Astrophysical upper limits on the photon rest mass

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The main ideas and methods currently used to obtain upper limits on the photon rest mass from astrophysical data are briefly reviewed. One method is based on the fact that if the photon has nonzero rest mass m magnetoacoustic waves cannot have frequencies lower than a certain critical frequency, which depends on m. If wisps in the Crab Nebula are interpreted as magnetoacoustic waves of extremely low frequency, then the data of many-year observations of the wisps put an upper limit on m which is much better than the one established under terrestrial conditions. An error is pointed out in a different method which has been proposed in the literature in which the contribution of the energy of the galactic magnetic field (or rather, the vector potential) to the mass of Galaxy is estimated on the basis of gravitational effects. The point is that in the general theory of relativity not only energy but also pressure has a weight. In the case under consideration, these two contributions cancel each other and the galactic magnetic field cannot produce anomalously strong gravitational fields. The most stringent upper limit on the photon rest mass, $m \lesssim 3 \cdot 10^{-60}$ g, is obtained from the analysis of the mechanical stability of magnetized gas in the galaxies with allowance for the specific pressure forces of the vector potential. This upper limit is 12 orders of magnitude better than the best upper limits obtained under terrestrial conditions. This result clearly demonstrates the effectiveness of astrophysical methods.

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CONTENTS

1. INTRODUCTION

The problem of the photon rest mass is of undoubted interest for both fundamental physics and applied electrodynamics.

Virtually all physicists, at least 99% of them, now believe that the photon rest mass is exactly zero. On the other hand, there is no doubt that experiment has the last word in this important question. All the experiments made to determine the photon rest mass give only upper limits for the mass. This means that the photon rest mass must be less than the experimentally obtained upper limit; in particular, it may be zero. The experimental possibilities under terrestrial condition have, it would seem, already been largely exhausted, and one has recently noted a tendency to use astrophysical data to reduce the upper limit on the photon rest mass. The aim of this note is to give a brief review of the main ideas and results of these investigations.

The best upper limit on the photon rest mass m obtained under terrestrial conditions (from measurements of the shape of the Earth's magnetic field) is $m \lesssim 4 \cdot 10^{-48}$ cm.[1] For the Compton wavelength $\lambda_C = h/cm$ this gives $\lambda_C \gtrsim 6 \cdot 10^{10}$ cm, i.e., the Compton wavelength of the photon, if it exists at all, is emphatically macroscopic, greater than the diameter of the Sun! This upper limit already restricts the photon mass to very small values, 20 orders less than the electron mass. One may reasonably ask: What is the reason for the unflagging interest in the problem of the photon rest mass? We can answer this by considering a static magnetic field. If one calculates the energy–momentum tensor in macroscopic electrodynamics with a nonzero photon mass, the total magnetic pressure P_M is found to consist of the pressure $P_B = B^2/8\pi$ of the field **B** and an additional pressure $P_A = \mu^2 A^2/8\pi$ of the vector potential **A**, where $\mu = 2\pi/\lambda_C$ is the Compton wave number of the photon.[2,3] If the magnetic field **B** varies over a characteristic distance l, then the definition $\mathbf{B} = \operatorname{curl} \mathbf{A}$ gives $A \sim (l/2\pi)B$. With allowance for this estimate, comparison of the magnetic pressure and the pressure of the vector potential shows that the effect of electrodynamics with finite photon mass—the pressure of the vector potential—is manifested over distances l greater than the Compton wavelength λ_C. Thus, the Compton wavelength of the photon is a kind of fundamental length in electrodynamics because the Maxwell equations must be replaced by the Proca equations[4] for scales greater than λ_C. This fundamental length is unusual in that it determines the region of applicability of the theory at large scales, and not at small, like the Compton wavelength $\lambda_{C,e} = 2 \cdot 10^{-10}$ cm[5] of the electron for classical electrodynamics. After these introductory remarks, let us turn to the actual investigations.

