Black holes and quantum processes in them

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Recent advances in the physics of black holes are discussed. The analogy between the laws of black-hole physics and thermodynamics revealed by the study of the interaction of black holes with one another and surrounding matter is discussed. Quantum pair creation in the gravitational field of black holes ic considered. Some cosmological consequences of the evaporation of small primordial black holes and the possibilities of observing such events are considered.

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INTRODUCTION

Modern astrophysicists are "spoiled" by the sensational discoveries of objects with unusual properties like pulsars and quasars. However, in 1973 the report of a possible new discovery made in the investigation of x-ray spectra obtained on the Uhuru satellite quickened the pulses once more. This time not only the astrophysicists but also the gravitational theorists were excited since it was the discovery of a so-called *black hole* that was claimed. At the present time, although certain doubts remain, the majority of specialists in this field are convinced that the first black hole in the Universe has already been found.

This discovery excited the imagination of physicists not so much through the existence of such an object being unexpected for theoreticians but rather the opposite. It is after all possible that objects have been found whose existence was predicted as early as the end of the thirties (Oppenheimer and Snyder, 1939) as one of the unusual but absolutely necessary consequences of Einstein's theory of gravitation. The direct astrophysical observations were preceded by much work on the part of the theoreticians. Particularly intensive theoretical investigations in black-hole physics have developed since 1965. As a result of this work it has been possible to advance rather far in the understanding of many previously completely obscure aspects of the structure of spacetime in strong gravitational fields. And although the methods of investigation in this subject have as yet a complicated mathematical nature, the results obtained have led to the construction of a fairly simple picture of the formation of black holes and their subsequent evolution. These theoretical results and the incipient astrophysical investigations demonstrate that during the last few years a new and remarkably interesting branch of physics has developed—the physics of black holes. In this review we wish to discuss some of the recent achievements in this direction.

Leaving on one side the details, scientists now have the following picture of the formation and evolution of black holes. Cooled massive stars with a mass greater than two or a few solar masses are, having completely exhausted their nuclear fuel, incapable of withstanding further the enormous gravitational forces compressing them. In these stars, therefore, instability develops sooner or later and this leads to collapse of the star. As a result of the collapse, which takes place fairly rapidly (in a time measured in minutes[1)]) a black hole is formed, i.e., a compact nonemitting body with radius equal to the star's gravitational radius (the gravitational radius of the Sun is about three kilometers). The black hole formed in this manner and its gravitational field are usually unstationary initially. However, fairly rapidly (in fractions of a second[2)]) the excited black hole goes over as a result of the emission of gravitational waves into the ground, stationary state.

All features of the internal structure of the collapsed star and the presence in it of sources of different physical fields except the electromagnetic become inaccessible to observation after the formation of the

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[1)]After the onset of collapse, the gravitational forces exceed the pressure forces by a finite amount, and one can therefore estimate the time of collapse of the star to a size comparable with the gravitational radius by considering the free fall of the star's matter in its own gravitational field. The characteristic "hydrodynamic" time obtained in this way is of order $T_0 \sim (R_0/c)\sqrt{R_0/R_g}$, where R_0 is the initial radius of the star and $R_g = 2GM/c^2$ is its gravitational radius.

[2)]The characteristic time for a black hole to go over into a stationary state is related in the following manner to the parameters of the collapsed star: $T \sim R_g/c \sim 10^{-5}(\text{sec}) \times M/M_\odot$, where M is the mass of the star and $M_\odot$ is the Sun's mass. This same time is characteristic time of "freezing" of the collapsing star around its gravitational radius from the point of view of a distant observer.

stationary black hole, and such black holes, irrespective of their origin, can be uniquely characterized by just the value of their mass and angular momentum (and in the case of charged black holes by the value of the electric charge contained in them).[3] Stationary black holes with the same parameters are completely indistinguishable.

In a sense, black holes have no other properties apart from the ability to attract. Therefore, it was originally assumed that these frozen "dead" objects are the natural end stage in the development of massive stars, and their detection appeared to be a rather hopeless task.[4] Later, however, it was realized that if a black hole is surrounded by a medium (interstellar gas or dust), it will pull this matter into it like a gigantic cosmic vacuum cleaner and the attracted (accreted) matter will be heated and become a source of strong x-ray radiation with very specific characteristics.

The same thing happens if a black hole is the companion to an ordinary star, with which it forms a binary system. In this case the black hole attracts the matter of the companion and can become an x-ray source with periodically varying emission. The period of variation of the intensity is equal to the period of revolution of the black hole around the star. The discovery of an x-ray source in Cygnus in a binary system and study of its properties and the properties of the visible component have led to the conclusion that a black hole has been discovered.[5]

Thus, the idea that black holes are "dead" objects has now been replaced by the idea that they are very active objects in the Universe. It is therefore natural that there is now even more interest in the detailed study of the interactions (both classical and quantum) in which black holes can participate. This study has revealed a number of interesting and rather unusual aspects.

It has been found in particular that although a black hole has mass and therefore internal energy this energy cannot be fully extracted. From an isolated rotating black hole one can extract only that part of its energy which is associated with rotation. From a nonrotating black hole one can extract energy only by making it interact with another black hole.[6] These results, along with many others, are consequences of a general theorem proved by Hawking[5,6] which asserts that the surface area of a black hole cannot decrease in any classical processes.

Besides black holes formed by the collapse of stellar objects, one can also have black holes formed from matter inhomogeneities during the early evolution of the Universe. These are called primordial black holes. Primordial black holes may have mass appreciably less than the Sun's. (We shall discuss primordial black holes in the final sections of this paper.)

The static gravitational field on the surface of a black hole is of order $\varkappa \sim GM/R_g^2 \sim c^4/GM$ and is therefore extremely large for black holes with small mass, so that one is naturally led to consider the possibility of quantum effects of pair creation in this strong static field. It is especially important to establish whether quantum pair creation processes in the black hole gravitational field could lead to observable effects.[7] Even if such processes are very weak, they would be decisive for a black hole in vacuum and over a vast time of the order of the age of the Universe, and they could lead to significant changes in the structure of the black hole or possibly even to its disappearance. Quantum processes in black holes, their role in the evolution of primordial black holes, and possible cosmological consequences of such effects will be discussed in the final sections of the paper.

§1. DOES A GRAVITATIONAL FIELD CREATE PARTICLES?

Before we consider the physics of black holes and quantum effects in curved spacetime, it is helpful to attempt to answer the following question: Why should one expect particle creation in a gravitational field and what conditions are necessary for this to occur? To answer this question, it is helpful to use the analogy

[3]Wheeler has picturesquely described the property of stationary black holes of requiring no further additional characteristics (such as multipole gravitational or electromagnetic moments, charge of massive vector field, weak charge and so forth[29–34,71]) by the statement: "Black holes have no hair." Markov has called the shedding of the "superfluous characteristics" during collapse "gravitational striptease." The global properties of collapsed matter are discussed in, for example, Markov's paper.[1] Penrose's interesting paper on black holes was published in[2].

[4]Such discovery would have required the detection of an invisible object with a mass greater than two solar masses, either by itself or as the second component in a binary system. Zel'dovich and Novikov in the book[3] characterized the situation as follows: "The appeal to invisibility as an argument sounds comic, rather like the proposition that the absence of telegraph poles and wires in archaeological excavations is a proof of ancient radio communications."

[5]In this paper we shall not consider further the properties of the accreted matter, the structure of the accretion disks, nor the detailed properties of the emission that enables one to identify a black hole, since all this can be read in detail in Thorne's paper.[4]

[6]This recalls the situation when one considers thermodynamic systems. The law of increase of entropy prevents the complete transformation of the internal energy of a thermodynamic system into work but enables one to extract this energy partly by using two systems with different temperatures. In black-hole physics, the surface area of the black hole plays a role analogous to entropy in thermodynamics. This analogy will be discussed in more detail in the various sections of the paper.

[7]We should emphasize that here and in what follows we are speaking about quantum processes in an external classical gravitational field. In principle, one could have other quantum effects due to the quantum nature of the gravitational field itself. But it would seem that these effects can be important only if the radius of curvature of spacetime (in the case of a black hole, its radius) becomes comparable with the characteristic quantum–gravitational (Planck) length $l_{Pl} = \sqrt{\hbar G/c^3} = 1.6 \cdot 10^{-33}$ cm.

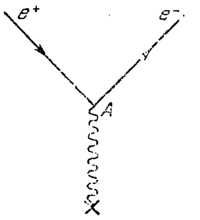

FIG. 1. Creation of an electron–positron pair in a variable electromagnetic field. The wavy line corresponds to a virtual photon of the electromagnetic field with mass greater than two electron masses. The virtual photon decays at the point A into an electron and a positron. The pair is formed in a space-time region of the order of the electron Compton wavelength.

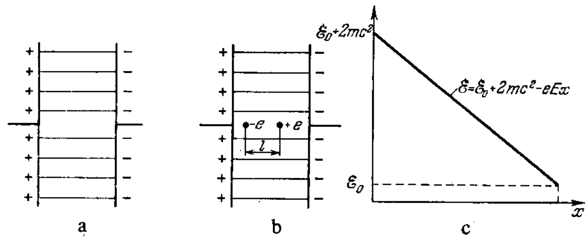

FIG. 2. Pair creation in the static electric field of a capacitor. The process is energetically possible if the energy of the capacitor without pair (a) is equal to the energy of the capacitor with the created pair (b). This condition means that a pair can be created if the particles appear at a distance greater than $l_0 = 2mc^2/eE$. Figure 2c shows the potential barrier which must be overcome by the virtual particle (electron) if it is to become real. The distance between the virtual particles of the pair is plotted along the x axis, while the corresponding energy of the system consisting of the capacitor and the virtual pair is plotted along the $\mathscr{E}$ axis.

that exists between the gravitational and electromagnetic fields.[8)] The creation of electron–positron pairs in an external electromagnetic field can be completely described by modern quantum electrodynamics. Although there is a unified description suitable for an external field of arbitrary form, it is helpful to consider pair creation effects in two extreme limiting cases.

As the first, let us consider a rapidly varying electromagnetic field generated by moving charges. Suppose this field contains Fourier components with frequencies ω such that $\hbar\omega \geqslant 2mc^2$ (m is the electron mass). Decomposition into Fourier components corresponds to the idea of a field as a superposition of individual modes, or photons. If the field is free, the mass of each of these photons is zero. In the general case when the variable field is associated with sources, the mass of the photons is not necessarily zero. In this case, the photons are called virtual. If the external field from the sources contains high frequencies or, which is the same thing, sufficiently heavy (with mass $M \geqslant 2m$) virtual photons, the conservation laws permit these virtual photons to decay into an electron–positron pair. Such a process is represented schematically in Fig. 1. In more usual language, the "decay" of heavy virtual photons every now and then can be described as processes of spontaneous creation of electron–positron pairs in an external variable high-frequency electromagnetic field.[9)]

The other situation arises when there is a homogeneous static electric field of strength E in, for example, the region between the plates of a capacitor (Fig. 2a). In order to establish whether this field can create pairs, let us proceed as follows. We attempt to understand first whether the law of conservation of energy (or other conservation laws) prohibit such effects. Suppose the energy of the field in the capacitor is $\mathscr{E}_0$. Suppose now that in this field an electron-positron pair is created and that the created particles are at a distance l from each other (Fig. 2b). It is readily seen that the energy of the system (the field of the capacitor and the created particles) is given by the expression $\mathscr{E} = \mathscr{E}_0 + 2mc^2 - elE$. Therefore, the energy conservation law allows pair creation only if the particles are at a distance greater than $l_0 = 2mc^2/eE$ when they are created.

