[118]Yu. G. Poltavtsev and V. P. Zakharov, Kristallografiya 18, 605 (1973) [Sov. Phys. Crystallogr. 18, 379 (1973)].
[119]Yu. G. Poltavtsev and V. P. Zakharov, Kristallografiya 19, 668 (1974) [Sov. Phys. Crystallogr. 19, 416 (1974)].
[120]Yu. G. Poltavtsev, V. P. Zakharov, I. M. Protas, T. V. Remizovich, and V. N. Chugaev, Ukr. Fiz. Zh. 18, 752 (1973).
[121]Yu. G. Poltavtsev, V. P. Zakharov, V. M. Pozdnyakova, and T. V. Temizovich, Izv. Akad. Nauk SSSR Ser. Neorg. Mater. 9, 907 (1973).
[122]Yu. G. Poltavtsev, V. P. Zakharov, and I. M. Protas, in: Poluprovodnikovaya tekhnika i mikroelektronika (Semiconductor technology and microelectronics), No. 16, Kiev, Naukova Dumka, 1974, p. 62.
[123]Yu. G. Poltavtsev, V. P. Zakharov, and V. I. Shvets, Ukr. Fiz. Zh. 18, 663 (1973).
[124]P. A. Young and W. G. Thege, Thin Solid Films 7, 41 (1971).
[125]Yu. G. Poltavtsev, Izv. Akad. Nauk SSSR Ser. Neorg. Mater. No. 12 (1976).
[126]V. A. Alekseev, A. A. Andreev, and V. Ya. Prokhorenko, Usp. Fiz. Nauk 106, 393 (1972) [Sov. Phys. Usp. 15, 139 (1972)].
[127]N. F. Mott and E. A. Davis, Electronic Processes in Non-Crystalline Materials, Oxford, 1971.
[128]P. W. Anderson, Phys. Rev. 109, 1492 (1958).
[129]A. S. Lashko and Yu. G. Poltavtsev, Ukr. Fiz. Zh. 13, 1578 (1968).
[130]R. Grigorovici, J. Non-cryst. Sol. 1, 303 (1969).
[131]H. Richter and S. Steeb, Zs. Metallk. 50, 369 (1959).

Translated by J. G. Adashko

Modification of a magnetic field by plasma mechanisms

S. I. Vaĭnshteĭn

Siberian Institute of Terrestrial Magnetism, Ionosphere, and Radiowave Propagation, Siberian Division, USSR Academy of Sciences, Irkutsk
Usp. Fiz. Nauk **120**, 613–645 (December 1976)

The review is devoted to the question of the rapid modification (generation, damping, etc.) of a magnetic field in a collision-dominated plasma, when the frequency γ of the process is lower than the electron-collision frequency ν. The universally employed approach—the dynamo theory—is discussed in Chap. 2. The simplest motions that lead to generation, to rapid annihilation of the "antidynamo", and to a modification of the topological-pumping type are indicated. The discussion concerns problems of the turbulent dynamos (where a functional approach is used) and of the nonlinear dynamo. In the latter case, the Gibbs ensemble is used; a reverse cascade of magnetic and kinetic energy into the region of small wave numbers is observed. Other plasma mechanisms are discussed, namely the modification of the field by ion sound, the weakly-ionized plasma, and the solid-state plasma (dynamo based on thermal effects)—Chap. 3. It is shown that the modification can be quite effective. Discussed in Chap. 4 are high-frequency oscillations (whistlers, Langmuir oscillations) and their role in the modification of the field. The indicated mechanisms can be effective under conditions of the solar chromosphere.

PACS numbers: 52.35.Ck

CONTENTS

1. INTRODUCTION

The question of modification of magnetic fields is important for many applications. Thus, after the discovery of the magnetic fields of the sun, the stars, and the interstellar gas, the question arose of the origin of these fields. Another aspect of this problem is the rapid destruction or annihilation of these fields (solar flare, the frontal point of the magnetosphere, the tail of the magnetosphere, z-pinch, etc.). As a rule, the plasma is not strongly collision-dominated here, so that the dissipation is low. The magnetic field is "frozen-in" into the medium, and the annihilation proceeds quite slowly. How are we then to explain the observed rather rapid modification of the field?

The "classical" approach to this problem is that of the dynamo theory, in which the entire responsibility for the modification is placed on the hydrodynamic motions. The latest review of this problem was published in our country in 1972.[1]

Abroad, reviews were published by Stix,[2] Gubbins,[3] Soward and Roberts,[4] and Rädler.[5] Since that time, major, principally qualitative, changes took place in the theory. In recent years, a tendency to "come down to earth" has been noted. On the one hand, papers were published using a simple velocity field that lends itself to laboratory simulation,[6–13] and on the other hand there are works that do not confine themselves to the magnetohydrodynamics approximation and use a larger group of plasma mechanisms for the generation and modification of the field. The range of applications has consequently widened to such an extent that it has become possible to verify the theory in laboratory experiments and even to use it in technology. Papers were also published on nonlinear MHD turbulence treated by using Gibbs ensembles.[14-16] All this is reflected in a most unsatisfactory manner in the foreign reviews. In addition, as a rule, the reviews are devoted to applications: the solar cycle, the terrestrial dynamo, etc. In the present article we attempt to describe new physical ideas. The applications will be described after the exposition of the physics itself: without applications, ideas become meaningless. The results contained in the review[1] will, of course, not be repeated except where necessary to make the text coherent and to clarify the exposition. In this case they will be reported quite concisely. Thus, the scope of this article is broader than in reviews devoted to applications. On the other hand, we confine ourselves only to collision-dominated plasma, when the frequency of the field is lower than the frequency of the collisions (the oscillation frequency can be even larger than the collision frequency). The procedure and the approach in the usual dynamo theory—the MHD approximation (Ch. 2)—has by now been well investigated and treated in the reviews. We shall therefore report only the results. To the contrary, the modification of a field by ion sound, thermomagnetic effects in a solid-state plasma (Ch. 3), and the generation by Langmuir oscillations (Ch. 4) are based on an approach developed only in recent years. This is precisely why the method and formulation of the problems of the second type will be treated in detail, even though the number of papers reviewed in Ch. 2 alone is incomparably larger than in Chs. 3 and 4, which constitute approximately half the paper.

2. NEW TOPICS IN THE "OLD" THEORY

A. Simple generators

Searches for simple models are necessitated by two circumstances. First, there are many restrictions on symmetrical models (Cowling,[17] Braginskii,[18] Zel'devich[19]), i.e., axisymmetric models and the like must be rejected. This immediately raised the question: what kind of not too complicated model can produce generation? Second, simple models are usually natural and frequently realizable in nature. Thus, the Herzenberg dynamo[20] is realized by two rotating pairs (the rotation axes of which are not parallel to each other) immersed in a conducting liquid. The most natural approach to dispensing with symmetry was used by Braginskiĭ in[18], where he considered models that deviate slightly from symmetry. In recent years this theory continued to develop effectively (see, e.g., [21-29]), and was used with particular success to explain the earth's magnetic field and its variations. It was noted that the motion that causes deflection from symmetry and produces rotation has helicity i.e., the velocity $\mathbf{v}$ correlates with curl $\mathbf{v}$. We shall have an opportunity to verify below that the correlation $\mathbf{v}\cdot\text{curl}\,\mathbf{v}$, i.e., helical motions, plays in general an important role in the theory of generation. Thus, Roberts[10] considers a velocity field $\mathbf{v}=\{\cos y-\cos z,\ \sin z,\ \sin y\}$, which, as can be easily verified, is helical: $\mathbf{v}\sim\text{curl}\,\mathbf{v}$. It is therefore not surprising that Roberts obtains generation. An exact stationary solution for a helix was obtained by Lortz.[9] Ponomarenko[11] obtained a nonstationary, i.e., an exponentially growing, solution for the helical model shown in Fig. 1. The considered models are by far not exotic in astrophysics. In fact, the rotation of a celestial body alone suffices to explain the correlation $\mathbf{v}\cdot\text{curl}\,\mathbf{v}\neq 0$. The idea of this explanation dates back to the work of Parker.[30] Let us consider the convective zone of a star; if we take into consideration the density gradient $\nabla\rho$ (the decrease of the density from the interior towards the surface), then it is clear that a rising object will tend to expand. The Coriolis force produces a torque for a given cell, and this cell should rotate, so that $\mathbf{v}\cdot\text{curl}\,\mathbf{v}\sim\omega\cdot\nabla\rho$, where ω is the angular velocity. It is easy to imagine that a similar relation $\mathbf{v}\cdot\text{curl}\,\mathbf{v}\sim\omega\cdot\nabla\rho$ is observed in the descending case, so that a predominant right-hand or left-hand helix is produced (depending on the sign of $\omega\cdot\nabla\rho$). The foregoing arguments give grounds for hoping that waves in a rotating compressible bounded medium (such as a celestial body) have helicity and are consequently of interest for the dynamo. Indeed, the Rossby waves, which are produced just in a rotating body, are invoked to explain the solar cycle and generation in general.[31-35] It was also natural to investigate[26,36-41] the convective instability of a rotating liquid (generally speaking, including a homogeneous superimposed magnetic field). The authors reach the conclusion that a rotating liquid, and the presence of a maintained temperature gradient, is convectively unstable precisely to such perturbations, which in turn generate the field. It is appropriate to mention here also the role of tidal motions, to which attention was called by Dolginov[42,43] as a possible field generator in binary stars and planets (the tidal action of satellites or of the sun). Finally, closely connected with the tidal mechanism is the precession mechanism. Incidentally, the authors of[44] have expressed doubts concerning the effectiveness of its action in the earth's core. There exist, however, non-helical models. For example, one toroidal Tverskoĭ vortex[45] excites a field (with an axisymmetric vortex exciting a non-axisymmetric field, so that no contradiction of the theorems of Cowling and Braginskii occur), and a system of two vortices[8] can likewise serve as a generator. The latter model comes closer to the convection model: it is assumed that the convective cells are analogous to toroidal vortices. A modification of two rotating Herzenberg spheres com-

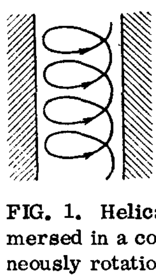


FIG. 1. Helical motion—field generator. A cylinder immersed in a conducting medium (shaded) executes simultaneously rotational and translational motions. The helical trajectory of an individual point of the cylinder is marked by the arrow.

prises two cylinders rotating about axes that are not parallel to each other. The departure from symmetry is realized in this manner (Fig. 2).[7] Another modification consists of two rotating spheres in a vacuum.[46] An application of this model suggests itself right away—binary stars. It appears that generation of this kind can also be effected in a laboratory and used in technology. One can attempt to depart from symmetry only in a magnetic field, i.e., using symmetric motion but an asymmetric field (just as in the Tverskoĭ model[45]). Let us simplify the motion to the utmost. The simplest non-rigid-body motion of a continuous medium is shear, for example, differential rotation. Assume that there is a rotation axis z and that the velocity in the cylindrical system is of the form $v_r = v_z = 0$, $v_\varphi = v(r)$. This motion is two-dimensional, so that there comes into play a theorem by Zel'dovich,[19] which states the following: no field can be amplified without limit by two-dimensional motion (there is no contradiction with the Cowling-Braginskiĭ theorems, since the field is not axisymmetric). This theorem,[19] however, was proved for an unbounded conducting medium. If we consider a medium that is bounded with respect to r, or in other words if it is assumed that σ depends on r, then the absence of dynamo instability cannot be proved.[13]

It is appropriate to recall here that all the restriction theorems are proved in the following manner: The induction equation

$$\frac{\partial \mathbf{H}}{\partial t} = \operatorname{rot}\, [\mathbf{v} \times \mathbf{H}] + \nu_m \Delta \mathbf{H}, \nu_m = \frac{c^2}{4\pi\sigma} \quad (1)$$

is written in a coordinate system that is natural for the given geometry, and an attempt is then made to separate one of the three equations for the field components from the others. If it is possible to separate at least one equation and its boundary conditions from the remainder, then it is possible to construct a restriction theorem in practically all cases. This rule is probably difficult to verify. It was found "empirically." In the two-dimensional case, the H_z component of the field is separated.[19] If now σ depends on r, then in the equation for the H_z component (1) there will be added a term $\partial \nu_m/\partial r\, (\partial H_z/\partial r - \partial H_r/\partial z)$, and now all the field components are interlinked. The $\sigma(r)$ dependence can be due to the very fact that the body is bounded, and one can consider in particular a model wherein $\sigma = \sigma_0$ inside the cylinder and $\sigma = 0$ (vacuum) outside the body. Figure 3 shows a solvable model: one cylinder rotates inside another. In this model, the angular velocity of the cylinder changes jumpwise.

However, if the whole situation is confined to inhomogeneous electric conductivity, then it is easiest to realize in experiment a model wherein vacuum (or an insulator) is located between cylinders—in which case there will be no friction between them (Fig. 4). For large $\mathbf{Rm} = \omega r_0^2/\nu_m$ and for a solution in the form

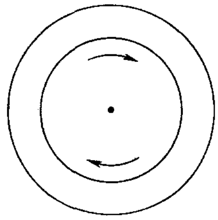


FIG. 3. Simplest generator. Two cylinders with axes parallel to the z axis; the inside cylinder rotates. The z axis is perpendicular to the plane of the figure.

$$\mathbf{H} = \mathbf{f}(r) \exp(Et + im\varphi + ikz)$$

$f(r)$ is expressed in terms of modified functions I and K of the argument βr in the internal cylinder and $\varkappa r$ in the external one; $\Delta r/r_0 \ll 1$, $\beta^2 = \varkappa^2 + (im\omega/\nu_m)$, $E = \nu_m(\varkappa^2 - k^2)$. Matching the solutions in the vacuum region, we obtain the following dispersion equation:

$$\left[4 + \frac{\beta - \varkappa}{r_0 \varkappa\beta}(4m^2 + 3)\right]\left[\operatorname{th} k\Delta r + \frac{k(\varkappa + \beta)}{\varkappa\beta}\right] + \frac{\Delta r}{4kr_0^2}\left[4 + (4m^2 - 1)\left(1 + \frac{2k^2}{\varkappa\beta}\right)\right] = 0.$$

We put $N = (4m^2 + 3)/4$, and then at $r_0\beta \approx N$, $\mathbf{Rm} \gg N^2/m$ and $N \gg 1$ we get an unstable solution:

$$\operatorname{Re} E = \nu_m N^2/r_0^2 = \omega N^2/\mathrm{Rm}.$$

Unfortunately, it cannot be regarded as proved that the two models considered above for the differentially rotating cylinders yield indeed an increasing solution. The point is that the author has attempted to find an unstable solution by using a continuous velocity field, i.e., without jumps, invoking the WKB method, which is applicable at $\mathbf{Rm} \approx 1$. The result turned out to be negative, and there were no growing solutions. The same is confirmed by the numerical experiments of M. Stix, who used a continuous velocity field. The asymptotic expansion for the Bessel functions at parameters corresponding to an increasing solution converge poorly, and this may explain the affirmative result of[9]. Actually, there is no generation at $\mathbf{Rm} \ll 1$, for in this case the problem becomes planar and does not "feel" the cylindrical geometry. In all probability, in this case the excitation sets in at $\mathbf{Rm} \approx 1$, when the cylindrical geometry must come into play. This possibility can be verified in the following manner. The equations for the H_φ and H_r components are made closed by the boundary conditions, so there is no need to consider the behavior of the H_z component. These equations are

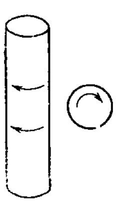


FIG. 2. Two rotating cylinders, the axes of which are perpendicular to each other, are capable of generating a field.

