Drift-dissipative instability of an inhomogeneous plasma in a magnetic field

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The drift-dissipative instability is one of the main instabilities of a plasma in a magnetic field. It is due to the density gradient of the plasma and can develop in the plasma in thermonuclear devices, in the magnetospheric plasma, in a gas-discharge plasma in a magnetic field, etc. The theory of the drift-dissipative instability was developed about ten years ago. In the subsequent years this theory has been convincingly confirmed in experiments with gas discharges in a magnetic field and also in experiments with Q machines. In the present review, the main theoretical and experimental results obtained in the investigation of the drift-dissipative instability are presented.

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INTRODUCTION

Instability of a plasma in a magnetic field has now become a common phenomenon. One of the first experiments that forced one to speak about instability of a plasma was described by Bohm *et al.* [1] In this experiment, they discovered an anomalously fast escape of plasma through a magnetic field to the wall of the vessel. To explain this phenomenon it was suggested in[1] that random electric fields are spontaneously excited in the plasma. In these fields, the charged particles move in a disordered manner, which leads to enhanced diffusion of the plasma through the magnetic field. For the diffusion coefficient, Bohm gave the empirical expression $D=(1/16)cT_e/eB$, which is called the Bohm diffusion coefficient; in it, T_e is the electron temperature and B the magnetic field. Later, anomalous diffusion of plasma with diffusion coefficient of the order of the Bohm coefficient was discovered in systems very different from the one described in[1]. Bohm diffusion is now attributed to the spontaneous excitation of so-called drift oscillations; see, for example, [2].

In this review we consider one of the possible mechanisms of excitation of drift oscillations in which an essential destabilizing (!) role is played by collisions of charged particles. [3] In[4], on the basis of analysis of the dimensions of the quantities that characterize the unstable oscillations, an attempt was made to derive the Bohm diffusion coefficient. [1)]

The origin of this instability is the fact that the plasma is not in thermodynamic equilibrium—there is a pressure gradient in the direction at right angles to the magnetic field. This disequilibrium is unavoidable in all systems that use magnetic plasma containment, and therefore the instability was called universal in[5]. It should be noted however that for an instability to develop the pressure gradient must exceed a certain critical value, which is determined by the length of the plasma along the magnetic field, the concentration of neutral particles, and other factors. Therefore, the instability will not develop in all devices by any means. In[6] the term inertial-dissipative instability was used for the phenomenon. This name reflects the circumstance that although the electrons move as in a viscous medium as the instability develops because of their frequent collisions with heavy particles (ions in a fully ionized plasma and neutral atoms in a weakly ionized plasma[2)]) while the motion of the ions must be free—inertial. At the present time, a third name is the one that

[1)] From the frequency and wave vector one can form a single combination, ω/k^2, which has the dimensions of a diffusion coefficient. The frequency of the unstable oscillations is $\omega \approx (cT_e/eB^2)\mathbf{B}\cdot(\mathbf{k}\times\boldsymbol{\varkappa})$ (see[2–4]), where $\boldsymbol{\varkappa}=-(1/n_0)\nabla n_0$, and n_0 is the initial plasma density unperturbed by the oscillations. If we take the smallest possible value $k\approx\varkappa$ for the wave vector, we really do obtain the required dependence $\omega/k^2\approx cT_e/eB$.

[2)] *A plasma is said to be weakly ionized if the frequency of* collisions of the charged particles with neutral particles exceeds the frequency of Coulomb collisions. Note that it would be more correct to speak of a weakly ionized and fully ionized gas (see, for example, [7]), though we shall not depart from the established tradition.

is the most widely used: the drift-dissipative instability, which was proposed in[8] by one of the authors. This name emphasizes the important role of drift motions in the excitation of the oscillations. Originally, the term drift-dissipative instability was used in a generalized sense, so that collisionless Landau damping was also included in the dissipative processes. Accordingly, the class of drift-dissipative instabilities included those of an inhomogeneous low-density plasma in a magnetic field (see, for example, [8,9]). However, with the course of time the term drift-dissipative instability has come to be used only to designate the instabilities of a collisional plasma. It is in this narrower sense that we shall use the name.

The main results of the theory of the drift-dissipative instability were established in the first analyses made of it. Subsequent investigations added little new to these results. This very rapid development of the theory was due in large part to the use of the so-called local quasiclassical approximation. In this approximation the inhomogeneity of the plasma is taken into account parametrically, which enables one to obtain a local algebraic equation determining the frequency of the plasma oscillations. Originally, doubts were expressed about the validity of this method (see, for example, [10]). However, further investigations (see, for example, [9,11]) established its adequacy for a large class of oscillations of an inhomogeneous plasma in a magnetic field in which drift motions of charged particles play an important role. This class includes the drift-dissipative instability.

The drift-dissipative instability is a fairly "weak" instability. If other instabilities develop simultaneously in the plasma, it is difficult to distinguish it on their background. It is therefore desirable to carry out an experimental investigation of the instability under conditions that most closely approach equilibrium. Frequently, disequilibrium of the plasma is due to the very method by which it is created (passage of a strong current through a gas, injection of beams of charged particles, etc). In this sense, the most "quiescent" are systems of two types: currentless gas discharges and Q machines. The term currentless gas discharge is sometimes used (see, for example, [12]) to designate the types of discharge which can be sustained without passing a direct current through the gas. This class includes high-frequency discharges, afterglow discharges, Penning discharges, etc. A direct current gas discharge in a magnetic field is subject to a "stronger" instability known as the current-convective instability. The discovery of this instability[13,14] and its successful identification in[15] very greatly stimulated, in conjunction with[1], the investigations of oscillations of all types of gas discharge in a magnetic field. It should be noted that although a current-convective instability can develop in a direct current discharge in a magnetic field this does not present insuperable obstacles for observing the drift-dissipative instability (see below).

From the very start Q machines were conceived as systems for studying a quiescent plasma. In them the plasma is created by the thermal ionization of a beam of neutral atoms that impinges on a heated slab.

One can distinguish two stages in the investigation into the oscillations of plasmas in Q machines and currentless gas discharges in a magnetic field. In the first stage there was a tendency to attribute all instabilities observed in collisional regimes to the drift-dissipative mechanism of excitation of oscillations. However, a more careful examination of the problem showed that other mechanisms can operate as well. Gradually, the conviction grew that only a direct verification of the dispersion relation would settle the question of the nature of the observed oscillations (the dispersion relation determines the oscillation frequency as a function of the components of the wave vector of the oscillations and the plasma parameters). This verification has now been carried out for the majority of forms of the drift-dissipative instability. The positive results of the verification now justify the assertion that the drift-dissipative instability is a firmly established physical phenomenon.

1. GENERAL CHARACTERIZATION OF THE DRIFT-DISSIPATIVE INSTABILITY

A. Basic equations

The processes leading to the development of the drift-dissipative instability can be analyzed in the approximation of two-fluid hydrodynamics. When this approximation is used, the electron and ion components of the plasma are represented in the form of two fluids, or, more precisely, gases, which penetrate each other. For isothermal processes, which will be considered below, the system of hydrodynamic equations reduces to the continuity equations and the equations of motion of each of the plasma components. The continuity equations have the standard form

$$\frac{\partial n}{\partial t}+\nabla(n\mathbf{v}_j)=0, \tag{1.1}$$

where the subscript j takes the two values $j=e,\ i$ for electrons and ions, respectively. The plasma is assumed to be quasineutral: $n_e=n_i=n$.

The equations of motion must be discussed in more detail. We shall be interested in comparatively slow processes whose frequency ω is low compared with ν_e, the frequency of collisions of the electrons with the heavy particles (neutral atoms in a weakly ionized plasma and ions in a fully ionized plasma). In the study of these processes, the force of inertia can be omitted in the equation of motion of the electrons:

$$0=-T_e\nabla n-en\left(-\nabla\varphi+\frac{1}{c}[\mathbf{v}_e\times\mathbf{B}]\right)+\mathbf{F}_e; \tag{1.2}$$

here, the magnetic field is assumed to be constant and homogeneous and the electric field to be electrostatic, $E=-\nabla\varphi$, which is valid for a low-pressure plasma: $8\pi nT/B^2\ll 1$. In what follows we shall consider the oscillations of a plasma with magnetized electrons, when the electron cyclotron frequency ω_e appreciably exceeds ν_e. It can be shown that in this case the requirement affects only the motion of the electrons along the magnetic field. Therefore, we set $\mathbf{F}_{\perp e}=0$, $F_{\parallel e} = -m_e n v_{\parallel e}\nu_e$, where here and in what follows the longitudinal and transverse symbols are appended to the directions relative to the magnetic field. Generally speaking, instead of $v_{\parallel e}$ in the expression for the force of friction one should have the difference $v_{\parallel e}-v_{\parallel n}$ in the case of a weakly ionized plasma or $v_{\parallel e}-v_{\parallel i}$ in the case of fully ionized plasma. However, in the former the charged particles are a small fraction $\lesssim 10^{-4}$ of the total number of particles. Therefore, the charged particles cannot bring the neutral component of the plasma into motion, and one can assume that the plasma is at rest and set $v_{\parallel n}=0$. Since the longitudinal electron velocity $v_{\parallel e}$ appreciably exceeds the ion velocity $v_{\parallel i}$, the same expression for the force of friction remains approximately true in the case of a fully ionized plasma as well.

We write the ion equation of motion in the form

$$m_i n\frac{d\mathbf{v}_i}{dt} = -T_i\nabla n + en\left(-\nabla\varphi+\frac{1}{c}[\mathbf{v}_i\times\mathbf{B}]\right)+\mathbf{F}_i; \qquad (1.3)$$

here, in the case of a weakly ionized plasma $\mathbf{F}_i=-m_i \times n\mathbf{v}_i\nu_i$, where ν_i is the frequency of ion-neutral collisions. In a fully ionized plasma the friction of the ions on the electrons is unimportant on account of the small electron mass, and a more important role is played by the viscosity due to the collisions of the ions with one another: $F_{i,\alpha}=-(\partial/\partial x_\beta)\pi_{i,\alpha\beta}$. In what follows, we shall require the following values of the components of the viscosity tensor: $\pi_{xx}-\pi_{yy}=-\eta_1(W_{xx}-W_{yy}) - 2\eta_3 W_{xy}$, $\pi_{xy}=\pi_{yx}=-\eta_1 W_{xy}-\eta_3/2(W_{xx}-W_{yy})$, where

$$W_{xx}-W_{yy}=2\left(\frac{\partial v_x}{\partial x}-\frac{\partial v_y}{\partial y}\right),\quad W_{xy}=\frac{\partial v_x}{\partial y}+\frac{\partial v_y}{\partial x},$$
$$\eta_1=0.3\frac{nT_i\nu_i}{\omega_i^2},\quad \eta_3=0.5\frac{nT_i}{\omega_i}$$

(see, for example, [16]). Here and in what follows we use a Cartesian coordinate system whose Oz axis is directed along the magnetic field and Ox axis along the density gradient of the plasma.

Although the behavior of a fully ionized and a weakly ionized plasma is described by the same equations, their physical meaning is not entirely the same. The hydrodynamic system of equations for a fully ionized plasma is derived by a regular procedure from the kinetic equation (see, for example, [16]). This procedure is based on the fact that the form of the distribution function of each of the components of the plasma is essentially determined by collisions between identical particles (electron–electron and ion–ion). Under the influence of collisions a Maxwellian distribution is established and the parameters of this distribution are the density, velocity, and temperature. The hydrodynamic equations determine the evolution of these parameters. In a weakly ionized plasma the distribution function of the charged particles is determined by external electric fields that sustain the discharge and by collisions with neutrals. Therefore, the energy and velocity of each of the components are uniquely determined, and accordingly the hydrodynamic system of equations reduces to a single equation for the density. [17,18] The region of applicability of this modified hydrodynamics is fairly restricted, and one must therefore use the moments of the kinetic equation, which have the meaning of balance equations. Truncation of the system of moments can lead to appreciable errors if one is considering processes in which the higher moments play an important role. Fortunately, the drift-dissipative instability is due to a deviation from a state of thermodynamic equilibrium as crude as a density gradient. Therefore, its development can be described with adequate accuracy by the first two moments of the kinetic equation (see however Sec. 4 of this review).