It would seem that this imposes an absolute veto on the possibility of pair creation since the pair must be formed in a region of the order of the electron Compton wavelength $\lambda = h/mc$ and for reasonable field strengths $\lambda \ll l_0$. However, it is sufficient to recall the phenomenon of sub-barrier tunneling—which is so characteristic of quantum theory—to realize that the veto is not absolute. There is a small but finite probability of pair creation. The small value of this probability is associated with the presence of the potential barrier (Fig. 2c) which must be overcome by the created (virtual) particle before it reaches the region in which its energy has an allowed value (i.e., where the particle can finally become a real particle). Therefore, if the probability of pair creation in a static field is not to be negligibly small, one requires fields with a high strength.

The usual quasiclassical approximation enables one to obtain the value $D = \exp(-\pi m^2c^3/eE\hbar)$ for the transmission factor of the potential barrier. The exact expression for the probability of creation of an electron-positron pair in unit volume in unit time by a constant homogeneous electric field has the form[7]

$$w = \frac{(eE)^2}{\pi^2\hbar^2c} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{\pi m^2c^3}{eE\hbar}\right).$$

The terms in the sum with $n \geqslant 2$ describe creation processes of several pairs at once. The first term of the sum agrees exactly with the result of the quasiclassical approximation if one bears in mind that the probability of pair creation of real particles is given by the product of the probability $(eE)^2/\pi^2\hbar^2c$ of creation of a virtual pair in the field and the probability D of tunneling through the barrier which is necessary to transform the virtual pair into a real one.

8)We should say that the analogy is of course incomplete. The gravitational field, in contrast to the electromagnetic, is an attractive field. Therefore, the analogy enables one to understand only some of the features of particle creation processes in a gravitational field.

9)It should be emphasized that the single-photon process of pair creation described above is encountered rather seldom in real experimental situations (the process occurs, for example, on the transition from excited states of the nuclei of carbon C^{12} and oxygen O^{16} to the ground state by the emission of an e^+e^- pair. Two-photon creation processes (for example, creation of an e^+e^- pair by a photon in the Coulomb field of a nucleus) are much more common.

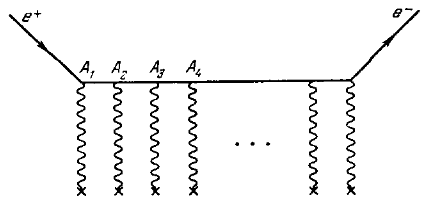

FIG. 3. Diagram showing the creation of an electron–positron pair in a static electric field. The wavy lines show the external electric field on which the virtual particles of the pair are scattered at the points (vertices) A_1, A_2, A_3, ... The points mean that it is necessary to take into account all such diagrams with arbitrarily large number of vertices.

In diagram language, one can describe the process of pair creation in a static field as in Fig. 3. The points in the figure show that the virtual particles undergo infinitely many scatterings on the static field before they become real. One can also regard a static field as the limit of a slowly varying field with frequency ω_0 as ω_0 tends to zero. In this case, if an electron is scattered once by such a field it can acquire the energy $\hbar\omega_0$ from the field. Therefore, before the virtual particles accumulate the energy $2mc^2$, which enables them become real particles, they must be scattered $N=2mc^2/\hbar\omega_0$ times. On the transition to a constant field, the number of interactions tends to infinity and the creation probability to the value given above.

Summarizing, we can say that the principal role in pair creation in a sufficiently rapidly varying field is played by the frequency of the field, whereas in the static case it is the field strength that is decisive. Another extremely important difference is that in the variable field pair creation is a local event (the created particles are at a distance of the order of the Compton wavelength from each other), whereas in the constant field the particles may be created at a fairly large *space-like distance* from each other.[8]

Returning to our discussion of pair creation in a gravitational field, we must bear in mind two possibilities. In the early stages of the development of the Universe, characterized by a rapid variation of the gravitational field, pairs are created in complete analogy with the case of a rapidly varying electromagnetic field. Such processes, which are of considerable interest for cosmology, are currently being intensively studied.[10)] The analogy with the electromagnetic field suggests that in the case of a stationary gravitational field one can expect appreciable effects of pair creation only in the case of very strong fields. Such fields are described in the general theory of relativity by curvature of spacetime. Strong fields can exist only near bodies that have large mass and high density. As we mentioned in the introduction, such objects are usually unstable and black holes are formed by the collapse of massive dense bodies. Therefore, when one studies the problem of pair creation by a constant gravitational field it is natural to take black holes as the most natural astrophysical objects for which these effects could be important.

§2. WHAT IS A BLACK HOLE? COLLAPSE OF STARS

What is a black hole? In the introduction we have explained that this is an object formed by the irreversible collapse of a massive body, and that it is characterized by a strong gravitational field. To give a more accurate definition of a black hole, it is simplest to begin with arguments due to Laplace.[25] If a particle has velocity v, it can, by overcoming the gravitational field of a mass M, escape from the surface of such a body to infinity only if the radius R of the body exceeds $R_0 = 2GM/v^2$ (G is the gravitational constant). Applying this result to light, we find that there is a critical size of a body of mass M: $R_g = 2GM/c^2$ (called its *gravitational radius*) such that if the body has radius less than R_g even light cannot escape from the surface of this body to infinity but is prevented from doing so by the strong gravitational field.

This qualitative Newtonian treatment gives the same result as the general theory of relativity, although in the latter the effect is described differently. According to the general theory of relativity, a gravitational field is manifested through the geometrical properties of spacetime. The propagation of light in curved spacetime is characterized by the position of the local light cones. Therefore, the fact that the gravitational field of a gravitating body curves a light ray and light, like all other matter, is attracted by a gravitating object means that in a gravitational field the local light cones are turned in the direction of the heavy body (Fig. 4).

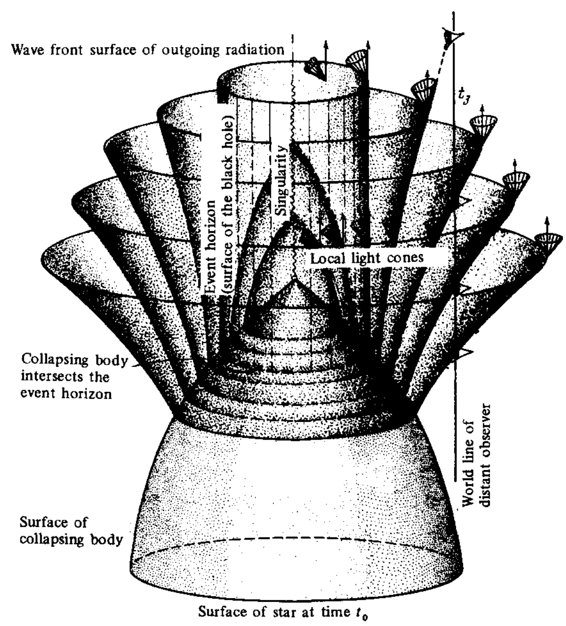

FIG. 4. Gravitational collapse of a spherically symmetric star. The time axis points upward. The spherical surface of the star at a given time is shown schematically as a circle.

[10)]The creation of scalar particles in isotropic models of the Universe was considered in[9–11]. In[12,13] the important role of anisotropy of the expansion of space in particle creation processes was demonstrated. The investigations[12–24] are devoted to quantum creation effects of different particles in different cosmological models.

Let us consider in more detail the process of gravitational collapse of a massive spherically symmetric star leading to the formation of a black hole. In Fig. 4, the time axis is directed upward, and the position and inclination of the local light cones corresponds to the situation obtained when the problem is treated exactly in the general theory of relativity. The picture of the local light cones enables one to understand how light propagates in the gravitational field of a collapsing body and to construct, in particular, the wave front surface of the emerging radiation. In Fig. 4 we also show the surfaces for light emitted outward from the surface of the body after different intervals of time. The gravitational effect of the collapsing star is manifested through the fact that the rays of light that leave the surface of the star find it harder and harder to reach infinity and, finally, as can be seen from the figure, there is an instant of time for which the light emitted outward never reaches infinity. This occurs when the surface of the collapsing body intersects the gravitational radius (i.e., the surface of radius $R_g = 2GM/c^2$).

The surface formed by the wave front of the light wave emitted at the instant of intersection of the collapsing body with the gravitational radius is called an *event horizon*. An event horizon also occurs for a nonspherical collapse and is always a light-like surface (i.e., a surface formed by light rays). The name "event horizon" is used because the surface of the horizon divides spacetime into two parts: an exterior part, from which a signal can reach infinity, and an interior part, from which escape is impossible. It is this property of an event horizon that is used to define it in the most general case. The region of space below the event horizon is called a *black hole*. At time τ, the boundary of the black hole is the two-dimensional surface of the intersection of the space-like hypersurface $\tau = \text{const}$ and the event horizon. The formation of an event horizon and, therefore, the formation of a black hole, always indicates that in some region in space the energy density increases so much that the gravitational field does not permit even light rays to leave this region.

The existence of an event horizon has some important and unexpected consequences, and to reveal these we examine again the picture of a spherically symmetric collapse shown in Fig. 4. First of all, from the very definition it is clear that no information about what occurs below the event horizon can reach the external observer. One therefore says that the event horizon is a one-sided membrane through which energy and information can pass inward but never outward.

What will an external observer see of a gravitational collapse if information about the motion of the body after it passes below the gravitational radius (below the event horizon) is inaccessible to him? It can be seen from the figure that the light rays emitted from the surface of the body at equal intervals will reach the distant observer with a delay that is the greater the nearer is the emission to the gravitational radius. Under the influence of the gravitational field, these signals are "reddened" more and more, so that the total luminosity of the star is correspondingly reduced. Therefore, if an emitting star collapses, the observer will see a rapid (exponential, as the calculations show) decrease in its luminosity with a simultaneous reddening; the visible dimension of the star will tend (also exponentially) to the gravitational radius.[11)] If a star with a few solar masses collapses, it already becomes invisible fractions of a second after the onset of appreciable reddening and forms a black hole, i.e., a compact nonemitting object with radius equal to the gravitational radius. Since the gravitational field outside a spherically symmetric distribution of matter is always static (Birkhoff[26]), the black hole formed by the collapse of this matter is also static.

In connection with the essential irreversibility in the processes that lead to the formation of an event horizon, one naturally asks how this irreversibility can be reconciled with the time-reversibility of Einstein's equations. The answer is that the initial conditions are significantly changed by the replacement of t by $-t$. Under such a replacement, the solution describing collapse becomes a new solution corresponding to a completely different physical situation (anticollapse). The new solution describes a system in which matter with kinetic energy greater than the gravitational energy of attraction expands irreversibly and emerges from under the gravitational radius. An external observer cannot influence events occurring below the sphere $R = R_g$, although what happens below R_g is open to observation. In[27,28] it was shown that if the "explosion" that started the expansion of matter were delayed in individual parts of the Universe these regions could give rise to anticollapsing bodies. Such objects have been called white holes. (White holes are discussed in more detail in[31].)

