Possible lines of research into weak-interaction effects in atomic physics

A. N. Moskalev, R. M. Ryndin, and I. B. Khriplovich

B. P. Konstantinov Institute of Nuclear Physics, USSR Academy of Sciences, Leningrad

Usp. Fiz. Nauk **118**, 409–451 (March 1976)

This review is concerned with physical phenomena which have not as yet been observed, namely, effects associated with the nonconservation of parity in atomic transitions. The observation of some of these effects at the current level of optical research appears to be within the range of realistic expectations. Experiments of this kind should throw light on the weak interaction between electrons (or muons) and nucleons, due to the so-called neutral currents, which has not as yet been observed. The general form of this interaction, which does not conserve parity but is CP-invariant, is suggested. Effects associated with the nonconservation of parity in hydrogen and hydrogen-like atoms, heavy atoms, and μ-mesic atoms are considered. Possible experiments designed to verify the existence of these effects are discussed (measurement of the circular polarization of radiation emitted by unpolarized atoms, rotation of the plane of polarization in optically inactive media, and so on).

PACS numbers: 32.10.–f

CONTENTS

1. INTRODUCTION

In this review, we shall be concerned with physical phenomena which have not as yet been observed experimentally, namely, the effects associated with the nonconservation of parity in atomic transitions. At the present level of optical research, the observation of at least some of these effects is within the reach of reasonable expectations. Such experiments are exceedingly interesting if only because they may lead to the discovery of an interaction between elementary particles which has not as yet been observed, namely, the weak interaction between electrons and protons or neutrons, which is due to the so-called neutral currents.

A. What are neutral currents?

One of the central problems in contemporary physics of weak interactions is the question of the existence of neutral currents. Let us illustrate the situation with some simple examples. Consider the usual β decay of the neutron

$$n \to p + e^- + \bar{\nu}_e$$

which is a well-known weak interaction process, and the closely associated reaction

$$p + e^- \to n + \nu_e,$$
$$\bar{\nu}_e + p \to n + e^+.$$

All these processes (Fig. 1) are accompanied by a change in the electric charge of the strongly-interacting particle (the nucleon) and a transfer of this charge to the electron or positron. Both processes and, generally, all weak-interaction processes which involve the transfer of charge from hadrons (strongly-interacting particles) to leptons (e, ν_e, μ, and ν_μ) are said to be due to the weak interaction of charged currents. The interaction of charged currents can also give rise to purely leptonic processes, for example, the muon decay

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu.$$

Moreover, known conservation laws do not forbid processes that are not accompanied by the transfer of charge from hadrons to leptons, for example, elastic scattering of a neutrino by an electron, or a muon by a nucleon, due to the weak interaction. Processes of this kind are usually said to be due to neutral weak currents. Neutral currents should also give rise to purely leptonic processes such as, for example, the elastic scattering of an electron by an electron, or a muon neutrino by an electron. (The concept of charged and neutral currents will be discussed in greater detail in the second chapter of the present review.)

The existence of neutral currents has frequently been discussed in the past.[1–8] They attracted particular attention after the emergence of renormalizable models which provided a unified description of electromagnetic

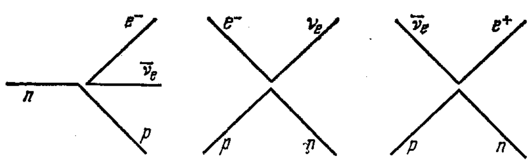


FIG. 1.

and weak interactions (see, for example, the review in[9]). In the most popular of these models, the neutral weak currents appear in a natural fashion.

The first experimental evidence for the existence of neutral weak currents was obtained in 1973 at CERN and Batavia.[10-11] These experiments resulted in the observation of the scattering of muon neutrinos and antineutrinos by nucleons and electrons, apparently due to weak interactions of neutral currents. The probabilities of these processes were found to be comparable with the probability of processes due to charged currents. However, no experimental data are available as yet on the weak interaction between electrons and muons, and between electrons and nucleons, due to neutral currents. The point is that it is exceptionally difficult to separate the contribution of the weak interaction from the background of the much stronger electromagnetic interaction. One way of obviating this difficulty is to work with high energies because, in contrast to the electromagnetic interaction, weak interactions are found to increase with increasing energy (see the review in[12]), and there is an accompanying increase in their relative contribution to the scattering cross section.

On the other hand, attempts to detect effects due to neutral currents in atomic physics, where the precision of measurements is high, are by no means hopeless.[1)] We shall consider these problems in the present review. We note that Ya. B. Zel'dovich discussed some effects associated with the nonconservation of parity in atomic physics in connection with the existence of neutral currents as far back as 1959.[2]

B. Possible manifestations of neutral currents in atomic physics

We shall now consider, in a purely qualitative fashion, the effects which may result from the existence of a weak interaction between electrons and nucleons in atoms. As in the case of the usual weak interaction, we shall suppose that this particular interaction has a very short range, i.e., in practice, it occurs only when the electron and nucleon are located at the same point. For the purpose of very approximate estimates, we shall suppose that the eN weak-interaction constant is of the same order as the Fermi β-decay constant

$$G = 10^{-5} \frac{1}{m_p^2}, \tag{1.1}$$

where m_p is the proton mass.

To establish the scale of these phenomena, let us estimate the energy-level shift δE due to the weak interaction in the case of the hydrogen atom. Since the interaction has a short range, this shift should be proportional to the probability density for finding the electron near the proton, i.e., $|\psi(0)|^2 = 1/\pi a^3$, where a is the Bohr radius. Thus,

$$\delta E \sim G \frac{\hbar^3}{\pi a^3 c} \sim 10^{-4} \text{ MHz} \tag{1.2}$$

The level shift due to the weak interaction will, in general, depend on the angular momentum of the atom. Hyperfine splitting of the ground state of the hydrogen atom will therefore change as a result of the weak interaction by an amount of the same order, i.e., 10^{-4} MHz. Hyperfine splitting in the hydrogen atom can be measured to within $\pm 1.7 \times 10^{-9}$ MHz[16] which, in itself, is more than sufficient for the detection of the weak-interaction effect. However, in the present case, this precision is of little use. The point is not only the uncertainty of the theoretical calculations of the hyperfine splitting ($\sim 10^{-2}$ MHz), resulting from proton polarizability and proton structure due to the strong interactions (see, for example, [17]). Even the magnetic moment of the proton, which is present as a factor in the expression for the hyperfine splitting, has been measured to only six significant figures,[18] which corresponds to an uncertainty of about 10^{-3} MHz in the interpretation of the hyperfine splitting in hydrogen.

The use of a hydrogen-like system such as muonium (μe) would facilitate the interpretation of experimental data because the muon does not exhibit strong interactions, and its magnetic moment has been measured with a relative precision of about 3×10^{-6}.[19] However, the precision of hyperfine-splitting measurements in the case of muonium is not, as yet very high[20] ($\sim 10^{-2}$ MHz). The accuracy is lower still in the case of positronium.[21]

This is a pretty general situation. Even when the spectroscopic accuracy is, at least in principle, sufficient for the detection of the weak interactions, this cannot, in fact, be done because of uncertainties in the interpretation of the results of measurement.

We must therefore resort to effects the very observation of which would indicate the presence of the weak interaction. The characteristic feature of weak interactions, which is exclusive to them, is the nonconservation of spatial parity. In our view, the observation of nonconservation of parity in atomic physics would place at our disposal an important tool for investigating weak interactions. We shall discuss effects of this kind in the present review.

They include, for example, the circular polarization of the radiation emitted by unpolarized atoms. Essentially, the point is that the radiation intensity I depends on the component of the photon spin $\mathbf{s}_\gamma$ along the direction $\mathbf{n}$ of motion [$\mathbf{s}_\gamma = -i\mathbf{e}^* \times \mathbf{e}$, where $\mathbf{e}$ is the photon polarization vector]. This correlation is characterized by the product $\mathbf{s}_\gamma \cdot \mathbf{n}$, which is a pseudoscalar that changes sign under inversion of the coordinate frame, i.e., $\mathbf{n} \to -\mathbf{n}$, $\mathbf{e} \to -\mathbf{e}$. It follows that a relationship of the form $I = I_0 + I_1(\mathbf{s}_\gamma \cdot \mathbf{n})$, i.e., $I_\pm = I_0 \pm I_1$, where $\pm$ refers to the two directions of circular polarization, would be an indication of the nonconservation of parity.

Let us now consider in detail the mechanism whereby pseudoscalar correlations may arise in atomic transi-

[1)]It is important to note that, some years ago, the methods of atomic spectroscopy yielded a result of interest for weak-interaction physics. Thus, measurements of the dipole moments of cesium,[13] xenon,[14] and thallium[15] show that the electric dipole moment d of the electron is subject to the restriction $d/e < 2\times 10^{-24}$ cm. However, this particular set of problems will not be discussed in the present review.

tions. The usual state of atoms is characterized not only by definite values of the total angular momentum and its component, but also by definite parity. When weak interaction is included, parity is no longer an exact quantum number, and the mixing of levels takes place, i.e., the stationary states acquire small admixtures of states with the same angular momentum but opposite parity. At the same time, if only one of the levels participating in an electromagnetic transition does not have definite parity, the probability of emission of right and left polarized photons (they transform one into the other under space reflection) will, in general, be different. This will mean the presence of circularly polarized emission.

The size of the admixture to the station of opposite parity is measured by $\langle V\rangle/\Delta E$, where $\langle V\rangle$ is the matrix element of the parity nonconserving potential (see Chap. 2) evaluated between the mixing states and ΔE is the energy difference for these states. The matrix element $\langle V\rangle$ contains an additional, as compared with δE [see (1.2)], small factor $\sim\alpha$ ($\alpha=1/137$ is the fine structure constant) which is connected with the fact that the parity nonconserving interaction contains the pseudoscalar factor $\mathbf{s}\cdot\mathbf{p}/mc\sim v/c\sim\alpha$, where $\mathbf{s}$, $\mathbf{p}$, and m are the spin, momentum, and mass of the electron, respectively. The mixing factor for states separated by the normal energy interval $\Delta E\sim E\sim\alpha^2mc^2$ is, therefore

$$F=\left|\frac{\langle V\rangle}{\Delta E}\right|\sim Gm^2\frac{\alpha^2}{\pi}\sim 10^{-16}. \qquad (1.3)$$

This is, of course, a fantastically small quantity. However, there are mechanisms in the atom which can enhance effects associated with the nonconservation of parity by many orders of magnitude.

C. Mechanisms for the enhancement of the effects of nonconservation of parity in atoms

The energy interval between the mixing states of opposite parity may turn out to be much less than the characteristic energy of an electron in the atom. From this point of view, a unique situation occurs in the hydrogen atom, [2,8,22] where the energies of the $nS_{1/2}$ and $nP_{1/2}$ levels with the same principal quantum number n differ from one another only by the Lamb shift. For the $2S_{1/2}$ and $2P_{1/2}$ levels, this splitting amounts to about 10^{-6} of the characteristic energy of the atom, so that the mixing factor is approximately 10^{-10}. The mixing of these levels ensures that the amplitude for the single-photon $M1$ transition $2S_{1/2}\to 1S_{1/2}$ will contain an admixture due to the amplitude of the $E1$ transition $2P_{1/2}\to 1S_{1/2}$. Moreover, the amplitudes $A_\pm$ turn out to be different: $A_\pm=A(M1)\pm FA(E1)$. The result of this is that the emitted radiation is circularly polarized, and the degree of this polarization is

$$P=\frac{I_+-I_-}{I_++I_-}=\frac{|A_+|^2-|A_-|^2}{|A_+|^2+|A_-|^2}\approx 2F\frac{A(E1)}{A(M1)}. \qquad (1.4)$$

In the present case, the degree of circular polarization may exceed 10^{-4}. The reason for this substantial additional enhancement of the effect lies in the high degree of forbiddenness of the single-photon $M1$ transition $2S_{1/2}\to 1S_{1/2}$, the matrix element for which is $\sim\alpha^2\mu$ (μ is the Bohr magneton), so that $A(E1)/A(M1)\sim ea/\alpha^2\mu\sim\alpha^{-3}$.

Unfortunately, there are considerable experimental difficulties in observing this effect in hydrogen. Thus, on the one hand, the above strong suppression of the $2S_{1/2}\to 1S_{1/2}$ transition seriously impedes observation of the corresponding spontaneous emission. On the other hand, the extreme proximity of the allowed $2P_{1/2}\to 1S_{1/2}$ transition (the frequency difference between these transitions, i.e., the Lamb splitting, exceeds the natural width of the $2P_{1/2}$ level by a factor of only 10) means that stimulated single-photon excitation of the $2S_{1/2}$ state, which may also exhibit parity violation, is not a very realistic prospect.

Effects associated with the nonconservation of parity in the hydrogen atom will be discussed in greater detail in Chap. 3.

Similar mechanisms, producing enhancement of parity nonconservation effects, occur in two-electron ions. [23] These problems are discussed in Chap. 4.

Another mechanism for the enhancement of parity-violation effects exists in heavy atoms. The modulus of the wave function for a valence electron near the nucleus increases with atomic number Z, and this leads to an increase in the mixing of levels with opposite parities due to the weak interaction between the electron and the nucleons. This was first noted by Bouchiat[24] and means that parity nonconservation effects in heavy atoms are enhanced so much that their experimental detection is a more or less realistic prospect.

We must now estimate, at least qualitatively, the dependence of level mixing for outer electrons in heavy atoms as a function of Z.

In the region $r\sim a$ (a is the Bohr radius), where the nucleus is screened by the other electrons, the potential energy of an outer electron is $U\sim -e^2/a$. Since the electron spends most of its time in this region, the total energy is $E\sim -e^2/a$.

On the other hand, when $r\ll aZ^{-1/3}$, the screening of the nucleus is negligible, and the potential energy is $U(r)\approx -Ze^2/r$ and its modulus is much greater than the modulus of the total energy. Finally, for $r\ll aZ^{-1}$, the wave function ψ of the outer electron is quasiclassical so that, for $aZ^{-1}\ll r\ll aZ^{-1/3}$, it is approximately given by

$$\psi(r)\sim\frac{1}{r\sqrt{p(r)}}\sim\frac{1}{r\sqrt[4]{U(r)}}. \qquad (1.5)$$

The coefficient in this expression is independent of Z because, when $r\sim a$, the wave function must become identical with the quasiclassical solution in the outer region, and does not contain Z explicitly. Using (1.5), we find that, when $r\sim aZ^{-1}$, we have[2)]

$$\psi(aZ^{-1})\sim Z^{1/2}. \qquad (1.6)$$

Since for $r\ll aZ^{-1/3}$, the electron moves in the field of the unscreened nucleus of charge Z, its wave func-

[2)]These ideas are borrowed from the book by Landau and Lifshitz. [25]

tion in this region differs from the hydrogen-type function only by the normalizing factor, and its argument is rZ/a. Hence, since for $r \to 0$ the s-electron wave function ψ_s tends to a constant, and the p-electron function is $\psi_p \sim r$, we have, using (1.6)

$$\psi_s|_{r\to 0} \sim Z^{1/2}, \quad \psi_p|_{r\to 0} \sim Z^{3/2}\frac{r}{a}. \tag{1.7}$$

This increase of the wave function for the valence electron with increasing Z near the nucleus is a well-established experimental fact. Thus, heavy elements exhibit large hyperfine splitting and isotopic level shifts, which are determined by the behavior of the wave function in this region.

Using (1.7), we obtain the following estimate for the matrix element which determines the degree of mixing:

$$\langle \delta(\mathbf{r})\,\sigma\mathbf{p}\rangle \sim |\psi_s \nabla \psi_p|_{r\to 0} \sim Z^2. \tag{1.8}$$

If we consider parity nonconservation effects which are not connected with the spin of the nucleon, we find that all the nucleons in the nucleus contribute to these effects. This results in an additional increase, proportional to Z, in the mixing factor. Finally, one other enhancing factor which increases rapidly with Z, and amounts to about 10 in the case of lead, appears because the motion of the electrons near the nucleus in a heavy atom is relativistic.

Thus, in heavy atoms, the parity nonconservation effects increase with Z more rapidly than Z^3 if all the nucleons in the nucleus contribute to them, i.e., if they are not connected with the nucleon spin. In particular, in the case of cesium ($Z=55$) and the highly forbidden $M1$ transition $6s_{1/2} \to 7s_{1/2}$, the amplitude for which is $\sim 10^{-4}$ (see Chap. 5 for details), one would expect a degree of circular polarization of the order of

$$\frac{Gm^2\alpha^2}{\pi} Z^3 \frac{ea}{10^{-4}\mu} \sim 10^{-4}. \tag{1.9}$$

In normal $M1$ transitions in thallium, lead, and bismuth ($Z \sim 80$, relativistic enhancement factor ~ 10), we have

$$\frac{Gm^2\alpha^2}{\pi} Z^3 \cdot 10 \frac{ea}{\mu} \sim 10^{-7}. \tag{1.10}$$

Parity nonconservation effects can be observed in heavy atoms during the excitation of highly forbidden $M1$ transitions by photons of different circular polarization.[24] Moreover, the detection of parity nonconservation through the rotation of the plane of polarization in metal vapor[26,27] is fairly realistic. It is quite possible that experiments with heavy atoms will result in the course of the next few years in the discovery of a weak interaction between electrons and nucleons. These problems are discussed in Chap. 5.

There is considerable interest in the detection of the weak interaction between muons and nucleons. This could be found through the effect of parity nonconservation in μ-mesic atoms, the main properties of which are similar to those of hydrogen. The muon wave function is large near the nucleus because the muon mass is large. Parity violation effects in the radiation emitted by μ-mesic atoms should therefore also be relatively large. In some cases, they may amount to a few percent.[28-31] Experimental searches for such effects would appear to be very promising. These questions will be discussed further in Chap. 6.

2. GENERAL FORM OF THE WEAK INTERACTION BETWEEN ELECTRONS AND NUCLEONS

A. Fermi theory of β decay

We begin by recalling the fundamentals of the theory of weak interactions describing processes such as, for example, the β decay of the neutron

$$n \to p + e^- + \bar{\nu}_e$$

or the decay of the muon

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu.$$

The first variant of β-decay theory was put forward by Fermi as far back as 1934.[32] Fermi based his theory on an analogy with electrodynamics.

The interaction between a proton at a space-time point x and the electromagnetic field described by the four-dimensional vector potential $A_\mu(x) = (\varphi, \mathbf{A})$ is known to be of the form:

$$V_{em}(x) = ej_\mu(x) A_\mu(x). \tag{2.1}$$

In this expression, e is the charge of the proton, the four-dimensional current vector $j_\mu = (\rho, \mathbf{j})$ is given by:

$$j_\mu(x) = \bar{p}(x)\gamma_\mu p(x), \tag{2.2}$$

where $p(x)$ is the proton wave function, and γ_μ are the Dirac matrices (we used the metric and the notation adopted in[33]).

This interaction can be given the following clear interpretation: the initial proton radiates (or absorbs) a photon at the point x and becomes the final proton (Fig. 2).