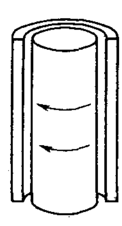


FIG. 4. Generator that can be easily realized in experiment. The inner cylinder, of radius r_0, rotates; a gap Δr exists between the inner and outer cylinders.

$$\frac{\partial H_r}{\partial t}=-\omega\frac{\partial H_r}{\partial\varphi}+\nu_m\left(\Delta H_r-\frac{H_r}{r^2}-\frac{2}{r^2}\frac{\partial H_\varphi}{\partial\varphi}\right),$$

$$\frac{\partial H_\varphi}{\partial t}=-\omega\frac{\partial H_\varphi}{\partial\varphi}+rH_r\frac{\partial\omega}{\partial r}+\nu_m\left(\Delta H_\varphi-\frac{H_\varphi}{r^2}+\frac{2}{r^2}\frac{\partial H_r}{\partial\varphi}\right),$$

where ω is the angular velocity. The conditions on the boundary with the vacuum at $kR \ll 1$, $H\sim \exp i(kz+m\varphi)$, where R is the radius of the cylinder (Fig. 3), are

$$H_r=iH_\varphi,\ H_r+\frac{R}{m+1}\frac{\partial H_r}{\partial r}=0 \quad \text{for} \quad r=R.$$

In the absence of rotation, the eigenfunctions break up into two types of harmonics:

$$1)\ H_r=-\frac{Dm}{\varkappa_j r}I_m(\varkappa_j r),\ H_\varphi=-iDI'_m(\varkappa_j r),\ I_{m-1}(\varkappa_j R)=0,$$

$$2)\ H_r=CI_{m+1}(\varkappa_i r),\ H_\varphi=-iCI_{m+1}(\varkappa_i r),\ I_m(\varkappa_i R)=0.$$

We now exclude the rotation, and let ω vary in continuous fashion; at $\mathbf{Rm}\approx 1$ the motion will influence only the lowest harmonics, while the higher ones attentuate too rapidly (i.e., the decrement γ_i due to the ohmic damping is much larger than the growth rate due to the motion).

We take into consideration only two harmonics at $m=1$: a lower one of type 1) with eigenvalue γ_0, and a lower one of type 2) with eigenvalue γ_1. The harmonics that are reciprocal (dual) to these two basic harmonics are

$$H_r=-I_{m+1}(\varkappa_1 r),\quad H_\varphi=iI_{m+1}(\varkappa_1 r)$$

and

$$H_r=I'_m(\varkappa_0 r),\quad H_\varphi=\frac{im}{\varkappa_0 r}I_m(\varkappa_0 r).$$

We expand the solution in the two basic harmonics, multiplying the scalar equations for H_r and H_φ by the dual harmonics and integrate with respect to r from 0 to R, and thus obtain

$$\frac{\partial D}{\partial t}=-(ima_+ +\gamma_0)D+iC\varepsilon_{12},\quad \frac{\partial C}{\partial t}=iD\varepsilon_{21}-(ima_- +\gamma_1)C,$$

$$a_\pm=\frac{1}{d}\left[\int_0^R \omega I_{m-1}(\varkappa_0 r)I_{m+1}(\varkappa_1 r)\,dr \pm \int_0^R \frac{1}{\varkappa_0}\frac{\partial\omega}{\partial r}I_m(\varkappa_0 r)I_{m+1}(\varkappa_1 r)\,dr\right],$$

$$d=\int_0^R I_{m-1}(\varkappa_0 r)I_{m+1}(\varkappa_1 r)\,dr,$$

$$\varepsilon_{12}=\frac{1}{d}\int_0^R r\frac{\partial\omega}{\partial r}I^2_{m+1}(\varkappa_1 r)\,dr,\quad \varepsilon_{21}=\frac{-1}{d}\int_0^R \frac{m^2}{\varkappa_0^2 r}\frac{\partial\omega}{\partial r}I_m^2(\varkappa_0 r)\,dr.$$

The described system has an unstable solution if $\omega\approx r\,\partial\omega/\partial r$ and $\gamma_0<\omega<\gamma_1$ (more accurately $\omega \gtrsim \sqrt{\gamma_0\gamma_1} \approx \nu_m/R^2$, i.e., $\mathbf{Rm}\gtrsim 1$).

In this case the growth rate will be close to ω. It appears that this is precisely the situation realized in cosmic electrodynamics, i.e., the growth rate is of the order of the reciprocal time of the cell rotation v/l, where l is the characteristic scale of variation of the velocity. In fact, if div $\mathbf{v}=0$ and $\mathbf{v}$ is independent of t, the eigenvalue problem takes the form $E\mathbf{H}=(\mathbf{H}\cdot\nabla)\mathbf{v}-(\mathbf{v}\cdot\nabla)\mathbf{H}+\nu_m\nabla^2\mathbf{H}$. For an increasing harmonic we have either $(\mathbf{H}\cdot\nabla)\mathbf{v}\approx\nu_m\nabla^2\mathbf{H}$ (then the scale of the field is $\delta\sim l\mathbf{Rm}^{-1/2}$, $E\approx v/l$), or else $(\mathbf{v}\cdot\nabla)\mathbf{H}\approx\nu_m\nabla^2\mathbf{H}$ (then $\delta\sim l\mathbf{Rm}^{-1}$, $E\approx v\mathbf{Rm}/l$), with $\mathbf{Rm}=vl/\nu_m$. The second case, however, is unphysical: neglect of $(\mathbf{H}\cdot\nabla)\mathbf{v}$ in comparison with $(\mathbf{v}\cdot\nabla)\mathbf{H}$ means in fact that we are dealing with the heat-conduction equation: $E\mathbf{H}=-(\mathbf{v}\cdot\nabla)\mathbf{H}+\nu_m\nabla^2\mathbf{H}$, and the dynamo is impossible. Therefore the unstable harmonic will vary slowly in a direction parallel to $\mathbf{v}$, $(\mathbf{v}\cdot\nabla)\mathbf{H}\approx Hv/l$, and will vary rapidly in the perpendicular direction, with a characteristic dimension δ (see Fig. 10 below); consequently, $E\approx v/l$ and does not depend on δ. We stipulate that this statement is still not universal. This is seen at least from the fact that the real situation is not characterized by a single scale, such as the dimension of the convective cell, of the entire body, of the convective zone, etc. In addition, this situation will certainly not take place if the field does not depend on the direction parallel to the motion of the plasma (e.g., differential rotation with axisymmetric field, or Hartmann flow with a field that is independent of direction parallel to the walls of the channel).

The example shown in Fig. 3 simulates the differential rotation of a cylinder and is close in a certain sense to the model of a differentially rotating galaxy. What is the situation with a sphere? Assume that we have a sphere and that v_φ depends on θ and r. Will generation take place? Unfortunately, no. The point is that in this case the component H_r is separated, and no $\sigma(r)$ dependence or even $\mu(r)$ dependence (μ is the magnetic permeability) will change this circumstance. Let now σ depend on the latitude θ (this may be due to the temperature difference between the equator and the pole, or, if σ is governed by turbulence, it may be due to the dependence of the intensity of the turbulence on θ). Now a term containing H_θ is added to the equation for H_r, so that the restriction is lifted. To be sure, no such problem was considered, and we can only expect that amplification of the field is possible. The already mentioned independence of the increment on σ suggests that the relative amplitude $|\nabla\sigma|/\sigma$ of the variation of σ is not essential. All that matters is that the restriction on the generation is lifted. The value of σ and its variation will affect only the form of the instability of the mode, which changes effectively over a scale $\sqrt{\nu_m\omega}$. The solution of this problem for a sphere would be of importance in the explanation of the magnetism of the sun (for which a dependence of v_φ on θ is directly observed) and for the earth's core. Despite the abundance of mechanisms that lead to generation, at the present time there exist only two universally accepted dynamo models (i.e., models used directly to explain the solar cycle, the terrestrial dynamo, etc.). These are the Parker model[30] in which the aforementioned helical convection serves to generate the poloidal component from the toroidal one (the toroidal component, on the other hand, is "drawn out" by the differential rotation from the poloidal one), and the Braginskiĭ model[18] of an almost-symmetrical dynamo.

B. Other types of modifications

The most essential feature of the modification is the decrease of the scalar field as a result of the motion. Can we attempt to use this circumstance to explain the annihilation of the large-scale fields? The motion leads to an effective decrease of the scale, and then the field is simply dissipated as a result of ohmic losses. The most convincing demonstration of this phenomenon is probably that of Weiss.[47] The induction equation was solved numerically for two-dimensional convective cells. The latter first tangles up the large-scale field, producing a "fine structure," and then expels the field from the region where such cells are present. To be sure, in this case the field is in final analysis merely expelled and not annihilated. Annihilation of the field can be obtained in fact by simple motion of the plasma: let $\partial/\partial z = 0$, $H_z = 0$, $v_z = 0$ (planar case). The equation for the vector potential A of the field is (curl A = H)

$$\frac{\partial A_z}{\partial z} + (\mathbf{v}\nabla) A_z = \nu_m \Delta A_z, \tag{2}$$

and the vector A has no other components. Let v be independent of t and have the form of shear motion, for example $\mathbf{v} = (v(y), 0)$, $v(y) > 0$ at $y > 0$ and $v(y) < 0$ at $y < 0$. Then, by specifying at $y = \pm L$ boundary conditions corresponding to the absence of an external field source, we can change over to the eigenvalue problem $A_z = A \exp(Et + ikx)$:

$$EA + vikA + \nu_m k^2 A = \nu_m \frac{\partial^2 A}{\partial y^2}$$

(Schrödinger equation with complex potential). So long as a system of eigenfunctions (3) is complete,[48] it remains for us only to find E_0 for the lowest harmonic, i.e., the one that attenuates most slowly (there are no growing solutions, as can be seen from the similarity between (2) and the equations for the temperature in a moving liquid). Of course, interest attaches to the situation when $\mathbf{Rm} \gg 1$. Let the initial scale of the field perturbation be $\approx l$, and then $k = 1/l$. It is useful to separate Eq. (3) into its real and imaginary parts, which are of the same order of magnitude (this is seen from the fact that the solution of (3) can be sought by the WKB method, and it exists). Further, in order for the right-hand side to cancel out the left-hand side (otherwise the order of the equation changes), the characteristic scale of the function A in terms of z must not exceed $\delta \sim \sqrt{\nu_m/vk}$, i.e., $|E| \gtrsim \nu_m/\delta^2 = vk$ and $|E_0| \approx v/l$. A rigorous solution[12] confirms this conclusion.

We have purposefully described the formal estimates, in considerable detail since the result is not self-evident. In fact, physically this result immediately contradicts the intuitive "freezing-in" concept. The point is that l/v is the time of deformation of a field with scale l, and it might seem that the field should be deformed by the motion during a time $t \gg l/v$. Figure 5a shows the initial fluctuations of the field while Fig. 5b shows their deformation in a time $\approx l/v$. Strictly speaking, the dissipation has already been included and leads to smoothing out of the "corners" of the field, where the gradients are maximal. Further stretching of the force lines leads to a new smoothing, etc. Thus, the "freezing-in" is violated immediately, during the initial stage. The importance and necessity of taking into account the finite electric conductivity in neutral-layer problems were indicated also in[49]. For a collisionless plasma

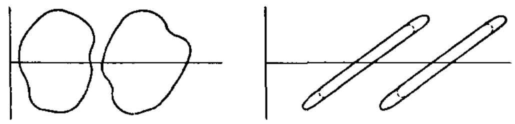


FIG. 5. Force lines of the field (solid lines). The dashed lines reflect the dissipation-induced smoothing of the field "corners"; the arrows indicate the plasma motion.

the field-dynamics equation also takes the form (1), but $\nu_m = 0$: there is no dissipation. Nonetheless, it is possible to use here, too, the fact that the scale of the field decreases. The appearance of steep field gradients turns on an anomalous dissipation: when the current velocity exceeds the velocity of the ion sound (if $T_e > T_i$, where T_e and T_i are the electron and ion temperatures), ion sound is excited and leads to the anomalous diffusion. In final analysis, the shear motions of the plasma lead to a decrease of the scale and to formation of a current layer. In[50], this process was calculated under the conditions of solar wind, where the velocity shears are observed, and the condition $T_e > T_i$ is also satisfied. In the region around which the earth's magnetosphere flows, there is a particularly steep velocity gradient (a decrease of the velocity from that of the solar wind to zero over a dimension comparable with the earth's magnetosphere, or even less). Figure 6 shows the origin of the magnetic inhomogeneity in the surrounding-flow region. If the magnetosphere-field vector is directed opposite to the field of the adjacent inhomogeneity, as shown in Fig. 6, then the current layer can be directly adjacent to the magnetosphere, and crossover of the force lines is possible at the point A.[51]

A principally new modification mechanism was proposed in[52]. Consider the convection produced by Benard cells. The plasma rises inside the cell (along the z axis) and descends along the edges. It is easy to visualize cells of this type, adjacent to one another, forming a horizontal row and filling the (x, y) plane. It must be borne in mind that the descending medium in the entire system is topologically connected, whereas the rising parts of the plasma are separated. We consider now the behavior of a large-scale magnetic field which is horizontal at the initial instant, in the presence of such an ensemble of cells. The upper force lines will be shifted towards the edges of the cells and then will glide entirely downward towards the base. On the other hand, the lower force lines have no place to go, since they will be drawn upward by the topologically unconnected parts of the liquid, where they will only bend into loops, so that on the whole they will not be displaced. This

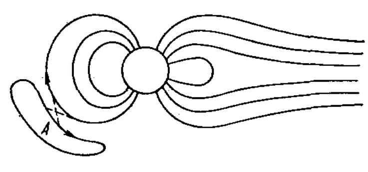


FIG. 6. Interaction of a passing inhomogeneity of the solar wind with the earth's magnetosphere. Crossover is possible at the point A.

mechanism operates as a pump, pumping the field into the lower part of the convective zone (the sun). We shall return to a discussion of this mechanism in Sec. (C) in connection with the question of turbulent convection.

C. Turbulence: Kinematic formulation

We shall ignore completely the electromagnetic forces, assuming the motion to be specified. This is in fact the kinematic formulation, which is justified if the fields are weak. Assume that turbulence is excited in the liquid. What happens to the large-scale field (i.e., whose scale is $L \gg l$)? There will operate the principal and therefore well known effect—mixing or turbulent diffusion. In other words, the average field satisfies the equation $\partial \langle \mathbf{H}\rangle/\partial t = \chi \nabla^2 \langle \mathbf{H}\rangle$, where χ, in analogy with physical kinetics, is estimated at $\chi = vl/3$ (it is more convenient to interpret l here as the "mean free path" of the convective element). Piddington[53,54] expressed doubts concerning the action of turbulent diffusion: first, in his opinion, diffusion according to (1) calls for annihilation of fields of opposite signs, in contrast to a scalar admixture in the turbulent stream; second, for this annihilation to occur the scale of the field must be small. With respect to the first objection, we note that the equation for the gradient of the scalar admixture is similar to (1), in the two-dimensional case they simply coincide, and the turbulent smoothing of the admixture gradient is an experimental fact. With respect to the second objection we were able to verify in Secs. (A)–(B) the extent to which the motion leads effectively to a decrease in the scale of the field.

The correlation tensor of the velocity field is of the form $\langle v_i v_j\rangle = A r_i r_j + B\delta_{ij}$, where $\mathbf{r}$ is the distance between the correlation points. Rotation of a celestial body leads, as already mentioned, to the appearance of the correlation $\langle \mathbf{v}\cdot \operatorname{curl}\mathbf{v}\rangle$, and the tensor $\langle v_r v_j\rangle$ acquires an additional rotation-invariant (odd, gyrotropic) term $\varepsilon_{ijf} r_f C(r)$ (helical turbulence); the equation for the field takes the form

$$\frac{\partial \langle \mathbf{H}\rangle}{\partial t} = \operatorname{rot} \alpha \langle \mathbf{H}\rangle + \chi \langle \Delta^2\rangle \langle \mathbf{H}\rangle. \tag{4}$$

The entire procedure reduces to the resolution $\mathbf{H} = \langle \mathbf{H}\rangle + \mathbf{h}$, to the solution of the linearized equation $\partial \mathbf{h}/\partial t = \operatorname{curl}[\mathbf{v}\times\langle \mathbf{H}\rangle] + \nu_m \nabla^2 \mathbf{h}$, and then to calculation of the quadratic correction. Credit for the development of this trend belongs to Steenbeck, Radler, and Krause.[55–57] An equation of the type (4) is derived also for a Markov process, when the velocity field is white noise.[58] This approach has been recently named mean field electrodynamics.

