Negative-energy waves and the anomalous Doppler effect

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CONTENTS

INTRODUCTION

In this article, as is evident from the title, we shall discuss two beautiful physical phenomena. Both were discovered long ago and have a fairly wide "range of action" in general physics,[1–3] plasma physics,[3–6] and electronics.[7] Up to now these phenomena have been treated independently. The author has called attention to the fact that there is a deep physical analogy between the buildup of waves having negative energy (in media in which there are charged-particle beams) and the anomalous Doppler effect. The purpose of the present article is to demonstrate this analogy, using electron beams in plasmas and structures employed in microwave electronics as examples. Of course the larger part of the article is of the nature of a review.

1. POSITIVE- AND NEGATIVE-ENERGY WAVES IN ELECTRON BEAMS IN PLASMAS

The concept of negative-energy waves in dispersive media and of their instability mechanisms was first advanced from the points of view of general and plasma physics by Kadomtsev, Mikhaĭlovskiĭ, and Timofeev,[3] although in a more limited sense it was known earlier in connection with electron beams in microwave oscillators and amplifiers such as the traveling wave tube.[7] Let us recall the physical meaning of this concept.

Let us suppose that a "monochromatic" wave propagates in some direction z in a uniform medium of dielectric constant ε, so that the electric field $E(z, t)$ of the wave varies according to the law

$$E = E_0 \exp [i(kz - \omega t)]. \tag{1.1}$$

If ε is independent of the frequency ω, i.e., if there is no dispersion, the energy density W of the wave will be given by $W = \varepsilon(\overline{E^2}/8\pi)$, where the bar indicates averaging over the period of the oscillations and W is the sum of the electrical energy of the field and the kinetic energy of the oscillating particles. In the presence of dispersion, we have, according to[8],

$$W = \frac{\overline{E^2}}{8\pi}\frac{d}{d\omega}(\varepsilon\omega), \tag{1.2}$$

where by ε and ω we mean their respective real parts $\mathrm{Re}\,\varepsilon$ and $\mathrm{Re}\,\omega$. The wave energy W will always be positive if the medium is in thermodynamic equilibrium,[8] but, as was shown in the pioneering study of Kadomtsev, Mikhaĭlovskiĭ, and Timofeev,[3] if the medium is not in equilibrium W may have either sign—it all depends on the nature of the dispersion. When $d\varepsilon\omega/d\omega < 0$ we say that the wave carries negative energy, meaning thereby that the energy of the medium is lower when the wave is present than when it is absent.[3]

Now let us suppose that the medium is a cold plasma consisting of electrons and "stationary" positive ions, the electron and ion concentrations both being n_e. For simplicity we shall assume that there is no magnetic field (although, as will be shown below, all the results will remain valid in the presence of a strong magnetic field). In this case the dielectric constant for a high-frequency electromagnetic wave is given by the well-known formula

$$\varepsilon = 1 - \frac{\omega_e^2}{\omega^2}, \tag{1.3}$$

in which $\omega_e = \sqrt{4\pi n_e e^2/m}$ is the Langmuir frequency of the longitudinal (parallel to $\mathbf{E}$) natural oscillations of the electrons due to their (initial) displacements with respect to the ions. For Langmuir oscillations ($\omega = \omega_e$), Eq. (1.3) yields the dispersion equation

$$\varepsilon = 0. \tag{1.4}$$

This equation is valid not only for the special case now under discussion, but in general for all space-charge potential oscillations (waves). It follows at once from Poisson's equation div $\mathbf{D} = 0$ (here $\mathbf{D} = \varepsilon\mathbf{E}$, there being no uncompensated charges in the plasma), which yields $\varepsilon \cdot \mathrm{div}\,\mathbf{E} = 0$, i.e., $\varepsilon = 0$, since div $\mathbf{E}$ does not vanish for volume waves.

Now let us assume that all the electrons move with respect to the ions in the z direction with the same constant velocity $\mathbf{u}$, i.e., that they constitute a monoenergetic beam; we shall denote the electron concentration in the beam by n_1. On transforming to the rest system of the beam we obtain the same expression (1.3) for ε, but with ω_e replaced by the Langmuir frequency $\omega_1 = \sqrt{4\pi n_1 e^2/m}$ of the beam and ω replaced by the frequency $\omega' = \omega - \mathbf{k}\cdot\mathbf{u}$, where $\mathbf{k}$ is the wave vector of the oscillations. In fact, as a result of the Doppler effect the frequency of the oscillations in the lab system (in which the ions are at rest) is $\omega = \omega'/(1 - ku/\omega)$, and this leads to the expression just given for ω'. For the beam, therefore,