In contrast to the electromagnetic interaction, the β decay of the neutron involves the participation of four particles. Assuming that the interaction $\mathscr{H}_\beta$ describing the β decay is local (as is the electromagnetic interaction), i.e., it occurs only when all four particles are at the same point, Fermi postulated that

$$\mathscr{H}_\beta = \frac{G\hbar^3}{c\sqrt{2}} [\bar{p}(x)\gamma_\mu n(x)][\bar{e}(x)\gamma_\mu \nu_e(x)] + \text{H.c.}, \tag{2.3}$$

where $p(x)$, $n(x)$, $e(x)$, and $\nu_e(x)$ are the wave functions for the corresponding particles and G is a constant characterizing the strength of the interaction.

This expression is the scalar product of two four-dimensional vectors. The first of them consists of the nucleon wave functions and is analogous to the proton electromagnetic current (2.2). It differs from the latter only by the replacement of the initial proton by the neutron, and a corresponding change in the nucleon

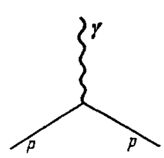


FIG. 2.

charge in the course of the β decay. In this sense, the quantity $j_\mu^N(x) = \bar{p}(x)\gamma_\mu n(x)$ can be referred to as the charged vector current. Instead of the potential $A_\mu(x)$, the interaction given by (2.3) contains the charged lepton current $j_\mu^l = \bar{e}(x)\gamma_\mu \nu_e(x)$, so that the interaction is the product of the nucleon and lepton charged currents. The interaction given by (2.3) is essentially that illustrated in Fig. 1.

Soon after Fermi postulated the interaction given by (2.3), it was noted that it was not the most general one possible. There are four other types of interaction which are invariant under the Lorentz transformation and space inversion (parity nonconservation was not known at the time), namely, the product of two scalars $\bar{p}n$ and $\bar{e}\nu_e$, pseudoscalars $\bar{p}\gamma_5 n$ and $\bar{e}\gamma_5\nu_e$ $(\gamma_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3)$, axial vectors $\bar{p}\gamma_\mu\gamma_5 n$ and $\bar{e}\gamma_\mu\gamma_5\nu_e$, and antisymmetric tensors $\bar{p}\sigma_{\mu\nu}n$ and $\bar{e}\sigma_{\mu\nu}\nu_e$ $[\sigma_{\mu\nu} = (\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2]$. The most general interaction (without using the derivatives of wave functions) would be a linear combination of all five variants.

However, Fermi was very close to the truth. After the discovery of parity nonconservation in weak-interaction processes, it was established that the β-decay interaction did, in fact, have the form

$$\mathscr{H}_\beta = \frac{G\hbar^3}{c\sqrt{2}}\bar{p}(x)\gamma_\mu(1+\gamma_5)n(x)\bar{e}(x)\gamma_\mu(1+\gamma_5)\nu_e(x) + \text{H.c.}, \tag{2.4}$$

which differs from (2.3) by the replacement of the charged vector current (nucleon and lepton) with superpositions of vector and axial vector currents. The interaction given by (2.4) is known as the V–A variant.[3] As already noted, the constant G is equal to $10^{-5}/m_p^2$. As in (2.3), the interaction given by (2.4) is the product of the nucleon and electron charged currents.

The interactions describing other weak processes, for example, muon decay or muon capture by the nucleus, can also be written as a product of two charged currents.

B. Weak interaction of neutral currents

We shall now discuss neutral currents by considering the example of the weak interaction responsible for electron-proton scattering. As before, we shall assume that the interaction is local, i.e., that the particles participating in the scattering process interact only when they are located at the same space-time point x. We shall also assume that the interaction is not necessarily a scalar product of the lepton and hadron vector and axial vector currents but can contain other variants, for example, the product of the electron scalar $\bar{e}(x)e(x)$ by the proton scalar $\bar{p}(x)p(x)$.

Finally, we assume that the weak interaction between neutral currents is similar to the ordinary interaction in the sense that it is invariant under the combined inversion of the CP transformation, which consists of the simultaneous application of charge conjugation C, i.e., the replacement of particles by antiparticles, and space inversion P.

The most general expression for the weak ep interaction $[p(x), e(x)]$ which satisfies all the above requirements and does not contain derivatives of wave functions can be written in the form

$$\begin{aligned}\mathscr{H}_n(x) = {} & G_S\bar{p}(x)p(x)\bar{e}(x)e(x) + G_P\bar{p}\gamma_5 p\bar{e}\gamma_5 e \\ & + G_T\bar{p}\sigma_{\mu\nu}p\bar{e}\sigma_{\mu\nu}e + G_V\bar{p}\gamma_\mu p\bar{e}\gamma_\mu e \\ & + G_A\bar{p}\gamma_\mu\gamma_5 p\bar{e}\gamma_\mu\gamma_5 e + G_V'\bar{p}\gamma_\mu p\bar{e}\gamma_\mu\gamma_5 e + G_A'\bar{p}\gamma_\mu\gamma_5 p\bar{e}\gamma_\mu e.\end{aligned} \tag{2.5}$$

The constants G_i and G_i' (the subscript i identifies the type of interaction, i.e., $i = S, P, V, A, T$) in this expression must be real. This follows from the requirement that the interaction $\mathscr{H}_n(x) = \mathscr{H}_n^+(x)$ must be Hermitian.[4]

In (2.5) only the last two terms are pseudoscalars, so that the parity conserving interaction is characterized by the two constants G_V' and G_A', and is equal to the sum of scalar products of the vector and axial proton and electron currents:

$$\begin{aligned}\mathscr{H}_n^{pv}(x) = {} & G_V'\bar{p}(x)\gamma_\mu p(x)\bar{e}(x)\gamma_\mu\gamma_5 e(x) \\ & + G_A'\bar{p}(x)\gamma_\mu\gamma_5 p(x)\bar{e}(x)\gamma_\mu e(x).\end{aligned} \tag{2.6}$$

The absence of other parity nonconserving variants of the interaction is connected with the assumption of CP-invariance of $\mathscr{H}_n(x)$. Consider, for example, the pseudoscalar $i\bar{p}\gamma_5 p\bar{e}e$. The scalar $\bar{e}e$ does not change under space inversion (P-even) and charge conjugation (C-even)[5] and, consequently, is CP-even. The pseudoscalar $i\bar{p}\gamma_5 p\bar{e}e$ changes sign under inversion, but does not change under charge conjugation, i.e., it is CP-odd. Consequently, the above expression is CP-odd as a whole and should, therefore, be absent from the interaction. Similarly, we can exclude all the other P-odd variants of the interaction other than those shown in (2.6).

C. Nonrelativistic potential for the P-odd electron-proton interaction

We shall now start with (2.6) and derive a nonrelativistic expression for the parity nonconserving potential for the electron-proton interaction.

In the approximation of an infinitely heavy proton, when the latter retains only its spin degrees of freedom, the expressions for the components of the vector and axial vector proton current have the form

$$\begin{aligned} & \bar{p}(x)\gamma_0 p(x) = \varphi_f^+\varphi_i\delta(\mathbf{r}), \quad \bar{p}(x)\boldsymbol{\gamma}p(x) = 0, \\ & \bar{p}(x)\gamma_0\gamma_5 p(x) = 0, \quad \bar{p}(x)\boldsymbol{\gamma}\gamma_5 p(x) = -\varphi_f^+\boldsymbol{\sigma}_p\varphi_i\delta(\mathbf{r});\end{aligned} \tag{2.7}$$

where φ_f and φ_i are two-component spin functions for the final and initial proton and $\boldsymbol{\sigma}_p$ are the proton Pauli matrices.

The components of the vector and axial vector elec-

[3]More precisely, in the nucleon current (2.4), we should replace $1+\gamma_5$ by $1+\lambda\gamma_5$. The appearance of the factor $\lambda = 1.25$ in front of γ_5 is due to the renormalization of the axial-vector constant by the strong interactions.

[4]For example, the term $G_S\bar{p}p\bar{e}e$ becomes $G_S^*[\bar{p}p\bar{e}e]^* = G_S^*\bar{p}p\bar{e}e$ when the interaction is Hermitian, so that it follows that $G_S = G_S^*$.

[5]The transformation properties of Dirac covariants under charge conjugation are described in detail, for example, in[33].

tron current in the first approximation in its momentum are as follows:

$$\left.\begin{aligned}
&\bar{e}(x)\gamma_0 e(x) = \psi_f^\dagger(\mathbf{r})\psi_i(\mathbf{r}),\\
&\bar{e}(x)\gamma e(x) = -\frac{i\hbar}{2mc}\psi_f^\dagger(\mathbf{r})\{\nabla_i-\nabla_f+i[(\nabla_i+\nabla_f)\sigma]\}\ \psi_i(\mathbf{r}),\\
&\bar{e}(x)\gamma_0\gamma_5 e(x) = \frac{i\hbar}{2mc}\psi_f^\dagger(\mathbf{r})\sigma(\nabla_i-\nabla_f)\psi_i(\mathbf{r}),\\
&\bar{e}(x)\gamma\gamma_5 e(x) = -\psi_f^\dagger(\mathbf{r})\sigma\psi_i(\mathbf{r});
\end{aligned}\right\} \quad (2.8)$$

where $\psi_f(\mathbf{r})$ and $\psi_i(\mathbf{r})$ are the two-component wave functions of the electron, m is its mass, and $\boldsymbol{\sigma}$ are the electron Pauli matrices. The operators ∇_i and ∇_f act on ψ_i and ψ_f, respectively.

Using (2.7) and (2.8), we can readily show that the expression given by (2.6) in the first order in the electron momentum can be reduced to the matrix elements of the following interaction potential.

$$V(\mathbf{r}, \sigma, \sigma_p) = \frac{G\hbar^3}{2\sqrt{2}\,mc^2}\{\varkappa_1[\sigma\mathbf{p}\delta(\mathbf{r})+\delta(\mathbf{r})\sigma\mathbf{p}] + \varkappa_2[(\sigma_p\mathbf{p}\delta(\mathbf{r})+\delta(\mathbf{r})\sigma_p\mathbf{p}) - i[\sigma\times\sigma_p](\mathbf{p}\delta(\mathbf{r})-\delta(\mathbf{r})\mathbf{p})]\}, \quad (2.9)$$

where $\mathbf{p} = -i\hbar\nabla$ is the momentum operator and the dimensionless constants $\varkappa_1$ and $\varkappa_2$ are related to G'_V and G'_A as follows: $G'_V = -(G\hbar^3/c\sqrt{2})\varkappa_1$, $G'_A = (G\hbar^3/c\sqrt{2})\varkappa_2$.

The characteristic feature of (2.9) is the presence of the function $\delta(\mathbf{r})$. The potential is nonzero only when the coordinates of the electron and proton coincide, in accordance with the original assumption that the four-fermion interaction was local.

Another characteristic feature of (2.9) is that this expression depends only on the two constants $\varkappa_{1,2}$ although it contains (in the linear approximation in the electron velocity $\mathbf{p}/m$) the three independent CP-invariant spin structures

$$\sigma\mathbf{p}, \quad \sigma_p\mathbf{p} \quad \text{and} \quad i[\sigma\cdot\sigma_p]\cdot\mathbf{p}.$$

The more general form of the CP-invariant Hermitian potential in the linear approximation in $\mathbf{p}/m$ contains three independent real constants:

$$V(\mathbf{r}, \sigma, \sigma_p) = \frac{G\hbar^3}{2\sqrt{2}\,mc^2}\{\varkappa_1\sigma(\mathbf{p}\delta(\mathbf{r})+\delta(\mathbf{r})\mathbf{p}) + \varkappa_2\sigma_p(\mathbf{p}\delta(\mathbf{r})+\delta(\mathbf{r})\mathbf{p}) + i\varkappa_3[\sigma\times\sigma_p](\mathbf{p}\delta(\mathbf{r})-\delta(\mathbf{r})\mathbf{p})\}. \quad (2.10)$$

The presence of only two independent constants in the potential given by (2.9) is due to our assumption that the interaction $\mathscr{H}_n(x)$ is made up only of the wave functions and does not depend on their derivatives.

It is readily shown that the only possible ep interaction with parity conserving but CP-invariant derivatives is

$$\frac{i\hbar}{mc}[G''_V(\bar{p}\sigma_{\mu\nu}\partial_\nu p+\partial_\nu\bar{p}\sigma_{\mu\nu}p)\bar{e}\gamma_\mu\gamma_5 e + G''_A\bar{p}\gamma_\mu\gamma_5 p(\bar{e}\sigma_{\mu\nu}\partial_\nu e+\partial_\nu\bar{e}\sigma_{\mu\nu}e)], \quad (2.11)$$

where the factor $\hbar/mc$ is introduced to ensure that the new constants $G''_{V,A}$ have the same dimensions as the constants G_i, G'_i in (2.5) and (2.6). In the limit of an infinitely heavy proton, the first term in (2.11) is obviously zero and the second leads to the appearance of a third independent constant in the potential (2.10). The addition to (2.6) of terms of the form (2.11) corresponds to the inclusion of terms of the form of an anomalous magnetic moment in the vector proton and electron currents.

The parity nonconserving potential (2.10) is also valid in the case of the electron-neutron and muon-nucleon interactions.

The constants $\varkappa_i$ are still unknown. The aim of the experiments described below is, in fact, to determine their values. If the interaction between neutral currents has a $V-A$ structure (in the same way as the interaction between charged currents) with a Fermi coupling constant ($G_s = G_P = G_T = 0$, $G_V = G_A = G'_V = G'_A = G\hbar^3/\sqrt{2c}$), the constants $\varkappa_i$ are given by

$$\varkappa_1 = -\varkappa_2 = \varkappa_3 = -1. \quad (2.12)$$

In the Weinberg model[34] (unified renormalizable theory of electromagnetic and weak interactions), which is the most popular at the present time, the constants $\varkappa_i$ are given by

$$\varkappa_{1p} = \frac{1}{2}(1-4\sin^2\theta), \quad \varkappa_{1n} = -\frac{1}{2},$$
$$\varkappa_{2p} = -\varkappa_{3p} = -\varkappa_{2n} = \varkappa_{3n} = -\frac{1}{2}(1-4\sin^2\theta)\lambda, \quad (2.13)$$

where $\lambda \approx 1.25$ is the axial current renormalization constant and the mixing angle θ is an independent parameter of the model. Analysis of the neutrino experiment on the properties of neutral currents[10,11] within the framework of the Weinberg model yields $\sin^2\theta \approx 0.35$.

A few words now about the parity-conserving weak potential. In the nonrelativistic approximation, i.e., if we neglect the momentum dependence of not only the proton but of the electron as well, we can construct only two scalars, namely, the unit matrix and the scalar product of the Pauli matrices $(\sigma\cdot\sigma_p)$. Since the interaction is local, we arrive at the following expression for the potential:

$$V(\mathbf{r}, \sigma, \sigma_p) = \frac{G\hbar^3}{\sqrt{2}c}(\varkappa_4+\varkappa_5\,\sigma\times\sigma_p)\,\delta(\mathbf{r}). \quad (2.14)$$

In the case of the $V-A$ variant, $\varkappa_4 = -\varkappa_5 = 1$ and the interaction occurs only in the singlet state.

3. HYDROGEN ATOM

A. $2S_{1/2}\to 1S_{1/2}$ transition

We shall begin our quantitative study of parity nonconservation effects in atoms by considering the single-photon transition $2S_{1/2}\to 1S_{1/2}$ in atomic hydrogen. By considering a simple example, we can readily see how the effects will arise and become enhanced.[6)]

Let us recall the situation which occurs in the absence of weak interaction. The $2S_{1/2}$ state is metastable. The electric dipole transition $2S_{1/2}\to 1S_{1/2}$ is strongly forbidden because the initial and final states have the same parities. On the other hand, the electric dipole transition to the $2P_{1/2}$ level, which differs from the $2S_{1/2}$ level only by the Lamb shift, is negligible because of the negligible energy difference (the lifetime for this transition is 39.6 years). The single-photon magnetic dipole

[6)]We shall neglect hyperfine splitting throughout this chapter with the exception of formula (3.13).

transition $2S_{1/2} \to 1S_{1/2}$ is also forbidden in the nonrelativistic approximation. The point is that the matrix element of the magnetic dipole-moment operator $\mu = \mu\sigma = (e\hbar/2mc)\sigma$ vanishes because radial wave functions with different principal quantum numbers are orthogonal.[7)] The transition is no longer forbidden when relativistic corrections to the wave functions for the electron and delayed effects are included. This leads to the appearance of the additional factor $\sim v^2/c^2$ in the amplitude for the $M1$ transition, which then becomes $\sim (v/c)^3 \sim \alpha^3$, as compared with the $E1$ transition $2P_{1/2} \to 1S_{1/2}$, and not $v/c \sim \alpha$, as in the case of the usual $M1$ transitions. The total probability of the $M1$ transition $2S_{1/2} \to 1S_{1/2}$ is[35–37]:

$$W_S = \frac{1}{2 \cdot 3^5} \alpha^{11} \frac{mc^2}{\hbar} = 0.25 \cdot 10^{-5} \text{ sec}^{-1}. \quad (3.1)$$

and the probability of the $E1$ transition $2P_{1/2} \to 1S_{1/2}$ is

$$W_P = \left(\frac{2}{3}\right)^8 \alpha^5 \frac{mc^2}{\hbar} = 0.63 \cdot 10^9 \text{ sec}^{-1}. \quad (3.2)$$

The main mode of decay of the $2S_{1/2}$ state in the presence of perturbations (external electric field, atomic collisions, and so on) is the $2S_{1/2} \to 1S_{1/2}$ transition with the emission of two electric dipole photons. Its probability[39] is 8.23 sec^{-1}.

B. Mixing of the $2S_{1/2}$ and $2P_{1/2}$ levels and circular polarization of the emitted radiation

If we ignore hyperfine splitting, the only contribution to the mixing of levels is provided by the first term of the P-odd potential (2.10), which is independent of the spin of the proton:

$$V(\mathbf{r}) = \frac{G\varkappa_1 \hbar^3}{2\sqrt{2}\, mc^2} \sigma \cdot [\mathbf{p}\delta(\mathbf{r}) + \delta(\mathbf{r})\mathbf{p}], \quad (3.3)$$

whereas the second and third terms are removed when the average over the proton spin is evaluated. The interaction given by (3.3) conserves the total angular momentum of the electron, so that it mixes $2S_{1/2}$ with only the $nP_{1/2}$ levels, and the amount of admixture does not depend on the component of the total angular momentum.