It can be shown that Eq. (4) has growing solutions (the so called α effect). Doubts concerning these results were advanced by Lerche,[59,60] but this is of course due to a misunderstanding: Equation (4) can be derived rigorously, as a theorem, in two cases: 1) small **Rm**, but large dimension of the body; 2) a Markov process (more concretely, $\tau \ll l/v$, where τ is the correlation time). All that are used here are the most general and fundamental properties of correlation or spectral functions.[61,62] We recall that according to Sec. (A) the α effect manifests itself not only in rising cells, but also in the descending ones. This is seen formally from the derivation of (4): what is important is the very predominance of the left-hand or right-hand screw, and not the direction of motion of the cell. In this connection we not note that a number of workers[53,63] have stated that the α effect is produced if the number of rising cells is larger than that of the descending cells, or vice versa, which raises, in Piddington's opinion,[53] a difficulty when it comes to applications. This statement is, of course, also a misunderstanding.

The most important for applications is the case **Rm** $\gg 1$. At the same time, as a rule, the condition $\tau \ll l/v$ is not satisfied, and this not a Markov process. It is therefore natural to attempt to dispense with the representation $\tau = 0$ (δ-correlation in time) and to consider the next approximation in the parameter $\tau/(l/v)$. An analogous formulation of the problem arises in the theory of the scattering of electromagnetic waves by inhomogeneities.[64] The Markov approximation is obtained if one sums selectively the perturbation-theory diagrams of Eq. (1). It is therefore natural to sum first a more extensive aggregate of diagrams, obtaining by the same token the next approximation. There is a more concise functional approach, but too unwieldy to be reported here; we present only its main ideas and the results, referring the reader to[65] for details. The basis of the approach is the introduction of the "Fourier transform" of the probability distribution function

$$G = \langle \exp (i \int u_j \theta_j \, d\mathbf{x}\, dt)\rangle,$$

where u_j is the velocity and θ_j is the argument, and analogously for the magnetic field. Taking the variational derivatives, we obtain different correlation moments, and in place of (1) we have an equation for G in terms of variational (and ordinary) derivatives. The solution is sought in the form of an expansion in a functional series. If we restrict this series to the minimum number of terms, we obtain directly the Markov approximation. In the next approximation we obtain a new result. Qualitatively, however, this result confirms the old one: without gyrotropy generation is impossible, the coefficient of turbulent diffusion does not differ in order of magnitude from the old value $\chi = vl/3$. We note that the stochastic model considered in[59] is incorrect (see[13]).

We turn now to the topological pumping of the field described in Sec. (B). We consider turbulent convection formed by Benard cells. In other words, let the convective cells with the topology described in Sec. (B) be not a stable but a long-lived formation. An example of such a convection is the convective zone of the sun: sometimes the upper convective cells are set in correspondence with granulation and supergranulation, and the lifetime of such a cell is close to the lifetime of the rotation of the cell, i.e., $\approx l/v$. This can be called arbitrarily "turbulence," since the process is random in time (although it exhibits a regular structure in space). One more feature of the convective zone is the presence of an entire spectrum of cell dimensions, starting with the largest ones, the heights of which are comparable with the depth of the entire zone, and ending with small ones, which are difficult to resolve by modern instruments. We assume that the topology of most cells is of the form described in Sec. (B). Will there be pumping of the field at the bottom of the zone? A formal answer to this question can be obtained by describing this process by means of anisotropic turbulence (the selected direction is in this case the vertical one). The spectral tensor of the velocity field for this process is known, and its knowledge suffices to explain the behavior of the magnetic field; this was done long ago.[58] The answer is negative: the anisotropic turbulence results only in anisotropic diffusion of the field and does not cause pumping. Wherein lies the matter? One might seem that the principal factor is the topology, and pumping should take place. First, the motion in[52] has the described topological property only for one series of cells. If we continue uninterruptedly the velocity field under this series and above it, then the topology will inevitably be reversed there: the topologically connected motions will be those in the upward direction. It is impossible to place cells of identical topology one under the other; and a discontinuity in the velocity field is bound to appear. Figure 7 shows a section through the cell of another type, below and above which it is possible to place a cell of the same topology: the velocity field decreases to zero from below and from above (the field lines become more widely spaced downward and upward). But a horizontal series of such cells causes pumping of the field not at the very bottom of the cells, so that many layers will not produce a common effect, and the pumping takes place only within each layer. Ultimately the picture will be the following: the largest cells, the vertical dimension of which is not smaller than the thickness of the zone, will cause topological pumping. The smaller cells will cause turbulent diamagnetism,[1,19,66–69] which also leads to a crowding out of the field under the convective zone, and the intensity of the field below the convective zone is larger by a factor $\sqrt{\mathbf{Rm}}$ than in the convective zone.

The crowding-out of the field under the convective zone is of importance, in particular, for the following circumstance. Many authors note that calculation of the period of the solar cycle by using Eq. (4) with addition of the term $\mathrm{curl}[\mathbf{v}_T \times \mathbf{H}]$, where $\mathbf{v}_T$ is the differential rotation, leads to too small a period (see, e.g.,[2]). Actually, the time is determined by the diffusion $\chi\, \nabla^2 \mathrm{H}$, and $t_0 = L^2/\chi$, where L is the thickness of the zone. The terms $\mathrm{curl}[\mathbf{v}_T \times \mathrm{H}]$ and $\mathrm{curl}\, \alpha \mathrm{H}$ enter in the form of sources in the equations for the toroidal and the poloidal components and influence only their relative amplitudes, but not the settling time. It turns out that $t_0 \sim 1$ year or less, whereas the solar cycle lasts 22 years. If we take the crowding-out into account, then the period increases,[70] owing to the high inductance of the sub-convective plasma, which prevents too rapid a change of the field.

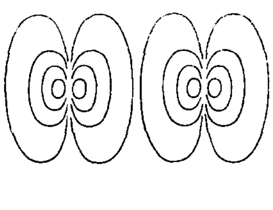


FIG. 7.

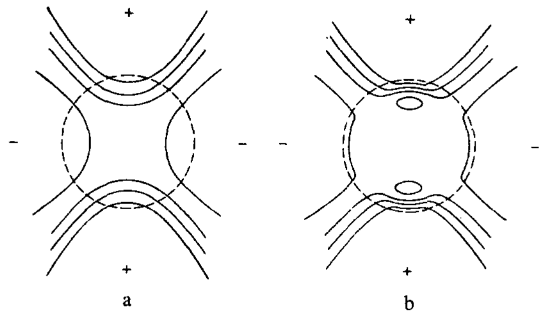


FIG. 8. Schematic representation of the configuration of the magnetic field corresponding to a sector structure of the interplanetary field. The signs + and − correspond to the polarities in the interplanetary space. The figure shows a section of the sun (dashed line) at the equator. The field having an intial configuration of the type shown in Fig. a is crowded out by the diamagnetism of the convective zone under the surface and under the convective zone of Fig. b.

An interesting application of the diamagnetic effect is the so-called sector structure of the interplanetary magnetic field, revealed by rocket observations. This field is shown schematically in Fig. 8, and the configuration of the field inside the sun is shown in Fig. 8a, of course, arbitrarily. Only one thing is clear: the field must be closed in some way or another under the surface of the sun. The presence of a convective zone under the very surface leads to a crowding out of the field from this zone, so that part of the field goes downward, while the main flux is closed under the surface of the sun (Fig. 8b). The resultant appreciable (as shown by elementary estimates) magnetic flux will interact with the field of the spots. This can lead to the phenomenon of active longitudes, wherein an increased activity is observed at certain longitudes: the field shown in Fig. 8b produces a distinct nonequivalence of the different longitudes.

So far we have dealt with the behavior of a large-scale field. How do things stand with the small-scale magnetic field? The problem is solved exactly for a Markov process or for a special wave turbulence (acoustic). The general conclusion is that the turbulence is unstable to perturbations of the magnetic field and that the fluctuations increase exponentially. As to the most typical case $\tau \approx l/v$, there exist here semi-empirical equations which a number of workers believe to be correct.[71–74] The situation is made difficult by the absence of a small parameter; numerical or laboratory simulation for large values of $\mathbf{Rm}$ is quite difficult (we are dealing with a three-dimensional random process, which is stationary only in the statistical sense). One can only speculate on the directions in which this problem will develop further. 1) We can use a functional approach and the methods described in[65] (it is even simpler to use the more recent method proposed in[75,76], which yields less cumbersome expressions) to go outside the framework of the Markov approximation. 2) We can attempt to approach the problem from the opposite direction: assume that in first-order approximation $\tau=\infty$, i.e., the field is stationary, and then we have an eigenfunction problem. The eigenfunction can be sought by the WKB method (the higher-order derivative $\nabla^2 H$ is preceded by a small quantity ν_m), although to be sure serious formal difficulties are encountered here, since it is not clear how to join together the solutions in the vicinities of the turning points. If this difficulty is overcome, then it is meaningful to consider in the next-order approximation slow adiabatic change of the velocity $\tau \gg l/v$—the inverse of the Markov process.[13] Can the α effect be used on turbulence scales? A large cell of scale l_1 rotates, and the small-scale motions (scale l_2) generate in it a field as a result of the α effect.[77] Unfortunately, this reasoning does not lead to anything: the instability growth rate[1] is $\alpha/l_1 \approx \omega_1 l_2^2/l_1^2$, $\omega_1 = v_1/l_1$ and the decrement due to the turbulent diffusion $\sim l_2 v_2/l_1^2$ is always larger than the growth rate.

Another aspect of the same problem is the generation of vortices by potential motions: the point is that the equation for the vortex curl **v** coincides with (1), and essentially α plays here the role of negative viscosity,[78] although it has a different dimensionality. The energy transfer from small scales to large scales is important in the physics of the earth's atmosphere. Generation of small-scale vortices is important in cosmology, and the creation of the moment that rotates the galaxies is connected with it. Unfortunately, relativistic effects (large velocities, gravitation) introduce nothing fundamentally new in this problem,[79] and at any rate we know only the generalization of the usual theory. Thus, the 4-curl $\omega^i = (-g)^{-1/2} e^{iklm} u_k u_{l,m}$ obeys, in the main, the same relation as in the classical theory, and can be generated in the same manner.

D. Turbulence: Nonlinearity

On this subject much is unclear, but nevertheless there are some clear-cut results. We have in mind MHD turbulence in the case of magnetic fields that are not weak (i.e., capable of influencing the motion) and $\mathbf{Rm} \gg 1$. We must first mention Kraichnan's idea of using a statistical-equilibrium ensemble, more accurately a Gibbs ensemble. Of course, turbulence is quite far from absolute equilibrium. Nonetheless, we consider an advanced turbulence. We turn on the dissipation: $\nu_m = \eta = 0$ (η is the ordinary viscosity), external force that excites the turbulence, and also large wave numbers, i.e., we deny the existence of perturbations with $k > k_{max}$ (thus, we deal so to speak with excitation of a lattice, where k_{max} is equal to the reciprocal-lattice vectors), while the small wave numbers are bounded by the dimensions of the system. Then, after a certain time the system arrives at a state of thermodynamic equilibrium, conserving the energy of the fluctuations. Of course, this situation is physically not realizable: there is no k_{max} in a continuous medium; nonetheless, this hypothetical equilibrium state will reflect the direction of the transport of the perturbations in phase space into regions of small dissipation.[80, 81] In fact, the equilibrium for nonmagnetic turbulence has a fluctuation spectrum $E(k) = ak^2$, and the energy is concentrated at k_{max}, thus indicating a tendency of transporting the perturbations into the short-wave region, i.e., the transfer of the energy to large k. The equilibrium two-dimensional state reflects the reverse transport into the region of small k, which is in accord with the known results. Nonmagnetic hyrotropic turbulence has been analyzed in in[81] and, again, the transfer of the characteristic features of the equilibrium state to a dynamic state is confirmed by numerical experiments and by acceptable arguments.

The approach to the magnetic case calls for a certain caution. Thus, in the simplest (not gyrotropic) case there is an equipartition of the magnetic and kinetic energies,[82] with $E_M \sim E = ak^2$, where E_M is the spectrum of the magnetic fluctuations. This result, while indicating a transfer of magnetic and kinetic energies into the region of large k (a fact which is trivial to a great degree), is not an indication of the fact that in the real case the magnetic energy tends to become comparable with the kinetic energy, i.e., that a turbulent dynamo of random fields takes place.[1] In fact, as already emphasized many times above, the dynamo itself is closely connected with dissipation, i.e., even if at initial instant of time the field is large-scale, it subsequently becomes rapidly modified, acquires a fine structure that "feels" the dissipation, and then either grows or attenuates rapidly. Therefore the situation $\nu_m = 0$ is physically quite far from the question of the turbulent dynamo. The very form of the spectrum $\sim k^2$ has a very simple interpretation: the Fourier amplitudes of the velocity and of the field are the degrees of freedom of the system, the spectrum $\sim k^2$ and the energy $\sim E(k)dk$ denote equipartition over the degrees of freedom (bosons at high temperatures).

Qualitatively new results are obtained if gyrotropic turbulence is considered,[15] and we now proceed to discuss these results. It is first necessary to find conserved quantities, for which purpose we shall need, besides (1), the equation of motion

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p - \frac{1}{4\pi\rho}[\mathbf{H}\times \mathrm{rot}\,\mathbf{H}] + \eta\nabla^2\mathbf{v}, \qquad (5)$$

($\mathrm{div}\,\mathbf{v} = 0$) and an equation for the vector potential $\partial\mathbf{A}/\partial t = [\mathbf{v}\times \mathrm{curl}\,\mathbf{A}] + \nu_m \nabla^2\mathbf{A} + \nabla^2\varphi$. Using these equations, we can easily show that if all the fields vanish at infinity and $\nu_m = \eta = 0$, the magnetic energy $\int[(H^2/8\pi) + (\rho v^2/2)]d^3r$, the magnetic gyrotropy

$$\int \mathbf{AH}\, d^3r \qquad (6)$$

and the "cross gyrotropy" $\int \mathbf{vH}\, d^3r$ are all conserved. For a homogeneous isotropic turbulence, the integrals are replaced by mean values. We note that the usual gyrotropy $\langle \mathbf{v}\cdot \mathrm{curl}\,\mathbf{v}\rangle$ is not conserved in the general magnetic case. We put

$$\frac{1}{8\pi} H_i(\mathbf{k}) H_i^*(\mathbf{k}) = E_M, \quad A_i(\mathbf{k}) H_i(\mathbf{k}) = H_M, \quad v_i(\mathbf{k}) H_i(\mathbf{k}) = H_c$$