Using (1.2), let us consider the motion of electrons perpendicular to the magnetic field in the absence of oscillations. We assume that in the initial state there is no electric field. Assuming also that $\mathbf{F}_{\perp e}=0$ (see above), we obtain the following expression for the electron velocity perpendicular to the magnetic field:

$$\mathbf{V}_e^* = -\frac{cT_e}{eB^2}\left[\mathbf{B}\times\frac{\nabla n}{n}\right]. \qquad (1.4)$$

It follows from (1.4) that if the plasma density changes in the direction perpendicular to the magnetic field then the electron component of the plasma is in a state of motion. This motion is called gradient or Larmor drift. It is interesting in that the hydrodynamic macroscopic velocity is not associated with displacements of individual electrons, each of which is at rest on the average revolving around a fixed Larmor circle. [15,19]

B. Drift oscillations

We now consider the oscillations of an inhomogeneous plasma in a magnetic field. Let us first analyze their spatial dependence. It is determined by the geometry of the system. Usually, drift oscillations are studied in systems that have the form of a long cylinder with length one or two orders of magnitude greater than its diameter. The magnetic field is generated by coaxial coils and is parallel to the axis of the system. A typical experimental arrangement used to study the oscillations of a gas-discharge plasma in a magnetic field is shown in Fig. 1.

Since the plasma parameters vary weakly along the axis, the system can be assumed to be approximately homogeneous in this direction. The greatest difficulty is in the analysis of the radial dependence since this requires one to solve a system of differential equations with variable coefficients. It can however be shown that in the oscillations in which we are interested the most

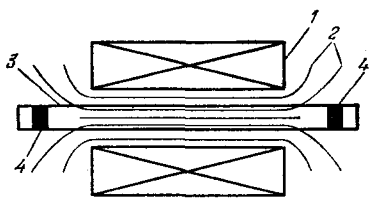

FIG. 1. Arrangement of experimental apparatus. 1) Solenoid; 2) lines of force of the magnetic field; 3) glass tube; 4) external ring electrodes.

important role among all the radial motions of the plasma is played by drift in crossed fields: the constant axial magnetic field and the azimuthal electric field of the oscillations. This drift leads to displacements of the plasma in the radial direction, i.e., in the direction of the density gradient, which gives rise to the oscillations of the density. To take into account this effect, it is sufficient to consider perturbation traveling azimuthally and independently of the radius. Detailed analysis of the problem confirms the validity of this approach (see, for example, [3-6,8-11]). Since the radial dependence of the perturbations is unimportant, instead of an axisymmetric system one can consider a system with planar symmetry. (The direction along Ox is equivalent to the radial direction and that along Oy to the azimuthal direction.) With allowance for all we have said, the expressions for the perturbations of the density and the potential in the oscillations take the form $\sim\exp(-i\omega t+ik_z z+ik_y y)$.[3)]

Let us also make the following simplifications: we set $T_i=0$; we ignore the motion of the ions along the magnetic field (see above), and we shall assume that their transverse motion is collisionless, $\mathbf{F}_i=0$. Assuming also that the frequency of the oscillations is not too high, in the ion equation of motion we omit the force of inertia. Under these assumptions we find from (1.2) and (1.3)

$$v_{\|e}=b_e\frac{\partial\varphi}{\partial z}-D_e\frac{1}{n}\frac{\partial n}{\partial z}, \tag{1.5}$$

$$v_{\|i}=0, \tag{1.6}$$

$$\mathbf{v}_{\perp e}=\frac{c}{B^2}[\mathbf{B}\times\nabla\varphi]-\frac{cT_e}{eB^2}\left[\mathbf{B}\times\frac{\nabla n}{n}\right], \tag{1.7}$$

$$\mathbf{v}_{\perp i}=\frac{c}{B^2}[\mathbf{B}\times\nabla\varphi]; \tag{1.8}$$

where $b_e=e/m_e\nu_e$ and $D_e=T_e/m_e\nu_e$ are the mobility and diffusion coefficients of the electrons, respectively.

Substituting (1.5)–(1.8) into the continuity equations and linearizing them with respect to the small perturbations, $n_1\ll n_0$, of the density and the potential φ_1, we obtain

$$-i\omega n_1-ik_y\frac{c}{B}\frac{dn_0}{dx}\varphi_1-b_e k_z^2 n_0\varphi_1+D_e k_z^2 n_1=0, \tag{1.9}$$

$$-i\omega n_1-ik_y\frac{c}{B}\frac{dn_0}{dx}\varphi_1=0. \tag{1.10}$$

Besides analyzing Eqs. (1.9) and (1.10), let us attempt, following[23], to present a clear explanation of the mechanism of propagation of the oscillations. For this, we consider Fig. 2. It shows the instantaneous

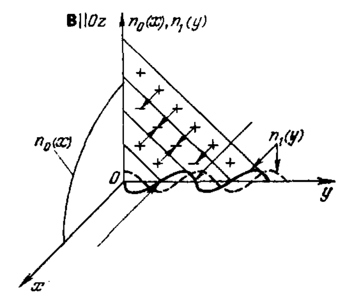

FIG. 2. Propagation mechanism of drift oscillations. The regions of enhanced and depleted concentration in the wave are denoted by the plus and minus signs, respectively. The sinusoids represent the instantaneous distributions of the plasma density $n_1(y)$ in the wave. The direction of the drift of the charged particles in the wave is indicated by the long arrows and that of the electric field of the wave is indicated by the short arrows.

distribution of the density in the wave $\sim\exp(-i\omega t+ik_y y+ik_z z)$, the regions of enhanced density being distinguished by the plus sign. Equations (1.9) and (1.10) are compatible if the last two terms in (1.9) cancel each other. This means that the pressure gradient of the electron component is compensated by the electric field $\varphi_1=(D_e/b_e)n_1/n_0=(T_e/e)n_1/n_0$. Therefore, the regions of enhanced concentration must be positively charged. In Fig. 2, the direction of the electric field is indicated by short arrows. Since the electric field has a y component, the plasma drifts along Ox (long arrows). In the region to the right of the maximum, plasma arrives from deeper levels, where its density is higher (in Fig. 2 the density decreases along Ox). In the region to the left of the maximum, plasma arrives from layers situated nearer the surface, where the plasma density is lower. As a result, the complete picture is shifted to the right (dashed curve). The continuous repetition of this process means that a density and potential wave travels through the plasma. This wave neither grows nor decays ($\mathrm{Im}\,\omega=0$). The real part, as follows from (1.9) and (1.10), is equal to $\omega^*=k_y\varkappa cT_e/eB$, where $\varkappa=-(1/n_0)dn_0/dx$. This is called the drift frequency and the oscillations are therefore called drift oscillations. They were first considered in[24] for a fully ionized plasma. The further development of the theory of drift oscillations is reflected in the reviews[2,6,9,11].

Let us dwell on two interesting features.

1) The drift oscillations are not damped although $\omega\ll\nu_e$. The reason for this is clear. Since the electrons are distributed in accordance with Boltzmann's law, their mean macroscopic velocity is zero, and the force of friction therefore disappears as well.

2) The phase velocity of the drift oscillations in the OY direction has been found to be equal to the unperturbed velocity of the electron Larmor drift (1.4). At the first glance one therefore gets the impression that we are here dealing with the transfer of perturbations by an electron stream. However, since each individual electron in its Larmor drift is at rest on the average, such transfer is obviously impossible. Larmor drift cannot at all lead to a change in the density, and there-

[3)]In a number of investigations (see, for example,[20-22]) an attempt was also made to take into account the dependence of the perturbations on the coordinate x (the radius) by choosing the perturbations in the form $\exp(-i\omega t+ik_x x+ik_y y+ik_z z)$. If characteristic oscillations are studied, this choice can lead to misunderstanding. The point is that in bounded systems characteristic oscillations must be more like standing waves. But if the dependence on the x coordinate is taken in the form $\exp(ik_x x)$, then, because of the inhomogeneity of the plasma, the dispersion relation between the frequency of the oscillations and the components of the wave vector is found to contain imaginary terms proportional to odd powers of ik_x, and this, depending on their sign, will be equivalent to the introduction of additional damping or excitation.

fore the corresponding term $\nabla(n\mathbf{V}_e^*)$ drops out of the resulting continuity equation (1.9). Evidently, the phase velocity of the oscillations is equal to the velocity of Larmor drift because the latter is the simplest quantity with the dimensions of a velocity characterizing the inhomogeneous plasma in the magnetic field. Indeed, since the macroscopic velocity can be regarded as a measure of the disequilibrium of the plasma, it is natural to assume that it must be proportional to $\varkappa T$. The simplest combination of $\varkappa T$, e, B, c, m_i, and m_e with[4)] the dimensions of velocity is evidently equal to the Larmor drift velocity $(cT/eB)\varkappa$ [cf. (1.4)].

C. Drift-dissipative instability

Drift oscillations neither decay nor grow ($\mathrm{Im}\,\omega = 0$). In a certain sense one can say that a plasma with drift oscillations is a new type of equilibrium state. Since the "crude" forces (the pressure and the force exerted by the electric field) are in equilibrium in drift oscillations, more subtle effects can be decisive. It is for this reason that the correct treatment of drift oscillations in a fully ionized plasma may in a number of cases require an extension of the scheme of ordinary hydrodynamics to take into account higher moments of the distribution function.[6] At this point we shall take into account the ion inertia. Its influence is very important even at a low oscillation frequency $\omega \ll \omega_i$.

Using Eqs. (1.3) we determine the ion velocity in the direction perpendicular to the magnetic field:

$$\mathbf{v}_{\perp i1} = \frac{c}{B^2}(\mathbf{B}\times\nabla\varphi_1) + \frac{c}{B}\frac{i\omega}{\omega_i}\nabla_\perp\varphi_1. \qquad (1.11)$$

As before, we ignore the motion of ions along the magnetic field. Substituting (1.11) into the continuity equation and taking into account the expression for the frequency of drift oscillations (see the previous subsection), we find that allowance for the force of inertia decreases the second term in (1.10) by the factor $(1 - k_y^2\rho_{ie}^2)$; here $\rho_{ie} = \omega_i^{-1}\sqrt{T_e/m_i}$ is the ion Larmor radius calculated with the electron temperature. Thus, the inertia of the ions effectively decelerates the drift in the crossed fields, and as a result the drift displacement of the ions is less than the electron displacement and the amplitude of the oscillations of the electron density is greater than that of the ion density. The quasineutrality of the plasma can be maintained by redistribution of the electrons along the magnetic field. The excess electrons, going over from the regions with higher density to those with lower density, must do work against the electric field (see Fig. 2). As a result, the amplitude of the oscillations increases. Since energy is taken from the electrons, they are cooled. Ultimately the energy is expended on bringing the heavy ions into motion. These processes do not contradict the second law of thermodynamics since the oscillations of the plasma in the direction of the density gradient smooth out the gradient (the mean value over the oscillations), taking the system nearer to equilibrium.

What is the role of the friction of the electrons on the heavy particles in this oscillation excitation mechanism? If the electrons were to have a free inertial motion along the magnetic field, then, as is readily seen, the change of their density as they move under the influence of the excess pressure would be shifted in phase relative to that of the original perturbation by $\pi/2$. We should then obtain a purely oscillatory regime without growth of the original perturbations. If we examine the matter more deeply, we find that the role of friction consists of introducing an element of irreversibliity. As a result, the process of smoothing the density profile of the plasma in the oscillations acquires an irreversible nature. Ultimately, it is this that allows the oscillations to take thermal energy from the electrons without contradicting the second law of thermodynamics.

We shall show that the conclusion that the plasma is unstable also follows from a formal analysis. Allowance for the inertia of the ions leads to the appearance in (1.10) of the additional term $-ik_y^2(c/B)(\omega/\omega_i)n_0\varphi_1$. As a result, the dispersion relation for determing the frequency of the characteristic oscillations of the plasma—it is obtained from the consistency condition of Eqs. (1.9) and (1.10)—takes the form

$$\omega - \omega^* = i\frac{\omega^2}{D_e k_z^2}k_y^2\rho_{ie}^2 - \omega k_y^2\rho_{ie}^2. \qquad (1.12)$$

Ignoring first the small right-hand side of (1.12), which is due to the allowance for the force of inertia, we obtain $\omega = \omega^* = (cT_e/eB)k_y\varkappa)$ (see above). In the following approximation we find that the oscillations are unstable ($\mathrm{Im}\,\omega > 0$).

2. DRIFT INSTABILITY IN A WEAKLY IONIZED GAS-DISCHARGE PLASMA IN A MAGNETIC FIELD

A. Detection of the instability

The investigations into the oscillations of currentless gas discharges in a magnetic field began with the experiments described by Bohm (see the introduction). These experiments were continued in[25–28]. It was found that anomalies in the behavior of the plasma occur only if the magnetic field exceeds a certain critical value B_{cr}. When this happens, one or several plasma "faculae" are drawn out of the region occupied by the beam of primary electrons and rotate around it. As the magnetic field is further increased, faculae of different spatial scales appear and they rotate with different velocities. The simultaneous presence in the discharge of several faculae leads to a disordered random picture, i.e., to turbulence.