The main contribution to the admixture is provided by the $2P_{1/2}$ level which is separated from the $2S_{1/2}$ level only by the Lamb shift:

$$\Psi_m(2S_{1/2}) \to \Psi_m = \Psi_m(2S_{1/2}) + iF\Psi_m(2P_{1/2}); \quad (3.4)$$

where $\Psi_m(2S_{1/2})$ and $\Psi_m(2P_{1/2})$ are the nonrelativistic wave functions for the electron with angular momentum component m:

$$\Psi_m(2S_{1/2}) = \frac{1}{\sqrt{4\pi}} R_{20}(r)\chi_m,$$
$$\Psi_m(2P_{1/2}) = -\frac{1}{\sqrt{4\pi}} R_{21}(r)\left(\sigma \frac{\mathbf{r}}{r}\right)\chi_m, \quad (3.5)$$

R_{nl} and χ_m are the radial and spin functions, respectively, and

$$iF = \frac{\langle 2P_{1/2} | V | 2S_{1/2} \rangle}{E(2S_{1/2}) - E(2P_{1/2})}. \quad (3.6)$$

The matrix element $\langle 2P_{1/2} | V | 2S_{1/2} \rangle$ can be evaluated with the aid of (3.3) and (3.5) in an elementary fashion, as follows:

$$\langle 2P_{1/2} | V | 2S_{1/2} \rangle = -\frac{3iG\varkappa_1 \hbar^4}{8\pi\sqrt{2}\, mc^2} R_{20}(0)\left(\frac{R_{21}}{r}\right)_{r=0} = -\frac{i\sqrt{3}\, G\varkappa_1}{32\pi\sqrt{2}} m^3\alpha^4 c^2. \quad (3.7)$$

Since the Lamb shift is $\Delta_L = E(2S_{1/2}) - E(2P_{1/2}) = 7.8\alpha^5 mc^2/6\pi$, we have

$$F = -0.029 Gm^2\alpha^{-1}\varkappa_1 = -1.2 \cdot 10^{-11}\varkappa_1. \quad (3.8)$$

The mixing of the $2S_{1/2}$ and $2P_{1/2}$ levels leads to the circular polarization of the emitted photons because of interference between the amplitudes for the main $M1(2S_{1/2} \to 1S_{1/2})$ and the added $E1(2P_{1/2} \to 1S_{1/2})$ transitions. To calculate it, we note that the $M1$ and $E1$ transition amplitudes can be written in the form $A_S\sigma \cdot [\mathbf{e}^* \cdot \mathbf{n}]$ and $A_P\sigma \cdot \mathbf{e}^*$, respectively, where $\sigma/2$ is the total angular momentum operator for the electron, $\mathbf{e}$ is the polarization vector of the emitted photon, and $\mathbf{n}$ is the direction of emission of the photon. The quantities A_S and A_P are normalized so that the transition probabilities (3.1) and (3.2) become $W_S = 8\pi|A_S|^2$ and $W_P = 8\pi|A_P|^2$. The amplitude for the transition between the state described by the wave function (3.4) to the ground $1S_{1/2}$ state is

$$A = A_S\sigma \cdot [\mathbf{e} \times \mathbf{n}] + iFA_P\sigma \cdot \mathbf{e}^*. \quad (3.9)$$

The probability of emission of a photon with polarization $\mathbf{e}$ in the direction $\mathbf{n}$ is[8)]

$$W(\mathbf{n}, \mathbf{e}) = \frac{1}{2} \mathrm{Sp}\, AA^+ = |A_S|^2 (1 - 2Fr\mathbf{s}_\gamma \cdot \mathbf{n}); \quad (3.10)$$

where

$$r = \left|\frac{A_P}{A_S}\right| = \sqrt{\frac{W_P}{W_S}} = \frac{2^5}{3\sqrt{3}} \alpha^{-3} = 1.59 \cdot 10^7, \quad (3.11)$$

and $\mathbf{s}_\gamma = -i[\mathbf{e}^* \times \mathbf{e}]$ is the photon spin. The spin of the photon is in the direction of its momentum for right-handed polarization ($\mathbf{s}_\gamma \cdot \mathbf{n} = +1$) and in the opposite direction ($\mathbf{s}_\gamma \cdot \mathbf{n} = -1$) in the case of left-handed polarization. The degree of circular polarization is therefore given by[22]

$$P = \frac{W(\mathbf{n}, \mathbf{e}_R) - W(\mathbf{n}, \mathbf{e}_L)}{W(\mathbf{n}, \mathbf{e}_R) + W(\mathbf{n}, \mathbf{e}_L)} = -2Fr \approx 3.8 \cdot 10^{-4}\varkappa_1. \quad (3.12)$$

The expression for the circular polarization with allowance for hyperfine splitting can be obtained[40] just as simply if we use the potential given by (2.10) and the wave functions which include the hyperfine structure. The result is

$$P = \frac{\sqrt{3}\, Gr\hbar^4}{64\pi\sqrt{2}\, mc^2 a^4}\left(\frac{3\varkappa_1 + \varkappa_2 - 2\varkappa_3}{E_{S1} - E_{P1}} + \frac{\varkappa_1 - \varkappa_2 + 2\varkappa_3}{E_{S0} - E_{P0}}\right). \quad (3.13)$$

In this expression, E_{SF} and E_{PF} are the energies of the $2S_{1/2}$ and $2P_{1/2}$ levels with definite total angular momentum F of the atom. When hyperfine splitting is neglected [$E_{S1} - E_{P1}$ and $E_{S0} - E_{P0} \to \Delta_L = E(2S_{1/2})$

---

7)The operator for the orbital angular momentum $\mathbf{l}$ does not contribute to μ since $l = 0$ in both states.

8)The invariance of the electromagnetic field under time reversal and the fact that the transition matrix is Hermitian in the Born approximation ensure that the phases of the amplitudes A_S and A_P are equal. We shall use this when we evaluate the trace in (3.10). Moreover, we have neglected in (3.10) all terms that are quadratic in F.

$-E(2P_{1/2})]$, we again have the expression given by (3.12). In the case of the $V-A$ variant, we have $3\varkappa_1 + \varkappa_2 - 2\varkappa_3 = 0$, and only the energy difference for the singlet levels is present in (3.13).

C. Parity nonconservation effects in an external magnetic field

The Zeeman effect ensures that each of the $2S_{1/2}$ and $2P_{1/2}$ levels splits in a magnetic field into two sublevels with $m=\pm 1/2$. Because of the difference between the g-factors of the $2S_{1/2}$ and $2P_{1/2}$ states ($g_S=2$, $g_P=2/3$), the $2S_{1/2}$ and $2P_{1/2}$ levels with $m=-1/2$ are found to cross when the magnetic field reaches 1.2 kG. Neglecting the natural linewidths, one would expect complete mixing of the $(2S_{1/2})_{m=-1/2}$ and $(2P_{1/2})_{m=-1/2}$ states in this case. However, when the linewidth is taken into account, it is found that the mixing is much less than unity. In the expression for the circular polarization, the energy factor $1/\Delta_L$, which is present in the absence of the magnetic field, is now replaced by the typical dispersion factor

$$\frac{1}{2}\frac{\Delta_L-E}{(\Delta_L-E)^2+(\Gamma^2/4)};$$

where E is the difference between the $(2S_{1/2})_{m=-1/2}$ and $(2P_{1/2})_{m=-1/2}$ Zeeman shifts, Γ is the natural width of the $2P_{1/2}$ state (the width of the $2S_{1/2}$ state is negligible), and the factor 1/2 appears because only levels with $m=-1/2$ are involved in the crossing effect. If we consider the polarization as a function of the magnetic field (or the quantity $\Delta_L - E$, which is a linear function of the magnetic field), we find that the polarization increases, reaching its maximum for $\Delta_L - E = \Gamma = \Gamma/2$. It vanishes at the point of level crossing. The maximum polarization is found to be enhanced, as compared with its value in the absence of the magnetic field, roughly by a factor of $(1/2)\Delta_L/\Gamma \sim 5$.

To avoid misunderstanding, we must introduce the following word of caution. Photons connected with the emission of a particular Zeeman line (for example, $m=-1/2$) and emitted in the direction of the magnetic field are, of course, 100% circularly polarized. In the present case, however, photons from the $2S_{1/2}$, $m=-1/2$ level are recorded together with photons from the $2S_{1/2}$, $m=+1/2$ level because the Zeeman energy difference is very small. Thus, despite the presence of the magnetic field, we are dealing with an unpolarized initial state, so that the polarization of the photons is connected exclusively with the weak interaction.[9)]

The external magnetic field leads to the appearance of one further effect connected with parity nonconservation, namely, an asymmetry in the emission of photons relative to the direction of the magnetic field **H**, i.e., a correlation of the form $1 + A(H)\cos\vartheta$. The coefficient $A(H)$ vanishes for $H=0$, and is almost equal to the circular polarization of the photons in the neighborhood of the level crossing.

In addition to the above effects, the inclusion of the magnetic field leads to the following new possibility. When the S and P levels with $m=-1/2$ approach one another sufficiently closely, hyperfine splitting must be taken into account. It then turns out that if the weak interaction is of the $V-A$ form, we have the mixing of only the S and P states with total angular momentum component $m_F=0$, and if the interaction is of the form $V+A$, then states with $m_F=-1$ are found to mix. If the weak interaction is a mixture of $V-A$ and $V+A$, or has some more complicated matrix form, then S and P states with both $m_F=0$ and -1 are found to mix. Thus, by studying the polarization or the asymmetry as functions of the magnetic field, we can, at least in principle, establish the form of the weak interaction (magnetic-field effects are discussed in greater detail in[40]).

D. Electric dipole moment of a metastable state

It is well known that CP (or T)-invariance ensures that a stable particle of spin **s** cannot have an electric dipole moment even when parity is not conserved. The usual argument can be reduced to the following. When parity is not conserved, the electric dipole moment **d** can only be parallel to the particle spin, i.e., $\mathbf{d}=el\mathbf{s}/s$, where s is the maximum component of the spin and l is a parameter with the dimensions of length. However, interaction with the electric field, $\mathbf{d}\cdot\mathbf{E}=el(\mathbf{s}/s)\cdot\mathbf{E}$, violates not only parity ($\mathbf{s}\cdot\mathbf{E}$ is a pseudoscalar) but also T-invariance in the same way as in the case of time inversion $t\to -t$: $\mathbf{E}\to\mathbf{E}$ and $\mathbf{s}\to-\mathbf{s}$.

Ya. B. Zel'dovich[41] was the first to point out, however, that these arguments could not be extended to unstable particles. An excellent example of this is the fact that the metastable $2S_{1/2}$ state has an electric dipole moment if the $2P_{1/2}$ level mixes with it as a result of parity nonconservation.

In the case of the mixing of the $2S_{1/2}$ and $2P_{1/2}$ states which we considered above [see (3.3)–(3.8)], we regarded these states as stationary and ignored the fact that they might be unstable. To take this into account, we must introduce the substitution $E(2P_{1/2}) \to E(2P_{1/2}) - i\Gamma/2$ in the mixing factor given by (3.6), where Γ is the width of the $2P_{1/2}$ state (the width of the $2S_{1/2}$ level can be neglected).[10)] As a result, the mixing factor iF [see (3.6)–(3.8)] is modified as follows:

$$iF \to i\tilde{F} = iF\frac{\Delta_L}{\Delta_L + i(\Gamma/2)} \qquad (3.14)$$

and the perturbed states Ψ_m have the form

$$\Psi_m = \Psi_m(2S_{1/2}) + i\tilde{F}\Psi_m(2P_{1/2}).$$

The dipole moment of the electrons in this state is

$$\langle e\mathbf{r}\rangle_m = \int d^3\mathbf{r}\,\Psi_m^* e\mathbf{r}\Psi_m = i(\tilde{F}-\tilde{F}^*)\sqrt{3}\,ea\chi_m^+\sigma\chi_m. \qquad (3.15)$$

9)More precisely, the probabilities of the different Zeeman transitions will be slightly different because of the difference between their frequencies. This leads to a circular polarization of about 10^{-6}.

10)The rigorous approach involves the application of perturbation theory in the weak interaction in the continuum of stationary states. The $2P_{1/2}$ state is then looked upon as a resonance in the system consisting of the stable $1S_{1/2}$ state plus a γ photon.

In evaluating this integral, we use the explicit expressions given by (3.5) for the wave functions. From (3.14), we have

$$\mathbf{d}=eF\frac{\Gamma}{\Delta_L}\sqrt{3}a\sigma. \tag{3.16}$$

The corresponding length is $l=\sqrt{3}F(\Gamma/\Delta_L)a\approx-1.0\times10^{-20}\varkappa_1$ cm. The factor Γ/Δ_L is a characteristic feature of (3.16). The dipole moment vanishes during the transition to the stable state. The existence of the dipole moment (3.16) leads to a linear Stark effect because of the interaction $\mathbf{d}\cdot\mathbf{E}$, i.e., to the removal of degeneracy with respect to the sign of the spin component in the electric field.[42] Unfortunately, this effect is exceptionally difficult to observe in hydrogen because of the rapid deexcitation of the $2S_{1/2}$ state in the external electric field, which is also due to the closeness of the $2P_{1/2}$ level.

E. What prevents the observation of parity nonconservation?

We shall now consider some of the factors which impede the experimental investigation of parity nonconservation in the hydrogen atom. We shall be mainly concerned with the spontaneous $2S_{1/2}\to1S_{1/2}$ transition because, as we have already noted in Chap. 1, the stimulated $2S_{1/2}\rightleftarrows1S_{1/2}$ transition in hydrogen is practically impossible to investigate because of the closeness of the $2P_{1/2}$ level. When the $2S_{1/2}\to1S_{1/2}+\gamma$ *spontaneous emission process is observed* (for example, with the aid of the atomic-beam technique), the influence of the $2P_{1/2}\to1S_{1/2}+\gamma$ transition is unimportant because the $2P_{1/2}$ state, which has a short lifetime 10^{-9} sec) is rapidly deexcited.

One of the main practical difficulties preventing the observation of parity nonconservation is the two-photon transition $2S\to1S$, the probability $W_{2\gamma}$ of which exceeds the probability W_S of the single-photon transition by a factor of 3×10^6. This means that the necessary statistics is difficult to achieve. Moreover, the photons from the two-photon transition, the energy of which is close to the energy difference between the $2S$ and $1S$ states, produce a background which reduces the degree of circular polarization. To avoid this background, we must isolate the radiation in a relatively narrow frequency band, defined by $\Delta\omega/\omega\sim\sqrt{W_S/W_{2\gamma}}\sim10^{-3}$.

Another serious problem is the necessity for screening from random external electric fields which produce Stark mixing of the $2S$ and $2P$ states. In contrast to the mixing by weak interaction, Stark mixing does not in itself lead to circular polarization. However, it does increase the probability of the single-photon transition which, in the end, leads to a reduction in the degree of polarization. To avoid this, the probability of deexcitation of the atoms as a result of the admixture of the P state should not exceed the probability W_S of the single-photon transition. Hence, it follows[40] that the external electric field must satisfy the condition $E\lesssim10^{-5}$ V/cm.

If, in addition to the electric field, there is also an external magnetic field H, the parity nonconservation effects may be simulated, for example, by the appearance of circular polarization proportional to the pseudoscalar $(\mathbf{E}\cdot\mathbf{H})$. If the electric field is restricted by the condition $E<10^{-5}$ V/cm, the restriction on the magnetic field[40] is $H\lesssim1$ G. If the proposed experiment is to involve a high external magnetic field ~1 kG (see Chap. 3), the restriction on the random external electric field becomes very stringent: $E\lesssim10^{-9}$ V/cm.

F. Hydrogen-like ions

The above discussion of parity nonconservation effects in the hydrogen atom can readily be extended to the case of hydrogen-like ions. The dependence of the various quantities defining the effect on the nuclear charge Z (in the lowest approximation in $Z\alpha$) is

$$\langle V\rangle\sim Z^5g,\quad \Delta_L\sim Z^4,$$
$$W_S\sim Z^{10},\quad W_P\sim Z^4,\quad W_{2\gamma}\sim Z^6\quad \left(g=\varkappa_{1p}+\frac{N}{Z}\varkappa_{1n}\right). \tag{3.17}$$

Hence, in hydrogen-like ions, the circular polarization of radiation emitted as a result of the $2S_{1/2}\to1S_{1/2}$ transition will decrease with increasing Z:

$$P\sim\frac{\langle V\rangle}{\Delta_L}\sqrt{\frac{W_P}{W_S}}\sim Z^{-2}.$$

However, the observation of the $2S_{1/2}\to1S_{1/2}+\gamma$ transition itself is then a more realistic proposition because its probability rapidly increases (more rapidly than the probability of the background two-photon transition). At the same time, the restrictions imposed on the external field become less stringent. Figure 3 shows the circular polarization P and the probabilities W_S, $W_{2\gamma}$ for hydrogen-like ions.[29]

4. TWO-ELECTRON IONS

A. Mechanisms for the appearance of circular polarization

The mechanisms that enhance parity nonconservation effects, which we discussed in the case of the hydrogen atom (highly forbidden main transition and closeness of levels with opposite parities), can also be used for two-electron systems[23] such as, for example, the helium atom and helium-like ions. Electromagnetic transitions in such systems have recently been investigated with great success as a result of the application of the beam-foil technique.[43] We shall consider electromagnetic transitions from singly excited states of the form (1s, *ns*) and (1s, *np*). We shall designate these states as $n^{2s+1}L_J$, where n and L are the principal quantum number and the orbital angular momentum of the excited

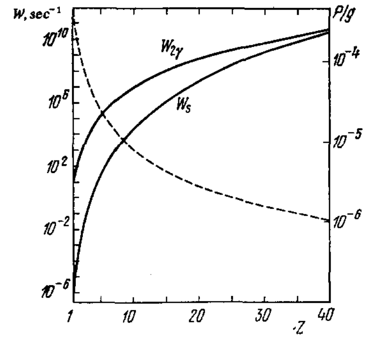


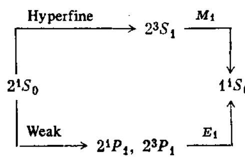
FIG. 3. Probabilities of the $2S\to1S$ transitions (solid lines) and circular polarization of photons (dashed line) in hydrogen-like ions.

electron, respectively, and s and J are the resultant spin and the total angular momentum of the two electrons.

Consider a single-photon transition from the metastable 2^1S_0 state to the ground 1^1S_0 state. If the nucleus has zero spin (as, for example, in the case of the He^4 atom), the total angular momentum J of the electron shell is a good quantum number, and the single-photon transition $2^1S_0 - 1^1S_0$ is strictly forbidden because it is a 0–0 transition. However, if the nucleus does not have zero spin, the quantity which will be conserved will be the total angular momentum of the ion $\mathbf{F} = \mathbf{J} + \mathbf{I}$, where $\mathbf{I}$ is the nuclear spin. Because of the hyperfine interaction between the magnetic moments of the nucleus and of the electron, the wave functions for the stationary states will be superpositions of wave functions with the same total angular momentum F, the same parity, and different J. In particular, the 2^1S_0 $(F=I)$ state will acquire an admixture due to the nearest state of the same parity, i.e., the 2^3S_1 $(F=I)$ state, which may go over to the ground 1^1S_0 state with the emission of a single photon. This transition is an $M1$ transition which is doubly forbidden for the same reasons as the $2S - 1S$ transition in hydrogen-like systems. Its amplitude is of the order of $\sim(\alpha Z)^2$ as compared with the amplitude for the allowed $M1$ transition.[37,44,45] Because of this and because of the small admixture of the 2^3S_1 state, the single-photon transition $2^1S_0 - 1^1S_0$ has an exceedingly low probability.