What happens to the above picture if the collapsing body is not spherically symmetric? If it collapses in such a way that its dimensions in all directions decrease, the qualitative picture of the collapse remains the same. If there is sufficiently strong contraction of the body, the gravitational field increases so much that light from its surface can no longer reach infinity and an event horizon is formed (Fig. 5). However, the surface of this horizon is no longer spherically symmetric. An important difference from spherical collapse is that the collapsing body now has quadrupole and higher moments, whose variation during collapse necessarily leads to the emission of gravitational radiation.[12)] Therefore, the resulting black hole is nonstatic. However, in a fairly short time, because of the emission of gravitational waves to infinity and also because of partial absorption of them by the black hole, the gravitational field becomes static. If as a result of these processes no unphysical singularities arise in the observed region of space (and there are reasons for believing that such "naked" singularities do not arise),

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11) The characteristic time of "freezing" of the star and "reddening" of its radiation is $T \sim R_g/c \sim 10^{-5}\,(\text{sec})M/M_\odot$.

12) Collapse with small deviations from spherical symmetry has been considered in[29–33]. The review[34] is devoted to nonspherical collapse.

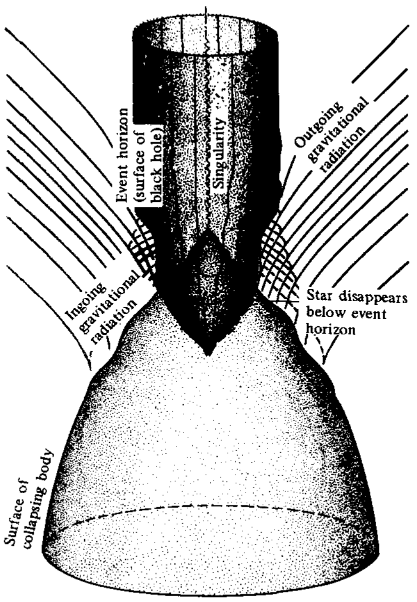

FIG. 5. Nonspherical gravitational collapse. The time axis points upward.

then, as Israel has shown,[35,36] the resulting static black hole must necessarily be spherically symmetric, and such a black hole is completely undistinguishable from one formed by spherical collapse.

We mention one other circumstance associated with the formation of a black hole. The formation of an event horizon means that if light cannot escape from the restricted region of space then much less so can the material particles retained by the gravitational field. Moreover, under the influence of the gravitational field the matter in this region will be compressed unboundedly, and, as Penrose has shown,[37] a singularity will unavoidably arise sooner or later, i.e., a physically singular region in which our ordinary ideas about spacetime cease to be valid. And although these singularities, like everything below the event horizon, are inaccessible to observers outside, the actual fact of their inescapable existence is rather unpleasant for theory and forces one to invoke quantum ideas, in particular, with a view to avoiding such a situation.

§3. ROTATING BLACK HOLE

Hitherto, in considering collapse, we have assumed that the collapsing body does not rotate. It is clear that if the rotation of a body is small and the corresponding centrifugal forces are incapable of breaking up the body or preventing collapse, then at some stage of the collapse an event horizon will form, as before. The resulting black hole again becomes stationary after a farily rapid process of emission of gravitational waves. A new feature however is that the gravitational field of this black hole "feels" the rotation of the collapsed star.

In order to see how the rotation of a gravitating body can affect the properties of its gravitational field, it is helpful to consider the case of a weak field for which the deviation of the metric $g_{\mu\nu}$ from the metric $\eta_{\mu\nu}$ of flat spacetime is small: $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$. In this case, Einstein's equations reduce to linear equations for $h_{\mu\nu}$[38][13)]

$$\Box h_{\mu\nu}=\frac{16\pi G}{c^4}\left(T_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}T^{\alpha}_{\alpha}\right),$$

where $T_{\mu\nu}$ is the energy-momentum tensor of the matter. In the stationary case, the solution of these equations can be obtained in the usual manner in the form

$$h_{\mu\nu}(\mathbf{r})=-\frac{4G}{c^4}\int\frac{T_{\mu\nu}(\mathbf{r}')-(1/2)\,\eta_{\mu\nu}T^{\alpha}_{\alpha}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\,d\mathbf{r}'.$$

If one chooses a cylindrical coordinate system with z axis along the direction of rotation of the body, the component $T_{t\varphi}$ of the energy-momentum tensor, which describes the matter flux due to the rotation, is nonzero, and there is therefore a nonzero component $g_{t\varphi}$ of the gravitational field. By means of a coordinate transformation that preserves the time-independence of $g_{\mu\nu}$ one can make this component vanish only if the central body does not have angular momentum.

Thus, rotation of the gravitating body leads to the appearance of nondiagonal components of the metric g_{ti}. These new components of the field appear in the form of additional forces[14)] acting on test bodies that move in the gravitational field near the rotating body. If a gyroscope is placed in such a gravitational field, the additional (Coriolis) force causes it to precess, and if $\mathbf{g}$ is this vector having components g_{ti}, the angular velocity of the precession is $\mathbf{\Omega}=(c/2)\,\mathrm{curl}\,\mathbf{g}$. As calculations show, if the central rotating body has angular momentum $\mathbf{J}$, then at a certain distance R from the body the angular velocity of precession of the gyroscope is equal to

$$\Omega=\frac{G}{c^2R^3}\,[\mathbf{J}-3\mathbf{n}\,(\mathbf{J}\mathbf{n})],$$

where $\mathbf{n}$ is the unit direction vector of the gyroscope axis.

This equation can be used to determine the angular momentum of the rotating gravitating body from a measurement of the rate of precession of a gyroscope far from the body even in the case when the gravitational field near the body is large and the spacetime in the region next to the rotating body is strongly curved. The exact solutions of Einstein's equations describing such an axisymmetric stationary field of a rotating

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[13)]To eliminate the coordinate arbitrariness in the derivation of this equation we have imposed an additional condition on the $h_{\mu\nu}$ that recalls the Lorentz condition in Maxwell theory: $\partial_\mu(h^{\mu}_{\nu}-\frac{1}{2}\delta^{\mu}_{\nu}h^{\alpha}_{\alpha})=0$.

[14)]To avoid possible misunderstanding, note that a free particle in a gravitational field always moves along a corresponding geodesic. In a stationary field, it is convenient to describe the motion of a test particle by a trajectory in three-dimensional space. These trajectories in the three-dimensional space need not be shortest lines from the point of view of the three-geometry of this space. It is convenient to explain the departure of the three-dimensional trajectories from three-geodesics by the influence of gravitational forces (for more detail see[38], p. 323). It is forces of this kind that we mean here and in what follows.

source must contain two constants: the mass M and the angular momentum J of the body. An approximate solution of Einstein's equations describing the gravitational field far from rotating bodies has long been known (the Thirring–Lense solution[39]). However, it was only in 1963 that Kerr succeeded in obtaining an exact solution[40] describing the gravitational field outside a rotating body and containing constants M and J. As in the case when the central body does not rotate, the spacetime described by the Kerr solution has an horizon whose radius is given by

$$R_g=\frac{GM}{c^2}+\sqrt{\frac{G^2M^2}{c^4}-\frac{J^2}{M^2c^2}}.$$

It was shown later[5,41,42] that the Kerr solution is the only possible solution without singularities describing a stationary gravitational field in a vacuum possessing an event horizon.

Therefore, if an event horizon arises as a result of the collapse of a rotating star, after rapid emission of gravitational waves the "excited" rotating black hole becomes stationary and it is uniquely determined by the values of its mass M and angular momentum J. Rapid rotation of the star prevents the formation of an horizon. As can be seen from the expression for R_g, the maximal angular momentum of the body at which an horizon can still be formed is $J_{max}=GM^2/c$. A black hole having the maximal angular momentum is called an *extremal black hole*. The surface of a rotating black hole is no longer a sphere, as was the case for a black hole without rotation, but a more complicated surface of revolution,[15)] and the area of the surface of the rotating black hole is

$$S\begin{pmatrix}\text{surface area}\\ \text{of black hole}\end{pmatrix}=8\pi\frac{GM}{c^2}\left(\frac{GM}{c^2}+\sqrt{\frac{G^2M^2}{c^4}-\frac{J^2}{M^2c^2}}\right).$$

As in the case of nonrotating black holes, the remarkable feature here is the complete identity of black holes with rotation if the parameters M and J describing them are the same, and the way in which the black hole completely "forgets" its prehistory and hides its internal "structure" after it has been formed.

§4. GRAVITATIONAL FIELD OF A ROTATING BLACK HOLE. ERGOSPHERE

We now consider in more detail the properties of the gravitational field generated by a black hole. If the hole does not rotate, a test body is subjected to a force of attraction that depends only on the position of the test body and not on the magnitude or direction of its velocity. As the surface of the black hole is approached, this attraction increases unboundedly. Therefore, no finite force can hold up a particle on the surface of a black hole or retrieve it from below the event horizon.

[15)]To simplify the following diagrams, we shall, as is customary, represent the surface of a black hole in the form of a sphere $R=R_g$, "forgetting" that R is not a spherical but rather an elliptic coordinate. Since the aim of the figures is to exhibit schematically the mutual disposition of the different regions of spacetime, this simplification does not alter anything qualitatively.

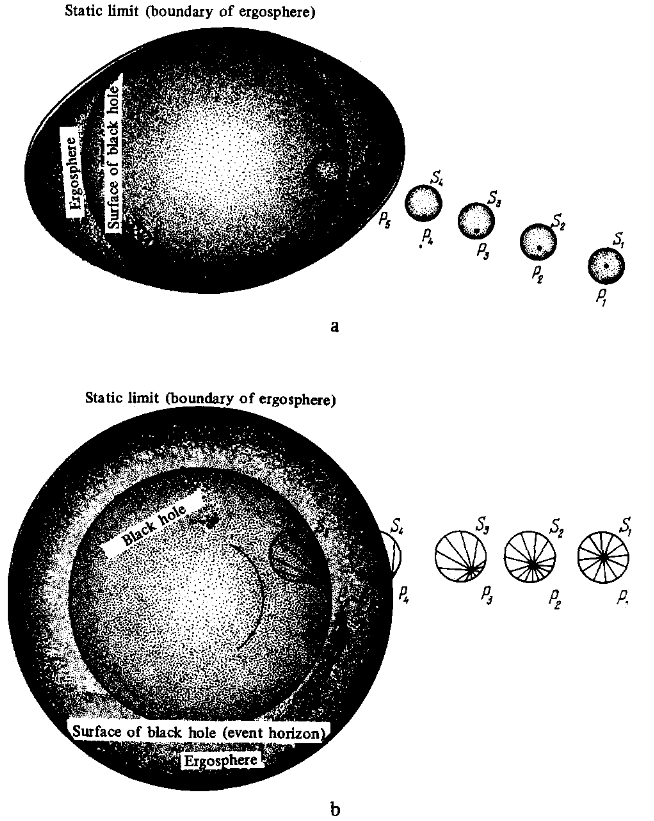

FIG. 6. Structure of the gravitational field of a rotating black hole. A three-dimensional schematic representation of the rotating black hole is shown in Fig. a. The black hole is seen "from above" along the axis of rotation of the same black hole in Fig. 6b. The spherical surfaces S_i represent the position of the wave front of emission short intervals of time after the emission of a wave at the points P_i.

