concepts and quantities to include the light ray, if the latter is taken to mean a stochastic mixture of wavelet trains that hardly differ in frequency and in propagation direction, and is received by the measuring devices as a single formation

$$S_i^{\text{ray}}(\omega, \mathbf{l}) = \sum_j S_i^j(\omega, \mathbf{l}), \tag{78}$$

where j is the number of the train.

However, the trains that form the light ray can be in different polarization states that depend on their origin or prior history.[22,23,107] Therefore a complete statistical description of the light ray cannot be confined to indication of the degree r of its polarization coherence [see (53)], which characterizes, as shown in[22], only the degree of homogeneity of the polarization of the different wavelet trains, or, in other words[37,38,41,43], the fractional contribution to the brightness of the light and to its fully polarized (i.e., coherent) component, if the latter is separated from the fully incoherent component, as is evident from (54).

A similar description with the aid of the quantity S_i (which includes r) reduces, as can readily be seen, to allowance for only the second statistical moment with respect to the intensity $\mathbf{E}$ of the radiation field—see[37]. On the other hand, a complete description of the state of the ray should contain also moments of higher order (say of the fourth), describing the fluctuations of the quantities S_i (i.e., the so-called "intensity interference" phenomena), and in the ideal case—the three-dimensional distribution function $W(\mathrm{p})$ of the wavelet trains in terms of the polarization states.[107]

By way of example let us consider single scattering of light by a colloidal system. On the one hand, the smearing of the frequency of the scattered light as a result of the Doppler broadening due to the Brownian motion or to the turbulent mixing produces brightness fluctuations (see, e.g.,[103–106]), and consequently also fluctuations of the polarization of the scattered light, which make it possible, in particular, to determine the distribution of the scattering particles by size.

On the other hand, a colloid can be regarded as an aggregate of particles distributed over states p with a probability density $n(p)$—this may be, say, the distribution of the molecules with respect to the orientations or the distribution of spherical particles in dimensions, etc. For each particle, the Stokes parameter of the radiation scattered by the particle in the direction $\mathbf{l}$ is written in the form[2]

$$S_i^{\text{scat}}(\mathbf{l};p) = D_{ik}(\mathbf{l}, p)\, S_k^0, \tag{79}$$

where $D_{ik}(\mathbf{l}, p)$ is the matrix of the scattering of the light by a particle in the state p, and S_k^0 are the Stokes parameters of the light incident on the colloid (see also below).

Owing to the incoherence of the scattering of the light by the stochastically distributed particles, i.e., owing to the additivity of $D_{ik}(p)$,[2,22,23] we have for a scattered-light ray produced by the entire aggregate of particles

$$S_i^{\text{ray}}(\omega, \mathbf{l}) = Ic_i^{\text{луч}} = S_k^0 \int D_{ik}(\mathbf{l}, p)\, n(p)\, dp = \int S_i^1(\mathbf{l}, p)\, n(p)\, dp = I \int c_i^1(p)\, w(p)\, (dp), \tag{80}$$

where according to (54) $\vec{c} \equiv (1, rP_1, rP_3, rP_4)$, S_i^1 and c_i^1 pertain to the coherent act of scattering of the light by one particle in the state p, and $Iw(p)\,dp = I^1(p)\,n(p)\,dp$ is the brightness of the light scattered by the entire aggregate of particles in states from p to $p + dp$, from which it follows, in particular, that $c_i^{\text{ray}} = \langle c_i^1(p)\rangle$.

Any polarimetric device based on the use of dichroism, birefringence, Fresnel reflections, and other phenomena that constitute splitting of each train into an aggregate of two alternately polarized components and their independent transformation (selective attenuation, removal, refraction, etc.) operates as a filter that separates a component with parameters $c_i = s_i$ (see Chap. 2), and in accordance with (55) the brightness of the light transmitted by this device is

$$I_H = \frac{1}{2} I c_i s_i. \tag{81}$$

Thus, under arbitrary variation of the polarization device (i.e., of the parameters s_i), the information on the measured light ray is restricted to the quantities $I = \langle I^1(p)\rangle$ and $c_i = \langle c_i^1(p)\rangle$, i.e., to the first moment with respect to the quantities of the form $\langle E_i E_k\rangle$. The higher moments remain certainly inaccessible to measurements by similar methods and call for the use of either fluctuation phenomena or fundamentally different means. This is quite evidently the reason why the question of determining the distribution function $W(\mathbf{p})$ of the wavelet trains with respect to the polarization state has not been raised in the literature until recently.

One of the possible means of measuring $W(\mathbf{p})$, as shown in[107], is to use the dependence of the direction of the Poynting vector of a propagating inhomogeneous plane wave on the state of its polarization. Indeed, according to (15), in an isotropic medium (including also vacuum) the radiation power is in general transported by a wavelet train of inhomogeneous waves with practically identical imaginary part $\mathbf{j}$ of the complex waves normal—see (9)—not in the direction of the real axis $\mathbf{l}$ of the train, but in the direction[56]

$$\vec{\mathscr{L}} = \mathbf{l} + q\,[\mathbf{l} \times \mathbf{j}], \tag{82}$$

where $q = S_4/S_1$ is the degree of ellipticity of the polarization of the wavelet train, defined by relation (11).[2] In other words, the direction of motion of the wavelet train deviates from the direction of the phase vector towards the perpendicular to the $(\mathbf{l}, \mathbf{j})$ plane, by an angle

$$\vartheta = q\,|\mathbf{j}|, \tag{83}$$

as was qualitatively pointed out back in 1912 by Boguslavskiĭ.[108]