The inclusion of the weak interaction between the electrons and the nucleus opens another possibility of a single-photon transition from the 2^1S_0 state. The weak interaction will mix the 2^1S_0 state with other states having the same total angular momentum $F=I$ but different parity (P states), so that the latter can participate in transitions to the ground state with the emission of $E1$ photons. Altogether there are four P states with $n=2$ and $F=I$ ($^1P_1$, $^3P_0$, and $^3P_2$), and the weak interaction can mix all of them with the 2^1S_0 state. However, the admixture of the $^3P_0$ state is of no interest to us because this state decays to the 1^1S_0 level by a single-photon transition (0–0 transition). The admixture of the $^3P_2$ state is also of little interest because the $2^3P_2 - 1^1S_0$ transition is an $M2$ transition which does not interfere with the main $M1$ transition and does not lead to parity nonconservation effects in the first order in the constant G. Of the two remaining states, 2^1P_1 and 2^3P_1, the admixture of the former plays the main role in most ions. The transition probability between this state and the ground state is high[46] (this is an allowed $E1$ transition). The 2^3P_1 admixture usually plays a minor role since the $E1$ transition $2^3P_1 - 1^1S_0$ is highly forbidden because the $^3P_1$ state has a spin $s=1$, whilst, in the ground state, $s=0$. The dipole-moment operator does not act on the spin functions and, in the nonrelativistic approximation, the amplitude for this transition is zero. The inclusion of relativistic corrections (spin-orbit coupling) ensures that this transition is possible[47,48] but, roughly speaking, its amplitude is of the order of $(\alpha Z)^2$ as compared with the amplitude for the allowed $E1$ transition $2^1P_1 - 1^1S_0$. Nevertheless, for some ions, for example, C^{13} V, Ni^{61} XXVII, $Cu^{63,65}$ XXVIII, the admixture of the 2^3P_1 state is anomalously large because of the closeness of the 2^3P_1 and 2^1S_0 levels (see below) which compensates the suppression of the $2^3P_1 - 1^1S_0$ transition. In these and the neighboring ions, allowance for the 2^3P_1 mixture is important.

Thus, hyperfine and weak interactions ensure that there are two possible branches of the single-photon transition from the 2^1S_0 state to the 1^1S_0 state. They differ by the multipolarity of the emitted photon:



Interference between these transitions leads to circular polarization of the emitted radiation.

B. Estimated size of the effect

We shall write the amplitude for the single-photon $2^1S_0 - 1^1S_0$ transition in the form

$$A = HA_M + iF_0A_E^0 + iF_1A_E^1, \tag{4.1}$$

where A_M is the amplitude for the $M1$ transition $2^3S_1 - 1^1S_0$ and A_E^s $(s=0, 1)$ is the amplitude for the $E1$ transitions from the singlet $(s=0)$ and triplet $(s=1)$ states $2^{2s+1}P_1 - 1^1S_0$. The quantity H characterizes the admixture of 2^3S_1 to the original 2^1S_0 state due to the hyperfine interaction, and the constants iF_s represent the admixture of the $2^{2s+1}P_1$ states due to the weak interaction. The probability W of the single-photon $2^1S_0 - 1^1S_0$ transition and the degree of circular polarization P of the emitted photons are given by the following formulas:

$$W = H^2W_M, \quad P = P_0 + P_1,$$
$$P_s = \frac{2F_s}{H}\sqrt{\frac{W_E^s}{W_M}} \quad (s=0, 1), \tag{4.2}$$

where W_M, W_E^0, and W_E^1 are the probabilities of the transitions from the 2^3S_1, 2^1P_1, and 2^3P_1 states, respectively. Let us consider the various quantities in (4.2), and begin with the mixing factors H, F_0, and F_1. We shall confine our attention to simple estimates of these factors and will write the wave functions for the ions in the form of suitably symmetrized products of hydrogen-like functions. This approximation is known to be valid when effects associated with the screening of the nuclear charge are neglected, i.e., if we ignore corrections $\sim 1/Z$.

In the nonrelativistic approximation, the potential for the hyperfine interaction between electrons and the nucleus can be written in the form

$$V_H = \frac{2\pi}{3}\frac{\alpha g\hbar^3}{mm_pc}(\sigma_1\mathbf{I}\delta(\mathbf{r}_1) + \sigma_2\mathbf{I}\delta(\mathbf{r}_2)), \tag{4.3}$$

where σ_i are the Pauli matrices for electrons $(i=1, 2)$, $\mathbf{I}$ is the nuclear-spin operator, g is the gyromagnetic ratio for the given nucleus, and m, m_p are the electron and proton masses. The admixture of the 2^3S_1 state can be calculated from the formula

$$H = \langle 2^3S_1,\ F=I\,|\,V_H\,|\,2^1S_0,\ F=I\rangle\,(\Delta E_H)^{-1}$$
$$= -\frac{7}{12}\sqrt{I(I+1)}\,\alpha g\,(Z\alpha)^3\,\frac{m}{m_p}\,\frac{mc^2}{\Delta E_H},$$
$$\Delta E_H = E(2^1S_0) - E(2^3S_1). \tag{4.4}$$

The potential for the weak interaction between the electrons and the nucleus can, in view of (2.10), be written in the form

$$V_W = V_W(1) + V_W(2),$$
$$V_W(i) = \frac{G\hbar^3}{2\sqrt{2}\,mc^2}\{(Z\varkappa_{1p} + N\varkappa_{1n})\,\boldsymbol{\sigma}_i\,[\mathbf{p}_i\delta(\mathbf{r}_i) + \delta(\mathbf{r}_i)\,\mathbf{p}_i]$$
$$+ 2(\varkappa_{2p}\mathbf{s}_p + \varkappa_{2n}\mathbf{s}_n)\,[\mathbf{p}_i\delta(\mathbf{r}_i) + \delta(\mathbf{r}_i)\,\mathbf{p}_i]$$
$$+ 2i(\varkappa_{3p}\,[\boldsymbol{\sigma}_i\times\mathbf{s}_p] + \varkappa_{3n}\,[\boldsymbol{\sigma}_i\times\mathbf{s}_n])\,[\mathbf{p}_i\delta(\mathbf{r}_i) - \delta(\mathbf{r}_i)\,\mathbf{p}_i] \quad (i=1,\ 2); \tag{4.5}$$

here Z, N is the number of protons and neutrons in the nucleus and $\mathbf{s}_p$ and $\mathbf{s}_n$ are the sums of spins of all the protons and all the neutrons, respectively. The term in (4.5), which contains $\varkappa_{1p}$ and $\varkappa_{1n}$, is independent of the nuclear spins and conserves the total angular momentum J of the electron shell so that, in general, it does not contribute to the 2^1P_1 and 2^3P_1 admixtures to the 2^1S_0 state. The terms including $\varkappa_{2p}$ and $\varkappa_{2n}$ do not contain the electron spins and they do not, therefore, mix the triplet 2^3P_1 and singlet 2^1S_0 states. Conversely, terms including $\varkappa_{3p}$ and $\varkappa_{3n}$ do affect the electron spin and do not contribute to the 2^1P_1 admixture. Thus, the quantity F_0 depends only on $\varkappa_{2p}$, $\varkappa_{2n}$, and F_1 on $\varkappa_{3p}$ and $\varkappa_{3n}$[11]:

$$\left.\begin{aligned}
iF_s &= \langle 2^{2s+1}P_1,\ F=I\,|\,V_W\,|\,2^1S_0,\ F=I\rangle\,(\Delta E_W^s)^{-1},\\
\Delta E_W^s &= E(2^1S_0) - E(2^{2s+1}P_1),\\
F_0 &= -\frac{Gm^2}{16\pi\sqrt{2}}(\alpha Z)^4\,\frac{mc^2}{\Delta E_W^0}\,\bar{\varkappa}_2,\\
F_1 &= \frac{Gm^2}{16\pi}(\alpha Z)^4\,\frac{mc^2}{\Delta E_W^1}\,\bar{\varkappa}_3\\
\bar{\varkappa}_i &\equiv \varkappa_{ip}s_p + \varkappa_{in}s_n \qquad (i=2,\ 3);
\end{aligned}\right\} \tag{4.6}$$

where s_p and s_n are the reduced matrix elements of the spin operators $\mathbf{s}_p$ and $\mathbf{s}_n$ between the nuclear wave functions.[12] It depends not only on the nuclear spin I but also on the structure of the nucleus. By Pauli's principle, s_p is zero for nuclei with an even number of protons and of the order of unity (and not Z) for an odd number of protons. Similar qualitative conclusions can be drawn with regard to s_n.

Let us now consider the energy denominators in the expressions for the mixing factors given by (4.4) and (4.6). Figure 4 shows the level spacing as a function of the nuclear charge Z. It is based on calculations of the spectra of two-electron ions.[49–51] It is clear from this figure that the quantities $\Delta E_H = E(2^1S_0) - E(2^3S_1)$ and $\Delta E_W^0 = E(2^1S_0) - E(2^1P_1)$ are monotonic functions of Z, and $\Delta E_H \sim (0.5–1.2)\,Z$ eV, $\Delta E_W^0 \sim (0.3–1.5)Z$ eV. Hence, we find that the order-of-magnitude estimates for the mixing factors are $H \sim 10^{-6}Z^2$, $F_0 \sim 10^{-16}Z^3\,\bar{\varkappa}_2$.

An interesting situation occurs in the case of the energy difference ΔE_W^1 between the 2^1S_0 and 2^3P_1 states. These levels are always close to one another. In the helium atom, the 2^1S_0 state lies below the 2^3P_1, and, as Z increases, these levels cross twice. This occurs for the first time near $Z=6$ (the C V ion), and the second time between $Z=28$ (Ni XXVII ion) and $Z=29$ (Cu XXVIII ion). The energy difference ΔE_W^1 for these ions is anomalously low, and the factor F_1 relatively large

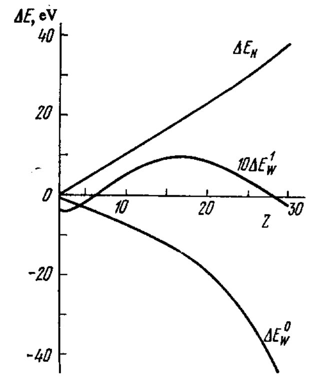


FIG. 4. Energy difference between levels with $n=2$ in two-electron ions.

$$\left.\begin{aligned}
&\mathrm{C^{13}\ V}: && \Delta E_W^1 \sim -1.6\cdot 10^{-2}\ \text{eV}, && F_1 \sim -0.4\cdot 10^{-11}\bar{\varkappa}_3,\\
&\mathrm{Ni^{61}\ XXVII}: && \Delta E_W^1 \sim 4\cdot 10^{-2}\ \text{eV}, && F_1 \sim 10^{-9}\bar{\varkappa}_3,\\
&\mathrm{Cu^{63,65}\ XXVIII}: && \Delta E_W^1 \sim -8\cdot 10^{-2}\ \text{eV}, && F_1 \sim -7\cdot 10^{-10}\bar{\varkappa}_3.
\end{aligned}\right\} \tag{4.7}$$

For values of Z well away from the crossing points, the factor F_1 is greater by one or two orders of magnitude as compared with F_0. To estimate the parity nonconservation effects, all that now remains is to consider the probabilities of transitions from the admixture states (W_M, W_E^0, and W_E^1). These probabilities were calculated in[37,44–48] and are plotted in Fig. 5. For approximate estimates of W_M and W_E^0, we can use the formulas obtained with the aid of the hydrogen-like functions[13]:

$$W_M \approx \frac{1}{2\cdot 3^6}\,\alpha\,(\alpha Z)^{10}\,\frac{mc^2}{\hbar} \approx 1.66\cdot 10^{-6}Z^{10}\ \text{sec}^{-1}$$
$$W_E^0 \approx \frac{2^9}{3^8}\,\alpha\,(\alpha Z)^4\,\frac{mc^2}{\hbar} \approx 1.25\cdot 10^9 Z^4\ \text{sec}^{-1} \tag{4.8}$$

The probability W_E^1 is approximately proportional to Z^8 for large Z.

We can now use (4.2)–(4.6) to calculate the probability of the single-photon $2^1S_0 \to 1^1S_0$ transition and the circular polarization of photons emitted as a result of this transition. The results of the calculations for some two-electron ions are shown in Table 1. As can be seen, the probability of the $2^1S_0 \to 1^1S_0$ transition increases rapidly with increasing Z (roughly as Z^{14}). The circular polarization due to the 2^1P_1 admixture falls in accordance with the formula $P_0 \sim 2\times 10^{-3}Z^{-2}\varkappa_2$. The circular polarization P_1 associated with the 2^3P_1 admixture varies irregularly with Z. For the C V ions

---

11) This distinguishes two-electron ions from hydrogen-like systems or heavy atoms, where most of the parity nonconservation effects are determined by the constants $\varkappa_{1p}$, $\varkappa_{1n}$. Experiments with helium-like ions are therefore of independent interest.

12) The reduced matrix element is defined by $\langle IM'|(\mathbf{s}_{p,n})_\mu|IM\rangle = s_{p,n}\langle IM, 1\mu\,|\,IM'\rangle$.

13) For more accurate estimates, Z in (4.6)–(4.8) must be replaced by $Z_{eff} = Z - \sigma$, where σ represents the screening of the nuclear charge by the inner $1s$ electron ($\sigma \sim 0.8–0.9$). This correction is important for small Z. In (4.4), we have $Z_{eff} \sim Z$ because the main contribution to H is provided by the $1s$ electron for which the screening by the outer electron is small.

and heavy ions with $Z \sim 30$, we have $P_1 \sim 10^{-4} \varkappa_3$ and this determines the total circular polarization of the radiation.

Since the energy difference between the S and P states is relatively large, the restrictions on the external electric and magnetic fields, which impede the observation of parity nonconservation effects, are much less stringent in two-electron ions than in the hydrogen atom. On the other hand, for the same reason, it is practically impossible to use the Zeeman level shift in a magnetic field in order to enhance the parity nonconservation effects (see Chap. 3). In the most favorable case (the C^{13} V ion), the crossing of the 2^1S_0 and 2^3P_1 ($m_J = -1$) levels would require a magnetic field of $\sim 10^6$ G. It is true that, since the width of the 2^3P_1 level in C^{13} V is small in comparison with E_W^1 ($\Gamma_p \sim 10^{-6} \Delta E_W^1$), the degree of circular polarization P_1 in this field can, at least in principle, be made to be of the order of unity.[23]

Measurements of parity nonconservation effects encounter serious difficulties in the case of two-electron ions because the single-photon $2^1S_0 \to 1^1S_0$ transition is exceptionally highly suppressed. The lifetime of the metastable 2^1S_0 state in two-electron ions is determined mainly by the two-photon transition, the probability of which is estimated from the formula[37,44] $W_{2\gamma} \approx 16.4 Z_{eff}^6$ sec^{-1} (see Fig. 5). The relative probability of the single-photon $2^1S_0 \to 1^1S_0$ transition, i.e., $W/W_{2\gamma}$, for ions with $Z \lesssim 10$ does not exceed 10^{-10} (see Table 1), which presents a serious difficulty for the observation of this transition. Moreover, to ensure that the two-photon transition background does not reduce the circular polarization of the photons, the radiation must be observed in an exceedingly narrow frequency interval defined by $\Delta\omega/\omega \sim [W/W_{2\gamma}]^{1/2}$. For example, in the case of the C V ions, we must have $\Delta\omega/\omega \lesssim 10^{-6}$. For ions with large Z, these restrictions become less stringent.

5. HEAVY ATOMS

A. Calculation of the mixing of levels with opposite parities

As already noted in the Introduction, the mixing of levels with opposite parities due to the presence of the weak interaction between electrons and nucleons increases rapidly with Z in heavy atoms. We must now estimate this mixing quantitatively. Since, in this case, relativistic effects are important, the parity nonconserving interaction between a relativistic electron and the nucleus will be written in the form

$$\mathcal{H} = -\frac{G\hbar^3}{\sqrt{2}c}\,\delta(\mathbf{r})\,(Zq\gamma_5 - \Sigma\boldsymbol{\alpha}),$$

$$\left.\begin{aligned} Zq &= \varkappa_{1p} Z + \varkappa_{1n}(A - Z), \\ \Sigma &= \varkappa_{2p}\sum_p \boldsymbol{\sigma}_p + \varkappa_{2n}\sum_n \boldsymbol{\sigma}_n. \end{aligned}\right\} \quad (5.1)$$

This expression is obtained from (2.6) in the approximation of infinitely heavy nucleons and a point nucleus.[14)] The summation in (5.1) extends over all the Z protons and the $A - Z$ neutrons in the nucleus.

To evaluate the matrix elements for the interaction (5.1), we shall need the electron wave functions in the neighborhood of the nucleus. The relativistic wave function for an electron with total angular momentum j, orbital angular momentum l, and component of total angular momentum m has the form

$$u_{njl} = \begin{pmatrix} g_{njl}(r)\,\Omega_{jlm} \\ if_{njl}(r)\,\Omega_{j,\,2j-l,\,m} \end{pmatrix}; \quad (5.2)$$

where Ω_{jlm} is the spherical function including spin. For distances $r \ll aZ^{-1/3}$, where the Coulomb field of the nucleus may be looked upon as unscreened, and the total energy can be neglected in comparison with the potential, the radial wave functions can be expressed in terms of Bessel functions, as follows:

$$f_{njl}(r) = \frac{c_{njl}}{r}\left[(\gamma + \varkappa) J_{2\gamma}(x) - \frac{x}{2} J_{2\gamma-1}(x)\right],$$

$$f_{njl}(r) = \frac{c_{njl}}{r} Z\alpha J_{2\gamma}(x). \quad (5.3)$$

In these expressions, we use the notation:

$$x = \left(\frac{8rZ}{a}\right)^{1/2}, \quad \varkappa = \begin{cases} -\left(j + \frac{1}{2}\right) & \text{for} \quad j = l + \frac{1}{2}, \\ j + \frac{1}{2} & \text{for} \quad j = l - \frac{1}{2}, \end{cases}$$

$$\gamma = \sqrt{\varkappa^2 - Z^2\alpha^2}.$$

As $r \to 0$, we have

$$g_{njl}(r) = -\frac{c_{njl}}{r}\,\frac{\gamma - \varkappa}{\Gamma(2\gamma + 1)}\left(\frac{2rZ}{a}\right)^{\gamma},$$

$$f_{njl}(r) = \frac{c_{njl}}{r}\,\frac{Z\alpha}{\Gamma(2\gamma + 1)}\left(\frac{2rZ}{a}\right)^{\gamma}. \quad (5.3a)$$

To determine the normalizing constant c_{njl}, we note that when $r \gg aZ^{-1}$, the radial function $g_{njl}(r)$ must become identical with the usual quasiclassical solution of the nonrelativistic Schrödinger equation

$$\psi_{nl}(r) = \frac{b_{nl}}{r\sqrt{k(r)}} \sin\left(\int_{r_1}^{r} dr' k(r') + \varphi\right); \quad (5.4)$$

TABLE 1. Parameters of the $2^1S_0 \to 1^1S_0$ transition in two-electron ions.