(H_i and v_i are the Fourier amplitudes of the magnetic field and of the velocity). Making use of the Gibbs ensemble (see, e.g.,[83]), we obtain the distribution function $\rho = z^{-1}\exp[-a(E_M+E_k)-\beta H_M-\gamma H_c]$, where z is a normalization constant and α, β, and γ are the thermodynamic constants. It is now easy to calculate $\langle E_M\rangle$, $\langle E_k\rangle$, $\langle H_M\rangle$ and $\langle H_c\rangle$, as well as $\int_{k_{\min}}^{k_{\max}}\langle E_M\rangle dk$ etc. The result is the following. If $\beta=\gamma=0$, then we obtain the result[82] $\langle E_M\rangle\sim\langle E_k\rangle\sim k^2$, and if $\beta\neq 0$, then the energy is concentrated mainly at $k_{\min}$, a fact that has led the authors of[15] to the conclusion that there exists a reverse cascade towards small wave numbers. These results are confirmed by numerical experiments[74] and by calculation in which the hypotheses of[71] are used. These conclusions are to a certain degree not unexpected. Indeed, we have already mentioned that gyrotropy on a turbulent scale gives rise to generation of a large-scale field, a fact that reflects to a certain degree a reverse cascade. It is stated, however, in Ch. 3 that inside the turbulent scales the α effect does not give a reverse cascade. Does not this contradict the results of[15]? No, since in the latter case we are dealing in fact with generation of a field in the presence of a correlation $\langle \mathbf{A}\cdot\mathbf{H}\rangle$ or $\langle \mathbf{H}\cdot\mathrm{curl}\,\mathbf{H}\rangle$, while $\langle \mathbf{v}\cdot\mathrm{curl}\,\mathbf{v}\rangle$ can be equal to zero. The authors of[15] called this the β effect; it can be intuitively understood in the following manner.[84] The action of a magnetic field on the motion can be followed by using the simplified equation of motion

$$\frac{\partial\mathbf{v}}{\partial t}=-\frac{1}{4\pi\rho}[\mathbf{H}\times\mathrm{rot}\,\mathbf{H}]\approx-\frac{1}{4\pi\rho}[\langle\mathbf{H}\rangle\times\mathrm{rot}\,\mathbf{h}]; \tag{7}$$

here $\mathbf{H}$ is represented in the form $\mathbf{H}=\langle\mathbf{H}\rangle+\mathbf{h}$, and the simplest linearization is carried out. We now substitute (7) in (1) and average:

$$\left\langle\frac{\partial\mathbf{H}}{\partial t}\right\rangle=-\frac{1}{4\pi\rho}\int_0^t\langle\mathrm{rot}\{[(\langle\mathbf{H}\rangle\times\ \mathrm{rot}\,\mathbf{h}(t_1)]\,\mathbf{h}(t)\}\rangle dt_1$$
$$=\frac{2}{3}\frac{1}{4\pi\rho}\mathrm{rot}\int_0^t\langle\mathbf{h}\,\mathrm{rot}\,\mathbf{h}\rangle\langle\mathbf{H}\rangle\,dt_1. \tag{8}$$

We see that the correlation $\langle\mathbf{H}\cdot\mathrm{curl}\,\mathbf{H}\rangle$ acts on the motion in such a way that the β effect is produced:

$$\beta=\frac{2}{3}\frac{1}{4\pi\rho}\int_0^\infty\langle\mathbf{H}(t)\,\mathrm{rot}\,\mathbf{H}(t_1)\rangle\,dt_1,$$

which is analogous to the α effect. The magnetic gyrotropy (6) may turn out to be important for the dynamics of the field in connection with the problem of nonlinear stopping or stabilization of the dynamo instability. We refer here to the fact that unlimited generation of a field is, of course, impossible, and in definite cases stabilization at a weakly nonlinear level is possible.[1,85–87] The idea consists in the fact that the growing large-scale field suppresses first of all the cause of its growth, i.e., the magnitude of the α term. A numerical calculation (which is quite complicated and cumbersome) confirms this idea. However, using magnetic gyrotropy (6), this stabilization can be illustrated quite simply. Let the invariant (6) (magnetic gyrotropy) vanish at $t=0$. Since it can only attenuate, it will remain equal to zero. In the presence of helicity $\langle\mathbf{v}\cdot\mathrm{curl}\,\mathbf{v}\rangle\neq 0$ the large-scale field begins to be excited and, as can be easily verified, a field of the spiral type is excited. To this end, we neglect dissipation in (4):

$$\begin{gathered}\frac{\langle\mathbf{A}\rangle\,\partial\langle\mathbf{H}\rangle}{\partial t}=\langle\mathbf{A}\rangle\,\mathrm{rot}\,\alpha\langle\mathbf{H}\rangle,\\ \frac{\langle\mathbf{H}\rangle\,\partial\langle\mathbf{A}\rangle}{\partial t}=\alpha\langle\mathbf{H}\rangle^2+\langle\mathbf{H}\rangle\nabla\varphi,\\ \frac{\partial}{\partial t}\int\langle\mathbf{A}\rangle\langle\mathbf{H}\rangle\,d^3r=2\int\alpha\langle\mathbf{H}\rangle^2\,d^3r,\\ \int\langle\mathbf{A}\rangle\langle\mathbf{H}\rangle\,d^3r\approx\frac{\alpha}{|\alpha|}L\int\langle\mathbf{H}\rangle^2\,d^3r,\\ \int\langle\mathrm{rot}\,\mathbf{H}\rangle\langle\mathbf{H}\rangle\,d^3r\approx\frac{\alpha}{|\alpha|}\frac{1}{L}\int\langle\mathbf{H}\rangle^2\,d^3r.\end{gathered} \tag{9}$$

It is seen from (9) that the helicity of the large-scale field is almost maximal $\langle\mathrm{curl}\,\mathbf{H}\rangle\parallel\langle\mathbf{H}\rangle$. The word "almost" has been added because actually it is also necessary to take dissipation into account. From the fact that the invariant (6) was equal to zero it follows that in turbulent scales there is generated a gyrotropy $\langle\mathbf{H}\cdot\mathrm{curl}\,\mathbf{H}\rangle$ which is opposite in sign to (9) and cancels the latter completely. Hence

$$\beta=\frac{-1}{4\pi\rho}\tau\langle\mathbf{H}\rangle\,\mathrm{rot}\,\langle\mathbf{H}\rangle\approx-\frac{1}{4\pi\rho}\frac{\tau}{L}\langle\mathbf{H}\rangle^2. \tag{10}$$

The dynamics of the field $\langle\mathbf{H}\rangle$ is now determined by the equation $\partial\langle\mathbf{H}\rangle/\partial t=\mathrm{curl}(\alpha+\beta)\,\langle\mathbf{H}\rangle+\chi\nabla^2\langle\mathbf{H}\rangle$ and, as can be seen from (10), the β effect acts opposite to the α effect. At a sufficient intensity of the average field, the instability becomes stabilized and generation stops, roughly speaking, at the instant when $\alpha+\beta=0$.

In a vibrational dynamo (when the magnetic field oscillates but is not attenuated as a result of the action of the motion of the motions on this field), for example, in the solar cycle, the nonlinear stabilization determines the level of the oscillations themselves.[88]

The incomplete spirality of the field, which was noted in the preceding paragraph, is quite essential. In fact, if we have exactly $\mathrm{curl}\langle\mathbf{H}\rangle\parallel\langle\mathbf{H}\rangle$, then the large-scale harmonic is force-free. But it is known that the field cannot be force-free in all of space (or in a bounded body, with the boundary conditions corresponding to the absence of external sources of the field). Therefore the problem (4) with $\chi=0$ cannot be posed as an eigenvalue problem, and the correct solution in the general case $\chi=0$[89] shows that the field is actually not force-free. It is interesting to note that the nonlinear effect not only brings the growth of the field to a halt. The field can be generated even in the absence of rotation.[90] In fact, since we are dealing with nonlinearity, α itself can depend on $\langle\mathbf{H}\rangle$. In the presence of one more physical vector $\mathbf{q}$, for example the density gradient, the pseudoscalar α can be combined in the form $\alpha\sim\langle\mathbf{H}\rangle\,\mathrm{curl}\,\langle\mathbf{H}\rangle\,q^2$, $(\langle\mathbf{H}\rangle\mathbf{q})$, $\mathrm{div}(\langle\mathbf{H}\rangle\mathbf{q})\,[\langle\mathbf{H}\rangle\times\mathbf{q}]$, $([\langle\mathbf{H}\rangle\times\mathbf{q}]\,\mathrm{curl}[\langle\mathbf{H}\rangle\times\mathbf{q}])$. Equation (4) becomes complicated and nonlinear in this case, but it can be shown that a field can be excited also under these conditions. The effect itself, in view of its nonlinearity, does not come into play at arbitrarily weak fields (as in the ordinary dynamo theory). Whether this nonlinearity gives rise to generation or simply to modification of the field, these terms must be included in Eq. (4) if the field intensity is no longer small. It can be noted here that the described systems become quite cumbersome in a real situation, when we have the combination of rotation, of a selected direction, of non-weak fields, and of differential rotation (the solar convective zone).

In sufficiently complicated nonlinear problems it is natural to invoke waves, their interactions, etc. Many papers have been devoted to just such a formulation of the problem. It is most natural to pose the problem in the following manner. We consider a rotating system with a definite configuration of the magnetic field. We find the magnetic perturbations, the waves, and see whether these wave perturbations react on the field in such a way as to prevent its ohmic damping or even enhance the field.[37–41] Braginskii[36] has considered waves in the earth's core and has shown that the essential role is played by the magnetic, Archimedean (buoyancy), and Coriolis forces (the so called Braginskii MAC waves). These waves not only represent that very deviation from symmetry referred to above in Sec. (A), which maintains the earth's field, but also casts light on the nature of the western drift of the field and explains the difference between the magnetic and geographic axes of the earth.

The theory of an almost symmetric dynamo becomes nonlinear: in[28] is specified not the entire velocity field, but only the field that deviates from symmetry, and what is thought is the principal symmetric motion that produces generation, while in[29] an investigation is made of the motion that results in stationary generation (i.e., $\partial \mathbf{H}/\partial t = 0$).

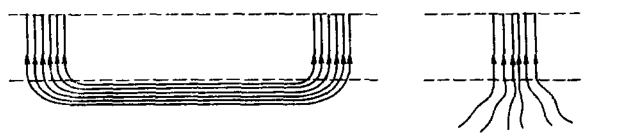
Wave motions, just like turbulent motions, are capable of stirring the field and lead to anomalous diffusion. If the magnetic field is weak, then in first-order approximation its action on the motion can be neglected, i.e., only sound waves remain in the hydrodynamics—acoustic turbulence. On the other hand, if the field is amplified, then the perturbations turn into Alfvén perturbations, both accelerated and decelerated. Their interaction with the magnetic field was considered by Ivanov.[91] It is clear that pure wave motions do not stir up the field, since they correspond only to oscillations of each particle. The interaction of the wave leads to a random walk of the particles, since each oscillation has a finite "memory time" as a result of the loss of phase coherence.[1] Actually the matter reduces to calculation of an integral of the type $\int_0^\infty \langle v_i(t)\, v_j(t_1)\rangle dt_1$, and this is just the diffusion tensor. For non-interacting waves, this integral has no meaning: it takes the form $\int_0^\infty \exp(i\omega)\,(t-t_1)dt_1$. The interaction leads to an integral over a "damped sinusoid" of the type $\exp(-a + i\omega_A)t$, $a \approx \omega_A v^2/v_A^2$, so that the integral can be estimated: $v^2\Delta t$, $\Delta t = (1/\omega)(v/v_A)^2$, where v_A is the Alfvén velocity, and ω_A is the frequency of the Alfvén wave. Ultimately we have $\chi = v^4/\omega_A v_A^2$.

We consider the application of this phenomenon to sunspots. There exists a problem of the rapid decay of the spots, which cannot be attributed to simple ohmic damping. If we interpret the motions observed in the spots as MHD, which is natural, since the observed $v < v_A$, then the damping time of the spot will be L^2/χ, where L is in this case the smallest dimension of the spot. The observed dimension of the spot is a large quantity, and a smaller quantity is its vertical dimension. In fact, at a depth of several hundred kilometers, where the convective zone begins, the kinetic energy of the convective motions already exceeds the magnetic energy in the spot. The field is effectively "churned" by the convection. More accurately, the field will be crowded out of the convective zone into the subphotospheric zone as a result of the already mentioned diamagnetism. Figure 9a shows a "section" through a bipolar group of spots. We note that in the literature they frequently use the concept of "complete entanglement" of the field of the spot under the photosphere (Fig. 9b), in which no account is taken of the diamagnetism. If L is now taken to mean the depth of the spot, then the resultant time is comparable with the lifetime of the spot.

Very strong magnetic fields, at a given source of turbulence, lead to a weakening of the interaction and to a decrease of the wave amplitudes. It might seem that such fields should suppress the turbulence completely, but this is not the case. Experiment shows that the turbulence degenerates into two-dimensional.[92,93] This can be understood in the following manner. Assume that a homogeneous field $\mathbf{H}_0$ parallel to the z axis is applied on a conducting medium. If there are no perturbations of the magnetic field at the initial instant, then in the presence of two-dimensional motion $\mathbf{v} = \{v_x(x, y), v_y(x, y), 0\}$ they will not occur: the term $\mathrm{curl}[\mathbf{v}\times\mathbf{H}_0]$ of (1) vanishes. Once there are no field perturbations, this means that there are neither currents nor electromagnetic forces acting on the liquid and suppressing the motions. Such a two-dimensional turbulence no longer "feels" the field in the sense that even if the field is further amplified, this will not affect its properties at all. Further discussion of this unusual phenomenon (in particular, the energy transfer into the region of small wave numbers etc.) is beyond the scope of this article. On the other hand, modification will take place if the field $\mathbf{H}_0$ depends on x and y but, as before, is parallel to the z axis. It is interesting that in this case it is possible to disregard the electromagnetic forces.[94] In fact, the equation for the field is

$$\frac{\partial H_z}{\partial t} = -(\mathbf{v}\nabla)H_z + \nu_m \nabla^2 H_z, \tag{11}$$

and the other components vanish, but in this case $[\mathbf{H}\times\mathrm{curl}\,\mathbf{H}] = \nabla H^2/2$, i.e., the forces are potential and can be compensated for by pressure. This model may turn out to be useful for the understanding of processes in sunspots. If the turbulence in the sunspots degenerates into a two-dimensional turbulence and therefore enters it only in the vertical field, then its dynamics will be described by Eq. (11). Turbulent motion "breaks apart" the field; in final analysis, the equation obtained for the average field is $\partial\langle\mathbf{H}\rangle/\partial t = \chi\Delta\langle\mathbf{H}\rangle$, and the diffusion co-



a b

FIG. 9. Bipolar group of spots. The field between the spots is crowded out under the photosphere as a result of diamagnetism.

efficient was calculated in[94]. Of course, for large ν_m an exact calculation is possible only for a Markov process. At the same time, in view of the complete similarity between (11) and the temperature equation (except that ν_m is replaced by the coefficient of molecular thermal conductivity) or, in general, with an equation for a scalar admixture, it is possible to simulate this process experimentally. This simulation makes it possible to determine exactly the coefficient χ not only for a Markov process. We note that in the experimental model the motion need not necessarily be two-dimensional, it suffices only that the temperature be a function of x and y. A more direct simulation is possible, where one applies to the conducting liquid not only a strong field that degenerates the turbulence into a two-dimensional one, but also a temperature gradient or some other admixture. In this case the applied uniform magnetic field field serves only to produce two-dimensional turbulence, and the additional scalar admixture simulates the behavior of the large-scale inhomogeneous magnetic field.

Rädler[95] has considered the reaction of a weak homogeneous magnetic field on the motion. The turbulence has not yet degenerated into two-dimensional, since the intensity of the field is not strong enough. It is possible, however, to trace the following tendency: the motion acquires two-dimensional features. What was unexpected was a different result: the weak field did not suppress the turbulence (as had been customarily assumed), and in some cases even amplified it. This can be understood in the following manner. The magnetic field can lead, among other effects, to a weakening of the energy flux into the region of large wave numbers. In fact, a very strong field excludes this transport completely, since the perturbations are transformed into non-interacting waves. This leads to accumulation of energy at small wave numbers and to a certain increase of the energy in comparison with the case when there is no applied field.

So far we have dealt in this section only with the behavior of a large-scale field. One of the most important problems of turbulence theory is to obtain the spectrum, meaning the distribution of the energy with respect to the scales. In magnetohydrodynamics the situation is aggravated by the fact that the question of the interaction of the plasma motion with the field has not been finally solved even in the linear approximation, i.e., if the fields are assumed to be weak. There exist semi-empirical equations, which when the magnetic field is turned off produce a Kolmogorov spectrum, and also have other numerical favorable properties and are confirmed, in particular, by various numerical tests. Their use[72–74] leads to the following result. In the region of scales where neither the viscosity nor the ohmic dissipation comes into play as yet, a stationary energy flux is established into the region of large wave numbers with equal distribution of the magnetic and kinetic energies. The spectrum takes the form $E \sim k^{-3/2}$. Indeed, the energy flux is $\rho v^2/2\tau = H^2/8\pi\tau = \text{const}$. The time of interaction τ is determined from $1/\tau = \omega v^2/v_A^2$, where v_A is determined by the field with the largest scale. This time coincides with the time of the interaction of magnetohydrodynamic waves in a homogeneous field, and in this case it is a quasi-homogeneous field of large scale. From this it follows immediately that $v^2 \sim H^2 \sim k^{-1/2}$, $E \sim k^{-3/2}$.