Owing to the mutual incoherence of the wavelet trains making up the light ray, their Poynting vectors are additive and each of them undergoes an independent deviation from the phase-normal direction that is common to all the wavelet trains. Thus, if the light ray is made up of an aggregate of trains of inhomogeneous waves, then the individual wavelet trains will move in different directions that depend on q (i.e., on their spin angular momentum)[56] and should fan out and form a spatial sweep of the wavelet-train distribution function $W(q)$, i.e., the *spin-spectrum* of the light ray.[107]

Practical realization of spin-spectroscopy apparatus calls for conversion of the homogeneous light ray with $\mathbf{j} = 0$ into an inhomogeneous beam, and this can be effected in various ways.[107,109] One of them can be refraction in an absorbing medium,[56] another is passage of the light beam through a prism of absorbing material or else through any medium (or body, say a photometric wedge) with a constant transverse optical-density gradient. After passing through such a prism, medium, or body, a homogeneous plane wave is transformed into a weakly-inhomogeneous wave with

$$\mathbf{j} = \frac{\nabla D}{\lg e \cdot k_0 n}, \tag{84}$$

where e is the base of the natural logarithms.

The expansion of the light ray into an angular spectrum with respect to the quantity q takes place accordingly in such a way that the angular illumination distribution produced by the aggregate of the diverging wavelet trains reproduces directly the distribution function $W(q)$, while the expansion into a spin spectrum occurs in a direction perpendicular to ∇D. In particular, when a prism of absorbing material is used, a simultaneous double expansion takes place—perpendicular to the edge of the prism in the ordinary frequency spectrum and parallel to the edge of the prism in the spin spectrum.

A third possibility of converting a homogeneous wave into an inhomogeneous one and accordingly of spatial selection of wavelet trains by the values of their spin angular momentum q is connected with the use of total internal reflection. The very effect of the transverse displacement of the wavelet train in total internal reflection as a function of q, first predicted by Fedorov,[110] was investigated theoretically quite some time ago (see, e.g.,[61,111,112]) and was recently confirmed experimentally.[113] Its use for the selection of trains and for the determination of $W(q)$, however, was first proposed in[107,109].

If a rotating compensator is placed in front of the device that makes the wave inhomogeneous, then the registration of $W(q)$ at three different positions of the compensator that transforms in a known manner the state of polarization of each wavelet train will be equivalent to a determination of the total three-dimensional distribution function $W(\mathbf{p})$ of the wavelet trains with respect to all the possible polarization states.[107,109]

From (83) and (84), under reasonable assumptions concerning $|\nabla D|$ and the geometrical dimensions of the wedge or the prism (i.e., the beam cross section), it can be easily seen that at $q = \pm 1$ the angle is $\vartheta \approx 10''-30''$, i.e., the effect of the polarization selection of the trains is relatively small. Spatial selection of the trains with $q = \pm 1$, and in the case of total internal reflection, it is also small and reaches only (10–30) λ, i.e., about 5–15 μ for visible light.[113]

Therefore the real resolution of the spin spectroscopy is determined primarily by diffraction noise in the case of transverse displacements of the Poynting vector. The corresponding estimates, carried out on the basis of a generalization, specially developed for this purpose,[114] of classical diffraction theory to include inhomogeneous waves, have shown that at reasonable parameters of the apparatus this noise is negligible and does not stand in the way of attaining an acceptable resolution.

Returning to the distribution function of the wavelet trains over the polarization states for light scattering by a colloidal system—see (80)—we note that the quantities c_i, which characterize the state of the scattered ray, are connected with the quantities c_i^1 by the relation

$$c_i = \int c_i^1(p)\, W(c_i^1)\, dc_i^1, \tag{85}$$

where $W(c_i^1)$ is the probability density for the realization of the quantity c_i^1 in the scattering act (or emission act, if we are dealing with an aggregate of incoherent radiators). The use of spin-spectroscopy makes it possible to determine $W(c_i^1)$ directly, i.e., to obtain information on the distribution function $w(p)$ of the scatters (or radiators), and consequently also on $n(p)$ (see (80)). For example, if the scattering is by spherical particles of radius $a \lesssim 2/k_0$, then $c_4^1 \equiv q^1$ depends monotonically on a and

$$n(a) = \frac{w(a)}{I^1(a)} = \frac{W(q^1)}{I^1(a)} \frac{dq^1}{da}, \tag{86}$$

where $I^1(a)$ and $q^1(a)$ are known from the Mie theory of the scattering of light by a sphere.[17]

B. Differential transformation of a ray

Within the framework of the validity of the light-ray concept, the propagation of a wavelet train in an inhomogeneous medium represents a consecutive sequence of acts of local transformation of the wavelet train (reflection, refraction, scattering, etc. by the interfaces or by local inhomogeneities of the medium), interspersed by acts of propagation of the wavelet train from one local transformation to another.[22] Each local transformation constitutes a direct change of the wavelet train parameters, whereas the transformation occuring during the propagation is characterized by a gradual differential transformation of the parameters.[2,23]

Since, furthermore, the very concept of the wavelet train is valid only in a quasi-homogeneous medium, the group of possible differential transformations is limited to the phenomena of refraction, absorption, dichroism, birefringence, and the radiation of the medium proper (including induced emission). Each train is transformed here independently, and it is this which determines the result of the transformation of the ray as a whole.