In[29], Schlüter investigated a high-frequency discharge in a magnetic field. He found that the load on the generator had an anomalous dependence on the magnetic field. Much earlier, in[30], Davies had noted that in such a discharge an anomaly is observed in the behavior of the electron temperature in a magnetic field, the anomaly increasing with increasing field. A more detailed study[31] of a high-frequency discharge revealed that when the magnetic field is increased above the

[4)]This list does not contain the plasma density since a dependence on the density must disappear in the limiting case of a dense quasineutral plasma ($n_0 \to \infty$).

critical value the flux of positive ions across the magnetic field increases and simultaneously high-frequency electric oscillations of noise type begin to be generated in the plasma. All these results indicated indirectly that an instability occurs and the diffusion coefficient increases.

It would be natural to expect to find the most quiescent conditions in the decaying plasma that remains in the discharge after the ionization source has been switched off; this is the so-called afterflow plasma. And, indeed, in these plasmas, unlike high-frequency plasmas, the electron temperature is near that of the neutral gas, and the stationary electric field that usually arises spontaneously in a gas discharge (ambipolar field) is fairly weak. However, investigations showed that in an afterflow plasma the diffusion coefficient has an anomalously large value. [32,33] Attempts were made to attribute the anomaly to the influence of impurities, misalignment of the axis of the discharge tube relative to the magnetic field, recombination, etc. (see, for example, [34,35]). These effects were analyzed in[36–38] (see also[7]). The conclusion reached was that only the development of unstable oscillations could explain the increase in the diffusion coefficient.

Anomalous diffusion accompanied by the excitation of electric noise was also observed in PIG reflex discharges. [39,40]

B. First attempts of identify the instability

Since similar phenomena were observed in discharges of different types, it was natural to assume that we are confronted here with some universal instability mechanism that is not associated with the details of a particular discharge. To establish whether it is the drift-dissipative mechanism, it was first of all necessary to find the conditions of the instability and compare them with the experimental conditions. For this, it is necessary to augment the idealized scheme used in the previous section by various factors manifested in real systems. For a weakly ionized gas-discharge plasma these are ion collisions with neutrals, the finite ion temperature ($T_i \neq 0$), the boundedness of the system in the direction of the magnetic field, and various others. In[41], only one of these factors (ion collisions with neutrals) was taken into account. This factor was found to be the most important. In order to include ion collisions with neutrals in the treatment, it is necessary to make the substitution $\omega \to \omega + i\nu_i$ in (1.11). Then the dispersion relation for the frequency takes the form

$$\omega^2 + i\omega\left(\nu_i + D_e k_z^2\left(1 + k_y^{-2}\rho_{ie}^{-2}\right)\right) + D_e k_z^2\left(-i\omega^* k_y^{-2}\rho_{ie}^{-2} - \nu_i\right) = 0. \tag{2.1}$$

If $\nu_i = 0$, it is readily seen that this goes over into (1.12).

Analysis of (2.1) gives the following approximate conditions of instability:

$$\left(\frac{\omega_i}{\nu_i}\right)^4 \frac{\alpha^4}{\beta} \gg \xi^2 \gg \max\left(\beta; \frac{1}{\beta}\right). \tag{2.2}$$

Here

$$\alpha = \frac{\varkappa}{k_y}, \beta = \frac{k_z^2}{k_y^2}\frac{\omega_e\omega_i}{\nu_e\nu_i} + \frac{\nu_e\omega_i}{\nu_i\omega_e}, \xi = \frac{\varkappa}{\nu_i}\sqrt{\frac{T_e}{m_i}}.$$

It is readily seen from (2.2) that individual modes with fixed values of k_y and k_z can be excited only in definite ranges of magnetic field values:

$$\frac{k_y}{\varkappa} \leqslant \frac{\omega_i}{\nu_i} \leqslant \xi\sqrt{\frac{b_i}{b_e}}\frac{k_y}{k_z}. \tag{2.3}$$

The instability region becomes larger with increasing ξ (decreasing pressure or increasing gradient of the plasma density) and decreasing longitudinal wave number. The instability regions of modes with different k_y values can overlap, a stronger magnetic field being required to excite oscillations with larger k_y.

In[41], the relation (2.2) was used to determine the instability region of a plasma on the plane of ξ and $\eta = (\omega_i/\nu_i)\xi^{-1} = 1/\varkappa\rho_{ie}$. It is convenient to use these coordinates to characterize the state of the plasma since they are related in a simple manner to the pressure of the neutral gas, $\xi \sim p^{-1}$, and the magnetic field, $\eta \sim B$, i.e., they are the quantities that can be most readily varied in an experiment. It was found that instability requires fairly large values of $\xi (\xi \gtrsim 1)$. This is a natural restriction since the instability is due to the plasma disequilibrium, which is characterized by the gradient, $\xi \sim \varkappa$. More accurate calculations (see[8,42]) give

$$\xi_{cr} = 2. \tag{2.4}$$

In Fig. 3 this boundary is indicated by the numbers (4)–(5). The boundary section (3)–(4) ($\xi\eta = \omega_i/\nu_i \approx 1$) is determined by the condition of excitation of oscillations with the smallest possible value $k_y \approx \varkappa$ (see the left inequality in (2.3)). With further increase in the magnetic field, the critical values of ξ continue to increase. On the section (2)–(3) we have $\xi_{cr} > \eta^2\sqrt{b_i/b_e}$. Finally, for $\eta < \sqrt{m_i/m_e}$ the plasma is stabilized since the electrons cease to be affected by the magnetic

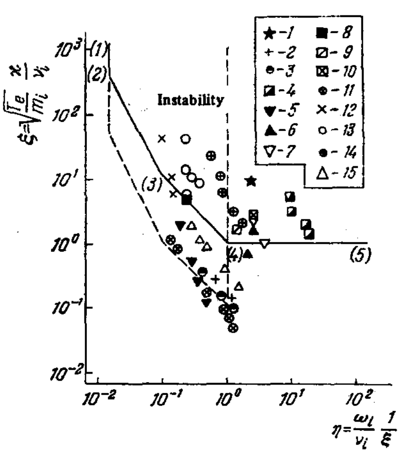

FIG. 3. The continuous and the dashed lines are the boundaries of the regions of the drift and ion-acoustic instabilities, respectively (see[41]). 1) Ar([1]); 2) H_2 ([47]); 3) He([86]); 4) H_2([50]); 5) He([92]); 6) He([22]); 7) H_2([22]); 8) He([67]); 9) H_2([67]); 10) He([51]); 11) Ar([43]); 12) Ar([60]); 13) Ar([60]); 14) Ar([59]); 15) Ar([59,77]).

field ($\varkappa\rho_e > 1$).

In[41], Timofeev also determined the boundaries of the region of the drift-dissipative instability of high-frequency oscillations with $\omega \gg \omega_i$; this is the so-called ion-acoustic instability (see Sec. 5 of this review). In Fig. 3 the boundaries are distinguished by the dashed line.

It can be seen in Fig. 3 that the values of the parameters ξ and η at which anomalies were observed in the behavior of the plasma in the experiment lie in the region of the drift-dissipative instability or near it. (So as not to clutter the figure, we have indicated the data of only some typical experiments.) This led Timofeev in[41] to assert that it is the drift-dissipative instability that was excited in the above experiments. Originally, this conclusion was not doubted. However, data gradually accumulated that could not be fitted into the scheme considered in[3,4,41]. For example, frequency analysis of the unstable oscillations[43] showed that their spectrum extends from very high frequencies—above the ion cyclotron frequency—to very low ones, below the frequency of ion collisions with neutrals. Since the frequency of drift oscillations is less than ω_i, in [43] the high-frequency part of the spectrum was attributed to the excitation of the ion-acoustic branch of the drift-dissipative instability (see Sec. 5). But the appearance of the low-frequency oscillations remained a puzzle since the drift-dissipative mechanism can lead to excitation only if $\omega > \nu_i$. (The conclusion drawn in[44] that oscillations with $\omega \ll \nu_i$ are unstable was the result of erroneous calculations.) Low-frequency oscillations with $\omega \ll \nu_i$ were also discovered in[45,49]. In the majority of cases, the values of the plasma parameters at which excitation of low-frequency oscillations was observed lie in the region of the drift-dissipative instability. But at the same time it was found (for example, in[47]) that the instability condition has the form $c_s/V_e^* = \eta > 1$, where $c_s = \sqrt{T_e/m_i}$ is the velocity of ion sound. Since at the same time ξ satisfies the inequality $\xi < 1$, the region of this instability on the $\xi\eta$ plane must have lain outside the region of the drift-dissipative instability. Thus, it follows from the experiments of[43,45-49] that in a weakly ionized gas-discharge plasma the drift-dissipative mechanism is not the only one operative. And one can then ask whether the additional mechanisms are not stronger in the frequency region $\omega > \nu_i$ as well.

On top of this, in[10,47], doubt was expressed in the validity of the local quasiclassical approximation, in which, despite the inhomogeneity of the system, the perturbations are chosen in the form of plane waves. A convincing answer to all these problems can be given only by a direct experimental verification of the dispersion relation (2.1).

C. Separation of individual modes of unstable oscillations

It is obvious that the dispersion relation can be verified only if the plasma is in a laminar regime, i.e., only a few—preferably only one—modes are excited in it. At the same time the amplitude of the oscillations must be sufficiently small for the linear approximation used in the derivation of (2.1) to be valid.

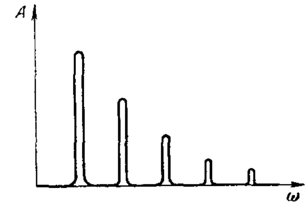

FIG. 4. Spectrum of drift oscillations.[50]

Individual modes of drift oscillations in a collisional plasma were apparently observed for the first time in[50], in which a study was made of unstable oscillations of the hydrogen plasma of a high-frequency discharge in a magnetic field. The results obtained from analysis of the oscillation spectrum are shown schematically in Fig. 4. Besides the fundamental frequency $\omega_1 = 9.4 \cdot 10^4$ sec^{-1}, the harmonics $\omega_n = n\omega_1$ up to $n = 5$ also appear. Essentially, the spectrogram is the Fourier expansion of nonlinear oscillations that are well correlated in time and space. These oscillations propagated both azimuthally and along the magnetic field. Since the system had a finite length along the magnetic field, one might expect standing waves to be formed. However, in these experiments the plasma was formed at one end of the discharge tube. As the plasma moved along the magnetic field it was lost on the tube walls, and only a small fraction of the charged particles reached the other end of the tube. It was apparently the pronounced inhomogeneity of the plasma which was responsible for the oscillations propagating along the magnetic field. The phase velocity of the oscillations was measured. Its projections onto the azimuthal direction and along the magnetic field, at gas pressure $2 \cdot 10^{-3}$ mm Hg, were $2 \cdot 10^5$ and $2 \cdot 10^6$ cm/sec, respectively. The longitudinal wavelength was approximately 100 cm, while the structure of the oscillations perpendicular to the magnetic field was not investigated.

In[50], the oscillation frequency was investigated as a function of the magnetic field. It was found that, in accordance with (1.2), the frequency decreases with increasing magnetic field, $\omega \approx \omega^* = k_y \varkappa c T_e/eB$. It was also found that the frequency decreases when the pressure of the neutral gas is increased.

Although interesting results were obtained in[50], they cannot be regarded as a complete vindication of the dispersion relation for the following reasons. 1) The azimuthal wave number m was not measured. 2) The comparison was made with the asymptotic value ω^* of the frequency of drift oscillations; this value is approximately valid only if one is far from the boundary of the instability region. 3) The influence of the inhomogeneity of the plasma column in the direction along the magnetic field was not analyzed. 4) The data correspond to the nonlinear regime, as is indicated by the presence in the spectrum of oscillations with a large number of harmonics of the fundamental frequency.

In[51], individual oscillation modes were investigated in the plasma column of a high-frequency discharge that was homogeneous in the direction along the magnetic

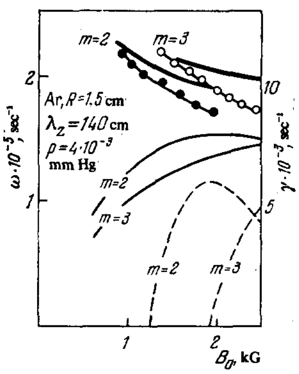

FIG. 5. Frequency ω and growth rate γ of drift oscillations as functions of the magnetic field B_0 ([63]). Ar, $p=4\cdot10^{-3}$ mm Hg, $\lambda_z=140$ cm. The black and open circles correspond to the azimuthal wave numbers $m=2$ and 3, respectively. The heavy solid curves are the calculated values of the frequencies with allowance for the Doppler effect; the thin curves, are without allowance for the effect. The dashed curves are the calculated values of the growth rates.

field. It was found that the oscillations, as required by theory, traveled azimuthally in the electron direction, and that their frequency was of the order of the drift frequency. On the $\xi\eta$ plane (see Fig. 3) the unstable state of the plasma corresponded to the region lying within the drift instability region bounded by the lines 1–5.