E. General discussion of magnetohydrodynamic modification

1) We have seen that the motion gives rise to a rather effective decrease of the scale—of the field, followed by generation or else by rapid annihilation of the field. It is precisely the failure to take this circumstance into account which led Piddington to the criticism of the dynamo mechanisms; he indicated a rather low rate of dissipation of the fields because of their large dimensions. What is the situation here with the usual concept of "freezing in"? After all such a shear motion, which at first glance is harmful, or else differential rotation, is capable of violating the "freezing-in" condition. These examples illustrate the reefs on which such a representation can founder.[95]

2) Further progress in the research will apparently involve with examination of the simplest motions and their role in the dynamo, in particular that of a differentially rotating sphere with electric conductivity that depends on the latitude.

3) The simplicity of the mechanisms described in Ch. 1 make it possible to verify the theory in a laboratory experiment, and also to use it in technology. Figure 4 shows in fact a dynamo without windings. In addition, the following hypothesis can be advanced: generation takes place in all cases when it is impossible to prove the opposite by using the rules developed in Sec. (A).

4) Allowance for the nonlinear electromagnetic forces that act on the plasma should at first glance greatly complicate the analysis. It appears, however, that this is in fact not so. The instability growth rate $\gamma \sim v/l$ obtained in all the problems follows from the simple estimate (1) under the condition that **H** is almost parallel to **v** (see the reasoning in Sec. (A)). An approximate form of an unstable harmonic is shown in Fig. 10. It is seen from Fig. 10 that the electromagnetic force will, in the main, be parallel to the y axis and will rapidly reverse sign, but this force can be cancelled by the pressure $p(y)$. Consequently, it will act over a scale l (and not δ) and cause only a simple slowing down of the fundamental velocity. The validity of this statement can be verified also from energy considerations:

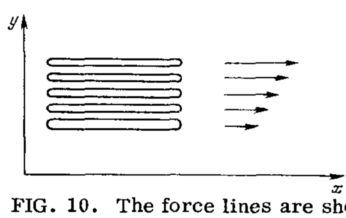


FIG. 10. The force lines are shown by the solid lines, and the arrows indicate the plasma velocity. It is seen that the field is almost parallel to the velocity and is small in scale. A large field gradient, $\sim H/\delta$, exists mainly in a direction perpendicular to **v**.

$$\frac{4\pi\rho}{2}\frac{\partial}{\partial t}\int \mathbf{v}^2 d^3r = -\int \mathbf{v}[\mathbf{H}\times \mathrm{rot}\,\mathbf{H}]d^3r$$

$$= -\int \mathbf{H}\,\mathrm{rot}\,[\mathbf{v}\times\mathbf{H}]d^3r \approx -\frac{v}{l}\int \mathbf{H}^2\,d^3r. \qquad (12a)$$

This expression does not contain the small scale δ at all.

5) The aforementioned decrease in the scale of the field can be of independent interest, for example, when it comes to explaining the fine structure of the field at the sun.[97] The dimension δ calculated for granulation and supergranulation is quite small and has so far not been resolved by the apparatus. Such minute formations are capable of heating the gas via Joule dissipation. An estimate of the largest field intensity yields $H_{max}=H_0\,\mathrm{Rm}^{1/2}$, H_0, where H_0 is the initial field. It follows from (12a) that the reaction of the field on the motion must be taken into account if $v_A\approx v$, and consequently v_A cannot exceed v. Therefore formations with scale δ and with field intensity $H_0\sqrt{\mathrm{Rm}}$ are realized if $H_0\sqrt{\mathrm{Rm}}<v\sqrt{4\pi\rho}$ (which yields in the case of granulation motions an upper bound of the initial field $H_0\lesssim 0.3$ G). On the other hand, if $H_0\sqrt{\mathrm{Rm}}>v\sqrt{4\pi\rho}$ but $H_0<v\sqrt{4\pi\rho}$ (for grunulation $0.3\leqslant H\leqslant 100$ G), then the scale will decrease already not to such small dimensions δ, but still a fine structure of the field will be produced. Finally, if $H_0>v\sqrt{4\pi\rho}$ (sunspot), the action of the indicated mechanism becomes impossible.

3. LOW FREQUENCIES

A. General remarks

The idea of the α effect can be transferred from magnetohydrodynamics to a plasma in which oscillations are excited. In fact, in principle it is possible to produce a pseudoscalar $\alpha\sim(\mathbf{k}\cdot\mathbf{H})$ ($\mathbf{k}$ is the wave vector), which follows from the gyrotropic properties of the plasma in a magnetic field. Formally, the gyrotropy of the plasma manifests itself in the appearance of terms of the type $i\varepsilon_{ijf}\omega_f^{(e)}/\omega$ ($\omega^{(e)}=e\mathrm{H}/mc$, is the wave frequency) in the dielectric tensor. It is clear beforehand, however, that the apparatus here is not at all similar to that used in the magnetohydrodynamics approximation.

We shall show that the correlation $\langle\mathbf{v}\cdot\mathrm{curl}\,\mathbf{v}\rangle$ appears in a plane wave. Here $\mathbf{v}$ should now be taken to mean the electron velocity. We consider longitudinal oscillations; if $\mathbf{H}=0$, then $\mathbf{v}=\mathbf{k}\varphi$. In the presence of a magnetic field in the general case we have

$$\mathbf{v}=\varphi\left\{\frac{\mathbf{k}+ia[\mathbf{k}\times\omega^{(e)}]}{\omega}\;\frac{b(\mathbf{k}\omega^{(e)})\,\omega^{(e)}}{\omega^2}\right\}, \qquad (12b)$$

where a and b are constants. Furthermore

$$\langle\mathbf{v}\,\mathrm{rot}\,\mathbf{v}\rangle=\frac{\varphi\varphi^* ab(\mathbf{k}\omega^{(e)})[\mathbf{k}\times\omega^{(e)}]^2}{\omega^3}+\mathrm{c.c.} \qquad (13)$$

The averaging is over the period. Thus, we actually have $\langle\mathbf{v}\,\mathrm{curl}\,\mathbf{v}\rangle\sim(\mathbf{k}\cdot\mathbf{H})$. In addition, it is clear that the effect will take place only in the case of anisotropy of the oscillations, i.e., when there is a selected phase velocity and the amplitude of the oscillations with this selected velocity is the largest. In the purely isotropic case it is no longer possible to form a pseudoscalar of the type $(\mathbf{k}\cdot\mathbf{H})$, and $\langle\mathbf{v}\cdot\mathrm{curl}\,\mathbf{v}\rangle=0$.

B. Formulation of the problem. Method

Two formulations of the problem are possible. The first is perfectly analogous to the dynamo problem. Assume that oscillations are excited in the plasma and there are no external field sources. What happens with the fluctuations of the magnetic field? We shall assume *henceforth throughout that the plasma is collision-dominated*, or more accurately that the characteristic frequency γ of the field variation is lower than the collision frequency. Consequently, without oscillations the field would simply attenuate in accordance with the equation

$$\frac{\partial\mathbf{H}}{\partial t}=\nu_m\nabla^2\mathbf{H}-\frac{c}{4\pi ne}\,\mathrm{rot}\,[\mathbf{H}\times\mathrm{rot}\,\mathbf{H}], \qquad (14)$$

where n is the electron density. The waves in the plasma cause oscillations of the charges, and in the linear approximation they have no influence whatever on the field. The nonlinear current averaged over the period gives a nonvanishing contribution. The magnetic field is assumed to be weakly inhomogeneous: $L\gg\lambda$, where L is the scale of the field and λ is the wavelength, and is assumed to be quasistationary: $\omega\gg\gamma$. Since the motions depend on the field (see (13)), the nonlinear current also depends on the field and, just as the field, it is weakly inhomogeneous and generally speaking contains a vortical component. The latter excites a field and can compete with the ohmic damping in accordance with (14).

The presence in the plasma of high-intensity oscillations (e.g., ion sound) can cause anomalous electric conductivity, so that in this case it is necessary to take σ in (14), as well as in all other formulas, to mean the *anomalous electroconductivity*. We recall that in the dynamo theory the plasma is assumed to be collision-dominated and, although the collision frequency does not enter in the growth rate $\gamma\approx v/l$, the very form of the instability of the harmonic (the scale) is determined by σ, meaning also by the frequency ν electron of the collisions with the other particles. The situation here is perfectly analogous: the final formulas contain the collision frequency. The resultant field perturbations are in general not waves; they are relatively large-scale ($L>\Lambda$, where Λ is the electron mean free path) and slowly growing ($\nu>\gamma$) formations. They cannot be regarded as a consequence of a direct interaction between the high-frequency oscillations with these perturbations (e.g., decay instability of the oscillations etc.) precisely because the result contains the collision frequency.

We note that the plasma is still "not quite" collisional. This follows from the very existence of the oscillations: $\nu<\omega_p$ (ω_p is the plasma frequency) if we deal with Langmuir oscillations; $\nu_i<\omega$, where ν_i is the frequency of the ion-ion or ion-neutral collisions, and ω is the frequency of the ion sound, if the latter is considered.

The nonlinear current can be calculated by standard methods. We first obtain the linear current $\mathbf{j}'$ and the linear velocity $\mathbf{v}'$; using the known dielectric tensor of the plasma ε_{ij} in a magnetic field, we can change over to σ_{ij}, and from it to $\mathbf{j}'$ (assuming that $\mathbf{E} \sim k$). The linear velocity is already simply expressed in terms of $\mathbf{j}'$: $\mathbf{v}' = \mathbf{j}'/n_0 e$, the density perturbations are $n' = n_0(\mathbf{k}\cdot\mathbf{v})/\omega$. We next calculate the nonlinear current $\langle n' e\mathbf{v}'\rangle + \langle n_0 e\mathbf{v}''\rangle$ averaged over the period. The correction $\mathbf{v}''$ is obtained from the nonlinear equation of motion, and in all the nonlinear terms we substitute the linear velocity $\mathbf{v}'$. The procedure itself is straightforward but cumbersome, so that in this article we shall write out only the fundamental expressions.

C. Modification of field by ion sound

For purposes of methodology, we shall first describe the effect qualitatively. Without a field, the motions of the electrons and ions are almost identical and $\mathbf{v}' \sim \mathbf{k}$. In a magnetic field, the linear velocity of the electrons takes the form (13). The nonlinear current $\langle n' e\mathbf{v}\rangle$ also is similar to (13), with greatest interest attaching to the last term in the bracket of (13): when we take the curl of this term in order to introduce it into the equation (14) for the field, it takes the form curl $\alpha\mathbf{H}$, so that in analogy with (4) it is precisely because of this term that we should expect the appearance of instability.

The current $\langle n' e\mathbf{v}\rangle$, due to this term, can be estimated at $n' \approx n_0 v'/s$, s is the speed of sound and $\langle n' e\mathbf{v}'\rangle = n_0 e \langle v'^2\rangle \omega^{(e)} (\omega^{(e)}\varkappa)/s\omega^2$, $\varkappa$ is a unit vector in the wave propagation direction. We now introduce this current into the electrodynamics equations:

$$\mathrm{rot}\,\mathbf{H} = \frac{4\pi}{c}(\sigma\mathbf{E} + \langle n' e\mathbf{v}'\rangle), \quad \mathrm{rot}\,\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t},$$

here $\mathbf{E}$ is the self-induction field. We then obtain an expression of the type (4), where

$$\alpha = \frac{\langle v'^2\rangle (\omega^{(e)}\varkappa)\, \nu}{s\omega^2}. \tag{15}$$

The equation turns out to be nonlinear, since α depends on $\mathbf{H}$; nonetheless, the same physical considerations that lead to the conclusion that a field can be generated in the linear equation (4) are applicable here, too. It suffices, for example, to consider the two-dimensional field $\{H_x(x, y),\ H_y(x, y),\ H_z(x, y)\}$ (or, accordingly in a spherical coordinate system, the axisymmetric field $\{H_r(r, \theta),\ H_\varphi(r, \theta),\ H_\theta(r, \theta)\}$. Now the planar component (H_x, H_y) (or the poloidal (H_r, H_θ) for a sphere) generates H_z (the toroidal H_φ) and vice versa. A rigorous proof can be obtained for a definite example.[98] As to the second formulation of the problem, of course, it is much simpler: the field should be represented in the form $\mathbf{H}_0 + \mathbf{h}$, the equation should be linearized, and the problem should be tested for stability. Analysis shows that under the conditions indicated below, the instability does indeed take place.

In a real situation, ion sound is excited at $T_e > T_i$ and if the current velocity $v_d > s$. If we assume that v_d slightly exceeds s, then excitation takes place in a narrow cone, and a broad wave spectrum (relative to k) is excited.[99] In this case (15) is replaced by an integral that depends weakly on $(\mathbf{k}\cdot\omega^{(e)})$.

Correct allowance for all the nonlinear terms leads to the equation

$$\frac{\partial\mathbf{H}}{\partial t} = -\nu_m\,\mathrm{rot}^2\,\mathbf{H} - \frac{c}{4\pi ne}\,\mathrm{rot}\,[\mathbf{H}\times\mathrm{rot}\,\mathbf{H}] + \mathrm{rot}\,\{\alpha\mathbf{H} + b\,(\omega^{(e)}\mathbf{H})\,\varkappa + d\,(\varkappa\omega^{(e)})\,(\varkappa\mathbf{H})\,\varkappa + e\,[\varkappa\times\mathbf{H}]\}, \tag{16}$$

where b, $d = \langle v'^2\rangle\,\nu/s\omega^2$, $e = \beta s$, β is the ratio of the energy of the oscillations to the energy of the plasma.[99–101] It is clear that in the curly bracket of (16) are gathered all the combinations that constitute true vectors and contain the field linearly and quadratically.

A comparison of the generation α term with the dissipation ν_m curl$^2\mathbf{H}$ leads to the instability criterion and to the growth rate

$$L\alpha > \nu_m, \quad \gamma = \frac{\alpha}{L}. \tag{17}$$

The unstable fluctuations drift, as can be seen from (16), along at a rate $e = \beta s$. An analysis of (16) shows that not only growing solutions are possible, but also solutions connected with the modification of the field, particularly solutions that attenuate rapidly.

It is seen from (15) that in the presence of an oscillation spectrum the largest contribution is made by the low-frequency oscillations. The question is: at what values of ω should the spectrum be cut off? The analysis itself is valid if $\omega^{(e)} < \omega$, therefore, if the frequency $\omega^{(e)}$ is higher than the lowest turbulence frequency ω' due to ion-ion or ion-neutral collisions,[99] then the spectrum is cut off precisely at the frequency $\omega^{(e)}$. In the opposite case the spectrum is cut off at ω'. There is one more limitation on the low frequency. The point is that for low frequencies the nonpotential electric fields $\mathbf{E}_b$ assume an important role, and the oscillations cease to be longitudinal. To estimate $\mathbf{E}_b$ we introduce the electronic part of the current $ne\mathbf{v}'$ into the electrodynamic equations, where $\mathbf{v}'$ is determined from (13)

$$\begin{aligned} \mathrm{rot}\,\mathbf{H} &= ne\mathbf{v}'\frac{4\pi}{c} + \frac{1}{c}\frac{\partial\mathbf{E}_b}{\partial t}, \\ \mathrm{rot}\,\mathbf{E}_b &= -\frac{1}{c}\frac{\partial\mathbf{h}}{\partial t}, \\ \mathrm{rot}^2\,\mathbf{h} &= n_0 e\,\mathrm{rot}\,\mathbf{v}'\frac{4\pi}{c} - \frac{1}{c^2}\frac{\partial^2\mathbf{h}}{\partial t^2}, \end{aligned} \tag{18}$$

$\mathbf{h}$ is the fluctuating magnetic field; the ion current is immaterial, since its nonpotential part $\sim \omega_i/\omega$ (ω_i is the ion cyclotron frequency)—this parameter is negligibly small. Since $k^2 \gg \omega^2/c^2$, the displacement current is negligible, and therefore $\mathbf{h} \approx (4\pi/c)(ne\mathbf{k}\times\mathbf{v}/k^2) \times (\omega^{(e)}/\omega)^a$, where a depends on the term of (13) of which the curl is taken. If it is taken of the second term, then $a = 1$, and if of the third then $a = 2$, and the curl of the first term vanishes. Hence $E_b = (\omega^{(e)}/\omega)^a \Omega_p^2 |\nabla\varphi| / c^2 k^2$, where Ω_p is the plasma frequency of the ions and $\nabla\varphi$ is the potential electric field. The quantity $(\omega^{(e)}/\omega)^a$ is a small parameter, but nonetheless we take into account in (13) the terms with this parameter; the expression for E_b contains an additional parameter Ω_p^2/c^2k^2, which is small if $\omega > \Omega_p s/c$. Thus, $\Omega_p s/c$ is the second lower bound of the frequency.