The notion of a quasi-homogeneous medium as applied to the conditions of the propagation of a plane monochromatic wave in it can be generalized also to include a medium containing local inhomogeneities, provided that the dimensions of these inhomogeneities are small in comparison with the photon mean free path,[23,30,31,36] and the presence of the scattering particles makes its own contribution to the refractive index of the medium, i.e., a change in the wave propagation velocity and the phenomena of absorption, dichroism, and birefringence.[2,12,23,36,44] It is necessary, in addition, that the averaging extend over a volume element with a sufficiently large cross section.[12,23,31]

In the opposite case of close packing of the inhomogeneities (for example, powders, precipitates), as shown by Ivanov's experimental investigations,[115] the concept of a light ray, and with it also the fundamentals of photometry (including radiation-transport theory) no longer correspond to reality, since an important role is assumed by essentially inhomogeneous components of the Fourier expansion of the radiation field in the medium.

If we consider a quasi-homogeneous medium in spatial scales that correspond to the realization of continuous differential transformations of the train, then we can assume[56,116]

$$m = n_0(1 + \mu - i\nu), \quad \mu = \frac{n - n_0}{n_0} \ll 1, \quad \nu = \frac{\varkappa}{n_0} \ll 1, \tag{87}$$

whence, in particular,

$$\frac{\nabla m}{m} \approx \nabla\mu - i\nabla\nu, \tag{88}$$

and we can also use the approximate relations (9), in view of the assumed slight inhomogeneity of the train ($\mathbf{j} \ll 1$). The general theory of refraction of inhomogeneous waves[56] leads then to approximate equations for the real and imaginary parts of the complex wave normal $\mathbf{l} = \mathbf{l} + i\mathbf{j}$ of the axial wave of the wavelet train:

$$\begin{gathered} \operatorname{rot}\mathbf{l} = [\mathbf{l}\times\nabla\mu], \quad \operatorname{rot}\mathbf{j} = [\nu\mathbf{l} + \mathbf{j}]\times\nabla\mu] - [\mathbf{l}\times\nabla\nu], \\ \operatorname{div}[\mathbf{l}\times\mathbf{j}] = -2[\mathbf{l}\times\mathbf{j}]\nabla\mu, \quad (\mathbf{l}\nabla)\mathbf{l} = \nabla\mu - (\mathbf{l}\nabla\mu)\mathbf{l}, \end{gathered} \tag{89}$$

and of the direction of the phase normal $\mathbf{N}/N = \mathbf{l} + \nu\mathbf{j} \ll \mathbf{l}$. The wavelet-motion direction is described by (82), the operator of differentiation along the wavelet train trajectory being

$$(\vec{\mathscr{L}}\nabla) = (\mathbf{l}) + (\{\nu\mathbf{j} + q[\mathbf{l}\times\mathbf{j}]\}\nabla) \approx (\mathbf{l}\nabla) + q([\mathbf{l}\times\mathbf{j}]\nabla). \tag{90}$$

Since the light-ray concept is based on the assumption that j and ν are extremely small, the curvature radius $\mathscr{R}$ and the torsion $\mathscr{T}$ of the real trajectory of the wavelet train do not differ in practice from those defined by the relations

$$\frac{1}{\mathscr{R}^2} \approx (\nabla\mu)^2 - (l\nabla\mu)^2, \quad \frac{1}{\mathscr{T}^2} \approx \frac{[\mathbf{l}\nabla\mu](\mathbf{l}\nabla)\nabla\mu}{(\nabla\mu)^2 - (\mathbf{l}\nabla\mu)^2}. \tag{91}$$

On the other hand, the principal normal $\mathbf{p}$ and the binormal $\mathbf{s}$ hardly differ from the real vectors

$$\mathbf{p} = \mathscr{R}[\nabla\mu - (\mathbf{l}\nabla\mu)\mathbf{l}], \quad \mathbf{s} = \mathscr{R}[\mathbf{l}\times\nabla\mu], \quad \mathbf{p}^2 = \mathbf{s}^2 = 1, \tag{92}$$

with

$$(\vec{\mathscr{L}}\nabla)\,\mathbf{p}\approx-\frac{1}{\mathscr{R}}-\frac{\mathbf{s}}{\mathscr{T}},\quad (\vec{\mathscr{L}}\nabla)\,\mathbf{s}\approx\frac{\mathbf{p}}{\mathscr{T}}. \tag{93}$$

When considering the transformation of the field intensity of the train by refraction in a quasi-homogeneous isotropic medium, it is convenient to choose as the reference plane the refraction plane $(\mathbf{p}, \vec{\mathscr{L}})$ itself, and to choose as the basis vectors

$$\mathbf{e}_1=\cos\gamma\mathbf{p}-i\sin\gamma\mathbf{s},\quad \mathbf{e}_2=i\sin\gamma\mathbf{p}+\cos\gamma\mathbf{s}, \tag{94}$$

whence

$$(\vec{\mathscr{L}}\nabla)\,\mathbf{e}_1=-\frac{\cos\gamma}{\mathscr{H}}\vec{\mathscr{L}}-\frac{\mathbf{e}_2^*}{\mathscr{T}},\quad (\vec{\mathscr{L}}\nabla)\,\mathbf{e}_2=-\frac{\sin\gamma}{\mathscr{H}}\vec{\mathscr{L}}+\frac{\mathbf{e}_1^*}{\mathscr{T}}. \tag{95}$$