Individual oscillation modes were also observed in a reflex-discharge plasma in a magnetic field. The collisional regime was studied in[52].[5)] It was shown that the oscillation frequency is of the order of the frequency calculated in accordance with (2.1).

Attempts were also made to observe drift oscillations in a direct current discharge in a magnetic field.[56–58] However, because of the comparatively low frequencies of the drift oscillations their characteristics could have been significantly modified by the directed motion of the electrons. (A direct current discharge is more favorable for investigating ion-acoustic oscillations with $\omega\gg\omega_i$; see Sec. 5.) It may have been for this reason that the oscillations discovered in[56] traveled azimuthally in the ion direction, while the oscillations investigated in[58], in which they were called pseudoion-cyclotron oscillations, formed a wave traveling along the magnetic field.

D. Verification of the dispersion relation

A detailed study of drift oscillations in a high-frequency discharge plasma in a magnetic field was made in[59–64]. In[63], the frequencies of oscillations with azimuthal wave numbers $m=2$, 3 were investigated as functions of the magnetic field. In Fig. 5, the experimental data are plotted in the form of black and open circles. Figure 5 also shows the results of calculations in accordance with Eq. (2.1) of the real part (continuous curves) and imaginary part (dashed curves) of the frequency. Only the part of ω as a function of B that corresponds to instability of the plasma ($\gamma>0$) is shown. The range of variation of the magnetic field in Fig. 5 is bounded above by 2.5 kG, the maximal value achieved in the experiments of[63].

It can be seen from Fig. 5 that in the plasma, as expected from theory, the instability region has boundaries at both low and high magnetic fields, the oscillations in the experiment being found within the calculated region. As the magnetic field is increased, oscillations with smaller m are the ones first excited. In higher magnetic fields, also in agreement with theory, oscillations with different m values ($m=2$ and 3) are simultaneously excited in a certain range of variation of the magnetic field B in the plasma. In[63] the real part of the frequency was calculated both with and without allowance for a radial electric field (heavy and thin curves, respectively). A radial field causes the plasma to rotate and the Doppler effect then shifts the frequency of the drift oscillations. The values of the radial field used in the experiment lead, as can be seen from Fig. 5, to fairly good agreement between theory and experiment (compare the thick continuous curves and the curves with black and open circles).

In[64], the oscillation frequency was studied as a function of the projection k_z of the wave vector onto the direction of the magnetic field. The characteristic dependence of ω on k_z is shown in Fig. 6. It can be seen from the figure that the experimental data, which are indicated by the open circles, agree well with the calculated values obtained with allowance for the radial electric field. In the same figure, the dashed curve shows the growth rate γ calculated as a function of k_z (the part of the curve with $\gamma>0$ is shown). The instability region has boundaries at both short and long wavelengths, as was found in the experiment. Since not less than half the wavelength fits into the plasma along the magnetic field ($k_z\geq\pi/L$), it follows from Fig. 6 that drift oscillations can be excited in a plasma only if its extension exceeds a certain minimal value determined by the parameters of the discharge and the tube radius.

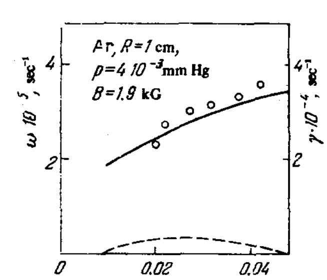

FIG. 6. Frequency ω and growth rate γ of drift oscillations as functions of the wave vector projection onto the direction of the magnetic field ($k_z=2\pi/\lambda_z$) ([64]). Ar, $p=4\cdot10^{-3}$ mm Hg, $m=2$, $B=1.9$ kG. The dashed curve gives the calculated growth rates.

[5)]It should be noted that a PIG discharge is distinguished among other types of gas discharge by the complexity of the physical processes occurring in it. Besides the electrode effects that make it difficult to establish the general features characterizing the drift-dissipative instability, it is also necessary to take into account the "short-circuit effect"[53] and "reversed-field" instability[54–55].

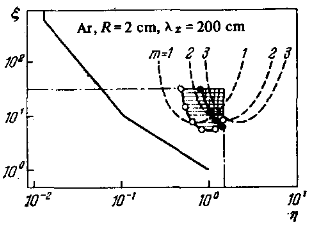

FIG. 7. Region of drift instability.[64] The open and black circles correspond to the azimuthal wave numbers $m=2$ and 3, respectively. The dashed curves are the calculated regions of excitation of oscillations with $m=1$, 2, and 3. The dot-dash-dot lines bound the region investigated in this paper.

In[63] the excitation regions for drift oscillations on the $\xi\eta$ plane calculated in[41] were tested. In Fig. 7, the open and black circles show the experimentally obtained boundaries of the instability regions for the individual oscillation modes. The excitation regions of oscillations with $m=2$ (open circles) are shown by horizontal hatching and those for $m=3$ (black circles) by vertical hatching. The dot-dash-dot lines in Fig. 7 bound the region of values of the parameters ξ and η investigated in[63]. It can be seen from Fig. 7 that the instability region of the individual oscillation modes lies within the range of ξ and η parameters predicted by the theory, although it does occupy only part of the region. In the calculations in[41] it was assumed that the ions in the plasma have zero temperature, $T_i=0$. It was shown in[59] that allowance for $T_i\neq 0$ can reduce the instability region. In fact, drift oscillations are possible if the ion Larmor radius satisfies $\rho_i<\varkappa^{-1}$, and if $T_i\approx 0.01T_e$ this leads to the condition $\eta>0.1$. If this condition is used, the instability region is restricted on the side of low magnetic fields. However, even after this correction a discrepancy between theory and experiment remains, as can be seen from Fig. 7. A discrepancy is also found in the corresponding boundaries at small ξ (high gas pressures). The best agreement between the experimental and the calculated values is obtained with allowance for the finite length of the system along the magnetic field, which leads to a restriction on k_z ($k_z\geq\pi/L$). The instability regions for the oscillations of the three first azimuthal modes calculated for a concrete value of k_z are shown in Fig. 7 by the dashed curves. (The complete instability region is the sum of the regions of the individual oscillation modes as $m_{max}\to\infty$.) It can be seen from Fig. 7 that the boundaries at small ξ and η of the instability regions of the individual oscillation modes agree fairly well with those established experimentally. At the same time, it should be noted that the first oscillation mode, which should be excited at the smallest values of the magnetic field in accordance with theory, was not observed experimentally.

The characteristics of individual modes of drift oscillations were also investigated in a reflex-discharge plasma in a magnetic field.[65] In the interpretation of the experimental data it was assumed that the oscillations with maximal growth rate must be established in the plasma. It follows from (2.1) that for this the components k_z and k_y of the wave vector must satisfy the relation $\omega^*=\omega_s$, where $\omega_s=D_e k_z^2(k_y\rho_{ie})^{-2}$. In the drift instability, as a rule, long-wave modes with few nodes are excited. Therefore, with varying magnetic field, pressure of the neutral gas, etc, the modes must be rearranged discretely at fairly large intervals of variation of these parameters. Under these conditions, the simplified equivalent of the dispersion relation in the form $\omega^*=\omega_s$ can be verified only at individual points. At the same time, interesting data were obtained[66] on the rearrangement of the azimuthal modes (Fig. 8). It follows from Fig. 8 that an increase of the magnetic field increases the azimuthal wave number of the unstable oscillations, and, in accordance with the left-hand approximate inequality (2.3), mode m is unstable for $\omega_i/\nu_i\approx m\approx k_y/\varkappa$.

It is necessary to dwell in detail on the experiments of[22,66,67], in which a study was made of the oscillations of a high-frequency discharge plasma and an afterglow plasma in a magnetic field. Individual modes were observed, and these were identified as drift modes, and the dispersion law was verified. Good agreement with the theoretical calculations was obtained. However, it was found that the oscillation frequency could take values appreciably lower (by an order of magnitude) than the frequency of ion collisions with neutrals. The result contradicts the basic assumptions of the theory of the drift instability.

In the experiments of[22,66] with an afterglow plasma the late stage of the discharge, when the plasma density is fairly low, was studied. One can therefore suppose that in these experiments a low-frequency variant of the drift-dissipative instability of a low-density plasma was actually observed (see Sec. 4 of this review). One can attempt to explain the agreement with the theory of the drift instability by the circumstance that in[22,66] in the theoretical calculations an additional dependence of the perturbations on the x coordinate in accordance with the law e^{ikx^x} was introduced. Then, as we noted in the third footnote in this paper, an additional excitation effect can be artificially introduced, and this evidently led to the conclusion that there was instability of drift oscillations with $\omega\ll\nu_i$.

3. DRIFT INSTABILITY IN THE FULLY IONIZED PLASMA OF Q MACHINES

Usually, electric fields or bunches of fast particles are used to ionize a gas. The plasma obtained in this way is in a state that is very far from thermodynamic equilibrium, and it is therefore electrostatically un-

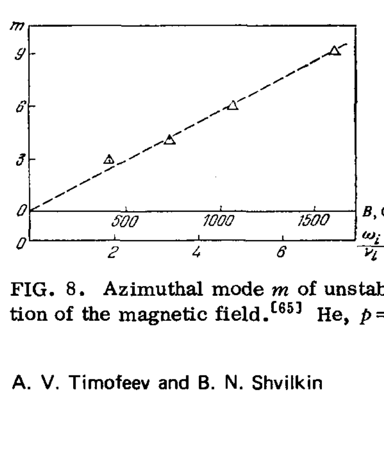

FIG. 8. Azimuthal mode m of unstable oscillations as a function of the magnetic field.[65] He, $p=2.2\cdot 10^{-2}$ mm Hg.

stable. The Q machines were conceived as a means to study a quiescent fully ionized plasma. In them, the plasma is produced by the thermal ionization of a beam of neutral atoms that impinge on a heated metallic slab. After it has been formed, the plasma flows freely from the plate along the magnetic field. However, even this "quiescent" plasma sprang a fair number of surprises. In particular, the development of several types of instability was established. Some time was required to analyze the observed phenomena and distinguish the drift-dissipative instability from the others. We shall not dwell on this original stage of the investigations, which has been fairly well covered in[68,69]. We turn directly to the derivation and verification of the dispersion relation of the drift-dissipative instability (see[70–72]).

In the plasma of Q machines, the ions have a temperature near that of the electrons. In considering the oscillations in such a plasma, one must take into account pressure and viscosity in the ion equation of motion. The components of the viscous stress tensor given in Sec. 2 contain the two viscosity coefficients $\eta_1 = 0.3 n T_i \nu_i / \omega_i^2$ and $\eta_3 = 0.5 n T_i / \omega_i$. The terms proportional to η_3 take into account the so-called collisionless viscosity, which is due to the effects of the finite Larmor radius, while the terms proportional to η_1 take into account the collisional viscosity. Use of the hydrodynamic approximation presupposes that the frequency of the processes is low compared with the frequency of ion collisions: $\omega \ll \nu_i$. We shall see below that these oscillations may be unstable. In this respect we have a radical difference between a fully ionized and a weakly ionized plasma. The reason for this is that in a weakly ionized plasma ion collisions with neutral atoms do not equalize the velocity gradients (viscosity) but rather decelerate the ion component (friction). Friction is a much more effective stabilizing factor. It is for this reason that oscillations with $\omega \ll \nu_i$ in a weakly ionized plasma are stable (see the previous section).

In the derivation of the dispersion relation of the drift oscillations we shall, as in Sec. 1, ignore the displacements of the ions along the magnetic field. Using the ion equation of motion (1.3) and the continuity equation, we obtain

$$-i\omega n_1 + i k_y \varkappa \frac{c}{B} n_0 \varphi_1 + \frac{c}{B} \frac{k_y^2}{\omega} [-i\omega + 0.3\nu_i (k_y \rho_i)^4] \left(n_0 \varphi_1 + \frac{T}{e} n_1 \right) = 0. \qquad (3.1)$$

The dispersion relation, which is determined from the condition of compatibility of (1.9) and (3.1), can be conveniently represented in the form

$$(\omega - \omega^*) \left(1 - \frac{\Omega_i}{\Omega_e} \right) = i \frac{\omega}{\Omega_e} (k_y \rho_i)^2 (\omega + \omega^*) - 2i\Omega_i - 2\omega (k_y \rho_i)^2. \qquad (3.2)$$

Here, for brevity, we have introduced the notation $\Omega_e = D_e k_z^2$, $\Omega_i = 0.3 \nu_i (k_y \rho_i)^4$. If collisional viscosity is ignored in (3.2) by setting $\Omega_i = 0$, the difference between (1.12) and (3.2) will be due solely to the effects of the finite Larmor radius (collisionless viscosity). For $(k_y \rho_i)^2 \ll 1$, the right-hand side of (3.2), as in Subsection C of Sec. 1, can be taken into account as a small correction. In the zeroth approximation in $k_y \rho_i$, we have $\omega = \omega^*$. Substituting this value of the frequency into the right-hand side of (3.2), we find that it is twice the right-hand side of (1.12), so that the growth rate of the oscillations is doubled. This means that in an isothermal plasma the effects of the finite Larmor radius influence the drift oscillations in exactly the same way as inertial effects. Inertia leads to an effective deceleration of the ions in the oscillations (see Sec. 1), and according to[3] the finite Larmor radius must have the same effect. The point is that charged particles revolving around a Larmor circle of finite radius are subjected on the average to a weaker electric field $E_{eff} = E(1 - k_y^2 \rho_i^2)$ (see[73]). Because of the equivalence of inertial effects and the effects of a finite Larmor radius in a plasma with hot ions, even oscillations with a very low frequency, whose development is quite unaffected by inertial effects, may be unstable.