Ions	E eV	W, sec^{-1}	$\lvert P_0 \rvert/\tilde{\varkappa}_2$	$\lvert P_1 \rvert/\tilde{\varkappa}_3$	$W/W_{2\gamma}$
He^3 I	20.61	$2.0 \cdot 10^{-14}$	$2 \cdot 10^{-4}$	10^{-7}	$4 \cdot 10^{-16}$
C^{13} V	304.3	$2.2 \cdot 10^{-8}$	10^{-4}	$3 \cdot 10^{-4}$	$7 \cdot 10^{-14}$
F^{19} VIII	731.8	$1.0 \cdot 10^{-4}$	$2 \cdot 10^{-5}$	10^{-5}	$2 \cdot 10^{-11}$
Cu^{63} XXVIII	8347	$5.1 \cdot 10^{2}$	$2 \cdot 10^{-6}$	$3 \cdot 10^{-4}$	$6 \cdot 10^{-8}$

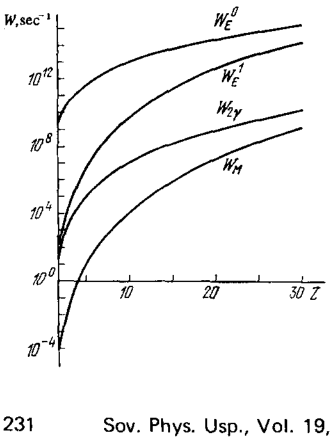


FIG. 5. Probabilities of radiative transitions in two-electron ions.

14)The four-fermion interaction (2.11), which includes the derivatives, will not be considered in this chapter.

where

$$k^2(r)=\frac{2m}{\hbar^2}\left[E_{nl}-U(r)-\frac{(l+1/2)^2}{r^2}\right],$$

r_1 is the turning point and φ is the constant phase. The constant b_{nl} can readily be found by recalling that the main contribution to the normalizing integral is provided by the quasiclassical region between the two turning points r_1 and r_2, so that

$$b_{nl}^2\int_{r_1}^{r_2}\frac{dr}{k(r)}\sin^2\left(\int_{r_1}^{r}dr'k(r')+\varphi\right)\approx\frac{b_{nl}^2}{2}\int_{r_1}^{r_2}\frac{dr}{k(r)}=1. \tag{5.5}$$

Differentiating the Bohr quantization rule for the radial motion

$$\int_{r_1}^{r_2}dr\,k(r)=\pi(n_r+\beta) \tag{5.6}$$

with respect to n_r, and remembering that $k(r_{1,2})=0$, we find that[52]

$$\frac{m}{\hbar^2}\frac{dE_{nl}}{dn_r}\int_{r_1}^{r_2}\frac{dr}{k(r)}=\pi. \tag{5.7}$$

Hence,

$$b_{nl}^2=\frac{2m}{\pi\hbar^2}\frac{dE_{nl}}{dn_r}=\frac{2}{\pi\nu^3}\frac{1}{a^2}. \tag{5.8}$$

Here, we have used the phenomenological expression for the energy of the outer electron

$$E_{nl}=-\frac{me^4}{2\hbar^2\nu^2}, \tag{5.9}$$

where ν is the effective principal quantum number: $\nu=n_r+l+1-\sigma_l=n-\sigma_l$, and the quantum defect σ_l is a slowly varying function of n_r for given l. It is readily seen that, as n_r increases, the atom becomes increasingly hydrogen-like in character, so that σ_l must fall. As a result, allowance for this dependence should lead to an increase in the calculated degree of level mixing.

Comparing (5.3a) with (5.4) for $aZ^{-1}\ll r\ll aZ^{-1/3}$, we see that

$$c_{njl}=\frac{\varkappa}{|\varkappa|}\sqrt{\frac{\pi a}{2}}\,b_{nl}=\sqrt{\frac{1}{a\nu^3}}\,\frac{\varkappa}{|\varkappa|}. \tag{5.10}$$

The wave functions found in this way can be used to show that the only nonzero matrix element which does not conserve parity in the interaction between the electron and the nucleon (other than the Hermitian conjugate) is given by

$$\langle s_{1/2}|H|p_{1/2}\rangle=i\,\frac{Gm^2\alpha^2Z^2R}{\sqrt{2}\,\pi}(\nu_s\nu_p)^{-3/2}\frac{me^4}{2\hbar^2}\times\left\{Zq+g_I\,\frac{2\gamma+1}{3}\left[F(F+1)-I(I+1)-\frac{3}{4}\right]\right\}; \tag{5.11}$$

where $\mathbf{F}$ is the total angular momentum of the atom and g_I is the proportionality factor between the nuclear matrix elements of the operator Σ and the nuclear spin $\mathbf{I}$. It is related to the quantity $\bar{\varkappa}_2$ [see (4.6)] by the formula $g_I=2[I(I+1)]^{-1/2}\bar{\varkappa}_2$. The divergence which appears in the course of the calculation of the matrix element of $\delta(\mathbf{r})$ with the relativistic functions is removed by introducing a finite nuclear radius $r_0=1.2\times10^{-13}A^{1/3}$ cm. The relativistic enhancement factor R is then given by

$$R=\frac{4(2r_0Z/a)^{2\gamma-2}}{\Gamma^2(2\gamma+1)}. \tag{5.12}$$

It can be shown that more accurate allowance for the finite size of the nucleus has little effect on R. This factor increases rapidly for large Z, varying between 2.8 for cesium ($Z=55$) and 9 for bismuth ($Z=83$).

Finally, the quantity q is determined by the form of the weak interaction between the electron and the nucleon. To be specific, calculations of this parameter will be carried out within the framework of the Weinberg model[34] in which

$$q=1-A/2Z-2\sin^2\theta. \tag{5.13}$$

For heavy atoms and $\sin^2\theta=0.35$, it turns out that q amounts to $-(0.8-0.9)$.

We note that it is readily shown that not only the first but also the second term in the Hamiltonian given by (5.1) will not lead to the mixing of the $s_{1/2}$ and $p_{3/2}$ states.

A more sophisticated but also more complicated calculation of the mixing of levels of different parity is given in[31] and leads to results which differ from (5.11) by only a few percent. A method of calculation close to that which we have used here is given in the review.[53]

After this general review, we must now consider possible experiments on the detection of parity nonconservation in heavy atoms.

B. Nonconservation of parity in stimulated doubly-forbidden *M*1 transitions

The first experiment on the nonconservation of parity in heavy atoms was proposed by Bouchiat.[24] It involves a search for the circular polarization of photons in the doubly forbidden $M1$ transition $6s_{1/2}\to7s_{1/2}$ in cesium ($\lambda=5395$ Å). The cesium level scheme is shown in Fig. 6.

We must now estimate the expected magnitude of the effect. Let us begin with the matrix element for the $M1$ transition. It is shown in[31,54] that neither relativistic effects similar to those which occur in the hydrogen atom (see Chap. 3) nor the mixing of terms due to the hyperfine interaction are important in this case. The main mechanism for the $6s-7s$ transition is configuration mixing. On the other hand, this phenomenon ensures that the g-factor for the $6s$ electron in cesium differs from the g-factor for the free electron.[55] The residual Coulomb interaction between the electrons, which is not taken into account in the effective potential, ensures that the $6s_{1/2}$ and $7s_{1/2}$ states contain an admixture of states with an additional $6p$ electron and a

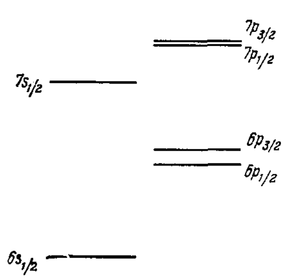


FIG. 6. Low-lying levels of Cs.

hole in the $5p$ shell. The large spin-orbit interaction of this hole leads to corrections to the g-factor for the $6s_{1/2}$ and $7s_{1/2}$ terms, and ensures that the $6s_{1/2}-7s_{1/2}$ transition is allowed. The matrix element for this transition is[31,54]

$$\mathbf{M}=\beta\mu\langle\sigma\rangle,$$
$$\beta=-\frac{\zeta^2(5p)}{3E^4}(F_{66}G_{76}+F_{67}G_{77}+F_{76}G_{66}+F_{77}G_{67}); \tag{5.14}$$

where $\zeta(5p)$ is the spin-orbit interaction parameter for the hole in the $5p$ shell, E is the excitation energy for the $5p$ electron, which is much greater than the difference between the $6s_{1/2}$ and $7s_{1/2}$ levels (so that the latter difference and the hyperfine splitting of the hole level can be neglected), and F and G are the Slater integrals

$$\left.\begin{array}{ll} F_{66}=F_0(6s,\ 5p;\ 6s,\ 6p), & G_{66}=G_1(6s,\ 5p;\ 6s,\ 6p),\\ F_{67}=F_0(6s,\ 5p;\ 7s,\ 6p), & G_{67}=G_1(6s,\ 5p;\ 7s,\ 6p),\\ F_{76}=F_0(7s,\ 5p;\ 6s,\ 6p), & G_{76}=G_1(7s,\ 5p;\ 6s,\ 6p),\\ F_{77}=F_0(7s,\ 5p;\ 7s,\ 6p), & G_{77}=G_1(7s,\ 5p;\ 7s,\ 6p). \end{array}\right\} \tag{5.15}$$

In the foregoing expressions we use the notation adopted in[56]. To estimate β, it is convenient to compare it with the correction to the g-factor for the valence electron in cesium. Experiment shows that[57]

$$\frac{\Delta g_{Cs}}{g}=1.1\cdot 10^{-4}. \tag{5.16}$$

The theoretical prediction for this is[31,54,55]

$$\frac{\Delta g_{Cs}}{g}=-\frac{2\zeta^2(5p)}{3E^4}(F_{66}G_{66}+F_{67}G_{67}). \tag{5.17}$$

To find the relation between the different Slater integrals, we note that, at distances that are characteristic for the electron, the energies of the $6s$ and $7s$ electrons can be neglected in comparison with the potential, so that the ratio of the wave functions for the valence electron in this region is equal to the ratio of the normalizing coefficients [see (5.4) and (5.8)]:

$$\frac{\psi_{70}}{\psi_{60}}=\left(\frac{\nu_6}{\nu_7}\right)^{3/2}=\left(\frac{1.9}{2.9}\right)^{3/2}=0.51. \tag{5.18}$$

Accordingly,

$$\begin{array}{ll} F_{76}\approx F_{67}\approx 0.51F_{66}, & F_{77}=0.26F_{66},\\ G_{67}\approx 0.51G_{66}, & G_{77}\approx 0.51G_{76}. \end{array} \tag{5.19}$$

Finally, there is no reason to suppose that the integrals G_{66} and G_{76} are essentially different. Assuming that they are equal, and using (5.14), (5.16), (5.17), and (5.19), we find that[31,54]

$$\beta\approx 10^{-4}. \tag{5.20}$$

Once the admixture of the $n'p_{1/2}$ states to the $6s_{1/2}$ and $7s_{1/2}$ states has been calculated with the aid of (5.11)–(5.13) [the term independent of Zq in (5.11) can be neglected], all that remains is to determine the matrix elements for the $E1$ transitions $ns-n'p$. This can be done by the Bates–Damgaard method,[58,59] which reduces to the use of hydrogen-like wave functions for the outer electrons. It is well known[60] that, in the case of cesium, this approximation results in excellent agreement with experiment.

It is important to remember, however, that the Bates–Damgaard tables were compiled on the assumption that all the radial wave functions were positive for $r\to\infty$. Moreover, in the calculation of the s and p level mixing, we assumed that the corresponding wave functions were positive at the origin. It is clear that their sign for $r\to\infty$ is then determined by the factor $(-1)^{n_r}$, where n_r is the radial quantum number. The numbers in the Bates–Damgaard tables must therefore be multiplied by the further factor $(-1)^{n_r+n'_r}$.

Since the dipole matrix elements decrease rapidly with increasing difference between the effective principal quantum numbers of the initial and final states, the main contribution to the effect is, in practice, given by the admixture of the $6p_{1/2}$ and $7p_{1/2}$ states to the $6s_{1/2}$ state, and the admixture of the $6p_{1/2}$ to the $7s_{1/2}$ state. Simple estimates based on experimental values of the oscillator strength for cesium show that allowance for all the other states, including the continuum, can hardly affect the result by more than a few percent. The so-called autoionization states, which appear during the excitation of closed-shell electrons, require separate analysis. Experimental data on the corresponding transitions in cesium are insufficient for any meaningful estimates of their contribution. Theoretical estimates indicating that the contribution of the autoionization states is small are reported in[31]. The same conclusion follows from analysis of the polarizability of xenon, which is determined by the dipole matrix element for the excitation of an electron in the filled $5p$ shell.

We have thus outlined the essence of the calculation which leads to the following result for the circular polarization of photons from the $6s-7s$ transition in cesium:

$$P\approx 10^{-4}. \tag{5.21}$$

Bouchiat has proposed the following method for measuring this effect.[24] A tunable laser operating at $\lambda=5395$ Å is used to excite the $6s_{1/2}-7s_{1/2}$ transition in cesium vapor. The fluorescence photons from the $7s_{1/2}-6p_{1/2}$ transition are recorded (see Fig. 6). If parity is not conserved, the probability of excitation of the $6s-7s$ transition and, consequently, the number of fluorescence photons will vary with the sign of the circular polarization of the incident radiation. The relative magnitude of this effect will, clearly, be equal to the degree of circular polarization P [see (1.4)].

Of course, a necessary preliminary stage for this very complicated experiment is the experimental determination of the amplitude for the doubly-forbidden $M1$ transition under consideration. So far, the very ingenious experiment described in[61] has succeeded in yielding only the following upper limit for this amplitude [see (5.14) and (5.20)]

$$\beta<3\cdot 10^{-4}. \tag{5.22}$$

A curious situation, resembling that discussed in Chap. 4, occurs[31] in the odd isotope Pb^{207}. Because of the hyperfine interaction, the $6p^2\,{}^3P_0$ and $6p^2\,{}^1S_0$ states contain an admixture of the $6p^2\,{}^3P_1$ state, so that the $M1$ transition between the perturbed levels, ${}^3\tilde{P}_0-{}^1S_0$ ($\lambda=3394$ Å), becomes possible. The parity nonconserving interaction, which depends on the nucleon spin [see the second term in (5.1)], leads to a circular polarization of the radiation emitted as a result of the transition and, according to the Bouchiat estimates,[51] this amounts to $P\sim 6\times 10^{-4}$. The parity nonconservation effects in the highly forbidden $M1$ transition are considered in[53].

In conclusion of this section, we emphasize that we have been concerned with searches for small circular polarizations in transitions which are so highly forbidden that they have not as yet been observed experimentally.

C. Rotation of the plane of polarization in heavy-metal vapors

A more realistic possibility from the point of view of the detection of parity nonconservation in atomic transitions is provided by the rotation of the plane of polarization in heavy-metal vapor.[28] The fact that parity nonconservation results in the optical activity of a medium was first noted by Zel'dovich.[2]

Let us now consider some possible experiments on the rotation of the plane of polarization. The refractive index for right- and left-handed photons near resonance at frequency ω_0 will be written in the form

$$n_{\pm}=1-\frac{2\pi N\overline{|M_{\pm}|^2}}{\hbar}\left\langle\frac{1}{\omega-\omega_0-(v/c)\,\omega_0+i\,(\Gamma/2)}\right\rangle. \tag{5.23}$$

In this expression, N is the density of atoms in the medium and Γ is the width of the excited state. The operator $M_{\pm}$ for dipole transitions is equal to the corresponding component of the dipole moment. The bar over the symbol in (5.23) represents summation of the square of the matrix element over the final polarizations of the atoms and averaging over the initial polarizations. Angle brackets represent averaging over v (the projection of the velocity of the atom onto the direction of the light beam).

If parity is not conserved, the matrix elements are not equal and can be written in the form

$$M_{\pm}=M\pm FM_1=M\left(1\pm\frac{P}{2}\right), \tag{5.24}$$

where $M1$ is the admixture matrix element corresponding to the "improper" parity. The plane of polarization is then rotated over a length l through an angle given by

$$\begin{aligned}\psi&=\frac{1}{2}\frac{\omega}{c}\,l\,\mathrm{Re}\,(n_+-n_-)\\&=-\frac{2\pi N\omega lF}{\hbar c}\,(\overline{M^*M_1}+\overline{M_1^*M})\left\langle\frac{\omega-\omega_0-(v/c)\,\omega_0}{[\omega-\omega_0-(v/c)\,\omega_0]^2+(\Gamma^2/4)}\right\rangle\\&=-\frac{2\pi N\overline{|M|^2}\omega}{\hbar c}\,lP\left\langle\frac{\omega-\omega_0-(v/c)\,\omega_0}{[\omega-\omega_0-(v/c)\,\omega_0]^2+(\Gamma^2/4)}\right\rangle.\end{aligned} \tag{5.25}$$

Moreover, the absorption coefficients $\alpha_{\pm}$ also turn out to be different for right- and left-handed photons:

$$\begin{aligned}\alpha_{\pm}&=2\,\frac{\omega}{c}\,\mathrm{Im}\,n_{\pm}\\&=\frac{4\pi N\omega}{\hbar c}\,[\,\overline{|M|^2}\pm F\,(\overline{M^*M_1}+\overline{M_1^*M})]\left\langle\frac{\Gamma/2}{[\omega-\omega_0-(v/c)\,\omega_0]^2+(\Gamma^2/4)}\right\rangle\\&=\frac{4\pi N\omega}{\hbar c}\,\overline{|M|^2}(1\pm P)\left\langle\frac{\Gamma/2}{[\omega-\omega_0-(v/c)\,\omega_0]^2+(\Gamma^2/4)}\right\rangle.\end{aligned} \tag{5.26}$$

The polarization will therefore be transformed from linear to elliptic. The ratio of the minor to major semiaxes of the ellipse is

$$\chi=\frac{1}{2}\frac{\omega}{c}\,l\,\mathrm{Im}\,(n_+-n_-)=\frac{l}{4}\,(\alpha_+-\alpha_-). \tag{5.27}$$

It is important to emphasize that the quantities ψ and χ, which, in this case, characterize the parity nonconservation effects, are proportional to the product of the main and the admixture matrix elements [in contrast to the degree of circular polarization P which is proportional to their ratio; see (1.4)]. It is therefore definitely inconvenient to look for the optical activity in the neighborhood of the highly forbidden $M1$ transition. The situation in which the main transition corresponding to the matrix element M is allowed is also inconvenient. Thus, for allowed transitions, the absorption coefficient α is very large, and since the path length l cannot appreciably exceed α^{-1}, the possible rotation ψ and ellipticity χ turn out to be exceedingly small.