Assuming, in particular, $\gamma=0$ and returning to (51), i.e., assuming

$$\mathbf{E}=E_1\mathbf{p}+E_2\mathbf{s}, \tag{96}$$

we obtain from the conditions of the solvability of the system of first-approximation equations of geometrical optics[55,56] relations that govern the variation of $\mathbf{E}$ along the ray trajectory

$$\operatorname{div}\{n(\mathbf{E}\mathbf{E}^*)\,\mathbf{l}\}=0,\quad \mathbf{l}\,\frac{E_1\nabla E_2-E_2\nabla E_1}{\mathbf{E}\mathbf{E}^*}=\frac{1}{\mathscr{T}}, \tag{97}$$

from which we have omitted small trajectories due to the fact that m and $\mathbf{l}$ are complex, and also due to the difference between $\mathbf{l}$ and $\vec{\mathscr{L}}$. Solving (92) with respect to $(\mathbf{l}\cdot\nabla)E_i$ we find after simple transformations

$$d\mathbf{E}=-\nu\mathbf{E}\,dl, \tag{98}$$

where

$$\nu=\frac{1}{\mathscr{T}}\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}+\left[\frac{1}{2}\left(\frac{d\ln m}{dl}+\operatorname{div}\mathbf{l}\right)+ik_0m\right]\delta_{ik}, \tag{99}$$

with the first term, which depends on the torsion radius, due explicitly to the rotation of the basis vectors $\mathbf{p}$ and $\mathbf{s}$ themselves in the space,[55,56] while the term ik_0m takes into account the advance of the complex phase along the wave normal.[2]

In addition to the smooth inhomogeneity of the medium, which produces the refraction and expresses itself in the coordinate dependence of the refractive index, the wavelet train may encounter in its path also local inhomogeneities (for example, light-scattering particles), which either remove quanta from the wavelet train, or else transform them coherently. The ensuring changes in the wavelet-train can be regarded as the result of interference between the incident wave and the wave scattered by the particles (see, e.g.,[33,44]). This, as is well known, is the basis of dispersion theory.[44] Generalization of these ideas to non-molecular scattering[23] (see also[2,12]) has shown that the presence of inhomogeneities along the path of the wavelet train is equivalent to the appearance of an additional term in the matrix ν, namely

$$\nu_{ik}^{\text{add}}=\frac{2\pi}{k^2}\sum_s\mu_{ik}^s(\mathbf{l},\mathbf{l})=\frac{2\pi N}{k^2}\mu_{ik}(\mathbf{l},\mathbf{l}), \tag{100}$$

where $\mu_{ik}^s(\mathbf{l},\mathbf{l})$ is the component of the amplitude scattering matrix of the light by the s-th particle in the forward direction, and N is the concentration of the particles.

If we confine ourselves to the case when there is no torsion of the ray ($\mathscr{T}\to\infty$), then we get from (96) and (97) for the matrix of the coherent differential transformation

$$\nu=\left[\frac{1}{2}\left(\frac{d\ln m}{dl}+\operatorname{div}\mathbf{l}\right)+ik_0m\right]\delta_{ik}+\frac{2\pi N}{k^2}\mu_{ik}(\mathbf{l},\mathbf{l}), \tag{101}$$

(torsion complicates predominantly the forms of the expressions, but not the gist of the phenomena); this corresponds in the case of isotropic particles ($\mu_{ik}=\mu_{11}\delta_{ik}$) to an effective change of the refractive index

$$m_{\text{eff}}=m\left(1+\frac{2\pi N}{ik^3}\mu_{11}\right). \tag{102}$$

Examples of an actual calculation of the form of the matrix for a magnetoactive plasma can be found, for example, in[62–64]. We note incidentally that the remark made on page 405 of[63], concerning the determination of the components of the tensor J_{ik}, is based simply on a misunderstanding. This choice is not arbitrary and is dictated by other considerations than those discussed in[63]. As seen from the foregoing, the photometric approximation operates essentially with the Stokes parameters defined by relation (48), while the variation of these parameters in the direction of the group velocity can be traced in accordance with (90) and (85).[56]

Owing to the linearity and homogeneity of Maxwell's equations, the transformation of the correlation functions that characterize the state of the wavelet train (and consequently also of the Stokes parameters) along its trajectory will also be continuous and can be written in the form[2,23]