In the gravitational field of a rotating black hole, the force acting on a test particle depends not only on the position of the body but also on its velocity (besides the ordinary force of attraction, there is an additional "Coriolis" force due to the rotation of the gravitating body). As the black hole is approached, the force of attraction acting on a *body at rest* increases and becomes infinite outside the event horizon on a surface that is called the *static limit* or *boundary of the ergosphere*, which is *defined* by the equation

$$\begin{pmatrix}\text{equation of}\\ \text{boundary of}\\ \text{ergosphere}\end{pmatrix} R_{be}=\frac{GM}{c^2}+\sqrt{\frac{G^2M^2}{c^4}-\frac{J^2}{M^2c^2}\cos^2\vartheta}.$$

Under this surface, no force can hold the particle at rest. However, the dependence of the gravitational force on the velocity of the test particle means that a particle with angular momentum in the same direction as the black hole's is subject to a weaker force than a body at rest. Therefore, if a particle moves in the direction of rotation of the black hole, the gravitational force on the boundary of the horizon remains finite and such particles can be retrieved. This "compensation" of the force of attraction by the Coriolis force is possible in the region between the surface of the static limit and the surface of the black hole. This region is called the *ergosphere*. The positions of the two surfaces are shown schematically in Fig. 6. Any particle in the ergosphere must necessarily rotate around the black hole. Even light must do this, as can be seen from the arrangement of the local light cones shown in Fig. 6. The angular velocity of rotation of different particles in the ergosphere far from the surface of the black hole may differ, but all particles that intersect the event horizon have the same angular velocity. This velocity is called the angular velocity of rotation of the black hole, and it is equal to[43]

$$\Omega \left(\begin{smallmatrix}\text{angular velocity}\\ \text{of black hole}\end{smallmatrix}\right) = \frac{4\pi J}{MS}.$$

All points on the surface of the black hole rotate with the same angular velocity, so that the black hole as a whole rotates like a rigid body.

As the angular velocity of rotation of a black hole decreases, the volume of the ergosphere decreases as well. Therefore, in a nonrotating black hole the event horizon surface and the static limit coincide. Both in the case of rotation and in the absence of it, the event horizon is defined as the surface below which no particle can be retrieved by the application of any force whatever the velocity (less than the velocity of light) the particle may have.

If a test body which emits radiation periodically falls into a rotating black hole, the gravitational field has the effect of making a distant observer receive these signals with ever greater time delay. From the point of view of the distant observer, the falling body comes to a stop near the surface of the black hole and then rotates with it at an angular velocity Ω.

The emission from a particle falling into a rotating black hole suffers qualitatively the same red-shift effect as in the case of a black hole without rotation. A difference is that even particles without angular momentum are caught up in rotation when they fall into a rotating black hole. All falling particles in the ergosphere rotate in the direction of rotation of the black hole, and therefore a periodic Doppler due to this motion is superimposed on the ordinary red shift. The magnitude of the additional Doppler effect is maximal in the equatorial plane and becomes zero on the polar axis. If we leave out the factor describing the Doppler effect, then as the particle approaches the horizon the frequency shift in the gravitational field is determined by

$$\frac{\omega_{\text{obs}}}{\omega_{\text{em}}} \left(\begin{smallmatrix}\text{ratio of observed}\\ \text{frequency of emitted}\\ \text{frequency}\end{smallmatrix}\right) \sim \exp\left(-\frac{\varkappa t}{c}\right),$$

where $\varkappa$, which is called the *surface gravity*, is equal to

$$\varkappa \left(\begin{smallmatrix}\text{surface}\\ \text{gravity}\end{smallmatrix}\right) = \frac{4\pi c^2}{S}\sqrt{\frac{G^2M^2}{c^4} - \frac{J^2}{M^2c^2}}.$$

If the black hole does not rotate, then $\varkappa = c^4/4GM = GM/R_g^2$, and the surface gravity is simply the attraction of the gravitational field (the acceleration of free fall) on the surface of the black hole.[16] Like the angular velocity, the surface gravity for a stationary black hole is the same for all points on the surface of the black hole.

Besides the above characteristics, the ergosphere has one more important property: Particles and light moving in the ergosphere can have negative total energy. Let us consider this effect in detail. In complete accordance with Noether's theorem, the fact that the gravitational field of a rotating black hole is stationary and axisymmetric ensures the existence of two conserved quantities: the energy and the angular momentum about the axis.

An expression for these quantities can be obtained as follows. The presence of symmetry enables us to choose a coordinate system in which the components of the metric do not depend on the time t or the angular variable φ. Then the conserved quantities are the projections of the four-momentum onto the coordinate lines t (energy) and φ (angular momentum). This same fact can be expressed in a more invariant form by introducing a pair of vector fields (called Killing vector fields) defined in the above coordinate system by the components $\xi^\mu_{(t)} = \delta^\mu_t$ and $\xi^\mu_{(\varphi)} = \delta^\mu_\varphi$. Then the energy and angular momentum of the particles are given by

$$E\ (\text{energy}) = g_{\mu\nu}P^\mu\xi^\nu_{(t)},\quad L\left(\begin{smallmatrix}\text{angular}\\ \text{momentum}\end{smallmatrix}\right) = g_{\mu\nu}P^\mu\xi^\nu_{(\varphi)}.$$

The expression for the energy of the particle in the gravitational field:

$$E\left(\begin{smallmatrix}\text{energy of}\\ \text{particle}\end{smallmatrix}\right) = mg_{0\nu}\frac{dx^\nu}{d\tau}$$

casts light on a number of circumstances. If the black hole does not rotate, the nondiagonal components of the metric are absent and the energy of the particle does not depend on the direction of motion but is determined solely by the modulus of the velocity (kinetic energy) and the position of the particle in the gravitational field (potential energy and gravitational mass defect). If a particle moves in the field of a rotating black hole, the nonzero nondiagonal components of the metric have the effect that the energy is influenced not only by the magnitude of the velocity but also by the direction of the motion (the value of the angular momentum). The energy of a particle whose angular momentum is directed parallel to the black hole's is greater than that of a particle with opposite direction of the angular momentum.[17]

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[16]The quantity $\varkappa = GM/R_g^2$ is the strength of the gravitational field of a body with mass M at distance R_g in Newtonian theory. In the general theory of relativity, $\varkappa$ determines the strength of the gravitational field in the following sense. Suppose a body is at rest near the surface of a nonrotating black hole and its four-dimensional velocity is $u^\alpha = (\sqrt{g_{00}}, 0, 0, 0)$. The world line of such a particle is obviously not a geodesic, and therefore its four-acceleration is nonzero. The change in the four-dimensional velocity in unit time t measured by the clocks of a distant observer is $a^\alpha = (Du^\alpha/d\tau)d\tau/dt$. As the test body at rest approaches the surface of the black hole, $|a| = \sqrt{-a_\alpha a^\alpha}$ tends to the value $\varkappa$. In invariant form in the general case of rotating black hole, $\varkappa$ is defined as follows.[44] Let $\xi_{(t)}$ and $\xi_{(\varphi)}$ be the fields of the Killing vectors associated with the time and axial symmetry of the black-hole field, and $l^\alpha = \xi^\alpha_{(t)} + \Omega\xi^\alpha_{(\varphi)}$, where Ω is the angular velocity of rotation of the black hole. Then $\varkappa$ on the horizon is defined by the relation $l^\alpha_{;\beta}l^\beta = \varkappa l^\alpha$.

[17]If the spherical gravitational field recalls the Coulomb field of a charged sphere, the additional field resulting from the rotation of a gravitational field is very similar to the magnetic field due to the rotation of a charged sphere. The additional interaction of the angular momentum of the particle with the gravitational field is completely analogous to the splitting of the energy levels when a charged particle moves in a magnetic field.

Whereas the energy of a particle in the field of a nonrotating black hole is always positive (the gravitational mass defect of the particle outside the black hole can reduce the amount of the energy but not make it negative), in the field of a rotating black hole one can have a situation in which the energy shift due to the interaction between the angular momentum of the particle and the gravitational field is so large that the total energy of the particle becomes negative. This is possible only if the particle moves sufficiently near the surface of the black hole. It can be shown that it is precisely the ergosphere that is the region in which states with total negative energy are possible.

We note finally one important although somewhat more formal property of the ergosphere. Far from the black hole, the vector $\xi_{(t)}$ is time-like and has unit length, but inside the ergosphere it must be space-like. This can readily be understood by recalling that the energy–momentum vector is time-like, and if the total energy, which is equal to the scalar product of the energy–momentum vector and $\xi_{(t)}$ is to be negative, then $\xi_{(t)}$ must be a space-like vector. The boundary of the ergosphere is therefore determined by the condition $\xi^{\mu}_{(t)}\xi_{(t)\mu}=0$. In a nonrotating black hole, the event horizon and the boundary of the ergosphere coincide, and therefore the vector $\xi_{(t)}$ becomes light-like on the boundary of a nonrotating black hole. Inside the black hole, the vector $\xi_{(t)}$ is space-like.

§5. EXTRACTION OF ENERGY FROM A BLACK HOLE. HAWKING'S THEOREM

The presence of an ergosphere enables one (at least in principle) to extract energy from a rotating black hole. Let us consider first the following gedankenexperiment with a nonrotating black hole. Onto this hole, lower a weight on a strong weightless thread. If m is the mass of the weight, its total energy will differ from mc^2 by an amount equal to the gravitational mass defect. As the horizon is approached, this mass defect reaches mc^2 and the total energy of the weight becomes zero. The total work performed by the gravitational field on the weight is exactly equal to the original internal energy of the weight. Therefore, this mechanism enables one to liberate the complete internal energy in the body, but none of the energy in the black hole is expended and its parameters such as the mass and radius remain unchanged. If this experiment is carried out without taking energy from the body (for example, by letting it fall freely), then as a result of this the energy of the black hole increases by an amount equal to the energy carried into it by the falling body. The mass and radius of the black hole then increase.

If we recall that no bodies or radiation can escape from the black hole (from under the event horizon), we readily see that the mass and radius of a nonrotating black hole do not decrease with the time. In other words, the surface area of a nonrotating black hole either remains constant or increases. It can be shown[5] that this occurs not only in the case of nonrotating black holes but also in the most general situation. Namely, *the surface areas of a black hole cannot decrease. If there are several black holes, the total area of their surfaces does not decrease in an arbitrary (classical) interaction between black holes and matter and with one another.* This result is known as *Hawking's* theorem (Fig. 7). The rigorous proof of this theorem requires the use of rather complicated mathematics. The key idea in the proof is the use of a property found previously by Penrose: Light rays forming an horizon never intersect. If one assumes that light rays forming an horizon begin to converge and the energy density everywhere in space is non-negative, then the focusing effect of the gravitational field means that this convergence must lead to the intersection of light rays. It is therefore concluded that the light rays forming an horizon cannot converge, so that the surface area of an horizon cannot decrease.