$$\varepsilon = 1 - \frac{\omega_1^2}{(\omega - \mathbf{k}\mathbf{u})^2} \tag{1.5}$$

and from (1.2) we obtain the following expression for the energy of the oscillations:

$$W = \frac{\overline{E^2}}{8\pi} \frac{2\,\omega\omega_1^2}{(\omega - ku)^3}, \tag{1.6}$$

where k is the projection of $\mathbf{k}$ in the direction of $\mathbf{u}$ (for the longitudinal oscillations under consideration we have $\mathbf{k} \parallel \mathbf{E} \parallel \mathbf{u}$). It is evident that

$$\begin{aligned} W > 0 \quad &\text{if} \quad u < \frac{\omega}{k}, \\ W < 0 \quad &\text{if} \quad u > \frac{\omega}{k}. \end{aligned} \tag{1.7}$$

We obtain $(\omega - ku)$ from the dispersion equation (1.4), using Eq. (1.5):

$$\omega - ku = \pm\omega_1. \tag{1.8}$$

This means that the Langmuir oscillations of the beam produce two space-charge waves in the rest system of the ions: a fast wave with phase velocity $\omega/k > u$ (the positive sign in (1.8)) and a slow wave with phase velocity $\omega/k < u$ (the negative sign in (1.8)). According to Eq. (1.6), the energy densities of these waves are

$$W = \pm \frac{\overline{E^2}\omega}{4\pi\omega_1}, \tag{1.9}$$

where the plus (minus) sign is for the fast (slow) wave. Thus, the fast (slow) beam wave has positive (negative) energy. The fact that one of the waves has negative energy is due to the fact that the medium is not in thermodynamic equilibrium, owing to its sharp anisotropy resulting from the presence of the particle beam.

To clarify the physical meaning of the result just obtained we shall derive Eq. (1.9) in another way, without making use of Eq. (1.2). The energy W of the wave, as was noted above, is the sum of the energy $W_E = E^2/8\pi$ of the electric field and the kinetic energy W_K of the oscillating particles. W_K is equal to the change in the kinetic energy of the beam due to the effect of the wave:

$$W_K = \frac{m}{2}\overline{(u+v)^2(n_1+n)} - \frac{m}{2}u^2 n_1, \tag{1.10}$$

where v and n are the perturbations of the velocity and density of the beam electrons due to the wave ($\mathbf{v} \parallel \mathbf{u} \parallel \mathbf{E}$ in the longitudinal oscillations under consideration) and, as before, the bar indicates averaging over a period of the oscillations. We determine v and n from the equation of motion and Poisson's equation:

$$\frac{d}{dt}(u+v) \equiv \frac{dv}{dt} \equiv \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial z} = \frac{e}{m}E$$

and

$$\operatorname{div}\mathbf{E} \equiv ikE = 4\pi ne.$$

In view of the fact that v, n, and E vary harmonically in accordance with Eq. (1.1), we have

$$\begin{aligned} v &= i\frac{e}{m}\frac{E}{(\omega - ku)}, \\ n &= \frac{1}{4\pi e} ikE, \end{aligned} \tag{1.11}$$

whence, with the aid of Eq. (1.8), we obtain

$$\frac{v}{n} = \pm\frac{4\pi e^2}{mk}\frac{1}{\omega_1}, \tag{1.12}$$

where the plus (minus) sign refers to the fast (slow) wave. Thus, for the slow wave the velocity and density perturbations v and n of the beam are 180° out of phase, and that, as we shall now show, is why the kinetic energy W_K and the total energy $W = W_K + W_E$ of this wave are negative (for the fast wave, the perturbations v and n are in phase, so W_K and W are positive)[9].

According to (1.10) we have

$$W_K = \frac{m}{2}\left\{\left[\frac{1}{T}\int_0^T (n_1 - n_0\sin\omega t)(u + v_0\sin\omega t)^2\,dt\right] - n_1u^2\right\}, \tag{1.13}$$

for the slow wave, where n_0 and v_0 are the amplitudes of n and v (since the average in (1.13) is taken over a period T of the oscillations the result will of course remain unchanged if the addition and subtraction signs in the two expressions in parentheses under the integral sign are interchanged). From Eqs. (1.13), (1.11), and (1.12) we have

$$W_K^s = -\frac{E_0^2}{8\pi}\frac{ku}{\omega_1} + \frac{E_0^2}{16\pi}, \tag{1.14}$$

where E_0 is the amplitude of the field E that determines v_0 and n_0 via Eqs. (1.11) and the superscript "s" indicates that the corresponding quantity belongs to the slow wave. Hence, taking (1.8) into account, the total energy density of the slow wave is

$$W^s = W_K^s + \frac{E_0^2}{16\pi} = -\frac{E_0^2}{8\pi}\frac{\omega}{\omega_1} = -\frac{\overline{E^2}}{4\pi}\frac{\omega}{\omega_1} \tag{1.15}$$