$$d\vec{S}=Q(\omega,\mathbf{l})\,\vec{S}\,dl, \tag{103}$$

where, taking (101) into account,

$$\left.\begin{aligned}
&Q_{11}=Q_{22}=Q_{33}=Q_{44}=\frac{d\mu}{dl}+\operatorname{div}\mathbf{l}+2k_0\varkappa+\frac{2\pi N}{k^2}\operatorname{Re}(\mu_{11}+\mu_{22}),\\
&Q_{12}=Q_{21}=\frac{2\pi N}{k^2}\operatorname{Re}(\mu_{11}-\mu_{22}),\quad Q_{13}=Q_{31}=\frac{2\pi N}{k^2}\operatorname{Re}(\mu_{12}+\mu_{21}),\\
&Q_{14}=-Q_{41}=\frac{2\pi N}{k^2}\operatorname{Im}(\mu_{12}-\mu_{21}),\quad Q_{23}=-Q_{32}=\frac{2\pi N}{k^2}\operatorname{Re}(\mu_{12}-\mu_{21}),\\
&Q_{42}=-Q_{24}=\frac{2\pi N}{k^2}\operatorname{Im}(\mu_{12}+\mu_{21}),\quad Q_{34}=-Q_{43}=\frac{2\pi N}{k^2}\operatorname{Im}(\mu_{11}-\mu_{22}).
\end{aligned}\right\} \tag{104}$$

Since each of the wavelets making up the ray is transformed independently, the transformation matrix $Q(\omega,\mathbf{l})$ pertains not only to an individual wavelet, but to the entire ray as a unit.

If the local inhomogeneities are isotropic we have $\mu_{ik}=\mu_{11}\delta_{ik}$ and the matrix $Q(\omega,\mathbf{l})$ degenerates into a scalar $Q_{ik}=\varepsilon(\omega)\delta_{ik}$, where the extinction coefficient is

$$\varepsilon(\omega)=\frac{d\mu(\omega)}{dl}+\operatorname{div}\mathbf{l}+2k_0\varkappa(\omega)+\frac{4\pi N}{k^2}\operatorname{Re}\mu_{11}(\omega). \tag{105}$$

We note that, generally speaking, $\varkappa(\omega)$ can be negative, as is the case under certain limitations as a result of optical pumping.

From relations (100) and (104), in particular, it is obvious[12] that the fluctuations of N or $\mu_{11}(\omega)$ can cause fluctuations of the amplitude and phase of the light wavelet train, of the same degree as the fluctuations of the refractive index μ considered in the monograph[116]. A successful attempt to take this phenomenon into account was undertaken recently in[117].

C. Local transformation of a ray and the radiation transport equation

We have already mentioned that in each act of local transformation of the radiation by a medium, resulting in a jumplike change of the state, direction, or frequency of the light wavelet train (for example, upon reflection, refraction, scattering, etc), but preserving the linearity and homogeneity of the electrodynamics equations, each wavelet train is transformed independently and the results of the transformation depend on the initial state of the wavelet train.[2,22,23] Accordingly, the act of local transformation can be represented within the framework of ray optics in the form[2,22,23,50]

$$\mathbf{E}'(\omega', \mathbf{l}') = \mu(\omega, \mathbf{l}'; \omega, \mathbf{l})\, \mathbf{E}(\omega, \mathbf{l}), \tag{106}$$

where the local-transformation operator μ, defined in the frequency-momentum representation, describes the properties of the medium. (If we dispense with the photometric-ray concepts and turn to the general theory of coherence, this operator is replaced by a certain space-time operator of the action of the medium on the radiation—see, e. g.,[33,36].)

It is shown in[23] (see also[2]) that it is always possible to find representations $\mathbf{e}_1$, $\mathbf{e}_2$ prior to the act and $\mathbf{e}_1'$, $\mathbf{e}_2'$ after the act such that the matrix μ is diagonalized. This means that each wavelet breaks up so to speak into two alternative spin components that are independently transformed by this action of the medium. It is precisely for this reason, in particular, that no polarization devices of the traditional type, based on subjecting the ray to a sequence of operations of splitting, filtration, and transformation of each wavelet train making up the ray, is capable of providing information on the distribution function of the wavelet trains with respect to the states of the polarization, other than the first moments of this distribution, namely the Stokes parameters. As shown above, there are only two ways out of this dilemma, namely by changing over to wavelet selection based on a radically different principle, or else by resorting to an analysis of the fluctuations of the Stokes parameters, i.e., of the behavior of parameters of the type

$$\langle S_i(\omega, \mathbf{l})\, S_k(\omega + \Omega, \mathbf{l} + \boldsymbol{\rho}) \rangle_{\omega, \mathbf{l}}.$$

Turning to the Stokes parameters, i.e., to the photometric characteristics of the wavelet train (or the ray) as a whole, we obtain from (106) for a single act of local transformation (for example, scattering)[2,22,23,46,49,52,118]

$$d\vec{S}'(\omega', \mathbf{l}') = D^1(\omega', \mathbf{l}'; \omega, \mathbf{l})\, \vec{S}(\omega, \mathbf{l})\, d\omega\, dO, \tag{107}$$

where the dimensionless matrix D_{ik}^1, which has the meaning of the brightness coefficient, is made up of the matrix μ_{ik} with the aid of the relation[119]:

$$D_{ik}^1 = \frac{1}{2}(-1)^{\delta_{k4}} \mu_{sj} \mu_{tl}^* \sigma_{st}^i \sigma_{jl}^k, \tag{108}$$

in which σ_{st}^i is the i-th Pauli matrix—see (49), and $\vec{S}'(\omega', \mathbf{l}')$ are the Stokes parameters of the transformed ray, averaged over the visible area s of the inhomogeneity, located at a distance $\mathbf{l}$ from the observer.