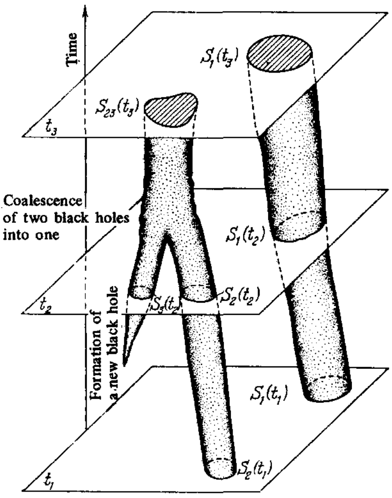

FIG. 7. Possible processes with black holes. Illustration of Hawking's theorem. The planes t_1, t_2, t_3 are spatial sections at the corresponding times; $S_a(t_i)$ is the surface area of black hole a at time t_i. Two black holes can coalesce into one, black holes can form, the surface area of an isolated black hole increases with the time. One black hole cannot break up into two or more black holes. The total surface area of the black holes at the time t is denoted by $S(t)$. Hawking's theorem asserts that $S(t)$ is a nondecreasing function of the time.

Because of its generality, Hawking's theorem has many helpful and interesting consequences. As the first, let us consider the application of this theorem to the case of one stationary black hole. If there is no rotation, the surface area of the horizon is proportional to the square of the mass of the black hole, and therefore, as we have already discussed, all processes can but increase the mass of the black hole, so that the extraction of energy from it is impossible.

The mass of a rotating black hole can be decreased without violating Hawking's theorem only if its angular momentum is simultaneously decreased. (This can be readily verified by analyzing the expression given previously for the surface area of a rotating black hole.) Penrose[45] proposed the following gedankenexperiment showing how one could draw off the rotational energy from a rotating black hole. Into a rotating black hole, toss (Fig. 8) an object in such a manner that it is carried into the ergosphere. Suppose that in the ergo-

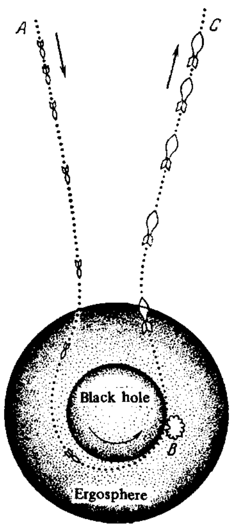

FIG. 8. The Penrose process. A body that falls from a certain distance (position A) enters the ergosphere of a rotating black hole and breaks up at point B near the surface of the black hole into two fragments. One of these is absorbed by the black hole. The parameters of the explosion are chosen in such a way that its energy is negative. The other fragment is ejected from the ergosphere (position C) with an energy greater than the original body's.

sphere this object splits into two parts at a definite point as a result of an explosion. One can choose the parameters of the explosion in such a way that one of the parts acquires an angular momentum in the opposite direction to the rotation of the black hole, so that its total energy is negative, while the other part is ejected out of the ergosphere. Then a simple application of the law of conservation of energy shows that the total energy of the ejected part is greater than the energy of the original object. The law of conservation of angular momentum means that the ejected body must carry away not only energy from the black hole but also some of its angular momentum. Calculations show that the maximal energy gain in this process is achieved when the original object explodes near the event horizon itself. In this case, the area of the horizon does not change. These processes, in which the surface area of the black hole does not increase, are said to be *reversible*. Hawking's theorem enables one to estimate simply the maximal amount of energy that can be extracted from a rotating black hole of mass M with angular momentum J. This energy is

$$E_{\text{rot}}\left(\begin{smallmatrix}\text{energy of rotation}\\ \text{of black hole}\end{smallmatrix}\right) = Mc^2\left[1-\sqrt{\frac{1}{2}\left(1+\sqrt{1-\frac{J^2c^2}{G^2M^4}}\right)}\right].$$

This expression, which determines the rotational energy reserve in a black hole and is equal to the energy which can be extracted from the black hole in a reversible process, can be obtained as follows. Let M_1 be the mass of a nonrotating black hole obtained in a reversible process from a rotating hole. The condition of reversibility enables us to find M_1 by equating the areas of the surfaces of the rotating and subsequently obtained nonrotating black holes. The value of E_{rot} is then found to be $E_{\text{rot}} = (M - M_1)c^2$.

Another consequence of Hawking's theorem is the *impossibility of decay, or breakup* of a black hole whatever may happen to it; for suppose that a black hole of mass M and angular momentum J breaks up into a pair of other black holes of masses M_1 and M_2 and angular momentum J_1 and J_2. From the law of conservation of energy

$$\left(\begin{smallmatrix}\text{energy}\\ \text{conservation law}\end{smallmatrix}\right)\quad M \geqslant M_1 + M_2.$$

(The inequality arises because some of the energy in the breakup may be carried away by gravitational radiation.) On the other hand, the condition of nondecrease of the surface area of black holes gives the inequality

$$\left(\begin{smallmatrix}\text{area of black holes}\\ \text{does not decrease}\end{smallmatrix}\right)\quad M\left(M+\sqrt{M^2-\frac{J^2c^2}{G^2M^2}}\right) \leqslant M_1\left(M_1+\sqrt{M_1^2-\frac{J_1^2c^2}{G^2M_1^2}}\right) + M_2\left(M_2^2+\sqrt{M_2^2-\frac{J_2^2c^2}{G^2M_2^2}}\right).$$

It is easy to show that these two inequalities are incompatible, and this shows that a black hole cannot break up.

One can verify that Hawking's theorem does not prohibit the opposite process—the coalescence of two black holes into one. In such a process, some of the energy within a black hole can be liberated. On the basis of Hawking's theorem, one can estimate that in the collision of two rotating black holes one can liberate energy up to $(1-2^{-3/2})(M_1+M_2)c^2$. Note that even in collisions of two nonrotating black holes some of their internal energy is liberated. But this does not exceed $(1-2^{-1/2})(M_1+M_2)c^2$.

§6. THE FOUR LAWS OF BLACK-HOLE PHYSICS. THE ANALOGY WITH THERMODYNAMICS

Hawking's theorem reveals an unexpected and, as it has been found, rather helpful analogy between black-hole physics and thermodynamics.

Just like a thermodynamic system, an arbitrary black hole, after relaxation processes (emission of gravitational waves), arrives at a state of equilibrium (stationary black hole) in which it is completely described by a finite number of parameters. The internal energy E of the stationary black hole $E = Mc^2$, which is determined by its mass M, can be found if one knows the surface area of the black hole and its angular momentum. (For this it is sufficient to use the expression for the surface area of the black hole.) Let us consider two stationary black holes whose surface areas differ by δS and whose angular momenta differ by δJ. Then calculations show that the internal energy of these black holes differs by an amount $\delta E = \delta(Mc^2)$ equal to[45]

$$\left(\begin{smallmatrix}\text{first law of}\\ \text{black-hole physics}\end{smallmatrix}\right)\quad \delta E = \frac{\varkappa c^2}{8\pi G}\,\delta S + \Omega\,\delta J.$$

In this equation, the quantity $\varkappa$ in front of δS is the surface gravity, while Ω is the angular velocity of rotation of the black hole. The second term on the right is the usual expression for the change in the energy of rotation. Superficially, this equation resembles the first law of thermodynamics: $\delta E = T\delta S + \Omega\delta J$, which gives the expression for the change in the internal energy of a body rotating with angular momentum Ω resulting from a change δS in its entropy and δJ in its angular momentum.

Bekenstein[46] suggested that a quantity proportional to $\varkappa$ could in a certain sense be regarded as the temperature of a black hole, while a quantity proportional to the area of the black hole could play a role analogous to entropy.

In complete agreement with this analogy, Hawking's theorem shows that the total entropy of a system of two black holes (which is proportional to the sum S of the surfaces of the black holes) does not decrease, i.e.,

$$\left(\begin{array}{c}\text{second law of}\\ \text{black-hole physics}\end{array}\right)\quad \delta S \geqslant 0.$$

It has been found that this analogy can be carried rather far. In both cases (thermodynamics and black-hole physics), the second law distinguishes a direction of time through its irreversibility. In thermodynamics, the law of increase of entropy means that a definite fraction of the internal energy is degraded, i.e., cannot be transformed into work. Similarly, the law of increase in the area of the black hole shows that the fraction of the internal energy of a black hole that cannot be extracted from the hole (for example, by the Penrose process) increases with the time. As in thermodynamics, the value of S is related to the impossibility of obtaining detailed information about the structure of the system (in the present case, about the interior of the black hole). The entropy of a thermodynamic system in a state of equilibrium measures the indeterminacy with which the external thermodynamic parameters (pressure and temperature) determine the internal configuration. Black holes, which in a state of equilibrium are determined by a few parameters (mass, angular momentum) can have very different origins and internal structures for the same values of these parameters. It is therefore natural to introduce the concept of the entropy of a black hole as a measure of the uncertainty of the internal structure for given external parameters.

Bekenstein[46,47] considered a process in which a small system possessing entropy falls onto a black hole and for a definite model he showed that the resulting increase in the surface of the black hole is proportional to the entropy of the falling body. In order to talk about the total entropy of a system consisting of black holes and ordinary matter, it is necessary to find the coefficient of proportionality between the surface of the black hole and the corresponding effective entropy. This coefficient was found by Hawking[48,49] by studying quantum processes in black holes (see the following sections of the paper).

It was found that

$$\tilde{S}\left(\begin{array}{c}\text{entropy of}\\ \text{black hole}\end{array}\right)=\frac{S}{4l^2_{\rm Pl}},\qquad \theta\left(\begin{array}{c}\text{temperature of}\\ \text{black hole}\end{array}\right)=\frac{\hbar\varkappa}{2\pi ck},$$

where $l_{\rm Pl}=\sqrt{\hbar G/c^3}$ is the Planck length and k is Boltzmann's constant.

If $\tilde{S}$ is the total entropy of the black holes in the system (i.e., the sum of the entropies of the individual black holes) and S_m is the entropy of the matter outside the black holes, then

$$\left(\begin{array}{l}\text{generalized}\\ \text{law of black-hole}\\ \text{physics}\end{array}\right)\quad \delta\tilde{S}+\delta S_m \geqslant 0.$$

The expression for the entropy of a black hole can be readily "obtained" from dimensional considerations (to within a numerical coefficient). The entropy, as the logarithm of the number of states of the system, is dimensionless, while the surface area of the black hole has the dimensions of the square of a length. The area is made dimensionless by dividing it by the square of the universal (Planck) length. The appearance of Planck's constant in the expression for the entropy of a classical system should not suprise the reader. Indeed, $\hbar$ appears in the expression for the entropy of a classical Boltzmann gas in the same way. The value of the entropy is associated with the number of states of the system, and real states of a system are ultimately always quantum states. The expression for the effective temperature of the black hole can be obtained as the coefficient of $\delta\tilde{S}$ in the expression of the first law of black-hole physics.

In thermodynamics, it is well known that equilibrium is impossible if the different parts of the system have different temperatures. The presence of a state of thermodynamic equilibrium and the existence of temperature in thermodynamics are postulated by the zeroth law. In black-hole physics it has been found similarly that the surface gravity in a stationary black hole is a constant quantity[44]:

$$\text{(zeroth law of black-hole physics):}\quad \left(\begin{array}{c}\text{the surface gravity } \varkappa \text{ of a}\\ \text{stationary black hole is constant}\\ \text{everywhere on the horizon.}\end{array}\right).$$

If the surface gravity at different points on the surface of a black hole takes different values, the black hole is nonstationary and after a certain time arrives in a stationary state with constant $\varkappa$.