D. Applications

In experiment it is possible in principle to produce the most favorable conditions $\omega^{(e)} \approx \omega'$, and the criterion (17) takes the form $L\omega_p^2 \langle v'^2 \rangle / (\omega' c^2 s) > 1$. We cut off the spectrum at the ion-ion collisions, and then, according to [102] (assuming that the current velocity v_d slightly exceeds s) we have $\omega' = \nu_i \sqrt{M_i/m}$. In addition, we assume that $\beta = 10^{-2}$ and then at $n = 10^{15}$ cm^{-3}, at an electron thermal velocity $v_T = 10^9$ cm/sec (thermonuclear plasma), and at T_i only slightly lower than T_e, we obtain a critical dimension $L_k = 1$ cm. From the equality $\omega^{(e)} = \omega'$ we obtain $H \approx 10$ G and $\gamma = 2 \times 10^5$ sec^{-1}. We note that $\omega' > \Omega_p s/c$, so that it is precisely at this frequency that the spectrum should be cut off. However, this ion sound can be excited by a current, but can be excited with a beam, the concentration of which should be lower than the plasma density: the point is that a large current density does by itself produce a field exceeding 10 G, and the condition $\omega^{(e)} \approx \omega'$ is violated.

E. Weakly ionized plasma

If the electrons and ions collide only with neutrals and all the collision frequencies (including the lowest one—the reciprocal time of momentum transfer to the neutral atoms, $\sim \nu' n_i/n_n$, where ν' is the ion-neutral collision frequency, n_i is the ion density, and n_n is the density of the neutrals) are lower than the frequencies of the processes, just as in the solar photosphere, then the situation is particularly simple. In this case the three-fluid hydrodynamics reduces to ordinary magnetohydrodynamics, i.e., to Eqs. (1) and (5), where the density here must be taken to be the density of the neutrals. All this can be easily verified by adding to the equations of motion of the neutrals, ions, and the electrons terms of the type $\nu'(\mathbf{v}_i - \mathbf{v}_n)$. In particular, the Alfvén wave constitutes oscillations of all three components, and therefore its velocity is determined by the density of the neutral component.

The solar photosphere satisfies also the "freezing-in" condition, i.e., even though the collision frequency is high, we have $\omega_{\rm pr} < \nu_m/l^2$, where $\omega_{\rm pr}$ is the frequency of the process. In this case the field is "frozen-in" into the neutral component. Now it is the velocity of the neutrals which is specified in the kinematic formulation of the dynamo. This velocity is transferred via collisions to the ionized component, and the latter already influences the field. The reaction of the field takes place in the reverse order: the field acts on the ionized component, which transfers the velocity to the neutral component. Lerche[3] has considered the turbulent state of such a three-fluid hydrodynamics. To close the resultant infinite chain of equations, he uses the Millionshchikov-Chandrasekhar hypothesis, namely, that the fourth moments have a Gaussian dependence on the second moments.

F. Long-known mechanisms

It should be stated that excitations of a magnetic fluctuation by a beam or by high-frequency oscillations is a situation far from new in plasma physics. Thus, it is well known that in a plasma of not too low pressure p ($8\pi p/H^2$ not very small) magnetic sound can build up at $v_d > v_A$.[99] This includes also the hose instability, viz., excitation of magnetic perturbations due to anisotropy of the distribution function ($p_\parallel > p_\perp$).[104] Mention can also be made of the buildup, considered in[105], of Alfvén waves in the presence of high-frequency sound, and also Thirring instability[106] and the new mechanism of excitation of magnetohydrodynamic waves in a nonuniform plasma.[107] It should be noted that in contrast to the foregoing mechanism, we have dealt in the preceding sections with the field having the largest scale, i.e., the scale of the perturbation can be comparable with the dimension of the object, so that the perturbation does not represent waves. In the particular case it can be a wave, but one that "feels" (via the growth rate) the collision frequency, and at any rate $\gamma < \omega$.

G. Solid-state plasma. General remarks

It is well known that a solid-state plasma has many properties of an ordinary plasma. For this reason alone, one can expect analogous magnetic instabilities to appear in it. For the process to be effective, it is important to have a sufficiently intense current, and we therefore confine our analysis to metals, because of the large number of carriers they contain. Usually one considers the properties of a solid in a homogeneous magnetic field. It turns out if an inhomogeneous field is involved, then various possibilities of its modification appear. In the examples considered above, heat flow will inevitably be produced in the solid. In a homogeneous magnetic field, various thermal effects and currents are produced, and in an inhomogeneous field on the other hand modification is possible. This phenomenon recalls the same thermal effect, but with feedback. If the Seebeck effect is realized as a result of heat flow and the difference between the thermoelectric powers of two metals (inhomogeneity of the thermoelectric power), then we have here the same heat flow and inhomogeneity of the magnetic field. The feedback consists of the fact that the produced current strengthens the field, meaning also the inhomogeneity, which in turn leads to a strengthening of the field. This is precisely how the instability sets in.

Mention should be made here of the long-known electroacoustomagnetic effect. Assume that acoustic oscillations are excited in a piezoelectric semiconductor by an external electric field: the drift velocity of the electrons should for this purpose exceed the speed of sound. If the directivity pattern of the phonon emission inside the Čerenkov cone is asymmetrical with respect to the direction of the carrier drift (this being connected with the piezoelectric properties of the crystal), then the acoustic force causes a solenoidel current and a magnetic moment in the sample.[108]

H. Thermal dynamo

The effect indicated above manifests itself most clearly in the simplest case: there are no phonons and there is heat flow. In this case the coupling between the electric field and the flows of the heat and of the electrons is given in the general case in the form

$$\mathbf{E}=\frac{\mathbf{j}}{\sigma}+R[\mathbf{H}\times\mathbf{j}]+q\nabla T+[\boldsymbol{\omega}^{(e)}\nabla T]\frac{n_1}{\nu}+\boldsymbol{\omega}^{(e)}(\boldsymbol{\omega}^{(e)}\nabla T)\frac{n_2}{\nu^2} \qquad (19)$$

(see, e.g.,[109]), where j is the current and ∇T is the temperature gradient; we have written out the kinetic coefficients in such a way that q, n_1, and n_2 have the same dimensionality. In this section we neglect the fifth term which is small in comparison with the fourth if $\omega^{(e)} \ll \nu$, as will indeed be assumed. We take the curl of (19), change over to the equation for H with the aid of Maxwell's equations, and then obtain

$$\frac{\partial \mathbf{H}}{\partial t}=-\operatorname{rot}[\mathbf{H}\times\nabla\Phi]+\nu_m\nabla^2\mathbf{H},\ \nabla\Phi=\nabla T\frac{en_1}{m\nu}; \qquad (20)$$

we have neglected here the Hall current (this is possible, since $\omega^{(e)} \ll \nu$), and in addition, to exclude the Seebeck effect, q is assumed to be constant: no ordinary heat fluxes are produced. The similarity between (20) and (1) is obvious. The magnetic field is assumed to be weakly inhomogeneous, or more concretely, Eq. (19) is valid in any case if $\Lambda \ll L$, where L is the inhomogeneity scale. In addition, in order for (19) and (20) to be valid it is necessary that the frequency ω of the process be much less than $\omega^{(e)}$ or ν. The boundary conditions are different here from those used when thermomagnetic phenomena are investigated. In this case it is important that the field not be maintained by external sources, i.e., there are no currents normal to the boundary of the body, and a potential electric field that does not influence the magnetic field is produced by the presence of the surface charges.

According to[12], motion in an unlimited medium is incapable of generating a field, and can produce only an "antidynamo"—a rapid annihilation of the field. In this case the "motion" $\nabla\Phi$ is potential, but in the limited problem generation is possible. To verify this, we turn to Fig. 4. Now, of course, the cylinders do not rotate, but heat is made to flow through the internal cylinder, i.e., one end of the cylinder is maintained at a temperature T_1 and the other at T_2. In the internal cylinder $\nabla\Phi$ is constant, and (20) is an equation with constant coefficient. In the external cylinder $\nabla T = 0$. The gap is an insulator (vacuum), so that neither heat nor current penetrate from the internal cylinder into the external one. Only an electromagnetic field is produced. We can now solve the eigenfunction problem $\mathbf{H}=\mathbf{f}(r)\times\exp(Et+im\varphi+ikz)$, and the solution behaves differently in three regions: in the internal cylinder the magnetic inhomogeneities are dragged by the heat flux, in the external cylinder the currents are induced by the fields that penetrate from the internal cylinder, and finally in the gap between the cylinders and outside the cylinders the field is current-free and is described by a harmonic function. The joining together of the solutions gives the dispersion equation for E. Fortunately, the system of equations for the joining coincides fully with the corresponding system that arises when the cylinders rotate, except that $m\omega$ must be replaced by $k|\nabla\Phi|$. Therefore the result is obtained without calculations. The analog of the magnetic Reynolds number is $R_T = |\nabla\Phi| r_0/\nu_m$, and if $R_T > 1$ the fluctuations of the field are "frozen in" in the heat flux. By analogy with Sec. (A) of Ch. 2, we have $\beta r_0 \approx kr_0 \approx N$. The instability sets in at

$$R_T > N, \qquad (21)$$

and the instability growth rate is

$$\gamma=\frac{\nu_m N^2}{r_0^2}. \qquad (22)$$

We discuss now the extent to which the instability conditions written out above are realistic. For estimates we assume $n_1 = q = \pi^2 T/3eE_F$, where E_F is the Fermi energy (see[110]). Equation (21) can then be rewritten in the form

$$\frac{\pi^2}{3}\frac{T}{E_F}\frac{\omega_p^2}{\nu^2}\frac{r_0\nabla T}{mc^2} > N; \qquad (23)$$

here T is the average temperature $(T_1+T_2)/2$. We consider by way of example a copper conductor. Assuming by way of estimate $r_0\nabla T \sim T$, $\nu = 4.2\cdot10^{13}$ sec^{-1} at room temperature T_r, and $n = 8\times10^{22}$ cm^{-3}, we find that the criterion (23) is not satisfied at $T = T_r$. We assume that below the Debye temperature θ we have $\nu \sim T^5$; we see that (23) contains a very strong dependence on T, so that to satisfy the criterion (23) it suffices to lower the temperature somewhat, actually to $T = 87\,^\circ$K at $N = 26$.

The unstable harmonic is quite complicated in form: it can be neither axisymmetric nor two-dimensional, in order not to contradict the theorems that forbid the dynamo. It is clear only that the entire field will be concentrated in a skin layer of thickness $\sim 1/k$ on the surface of the internal cylinder and in the internal surface of the external cylinder. Saturation of the instability sets in when $\omega^{(e)}$ is no longer small in comparison with ν (n_1 begins to depend on $\omega^{(e)}$, and in addition the heat flux can no longer be regarded as specified, since the magnetic field will affect the flux). The estimate $\omega^{(e)} = \nu$ ($T = 87\,^\circ$K) yields $H = 4\times10^3$ G.

From the solution of the dispersion equation we see that not only $\gamma = \mathrm{Re}E \neq 0$, but also $\mathrm{Im}E \neq 0$, and consequently drift of the perturbations takes place. Analysis shows that they drift with a velocity $\nabla\Phi$ along the axis of the cylinders. Therefore, if we solve the dispersion equation with respect to k, assuming ω to be real, we can obtain a spatial amplification of the perturbations. An incident electromagnetic wave with frequency $\omega < \gamma$ will be transformed into an unstable harmonic and become amplified along the axes of the cylinders.

We note that the model in Fig. 4 can be realized not only in the laboratory. In fact, the gap between the cylinders can be filled with an insulator having a poor thermoconductivity and nothing changes. Now, however, this model imitates a continuous medium. This raises the interesting question: is it possible to excite a field in the solid mantles of planets (Mercury, moon, earth) as a result of heat flux and inhomogeneous electric conductivity and thermal conductivity.

I. Instability against the background of a uniform field

In the case of a simple homogeneous conductor, the excitation described in Sec. (H) is impossible, and it is necessary to take into account quantities of higher order of smallness in the parameter $\omega^{(e)}/\nu$. We shall therefore neglect the fifth term of the right-hand side of (19). Now we obtain in place of (20) an equation of type (16), with the pseudoscalar $\alpha \sim (\mathbf{H}\cdot\nabla T)$. The exact value of α is $\alpha = -en_2(\omega^{(e)}\nabla T)/m\nu^2$. Since α depends on the field, the second formulation of the problem in Sec. 2 is appropriate: we consider the instability against the background of a uniform field; in particular, the field can be parallel to ∇T. Satisfaction of the criterion (17) is possible in principle if $\omega^{(e)}$ is not much smaller than ν. In this case, if it is assumed that $n_2 = n_1 = q$, then it is easy to verify that this is the same criterion (23), with $N=1$, which, as stated above, is easily realized. The term $\mathrm{curl}[\mathbf{H}\times\nabla\Phi]$ will be present in the equation for the field and cause the perturbation to drift with velocity $\nabla\Phi$.

The models considered in Secs. (H) and (I) are none other than direct conversion of thermal energy into electromagnetic energy. For estimates we assume that $\omega^{(e)} \approx \nu$. We fix ω in the dispersion equation, and then we have spatial amplification. To realize the amplification indicated in Sec. (H), we must choose specially the conditions of the inhomogeneity of the sample, whereas in Sec. (I) the sample can be homogeneous, but on the other hand it is necessary to apply to it an external field such that $\omega^{(e)} \approx \nu$. According to (22), $\gamma \approx T^{-11}$ (we have replaced the inequality (21) by the approximate equality), therefore at low temperatures the condition $\omega < \gamma$ actually does not impose any limitations, and in contrast to the ordinary thermal effect in this case there is excited an *alternating current* with frequency ω. The power released in a unit volume is $P = \gamma H^2/8\pi \sim T^{-1}$. The possibility of heat removal also improves with decreasing temperature: the required flux through a unit area of the lateral surface of the external cylinder is $p/k \sim T^{-7}$. It is convenient to use for the cooling a stream of low-temperature liquid flowing in the gap between the cylinders—it is the metal layers of thickness $1/k$ adjacent to the gap which are heated. By way of example we consider copper at $T=\theta/5\approx 60\ ^\circ\mathrm{K}$, $r_0 = 10$ cm, and then $\gamma = 61\ \mathrm{sec}^{-1}$, $1/k = 2\times10^{-2}$ cm, and $\Lambda = 6\times10^{-3}$, so that the weak-inhomogeneity condition $L = 1/k > \Lambda$ is satisfied. The weak-nonstationarity condition $\omega < \gamma < \omega^{(e)}$ is satisfied already at $H > 3\times10^{-6}$ G. The collision frequency at this temperature is $\nu = 10^{10}$ sec^{-1} and $\gamma \ll \nu$, i.e., the process is relatively slow. The heat flux necessary for heat removal at these parameters is $p/k = 10^{-3}\ \mathrm{W/cm^2}$.