We note that in the American literature (including also its Russian translations and expositions—see, e.g.,[60,120]), the matrices D of the local transformation of radiation by matter are frequently credited to H. Mueller, for which there are no grounds whatever. Actually, the idea of introducing matrices of this type, which describe the transformation of the Stokes parameters of radiation by the medium, was advanced in 1929 by Soleillet[48] and was first realized in 1942 by Perrin[49] as applied to the act of scattering of light. Independently and in a more expanded form (including the introduction of the local-transformation operators μ and Q, the investigation of their form for different types of scattering, and formulation of the matrix equation for radiation transport), the concept of algebraic optics of light rays was developed in 1946 by the author of[22] (see also[2]), to whom the work of F. Perrin was unknown as a result of wartime conditions, as was also a publication by G. Jones,[50] who has a developed a coherent algebraic optics as applied to plane monochromatic waves. This was soon followed by a publication by H. Mueller[118] (1948) and N. Parke[51] (1949), who developed independently some of the ideas and methods contained in[22]. A generalization of these ideas to arbitrary representations for the Stokes parameters and continuous transformations of the ray was realized in[23] (see also[2]).

The light-scattering matrices were first measured for atmospheric air in 1957.[121,122] Subsequently, many authors have performed these measurements for various media—in the atomsphere, sea water, suspensions, latex, etc. (see, e.g.,[123–128]).

Relation (107) pertains to a single inhomogeneity (particle) visible to the observer in scattered light at a solid angle $\Delta O_s = s/\mathbf{l}^2$. If now many local inhomogeneities are located in the observers field of view $\Delta O \gg \Delta O_s$, and are located at a distance $\mathbf{l}$ in a layer of thickness $d\mathbf{l}$ with volume concentration N, then the Stokes parameters averaged over the field of view ΔO, in accordance with (30)–(33), are

$$d\vec{S}(\omega', \mathbf{l}') = N\, dV\, d\vec{S}(\omega', \mathbf{l}') \frac{\Delta O_s}{\Delta O}, \tag{109}$$

and since the volume is $dV = \mathbf{l}^2 d\mathbf{l} \Delta O$, we get

$$d\vec{S}(\omega', \mathbf{l}') = D(\omega', \mathbf{l}'; \omega, \mathbf{l})\, \vec{S}(\bar{\omega}, \mathbf{l}')\, d\omega\, dO\, d\mathbf{l}, \tag{110}$$

where the volume matrix D of the local transformation (scattering) of the light by the medium is connected with D^1 by the relation

$$D(\omega', \mathbf{l}'; \omega, \mathbf{l}) = N D^1(\omega', \mathbf{l}'; \omega, \mathbf{l}). \tag{111}$$

From the reciprocity relations for the components of the amplitude matrix μ pertaining to a single act, namely[119]

$$\mu_{ik}(\omega', \mathbf{l}'; \omega, \mathbf{l}) = \mu_{ki}(\omega, -\mathbf{l}; \omega', -\mathbf{l}') \tag{112}$$

follow reciprocity relations for the matrix D_{ik}[119] in the form

$$(-1)^{\delta_{i4}} D_{ik}(\omega', \mathbf{l}'; \omega, \mathbf{l})\frac{s}{n} = (-1)^{\delta_{k4}} D_{ki}(\omega, -\mathbf{l}; \omega', -\mathbf{l}')\frac{s'}{n'}, \tag{113}$$

where s and s' are the visible areas of the transforming radiation of the object (inhomogeneity) for the radiator (in the direction of $\mathbf{l}$) and observer (in the direction of $\mathbf{l}'$), while n and n' are the refractive indices of the medium at the locations of the radiator and observer, respectively.

Owing to the independence of the transformation of the incoherent trains and the additivity of the Stokes parameters for the transformed wavelets, the matrix D remains valid also in the case of an optical ray considered as a unit.

On the other hand, if the wavelet train experiences parallel uncorrelated transformations by various objects (for example, scattering particles or stochastic interfaces), and if the wavelet trains generated by these transformations are again unified in a single transformed light ray, then, owing to the additivity of the Stokes parameters, the matrix of their aggregate transformation D, is made up additively of the matrices D_i^1 of the individual transformations, i.e., $D=\sum_i D_i^1$. In this case, even though the parallel transformations D_i^1 are statistically independent, each of them taken separately is strictly coherent in accordance with (106) and (108). Therefore, as can be easily shown, the components of the matrix of the joint transformation should satisfy the following inequality:

$$\left(\sum_k c_k D_{1k}\right)^2 \geqslant \sum_{i=1} \left(\sum_k c_k D_{ik}\right)^2 \tag{114}$$

for arbitrary c_k ($c_1 = 1$, $c_2^2 + c_3^2 + c_4^2 \leq 1$), whence, in particular,

$$D_{11}^2 \geqslant D_{21}^2 + D_{31}^2 + D_{41}^2, \quad (D_{11} \pm D_{1k})^2 \geqslant \sum_{i=1} (D_{i1} \pm D_{ik})^2. \tag{114a}$$

For the series of successive transformations, as is obvious from (107) or (110), the matrix of the resultant transformation is formed by multiplying the matrices of the partial transformations and preserving the order in which they are experienced by the ray.[2,22,91,118]

Thus, regardless of the final fate of the ray, it goes through a series of alternating differential and local transformations, which are experienced in parallel by all the wavelets making up the ray. Each of them is described by the operators Q and D corresponding to one or the other type of action of the medium on the radiation.[2,22,23] The operators Q and D acquire a distinct meaning of the probability of one or another transformation of the wavelet train, and with it of the entire ray as a unit.