It is therefore natural to turn to the case where the $M1$ transition is the main one and the admixture is $E1$. It is well known that the $M1$ transition occurs without additional suppression only between terms corresponding to the same electron configuration. To observe the small optical activity effects, it is then very desirable for this transition to lie in the visible part of the spectrum or near this region. This situation occurs in heavy elements. Finally, the material must have an appreciable vapor pressure at a reasonable temperature. If by "appreciable" we mean a pressure of about 10 mm, and by "reasonable" temperature we mean ~1200 °C, the range of possible elements narrows down to tellurium, iodine, europium, thallium, lead, bismuth, and polonium.

Among these elements, tellurium has the lowest atomic number ($Z=52$) and should therefore exhibit the smallest effect. This disadvantage is not compensated by any advantages. The transitions in which we are interested lie in the infrared in the case of tellurium ($\lambda=9471$ Å, 21 048 Å). Moreover, the vapor of this element is mainly molecular (Te_2). The absorption of light through the molecular component of the tellurium vapor may turn out to be an additional complication.[15)] Similar considerations apply to iodine ($Z=53$, $\lambda=13\,152$ Å).

In the case of europium, the energy states corresponding to the required electron configuration are still unknown.

Finally, polonium ($Z=84$, $\lambda=4613$ Å, 5941 Å) is highly radioactive, which makes it difficult to work with. This is particularly disappointing because, in the case of polonium, the problem of a strong enough monochromatic source of light (see below) is solved by the fact that the second of the above lines is practically coincident with one of the lines generated by the helium-neon laser.

Thus, at present, the most suitable elements for our experiments are thallium, lead, and bismuth.

[15)] It is important to note that searches for the nonconservation of parity in the electronic spectra of molecules containing heavy atoms are also of considerable interest. However, reliable estimates of this effect are exceedingly difficult. Nonconservation of parity in molecular transitions will not, therefore, be considered in the present review.

We must now consider the conditions imposed on the frequency stability and linewidth of the light source. Let us return to (5.23). Averaging over the Maxwellian distribution of the atoms yields

$$\left\langle \frac{1}{\omega-\omega_0-(v/c)\,\omega_0+i\,(\Gamma/2)} \right\rangle = -i\,\frac{\sqrt{\pi}}{\Delta_D}\,e^{-w^2}\,[1-\Phi(-iw)]; \quad (5.28)$$

where

$$\Phi(z)=\frac{2}{\sqrt{\pi}}\int_0^z dz' e^{-z'^2}$$

is the error integral and the complex quantity w in the frequency detuning $\Delta=\omega-\omega_0$ and the Doppler broadening $\Delta_D=\omega_0\sqrt{2kT/Mc^2}$ (M is the mass of the atom) are related by

$$w=u+iv=\frac{\Delta}{\Delta_D}+i\,\frac{\Gamma}{2\Delta_D}.$$

We shall be interested in the situation where the width Γ of the upper level (due to the resonance transfer of excitation by collisions with atoms in the ground state) is much less than the Doppler broadening Δ_D, and Δ is comparable with Δ_D, i.e., $v\ll 1$, $u\sim 1$. The expression given by (5.28) then becomes much simpler and assumes the form

$$\left\langle \frac{1}{\Delta-(v/c)\,\omega_0+i\,(\Gamma/2)} \right\rangle = \frac{1}{\Delta_D}\,[g(u,\,v)+if(u,\,v)]$$
$$\approx \frac{1}{\Delta_D}\,\{g(u)-i\,[\sqrt{\pi}\,e^{-u^2}+2v\,(ug(u)-1)]\}. \quad (5.29)$$

The function $g(u)=2e^{-u^2}\int_0^u e^{z^2}dz$ has the maximum value of 1.1 for $u=0.9$, and $g(u)\approx 1/u$ when u when $u\gtrsim 3$.

We must have $\Delta\sim\Delta_D$ to ensure that the rotation of the plane of polarization ψ is not too small. Since ψ is an odd function of the detuning [see (5.25) and (5.29)], the stability of the source frequency and linewidth should at least be comparable with the Doppler broadening which, in our case, amounts to about $10^{-6}\,\omega_0$. This is less than the hyperfine splitting, so that the transition will occur only between definite hyperfine-structure components. As a result, the optical activity will be reduced since, firstly, not all the atoms in the ground state will participate in the transitions and, secondly, the probability of transition to a definite hyperfine state is, of course, less than the total transition probability.

We shall begin our discussion of the possible size of the effect in the above elements with thallium. The ground state in this element is $6s^26p_{1/2}$ and the $M1$ transition in which we are interested is $6p_{1/2}$–$6p_{3/2}$. The admixture of states of the form $6s^2ns_{1/2}$ to the ground state can be calculated from (5.11). The $E1$ transition amplitude can be deduced from experimental data on the oscillator strengths in thallium.[62,63] The signs of these amplitudes, on the other hand, can be found from the Bates–Damgaard tables.[58] A large contribution to the effect is provided by states corresponding to the $6s6p^2$ configuration and the $6s$ electrons which arise during the excitation. All of them except one have positive energies and are therefore resonances in the continuous spectrum. The expansion coefficients of the wave functions for these states over the Russell–Saunders functions are calculated in the intermediate-coupling approximation. The effective principal quantum numbers for the $6p$ electron in the $6s6p^2$ configuration and the $6s$ electron in the $6s^26p$ configuration can be found by assuming that the corresponding electrons are added to the Tl II ion in the $6s6p$ state. The dipole matrix elements of the $E1$ transitions can be determined from the oscillator strength for the $6s^26p_{1/2}$–$6s6p^{2\,2}D_{3/2}$ transition.[64,65]

In lead and bismuth, the intermediate-coupling approximation must be used even in the calculation of the main configurations ($6s^26p^2$ and $6s^26p^3$) and the usual excited configurations ($6s^26pns$ and $6s^26p^2ns$). Experimental data on $E1$ transitions in these atoms were taken from[66–69]. States belonging to the $6s6p^3$ and $6s6p^4$ configurations have not been observed. Nevertheless, their contribution to the effect can be estimated from the data on the Pb II and Bi II spectra. The corresponding dipole matrix elements were taken from the Bates–Damgaard tables. A comparable estimate for the analogous transitions in thallium is in good agreement with experiment.

The results of the calculations, a detailed account of which is given in[70], are summarized in Table 2. This table is concerned with transitions between states with the highest values of the total angular momentum F of the atom near which the rotation of the plane of polarization is at a maximum. The quantity $\overline{|M|^2}$ represents the result of summation of the square of the amplitude for the $M1$ transition over the components of F and F', divided by the total number of initial states [i.e., it replaces $\overline{|M|^2}$ in (5.23)]. The detuning Δ is chosen so that the absorption coefficient α is 1 m^{-1} at 1200 °C. It is only for the last transition in bismuth, in which the absorption is exceedingly small, that Δ was chosen from the requirement that the angle ψ was a maximum. The vapor pressure of thallium, lead, and atomic bismuth at this temperature is 100, 17, and 23 mm, respectively.[71] The values of the scattering cross section σ which we have adopted for thallium on thallium, lead on lead, and bismuth on bismuth are not inconsistent with the only known experimental results, according to which $\sigma<10^{-14}$ cm^2 for thallium.[72] The much smaller rotation angles in the case of bismuth are due to the fact that its vapor pressure is lower than that of thallium, the transition amplitudes are smaller than in lead, and the angular momentum of the bismuth nucleus is large, which leads to a complicated hyperfine structure. We note, finally, that the transitions in thallium and bismuth may also take the form of electric quadrupole transitions which will, in general, lead to additional absorption of light. However, estimates show that the amplitudes of the $E2$ transitions are small

TABLE 2. Parameters of $M1$ transitions in heavy atoms.

Atom	J	F	J'	F'	λ, Å	$\overline{\lvert M\rvert^2}/\mu^2$	$P\cdot 10^7$	$\sigma\cdot 10^{14}$, cm^2	Δ/Δ_D	$\psi/l\cdot 10^7$, rad/m
Tl	1/2	1	3/2	2	12832.8	0.139	3.1	0.5	5.3	79
Pb^{208}	0	0	1	1	12788.93	0.572	2.1	0.5	2.6	78
Bi	3/2	6	3/2	6	8757.45	0.056	2.0	$\leqslant 1$	2.0	15.7
			5/2	7	6477.23	0.007	2.3	$\leqslant 1$	1.4	3.3
			1/2	5	4616.39	0.010	4.5	$\leqslant 1$	1.5	9.2
			3/2	6	3015.22	0.0007	7	$\leqslant 1$	0.9	1.2

enough to ensure that the quadrupole transition can be neglected. The only exception is the 3/2–5/2 transition in bismuth, where the $E2$ amplitude is responsible for about 16% of the absorption.

A few words, now, about the possibility of detecting the effects. Rotations of $\sim 10^{-6}$ rad can certainly be detected at the present time (see[73–75]). The most promising experiments are those involving bismuth vapor, in which two transitions ($\lambda = 6477$ Å and 4616 Å) lie in the region covered by modern tunable dye lasers. The $\lambda = 3015$ Å transition can be investigated with the aid of a similar tunable laser after frequency doubling. Finally, the long-wave transition $\lambda = 8757$ Å lies in the working region of tunable semiconductor lasers based on gallium arsenide.

It is important to note, however, the relatively complicated problem presented by the random external magnetic field which is also known to produce a rotation of the plane of polarization. There are a number of mechanisms through which a magnetic field will produce optical activity. We shall briefly consider two of the most hazardous from our point of view.

Firstly, an external magnetic field leads to the mixing of different hyperfine states. The longitudinal magnetic field H_1 which will simulate the effect can be estimated from the formula $\mu H_1/\Delta_{HF} \sim P$, where Δ_{HF} is the hyperfine splitting energy. Hence, we have the condition $H < H_1 \sim 10^{-2}$–10^{-3} G.

Another mechanism which leads to more stringent limitations on the magnetic field is the difference between the resonance frequencies for right- and left-handed photons, which is due to the Zeeman splitting of the lines by the longitudinal field.[16] The restriction on this field, which follows from the formula $\mu H_2/\Delta \sim P$, is: $H < H_2 \sim 10^{-4}$–10^{-5} G. We note that the rotation of the plane of polarization in a magnetic field due to the second mechanism is an even function of the detuning Δ, in contrast to the rotation due to parity nonconservation. This result can be used as a means of reducing the background.

D. Rotation of the plane of polarization in the radiofrequency region

Parity nonconservation effects in heavy atoms, which depend on the nucleon spin, are equally interesting. They appear in transitions between hyperfine structure components of a given electronic term. These transitions lie in the radiofrequency range and are also magnetic dipole transitions, so that they are convenient for observation of optical activity due to parity nonconservation.

Since, in this case, the effect is produced by the interaction between an electron and one unpaired nucleon rather than all the nucleons in the nucleus, as in the case of optical transitions, the circular polarization is, roughly speaking, smaller by a factor of Z than in the optical region. However, the precision that can be achieved in modern centimeter-wave techniques is much higher than in optics,[17] so that consideration of parity nonconservation in the radiofrequency region can be regarded as justified. The degree of circular polarization in the case of transitions between hyperfine-structure components has been calculated for cesium and thallium.[27,70] We shall not consider it here because it is not very different from the calculations discussed in the preceding section. We merely reproduce the results:

1) Cesium ($\lambda = 3.26$ cm): $P = -0.6\times 10^{-9}\varkappa_{2p}$.
2) Thallium ($\lambda = 1.42$ cm): $P = 1.3\times 10^{-8}\varkappa_{2p}$.

Linearly polarized radio waves propagating through cesium or thallium vapor should exhibit rotation of the plane of polarization. Moreover, the polarization itself should become elliptic. All these effects are described by the same formulas as in the optical region, namely, (5.23)–(5.27). The only difference is that, in the present case, the Doppler broadening Δ_D for pressures of about 10^{-2} mm or more is small in comparison with collisional broadening of the upper level. Averaging over the velocity distribution of the atoms is therefore unimportant in this case, and the term $-(v/c)\omega_0$ in the denominator of (5.23) and the symbol $\langle\ldots\rangle$ can simply be omitted. We shall confine our attention to this case henceforth.

The other difference which leads to a considerable complication in the experiments which we are considering is that the upper and lower hyperfine-structure levels must have different populations for optical activity. The symbol N in the above formulas must be interpreted as the difference between the densities of atoms occupying the upper and lower states. If we confine our attention to the natural temperature difference between the populations, we find that the effects are additionally reduced by a factor of 100–1000. However, a population difference much greater than the difference due to the temperature alone can be achieved by laser excitation of one of these levels. Steps must then be taken to exclude the possibility of optical orientation of the atoms. We shall confine our attention to this particular case, i.e., we shall assume that the probability Γ_e of excitation of the upper level by the laser beam is much greater than the probability of inelastic collisions which tend to equalize the populations of the hyperfine-structure components.

The rotation of the plane of polarization is a maximum when the detuning is equal to the half-width of the line which, in the present case, is $\Gamma + \Gamma_e$. Calculations lead to the following results:

1) *Cesium.* (The dephasing cross section which is

[16]Since the laser frequency is unavoidably different from the the resonance frequency, this mechanism must also be taken into account in the experiment proposed by Bouchiat.[24]

[17]Even when modulation methods are not employed, the sensitivity with which rotations of the plane of polarization can be measured in the radiofrequency region can reach 10^{-6}–10^{-8} rad.[76–78] In optics, the modulation technique can be used to increase the sensitivity of ellipsometric measurements by at least three orders of magnitude.

responsible for the collisional linewidth is $\sigma = 2.3\times10^{-14}$ cm^2.[79]) The absorption coefficient is $\alpha = 0.08[\Gamma/(\Gamma+\Gamma_e)]$m^{-1}

$$\frac{\psi_{\max}}{l}=\frac{1}{2}\alpha P\approx-0.25\cdot10^{-10}\varkappa_{2p}\frac{\Gamma}{\Gamma+\Gamma_e}\ \text{rad/m}.$$

2) *Thallium*. (We adopt the same value for σ as in the case of cesium.) $\alpha = 0.03[\Gamma/(\Gamma+\Gamma_e)]$m^{-1}

$$\frac{\psi_{\max}}{l}\approx1.8\cdot10^{-10}\varkappa_{2p}\frac{\Gamma}{\Gamma+\Gamma_e}\ \text{rad/m}$$

It is convenient to work with a detuning equal to the half-width of the line because the most hazardous, second background mechanism for the rotation of the plane of polarization due to the random external magnetic field (see preceding section) is unimportant in the present case. To avoid the simulation of the effect by the first mechanism, the mean longitudinal magnetic field must be less than $2\times10^{-6}\varkappa_{2p}$ G in cesium and $3\times10^{-4}\varkappa_{2p}$ G in thallium.

E. Parity nonconservation effects in superconductors

The parity nonconserving interaction between electrons and nucleons can, in principle, be detected with the aid of certain effects[80] in superconductors with polarized nuclei.[18)]

Consider the weak interaction between electrons and nuclei in a superconductor of this kind. Since the electrons are unpolarized, the only term present in the potential (1.20) is that proportional to $\varkappa_2$. The parity nonconserving interaction between the electron and the nucleus takes the form

$$V=\frac{G\hbar^3 g_I I}{2\sqrt{2}\,mc^2}\sum_i\left[\xi_i\mathbf{p}\delta(\mathbf{r}-\mathbf{r}_i)+\delta(\mathbf{r}-\mathbf{r}_i)\xi_i\mathbf{p}\right],\qquad(5.30)$$

where ξ_i is a unit vector indicating the direction of the spin of the i-th nucleus, $\mathbf{r}_i$ is the position vector, and the constant g_I is introduced in (5.11).

If we average (5.30) over the electron wave functions of a Cooper pair, we obtain the following expression for the P-odd addition to the effective Hamiltonian describing the motion of the pair as a whole:

$$V_{\text{eff}}=\frac{G\hbar^3 g_I IKN}{2\sqrt{2}\,m_e c^2}\left[\mathbf{p}\zeta(\mathbf{r})+\zeta(\mathbf{r})\mathbf{p}\right];\qquad(5.31)$$

where m_e is the effective mass of the electron, N is the density of nuclei, $\zeta(\mathbf{r})$ is the nuclear polarization vector (its modulus is equal to the degree of polarization), and the factor K represents the difference between the electron current in the neighborhood of the nucleus and the average current within the crystal:

$$K=\frac{|\operatorname{Im}\psi_{\mathbf{k}}^*\nabla\psi_{\mathbf{k}}|_{\mathbf{r}=\mathbf{r}_{\text{nucleus}}}}{|\operatorname{Im}\psi_{\mathbf{k}}^*\nabla\psi_{\mathbf{k}}|_{\text{av}}}\qquad(5.32)$$

In this expression, $\psi_{\mathbf{k}}(\mathbf{r})$ is the electron wave function in the crystal and the crystal momentum of the electron $\mathbf{k}$ is assumed to be small. Crystal anisotropy is neglected in (5.31) and (5.32).

The interaction given by (5.31) has a form analogous to the electromagnetic interaction, and we can readily show that it can be included (in the first order in G) by introducing the substitution

$$\frac{2e}{c}\vec{\mathcal{A}}\to\frac{2e}{c}\vec{\mathcal{A}}-\frac{2G\hbar^3 g_I IKN}{\sqrt{2}\,c^2}\zeta.\qquad(5.33)$$

We must now consider the associated change in the electrodynamics of the superconductors. We shall confine our attention to London-type superconductors although it is clear that this restriction is unimportant insofar as our conclusions will be concerned.

The Maxwell equations for a constant magnetic field now have the form

$$\operatorname{curl}\mathbf{H}=\frac{4\pi}{c}\,\mathbf{j}=\frac{4\pi e\rho}{m_e c}\left(\hbar\nabla\varphi-\frac{2e}{c}\vec{\mathcal{A}}+\frac{2G\hbar^3 g_I IKN}{\sqrt{2}\,c^2}\zeta\right);\qquad(5.34)$$

where ρ is the density of Cooper pairs and φ is the phase of their wave function.

If we take the curl of the equation given by (5.34), we obtain;

$$\left(-\Delta+\frac{8\pi e^2\rho}{m_e c^2}\right)\mathbf{M}=\frac{8\pi e\rho G\hbar^3 g_I IKN}{\sqrt{2}\,m_e c^2}\operatorname{curl}\zeta.\qquad(5.35)$$

Thus, neither the magnetic field nor the current will, in general, be zero within the superconductors. However, when polarization is achieved in the usual way with the aid of an external magnetic field $\mathbf{H}_0(\mathbf{r})$, the polarization vector $\zeta(\mathbf{r})$ is obviously proportional to $\mathbf{H}_0(\mathbf{r})$, so that curl $\zeta(\mathbf{r})=0$, i.e., neither the magnetic field nor the current penetrate the superconductor. Henceforth, we shall restrict our attention to this case for simplicity.