2. INFLUENCE OF THE PHOTON MASS ON THE DISPERSION OF MAGNETOACOUSTIC WAVES

The dispersion relation between the frequency ω and the wave number k for sound propagating in a plasma with magnetic field is

$$\omega^2 = k^2u^2,$$

where the square of the velocity of sound u is of the order of the ratio of the total pressure in the medium to its density ρ. The total pressure is made up of the thermal pressure P_T of the plasma, the magnetic field pressure $P_B = B^2/8\pi$, and the pressure $P_A = \mu^2A^2/8\pi$ of the vector potential in electrodynamics with nonzero photon rest mass. This gives the estimate $u^2 \sim (P_T + P_B + P_A)/\rho$. For an acoustic wave, the relation $\mathbf{B} = \operatorname{curl}\mathbf{A}$ gives $A \sim B/k$, which enables us to write ω explicitly as a function of k and of μ: $\omega^2 = k^2u_0^2 + \mu^2v_0^2$, where $u_0^2 \sim (P_T + P_B)/\rho$ and $v_0^2 \sim B^2/8\pi\rho$. We see immediately that, in contrast to Maxwellian electrodynamics, the dispersion relation takes on a k-independent term when $\mu \neq 0$. As a consequence, there is a critical frequency $\omega_{cr}^2 \sim \mu^2B^2/8\pi\rho$ such that for $\omega < \omega_{cr}$ there are no periodic solutions, i.e., magnetoacoustic waves cannot propagate. The use of dispersion relations to establish upper limits on the photon mass[6] is ultimately based on this fact. In order to obtain a good upper limit, one need only find a physical or astrophysical situation in which the magnetoacoustic frequency is sufficiently low. Several such studies have been made.[1,6,7–10] Let us consider in more detail the recent[10], in which the best upper limit has been obtained. In[10], Barnes and Scargle investigated the propagation of the so-called wisps in the Crab Nebula. They emanate from a center which coincides with the pulsar in the Crab. The traveling wave compresses the magnetic field, and this enhances the synchrotron radiation, which appears in photographs in the pattern of wisps. On the basis of the observed good correlation between the fluctuations in the plasma density and the magnetic field, and also other indirect confirmations, these wisps really are magnetoacoustic waves. Analysis of numerous photographs of the Crab Nebula taken during nearly the last 20 years yields numerical values of the necessary parameters. It is found that the frequency of the wisps is very low: $\omega \sim 10^{-6}$ sec^{-1}, which corresponds to the generation of a few wisps in a year. This gives the following upper limit on the photon rest mass:

$$m \lesssim 10^{-53}\ \text{g}, \quad \lambda_C \gtrsim 2 \cdot 10^{16}\ \text{cm}. \tag{1}$$

This is approximately four orders of magnitude better than the upper limit obtained under terrestrial conditions.

This method does suffer from one weak point: Each wisp in a photograph need not be the individual crest of a low-frequency wave but merely the smooth envelope of a modulated high-frequency wave. High-frequency oscillations will not be visible in the photographs because of the restricted resolution of the telescopes. If this is the case, then the actual upper limit is much worse than (1). Barnes and Scargle[10] consider such a possibility but regard it as improbable for the following reason. On the one hand, the high-frequency wave must have a frequency less than the gyrofrequency of the particles (~ 1 Hz) if the modulation of the wave is not to be "washed out" by dispersion and the high-frequency wave itself is not to be damped too rapidly; this frequency is appreciably lower than the rotation frequency of the pulsar (30 Hz). On the other hand, there are no other motions known in the system that could excite a powerful wave with frequency less than 1 Hz except the wisps themselves with $\omega \sim 10^{-6}$ sec^{-1}.

3. TOTAL MATTER DENSITY AND THE PHOTON REST MASS

We have already pointed out that in electrodynamics with nonzero photon rest mass the energy–momentum tensor differs from the Maxwellian expression by an amount of order $\mu^2A^2/8\pi$. This is true of both the pressure and the energy density. In particular, for a static magnetic field the energy density ε is[2,3]

$$\varepsilon = \frac{B^2 + \mu^2A^2}{8\pi}. \tag{2}$$

The presence in ε of the correction proportional to μ^2 was used in[11] to obtain an upper limit on the photon rest mass. Byrne and Burman[11] argued as follows. It is well known that our Galaxy has a magnetic field $B \sim 2 \cdot 10^{-6}$ G with characteristic homogeneity scale $l \sim 300$ pc. In accordance with the relation $\mathbf{B} = \operatorname{curl} A$, the contribution ε_A to the density is $\varepsilon_A \sim \mu^2B^2l^2/8\pi$. On the other hand, it is well known that the total masses of the galaxies and the total matter densities ρ_{tot} corresponding to them can be determined from gravitational effects: either from the velocity of their differential rotation or from the orbital velocity of galaxies, if they occur in binary systems. The masses of galaxies obtained in this way exceed the sum of the masses of the observed stars by not more than an order of magnitude, and the mass of our Galaxy is certainly less than $10^{12}M_\odot$. It is obvious that the contribution to the mass due to a nonzero photon rest mass cannot exceed the total mass of the Galaxy. This condition leads to the inequality $\mu^2 \lesssim 8\pi\rho_{tot}c^2/B^2l^2$, which, after substitution of the numerical values, gives[11]

$$\mu \lesssim 3 \cdot 10^{-14}\ \text{cm}^{-1}, \quad m \lesssim 10^{-51}\ \text{g}. \tag{3}$$