At the same time, the trajectory of the wavelet train between the acts of its local transformation is described by the refraction equations (89)–(90), and the form of the operator Q is directly connected with the behavior of the unit vectors **p** and **s** and with the conservation laws (97). It should be noted here that a connection exists between the conservation laws (97) and (64)–(66) and leads therefore to a connection between the matrices Q and D.

The most important of them is the so-called "optical theorem" (see, e.g., [17,38]), which can be generalized with practically no changes to include nonmolecular scattering[36] and which establishes a direct connection between the matrices $\mu_{ik}(\mathbf{l}', \mathbf{l})$ and $D_{ik}(\mathbf{l}', \mathbf{l})$. Its particular case is the well known relation[2,12,23,36]

$$\frac{4\pi}{k^2}\operatorname{Re}\mu_{11} = \oint D_{11}(\omega, \mathbf{l}'; \omega, \mathbf{l})\, dO + \alpha(\omega), \tag{115}$$

the physical meaning of which reduces to the statement that each quantum taken out of the ray by the particle must be either scattered in some direction or absorbed by this particle—the probability of the latter is described then by the quantity $\alpha(\omega)$ in (115).

However, the real meaning of the optical theorem is much fuller than this statement (see[36]), since it is not confined to the connection between the conservation laws (64)–(66) and the form of the matrix D_{ik}, which is obligated to take into account the entire aggregate of the phenomena involving exchange of energy, momentum, and angular momentum between the radiation and the inhomogeneity during the scattering act.

In other words, *ray optics* confronts us with a unified indivisible concept, that combines geometrical optics, photometry, polarimetry, refraction theory, and, as we shall show, the theory of radiative transfer; it is an independent branch of physical optics dealing with the laws of formation of the light field, as well as with the theory of the measurement of this field. Extending over a wide range of practical applications to both purely scientific and applied problems, it has now attained a certain degree of internal completeness.

However, the real possibilities of its application are substantially limited by our insufficient knowledge of the structure and behavior of the operators Q and D for various types of action of a medium on radiation, as functions of different physical factors, such as the structure or the state of the medium. Their investigation remains the destiny of the related branches of optics and becomes, in view of the foregoing, one of the primary problems, including also the translation of many of the traditional results into the specific operator language of ray representations.

One of the examples of such an approach is the theoretical and experimental study of light-scattering matrices, initiated by F. Perrin,[49] U. Fano,[52] and the author.[22,23] Even during the early stages it led to experimental observation of the ellipticity of the scattering of light upon scattering,[121,122] optical anisotropy of the water in the ocean,[126] and the finely dispersed character of optically active atmospheric aerosol,[65,124,127] and has by now developed into an extensive and varied program, occupying a more and more prominent place in optical journals (see, e.g.,[17,18]), particularly in connection with the development of new methods of optical investigations of the states of sols and the study of processes that occur in the dispersed phase of matter.

Another branch of modern ray optics is the development of operator methods of calculating optical systems and devices, reported in the initial publications of Jones[50] and Parke,[51] which have developed into an independent technical discipline.[60,120]

Finally, as already mentioned several times, the concept of ray optics is the basis of a most extensive and thoroughly developed branch of modern mathematical physics, namely the theory of radiative transfer in scattering media. Since the light field can be treated as an aggregate of incoherent optical wavelet trains of all possible directions, the theory of its structure can be based on the elementary idea of tracing the fate of each wavelet in all its experiences.

This idea, advanced by Soret[128,129] about ninety years ago, can be written in modern language in the form[13,113]

$$S_i(\omega', \mathbf{l}'; B, t+\theta) = P_{ik}^{AB,\theta}(\omega', \mathbf{l}'; \omega, \mathbf{l}) S_k(\omega, \mathbf{l}; A, t), \quad (116)$$

where $S_k(\omega, \mathbf{l}; A, t)$ pertains to the initial light wavelet at an arbitrary point A at the instant of time t, while $S_i(\omega', \mathbf{l}', B, t+\theta)$ pertains to the family of wavelet trains generated by it at some other point B after the lapse of a time θ.

The *transfer matrix* $P_{ik}^{AB,\theta}$, which relates a pair of arbitrary points A and B, is the probability that a quantum emerging from a point A in the direction $\mathbf{l}$ reaches via arbitrary paths the point B during the time θ, by changing in suitable manner its polarization state, frequency, and direction of motion.

The task of the theory thus becomes the determination of already possible paths open to the quantum, and accordingly the probabilities of their realization. It is the procedure for realizing such an analysis which distinguishes between the variants of radiative-transfer theory proposed by various authors.[7-14,91,130-134]

In particular, the basis for the investigation of the transfer matrices $P_{ik}^{AB,\theta}$ can be the analysis of the process of the change of the light-ray parameters over a length element $d\mathbf{l}$ of its trajectory. This change consists of two processes—removal or transformation of the photons making up the ray, i.e., differential transformation of the type (133), or the incorporation by the ray of photons previously belonging to other rays, as a result of local scattering occurring on the same path segment $d\mathbf{l}$. This process is described by relation (110). By combining the two processes we arrive at a relation first obtained back in the forties independently by Chandrasekhar[8] and by the author[22,23] (see also[2,12]) and known as the radiative-transfer matrix equation

$$(\mathbf{l}\nabla) S_i(\omega, \mathbf{l}) = -Q_{ik}(\omega, \mathbf{l}) S_k(\omega, \mathbf{l}) + \int \oint D_{ik}(\omega, \mathbf{l}; \omega', \mathbf{l}') S_k(\omega', \mathbf{l}')\, d\omega'\, dO' + S_i^{\mathrm{rad}}(\omega, \mathbf{l}), \quad (117)$$

where S_i^{rad} pertains to the intrinsic incoherent (say, thermal) radiation of the medium.