According to Eq. (1.12), in calculating the kinetic energy density W_K^f for the fast wave one must take the same signs in the two expressions in parentheses under the integral sign in (1.13). Then we obtain

$$W_K^f = \frac{E_0^2}{8\pi}\frac{ku}{\omega_1} + \frac{E_0^2}{16\pi}, \tag{1.14}$$

$$W^f = W_K^f + \frac{E_0^2}{16\pi} = +\frac{\overline{E^2}}{4\pi}\frac{\omega}{\omega_1}. \tag{1.15}$$

It is evident that expressions (1.15) for the two beam waves, which were obtained from simple kinematic considerations, agree precisely with expression (1.9),

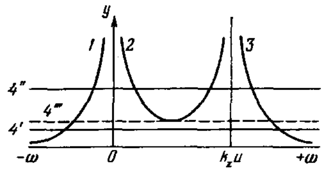

FIG. 1. Dispersion curves: 1–3—branches of the function $y = 1 - \varepsilon(\omega, k)$ at $k = \text{const.}$, 4—lines $y = 1$ (4′—instability, 4″—stability, 4‴—threshold (critical) regime).

which was derived with the use of the general definition (1.2) for the energy of a wave.

Now let us write down the energetic relationships for the total energy of the system (beam plus wave). This energy, which is the sum of the kinetic energy T of the beam electrons and the energy $E^2/8\pi$ of the electric field of the wave, changes as follows on excitation of a wave:

$$T^f + \frac{\overline{E^2}}{8\pi} = T^0 + W^f, \qquad T^s + \frac{\overline{E^2}}{8\pi} = T^0 - |W^s|, \tag{1.16}$$

where the superscripts "f" and "s" indicate the presence, and the superscript "0," the absence, of waves. It is evident that if a fast (slow) wave is excited, the total energy of the system (beam plus wave) will be higher (lower) than the initial energy of the beam; moreover, according to (1.16), this energy difference is precisely equal to the absolute value of the energy of the excited waves. Therein lies the physical meaning of positive and negative energies of waves.[3,7,14] As regards the energy of the electric field of the oscillations, we see that it, unlike the total energy W of the oscillations, is naturally positive and increases with increasing amplitude of the wave: $W_E^s = W_E^f = E_0^2/16\pi$.

The present derivation of the expression for W sheds light on still another fundamental fact: the energy of one of the Langmuir waves in a medium is negative only because the medium (in this case the beam) is moving. This can be seen directly from Eqs. (1.14) and (1.15): if we set $u = 0$, we find that both the kinetic and total energies of the slow wave are positive. For a plasma that is stationary as a whole, we find from Eqs. (1.14) and (1.15) that

$$W_E = W_K = \frac{\overline{E^2}}{8\pi}, \qquad W = W_E + W_K = \frac{\overline{E^2}}{4\pi}. \tag{1.17}$$

This means that the total energy of the Langmuir oscillations of a plasma at rest is positive and is the sum of two equal positive quantities: the kinetic energy of the oscillating electrons and the energy of the electric field. As is easily seen, this same result (1.17) can also be obtained in a different way: by calculating W from Eqs. (1.2) and (1.3).

Finally, we note that so-called moderating structures are used in microwave electronics. A familiar example of such a structure is the metallic helix surrounding the beam in devices such as the traveling wave tube.[7] The pitch of the helix is chosen so that the axial velocity of the electromagnetic wave excited in the helix by the beam will be close to the velocity of the beam particles. It is clear from what was said above that the energy of the wave in the helix will always be positive, since the helix does not move.

2. INSTABILITY IN THE INTERACTION BETWEEN WAVES HAVING ENERGIES OF OPPOSITE SIGNS

The result (1.16) obtained in Chap. 1 means that to excite the fast space-charge wave in an electron beam one must provide additional energy to the beam, while to excite the slow wave one must extract energy from the beam. In other words, in order for the slow wave to build up there must be some mechanism for dissipating its energy. For example, such a mechanism might be energy transfer from the wave to some other wave associated with the stationary medium and therefore having positive energy, e.g., to Langmuir oscillations of the stationary plasma or to an electromagnetic wave in a moderating microwave structure. We shall consider both these possibilities.