Let us now take into account collisional viscosity. It follows from (3.2) that the oscillations are stable at both very small and very large values of Ω_e. The range of values in which the plasma is unstable is determined by the inequalities $(\omega^* k_y \rho_i)^2 \Omega_i^{-1} \gtrsim \Omega_e \gtrsim \Omega_i$. The last inequality can be represented conveniently for what follows in the form (see[71])

$$\frac{B}{k_y} \gtrsim (1.2)^{1/4} \frac{c}{e} \left(\frac{m_i}{2k_z} \right)^{1/2} (T m_e \nu_e \nu_i)^{1/4} \sim \left(\frac{n_0}{T} \right)^{1/2} m_i^{3/8} k_z^{-1/2}. \qquad (3.3)$$

The dependence of the critical magnetic field on the ion mass can be conveniently verified by using a plasma which is a mixture of two elements. The mean mass in the expression for the critical magnetic field can vary continuously with the relative fractions $\langle m_i \rangle = \sum_\alpha n_{0\alpha} m_{i\alpha} / \sum_\alpha n_{0\alpha}$ of the elements. In the experiments of[71] a mixture of Cs and K was used. Figure 9 shows that the required dependence $B \sim \langle m_i \rangle^{3/8}$ was indeed obtained. At the same time, the coefficient of proportionality exceeds the calculated value by a factor of about 1.5. In[71], the dependence of the critical magnetic field on the plasma density calculated from (3.3) was also confirmed, and it was found that only the first mode with the smallest wave number $m = 1$ exhibited any significant deviation from the theory. This circumstance unambiguously indicates that the local quasiclassical approximation is a possible source of the discrepancies. Another source

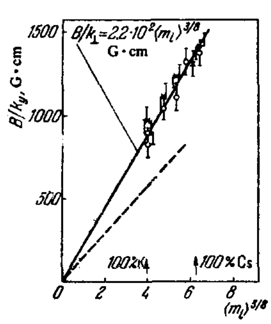

FIG. 9. Critical value of B/k_y as a function of the mean ion mass expressed in mass units for different mixtures of potassium and cesium. The theory gives $B/k_y = 1.5 \cdot 10^2 (m_i)^{3/8}$ G·cm for $\lambda_z = 2L$; $T = 2800$ °K. $n_0 = 10^{11}$ cm^{-3}, ([71]). The dashed line is the calculation for $\lambda_z = 4L$.

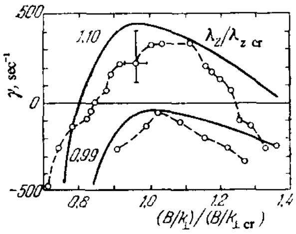

FIG. 10. Growth rate γ as a function of B/k_y for two different values of the ratio $\lambda_z/\lambda_{z.cr}$ ($B/k_{\perp cr} = B/k_\perp$ for $\lambda_{z.cr}$). $n_0 = 5\times10^{10}$ cm^{-3}, $k_\perp = 2.5k_y$, $m=1$, $T=2650$ °K, $\varkappa = 2.3$ cm^{-1}. The continuous curves are calculated.[72]

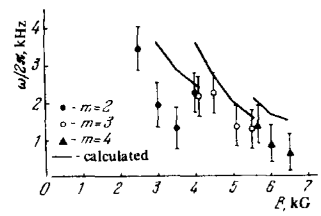

FIG. 11. Comparison of the frequency of unstable oscillations with the drift frequency in a potassium plasma.[71] The Doppler shift has been taken into account.

could be the use of the hydrodynamic equations to describe the oscillations despite the fact that the wavelength at right angles to the magnetic field is of the same order of magnitude as the ion Larmor radius.

The stabilization at sufficiently small values of the magnetic field is due to the influence of viscosity. If the length of the system in the direction along the magnetic field is not too great or the magnetic field itself is sufficiently strong, so that the condition $(\omega/\omega_i)k_y/k_z \leq 1$ is satisfied, it may be necessary to take into account another important stabilizing factor—the longitudinal motion of the ions (see[71,72]).[6)] We note in particular that, to eliminate the influence of the last factor, all theoretical studies of the drift-dissipative instability were made for oscillations with pronounced extension along the magnetic field. In[72], the growth rate of the drift-dissipative instability was calculated with allowance for the longitudinal motion of the ions (Fig. 10). It can be seen from Fig. 10 that oscillations with $\lambda_z < \lambda_{z,cr}$ are stable for all values of the magnetic field. If $\lambda_z > \lambda_{z,cr}$, then in a certain range of the magnetic field the oscillations become unstable. Usually, in the experiments on the drift-dissipative instability data are given only for the real part of the oscillation frequency. In[73], their growth rate was also measured. As follows from Fig. 10, the results of the measurements agree well with the theoretical calculations if it is borne in mind that $k_x \neq 0$ and one assumes $k_\perp = \sqrt{k_x^2 + k_y^2} = 2.5k_y$.

In Fig. 10, $(B/k_\perp)/(B/k_{\perp cr})$ is plotted along the abscissa. On the basis of this figure, we can conclude that for oscillations with large azimuthal wave number m the instability region must be shifted to higher magnetic fields. This feature is reflected in Fig. 11. We recall that the same effect is observed in a weakly ionized plasma; see the previous subsection. In[72] it was found that the frequency of unstable oscillations is approximately equal to $\omega^*/2$. This theoretical result agrees satisfactorily with the experimental data (see Fig. 11).

To get an idea about the conditions under which the various stabilizing mechanisms can be manifested, it is helpful to consider the phenomena on the plane of λ_z and $B/k_\perp$ (Fig. 12). In Fig. 12, the instability region is bounded on three sides. Below, i.e., in the region of weak magnetic fields, the stabilization is due to viscosity; above, to the influence of the longitudinal motion of the ions; finally, on the left, i.e., in the region of short wavelengths, diffusion of the electrons leads to stabilization. Indeed, as is shown in[72] (see also the foregoing subsection) the drift-dissipative instability at a sufficiently short wavelength is stabilized even without allowance for the longitudinal motion of the ions.

In this review we have discussed only those of the results obtained in[71,72] that appear to us the most important and interesting. In[71,72] investigations were also made of the longitudinal wavelength, the critical magnetic field on the plasma density, the oscillation frequency on the plasma temperature, the transverse wavelength on the magnetic field; the plasma flux outward due to the unstable oscillations was also measured. The complete set of results of[71,72] leave no doubt that the observed instability really is the drift-dissipative instability and that its properties are described perfectly satisfactorily in the framework of the above theoretical model. It appears to us that it is the experiments in Q machines that provide the fullest and most comprehensive verification of the theory of the drift-dissipative instability. At the same time, we cannot ignore the numerical discrepancies between the theory and the experiment (see Figs. 9 and 10). We have already discussed the possible origins of these discrepancies.

4. LOW-FREQUENCY DRIFT INSTABILITY OF A WEAKLY IONIZED PLASMA IN A MAGNETIC FIELD

A. Instability of a not fully magnetized plasma

As we have already mentioned, in a number of experiments low-frequency ($\omega \ll \nu_i$) oscillations were ob-

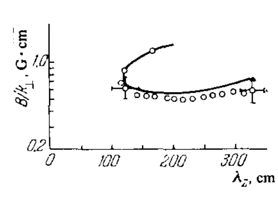

FIG. 12. Critical value of $B/k_\perp$ as a function of λ_z([72]). $k_\perp = 2.5k_y$, $m=1$, $T=2650$ °K, $n = 2\cdot10^{11}$ m^{-3}, $\varkappa = 1.8$ cm^{-1}. The continuous curve is calculated.

[6)]In a number of papers (see, for example, [75]) experimental results have been interpreted by introducing an effect of damping of the oscillations at the ends of the Q machine. However, the analysis of this question in[74,76] showed that the conditions at the ends affect only the longitudinal wavelength of the oscillations, while the stabilizing effect due to the drift of particles to the ends out of the volume occupied by the plasma is insignificant.

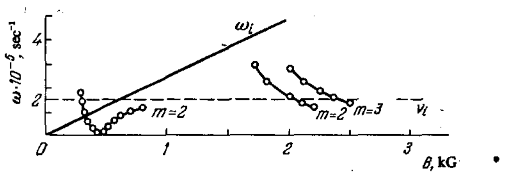

FIG. 13. Frequency ω of unstable oscillations as a function of the magnetic field B([59]). Ar, $p=1\cdot 10^{-2}$ mm Hg. The straight line through the origin is the ion cyclotron frequency ω_i. The dashed line is the frequency ν_i of ion-neutral collisions.

served whose excitation could not be attributed to the instability mechanism considered in the first section. So far, it has been possible to interpret only some of these experiments. The greatest clarity has been achieved in understanding the nature of the instability observed in[77,78]. As the magnetic field is varied, oscillations of two types are excited successively. The frequency of the oscillations excited at comparatively large magnetic fields exceeded ν_i (Fig. 13). The corresponding instability was identified as the drift instability. In the region of weak magnetic fields, low-frequency oscillations with $\omega < \nu_i$ were observed. These like the drift oscillations, had the form of a standing wave in the direction along the magnetic field (Oz) and of a traveling wave along the azimuthal direction. It is characteristic that at a certain magnetic field the sign of the azimuthal phase velocity was reversed. On the left-hand (decreasing) section in the curve of the frequency as a function of the magnetic field the oscillations traveled in the electron direction, while on the right-hand (increasing) section they traveled in the ion direction. The wavelength in the direction along the magnetic field was twice the length of the solenoid, so that half a wavelength was fitted into the plasma. The low-frequency oscillations were observed only in plasma with heavy ions (Ar, Kr, Xe), and then for magnetic field values for which the ion Larmor radius ($\rho_i \sim \sqrt{m_i}$) was comparable with the radius of the discharge tube. This circumstance suggested that the finite Larmor radius plays an important role in the excitation mechanism.[79] The influence of the effects of the finite Larmor radius on drift oscillations was considered in the previous section, but we cannot use these results because they were obtained in the hydrodynamic approximation, and in the case of a weakly ionized plasma the system of hydrodynamic equations is a collection of moments of the kinetic equation (see Sec. 1). Its use to describe effects as subtle as those due to the finite Larmor radius would require the additional introduction of many moments. In this case, it is simpler to have direct recourse to the kinetic equation:

$$\frac{\partial f}{\partial t}+(\mathbf{v}\cdot\nabla)f+\frac{e}{m_i}\left(\mathbf{E}+\frac{1}{c}(\mathbf{v}\times\mathbf{B})\right)\cdot\frac{\partial f}{\partial\mathbf{v}}=\mathrm{St}(f). \qquad (4.1)$$

Here, f is the ion distribution function. In[79], the collision term was taken in a model form which allows for the conservation of the ion number in collisions with neutral atoms (see[80]):

$$\mathrm{St}(f)=-\nu_i(f-f_0)+\nu_i\frac{f_0}{n_0}\int d\mathbf{v}\,(f-f_0). \qquad (4.2)$$

In (4.2) it is assumed that the collisions carry the ions component into thermal equilibrium with the neutral component: $f\to f_0=n_0(m_i/2\pi T_i)^{3/2}\cdot\exp(-mv^2/2T_i)$, where T_i is taken equal to the temperature of the neutral gas.