Finally, in complete analogy with the third law of thermodynamics one can formulate[44]:

$$\text{(third law of black-hole physics):}\quad \left(\begin{array}{l}\text{the surface gravity } \varkappa \text{ cannot be made}\\ \text{equal to zero by any finite number}\\ \text{of operations}\end{array}\right).$$

It should be emphasized that the temperature and entropy of black holes introduced here are in no way related to the temperature and entropy of the collapsed matter. Also, we should point out that quite apart from the analogy between black-hole physics and thermodynamics, the laws of black-hole physics are extremely helpful when one considers different processes in which black holes participate, just as the laws of thermodynamics enable one to obtain many general characteristics of thermodynamic processes.

§7. SUPER-RADIATION. VACUUM INSTABILITY IN THE FIELD OF A ROTATING BLACK HOLE

After these remarks on Hawking's theorem and its applications, let us return to the problem of extracting energy from rotating black holes. The method proposed by Penrose can be slightly modified if, instead of tossing an object into a rotating black hole, one chooses an appropriate electromagnetic wave or the wave of some other field.[50–52] Because the black hole field is stationary, the frequency ω of the incident and the reflected wave is the same. To consider the problem, it is convenient to use the axial symmetry and seek a solution as an expansion with respect to modes of the type $\exp[i(\omega t+m\varphi)]R_{\omega m}(t,\theta)$. Usually, the amplitude of the scattered wave is less than that of the incident wave since some of the energy is absorbed by the black hole. However, if the condition

$$\left(\begin{array}{c}\text{condition for}\\ \text{wave amplification}\end{array}\right) \quad \omega \leqslant m\Omega,$$

where Ω is the angular velocity of the black hole, is satisfied, then the reflected wave is amplified. Therefore, waves with appropriate frequency ω and azimuthal quantum number m can be used to draw energy from a rotating black hole.

The wave amplification condition can be obtained fairly simply by using the laws of black-hole physics formulated in the preceding section. To do this, we assume that the amplified wave has an additional energy ε and additional angular momentum j compared with the incident wave. It is easy to see that the ratio ε/j is equal to ω/m. Both energy and angular momentum are extracted from the black hole by the wave, and therefore the change in the energy of the black hole is $\delta E = -\varepsilon$ and the change of the angular momentum is $\delta J = -j$. On the basis of the first law of black-hole physics, we conclude that $(\varkappa c^2/8\pi G)\delta S = \Omega j - \varepsilon$. If we now recall the second law, $\delta S \geqslant 0$, we find that $j\Omega \geqslant \varepsilon$. Finally, using $\varepsilon/j = \omega/m$, we obtain the wave amplification condition in the above form.

The amplification condition is universal and does not depend on the spin of the radiation. The amplification coefficient (gain) of the wave does depend strongly on the spin of the field.[53] Whereas for an electromagnetic field the maximal increase in the energy of the wave is 4.4%, for a gravitational wave it is of order 138%. This amplification of a wave by a rotating black hole has been called *super-radiation*. The phenomenon is similar to the amplification that is well known in classical electrodynamics when electromagnetic waves are incident on a rotating absorbing cylinder.

It is interesting to note that the above derivation of the wave amplification condition and the amplification condition itself are completely reproduced when a wave is incident on a rotating absorbing cylinder if Ω is understood as the angular velocity of rotation of the cylinder and the thermodynamic analogs are substituted for the laws of black-hole physics. Note also that this effect is one of the phenomena that arise when the interface between media has a velocity exceeding the phase velocity of the radiation wave. In the case considered here, amplification occurs when the angular velocity Ω of the boundary exceeds the angular phase velocity of the wave, which is equal to ω/m.

Super-radiation is purely classical, as is shown, for example, by the gain's being independent of Planck's constant. Like other classical processes, super-radiation can be described in quantum-mechanical language. In this description, super-radiation is an increase in the number of quanta in the reflected wave compared with the original number; for the wave energy is proportional to the square of its amplitude and the total number of quanta in a wave can be found by recalling that the energy of a quantum of frequency ω is $\hbar\omega$. Therefore, an increase in the amplitude at unchanged frequency means an increase in the total number of field quanta.

If a black hole is surrounded by walls which reflect radiation completely, even a small signal having parameters satisfying the amplification condition will increase continuously. Such a gedankensystem could be a generator of this radiation.

Study of the classical phenomenon of super-radiation also helps one to understand a purely quantum effect associated with it—*spontaneous creation of particles from the vacuum in the gravitational field* of a rotating black hole. The point is that in the physical vacuum (in contrast to the vacuum of classical physics) only the mean value of the field is equal to zero, while the actual fields themselves fluctuate about these zero mean values (zero-point fluctuations). The amplitude of the zero-point fluctuations for which the amplification condition is satisfied increases continuously and this is manifested in the creation of real field quanta with quantum numbers ω and $m(\omega \leqslant \Omega m)$. For massless fields, the exact expression for the energy flux and angular momentum carried away by the quanta created from the vacuum is given by[52,54]

$$\frac{dE}{dt}\left(\begin{array}{l}\text{flux of energy to infinity}\\ \text{carried by quanta created}\\ \text{from the vacuum}\end{array}\right)=\frac{\mu}{\pi}\sum_{\substack{l,m\\ m\Omega>0}}\int_0^{m\Omega}(K_{lm}(\omega)-1)\,\hbar\omega\,d\omega,$$

$$\frac{dJ}{dt}\left(\begin{array}{l}\text{flux of angular momentum carried}\\ \text{by the quanta created from the}\\ \text{vacuum}\end{array}\right)=\frac{\mu}{\pi}\sum_{\substack{l,m\\ m\Omega>0}}\int_0^{m\Omega}(K_{lm}(\omega)-1)\,\hbar m\,d\omega,$$

where $\mu = 1$ for a scalar field and $\mu = 2$ for other fields, and $K_{lm}(\omega)$ is the gain for a wave of frequency ω with total angular momentum l and projection thereof onto the rotation axis m.

Qualitatively (to within numerical coefficients) one can obtain the above expressions from the following simple considerations. Suppose that a black hole is surrounded by a large bounding sphere of radius L (much greater than the radius of the black hole). If we fix the quantum numbers l and m, the problem of stationary states with zero boundary conditions on the outer sphere is a one-dimensional problem and therefore in the interval $d\omega$ the number of levels is of order $dn \sim L d\omega/c$. A stationary state with definite energy is a superposition of two waves, one traveling inward and the other outward, in each of which the probability flux has order $j \sim c/L$. The waves traveling inward toward the surface of the black hole are amplified. In the vacuum state, the state with frequency ω has energy $\hbar\omega/2$, and therefore if the gain is $K_{lm}(\omega)$ the contribution to the energy flux from the state with frequency ω has the order $(\hbar\omega/2)(c/L)(K_{lm}(\omega)-1)$. Recalling that the number of states with frequency in the interval $(\omega, \omega$

$+d\omega$) is $dn \sim Ld\omega/c$, we find the contribution to the energy flux from states in the interval $d\omega$ in the form $(\hbar\omega/2)(K_{lm}(\omega)-1)d\omega$. Integration of this quantity with respect to the frequencies for which the gain is greater than unity and summation over the quantum numbers l and m leads (to within a coefficient) to the above expression for the energy flux of the quanta created from the vacuum. In the same way, one can arrive at an expression for the flux of the angular momenta.

Pair creation of particles in the field of a rotating black hole can also be considered in a somewhat different manner, in which the role of the ergosphere is brought out more clearly. For this, we proceed as earlier in our discussion of pair creation in a constant electric field, i.e., we examine whether the conservation laws (in our case, of energy and angular momentum) allow the creation of particle pairs. If the total energy and angular momentum of the system is to remain unchanged by the creation of a pair, the total angular momentum and total energy of the created pair must be zero. This is possible if one of the created particles is in the ergosphere and has negative energy. Processes allowed by the conservation laws usually do take place in quantum theory. These general considerations are confirmed in the present case by concrete calculations which show that in the gravitational field outside the black hole the vacuum is not stable against pair creation processes. One of the created particles, having negative energy, falls into the black hole, decreasing its mass and angular momentum. The other particle of the created pair, having overcome the potential barrier of the gravitational and centrifugal forces, escapes to infinity. Therefore, outside the rotating black hole one observes a constant flux of created particles carrying away the energy and angular momentum of the black hole.

The characteristic frequency (measured by a distant observer) of this radiation is $\omega \sim \Omega$, and the total energy flux, as can be seen from the exact expression we have given, has order $dE/dt \sim \hbar\Omega^2$. The maximal angular momentum of a black hole of mass M is attained for an extremal black hole and is $\Omega_{\max}=c^3/2GM$. Therefore, the created particles have wavelength of the order of or exceeding the radius of the black hole. The characteristic order of the energy of the created particles in a maximally rotating black hole is $\hbar\omega \sim c^3\hbar/GM \sim 100$ (MeV)$\cdot 10^{15}$ g$/M$. The rate of escape of energy from such a black hole is of order $dE/dt \sim \hbar(c^3/GM)^2 \sim 10^{20}$ (erg/sec)$(10^{15}$ g$/M)^2$. These estimates show that for black holes formed by the collapse of stars (whose mass exceeds the Sun's, i.e., is greater than $2\cdot 10^{33}$ g) these quantum effects are extremely small even for rapidly rotating black holes and they are completely absent for black holes without rotation. All the above points have related solely to the creation of massless particles (photons, neutrinos, gravitons); the creation rate of massive particles is appreciably lower.

§8. PARTICLE CREATION IN NONROTATING BLACK HOLES. THE HAWKING EFFECT

The arguments of the previous sections show that pair creation outside a nonrotating black hole is energetically impossible (a particle outside a black hole without rotation always has positive energy), and there are therefore reasons for believing that the vacuum is completely stable in a nonrotating black hole.

A result recently obtained by Hawking[48,49][18] has made it necessary to give up this idea. In his paper, Hawking considered the apparently innocuous question of the number of particles created in a collapse leading to the formation of a nonrotating black hole. It would seem that the answer to this question must be as follows. During the collapse, the gravitational field is variable, and, like every variable field, it will create particles. However, from the point of view of an external observer the collapsing body fairly rapidly freezes near the gravitational radius and the resulting static external field is incapable of creating particles. Therefore, an observer studying the collapse detects a certain finite number of particles created during the collapse that escape outward. The total number of particles depends on the actual characteristics of the collapse and virtually all the created particles are formed during the active stage of the collapse.

Hawking however obtained a completely different result. He found that besides a finite number of particles created by the variability of the field and dependent on the details of the collapse there is also present a stationary flux of created particles after the active stage of the collapse has ended. The spectrum and intensity of this flux are determined solely by the parameters of the stationary black hole eventually formed.[19] Moreover, every nonrotating black hole creates and emits particles such as photons, neutrinos, or gravitons with the rate one would expect if the hole were a black body heated to temperature $\theta=\hbar\varkappa/2\pi ck$ (i.e., to the effective temperature of the black hole).