The above arguments appear to be without a flaw. But in fact they do not take into account an important circumstance that radically alters the conclusions of[11]. The point is that to calculate correctly the gravitational field φ of relativistic matter such as the electromagnetic field it is necessary to use the equations of general relativity. For weak fields $|\varphi| \ll c^2$, we have[5]

$$\Delta\varphi = 8\pi Gc^{-2}\left(T_0^0 - \frac{1}{2}T\right), \tag{4}$$

where T_0^0 and $T \equiv T_i^i$ are, respectively, the energy density and the trace of the energy–momentum tensor T_i^k. In the case of ordinary "dust," the only nonzero component of T_i^k is T_0^0, so that $T_0^0 = T = \rho c^2$, and Eq. (4)

goes over into the ordinary Poisson equation $\Delta\varphi = 4\pi G\rho$. In electrodynamics with nonzero photon mass[2,3]

$$T_i^k = (T_i^k)_0 + \frac{\mu^2}{4\pi}\left(A_i A^k - \frac{1}{2}\delta_i^k A_l A^l\right), \tag{5}$$

where $(T_i^k)_0$ is the corresponding Maxwellian expression, which depends only on **B** and **E**. Substituting (5) into (4), we readily see that in the case of a static magnetic field ($E = 0$, $A_0 = 0$) of arbitrary configuration the right-hand side of (4) does not depend explicitly on the photon mass! Therefore, the mass of the Galaxy determined from the differential rotation certainly cannot contain a contribution from effects associated with a nonzero photon mass. And then the main idea of [11] is incorrect.

4. STABILITY OF THE GALAXIES AND THE PHOTON REST MASS

The pressure $P_A \sim \mu^2 A^2/8\pi$ of the vector potential which we have discussed above may appear in the balance of forces in different equilibrium systems containing a magnetic field; for example, in galaxies. Following [12], let us consider the equilibrium conditions of magnetized interstellar gas. It follows from the virial theorem[13] written down with allowance for the effects of electrodynamics with nonzero photon rest mass that the pressure of the vector field tends on the average to compress the gas. Therefore, in an equilibrium system the forces of the thermal pressure, the pressure of the magnetic field, and the centrifugal force must exceed the forces due to the pressure of the vector potential. This criterion was used in[12] to analyze the equilibrium of the gas in the Magellanic Clouds. Since the magnetic field energy density $B^2/8\pi$ in the Magellanic Clouds exceeds the density of the thermal and mechanical energy, [14] the equilibrium criterion actually reduces to the inequality $\mu^2 A^2 \lesssim B$. With allowance for $\mathbf{B} = \mathrm{curl}\,\mathbf{A}$, this gives $\mu \lesssim l^{-1}$, where l is the characteristic dimension of the region occupied by the magnetized gas. For the Small Magellanic Cloud $l \sim 3$ kpc, and the inequality gives

$$m \lesssim 3\cdot 10^{-60}\ \mathrm{g}, \quad \lambda_C \gtrsim 6\cdot 10^{22}\ \mathrm{cm} \tag{6}$$

which is the best of the currently known upper limits on the photon rest mass.

5. THE ULTIMATE LIMIT

We have here discussed some of the upper limits obtained by different methods. The best of them shows that the photon's rest mass is at least 32 orders of magnitude less than the electron's. Do we really have to continue to infinity the succession of these upper limits in order to convince ourselves that the photon rest mass is zero? The answer is no. In the Universe, there is a maximal distance, called the horizon, up to which we can obtain information. The existence of the horizon is due to the finiteness of the maximal signal propagation velocity (c) and the age of the Universe (t). The distance to the horizon is $ct \sim 10^{28}$ cm. Since specific effects in electrodynamics with nonzero photon mass can appear only at distances greater than or of the order of the Compton wavelength λ_C, the very fact that there is an upper information radius shows that the inequality $\lambda_C > ct$ would be equivalent to a positive answer to the question of whether the photon mass is zero, since a nonzero mass satisfying this inequality could not in practice be manifested.

In a paper as brief as this, we cannot consider all the papers published recently (see, for example, [15,16]). We merely mention that the most radical improvement of the upper limit in the photon rest mass could be achieved in the near future if a uniform metagalactic magnetic field is discovered. This would give $\lambda_C \geq 10^{24}$–10^{25} cm. [12]

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