We assume as before that in the oscillations all the perturbed quantities vary in accordance with the law $\exp(-i\omega t+ik_y y+ik_z z)$. To determine the perturbed distribution function f_1, we linearize the kinetic equation (4.1) with respect to the small perturbations. Integrating the expression for f_1 with respect to the velocities, we find the perturbation of the ion density n_1. By analogy with (1.9), it is convenient to represent the expression for n_1 in the form

$$\left[-i\omega+ik_y u\frac{\omega_i^2}{\omega_i^2+\nu_i^2}(1+a_3)+D_i k_y^2\frac{\nu_i^2}{\omega_i^2+\nu_i^2}(1-a_2)\right]n_1$$
$$+\left[ik_y\varkappa\frac{\omega_i^2}{\omega_i^2+\nu_i^2}(1-a_1)+k_y^2\frac{\omega_i\nu_i}{\omega_i^2+\nu_i^2}(1-a_2)\right]\frac{c}{B}n_0\varphi_1=0; \qquad (4.3)$$

where $u=c/Bd\varphi_0/dx$, in which φ_0 is the unperturbed electric potential, which is usually present in a gas-discharge plasma and which, generally speaking, must be taken into account in the analysis of stability.

Accordingly, Eq. (1.9) must be augmented by the term $-ik_y un_1$. The effects of the finite Larmor radius are taken into account in (4.3) by means of

$$a_1=1-\frac{A_1}{A_2},\quad a_2=1-\frac{1}{z_i}\frac{1-A_1}{A_2},\quad a_3=\frac{1-A_2+A_3}{A_2},$$

where

$$A_1=\eta_0(z_i)+2\sum_{n=1}^{\infty}\eta_n(z_i)\frac{\nu_i^2}{\nu_i^2+n^2\omega_i^2},$$
$$A_2=1-4\sum_{n=1}^{\infty}\eta_n(z_i)\frac{\nu_i^2 n^2\omega_i^2}{(\nu_i^2+n^2\omega_i^2)^2},$$
$$A_3=\frac{\nu_i^2}{\omega_i^2}\left(1-\frac{2}{z_i}\sum_{n=1}^{\infty}\eta_n(z_i)n^2\frac{\nu_i^2+\omega_i^2}{\nu_i^2+n^2\omega_i^2}\right),$$
$$\eta_n(z_i)=I_n(z_i)e^{-z_i},\quad z_i=(k_y\rho_i)^2,$$

and I_n is a Bessel function of imaginary argument.

As usual, we find the dispersion relation for the frequency from the condition of solvability of the system (1.9) and (4.3). Analysis leads to the instability condition

$$D_e k_0^2\left(1+\frac{T_i}{T_e}\right)\frac{\nu_i}{\omega_i}(1-a_2)\left(1+\frac{\nu_i^2}{\omega_i^2}+a_4\right)$$
$$<-\varkappa u\left[-1+a_1\left(1+\frac{\nu_i^2}{\omega_i^2}-\frac{\omega_i^2}{\nu_i^2}\right)+a_2\left(1+\frac{\nu_i^2}{\omega_i^2}\right)-a_3\right.$$
$$\left.+a_4-a_1a_3\frac{\omega_i^2}{\nu_i^2}+a_1a_4\left(1+\frac{\omega_i^2}{\nu_i^2}\right)+a_2a_4\right]$$
$$+\omega^*\left(1-a_1-\frac{T_i}{T_e}a_4\right)\left(1+a_1\frac{\omega_i^2}{\nu_i^2}\right), \qquad (4.4)$$

where

$$k_0^2=k_z^2+\frac{\nu_e^2}{\omega_e^2}k_y^2,$$
$$a_4=\frac{k_y^2}{k_0^2}\frac{\nu_i\nu_e}{\omega_i\omega_e}(1-a_2).$$

Note that although a_1, a_2, a_3 cannot exceed unity, they do become comparable to it when $k_y\rho_i\approx 1$.

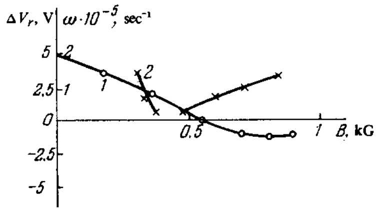

FIG. 14. Radial electric field $\Delta V_r(1)$ and frequency of unstable oscillations $\omega(2)$ as a function of the magnetic field B([81]). Ar, $p=8\cdot10^{-3}$ mm Hg, $\lambda_z=120$ cm, $m=2$.

It follows from (4.4) that oscillations traveling azimuthally in the ion direction ($k_y<0$) can be unstable even in the absence of an electric field ($u=0$). This conclusion is confirmed by the measurements of the radial electric field made in[81] (Fig. 14), from which it can be seen that when the electric field is zero the unstable oscillations are the ones traveling in the ion direction, whose frequency increases with increasing B.

Let us now consider the question of the direction of rotation of the oscillations. Simple estimates show that in a weakly ionized plasma the sign of the phase velocity of the drift oscillations depends on the ratio of $\Omega_e' = D_e[k_z^2+k_y^2(\nu_e^2/\omega_e^2)]$ and $\Omega_i'=D_i\{k_z^2+k_y^2[\nu_i^2/(\nu_i^2+\omega_i^2)]\}$. If electron diffusion is predominant ($\Omega_e'>\Omega_i'$), the oscillations travel in the electron direction, but if the ion diffusion is predominant ($\Omega_i'>\Omega_e'$) then they travel in the ion direction. If the wavelength of the oscillations in the direction of the magnetic field is sufficiently great and the ion temperature is not too low it may be that at very small and very large values of the magnetic field the condition $\Omega_e'>\Omega_i'$ is satisfied, while $\Omega_i'>\Omega_e'$ in the intermediate region. In this case, the dependence of the phase velocity on the magnetic field must have the form shown in Fig. 15. Of course, the experiment establishes only the part of the curve to which positive growth rates correspond. In Fig. 15 they are indicated by the thicker curves (cf. Fig. 13).

Low-frequency oscillations were studied in detail in[81]. Figure 16 shows typical results of measurements of the frequency of low-frequency drift oscillations as a function of the magnetic field obtained in an experiment with a weakly ionized argon plasma at different pressures (curves with the open circles). In the same figure, the dashed curves are the dependences of the frequency on the magnetic field calculated by means of (1.9) and

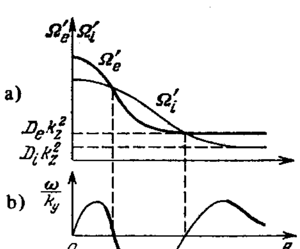

FIG. 15. Diffusion frequencies of electrons and ions, Ω_e' and Ω_i' (a), and phase velocity ω/k_y of drift oscillations (b) as functions of the magnetic field. The heavy curves are the sections corresponding to instability ($\gamma>0$).

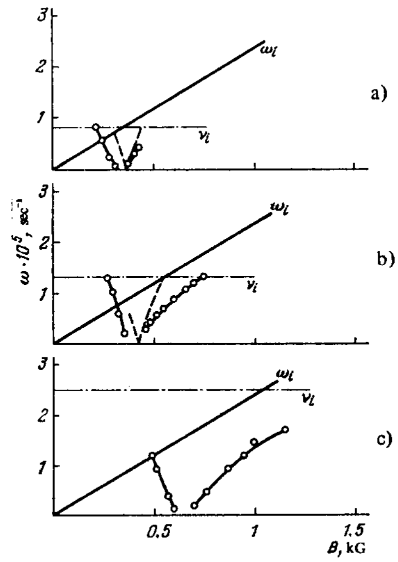

FIG. 16. Frequency of unstable oscillations ω as a function of the magnetic field B([81]). Ar, $\lambda_z=180$ cm, $m=2$; pressures (mm Hg): $5\cdot10^{-3}$ (a), $8\cdot10^{-3}$ (b), and $1.5\cdot10^{-2}$ (c). The dashed lines are the calculated dependences.

(4.3). It can be seen that there is good agreement between the calculations and the experiment.

The theory also reproduces satisfactorily the experimentally observed dependence of the oscillation frequency on the projection k_z of the wave vector onto the direction of the magnetic field B. This can be seen from Fig. 17, in which we have plotted the calculated and measured dispersion characteristics of the low-frequency drift oscillations with decreasing dependence of ω on B for two values of the magnetic field. The theory also enables one to determine the interval of unstable wavelengths as a function of the magnetic field. Figure 18 shows the experimentally determined (*a*) and calculated (*b*) regions of excitation of oscillations with different longitudinal wavelengths for three different pressures. Comparison of the data in this figure shows that, as predicted by theory, low-frequency drift oscillations are excited in the plasma only in a restricted range of longitudinal wavelengths.

In[79,81] the experimentally measured and calculated

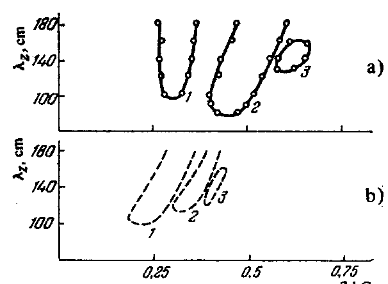

FIG. 17. Longitudinal wavelength λ_z of unstable oscillations as a function of the magnetic field B([81]). Ar, $m=2$. a) Experiment, pressures (mm Hg): $8\cdot10^{-3}$ (1), $1.5\cdot10^{-2}$ (2), and $3\cdot10^{-2}$ (3); b) calculation, pressures (mm Hg): $5\cdot10^{-3}$ (1), $8\cdot10^{-3}$ (2), and $1\cdot10^{-2}$ (3).

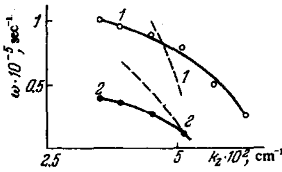

FIG. 18. Frequency ω of unstable oscillations as a function of $k_z = 2\pi/\lambda_z$([81]). Ar, $p = 8 \cdot 10^{-3}$ mm Hg, $m = 2$; magnetic fields (G): 300 (1) and 200 (2). The dashed curves are the calculated dependences.

instability regions of low-frequency drift oscillations in the magnetic field and the pressure were compared. The results of these comparisons are shown in Fig. 19. It can be seen that the regions overlap only partly. However, they both lie near the line $\omega_i/\nu_i = 1$. The fulfillment of the last condition is important for the excitation of low-frequency drift oscillations, [77] since the difference between the drift velocities of the electrons and ions in the crossed fields E_y and B needed for the occurrence of this instability is especially large when $\omega_i/\nu_i \approx 1$. Note that from the condition $\omega_i/\nu_i \approx 1$ one can estimate the smallest pressure at which instability is still possible; for with decreasing pressure of the gas the minimal magnetic fields needed for the instability to occur decrease (the critical magnetic field decreases). This, in its turn, leads to an increase of the ion Larmor radius, and when it exceeds the radius of the discharge tube the instability ceases. Estimates of the lower limiting pressure for the existence of the low-frequency drift instability[81] confirm the correctness of this approach.

B. Instability of a low-density plasma

Comparatively long ago it was noted theoretically that a low-density plasma may be more unstable than a dense one (see, for example[11]). Let us demonstrate this. If one is considering oscillations in a low-density plasma, one must replace the condition $n_e = n_i$ of quasineutrality by the Poisson equation

$$\Delta\varphi = 4\pi e\,(n_e - n_i). \tag{4.5}$$

Assuming that the perturbations of the electron density in (1.9) and the ion density in (1.10) are related through (4.5), we find the following dispersion relation for the

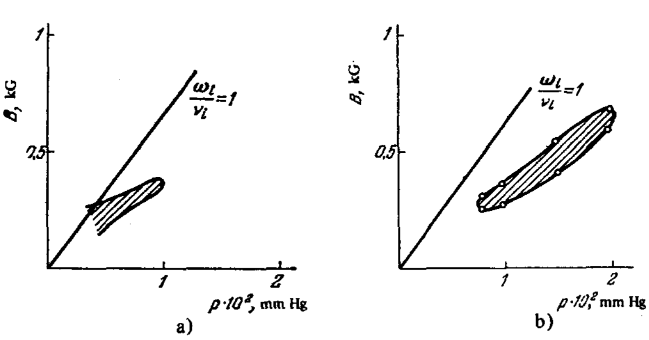

FIG. 19. Regions of excitation of oscillations with respect to the magnetic field B and the pressure p([81]). Ar, $\lambda_z = 120$ cm, $m = 2$. (a) Calculation; b) experiment.

