H. H. Oiglane. *Symmetry Properties of the Lagrangians of the Four-Fermion Interaction.* The Lagrangian of the four-fermion interaction has the general form

$$\mathscr{L} = \sum_I [C_I \mathscr{L}_I + C'_I \mathscr{L}'_I], \tag{1}$$

where

$$\mathscr{L}_I = \sum_{i \in I} \langle \bar{\psi}_1 \Gamma_i \psi_2 \rangle \langle \bar{\psi}_3 \Gamma_i \psi_4 \rangle \tag{2}$$

and

$$\mathscr{L}'_I = \sum_{i \in I} \langle \bar{\psi}_1 \Gamma_i \psi_2 \rangle \langle \bar{\psi}_3 \Gamma_0 \Gamma_i \psi_4 \rangle \tag{3}$$

are the scalar and pseudoscalar parts of the Lagrangian, respectively. Here the Γ_i are the elements of the Dirac group in the four-row representation (Dirac matrix), with $\Gamma = \gamma^5$ the pseudoscalar matrix and $I = S$, P, V, A, T. The coupling constants are generally complex. To determine the form of the Lagrangian of the four-fermion interaction, it is therefore necessary to measure 19 real parameters in the experiment. Only six of them have been measured thus far. The values of the measured parameters in weak interaction processes agree very closely with the Lagrangian of the so-called V–A interaction:

$$\mathscr{L}_{(V-A)} = C \langle \bar{\psi}_1 (1-\gamma^5) \gamma^\alpha \psi_2 \rangle \langle \bar{\psi}_3 (1-\gamma^5) \gamma^\alpha \psi_4 \rangle, \tag{4}$$

but from the experimental standpoint we may not assume that the V–A interaction Lagrangian is the only one possible.

The behavior of the scalar Lagrangian $\mathscr{L}_I$ under the action of the Fierz transformation

$$\Phi \langle \bar{\psi}_1 \Gamma_i \psi_2 \rangle \langle \bar{\psi}_3 \Gamma_i \psi_4 \rangle = \langle \bar{\psi}_1 \Gamma_i \psi_4 \rangle \langle \bar{\psi}_3 \Gamma_i \psi_2 \rangle. \tag{5}$$

has been studied in many papers. It is found that three anti-invariant and two invariant combinations with respect to the Fierz transformation can be formed from the scalar Lagrangians $\mathscr{L}_I$.[1]

Let us consider this problem in the more general case of the mixed Lagrangian (1). We shall require that the mixed Lagrangian be a characteristic state of the operation

$$\Phi \mathscr{L} = \pm \mathscr{L}. \tag{6}$$

We obtained six additional relationships among the constants C_I from the requirement of the anti-invariance of $\mathscr{L}$; only four complex coefficients remain arbitrary. This invariant contradicts experiment accurate to the CP-noninvariant effects. From the requirement of invariance of $\mathscr{L}$, we obtain four additional relations, and six constants remain arbitrary. Reality of the coupling constants follows from the requirement of T-invariance. In this case, the only possible Lagrangian of the weak four-fermion interaction that agrees with the experimental data is the Lagrangian of the V–A interaction. It seems to us that the requirement of Fierz invariance is of interest only in quark interactions, although the direct experimental data here are very few in number.

It was shown in[2] that in the case of the scalar Lagrangian, all invariants and anti-invariants of the Fierz transformation can be expressed in the form

$$\sum_{i=1}^{16} \chi_i^{(f)} (\bar{\psi}_1 \Gamma_1 \psi_2)(\bar{\psi}_3 \Gamma_i \psi_4). \tag{7}$$

Here the $\chi_i^{(k)}$ are elements of one-dimensional representations of the Dirac group:

$$\Gamma_i \longleftrightarrow \chi_i^{(k)}, \qquad k=1,\ 2,\ \ldots,\ 16. \tag{8}$$

We analyzed this problem for the case of the mixed Lagrangian (1).

Since $\chi_i^{(k)} = \chi_k^{(i)}$, [3] the Lagrangian of the $V-A$ interaction can be expressed in the form

$$\mathscr{L}_{(V-A)} = C \sum_{i \in V} K^{(i)} (\bar{\psi}_1 (1-\gamma^5) \gamma^i \psi_2)(\bar{\psi}_3 (1-\gamma^5) \gamma^i \psi_4), \tag{9}$$

where

$$K^{(i)} = \sum_{k \in V} [(1-\chi_0^{(i)}) \chi_k^{(i)}] [(1-\chi_0^{(i)}) \chi_k^{(i)}]; \tag{10}$$

here $\gamma^5 \leftrightarrow \chi_0^{(i)}$ is the element of the i-th one-dimensional representation that corresponds to the matrix γ^5.

The Lagrangian of the $V-A$ interaction is the only Lagrangian that is symmetrical with respect to the four-dimensional and one-dimensional representations of the Dirac algebra.

The materials of the paper will be published in Izvestiya Akademii Nauk EstSSR.

[1]H. Pietschmann, Formulae and Results in Weak Interactions, Vienna—New York, 1974, (Acta Physica Austriaca, Suppl. 12).

[2]E. de Vries and A. Y. van Zanten, Comm. Math. Phys. 17, 322 (1970).

[3]H. Oiglane, Predstavleniya gruppi Diraka (Representations of the Dirac Group). Preprint FI-39, Tartu, 1975.