This rather unexpected result naturally prompts some questions that need answering. Above all, one must understand how this result can be reconciled with the assertion that the gravitational field outside a nonrotating static black hole cannot create pairs. The apparent contradiction between Hawking's result and this assertion can be readily resolved by recalling that in the quantum creation of particles in static fields the created components of the pair may appear in the form of real

[18] Later, Hawking's results were confirmed by several other authors.[55-58]

[19] In studying the processes of quantum pair creation in an electric field, for example, in the field of a capacitor, one usually uses the following device. For a convenient definition of the vacuum state and the concept of a particle one first considers a capacitor without field, and then, increasing the field in some manner or another to the relevant maximal constant value, one finds the total number of created particles. This total number of particles can be divided into two parts. The first depends on the method and rate of switching on of the field, while the second is completely determined by the characteristics of the constant field and is a stationary flux of particles created by the constant static field. This situation is completely analogous to the case considered here in which the gravitational field of the black hole is "switched on" by the collapse.

particles at an appreciable *space-like distance* from each other. One can therefore have a situation in which one of the created particles is below the surface of the black hole while the other appears outside it. Under the surface of the black hole, the particle can have negative total energy (since the Killing vector field is there space-like, i.e., the situation is analogous to the one in the ergosphere). Therefore, the law of conservation of energy permits the creation of such pairs. An exterior observer never sees the particle created below the horizon. The second of the created particles, after it has overcome the potential barrier of the gravitational and centrifugal forces, can escape to infinity. In the same way one can see that if in the Penrose process the decay of a particle that has fallen below the surface of a black hole takes place quantum mechanically, so that one of the particles from the decay is outside the event horizon, this particle outside the horizon can have an energy greater than the incident particle's, i.e., in this quantum form of the Penrose process some of the energy of a nonrotating black hole can be extracted.

The second natural question concerns Hawking's theorem. As a result of quantum processes, the mass of a nonrotating black hole can decrease, and this reduces the surface area of the black hole, contradicting Hawking's theorem. It must however be recalled that the theorem was formulated and proved under the assumption that all particles and fields under consideration are classical. In the quantum case, the assumption that the local energy density is always positive need not be satisfied. [59] [20)]

Although Hawking's classical theorem does not hold in the quantum case, the generalized second law of black-hole physics remains true. Moreover, if black holes did not emit like black bodies with the effective temperature, the generalized second law of black-hole physics would necessarily be violated. This can be seen by considering such a black hole in a gas of radiation at a temperature lower than the effective temperature of the black hole.

Indeed, suppose that the effective temperature of the black hole, θ_1, is greater than the temperature θ_2 of the radiation gas and that there is no radiation from the black hole. Suppose that in unit time the black hole absorbs thermal radiation of energy δE. The entropy of the radiation gas is then decreased by $\delta S_2 = \delta E/\theta_2$, while the effective entropy of the black hole is increased by $\delta\tilde{S}_1 = \delta E/\theta_1$. In this process, the total change in the entropy of the system (of the black hole and the radiation gas outside it) is

$$\delta S = \delta\tilde{S}_1 - \delta S_2 = \delta E\left(\frac{1}{\theta_1} - \frac{1}{\theta_2}\right),$$

and, since $\theta_1 > \theta_2$, this is negative, which contradicts the generalized second law.

The next question to be considered is that of the relationship between Hawking's result and the phenomenon of spontaneous pair creation in the field of a rotating black hole considered in the previous section. As we have already said, the emission of a nonrotating black hole has a thermal spectrum with effective temperature θ, and therefore the intensity of energy emission from unit surface in frequency interval $d\omega$ is given by

$$\frac{dE}{dt\,dS}\left(\begin{array}{l}\text{intensity of emission from}\\ \text{unit surface of nonrotating}\\ \text{black hole in the frequency}\\ \text{interval } d\omega\end{array}\right) = \frac{\hbar\omega^3}{4\pi^2c^2}\left(e^{\hbar\omega/k\theta} \mp 1\right)^{-1} d\omega,$$

where the minus sign must be taken for bosons and the plus sign for the emission of massless fermions. If the black hole rotates with angular velocity Ω, the result for the emission of waves with projection m of the angular momentum onto the axis of rotation is[21)]

$$\frac{dE}{dt\,dS}\left(\begin{array}{l}\text{intensity of emission from}\\ \text{unit surface of rotating}\\ \text{black hole in the frequency}\\ \text{interval } d\omega\end{array}\right) = \frac{\hbar\omega^3}{4\pi^2c^2}\left| e^{\hbar(\omega - m\Omega)/k\theta} \mp 1\right|^{-1} d\omega.$$

If the effective temperature of the black hole is low, the main contribution to the emission comes from the frequencies ω for which the wave amplification condition $\omega < m\Omega$ is satisfied.[22)]

Hawking's result also remains valid for a nonspherical collapse. The only thing changed is the number of particles created by the variability of the field. Black holes emit not only massless particles but also any massive particles, just like a hot black body of temperature θ. However, this emission is extremely weak until the thermal energy $k\theta$ is comparable with the rest energy mc^2 of the particle. The thermal spectrum of the emission of a black hole is a consequence of the fact that the probabilities of emission of the different modes and different numbers of particles in one mode are completely uncorrelated.

Before we discuss the possible consequences of the quantum creation of particles in black holes, let us make some comments. It should be recalled that in all the previous discussion it has been assumed that the gravitational field of the black hole is purely classical and given. It would therefore be quite incorrect to understand the Hawking effect as a consequence of quantum fluctuations of the gravitational field leading to a "spreading" of the horizon permitting some of the energy within the black hole to escape. In principle, a quantum–gravitational effect of this kind is also possible but it is apparently important only if the radius of the black hole does not differ strongly from the Planck length.

[20)] The possibility that Hawking's theorem could be violated in quantum phenomena was first pointed out by Markov. [60]

[21)] It must be borne in mind that if the created particles are to escape to infinity they must overcome the potential barrier of the centrfugal and gravitational forces. If the transmission factor of this barrier for particles with quantum numbers ω, l, and m is denoted by $\Gamma_{lm}(\omega)$, the exact result is obtained by multiplying the right-hand side of the given expression by $\Gamma_{lm}(\omega)$.

[22)] In complete agreement with the thermodynamic analogy, this expression for the emission of a rotating black hole agrees with the expression for the emission of a rotating black body at temperature θ.

The Hawking effect has more similarity to the creation of particles in the field of a deep potential well, for which one of the created particles, with negative energy, falls to one of the levels in the "well" and the other is emitted outward. The potential well of the gravitational field of a black hole is so deep that even light cannot escape from it.

Another remark that should be made is the following: The quantum creation of particles in the case of collapse has been treated under the neglect of the back reaction of the created matter on the gravitational field and on the collapse dynamics. Although this question has not yet been investigated, one can expect[49] that the back reaction will be negligibly small if the black-hole radius is appreciably greater than the Planck length.

§9. PRIMORDIAL BLACK HOLES AND THEIR EVOLUTION

We now turn to quantitative estimates and consider what observable phenomena can result from the above quantum processes in black holes. For simplicity, we consider only nonrotating black holes. Since such a black hole emits as a black body with temperature $\theta = \hbar \varkappa / 2\pi c k = 10^{-6}(M_\odot/M)^\circ$ K ($M_\odot$ is the mass of the Sun, equal to $2 \cdot 10^{33}$ g), in complete agreement with the Stefan–Boltzmann law the intensity of emission from unit surface of the black hole is $dE/dt\, dS = \sigma\theta^4$, where $\sigma = 2\pi^5 k^4/15\hbar^3 c^2$ is the Stefan–Boltzmann constant. The total surface area of the nonrotating black hole is $S = 16\pi G^2 c^{-4} M^2$, and therefore in unit time the black hole loses the energy $dE/dt = (\hbar/15 \cdot 4^5 \pi)\,(c^3/GM)^2$. In this expression, it must also be remembered that some of the emission does not reach the distant observer because of scattering on the static gravitational field and it is absorbed after scattering by the black hole. The transmission coefficient, which determines the fraction of transmitted radiation, depends on the spin and mass of the emitted particles. In addition, the black hole may emit several different species of particle, and therefore the total emission intensity is made up of the intensities of each particle species. A black hole with mass M emits particles whose rest mass does not exceed $k\theta/c^2$. For black holes with mass greater than 10^{17} g, only massless particles can be emitted (photons, neutrinos, and gravitons). As the effective temperature of the black hole increases (and its mass decreases) it becomes possible for various massive particles to be emitted (electrons, mesons, baryons). Therefore, in the general case

$$\frac{dE}{dt}\left(\begin{array}{c}\text{energy emission by}\\ \text{black hole through}\\ \text{quantum processes}\end{array}\right) = 10^{46}\,\frac{\text{erg}}{\text{sec}}\, f(M)\,(M\,(\text{g}))^{-2}.$$

The factor $f(M)$ effectively takes into account the number of different species of emitted particle and the corresponding transmission coefficient for each species. For black holes with mass $M > 10^{17}$ g we have $f(M) \sim 1$, while for $M \sim 10^{14}$ g we have $f(M) \sim 10$ (see [61,62]).

These estimates, obtained from the exact treatment, can also be obtained by dimensional arguments. To determine the wavelength of the massless particles created by a black hole, there is only the single parameter with the dimensions of a length in the problem lem—the radius of the black hole (gravitational radius) R_g.[23)] Taking the wavelength of the radiation to be $\lambda \sim R_g$, we obtain the estimate $\hbar\omega \sim \hbar c/R_g \sim hc^3/GM$ for the characteristic energy of the emitted particles. If we recall that the mean energy of particles in thermal emission is proportional to the temperature of the black body, we arrive at the above expressions for the effective temperature and the emission intensity of the black hole.

For a black hole whose mass is of the order of the Sun, the intensity of the quantum emission is extremely low and is of order $3 \cdot 10^{-20}$ erg/sec. (For comparison we recall that the Sun emits about $4 \cdot 10^{33}$ erg/sec.) The effective temperature of such a black hole is 10^{-6} °K (which is very much lower than the temperature of the black body microwave background). During the whole life of the Universe this black hole would lose only 10^{-17} g. All this indicates that the quantum effects are completely unimportant in the life of massive black holes formed by the collapse of stars. The quantum processes can lead to observable astrophysical phenomena only in the case of black holes with mass appreciably less than the Sun's.

It can be shown[63,64] that although such small black holes do not arise as a result of collapse at the present epoch, they might well have been formed during the early stages in the evolution of the Universe. The very existence of galaxies in our epoch indicates that in the early development of the Universe there must have been appreciable inhomogeneities. The presence of these random perturbations and inhomogeneities suggests that at that time there were regions in which the matter was so strongly compressed that the gravitational forces could overcome the pressure force and the force due to the kinetic energy of the expanding matter. During the evolution of the Universe, the matter in these regions would have collapsed and black holes would have been formed (Fig. 9). These are called *primordial* black holes.[24)] The mass of the primordial black holes can be arbitrary, from 10^{-5} g up to the mass of the Sun or more, depending on the epoch at which they were formed. The earlier such a hole was formed, the smaller is its mass.

This can be readily understood on the basis of the following simple arguments. The matter density at the time when a black hole is formed can be determined by dividing the mass M of the black hole by the volume of the sphere with radius equal to the gravitational radius. The critical density ρ obtained in

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[23)] The Planck length l_{Pl}, which is constructed from fundamental constants, also has the correct dimensions. However, this quantity usually arises only when the gravitational field itself is quantized, and, since such processes are not considered here, the Planck length does not occur in the equations that describe the creation of particles in an external gravitational field.