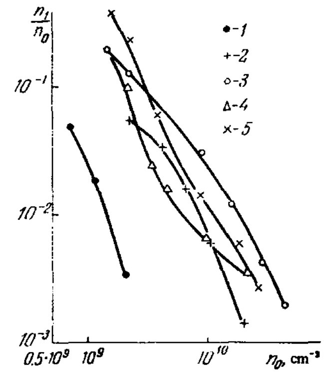

FIG. 20. Relative magnitude of the oscillations of the plasma density in the perturbations, n_1/n_0, as a function of the density n_0([83]). $2R = 1.2$ cm, $p = 0.15$ mm Hg. Magnetic fields (kG): 0.7 (1), 1 (2), 1.6 (3), 3 (4), and 5 (5).

oscillation frequency:

$$\omega - \omega^* = \frac{i\omega^2}{D_e k_z^2} k^2 d_e^2 - \omega k^2 d_e^2. \tag{4.6}$$

This equation differs from (1.12) by the substitution $\rho_{ie} \to d_e$, where $d_e = \sqrt{T_e/4\pi e^2 n_0}$ is the electron Debye radius. Therefore, the effects due to the plasma's being nonquasineutral play the same role as ion inertia, and low-frequency oscillations with $\omega \ll \nu_i$ can be excited. In[82,83] these ideas were invoked to explain the experiments in an afterglow plasma. The anomalously fast decay of such a plasma had been noted earlier in[32,33]. It was found later that the increase in the diffusion coefficient is accompanied by the occurrence of oscillations. [48,49] This circumstance forced one to look for the origin of the anomalies in an instability of the plasma. To establish its nature, it is necessary to take into account the following circumstances. 1) When the instability occurs, the dimensionless parameters ξ and η lie outside the region of the drift instability in Fig. 3 ($\omega_i < \nu_i$, $\xi < 1$). 2) Because the electron temperature in the afterglow plasma is near the ion temperature, ion-acoustic oscillations cannot be excited. 3) The instability develops only in the later afterglow, when the plasma density is fairly low. This is illustrated in Fig. 20, which is taken from[83], and in which the relative magnitude of the fluctuations in the plasma density, n_1/n_0, is plotted against the density n_0. Similar results were also obtained for other values of the pressure of the neutral gas and the discharge tube radius.

The dispersion relation (2.4) cannot be used to describe these experiments since it was obtained under the assumptions $T_e \gg T_i$, $\omega_i \gg \nu_i$, whereas the afterglow plasma is almost isothermal ($T_e \approx T_i$) and the instability region is characterized in fact by the opposite relation between the frequencies ω_i and ν_i ($\omega_i \ll \nu_i$). In this case, simple calculations give

$$\omega(1+s) - t\omega^* = \frac{i}{\Omega_i'}(\omega - \alpha t\omega^*)(\omega s - t\omega^*) - it\Omega_i'; \tag{4.7}$$

here we have denoted $s = k^2 d_e^2$, $t = T_i/T_e$,

$$\alpha=\frac{1-(1/t)(\nu_e\nu_i/\omega_e\omega_i)}{1+(\nu_e\nu_i/\omega_e\omega_i)},$$

$$\Omega_i'=D_i k^2,\quad \Omega_e'=D_e[k_z^2+k_y^2(\nu_e^2/\omega_e^2)].$$

In deriving (4.7), we have used the condition $\Omega_i'\gg\Omega_e'$. Since $T_e\approx T_i$ in an afterglow plasma, for oscillations with $k_y\gg k_z$ this condition is satisfied at comparatively small values of the magnetic field. We have also borne in mind that in a plasma there exists a stationary ambipolar electric field under the influence of which the electrons drift azimuthally (in the equivalent system which we consider, from the symmetry plane along OY).

From (4.7) we find that the plasma is unstable when

$$\alpha>\frac{1}{1+s}+t(1+s)\frac{\Omega_e'\Omega_i'}{\omega^{*2}}. \tag{4.8}$$

The value of α is positive if $\omega_e\omega_i/\nu_e\nu_i>T_e/T_i$. This necessary condition of instability agrees in order of magnitude with the one found experimentally: $\omega_e\omega_i/\nu_e\nu_i\approx 5$–$9$ (see[83]). Note that when $\alpha>0$ the plasma is positively charged relative to the walls.

Analysis of (4.8) shows that oscillations are excited only at a fairly low plasma density, when $(\varkappa d_i)^2\geqslant(m_e/m_i)\nu_e/\nu_i$. The most unstable oscillations are the ones with $s\approx(\varkappa d_i)^{2/3}(m_i\nu_i/m_e\nu_e)^{1/3}$ and $k_z\lesssim k_y\nu_e/\omega_e$. In[83], however, it was noted that the experimentally determined critical density is about two orders of magnitude less. Subsequent comparison of theory and experiment revealed agreement in some respects as well as some discrepancies. For example, in agreement with theory[83] [see also (4.7)] the frequency decreased with decreasing plasma density, while the noise intensity increased with the magnetic field, length of the discharge tube, and density gradient. At the same time, the phase velocity of the oscillations exceeded the calculated value by 1 to 2 orders of magnitude. In[83], the discrepancies between theory and experiment were attributed to the inaccuracy of the local quasiclassical approximation. But it then remains unclear why this approximation, which can be used successfully to describe other forms of the drift-dissipative instability, is unsatisfactory in the present case.

This discussion shows that the study of the instability observed in an afterglow plasma cannot be regarded as completed. However, the point of view expressed in[82,83] already enables us to understand the characteristic features of the instability such as the low frequency of the unstable oscillations ($\omega<\nu_i$) and also the occurrence of the instability in the later afterglow, when the plasma density has fallen below a certain critical level. At the same time, the possibility cannot be excluded that the physical phenomena that develop in the afterglow plasma have a more complicated nature. For example, in[84] the excitation of oscillations was observed even in the absence of a magnetic field in the later afterglow. This result was interpreted as the excitation of acoustic oscillations in which the neutral component of the plasma participated.

5. ION-ACOUSTIC INSTABILITY

A. Theory

In the preceding subsections we have analyzed the drift instability of an inhomogeneous plasma in a magnetic field. The frequency of drift oscillations is proportional to the density gradient $\omega\approx\omega^*=(cT_e/eB)k_y\varkappa$ and, therefore, in a homogeneous plasma the branch of drift oscillations itself is absent. However, inhomogeneity of the plasma may lead to the excitation of oscillations that are characteristic of a homogeneous plasma. For example, in a nonisothermal plasma with $T_e\gg T_i$ the branch of ion-acoustic oscillations exists. (We recall that, usually, in a gas discharge the electron temperature is appreciably higher than the ion temperature: $T_e/T_i\approx 10^2$. Only an afterglow plasma, in which $T_e\approx T_i$, is an exception.) In the absence of a magnetic field, ion-acoustic oscillations propagate with velocity $c_s=\sqrt{T_e/m_i}$ ($\omega=kc_s$). If the wavelength of the oscillations is sufficiently short, so that the condition $\omega\gg\omega_i$ is satisfied, a magnetic field does not affect the oscillations. In this case, even in the presence of a magnetic field, we can use the same expression $\omega/k=d\omega/dk=c_s$ for the phase (and group) velocity of the ion-acoustic oscillations. In terms of the velocity, a measure of the plasma inhomogeneity is the Larmor drift velocity $\omega^*/k_y=(cT_e/eB)\varkappa$. It is natural to assume that the inhomogeneity of the plasma has more influence on the ion-acoustic oscillations for $V^*>c_s$. We shall show that it is this condition which is the condition of instability of the ion-acoustic oscillations. Using the continuity equation and also the equations of motion, in which we ignore the influence of the magnetic field, we obtain

$$\omega n_1-\frac{e}{m_i}\frac{k^2}{\omega+i\nu_i}n_0\varphi_1=0. \tag{5.1}$$

The condition of solvability of the system (1.9) and (5.1) leads to the dispersion relation

$$\omega(\omega+i\nu_i)-k^2c_s^2\left(1-i\frac{\omega-k_y\Delta V_0}{D_ek_z^2}\right)\left(1-i\frac{\omega^*}{D_ek_z^2}\right)^{-1}=0. \tag{5.2}$$

To simplify the calculations, we have gone over in (5.1) and (5.2) to a coordinate system in which the ions are at rest: $\omega=\omega'-k_yV_{0i}$, where ω' is the frequency of the oscillations in the laboratory system,

$$V_{0i}=\frac{1}{k_y}\omega^*\left(1+\frac{\nu_i^2}{\omega_i^2}\right)^{-1}\left(1+\frac{\omega_i\omega_e}{\nu_i\nu_e}\right)^{-1}$$

is the ion drift velocity in the stationary ambipolar electric field. In (5.2), we have also denoted $\Delta V_0=V_{0e}-V_{0i}=V_{0i}\nu_i^2/\omega_i^2$.

Let us consider short-wave high-frequency oscillations with $k_y\gg\mathrm{Max}(k_z,\ \nu_i/c_s)$ for $c_s\gg\Delta V_0$. To determine the boundaries of the instability region, we equate to zero, assuming $\mathrm{Im}\,\omega=0$, the real and the imaginary part of (5.2) separately. We then find $\omega=k_yc_s$, $V^*=c_s$. For $V^*>c_s$, the plasma is unstable. This condition was obtained in[41] (see also[42]). It can be conveniently represented in the form $\varkappa\rho_{ie}=\eta^{-1}<1$. In Fig. 21 (see also Fig. 3) it restricts the instability

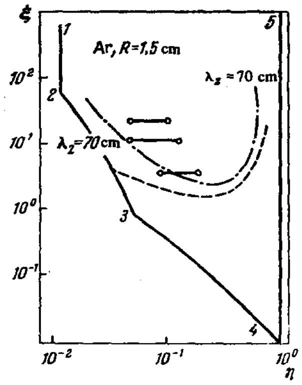

FIG. 21. Boundaries of the region of the ion-acoustic instability in an argon plasma.[2,41] The open circles are the results of the experiments in [64]. The dashed curve is the boundary of the instability region obtained from (5.2) for $m=1$; the dot-dash-dot curve is the boundary found from the condition (5.4).

region of the ion-acoustic oscillations at large values of the magnetic field. In[41] however it was shown that short-wave oscillations with $k_z \geqslant \nu_e/\sqrt{T_e/m_e}$ can be unstable even when $\eta > 1$. In the analysis of these oscillations, the behavior of the electron component must be described kinetically.

The region of the ion-acoustic instability is also bounded at low magnetic fields; for, first, the electron Larmor radius must be less than the tube radius: $\varkappa\rho_e \lesssim 1$. This condition can be represented in the form $\eta > \sqrt{m_e/m_i}$ and it corresponds to the section 1–2 of the boundary of the instability region. Second, it was noted in[2] that in the study of ion-acoustic oscillations in narrow tubes it is necessary to take into account the loss of the plasma to the tube walls. At comparatively low values of the magnetic field, the loss rate is approximately equal to $D_{\perp e}\varkappa$, where $D_{\perp e} = D_e \omega_e^2/\nu_e^2$. If it exceeds the group velocity of the oscillations, the instability has the form of a convective instability. This means that because of the drift of the plasma regular characteristic oscillations cannot be established in the system and the instability will be observed in the form of random noise. Estimates of the growth rate show that over the loss time the fluctuations do not succeed in growing at all appreciably, and therefore the plasma will be almost stable. Assuming that the group velocity of the oscillations is equal to the velocity of ion sound, we obtain an instability condition in the form[2]

$$c_s > D_{\perp e}\varkappa. \tag{5.3}$$

This condition determines the section 2–3 of the boundary of the ion-acoustic instability region.

Finally, drift of the electrons in the ambipolar electric field has a stabilizing influence. Calculations lead to the instability condition $\omega_i/\nu_i = \xi\eta > \sqrt{b_i/b_e}$ (see[41]). In Fig. 21, the corresponding section of the boundary is designated by 3–4.

We have analyzed the ion-acoustic instability of a weakly ionized plasma. In a fully ionized plasma the excitation of ion-acoustic oscillations in the collisional regime can occur in only a very restricted range of variation of T_e/T_i; for in order to eliminate Landau damping on ions it is necessary to require fulfilment of the condition $\sqrt{T_e/T_i} \gg 1$, whereas the inequalities $\nu_i \ll \omega \ll \nu_e$ entail $\sqrt{(m_e/m_i)(T_e/T_i)^3} \ll 1$. It is interesting to note that in an unmagnetized plasma the analogous inequalities are mutually exclusive.[85]

We have used the name ion-acoustic oscillations since the dispersion law of these oscillations when $D_e k_z^2 \gg \mathrm{Max}(\omega, \omega^*)$, $\omega \gg \nu_i$, has the form $\omega^2 = k^2 c_s^2$ [see (5.2)]. At the same time, when $D_e k_z^2 \ll \mathrm{Min}(\omega, \omega^*)$, $\omega \gg \nu_i$, we obtain from (5.2) the relation $\omega = \omega_i k^2/k_y\varkappa$. Since the density gradient then appears in the denominator, these oscillations have been called antidrift oscillations on a number of occasions (see, for example, [86]) to distinguish them from drift oscillations.