If an electron beam passes through a plasma in such a manner that the slow wave in the electron beam (the "beam wave") can transfer energy to a plasma wave, then both these waves will build up (at the expense of the kinetic energy of the beam), i.e., the so-called beam instability will arise. To formulate the condition for beam instability in terms of the concepts under discussion let us return to the expression for the dielectric constant of the medium. As is evident from Eqs. (1.3) and (1.5), we have

$$\varepsilon = 1 - \frac{\omega_1^2}{(\omega - ku)^2} - \frac{\omega_e^2}{\omega^2}. \tag{2.1}$$

for the system consisting of the beam and the plasma. If this system is to be unstable, the slow beam wave must have a sufficiently large "reservoir" from which the energy required for building up the plasma wave can be drawn. For this it is necessary that the (negative) contribution from the slow wave to the total energy (1.2) of the oscillations of the beam-plasma system be larger in absolute value than the (positive) contribution from the plasma wave, i.e., that

$$\frac{\omega\omega_1^2}{|(\omega - ku)^3|} > \frac{\omega_e^2}{\omega^2},$$

or

$$\frac{|\omega - ku|}{\omega} < \left(\frac{n_1}{n_e}\right)^{1/3}. \tag{2.2}$$

Beam instability will evidently set in at some threshold value of the beam density. It is not difficult to see that exactly the same expression for the beam-instability threshold can also be obtained directly from the dispersion equation $\varepsilon = 0$, i.e.,[10a]

$$1 - \varepsilon(\omega) \equiv \frac{\omega_1^2}{(\omega - ku)^2} + \frac{\omega_e^2}{\omega^2} = 1. \tag{2.3}$$

This equation, which is of the fourth degree in ω, is best solved graphically.[11] The curves $y = 1 - \varepsilon(\omega)$ (the left side of Eq. (2.3)) and $y \equiv 1$ (the right side of Eq. (2.3)) are sketched in Fig. 1. If these curves intersect in four points, then all the roots for ω will be real and there will be no instability; but if the curves intersect in two points, then two of the four roots will be complex and one of them will have a positive imaginary part ($\omega = \mathrm{Re}\,\omega + i\gamma$, with $\gamma > 0$), and this means, in accordance with Eq. (1.1), that the oscillations will build up. Instability sets in when the line $y \equiv 1$ becomes tangent to the central branch of the curve $y = 1 - \varepsilon(\omega)$. At the point of tangency we have

$$\frac{\partial \varepsilon}{\partial \omega} = 0, \tag{2.4}$$

and this, together with Eq. (2.1), gives the instability threshold. It is evident from Eqs. (2.3) and (1.2) that this threshold agrees precisely with the threshold given by (2.2). The critical regime (2.4) at which instability sets in corresponds, according to Eq. (2.3), to the oscillation frequency

$$\omega = \frac{ku}{1 + (n_1/n_e)^{1/3}}. \tag{2.5}$$

It is important for what will follow to point out that this frequency satisfies the condition

$$u > \frac{\omega}{k}: \tag{2.5'}$$

the velocity of the beam particles is higher than the phase velocity of the oscillations.

We have considered collisionless beam instability. Now let us complicate the problem somewhat: we assume that the plasma electrons collide with one another and with the ions and the neutral gas, and denote the collision frequency by ν. If there were no beam and Langmuir oscillations were excited in the plasma the oscillations would be damped as a result of the collisions (energy dissipation) since plasma oscillations have positive energy. In the presence of a beam, dissipation of the energy of the oscillations may lead to buildup of the slow beam wave (since this wave has negative energy) and thereby favor instability. To see this, let us write the dispersion equation for the oscillations with allowance for the collisions[10b]:

$$1 - \varepsilon \equiv \frac{\omega_1^2}{(\omega - ku)^2} + \frac{\omega_e^2}{\omega(\omega + i\nu)} = 1 \tag{2.6}$$

(it is the imaginary term added to the oscillation frequency that leads to the above mentioned damping of plasma oscillations in the absence of a beam). The instability threshold is determined by the condition $\partial(\omega\,\mathrm{Re}\,\varepsilon)/\partial\omega = 0$ according to the "energetic" approach under discussion and by the condition $\partial\,\mathrm{Re}\,\varepsilon/\partial\omega = 0$ according to Eq. (2.4). These two conditions are fully equivalent since $\mathrm{Re}\,\varepsilon(\omega) = 0$ at the instability threshold. It is evident from Eq. (2.6) that the condition for instability is

$$\frac{n_1}{n_e} \equiv \frac{\omega_1^2}{\omega_e^2} > \frac{\omega\,|\,\omega - ku\,|^3}{(\omega^2 + \nu^2)^2}, \tag{2.7}$$