The question now is—what is the effect of the interaction given by (5.31) on the physics of superconductors? It is clear from the analogy between $\vec{\mathcal{A}}$ and ζ [see (5.33)] that the quantization condition for the magnetic flux Φ through the superconducting ring is modified. It now takes the form

$$\frac{2e}{\hbar c}\Phi-2f=2\pi m\qquad(m=0,1,2,\ldots),$$
$$f=\frac{G\hbar^2 g_I IKN}{\sqrt{2}\,c^2}\oint d\mathbf{r}\zeta(\mathbf{r}).\qquad(5.36)$$

We now draw attention to the relatively profound analogy between the vector potential $\vec{\mathcal{A}}$ and the polarization ζ. In particular, the relation given by (5.36) differs from the usual quantization condition for a magnetic flux by the term representing the flux of the quasimagnetic field which is proportional to curl ζ (in the situation which we are considering, curl $\zeta\neq0$ on the boundary of the superconductor).

On the other hand, despite the presence of currents and magnetic fields near the boundary of the superconductor, which modify the orientation of nuclei in this region, the derivation of (5.36) requires only the existence of a closed circuit at all points of which $\mathbf{j}=0$, $\mathbf{H}=0$, and curl $\zeta=0$.

We now take as the second example the flow of current through two Josephson contacts connected in

---

18) The nuclei can be oriented by an external magnetic field, and this is followed by the establishment of the superconducting state. We assume that the nuclear spin relaxation time, which can reach a few seconds,[81,82] is sufficient for the observation of the effects we are discussing.

parallel. It is well known[83] that the expression for the maximum current is (for simplicity, we consider identical contacts)

$$I_{\max}=2I_c\left|\cos\frac{e\Phi}{\hbar c}\right|, \quad (5.37)$$

where I_c is the maximum current through one contact and Φ is the magnetic flux through the circuit. Inclusion of the P-odd interaction (5.31) results in the following modification of (5.37), just as in the case of the quantization of flux:

$$I_{\max}=2I_c\left|\cos\left(\frac{e\Phi}{\hbar c}-f\right)\right|. \quad (5.38)$$

Thus, the quantity $I_{\max}$ depends on the sign of Φ, i.e., it will change as a result of a change in the relative orientation of the frame and the external magnetic field. This can be used to make the effect a periodic function of time by rotating the system because the orientation of the nuclei in space will then remain the same.

Let us now estimate the magnitude of this effect. The contribution of weak interactions is determined by the dimensionless parameter

$$f\sim\frac{G\hbar^2 g_I IKN|\zeta|L}{\sqrt{2}c^2}. \quad (5.39)$$

where L is the length of the circuit. It is assumed that the nuclei are polarized in the direction of the circuit.

Estimates of the factor K given by (5.32) are not at all trivial in this case. The electron wave function $\psi_{\mathbf{k}}(\mathbf{r})$ can be found by the Wigner–Seitz method.[84] In the first order in $\mathbf{k}$, it is given by

$$\psi_{\mathbf{k}}(\mathbf{r})=e^{i\mathbf{k}\mathbf{r}}\left(u_0+\frac{1}{m}\sum_{\alpha\neq 0}\frac{\langle\alpha|\mathbf{k}\mathbf{p}|0\rangle}{E_0-E_\alpha}u_\alpha\right), \quad (5.40)$$

where $\mathbf{p}$ is the momentum operator and u_α is the wave function for the lowest state in the band. The functions u_α are such that their normal derivative vanishes on the boundary of the unit cell. The potential is usually chosen to be the same as in the case of the atomic problem, so that the only difference is in the boundary conditions. This difference can be neglected in estimates of the factor K.

It is clear from (5.40) and (5.32) that the factor K differs from zero in the small-$\mathbf{k}$ approximation only in the case of the s and p conduction bands (i.e., when u_0 describes the s or p state).

Since we are concerned with the interaction between electrons and one unpaired nucleon in the nucleus, the factor K is proportional to Z^2 [see (1.8) and (5.11)]. Recalling the relativistic enhancement factor R given by (5.12), we find that, for heavy metals and $|\zeta|\sim 1$, the parameter f may exceed 10^{-6} L/cm.

Although the precision that can be achieved at present is clearly insufficient for the observation of these phenomena, the discussion of parity nonconservation effects in superconductors is relevant, at least from the methodological point of view.

6. μ-MESIC ATOMS

A. The $2S\to 1S+\gamma$ transition

An interesting and evidently very realistic possibility of an application of the ideas discussed in Chap. 3 in connection with the hydrogen atom is afforded by the single-photon $2S-1S$ transition in μ-mesic atoms, i.e., atoms in which one of the electrons is replaced by a muon. The muon orbits lie much lower than any of the electronic orbits because of the large muon mass ($m_\mu = 206.8m_e$), and the screening effect of the electrons is negligible. This is why μ-mesic atoms are virtually ideal examples of hydrogen-like systems, whatever the atomic number Z. Studies of parity nonconservation effects in mesic atoms have a number of advantages as compared with the hydrogen atom. They include, above all, the large magnitude of the effect which is connected with the large mass of the muon, i.e., the small radius of the Bohr orbit $a_\mu=\hbar/(\alpha m_\mu c)\approx 2.6\times 10^{-11}$ cm. The use of mesic atoms with different Z should enable us to obtain a more favorable relationship between the probability for the $2S\to 1S+\gamma$ transition and any competing processes. Moreover, because of the large difference between the level energies in mesic atoms, there is practically no restriction on the random external electromagnetic fields, which were quite stringent in the case of the hydrogen atom. Experiments with mesic atoms are also interesting because comparison of the weak interaction constants for electrons and muons interacting with nucleons should enable us to establish whether the $\mu-e$ universality extends to the weak interaction between neutral currents. The principal difficulty in experiments involving mesic atoms appears to be the production of a sufficiently large number of such atoms, so that the effect can be observed with acceptable statistics.

Let us begin by considering parity nonconservation effects in mesic atoms, neglecting the hyperfine interaction between the muon and the nucleus.[28–30] In this case, the weak interaction potential between the muon and the nucleus need retain only terms proportional to the muon spin $\mathbf{s}=\sigma/2$:

$$V=\frac{G\hbar^3}{2\sqrt{2}m_\mu c^2}Zq_\mu\sigma[\mathbf{p}\delta(\mathbf{r})+\delta(\mathbf{r})\mathbf{p}],$$
$$q_\mu=\varkappa_{1p}+\frac{N}{Z}\varkappa_{1n}, \quad (6.1)$$

where m_μ is the reduced mass of the muon in the mesic atom, Z is the number of protons (nuclear charge), and N is the number of neutrons in the nucleus.[19)] The formulas that describe electromagnetic transitions and parity nonconservation effects in mesic atoms are analogous to the corresponding formulas for the hydrogen atom. In particular, the magnitude of the parity nonconservation effects is characterized by the single parameter $P=-2Fr$, where

$$F=-\frac{G\sqrt{3}}{32\pi\sqrt{2}}Zq_\mu\frac{m_\mu^3c^2(Z\alpha)^4}{E(2S)-E(2P)}, \quad (6.2)$$

$$r=\sqrt{\frac{W_P}{W_S}}=\frac{2^5}{3\sqrt{3}}(Z\alpha)^{-3}\approx 1.6\cdot 10^7 Z^{-3}, \quad (6.3)$$

and the probabilities of the $M1$ transitions $2S\to 1S+\gamma$, and the admixture of the $E1$ transition $2P\to 1S+\gamma$, are

[19)] If the $\mu-e$ universality occurs in the weak interaction between neutral currents, the constants $\varkappa_{1p}$ and $\varkappa_{1n}$ are equal to the corresponding constants in the electron-nucleon interaction and $q_\mu=q$ [see (5.1)].

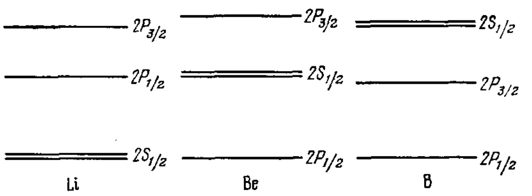


FIG. 7. Levels with $n=2$ in light μ-mesic atoms ($Z=3, 4, 5$).

given by

$$W_S=\frac{1}{2^2 3^5}\alpha(Z\alpha)^{10}\frac{m_\mu c^2}{\hbar}, \tag{6.4}$$

$$W_P=\left(\frac{2}{3}\right)^8\alpha(Z\alpha)^4\frac{m_\mu c^2}{\hbar}. \tag{6.5}$$

The most important difference between mesic atoms and the hydrogen atom is that the reason for the energy difference between the 2S and 2P levels in the mesic atoms is completely different from that in hydrogen.[20)] In the hydrogen atom, the main contribution to this difference is due to correction to the proper energy of the bound electron. In the case of the μ-mesic atoms, the energy difference between the 2S and 2P levels is determined largely by vacuum polarization and by the influence of the finite size of the nucleus. Vacuum polarization, i.e., the creation of virtual electron-positron pairs, distorts the field of the nucleus at distances of the order of the Compton wavelength of the electron ($r \lesssim \hbar/m_e c$), and this is much less than the radius of the electron orbits ($\hbar/\alpha m_e c$) but is comparable with the radii of the μ-mesic orbits ($\hbar/Z\alpha m_\mu c$). Vacuum polarization ensures that the effective charge on the nucleus at small distances is greater than Z (by definition, Z is the charge at large distances) and, consequently, the energy of all the levels and especially the S levels is reduced. Vacuum polarization plays a predominant role in the lightest mesic atoms (H, He, and Li). In these atoms, the $2S_{1/2}$ level lies below the $2P_{1/2}$ level. As Z increases from $Z=4$ onward, the main contribution to the energy difference between the 2S and 2P levels is due to corrections connected with the finite size of the nucleus. They lead to an effect which is opposite to vacuum polarization, i.e., they increase the S-level energy. The P-level energies are unaffected because the wave functions for these states are practically zero inside the nucleus. Owing to this effect, the $2S_{1/2}$ level in all mesic atom with $Z \geq 4$ lies above the $2P_{1/2}$ level (Fig. 7). In lithium and beryllium, the effect of vacuum polarization and of the finite linear dimensions of the nucleus are mutually compensated to a considerable extent. In these mesic atoms, the energy difference between the 2S and 2P levels is particularly small and, consequently, one would expect large parity nonconservation effects.

TABLE 3. Parameters of the $2S \to 1S$ transition in light mesic atoms.

Mesic atom	$E(2S)-E(2P_{1/2})$, eV	E_γ, keV	W_S, sec^{-1}	$P/q_\mu \cdot 10^2$
$^1H_1$	-0.201	1.90	$4.64\cdot10^{-4}$	-5.3
$^4He_2$	-1.37 ± 0.01	8.21	$5.14\cdot10^{-1}$	-4.0
$^6Li_3$	-1.1 ± 0.2	18.6	$2.99\cdot10^{1}$	-12
$^9Be_4$	1.3 ± 0.8	33.3	$5.34\cdot10^{2}$	17
$^{10}B_5$	10.6 ± 2.5	52.2	$4.98\cdot10^{3}$	3.4
$^{12}C_6$	30 ± 2	75.2	$3.10\cdot10^{4}$	1.7
$^{16}O_8$	162 ± 7	134	$5.50\cdot10^{5}$	0.6

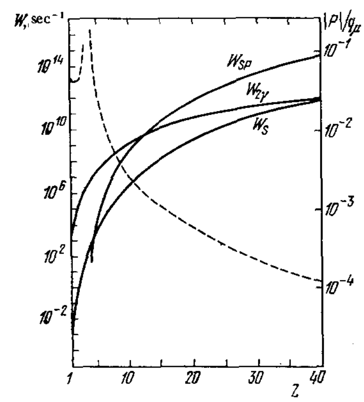


FIG. 8. Probabilities of radiative transitions from the 2S level (solid lines) and the circular polarization of gamma rays (broken line) in μ-mesic atoms.

Table 3 lists theoretical calculations of the $2S_{1/2} - 2P_{1/2}$ level-energy difference for light mesic atoms.[28,30,85,86] In these calculations, vacuum polarization was taken into account in the lowest order in α,[87] and the level shift due to the finite linear dimensions of the nucleus was estimated from the formula

$$\delta E_{2S}=\frac{1}{12\hbar^2}(Z\alpha)^4 m_\mu^3 c^4\langle r^2\rangle, \quad \delta E_{2P}=0, \tag{6.6}$$

where $\langle r^2\rangle^{1/2}$ is the root mean square radius of the charge distribution in the nucleus.[21)]

Using these data, one can calculate the parameter P which characterizes the parity nonconservation effects in the single-photon $2S \to 1S$ transition from (6.2) and (6.3). The values of this parameter and of some other parameters for the $2S \to 1S$ transition in light mesic atoms[28,30] are listed in Table 3. Estimates of the parity nonconservation effects in mesic atoms with large Z[29] show that the mixing factor F is a slowly varying function of Z, and is approximately given by

$$F \sim (1.0-1.3)\cdot10^{-7} q_\mu. \tag{6.7}$$

Hence, the value of P for heavy mesic atoms is approximately given by (see also Fig. 8)

$$P \sim (3-4)\, Z^{-3} q_\mu. \tag{6.8}$$

In particular, for mesic atoms with $Z \sim 30-60$, one would expect that $P \sim 10^{-4}-10^{-5}$. These estimates show

---

20) It is precisely for this reason that parity nonconservation effects in mesic hydrogen are greater than in ordinary hydrogen not by a factor of $(m_\mu/m)^2 \sim 4\times10^4$, as one would expect from simple dimensional considerations (according to which the effects are proportional to Gm_e^2), but, as will be seen from the ensuing analysis, they are greater by only two orders of magnitude.

21) We note that uncertainties in the experimental values of the nuclear radii lead to relatively large uncertainties in calculations of the energy difference between the 2S and 2P levels in light mesic atoms, which, in the case of lithium, beryllium, and boron, amount to a few tens of percent. The corresponding uncertainties arise also in estimates of P. Direct experimental determination of the energy difference between the 2S and 2P levels would substantially increase the precision of theoretical estimates of parity nonconservation effects.

that light mesic atoms should exhibit greater nonconservation effects and are therefore convenient for investigation.

B. Possible P-odd correlations

We must now consider correlations, the observation of which would enable us to determine experimentally the quantity P and hence the weak-interaction constants for the muon-proton and muon-neutron interactions. An important fact is that muons are always created in a polarized state and, in a number of mesic atoms (for example, in carbon), a relatively high degree of residual muon polarization persists even after capture of the mesons into the $2S_{1/2}$ orbit. This leads to new (as compared with the hydrogen atom) possibilities.

The probability of emission of γ rays in the direction n in the course of the $2S_{1/2}\to 1S_{1/2}+\gamma$ transition is given by

$$dW(\mathbf{n}) = W_S\,[1+(\boldsymbol{\xi}\cdot\mathbf{n})(\mathbf{s}_\gamma\cdot\mathbf{n})+P(\mathbf{s}_\gamma\cdot\mathbf{n})+P(\boldsymbol{\xi}\cdot\mathbf{n})]\,\frac{d\Omega}{8\pi}, \qquad (6.9)$$

where W_S is the total probability of the transition (6.4), $\mathbf{s}$ is the spin of the photon, and ξ is the polarization vector of the muon in the $2S_{1/2}$ state. The parity conserving correlation between the spins of the gamma ray and the muon $(\xi\cdot\mathbf{n})(\mathbf{s}_\gamma\cdot\mathbf{n})$ impedes the observation of the circular polarization of the photon which is connected with the nonconservation of parity and is proportional to P. In fact, according to (6.9), the circular polarization of the gamma rays traveling in the direction n is given by

$$P_\gamma(\mathbf{n}) = \frac{W_+-W_-}{W_++W_-} = \frac{P+(\xi\cdot\mathbf{n})}{1+P(\xi\cdot\mathbf{n})}. \qquad (6.10)$$

Special steps must therefore be taken in experiments on the circular polarization in mesic-atom transitions in order to reduce the effect of terms of the form $\sim(\xi\cdot\mathbf{n})$.

The polarization of muons in the initial state, on the other hand enables us to determine P from measurements of another P-odd correlation, namely, the angular asymmetry of the gamma rays relative to the muon polarization vector ξ. This correlation can be obtained by summing (6.9) over the polarizations of the gamma rays:

$$dW(\mathbf{n}) = W_S\,[1+P(\xi\cdot\mathbf{n})]\,\frac{d\Omega}{4\pi}. \qquad (6.11)$$

In principle, other more complicated correlations can also be observed. For example, nonconservation of parity may lead to the polarization of the meson in the final $1S$ state as a result of the $2S\to 1S+\gamma$ transition even when the initial $2S$ state is not polarized. The degree of this polarization is also given by the parameter P and can be determined by examining the angular asymmetry of electrons originating in the decay of polarized muons. Thus, the quantity to be determined in this experiment is the correlation between the directions of emission of the gamma ray in the $2S\to 1S$ transition and the direction of emission of the electron from the subsequent muon decay.

C. Competing processes

Despite the considerable magnitude of the parity nonconservation effects in the mesic-atom $2S\to 1S+\gamma$ transition, their observation is substantially complicated by the presence of two other transitions from the $2S$ state. They do not affect the magnitude of the parity nonconservation effects but they do impede the attainment of the necessary statistics. We shall now briefly consider estimates of the probabilities of the main competing processes in light mesic atoms where parity nonconservation effects are particularly large. The probabilities of these processes must be compared with the probability of the single-photon transition [see (6.4) and Table 3].

1) *Auger transitions*. Apart from the single-photon transition, there is another possible transition between the $2S$ and $1S$ states which does not involve the emission of a gamma ray but, instead, the emission of an electron from the K shell ($E0$ transition). The probability of this process is practically independent of Z, and is approximately equal to 2×10^9 sec^{-1}. For light mesic atoms (with $Z<12$), this process is the main competitor to the single-photon $2S\to 1S$ transition. The exception is the mesic atom of hydrogen, where the Auger transition is forbidden.[22] We note that, in principle, Auger electrons can also be used in searches for parity nonconservation effects which, in this case, take the form of an angular asymmetry in the distribution of the emitted electrons relative to the muon polarization.[89] The asymmetry coefficient for this process is practically independent of Z and is given by $P_e=-2F(m_\mu/m_e)^{1/2}$ which, for beryllium, yields $\sim 10^{-5}$.

2) *The two-photon* $2S\to 1S$ *transition*. In the case of the ordinary hydrogen atom, this process plays the dominant role. For the mesic atoms, the probability of the two-photon transitions can be estimated from the formula

$$W_{2\gamma}\approx 1.6\ 10^3 Z^6\ \mathrm{sec}^{-1} \qquad (6.12)$$

Figure 8 shows a graph of the probability of this process for different mesic atoms. The probability of the two-photon transition increases with increasing Z more slowly than the probability of the single-photon transition which is proportional to Z.[10] The relative role of the two-photon transition will therefore fall with increasing Z.

3) $E1$ *transition* $2S\to 2P+\gamma$. In mesic atoms with $Z>4$, the $2S_{1/2}$ level lies below the $2P_{1/2}$ and $2P_{3/2}$ levels and, therefore, the $E1$ transitions $2S\to 2P+\gamma$ are possible. The probability W_{SP} of this process is a very rapidly varying function of Z (roughly speaking, it is proportional to $Z^{10}A^2$, where A is the atomic number); numerical estimates of W_{SP}[29] are shown in Fig. 8. For mesic atoms with $Z>12$, this transition is the dominant one.