B. Experiment

Originally, any instability with excitation region on the $\xi\eta$ plane within the region of the ion-acoustic instability was taken as an ion-acoustic instability. For example, in[41] the anomalies observed in a high-frequency discharge in a magnetic field[31] were explained by excitation of ion-acoustic oscillations. Later,[45,78,87] an instability with boundary coinciding on the $\xi\eta$ plane with the section 2–3 of the boundary of the ion-acoustic instability region was found and called an ion-acoustic instability. In a more detailed study it was however found that the frequency of the unstable oscillations can be lower than the frequency of ion collisions with the neutrals.[12,46,88–91] This circumstance alone is sufficient to cast doubt on the identification of the instability with the ion-acoustic instability. At the same time, we have no reason to doubt the conclusions of[92], in which a study was made of a direct current discharge in a magnetic field at comparatively low pressures of the neutral gas. It was found that in a magnetic field somewhat greater than a critical value noise is excited in the discharge and the diffusion coefficient of the plasma increases. The critical value of the magnetic field was found to be appreciably less than that required for the excitation of the current-convective instability (Fig. 22). Since the characteristic frequency of the noise exceeded the frequency of ion-neutral collisions, and the critical

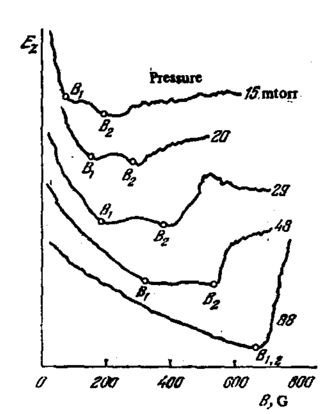

FIG. 22. Longitudinal gradient of the potential, E_z, as a function of the magnetic field B([91]). B_1 is the critical magnetic field for the excitation of the ion-acoustic instability, and B_2 is the one for the current-convective instability.

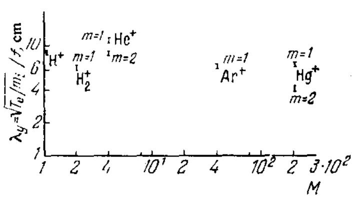

FIG. 23. Azimuthal wavelength λ_y as a function of the ion mass expressed in mass units.[93]

values of the dimensionless parameters ξ and η lay within the instability region of the ion-acoustic oscillations, the instability was identified in[92] as of the ion-acoustic type.

However, as in the case of the drift instability, a final conclusion about the nature of the oscillations can be drawn only after verification of the dispersion relation. For unstable ion-acoustic oscillations the approximate dispersion relation (which is true only in order of magnitude) has the form $\omega \approx kc_s$. If it were satisfied exactly, then in experiments with different gases and different electron temperatures but the same azimuthal wave number m and radius of the apparatus, we should obtain $\lambda_y = 2\pi/k_y = 2\pi c_s/\omega = \text{const}$ $(k_y = m/r,\ k_y \gg k_z)$. Figure 23 gives the results of the experiments of[93], which were made in an apparatus of the same type as is described in[50]. It follows from the figure that, despite the large differences between the oscillation frequencies in the different gases, c_s/ω for modes with the same azimuthal wave number m is approximately constant. In accordance with the theory, the oscillations investigated in[93] traveled azimuthally in the electron direction and formed a standing wave in the direction along the magnetic field. Their frequency satisfied the relation $\omega \gg \omega_i,\ \nu_i$. At the same time it should be noted that in these experiments the condition $D_e k_z^2 \gg \mathrm{Max}(\omega, \omega^*)$, which theory suggests guarantees proximity of the frequency of the unstable oscillations to the ion-acoustic frequency, was not satisfied.

In[62,64] it was not the very approximate relation $\omega \approx kc_s$ that was verified but the much more accurate dispersion relation (5.2). Figure 24 shows the experimental and calculated [in accordance with (5.2)] frequencies of oscillations with $m = 1$ $(k_y = \varkappa)$ as a function of the magnetic field for three different pressures of the neutral gas. It can be seen from the figure that the general nature of the experimental dependence $\omega = \omega(B, p)$ corresponds to the results of the theoretical calculations. However, the plasma is more stable than predicted by theory: The range of magnetic fields in which the instability was observed experimentally is appreciably narrower than the region in which the growth rate calculated by (5.2) (dashed curves in Fig. 24) is positive. This discrepancy can be partly removed by bearing in mind that because of the movement of the charged particles to the wall of the discharge tube one can in reality observe only those oscillations whose growth rate is greater than the reciprocal time $(D_{\perp_e} \varkappa^2)^{-1}$ of this motion:

$$\mathrm{Im}\,\omega > D_{\perp e}\varkappa^2. \tag{5.4}$$

Since one usually has $\mathrm{Re}\,\omega > \mathrm{Im}\,\omega$ (see Fig. 24), for large-scale oscillations with $k_y \approx \varkappa$ the condition (5.3) used above is less stringent than (5.4). If (5.4) is taken as the instability condition, then the interval of magnetic fields to the left of the vertical dashed arrows is eliminated from the instability region in Fig. 24. As a result, for weak magnetic fields (left-hand boundary) we obtain better agreement with theory, although the discrepancy remains on the right-hand boundary.

It is helpful to represent the results of the experiments in[64] on the $\xi\eta$ plane (see Fig. 21). To each value of the pressure $(p \sim \xi^{-1})$ in this figure there corresponds a segment of a horizontal straight line whose length and position is determined by the range of variation of the magnetic field $(\eta \sim B)$. We have plotted only the part of the segment that corresponds to instability of the plasma. As a result, we obtain three segments parallel to the η axis. At the same time, in Fig. 21 the dashed curve shows the instability region of the first oscillation mode found by solving (5.2) and the dot-dash-dot curve shows the region found with allowance for the condition (5.4). In the calculations, in agreement with the experiment, the longitudinal wavelength was taken equal to twice the length of the plasma. If the longitudinal wavelength is increased, the instability region is enlarged. Note that the total volume of the ion-acoustic instability, which is bounded in Fig. 21 by the continuous curve, is obtained by superimposing the instability regions of the individual modes in a system that is unbounded along the magnetic field $(\infty \geq k_z \geq 0)$ for $k_y \geq \varkappa$.

The frequency of the ion-acoustic oscillations was also investigated as a function of the magnetic field in a direct current discharge in a longitudinal magnetic field.[58] Approximately the same results as in the plasma of the high-frequency discharge were obtained.

An investigation of the frequency of unstable oscillations as a function of the pressure of the neutral gas and the wave vector was made in[64]. Figure 25 shows the experimental and calculated dependences of the oscillation frequency ω as a function of the gas pressure p obtained in[64] in an argon plasma. Here, as in the earlier figures, the dashed curve is the instability

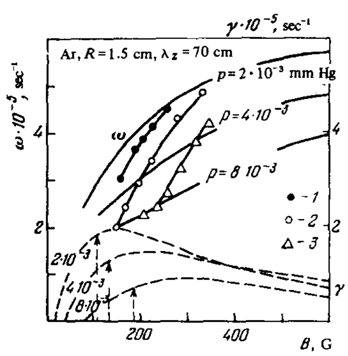

FIG. 24. Frequency ω and growth rate γ of oscillations with $m = 0$ as a function of the magnetic field B([64]). Pressures (mm Hg), temperatures (°K): 1) $2\cdot10^{-3}$, $6\cdot10^4$; 2) $4\cdot10^{-3}$, $5.2\cdot10^4$; 3) $8\cdot10^{-3}$, $4.2\cdot10^4$. The continuous curves are the calculated frequencies and the dashed curves are the calculated growth rates.

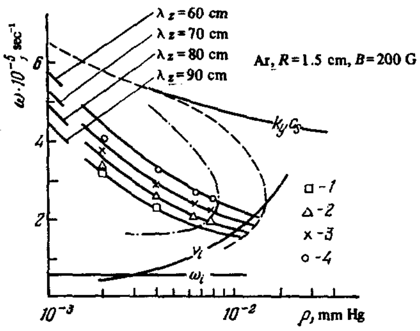

FIG. 25. Frequency ω of unstable oscillations with $m=1$ as a function of the gas pressure p([64]). The longitudinal wavelengths (cm): 90 (1), 80 (2), 70 (3), and 60 (4). The dashed curve gives the boundary of the instability region calculated by (5.2); the dot-dash-dot curve the boundary found from the condition (5.4). The continuous curves for $\lambda_z = 60$–90 cm are the calculated frequencies.

boundary calculated from the dispersion relation (5.2), while the dot-dash-dot curve is the boundary found from condition (5.4). It can be seen from the figure that at all pressures the conditions $\omega > \nu_i$, ω_i are satisfied. With decreasing longitudinal wavelength and gas pressure, an increase in the oscillation frequency is observed, in agreement with theory. Note that under the conditions of[64] the longitudinal wavelength decreased from $\lambda_{z\,\max} = 90$ cm to $\lambda_z = 60$ cm, this being governed by the possibilities of the experimental apparatus that was used. The frequency of the oscillations with $\lambda_z = 60$ cm at low gas pressure was close in magnitude to $k_y c_s$. In[64], the oscillation frequency ω was also studied as a function of the wave vector **k**. The oscillation frequency ω as a function of k_z (k_z is the projection of the wave vector onto the direction of the magnetic field) is shown in Fig. 26, from which it can be seen that, as follows from (5.2), the frequency increases with increasing magnetic field B and decreasing gas pressure p. The frequency is less than $k_y c_s$, approaching this value with decreasing longitudinal wavelength $\lambda_z = (\lambda_z = 2\pi/k_z)$.

As can be seen from Fig. 26a, the maximal pressure above which the plasma becomes stable was near $8 \cdot 10^{-3}$ mm Hg and comparable with the value obtained from the calculations (the dashed and the dot-dash-dot curves).

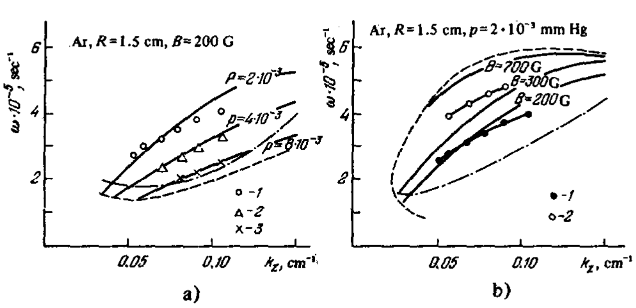

FIG. 26. Frequency ω of unstable oscillations with $m=1$ as a function of the wave vector projection k_z onto the direction of the magnetic field.[64] a) Pressures (mm Hg): $2 \cdot 10^{-3}$ (1), $4 \cdot 10^{-3}$ (2), and $8 \cdot 10^{-3}$ (3); b) magnetic fields (G): 200 (1) and 300 (2). The notation is the same as in Fig. 25.

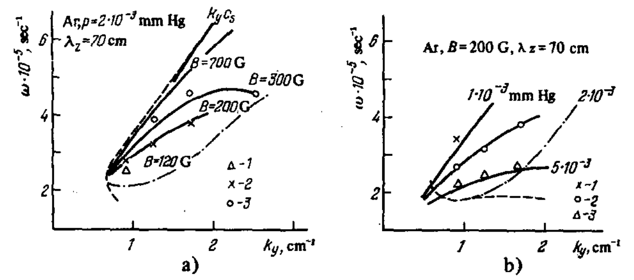

FIG. 27. Frequency ω of unstable oscillations as a function of the wave vector projection k_y onto the azimuthal direction.[64] a) Magnetic fields (G): 120 (1), 200 (2), and 300 (3); (b) pressures (mm Hg): $1 \cdot 10^{-3}$ (1), $2 \cdot 10^{-3}$ (2), and $5 \cdot 10^{-3}$ (3). The notation is the same as in Fig. 25.

Note that at low pressures the theory does not predict a restriction on the instability. In[64] the gas pressure was reduced to values $\lesssim 10^{-3}$ mm Hg and at these pressures unstable oscillations were still observed in the plasma. From Eq. (5.2) one cannot obtain restrictions on the instability region at weak magnetic fields (the dashed curve in Fig. 26b is open downward). A restriction is given by the condition (5.4); it is the dot-dash-dot curve.

Figure 27 shows ω as a function of k_y. It can be seen from this figure, as also from Fig. 26, that the experimental data in it agree fairly well with the calculated values.

CONCLUSIONS

Thus, the conditions have now been established under which the drift-dissipative instability must arise, and the dispersion properties of unstable oscillations of small amplitude have been analyzed. In these matters, theory and experiment agree perfectly satisfactorily. It is hoped that certain numerical discrepancies will be eliminated by a more complete allowance for the effects associated with inhomogeneity of the plasma and nonlinearity of the oscillations. It is possible that for this one must go over from the hydrodynamic to the kinetic description.

It must however be borne in mind that the physical origin of a number of oscillations that develop in a gas-discharge plasma in a magnetic field has not been established. It is possible that the investigation of these oscillations will reveal new aspects of the influence of inhomogeneity of a magnetized plasma on its oscillations that are not included in the drift-dissipative instability mechanism.

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