which is a weaker condition than the "collisionless" condition (2.2). Thus, collisions (energy dissipation) lower the instability threshold: instability sets in before it would if there were no collisions. This phenomenon is called dissipative instability. It is easily seen that the oscillation frequency at which instability sets in is lower than the frequency (2.5) and even better satisfies the condition

$$u > \frac{\omega}{k}. \tag{2.5'}$$

Now let us consider two more examples of beam instability. Let an electron beam propagate through a "background" of ions of finite mass M, which neutralize its space charge, and let us assume that there is no third component (plasma electrons). The oscillations of such a quasineutral beam are described by the familiar equation (2.3), but with the electron Langmuir frequency ω_e replaced by the ion Langmuir frequency $\omega_+ = \sqrt{4\pi n_+ e^2/M}$, where $n_+ = n_1$ is the ion concentration:

$$\frac{\omega_1^2}{(\omega - ku)^2} + \frac{\omega_+^2}{\omega^2} = 1. \tag{2.3'}$$

Accordingly,

$$\varepsilon = 1 - \frac{\omega_1^2}{(\omega - ku)^2} - \frac{\omega_+^2}{\omega^2}. \tag{2.1}$$

This problem is quite analogous to the one just discussed. The instability (it is usually called the Buneman instability[12]) develops because the slow beam wave, which carries negative energy, transfers energy to the ion wave, which carries positive energy, and therefore builds itself up. As in the preceding problem, the instability threshold can be determined either via the "energetic" approach, or by solving the dispersion equation directly; and of course both approaches lead to the same result.[13] Here it is important to note that the oscillation frequency corresponding to the instability threshold, which is given by

$$\omega = \frac{ku}{1 + (M/m)^{1/3}}, \quad \frac{\omega}{ku} \leqslant \left(\frac{m}{M}\right)^{1/3}, \tag{2.8}$$

again (and with a large margin) satisfies the condition

$$u > \frac{\omega}{k}. \tag{2.5'}$$

A quasineutral electron beam is subject to still another instability provided its transverse dimensions are finite, its transverse density profile is (inevitably) nonuniform, and it propagates along a strong magnetic field ($\mathbf{H} = \mathbf{H}_z$). This instability, which limits the attainable beam current,[13] is also described by the dispersion equation $\varepsilon = 0$, in which the contribution from the beam electrons to the dielectric constant is given by the expression (we omit unimportant details)[13]

$$\varepsilon_{\text{beam}} \approx 1 - \frac{\omega_1^2}{(\omega - k_z u)^2}\frac{k_z^2}{k^2} - \frac{\omega_1^2}{\omega_H(k_z u - \omega)}, \tag{2.9}$$

in which k is the magnitude of the wave vector $\mathbf{k}$ of the oscillations, k_z is the projection of $\mathbf{k}$ on the z axis (the longitudinal wave number), and $\omega_H = eH/mc$ is the electron Larmor frequency ($\omega_H > \omega$, ω_1, $k_z u$). It is evident that in this case, according to Eq. (1.2), the energy of the slow beam wave will be negative. The mechanism of this instability is (from the point of view adopted here) analogous to that of the Buneman instability. The growing wave again satisfies the condition

$$u > \frac{\omega}{k_z}: \tag{2.5'}$$

the velocity of the beam particles is higher than the phase velocity of the oscillations.

This is a very appropriate place to point out that, as the last example shows, the presence of a strong external magnetic field does not affect the conclusions that we reached above under the simplifying assumption that no magnetic field was present. In the presence of a (strong) field we have only to replace $\mathbf{k}$ by its projection k_z onto the x direction of the field $\mathbf{H} = \mathbf{H}_z$ in all the formulas given earlier.[1)]

Now let us turn to a few examples from the field of electronics, in which the instability of negative-energy waves is used to generate and amplify microwave oscillations.[7] In the traveling-wave tube mentioned earlier, an electromagnetic wave builds up in the helix surrounding the beam (this wave has positive energy) as a result of the action of the slow space-charge wave in the beam (which carries negative energy) under such conditions that the two waves are synchronous (have the same phase velocity)[7,14]:

$$v^h_{\text{ph}} \equiv \frac{\omega}{k_z} = v^s_{\text{ph}} \equiv u - \frac{\omega_1}{k_z}, \tag{2.10}$$

where v^h_{ph} is the phase velocity of the wave in the helix (measured along the axis of the helix), v^s_{ph} is the phase velocity of the slow beam wave, and k_z is the longitudinal wave number (measured along the beam). When condition (2.10) is satisfied the positive-energy wave in the helix is a "dissipative" load for the slow beam wave, and both waves grow with time. It is evident from (2.10) that this takes place under the condition

$$u > \frac{\omega}{k_z}: \tag{2.5'}$$

the beam "overtakes" the wave in the microwave structure surrounding it.

