equal to zero, i. e., the Einstein paradox has no classical analog. This is due to the fact that the internal energy of a classical ideal gas is independent of its density.

3. *The temperature paradox.* The Einstein paradox arises in the isothermal mixing of quantum ideal gases. In adiabatic mixing of such gases it is absent. However, in this case, a new paradox is displayed—a discontinuity in the temperature change on going from adiabatic mixing of arbitrarily close quantum ideal gases to mixing of identical gases.

We shall consider adiabatic mixing of N particles of each of the weakly degenerate gases A and B, with masses m_1 and m_2 respectively, in equal volumes V separated by a thermally conducting partition and having temperature T_0.

Before mixing, the internal energy of the gases is equal to

$$U_{\rm I}=3NkT_0\left[1+\frac{\delta}{32}\frac{Nh^3}{V(\pi kT_0)^{3/2}}(m_1^{-3/2}+m_2^{-3/2})\right].$$

After removal of the partition and adiabatic mixing of the gases, when each gas occupies a volume $2V$, for unchanged internal energy of the system the temperature of the gases will be different (T) and the expression for the internal energy of the mixture takes the form

$$U_{\rm II}=3NkT\left[1+\frac{\delta}{64}\frac{Nh^3}{V(\pi kT)^{3/2}}(m_1^{-3/2}+m_2^{-3/2})\right]. \tag{12}$$

For adiabatic mixing of arbitrarily close gases ($m_2\approx m_1 = m$) we have

$$U'_{\rm I}=3NkT_0\left[1+\frac{\delta}{16}\frac{Nh^3}{V(\pi mkT_0)^{3/2}}\right],$$
$$U'_{\rm II}=3NkT\left[1+\frac{\delta}{32}\frac{Nh^3}{V(\pi mkT)^{3/2}}\right]$$

and we find the resulting temperature change $T-T_0$ from the condition $U'_{\rm I}=U'_{\rm II}$, i. e, from the equation

$$T-T_0=\frac{\delta}{32}\frac{Nh^3}{V(\pi mk)^{3/2}}\left(\frac{2}{\sqrt{T_0}}-\frac{1}{\sqrt{T}}\right). \tag{13}$$

In the limiting case of adiabatic mixing of two portions of an identical gas ($m_2=m_1=m$) we find the expression for the internal energy of the system from (12) by taking into account the density discontinuity which occurs in this mixing; this leads to $T-T_0=0$.

Thus, on going from adiabatic mixing of arbitrarily close quantum ideal gases to mixing of identical gases the temperature change in the mixing experiences the discontinuity given by Eq. (13). This new paradox for the temperature in the adiabatic mixing of quantum ideal gases is also due to the discontinuity in the density of the gas when we go from mixing of arbitrarily close gases to mixing of identical gases.

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Translated by P. J. Shepherd

On the problem of the "retardation" of the relativistic contraction of moving bodies

S. I. Syrovatskiĭ

P. N. Lebedev Physics Institute, Academy of Sciences of the USSR
Usp. Fiz. Nauk. **118**, 545–547 (March 1976)

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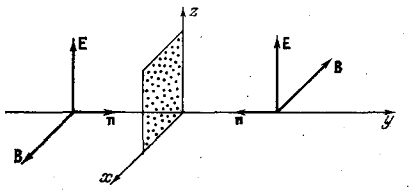
The relativistic contraction of moving bodies of systems of bodies sometimes leads to misunderstandings associated with the confusion of two essentially different phenomena. On the one hand we have the difference in the dimensions of a body as measured in two reference frames moving relative to each other, and on the other we have the change in dimensions of a body that is set in motion (or stopped) in a given coordinate frame. Whereas in the first case the answer is unique and is given by the Lorentz-transformation formulas, in the second case the answer depends essentially on precisely how the body under consideration has moved between two measurements. Despite the obviousness of these statements, it is useful to illustrate them by a simple example, especially as it shows in what conditions the difference disappears and the result can be expressed by the Lorentz transformation in both cases.