4. HIGH FREQUENCIES

A. Helicon frequencies

Helicons correspond to frequencies between ω_i and $\omega^{(e)}$ and constitute strongly gyrotropic oscillations. In fact, this is a helix in pure form. Therefore it might seem that helicons should serve as good generators of a large-scale field, by producing the α effect. Calculation shows, however, that they are capable only to lead to anomalous diffusion of the field (in exactly the same manner as magnetohydrodynamic waves; see Sec. (D) of Ch. 2).

In the presence of helicon sources, for example instabilities that cause their buildup, the helicons interact with one another or with other plasma oscillations. An equilibrium spectrum can be established in this case—a distribution of energy over the frequencies. There exists no universal oscillation spectrum, it depends on the various conditions of the plasma. Spectra in a collisionless plasma were first obtained by Lifshitz and Tsytovich[111] and were described in detail in the monograph[112].

In a collision-dominated plasma and in the absence of other oscillations, helicons interact with one another. It is simplest to start from Eq. (14): at $\omega^{(e)} \ll \nu$, the Hall term is not small and the equation

$$\frac{\partial \mathbf{H}}{\partial t} = -\frac{c}{4\pi n e}\,\mathrm{rot}\,[\mathbf{H}\times\mathrm{rot}\,\mathbf{H}] \tag{24}$$

is an exact nonlinear equation for the magnetic fluctuations at helicon frequencies. Linearization of Eq. (24) leads to a dispersion equation for the helicons $\omega = \pm|\cos\theta|\,\omega^{(e)}c^2k^2/\omega_p^2$, $\cos\theta = (\mathbf{k}\cdot\mathbf{H}_0)/kH_0$. There are two possibilities here: weak or strong turbulence. The weak turbulence can be defined in this case as follows: the fluctuations of the field $\mathbf{h}$ are much smaller than those of the homogeneous $\mathbf{H}_0$. It is possible to verify directly that the exact equation (24) conserves the energy $\int \mathbf{H}^2 d^3r$, or, in the case of homogeneous turbulence, $\langle \mathbf{H}^2\rangle$. Using the weak-coupling approximation (i.e., expressing the fourth moments in terms of the second moment with the aid of the random-phase approximation or, equivalently, with the aid of the Millionshchikov hypothesis), we can prove the presence of energy transfer into the region of larger wave numbers, due to the nonlinear interaction of the helicons.[113]

We can now formulate the problem in the same manner as when the Kolmogorov spectrum is determined in hydrodynamics. Let the helicons be excited at low frequencies $\omega < \omega_i$ and let them be attenuated as a result of ohmic losses at high frequencies. In the region $\omega_i < \omega < \omega^{(e)}$, a universal spectrum is established. Conservation of the energy flux in the region of large wave numbers yields $h^2/\tau = \mathrm{const}$, where τ is the helicon lifetime. If the turbulence is weak, then, just as in Ch. 2 [Sec. (5)], $1/\tau = \omega h^2/H_0^2$, and since $\omega \sim k^2$, we have $h^2 \sim k^{-1}$ and

$$E \sim k^3. \tag{25}$$

This spectrum was obtained in[114]. Since $1/\tau \sim k$ and the ohmic damping is $\sim \nu_m k^2$, the spectrum is ultimately cut off at large k by the damping at $h^2 = H_0^2\,\nu/\omega^{(e)}$. On the other hand, if the turbulence is strong, then the quantity $1/\tau$ must be estimated directly from the exact equation (24): $1/\tau = ck^2h/4\pi ne$. From the condition that the flux be stationary we obtain $k^2hh^2 = \mathrm{const}$

$$E \sim k^{-7/3}, \tag{26}$$

i. e., a steeper spectrum than for weak turbulence.[113] In this case, too, $1/\tau \sim k^{4/3}$ increases more slowly than $\nu_m k^2$. Using the complete equation (14), we easily understand that this spectrum is cut off at $eh/mc = \nu$.

Which of the spectra is actually established in a real situation of strong turbulence, when h at frequencies $\omega \approx \omega_i$ is not smaller than H_0 itself? In the region of the smallest k (defined by $\omega \gtrsim \omega_i$), the spectrum (26) is established. At large wave numbers, a deviation from locality of the turbulence is observed, since this region will interact also with the large scales. The large-scale fluctuations will play the role of the field H_0, and in the smaller scales the fluctuations are transformed into waves with the spectrum (25).

A large-scale weakly-inhomogeneous field (scale $L \gg 1/k$) will become modified in the presence of turbulent helicons. In fact, transfer into the region of large k acts also on the components with the largest scale. Just as in hydrodynamics, turbulent viscosity destroys the large-scale flow, and turbulent helicons cause dissipation of a field of scale L within a time L^2/χ, where the turbulent viscosity is $\chi = \sqrt{\langle h^2 \rangle}\, c/4\pi ne$. It is easy to verify that $\chi/\nu_m = e\sqrt{\langle h^2 \rangle}/mc\nu$, and since $\sqrt{\langle h^2 \rangle} \approx H_0$, we have $\chi/\nu_m = \omega^{(e)}/\nu$, ($\chi/\nu_m$ is the analog of the Reynolds number).

B. Langmuir oscillations

Excitation of large-scale fields by Langmuir oscillations was first considered by Tsytovich.[115] If Langmuir oscillations are excited in a plasma, then there is an instability of the type of an aperiodically growing second sound. Transverse waves are excited, for which $H = \varepsilon^t E$, where ε^t is the turbulent dielectric constant. Since $\varepsilon^t \gg 1$, consequently $H \gg E$, i. e., the principal energy of the oscillations is magnetic. Under definite conditions, the scales of the magnetic perturbations may turn out to be not small. This mechanism is described in detail in the monograph[116].

We proceed now to a collision-dominated plasma. The high-frequency oscillations of the electrons in the magnetic field can be obtained from the equation of motion

$$-i\omega\mathbf{v} = ie\varphi\mathbf{k}/m + [\mathbf{v}\times\omega^{(e)}] \quad -k^2\varphi = 4\pi n' e, \tag{27}$$

where $\omega n' = n_0(\mathbf{k}\cdot\mathbf{v})$. Solving (27) with respect to $\mathbf{v}$, we obtain

$$\mathbf{v} = \left\{\omega\mathbf{k} - \frac{(\boldsymbol{\omega}^{(e)}\mathbf{k})\,\boldsymbol{\omega}^{(e)}}{\omega\ !} + i\,[\mathbf{k}\omega^{(e)}]\right\} i[\mathbf{k}\times\omega^{(e)}]\ \psi,$$
$$\psi = -\frac{e}{m}\frac{\varphi}{\omega^2 - \omega^{(e)2}}. \tag{28}$$

Obviously, (28) is a particular case of (13). The magnetic field produces weak gyrotropy of the oscillations at $\omega^{(e)} \ll \omega_p$, as will be assumed from now on. In fact, $\langle \mathbf{v}\,\mathrm{curl}\,\mathbf{v}\rangle \sim (\mathbf{k}\cdot\omega^{(e)})$. Equation (27) is valid if the produced nonpotential electric field E_b is much less than $i\mathbf{k}\varphi$. The estimate of E_b is obtained in the same manner as for ion sound. We assume that $ck \gg \omega_p$, and then the displacement current in (18) can be neglected, the fluctuations of the magnetic field are

$$h \sim \frac{4\pi}{c}\,\frac{n_0 e[\mathbf{k}\times\mathbf{v}]}{k^2}\left(\frac{\omega^{(e)}}{\omega}\right)^2, \quad E_b = \left(\frac{\omega^{(e)}}{\omega_p}\right)^a \frac{\omega_p}{k^2c^2}\,|\mathbf{k}\varphi|,$$

so that for $k \gg \omega_p/c$ (this condition coincides with the condition that the displacement current be small) we can disregard E_b.

Calculation of the nonlinear current $\langle n' e\mathbf{v}\rangle$ leads to the expression

$$\mathbf{j}'' = \langle n' e\mathbf{v}'\rangle = n_0 e\boldsymbol{\omega}^{(e)}\int(\boldsymbol{\omega}^{(e)}\mathbf{k})\,(k^2/\omega)\,\Phi(\mathbf{k},\omega)\,d^3k\,d\omega,$$
$$\Phi(\mathbf{k},\omega) = \Phi_1(\mathbf{k})\,\delta(\omega-\omega_p) + \Phi_1(-\mathbf{k})\,\delta(\omega+\omega_p). \tag{29}$$

$\Phi_1(\mathbf{k}, \omega)$ is the spectral function of the oscillations. The main contribution to the integral in (29) is made by the large wave vectors, so that for estimates we can assume $k = k_j$, and k_d is the Debye wave vector. Of course, this is true if the oscillation spectrum does not decrease too steeply with large k, as is indeed assumed by us. On the other hand, for isotropic oscillations $\Phi = (|\mathbf{k}|, \omega)$, and the integral (29) vanishes. The anisotropy is ensured in the presence of two-stream instability; oscillations are excited with $(\mathbf{k}\cdot\varkappa) > 0$, where $\varkappa$ is a unit vector in the direction of the chosen phase velocity; if the oscillations are excited by a beam, then $\varkappa$ is parallel to the beam. The induced scattering of plasmons by particles causes the latter to become isotropic, but the chosen direction $\varkappa$ still remains. Estimating (29) with allowance for the foregoing, we obtain $\mathbf{j}'' = -(\varkappa\cdot\mathbf{H})\times Hev_T\beta/2mc^2$.

Substitution of the nonlinear currents in Maxwell's equations yields[117]

$$\frac{\partial\mathbf{H}}{\partial t} = \nu_m\nabla^2\mathbf{H} - \frac{c}{4\pi n_0 e}\,\mathrm{rot}\,[\mathbf{H}\times\mathrm{rot}\,\mathbf{H}] - \mathrm{rot}\,a\mathbf{H} + \mathrm{rot}\,b\,[\varkappa\times\mathbf{H}],$$
$$\alpha = \frac{(\varkappa\omega^{(e)})\,v_T\beta}{2\sigma}, \quad b = 2\pi\beta v_T. \tag{30}$$

Thus, we arrive again at the α effect. The criterion of the instability and the growth rate are given by (17) with (30) taken into account. If a homogeneous field is applied to the plasma, then the question of the stability is solved by linearizing Eq. (30). In the "classical" formulation of the dynamo theory, the initial weakly-inhomogeneous field is amplified to observable values. The initial intensity of the field itself is of no importance—it can be infinitesimally small. The situation here is different. The equation is essentially nonlinear: α depends on the field, and therefore the criterion (17) can be regarded as a condition on the initial field. Fields that are too weak are not excited. Another new feature connected with the nonlinearity is the non-exponential amplification. In fact, the model equation $\partial y/\partial t = y^2$, which reflects the fact that the α term is quadratic in the field, has a solution $y = y_0(1 - Et)^{-1}$ (see Fig. 11), which has a faster-than-exponential growth. The singularity in $1/E$ raises no difficulty: the growth of the field stops at $\omega^{(e)} \approx \omega_p$.

It is of interest to compare the emf field produced by the Langmuir oscillations with the result of the Weinreich formula.[118] The latter reflects the connection between the nonlinear current and the damping of the wave. Obviously, such a connection should exist, since the momentum transferred by the electrons and causing

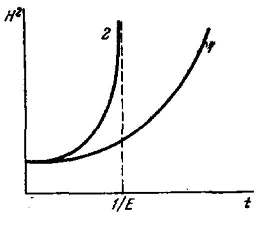


FIG. 11. Linear (1) and nonlinear (2) α effect at identical initial data. Solution 2 has a singularity at $t = 1/E$.

the emf is taken from the wave, and as a result the wave attenuates. The Weinreich relation contains the wave energy flux. In our case it has no meaning, inasmuch as in the dispersion equation for the plasmons we have neglected the thermal corrections, and there is no flux (the group velocity vanishes). We derive the Weinreich relation for this case. We multiply the density of the number of quanta $w/\hbar\omega$ (w is the oscillation energy density) by hk and obtain the momentum density wk/ω. Further, in view of the fact that the damping decrement of the plasmons, due to the collisions, is close to ν, the volume force acting on the electrons is given by $n_0\mathbf{F} = \omega\mathbf{k}\nu/\omega$. This force is potential, and to take into account deviations from potentiality it is necessary to multiply by $(\omega^{(e)}/\omega_p)^2$, which is a measure of the nonpotentiality of the oscillations. Consequently, $\mathbf{E} = \mathbf{F}/e = (w\mathbf{k}\nu/n_0\omega e)(\omega^{(e)}/\omega_p)^2$. This is the analog of the Weinreich formula. On the other hand, according to (30), we have $E = \alpha H/c$. It is easily seen that these expressions simply coincide at $k = k_a$.

C. Applications

The criterion of the instability and the increment in this problem are given by

$$2\pi\beta L\omega^{(e)}v_T > c^2, \quad \gamma = \frac{\omega^{(e)}v_T\beta}{2\sigma L}. \tag{31}$$

It is difficult to realize the criterion (31) in experiment at small β. Thus, if $\beta = 10^{-4}$, then it follows from the criterion (31) that $HL > 7.5\times10^7$ at $v_T = 10^9$ cm/sec (H is in gausses and L is in centimeters). This is a rather stringent condition. The instability is therefore realized at larger β.

We proceed to discuss astrophysical examples. We consider a relatively high-temperature plasma. For example, for solar-flare conditions we have $v_T = 10^8$ cm/sec and $\beta = 10^{-4}$, so that $HL > 7.5\times10^8$. At $H = 300$ G we have $L > 25$ km; the critical dimension 25 km, of course, is very small in comparison with the dimensions of the flare region. It is assumed that in flares σ is determined by the resonant interactions, or in other words, by collisions of electrons with ion-sound oscillations. If this is so, then it is necessary to substitute the turbulent electric conductivity in expression (31) for γ. The resultant fluctuations of the field can serve as a source of additional heating of the gas, since their scale is quite small. The nonlinearity of Eq. (30) leads to an ambiguity of the solution and to different possibilities. Thus, it is possible to have not only a rapid growth of the field, but also a rapid damping of the field (in comparison with the usual diffusion damping), as well as a modification of a different kind (see Sec. (C) of Ch. 3).

Maksimov[119] has considered an application of the given mechanism for the high-latitude ionosphere, where the beams definitely exist and are observed. If $\beta = (2/3)n_b v_b^2/n_0 v_T^2$, where n_b and v_b are the concentration and velocity of the beam, then the critical scale is $L_k = 100$ km at $n_b = 1$ cm^{-3}. The perturbations develop again against the background of the main magnetic field of the earth, constitute magnetohydrodynamic waves, and are registered in the form of definite variations.

The indicated mechanism permits a choice between two interpretations of the linearly polarized x rays from the flare loops on the sun.[120] In one of them the polarization is attributed to bremsstrahlung of anisotropic beams.[121–123] In the other it is attributed to Thomson scattering of the radiation in the photosphere. In the latter model, the distribution of the electrons in the beam is isotropic.[124] In the first interpretation, the mechanism proposed above is operative, while in the second it is not (the anisotropy of the beam and of the oscillations is of importance). If the mechanism is operative, then the excited fluctuations can be in principle registered, and this will make the choice possible.

Tomozov[125] has considered the action of the plasma mechanisms of the field enhancement in quasars and in active galactic cores. The presence of magnetic fields in the indicated objects is necessary to interpret their flare activity, as indicated by Shklovskiĭ.[126] The excitation of the field in such large scales calls for a long time, so that it is necessary to invoke turbulent electric conductivity due to ion-sound oscillations, thus greatly decreasing the characteristic time of the amplification. Estimates show that this mechanism is capable of producing a field on the order of several hundred gauss over reasonable times.

5. CONCLUSION

The "old" dynamo theory has a sufficient number of applications. At the same time, new ideas and mechanisms exist to a large degree only in the theory, and here the theory has overtaken the interpretation of the observations. Of course, this situation is not satisfactory. The author would consider the task of the present review to be fulfilled to a considerable degree if it were able to suggest a large number of applications of these ideas in astrophysics, geophysics, and in experiments.

I am grateful to Professor U. Frisch and Professor A. Pouquet for gracious explanations concerning their work and for preprints, to Professor P. H. Roberts, Professor A. M. Soward, and Professor J. A. Jacobs for preprints, and to Dr. K. H. Rädler for interest in the work.