[24)] In principle other processes could lead to the formation of small black holes; for example, accretion of matter onto a white hole[65] and the quantum explosion of a white hole.[66]

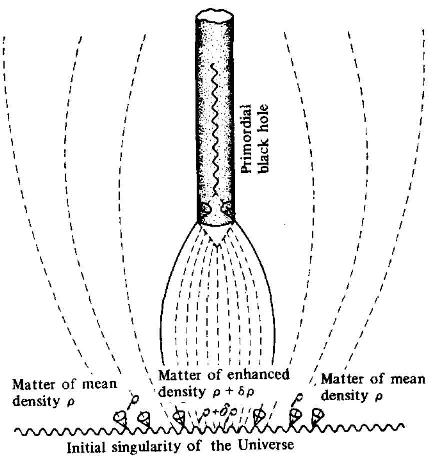

FIG. 9. Formation of a primordial black hole. If there were inhomogeneities of the matter density $\delta\rho/\rho \sim 1$ during the early evolution of the Universe (near the initial singularity), the denser matter (with density $\rho + \delta\rho$) could collapse subsequently. The result is the primordial black hole shown in the figure. (The time axis points upward.)

this way and the mass M are related by $M \sim \sqrt{c^6/\rho G^3} \sim 10 M_\odot \sqrt{\rho/10^{15}(\mathrm{g/cm^3})}$. If it is assumed that the characteristic density fluctuation of the matter in the Universe is $\delta\rho/\rho \sim 1$, it is then clear that at the same time when the matter density in the Universe was ρ there was a tendency for black holes to be formed preferentially whose mass satisfies the above relation between the mass and the density. On the basis of this expression one can conclude, in particular, that the formation of primordial black holes of mass 10^{-5} g was possible only when the matter had the colossal (quantum) density 10^{93} g/cm^3, i.e., when quantum-gravitational effects played the principal role.

Once produced, the primordial black holes will absorb matter from the surrounding space, and gradually increase their mass. However, the increase in the mass of primordial black holes by accretion (falling into the hole) of matter surrounding them cannot in all probability increase their original mass by more than order of magnitude. [67] Therefore, for small black holes of mass less than 10^{15} g their quantum decay is more important. As the emission of the created particles proceeds, the mass of such a small black hole decreases. At the same time, the effective temperature of the black hole and the radiation intensity rise. The last stage in the evolution of the black hole proceeds very rapidly and is essentially an explosion, in which the decay of the remaining black hole, with a mass of order $3 \cdot 10^9$ g, leads to the liberation of 10^{30} erg in the last 0.1 sec. Although this energy is not very great on astrophysical scales, the phenomenon is impressive and unique, since an energy equivalent to the explosion of one million one megaton hydrogen bombs is liberated in a region of space comparable in size with a nucleon.

If it is borne in mind that as the temperature of the black hole increases it becomes possible for heavier and heavier particles to be emitted, it will be seen that in the quantum explosion of a small black hole an amount of energy greater by five orders of magnitude can be liberated in an extremely short (compared with nuclear times) time.

Although the brief final stage in the life of the black hole is rather stormy, up till then the black hole emits energy steadily throughout its life. Using the expression for the intensity of energy emission by the black hole that we have given, we can readily obtain the total lifetime of a black hole:

$$T \left(\begin{smallmatrix}\text{lifetime of}\\ \text{black hole}\end{smallmatrix}\right) \approx 10^{-27}\,(\mathrm{sec})\,(M\,(\mathrm{g}))^3 \approx 10^{10}\,(\mathrm{years})\left(\frac{M}{10^{15}\,(\mathrm{g})}\right)^3.$$

In deriving this relation, we have set $f(M) = 1$, and the formula is therefore valid for black holes with mass greater than 10^{14} g. The lifetime of smaller black holes depends significantly on the mass spectrum of elementary particles. For small black holes, our expression is an upper bound for the lifetime.

This expression shows that primordial black holes with mass greater than 10^{15} g could not have survived to the present epoch, since they would have evaporated completely during the 10^{10} years which have elapsed since the expansion of the Universe began. Black holes with mass greater than 10^{15} g would have remained virtually unaltered. Primordial black holes whose mass was near 10^{15} g must be exploding from time to time at the present epoch. Such black holes were formed in the very early stage of development of the Universe (10^{-23} sec after the start of expansion from the singularity). It is difficult to estimate the mass spectrum and number of primordial black holes since they depend strongly on the detailed structure of the Universe at that time. Unfortunately, we do not yet know these details.

In discussing the possibility of observing quantum explosions of primordial black holes we run into a further difficulty. We have already pointed out that the reduction in the mass and the corresponding increase in the temperature of the black hole means that heavier and ever heavier elementary particles can be emitted. There is therefore an uncertainty in the determination of the time of explosion and the explosion products associated with our ignorance of the spectrum of elementary particles. It is very probable that although the explosion time of a primordial black hole is very short, the strong interaction between the created elementary particles will mean that they do not fly apart immediately but form a hot fireball, which then decays. [61,68] In this case, the quantitative and qualitative composition of the emitted particles can be obtained by thermodynamic methods as in the ordinary hydrodynamic theory of the collision of particles at high energies.

Despite the many uncertainties, one can already give some estimates concerning the number of small black holes in the Universe and the frequency of their explosions. This is because black holes lose the major part of their mass in the form of massless particles (photons, neutrinos, and gravitons). Figure 10 shows the results of calculation of the energy and composition of the emission of nonrotating black holes of different masses. [61,62] On the basis of these results one can assume that approximately 10% of the original mass of

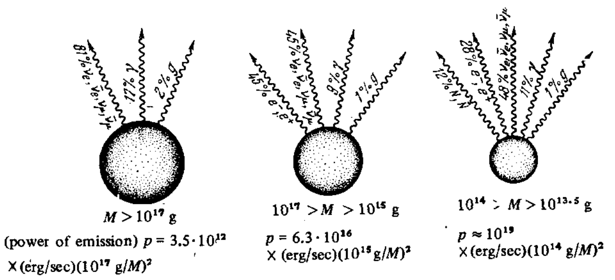

FIG. 10. Quantum evaporation of a nonrotating black hole. The fractions of gravitons (g), photons (γ), neutrinos (ν), and other elementary particles in the total number of particles emitted by black holes of different masses are shown.

a 10^{15}-gram primordial black hole is emitted in the form of photons. The mean characteristic energy of the photons produced by the decay of a 10^{15}-gram black hole is about 100 MeV. Observations show that the total density of photons with this energy in the Universe is of order 10^{-38} g/cm^3, and therefore the mean density of matter in black holes with mass of order 10^{15} g must be less than 10^{-8} of the critical density of matter in the Universe (i.e., the density 10^{-29} g/cm^3). This estimate shows that if the primordial black holes are distributed uniformly over the whole of the Universe, the frequency of their explosions must be less than 10^{-7} per year in one cubic parsec of space. [61]

We have already mentioned the strong dependence of the number of primordial black holes on the degree of inhomogeneity of the Universe during the early stages of expansion. It is therefore extremely important that restrictions on the mean density of primordial black holes of given mass existing now enable us to draw conclusions about the degree of inhomogeneity of the Universe in the very distant past.

It is also worth emphasizing once more that a very important question is whether the estimates made here remain valid when allowance is made for the back reaction of the created particles on the gravitational field of the black hole. Clearly, as long as the emission intensity is low the gravitational field of the black hole can "readjust" and the process can be assumed quasistatic, so one is justified in using the results obtained for a static black hole. However, during the last stage in the life of a small black hole the emission becomes so strong that further investigations are needed for the correct treatment of this stage in the evolution of small black holes. At present it is not at all clear whether a small black hole evaporates completely or whether this process stops at a certain stage. A very interesting possibility associated with the decay of small black holes is that the quantum explosion of a black hole could lead to the formation from it of a microscopic black hole with the Planck mass 10^{-5} g (a maximon[69]). If such elementary black holes are stable, then at the present epoch there could be a considerable number of them in the Universe, both as a result of their formation at the earliest epochs and as a result of the evaporation of small black holes.

Several fundamental questions relate to this phenomenon of evaporation of small black holes. The most interesting is the question of the conservation of baryon charge. In the collapse process, matter consisting of baryons falls below the horizon. After the formation of the black hole, an exterior observer can no longer "count" the number of baryons within the black hole. Moreover, outside the horizon there is no massive vector field whose source is baryons (a black hole has no "hair"). Therefore, baryons inside a black hole are inaccessible to an observer who studies the black hole from the outside. But there is still the possibility of verifying whether the baryons that fell into the black hole are still there. The observer himself must fall into black hole and see if they are.

The situation has been radically changed by the Hawking effect; for after a black hole has been formed it begins to evaporate as a result of the quantum pair creation. The black emits equal numbers of particles and antiparticles. If a small black hole evaporates completely as a result of this process, we are faced with a violation of the law of conservation of the baryon charge, since the baryons that formed the black hole have completely disappeared. Even if one assumes that a black hole emits preferentially baryons and not antibaryons when it evaporates, [25] we cannot achieve fulfillment of the law of conservation of baryon charge. The point is that a primordial black hole formed by the collapse of 10^{15} g of baryons loses almost 90% of its mass during evaporation by the emission of massless particles and leptons before its temperature increases sufficiently for it to begin to emit baryons. If small black holes do not evaporate completely but lead to the formation of a naked singularity or objects with mass 10^{-5} g (maximons), then to avoid violation of the baryon conservation law we are forced to assume that these remnants of the small black hole have the necessary baryon charge. But it still remains a rather difficult question how one could in principle measure this charge. Unfortunately, at the present time we do not know the answer to this question.

The evaporation of small black holes may strongly influence our ideas about the early stages in the development of the Universe. The possibility cannot be excluded that an appreciable fraction of the matter in the early stages was in the form of small primordial black holes, which then evaporated completely. If *CP* invariance was broken by this evaporation, there is a possibility of explaining the charge asymmetry of the Universe even it was originally charge symmetric.

Small primordial black holes (if they exist) are a remarkable phenomenon in nature, in which microscopic and macroscopic scales are combined. They have a macroscopic mass (10^{15} g) but a microscopic size (10^{-13} cm). The properties of these macroscopic systems depend strongly on the structure of spacetime in the small. Ginzburg[70] has pointed out the interesting fact that study of the evaporation of primordial black

[25] For example, because of the existence of some long-range field that interacts with baryons, as has been suggested by Wheeler, or by breaking of CP invariance.

holes may give information about the fundamental length, if a fundamental length does exist in nature. The existence of a fundamental length could radically affect the behavior of black holes during the later stages of their evolution and, in particular, the emission spectrum of the holes at high energies, and even the very possibility of their formation.

To conclude this review, I should like to emphasize that I have discussed only some of the questions related to black-hole physics. Many questions, for example, those dealing with the cosmological consequences of the evaporation of primordial black holes, the possibility of observing quantum explosions of black holes, and so forth, are only now being investigated. In this connection, I should like to recall that black-hole physics is still comparatively young. The classical physics of black holes has an age of hardly more than ten years. The quantum physics of black holes "saw the light of day" barely two or three years ago. The stormy development of black-hole physics and the intensive theoretical and astrophysical investigations currently being carried out suggest that we can expect new and perhaps completely unexpected discoveries in this interesting branch of physics.

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