It is precisely this integro-differential equation which constitutes a photometric (ray) approximation of the more general and much later formulated Bethe–Salpeter equation,[36] which usually serves as the theoretical basis for the investigation of the behavior of the transfer matrix $P_{ik}^{AB,T}$. By the same token this determines the position of *radiative transfer theory*, which is photometric in character, within the framework of the general statistical optics and within the concept of ray optics—in the latter case it serves precisely as the *photometric theory of the diffuse light field*.

We note incidentally that until recently extensive use was made of an initial scalar variant of the transfer equation, which is physically known to be incorrect; this variant was developed in the beginning of the century independently by Schwarzschild and Shuster. The incorrectness of this equation, which deals only with the brightness of the ray $I \equiv S_1$, is mathematically connected with the omission from (117) of terms of the same order of magnitude as those retained. Physically it follows from neglecting the influence of the polarization state of the wavelet train on the result of its transformation and, at the same time, the requirements imposed by the existence of conservation laws other than the energy conservation law.[2,22] On the other hand, the only arguments favoring the scalar equation are its relative simplicity, the extremely limited available data on the matrices D_{ik} of the local transformation of the ray, and perhaps the tendency to confine oneself to a qualitative analysis of the results, despite the highly laborious mathematical procedures.

We note in conclusion also that the central problem of the transition from the Bethe–Salpeter equation to the radiative-transfer equation (117), i.e., that of introducing the photometric approximation, is the definition of the material *volume element* over which *the averaging* of the correlation functions of the field are averaged.[30,31,36] It is in the course of this averaging that the field of the mutual irradiation of inhomogeneities splits up effectively into coherent and incoherent parts.[23] The first of them gives rise to a change in the effective field in which the inhomogeneities are located, and in final analysis manifests itself in the components of the dispersion matrix ν (or Q)—see (101), (104). In addition, the coherent part of the field of the mutual irradiation of the particles violates to a greater or lesser degree the additivity of the matrices ν, Q, and D in the volume element, and also changes the form of the matrix D (for example, the angular dependence of its coefficients).

The incoherent part of the field of the mutual irradiation of the particles, to the contrary, splits when the correlation functions over the volume element are averaged into an aggregate of independent rays, which produce precisely those multiple-scattering effects dealt with in the theory of radiative transfer.

This is precisely why the so-called cooperative effects caused by the coherent part of the mutual irradiation of particles are beyond the scope of the transfer theory and manifest themselves only in the behavior of those characteristics of the medium with which it operates.[30,31,36]

From the foregoing point of view, the most important task at present is the investigation of these characteristics (i.e., the matrices ν, Q, and D), including the role played by the coherent cooperative effects in their formation, as functions of the properties and state of the medium, but this is already outside the scope of ray optics.

* * *

If we turn to the structure of modern statistical optics, then we are immediately struck by the inseparable connection between its two aspects—measurement theory and the theory of the structure of the radiation field.[37–42] One aspect of this connection has been the subject of our attention.

We have observed that the use of an *optical* receiver inevitably leads to the *ray*-optics approximation, i.e., to a closed system of representations, concepts, and relations that constitute the real content of the modern *theory of the light field*. The latter, as we have seen, encompasses from unified positions such branches of optics as geometrical optics, photometry, polarimetry, transformation of the ray by matter, the theory of radiative transfer, etc.

The picture wherein a ray that seemingly is connected with the light source but is actually generated by the act of measurement with the aid of an optical receiver (such as the eye) takes us from the customary coordinate-time representation into the frequency-momentum representation of Fourier transforms, for which in fact the principal concepts and quantities of ray optics are defined, including also the specific formulations of the conservation laws and transformation laws. These in fact are the distinguishing feature of the optical treatment and the optical method of describing phenomena.

This pertains first of all to algebraic optics, which is an autonomous division of the theory of the optical field, and the task of which is to describe the processes of transformation of a light ray by a medium, be it the processes of propagation (refraction, birefringence, etc.) or local acts of reflection, refraction, scattering, etc. By resorting to the formalism of linear operators, which receive a physically lucid probabilistic treatment, we arrive immediately at the theory of radiative transfer (which previously had no electrodynamic foundation) with all its technical, geophysical, and astrophysical subdivisions. At the same time, the conditions under which optical receivers can be used become definite, and with them also the limits of the validity of the associated ray-optics approximation.

Although the individual branches of ray optics already have a long history and an extensive literature, the unified conception of the theory of the light field described above requires further dvelopment in various directions. This article will have done its duty if it stimulates interest in this theory and at the same time helps put to rest the still remaining notion that photometry and radiative transfer theory are independent of electrodynamics.

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