The one-dimensional problem of the motion of a plane layer of charges under the action of a plane electromagnetic wave serves as such an example. This problem arises in the problem of acceleration of particles in neutral current layers in a plasma.[1,2] We shall consider it in its simplest variant.

Suppose that a plane electromagnetic wave with a square front is incident on a thin layer of charged particles (in the (x, y)-plane; see the figure) along the normal to the layer. The problem is especially simple in



the symmetric case in which a similar wave arrives simultaneously from the opposite side of the layer. In this case the magnetic fields cancel each other and the electric fields add, setting the charges in motion in the direction of the z-axis (see the figure). The current that arises, with density $j = nev$, where n is the density of charges and v is their velocity, in turn generates radiation (a reflected wave). The exact solution of this self-consistent problem presents no difficulty and we shall not give it here (cf. [2]). We shall discuss only one question: how is the density n of the moving charges related to its initial value n_0?

The answer to this question follows automatically from the one-dimensionality of the problem (all quantities are independent of the coordinates x and z in the plane of the layer) and from the conservation of particle number: all the particles move simultaneously and equivalently, and, therefore, no change in density is possible. It would be an error to assume that the distances between the charges, and their density, will vary in accordance with the relativistic formulas

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}, \quad n = \frac{n_0}{\sqrt{1 - (v^2/c^2)}} \tag{1}$$

as the velocity of the charges increases. In fact, any two charges move equally and independently of each other, with conservation of the distance between them. In this case, application of the Lorentz transformation shows that in the proper reference frame (moving with the charges) the density of the latter is equal to

$$n' = n_0 \sqrt{1 - (v^2/c^2)}, \tag{2}$$

i. e., the layer is found to be "expanded," with a larger distance between the particles than in the initial state. This expansion is entirely real and its cause lies in the fact that, unlike in the original "laboratory" frame, in which the front or a fixed phase of the wave acts simultaneously on all the particles, in the "proper" frame we no longer have this simultaneity. In the proper frame the retardation of the front at the point z_2' as compared with the point z_1' is equal to

$$\Delta t = t_2' - t_1' = -\frac{v}{c^2}(z_2' - z_1'), \tag{3}$$

as follows from the Lorentz transformation and the simultaneity in the laboratory frame. In other words, in the proper frame of the layer the waves are no longer incident "head on"; they come together at a certain angle and their fronts are not parallel to the layer, although, as before, the whole physical picture is symmetric with respect to the layer.

In view of the expansion of the layer in the proper frame, the set of independent charges under consideration cannot be chosen as a measuring scale for the lengths and, correspondingly, cannot satisfy the transformations (1). In the theory of relativity, for measuring scales we choose real bodies in identical physical conditions in different systems, e.g., elastic rods in the unloaded state. By virtue of the principle of relativity their proper lengths will be the same in all systems.

We shall modify our problem by fixing the charges on an elastic plate. The motion considered above first gives the plate a velocity along the z-axis and, secondly, gives rise to expansion of the plate, i. e., takes it out of elastic equilibrium. In order that we can use it again as a measuring scale, we must wait until these stresses relax and elastic equilibrium is restored. This time, obviously, will be determined by the size of the plate along the z-axis and by the velocity of propagation of the interactions determining the equilibrium of the plate (in the case of an elastic plate, by the velocity of the elastic waves, i. e., sound waves). When this time has elapsed the size of the plate will take the equilibrium value and, by virtue of the principle of relativity, will obey the relations (1).

Thus, in those cases when the body being used as a measuring scale changes its state of motion (e.g., on transfer from one coordinate frame to another), the formulas for the relativistic transformation of the length will be valid only after the time necessary for establishment of the internal equilibrium state of this body (i. e., for removing the stresses which arise in the transfer); the conclusion is completely trivial, although it is not always taken into account. We point out here that, in a wider scheme, a discussion of the question of the Lorentz transformation in the process of establishment of equilibrium is contained in the article by Feĭnberg.[3]

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