In the so-called resistive microwave amplifier,[15] the beam is surrounded by walls of finite (relatively low) conductivity in order to provide a dissipative load for the slow beam wave. In this system the electric field of the slow beam wave induces conduction currents in the walls; the consequent Joule losses are supplied by the energy of the wave, and as a result the wave builds up (at the expense of the kinetic of the beam electrons). Let us formulate the dispersion equation for such a system. To do this we first find the current densities in the beam and the walls due to the field of the waves; these are

$$j_{\text{beam}} = n_1 ev = i\frac{n_1 e^2 E}{(\omega - ku)m}$$

(according to (1.11)), and $j_{\text{walls}} = \sigma E$, where σ is the conductivity. The space-charge densities in the beam and the walls are determined from the corresponding equations of continuity:

$$-i(\omega - ku)\rho_{\text{beam}} \equiv \frac{\partial\rho_{\text{beam}}}{\partial t} = -\operatorname{div}\mathbf{j}_{\text{beam}},$$
$$-i\omega\rho_{\text{walls}} \equiv \frac{\partial\rho_{\text{walls}}}{\partial t} = -\operatorname{div}\mathbf{j}_{\text{walls}}.$$

On substituting the total space-charge density $\rho = \rho_{\text{beam}} + \rho_{\text{walls}}$ into Poisson's equation $\operatorname{div}\mathbf{E} = 4\pi\rho$, we obtain the dispersion equation

$$\frac{\omega_1^2}{(\omega - ku)^2} + \frac{4\pi\sigma}{i\omega} = 1, \tag{2.11}$$

whence

$$\omega - ku = \pm\omega_1\left(1 - i\frac{4\pi\sigma}{\omega}\right), \tag{2.12}$$

where the plus (minus) sign on the right is for the fast (slow) wave. The imaginary part of the frequency of the slow wave is evidently positive, and according to (1.1), this means that the oscillations build up (are unstable). In this example, too, as in all the preceding ones, instability sets in when the condition

$$u > \frac{\operatorname{Re}\omega}{k}, \tag{2.5'}$$

is satisfied, i.e., when all the beam electrons move faster than the wave that they excite.

Finally, we mention still another microwave amplifier, in which two electron beams with unequal velocities pass through a background of "stationary" charge-compensating ions (the so-called electron-wave tube[16]). Here the beam with the lower velocity acts as a dissipative load for the slow wave in the beam with the higher velocity. It is easily seen (by transforming to the rest system of one of the beams) that the mechanism responsible for the instability (for the buildup of the oscillations) is quite the same in this case as in the cases discussed above of a beam in a plasma or in a microwave moderating structure. Condition (2.5′) is naturally also satisfied in this case.

3. THE ANOMALOUS DOPPLER EFFECT

Here we shall limit ourselves to a brief discussion of the physical meaning of this remarkable phenomenon, referring the reader to the fundamental works of Ginzburg and Frank,[1] to their reveiws,[2] and to the lectures of Tamm and Frank.[1]

First we recall the phenomenon known as "Vavilov-Čerenkov radiation,"[1] which is more "popular" just now. This (electromagnetic) radiation arises when a charged particle moves through a medium with a velocity ($\mathbf{u} = \mathbf{u}_z$) exceeding the velocity of light $v_{\text{ph}} = c/N$ in that medium:

$$u > \frac{\omega}{k}, \quad u = \frac{\omega}{k_z}, \tag{3.1}$$

where $\omega/k = c/N = v_{\text{ph}}$, $k_z = k\cos\theta_0$, and N is the refractive index of the medium (assumed to be homogeneous and isotropic). The radiation is observed only at an angle θ_0 to the direction of motion of the particle such that

$$\cos\theta_0 = \frac{c/N}{u} = \frac{\omega}{ku}, \tag{3.1'}$$

[1)]And multiply ω_1^2 and ω_e^2 by k_z^2/k^2; for more details see[13].

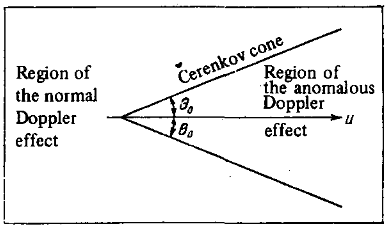

FIG. 2. (Drawn according to[2].)

i.e., only on the so-called Cerenkov cone (Fig. 2, from[2]).