1 S. I. Vaĭnshteĭn and Ya. B. Zel'dovich, Usp. Fiz. Nauk 103, 431 (1972) [Sov. Phys. Usp. 14, 104 (1972)].
2 M. Stix, Astron. and Astrophys. 37, 121 (1974).
3 D. Gubbins, Rev. Geophys. and Space Phys. 12, 137 (1974).

[4]A. M. Soward and P. H. Roberts, Magnit. gidrodin. No. 1, 3 (1976).
[5]K. H. Rädler, Mem. Soc. Roy. Sci. Liège, 6e sér., 8, 109 (1975).
[6]A. Gaĭlitis, Magnit. gidrodin. No. 6, 19 (1970).
[7]A. Gaĭlitis, *ibid.* No. 4, 12 (1973).
[8]A. Gaĭlitis and Ya. Freĭnberg, *ibid.* No. 1, 37 (1974).
[9]D. Lortz, Plasma Phys. **10**, 967 (1968).
[10]G. O. Roberts, Phil. Trans. Roy. Soc. **A271**, 411 (1972).
[11]Yu. B. Ponomarenko, Zh. Prikl. Mekh. Tekh. Fiz. No. 6, 47 (1973).
[12]S. I. Vaĭnshteĭn, Zh. Eksp. Teor. Fiz. **65**, 550 (1973) [Sov. Phys. JETP **38**, 270 (1974)].
[13]S. I. Vaĭnshteĭn, Zh. Eksp. Teor. Fiz. **68**, 997 (1975) [Sov. Phys. JETP **41**, 494 (1975)].
[14]J. Leorat, U. Frisch, and A. Pouquet, Rept. at the NORDITA Meeting, Copenhagen, June 1974.
[15]U. Frisch, A. Pouquet, J. Leorat, and A. Mazure, J. Fluid Mech. **68**, 769 (1975).
[16]A. Pouquet and G. S. Patterson, Jr., Numerical simulation of helical magnetohydrodynamic turbulence, Rept. of National Center for Atmos. Res., Boulder, Colorado, USA, March 1976.
[17]T. G. Cowling, Mon. Not. RAS **94**, 39 (1933).
[18]S. I. Braginskiĭ, Zh. Eksp. Teor. Fiz. **47**, 1084 (1964) [Sov. Phys. JETP **20**, 726 (1965)].
[19]Ya. B. Zel'dovich, Zh. Eksp. Teor. Fiz. **31**, 154 (1956) [Sov. Phys. JETP **4**, 460 (1957)].
[20]A. Herzenberg, Phil. Trans. Roy. Soc. **A250**, 543 (1958).
[21]A. M. Soward, J. Math. Phys. **12**, 1900 (1971).
[22]A. M. Soward, *ibid.* **12**, 2052 (1971).
[23]P. H. Roberts and A. M. Soward, Ann. Rev. Fluid Mech. **4**, 117 (1972).
[24]A. M. Soward, Phil. Trans. Roy. Soc. A **272**, 431 (1972).
[25]P. H. Roberts, *ibid.*, p. 663.
[26]P. H. Roberts and K. Stewartson, *ibid.* **277**, 287 (1974).
[27]S. Kumar and P. H. Roberts, Proc. Roy. Soc. **A344**, 235 (1975).
[28]S. I. Braginskiĭ, Geomagnet. i Aéronom. **15**, 149 (1975).
[29]E. P. Kropachev and S. N. Geoshkov, *ibid.*, 709.
[30]E. N. Parker, Astrophys. J. **122**, 293 (1955).
[31]Kato Shoji, *ibid.* **157**, 827 (1969).
[32]P. A. Gilman, Solar Phys. 8, 316 (1969).
[33]P. A. Gilman, *ibid.* 9, 3.
[34]Kato Shoji and Y. Nakagawa, *ibid.* **10**, 476.
[35]H. K. Moffatt, J. Fluid Mech. **44**, 705 (1970).
[36]S. I. Braginskiĭ, Geomagnet. i Aéronom. **7**, 851 (1967).
[37]S. Childress and A. M. Soward, Phys. Rev. Lett. **29**, 837 (1972).
[38]A. M. Soward, Phil. Trans. Roy. Soc. **275**, 611 (1974).
[39]P. H. Roberts, Rept. at the 16th General Assembly of I.U.G.G., Grenoble, August–September 1975.
[40]P. H. Roberts and S. Stewartson, J. Fluid. Mech. **68**, 447 (1975).
[41]A. M. Soward, J. Fluid Mech. **69**, 145 (1975).
[42]A. E. Dolginov, Pis'ma Zh. Eksp. Teor. Fiz. **18**, 67 (1973) [JETP Lett. **18**, 38 (1973)].
[43]A. E. Dolginov, Astron. Zh. **51**, 388 (1974) [Sov. Astron. **18**, 225 (1974).
[44]M. G. Rochester, J. A. Jacobs, D. E. Smylie, and K. F. Chong, Geophys. J. R. artr. Soc. **43** (1975); Rept. at the 16th General Assembly of I.U.G.G., Grenoble, August–September 1975.
[45]B. A. Tverskoĭ, Geomagnet. i. Aéronom. 6, 11 (1966).
[46]A. Z. Dolginov, Izv. Akad. Nauk SSSR Ser. Fiz. **39**, 354 (1975).
[47]N. O. Weiss, Proc. Roy. Soc. **A293**, 310 (1966).
[48]M. V. Keldysh, Dokl. Akad. Nauk. SSSR **77**, 11 (1951).
[49]E. N. Parker, J. Plasma Phys. **9**, 49 (1973).
[50]S. I. Vaĭnshteĭn and V. M. Tomozov, Gerlands Beitr. Geophys. (Leipz.) 85, 269 (1976); in: Isslodovaniya po geomagnetizmu, aeronomii i fizike Solntsa (Research on Geomagnetism, Aeronomy, and Solar Physics), No. 37, Nauka, 1975, p. 100.
[51]S. I. Vaĭnshteĭn and V. M. Tomozov, Paper at Symposium on "Problems of Magnetic Fields in Outer Space," Crimean Astron. Observ., April, 1976.
[52]E. M. Drobyshevski and V. S. Yuferev, J. Fluid Mech. **65**, 33 (1974).
[53]J. H. Piddington, Cosm. Electr. **3**, 60, 129 (1972).
[54]J. H. Piddington, Astrophys. and Space Sci. **37**, 183 (1975).
[55]M. Steenbeck and F. Krause, Zs. Naturforsch. **21a**, 1285 (1966); Astron. Nachr. **291**, 49 (1969).
[56]K.-H. Rädler, Mon. Ber. dt. Akad. Wiss, Berlin **11**, 194 (1969).
[57]F. Krause, Rept. at the 71th Symposium of I.A.U., Praha, August 1975.
[58]S. I. Vaĭnshteĭn, Zh. Eksp. Teor. Fiz. **58**, 153 (1970) [Sov. Phys. JETP **31**, 87 (1970)].
[59]I. Lerche, J. Math. Phys. **12**, 1858 (1971).
[60]I. Lerche, J. Math. Phys. **14**, 1381, 1579, 1588 (1973).
[61]P. H. Roberts and A. M. Soward, Astron. Nachr. **296**, 49 (1975).
[62]P. H. Roberts and A. M. Soward, J. Math. Phys. **16**, 609 (1975).
[63]S. B. Pikel'ner, Osnovy kosmicheskoĭ elektrodinamiki (Principles of Cosmic Electrodynamics), 2nd ed., Nauka, 1966, Ch. 5.
[64]V. I. Tatarskiĭ, Rasprostranenie voln v turbulentnoĭ atmosfere (Wave Propagation in the Turbulent Atmosphere), Nauka, 1967, Ch. 2 and 5.
[65]S. I. Vaĭnshteĭn, Zh. Eksp. Teor. Fiz. **62**, 1376 (1972) [Sov. Phys. JETP **35**, 725 (1972)].
[66]N. O. Weiss, Phil. Trans. Roy. Soc. **A256**, 99 (1964).
[67]K.-H. Rädler, Zs. Naturforsch. **23a**, 1851 (1968).
[68]S. I. Vaĭnshteĭn, Zh. Prikl. Mekh. Tekh. Fiz. No. 1, 12 (1971).
[69]N. N. Noskov, Magnit. gidrodin. No. 3, 41 (1973).
[70]T. S. Ivanova and A. A. Ruzmaĭkin, Preprint No. 43, Inst. of Appl. Mech. USSR Acad. Sci., Moscow, 1975.
[71]S. A. Orszag, J. Fluid Mech. **41**, 363 (1970).
[72]S. Nagarajan, in: Solar Mag. Fields, Ed. Howard IAU, Dordrecht, D. Reidel, 1971, p. 487.
[73]U. Frisch, J. Leorat, and P. Pouquet, Rept. at the 16th General Assembly of I.U.G.G., Grenoble, August–September 1975.
[74]A. Pouquet, U. Frisch, and J. Leorat, J. Fluid Mech. (1976) (in press).
[75]V. I. Klyatskin and V. I. Tatarskiĭ, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **14**, 1400 (1971).
[76]V. I. Klyatskin and V. I. Tatarskiĭ, *ibid.* **15**, 1433 (1972).
[77]A. Gailitis, Magnit. gidrodin. No. 2, 31 (1974).
[78]V. P. Starr, Physics of Negative Viscosity Phonomena, McGraw, 1968.
[79]S. I. Vaĭnshteĭn and A. A. Ruzmaĭkin, Astron. Zh. **50**, 12 (1973) [Sov. Astron. **17**, 7 (1973)].
[80]R. H. Kraichnan, Phys. Fluids **10**, 1417 (1967).
[81]R. H. Kraichnan, *ibid.* **59**, 745 (1973).
[82]T. D. Lee, Quart. Appl. Math. **10**, 69 (1952).
[83]L. D. Landau and E. M. Lifshitz, Statisticheskaya fizika (Statistical Physics), 2nd revised ed., Nauka, 1964, Ch. 3 [Pergamon, 1971].
[84]J. Leorat, Thèse (Observation de Paris, Meudon, 1975).
[85]K. H. Moffatt, J. Fluid Mech. **53**, 385 (1972).
[86]K. H. Moffaitt, *ibid.* **57**, 625 (1973).
[87]A. E. Dolginov, A. D. Kaminker, and Yu. A. Shibanov, Zh. Eksp. Teor. Fiz. 68, 69 (1975) [Sov. Phys. JETP **41**, 35 (1975)].
[88]T. S. Ivanova and A. A. Ruzmaĭkin, Preprint No. 47, Inst. Appl. Mech. USSR Acad. Sci., Moscow, 1976.
[89]N. N. Noskov, Geomagnet. i. Aéronom. **13**, 347 (1973).
[90]S. I. Vaĭnshteĭn and L. L. Vaĭhshteĭn, *ibid.*, 149.

[91]S. D. Ivanov, Zh. Prikl. Mekh. Tekh. Fiz. No. 2, 16 (1976).
[92]Yu. B. Kolesnikov and A. B. Tsinober, Magnit. gidrodin. No. 3, 23 (1972); Izv. Akad. Nuak SSSR, ser. Mekhanika zhidkosti i gaza No. 4, 146 (1974).
[93]A. B. Tsinober, Magnit. gidrodin. No. 1, 7 (1975).
[94]F. Krause and G. Rädiger, Prep. Zentralinstitut für Astrophysik, Potsdam, 1975; Solar Phys. **42**, 107 (1975).
[95]K.-H. Rädler, Astron. Nachr. **295**, 265 (1974).
[96]S. I. Vaĭnshteĭn, Rept. at the 71th Symposium of IAU, Praha, August 1975.
[97]S. I. Vaĭnshteĭn, G. V. Kuklin, and V. P. Maksimov, Paper at Symposium on Problems of Magnetic Fields in Outer Space, Crimean Astron. Observ., April 1976.
[98]S. I. Vaĭnshteĭn, Zh. Tekh. Fiz. **45**, 1623 (1975) [Sov. Phys. Tech. Phys. **20**, 1034 (1976)].
[99]B. B. Kadomtsev, in: Voprosy teorii plazmy (Problems of Plasma Theory), No. 4, Atomizdat, 1964, p. 188.
[100]S. I. Vaĭnshteĭn, in: Abstracts of Second Internat. Conf. on Plasma Theory, Kiev, October 1974.
[101]S. I. Vaĭnshteĭn, Paper at International Symposium, Eisenach, January 1975; Gerland Beitr. Geophys. (Lpz.) 85, 93 (1976).
[102]A. A. Galeev and R. Z. Sagdeev, collection cited in[99], No. 7, 1973, p. 3.
[103]I. Lerche, Plasma Phys. **15**, 417 (1973).
[104]R. Z. Sagdeev, collection cited in[99], No. 4, 1964, p. 20.
[105]A. Rogister, J. Plasma Phys. **6**, 309 (1971).
[106]A. A. Galeev and L. M. Zedenyi, Zh. Eksp. Teor. Fiz. **69**, 882 (1975) [Sov. Phys. JETP **42**, 450 (1976)].
[107]A. B. Mikhaĭlovskiĭ and O. A. Pokhotelov, Fiz. Plazmy **1**, 786 (1975) [Sov. J. Plasma Phys. **1**, 430 (1975)].
[108]V. I. Pustovoĭt, Usp. Fiz. Nauk **108**, 596 (1972) [Sov. Phys. Usp. **15**, 832 (1973)].
[109]J. M. Ziman, Electrons and Phonons, Oxford, 1960, Chap. 12.
[110]A. A. Abrikosov, Vvedenie v teoriyu normal'nykh metallov (Introduction to the Theory of Normal Metals), Nauka, 1972, Ch. 6.
[111]M. A. Lifshitz and V. N. Tsytovich, Zh. Eksp. Teor. Fiz. **62**, 606 (1972) [Sov. Phys. JETP **35**, 321 (1972)].
[112]V. N. Tsytovich, Teoriya turbulentnoĭ plazmy (Theory of Turbulent Plasma), Atomizdat, 1971, Ch. 4.
[113]S. I. Vaĭnshteĭn, Zh. Eksp. Teor. Fiz. **64**, 139 (1973) [Sov. Phys. JETP **37**, 73 (1973)].
[114]V. M. Yakovenko, Zh. Eksp. Teor. Fiz. **57**, 574 (1969) [Sov. Phys. JETP **30**, 315 (1970)].
[115]V. N. Tsytovich, Dokl. Akad. Nauk SSSR **181**, 60 (1968) [Sov. Phys. Dokl. **13**, 672 (1969)].
[116]V. N. Tsytovich, Teoriya turbulentnoĭ plazmy (Theory of Turbulent Plasma), Atomizdat, 1971, Ch. 8.
[117]S. I. Vaĭnshteĭn, Zh. Eksp. Teor. Fiz. **67**, 517 (1974) [Sov. Phys. JETP **40**, 256 (1975)].
[118]G. Weinreich, T. M. Sanders, and H. G. White, Phys. Rev. **114**, 33 (1959).
[119]V. P. Maksimow, Gerlands Beitr. Geophys. (Leipz.) **85**, 103 (1976); Paper at Internat. Symposium, Eisenach, January 1975.
[120]V. P. Maksimov, Astron. Zh. **53**, 215 (1976) [Sov. Astron. **20**, 117 (1976).
[121]I. P. Tindo, S. L. Mandel'stam, and A. I. Shurygin, Solar Phys. **32**, 469 (1973).
[122]A. A. Korchak, Astron. Zh. **49**, 137 (1972) [Sov. Astron. **16**, 107 (1972)].
[123]G. Elwert and E. Haug, Solar Phys. **15**, 234 (1970).
[124]I. L. Beĭgman, Astron. Zh. **51**, 1017 (1974) [Sov. Astron. **18**, 600 (1975)].
[125]V. M. Tomozov, Astron. Tsirkulyar Akad. Nauk SSSR No. 918, 5 1976).
[126]I. S. Shklovskiĭ, Astron. Zh. **47**, 742 (1970) [Sov. Astron. **14**, 594 (1970)].

Translated by J. G. Adashko