4) *Muon decay and muon capture by the nucleus*. The

[22]Unfortunately, in the hydrogen mesic atom, there is a specific mechanism for transitions from the $2S$ state. Because of the small size of the mesic hydrogen (the absence of electron shells in this atom), the metastability of the $2S$ level is violated mainly by collisions between the mesic hydrogen and other atoms.[88]

probability of muon decay is approximately 4.5×10^5 sec^{-1}. The probability of direct capture of a muon by the nucleus from the $2S$ state is very dependent on the nuclear structure. However, the order of magnitude of this probability can be estimated with the aid of the probability W_0 for capture from the $1S$ state in mesic hydrogen, using the formula

$$W_{\text{cap}} \approx \frac{1}{8} Z^4 W_0 \approx 30 Z^4 \ \text{sec}^{-1}.$$

Consequently, these processes have a low probability as compared with the other competitors to the $2S\to1S+\gamma$ transition.

The above estimates indicate that the main processes which impede the observation of the single-photon transition $2S\to1S$ are Auger transitions (for mesic atoms with $Z<12$) and electromagnetic $2S\to2P$ transitions (for mesic atoms with $Z>12$). The most favorable relationship between the $2S\to1S+\gamma$ transition and the competing processes is obtained for mesic atoms with a nuclear charge of about 10. In this case, the ratio $W(2S\to1S+\gamma)/W(2S\to\text{all})$ is 10^{-2}–10^{-3}, and parity nonconservation effects are expected at a level of about 10^{-3}. Further increase in Z is inconvenient mainly because of the reduction in the magnitude of the effects.

D. Allowance for hyperfine structure

So far, in our discussion of parity nonconservation effects in mesic atoms, we have neglected the hyperfine interaction between the muon and the nucleus. This approximation enables us to find the main, and nuclear-spin independent, contribution of the weak interaction, which is proportional to the constants $\varkappa_{1p}$ and $\varkappa_{1n}$. When the nucleus has nonzero spin, the hyperfine interaction leads to a splitting of the $2S_{1/2}$ level (just as in the case of the $1S_{1/2}$ level) into two sublevels with total angular momentum of the mesic atom $F=I\pm1/2$. Because of the weak interaction (2.10), in each of these $2S$ states there is an admixture of the $2P_{1/2}$ state (and, in general, the $2P_{3/2}$ state) with the same total angular momentum F. The contribution of terms in the weak potential, which contain the nucleon spin, to the magnitude of the admixture of states with different F is different. This in turn, leads to a dependence of the P-odd correlations in the $2S\to1S$ transition on the constants $\varkappa_2$ and $\varkappa_3$. Experimental studies of this dependence are of major interest because they should be able to establish the spin structure of the weak interaction between neutral currents. In particular, there is considerable interest in the verification of the result $\varkappa_{2N}=-\varkappa_{3N}$ (see Chap. 2).[23] We shall confine our attention to a qualitative description of effects associated with hyperfine interactions. These questions are discussed in greater detail in[90,91].

[23] If this result is valid, then the admixture of the $2P_{3/2}$ state to the $2S_{1/2}$ state is absent even when the hyperfine interaction is taken into account. The most convenient mesic atoms in this connection are B^{10} and B^{11} in which the separation between the $2P_{3/2}$ and $2S_{1/2}$ levels is relatively small, so that the role of the possible $2P_{3/2}$ admixture is relatively important.

The influence of the hyperfine interaction on parity nonconservation effects is particularly important in the light mesic atoms such as Li, Be, and B. This is so for two reasons.

Firstly, by Pauli's principle, the nucleon spins in the nucleus have a tendency to cancel each other out. In the case of nuclei with large Z, therefore, the terms in the weak interaction potential which depend on the nucleon spins lead to small corrections (of the order of Z^{-1}) in comparison with terms containing only the spin of the muon. Secondly, as indicated above, for mesic atoms such as lithium and beryllium, the energy difference between the $2S$ and $2P$ levels is anomalously low (because of the mutual cancellation of the contribution due to the vacuum polarization and the finite size of the nucleus) and is comparable with the hyperfine splitting of these levels.

The maximum information on the spin structure of the weak interaction can be obtained by measuring the circular polarization, the angular asymmetry of the emitted gamma rays, and so on, in transitions between individual hyperfine components of the $2S_{1/2}$ and $1S_{1/2}$ levels. These effects are the most sensitive to the constants $\varkappa_2$ and $\varkappa_3$. However, such measurements necessitate the energy separation of gamma rays corresponding to different hyperfine transitions. In the case of light mesic atoms, this requires an energy resolution $\Delta\omega/\omega\sim10^{-4}$–$10^{-5}$.

In simpler experiments, the gamma-ray energies need not be separated, and P-odd correlations averaged over all the hyperfine transitions can be determined. Different results are then obtained for the circular polarization of photons and the correlations between the muon spin and the direction of emission of the gamma rays. The mean circular polarization is practically independent of $\varkappa_{2N}$ and $\varkappa_{3N}$. This can be expected simply on the basis that the statistical averaging over hyperfine transitions is equivalent to averaging the effect over the nucleon spins.

At the same time, the asymmetry coefficient describing the angular distribution of the gamma rays retains the dependence on $\varkappa_{2N}$ and $\varkappa_{3N}$ even after summation over the hyperfine transitions. The point is that the hyperfine interaction has an important effect on the degree of polarization of the muon in the $2S_{1/2}$ state because of the considerable lifetime of this state[92] (the $2S$ level width is much smaller than its hyperfine splitting). As a result, the main contribution to the asymmetry is provided by transitions from the $2S$ state with total angular momentum $F=I+1/2$, in which the degree of polarization is a maximum, i.e., there is no statistical averaging.

The different dependence of the circular polarization and the asymmetry coefficients on the weak interaction constants $\varkappa_1$, $\varkappa_2$, $\varkappa_3$ can, in principle, be used to determine the spin structure of the weak interaction by comparing these correlations, even without separating the individual hyperfine transitions. However, this requires a knowledge of the degree of polarization of the mesic atoms in different hyperfine $2S$ states. This quantity is not well known at present.

7. CONCLUSIONS

The current situation thus seems to be that optical methods of investigation can be used to obtain answers to a number of topical and urgent questions in elementary-particle physics. Experiments of this kind are relatively cheap and can serve as an important addition to the methods that are traditional in high-energy physics.

We will have achieved our aim if we succeed in drawing the attention of physicists to this circle of problems.

We are greatly indebted to Ya. I. Azimov, A. A. Ansel'm, L. M. Barkov, A. I. Vaĭnshteĭn, V. G. Gorshkov, M. S. Zolotorev, V. N. Novikov, and O. P. Sushkov for many exceedingly useful discussions. We are also particularly indebted to L. B. Okun' for his interest in this work and for valuable advice.

[1]S. A. Bludman, Nuovo Cimento **9**, 433 (1958).
[2]Ya. B. Zel'dovich, Zh. Eksp. Teor. Fiz. **36**, 964 (1959) [Sov. Phys.-JETP **9**, 682 (1959)].
[3]S. B. Treiman, Nuovo Cimento **15**, 916 (1960).
[4]T. D. Lee and C. N. Yang, Phys. Rev. **119**, 1410 (1960).
[5]Ya. B. Zel'dovich and A. M. Perelomov, Zh. Eksp. Teor. Fiz. **39**, 1115 (1960) [Sov. Phys.-JETP **11**, 806 (1960)].
[6]V. N. Baĭer and I. B. Khriplovich, *ibid.*, p. 1374 [Sov. Phys.-JETP **11**, 990 (1960)].
[7]R. W. King, Phys. Rev. **121**, 1201 (1961).
[8]F. Curtis Michel, *ibid.* **138** B, 408 (1965).
[9]A. I. Vaĭnshteĭn and I. B. Khriplovich, Usp. Fiz. Nauk **112**, 685 (1974) [Sov. Phys. Usp. **17**, 263 (1974)].
[10]F. J. Haser *et al.*, Phys. Lett. B **46**, 138 (1973).
[11]A. Benvenuti *et al.*, Phys. Rev. Lett. **32**, 800 (1974).
[12]B. L. Ioffe, Usp. Fiz. Nauk **110**, 357 (1973) [Sov. Phys. Usp. **16**, 459 (1974)].
[13]T. S. Stein, J. P. Carrico, E. Lipworth, and M. C. Weisskopf, Phys. Rev. **186**, 39 (1969).
[14]M. A. Player and P. G. H. Sandars, J. Phys. B **3**, 1620 (1970).
[15]H. Gould, Phys. Rev. Lett. **24**, 1091 (1970).
[16]R. Vessot *et al.* IEEE Trans. Instr. Meas., **IM-15**, 165 (1966).
[17]H. Grotch and D. R. Yennie, Rev. Mod. Phys. **41**, 350 (1969).
[18]T. Myint, D. Kleppner, N. F. Ramsey, and H. G. Robinson, Phys. Rev. Lett. **17**, 405 (1966).
[19]CERN (Daresbury)-Mainz Group, CERN Courier **15**, 67 (1975).
[20]R. A. Brown and F. M. Pipkin, Phys. Rev. **174**, 48 (1968).
[21]E. D. Theriot Jr, R. H. Beers, and V. W. Hughes, Phys. Rev. Lett. **18**, 767 (1967).
[22]A. N. Moskalev, Pis'ma Zh. Eksp. Teor. Fiz. **19**, 229 (1974) [JETP Lett. **19**, 141 (1974)].
[23]V. G. Gorshkov and L. N. Labzovskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **19**, 768 (1974) [JETP Lett. **19**, 394 (1974)]; Zh. Eksp. Teor. Fiz. (1975).
[24]M. A. Bouchiat and C. C. Bouchiat, Phys. Lett. B **48**, 111 (1974).
[25]L. D. Landau and E. M. Lifshitz, Kvantovaya Mekhanika (Quantum Mechanics), Nauka, M., 1974.
[26]I. B. Khriplovich, Pis'ma Zh. Eksp. Teor. Fiz. **20**, 686 (1974) [JETP Lett. **20**, 315 (1974)].
[27]V. N. Novikov and I. B. Khriplovich, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 162 (1975) [JETP Lett. **22**, 74 (1975)].
[28]A. N. Moskalev, Pis'ma Zh. Eksp. Teor. Fiz. 19, 394 (1974) [JETP Lett. **19**, 216 (1974)].
[29]G. Feinberg and M. Y. Chen, Phys. Rev. D **10**, 190 (1974).
[30]J. Bernabeau, T. E. O. Ericson, and C. Jarlskov, Phys. Lett. B **50**, 467 (1974).
[31]M. A. Bouchiat and C. C. Bouchiat, J. Phys. (Paris) **35**, 899 (1974); **36**, 493 (1975).
[32]E. Fermi, Z. Phys. **88**, 161 (1934).
[33]L. B. Okun', Slaboe vzaimodeĭstvie elementarnykh chastits (Weak Interaction of Elementary Particles), Fizmatgiz., M., 1963.
[34]S. Weinberg, Phys. Rev. D **5**, 1412 (1972).
[35]G. Breit and E. Teller, Astrophys. J. **91**, 215 (1940).
[36]V. Ch. Zhukovskiĭ, M. M. Kolesnikova, A. A. Sokolov, and I. Kherrmann, Opt. Spektrosk. 28, 622 (1970) [Opt. Spectrosc. (USSR) **28**, 337 (1970)].
[37]C. Drake, Phys. Rev. A **3**, 908 (1971).
[38]H. A. Bethe and E. E. Salpeter, Quantum Mechanics One- and Two-Electron Atoms (Russ. Transl. Fizmatgiz., M., 1960).
[39]J. Shapiro and G. Breit, Phys. Rev. **113**, 179 (1959).
[40]Ya. I. Azimov, A. A. Ansel'm, A. N. Moskalev, and R. M. Ryndin, Zh. Eksp. Teor. Fiz. **67**, 17 (1974) [Sov. Phys.-JETP **40**, 8].
[41]Ya. B. Zel'dovich, Zh. Eskp. Teor. Fiz. **39**, 1483 (1960) Sov. Phys.-JETP **12**, 1030 (1961)].
[42]V. G. Gorshkov and L. N. Labzovskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **21**, 19 (1975) [JETP Lett. **21**, 8 (1975)].
[43]I. Martinson and A. Gaupp, Phys. Lett. C **15**, 113 (1974).
[44]G. Feinberg and J. Sucher, Phys. Rev. Lett. **26**, 681 (1971).
[45]W. R. Johnson and Chien-ping Ling, Phys. Rev. A **9**, 1486 (1974).
[46]B. Schiff, C. Pekeris, and Y. Acad., *ibid.* A **4**, 885 (1971).
[47]G. W. F. Drake and A. Dalgarno, Astrophys. J. **157**, 459 (1969).
[48]G. L. Klimchitskaya, U. I. Safronova, and L. N. Labzovskiĭ, Opt. Spektrosk. **38**, 838 (1975) [Opt. Spectrosc. (USSR) **38**, 480 (1975)].
[49]Y. Acad., C. L. Pekeris, and B. Schiff, Phys. Rev. A **4**, 516 (1971).
[50]A. M. Ermolaev, *ibid.* A **8**, 1651 (1973).
[51]G. L. Klimchitskaya and L. N. Labzovskiĭ, Opt. Spektrosk. **34**, 633 (1973) [Opt. Spectrosc. (USSR) **36**, 365 (1973)].
[52]I. I. Gol'dman and V. D. Krivchenkov, Sbornik zadach po kvantovoĭ mekhanike (Collection of Problems on Quantum Mechanics), Gostekhizdat, M., 1957).
[53]V. A. Alekseev, B. Ya. Zel'dovich, and I. I. Sobel'man, Usp. Fiz. Nauk **118**, 385 (1976) [Sov. Phys. Usp. **19**, 207 (1976)].
[54]I. B. Khriplovich, Yad. Fiz. **21**, 1046 (1975) [Sov. J. Nucl. Phys. **21**, 538 (1975)].
[55]M. Phillips. Phys. Rev. **88**, 202 (1952).
[56]E. U. Condon and G. H. Shortley, Theory of Atomic Spectra (Russ. Transl., IL, M., 1949).
[57]P. A. Vandenbout *et al.*, Phys. Rev. **165**, 88 (1968).
[58]D. R. Bates and A. Damgaard, Phil. Trans. A **242**, 101 (1949).
[59]I. I. Sobel'man, Vvedenie v teoriyu atomnykh spektrov (Introduction to the Theory of Atomic Spectra), Fizmatgiz., M., 1963.
[60]P. M. Stone, Phys. Rev. **127**, 1151 (1962).
[61]M. A. Bouchiat and L. Pottier, J. Phys. Lett. (Paris) **36**, L189 (1975).
[62]N. P. Penkin and L. N. Shabanova, Opt. Spektrosk. **14**, 167 (1963) [Opt. Spectrosc. (USSR) **14**, 87 (1963)].
[63]A. Gallagher and A. Lurie, Phys. Rev. A **136**, 87 (1964).
[64]G. V. Marr, Proc. R. Soc. Ser. A **224**, 83 (1954).
[65]G. V. Marr and R. Heppinstall, Proc. Phys. Soc. London **87**, 293 (1966).
[66]N. P. Penkin and I. Yu. Yu. Slavenas, Opt. Spektrosk. **15**, 154 (1963) [Opt. Spectrosc. (USSR) **15**, 83 (1963)].
[67]A. Baghdadi, J. B. Halpern, and E. B. Salomon, Phys. Rev. A **7**, 403 (1973).

[68]T. Andersen, O. H. Madsen, and G. Sörensen, J. Opt. Soc. Am. **62**, 1118 (1972).
[69]S. Svanberg, Thesis (Univ. of Gothenburg) (in Ref. 68).
[70]V. N. Novikov, O. P. Sushkov, and I. B. Khriplovich, Zh. Eksp. Teor. Fiz., in press.
[71]R. Hultgren *et al.*, Selected Values of the Thermodynamic Properties of the Elements, American Society for Metals, 1973.
[72]A. Flusberg, R. A. Weingarten, and S. R. Hartman, Phys. Lett. A **43**, 433 (1973).
[73]H. J. Mathieu, D. E. McClure, and R. H. Muller, Rev. Sci. Instrum. **45**, 798 (1974).
[74]E. F. I. Roberts and A. Meadows, J. Phys. E **7**, 379 (1974).
[75]Yu. A. Blyumkina, Yu. B. Algazin, A. V. Arkhipenko, and K. K. Svitashov, Opt. Spektrosk. **39** (1975).
[76]N. N. Neprimerov, Izv. Akad. Nauk SSSR Ser. Fiz. **18**, 368 (1954).
[77]A. M. Portis and D. Teaney, J. Appl. Phys. **29**, 1692 (1958).
[78]S. E. Bresler, É. N. Kazbekov, and V. N. Fomichev, Zh. Tekh. Fiz. **41**, 1237 (1971) [Sov. Phys. Tech. Phys. **16**, 978 (1972)].
[79]H. W. Moos and R. H. Sands, Phys. Rev. **135**, 591 (1964).
[80]A. I. Vaĭnshteĭn and I. B. Khriplovich, Pis'ma Zh. Eksp. Teor. Fiz. **20**, 80 (1974) [JETP Lett. **20**, 34 (1974)]; Zh. Eksp. Teor. Fiz. **68**, 3 (1975) [Sov. Phys. JETP **41**, 1 (1975)].
[81]Y. Masuda and A. G. Redfield, Phys. Rev. **125**, 159 (1962).
[82]R. H. Hammond and W. D. Knight, *ibid.* **120**, 769 (1960).
[83]R. P. Feynman, R. B. Leighton, and M. Sands, Feynman Lectures on Physics (Russ. Transl., Vol. 9, Mir, M., 1967, p. 252).
[84]C. Kittel, Introduction to Solid State Physics (Russ. Transl., Fizmatgiz, M., 1962, p. 321).
[85]A. Di Giacomo, Nucl. Phys. B. **11**, 411 (1969).
[86]E. Campani, Lett. Nuovo Cimento **4**, 982 (1970).
[87]A. L. Galanin and I. Ya. Pomeranchuk, Dokl. Akad. Nauk SSSR **86**, 251 (1952).
[88]G. Kodosky and M. Leon, Nuovo Cimento B **1**, 41 (1970).
[89]V. G. Gorshkov, A. I. Mikhaĭlov, and A. N. Moskalev, Zh. Eksp. Teor. Fiz. **69**, 1507 (1975) [Sov. Phys. JETP **42**, (1976)]
[90]G. Feinberg and M. Y. Chen, Preprint CO-2271-41 (1974).
[91]A. N. Moskalev and R. M. Ryndin, Yad. Fiz. **22**, 147 (1975) [Sov. J. Nucl. Phys. **22**, 71 (1976)].
[92]H. Uberall, Phys. Rev. **114**, 1640 (1959).

Translated by S. Chomet