Now let us replace the free charged particle by a system that has internal energy U in addition to its kinetic energy T (e.g., an arbitrary oscillator). If this system emits a photon of energy $\hbar\omega \ll T$ and momentum $\hbar\mathbf{k}$ in the direction θ it will "recoil," and as a result its kinetic energy will decrease by the quantity $|\Delta T| = \hbar k_z u = \hbar k u \cos\theta$. Consequently,

$$\frac{\Delta T}{\hbar\omega} = \frac{k_z u}{\omega} = \frac{u}{\omega/k_z} = \frac{u\cos\theta}{v_{\mathrm{ph}}}. \tag{3.2}$$

But now we have a curious situation:

$$|\Delta T| > \hbar\omega \text{ when } u > \omega/k_z; \tag{3.3}$$

the system loses more kinetic energy than the emitted photon carries off (!). This is one of the seeming paradoxes of "superluminal" motion (in optics this is the case in which $u\cos\theta > c/N$). If such emission is to be possible, the excess kinetic energy lost by the system must go to increase the internal energy:

$$\Delta U = \hbar\omega\left(\frac{k_z u}{\omega} - 1\right) = \hbar\omega\left(\frac{u\cos\theta}{v_{\mathrm{ph}}} - 1\right). \tag{3.4}$$

Thus, when condition (3.3) is satisfied the system emits energy and at the same time undergoes a transition to a more highly excited state (!). This phenomenon is called the anomalous Doppler effect. It differs from the normal ("ordinary," in particular, "subluminal") Doppler effect "only" in the sign of ΔU in formula (3.4): In the normal Doppler effect the inequality opposite to (3.3) is satisfied; then, according to Eq. (3.4), the photon is emitted "as usual" at the expense of decreasing the internal energy of the system. The normal and anomalous Doppler effects are separated by the situation in which the Čerenkov condition (3.1), (3.1′) is satisfied. Then $\Delta U = 0$, and the emission of a photon at the Čerenkov angle $\theta_0 = \arccos(v_{\mathrm{ph}}/u)$ takes place without change in the internal energy of the system. Hence a free charged particle having no internal degrees of freedom can emit Vavilov-Čerenkov radiation. Thus, all three effects are possible for a system moving at "superluminal" velocity ($u > v_{\mathrm{ph}}$), depending on the direction in which the radiation is emitted: for $\theta > \theta_0$, the normal Doppler effect; for $\theta = \theta_0$, the Vavilov-Čerenkov effect; and for $\theta < \theta_0$, the anomalous Doppler effect. Only the normal Doppler effect is possible for a system moving at "subluminal" velocity ($u < u_{\mathrm{ph}}$).

Now let us consider the particular case in which the oscillator is a charged particle (e.g., an electron) moving freely with velocity u in the direction of an external magnetic field $\mathbf{H} = \mathbf{H}_z$ and having a small transverse ($\perp\mathbf{H}$) velocity component. The rotational energy of the particle in the field H (the internal energy of the oscillator) can change by quanta of magnitude

$$\Delta U = n\hbar\omega_H, \quad \omega_H = \frac{eH}{mc},$$

where m is the mass of the particle and $n = 0, \pm 1, \pm 2, \ldots$. According to Eq. (3.4), which expresses the conservation of energy, we have

$$n\hbar\omega_H = \hbar(\omega - k_z u), \tag{3.5a}$$

or

$$\omega - k_z u = n\omega_H. \tag{3.5b}$$

The case $n > 0$ corresponds to the normal Doppler effect: the frequency of the emitted radiation in the rest system of the oscillator ($\omega - k_z u$) is equal to the frequency of the corresponding quantum transition. The case $n < 0$ corresponds to the anomalous Doppler effect; $\hbar k_z u = \hbar\omega + \hbar|n|\omega_H$: the change in the kinetic energy of the longitudinal motion of the oscillator goes to supply the energy $\hbar\omega$ of the emitted photon and to increase the internal energy of the oscillator, i.e., the energy of its rotation in the magnetic field. If the particle had no rotational energy before radiating, it begins to "revolve" as a result of emitting a photon (with $n < 0$). In the case of a particle beam, the anomalous Doppler effect tends to facilitate its becoming isotropic, and this has been observed experimentally.[17] Finally, the case $n = 0$ corresponds to the Vavilov-Čerenkov effect: the emission of the photon is not accompanied by any change in the rotational energy of the particle. Concerning the part played by these phenomena in plasma physics, see[2,4,11,18].

4. AN ANALOGY: THE LIMITING CONDITIONS FOR THE EFFECTS

Now we can finally turn to what, according to the title and introduction, is the basic purpose of this article. Let us compare three facts discussed above: 1) (Chap. 2) When a negative-energy wave excites (emits)[2] another wave having positive energy, the negative-energy wave goes over into a state in which the amplitude of its oscillations, and therefore the energy of its electric field, is higher; 2) (Chap. 3) If a system moves faster than the wave it emits, then, when it emits the wave it undergoes a transition to a more highly excited state—to a state of higher internal energy (the anamolous Doppler effect); and 3) (Chaps. 1–3). Both the phenomena discussed in Chaps. 1 and 2 take place under the same conditions as regards dispersion, namely when the condition (2.5′), $u > \omega/k_z$, is satisfied.[3] In particular, the

[2]Here we are using the word "emits" in a very conventional manner—from a purely energetic point of view and only to emphasize the analogy under discussion.

[3]In (2.5′), $k \equiv k_z$.

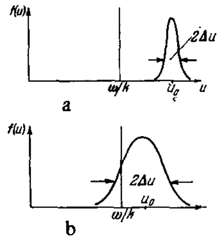

FIG. 3. Velocity distributions in beams: a) Hydrodynamic regime, $\Delta u < u_0 - \omega/k_z$; b) Kinetic regime, $\Delta u > u_0 - \omega/k_z$ (in this case $k = k_z$).

dispersion of the slow wave in an electron beam, which carries negative energy and (under suitable conditions) is unstable in a medium having positive energy, is described by the equation

$$\omega - k_z u = -\omega_1, \tag{1.8}$$

which is fully analogous to the dispersion equation

$$\omega - k_z u = -\mid n \mid \omega_H \tag{3.5}$$

in the anomalous Doppler effect.

To these facts we add another, which is associated with the so-called cyclotron waves of an electron beam, which have found application in microwave electronics and plasma physics.[5b,7] The dispersion of these waves, which are circularly polarized in the cross-section plane of the beam, is determined by the dielectric constant

$$\varepsilon = 1 - \frac{\omega_1^2}{(\omega - k_z u)^2 - \omega_H^2}; \tag{4.1}$$

the frequency of the wave is determined from the dispersion equation $\varepsilon = 0$. When $\omega_H \gg \omega_1$ this gives[4)]

$$\omega - k_z u = \pm \omega_H \pm \frac{\omega_1^2}{2\omega_H},$$

i.e.,

$$\omega - k_z u \approx \pm \omega_H, \tag{4.2}$$

where the plus (minus) sign is for the fast (slow) wave. Clearly, the dispersion formula (4.2) for cyclotron waves is virtually the same as condition (3.5) for radiation by a Larmor oscillator. It is directly evident from Eqs. (1.2), (4.1), and (4.2) that the fast wave, which corresponds to the normal Doppler effect ($\omega > k_z u$) carries positive energy, while the slow wave, which gives rise to the anomalous Doppler effect ($\omega < k_z u$) carries negative energy.

This group of facts permits us quite definitely to conclude that there is a direct physical analogy between the instability of a negative-energy "beam" wave in a positive-energy medium and the anomalous Doppler effect. The difference between the two phenomena is that the anomalous Doppler effect (in the form examined above) is an elementary process, while the instability is a collective process; in particular, in the case of Eq. (3.5) the internal energy of the system is the rotational energy of a single particle, while in the case of Eq. (1.8), it is the vibrational energy of the aggregate of beam particles. But the instability arises precisely because one elementary act induces the next one.[19] Hence the instability of a negative-energy wave under consideration can be treated as an induced anomalous Doppler effect. It is just this effect that lies at the basis of the instabilities of a monoenergetic electron beam discussed in Chap. 2. Let us list these instabilities: 1) (collisionless) instability of a beam in a plasma; 2) (dissipative) instability of a beam in a plasma; 3) Buneman instability of a beam moving through a background of stationary ions; 4) beam-drift instability; 5) the instability of glancing beams (the two-ray or electron-wave tube); 6) the instability of a beam in slow-wave structures—the generation and amplification of microwave oscillations in electronic devices such as the traveling-wave tube; and 7) dissipative instability of a beam in systems such as the resistive-type microwave amplifier.

Thus, if the dispersion in a "beam" system is such that the phase velocity of the waves is lower than the velocity of the beam (i.e., if the condition for the anomalous Doppler effect is satisfied) one can confidently expect the system to be unstable with respect to the growth of two waves, of which one carries negative energy, and the other, positive energy. On the other hand, if one explains the instability of the system from the point of view of the concept of negative-energy waves, it is the induced anomalous Doppler effect that is "responsible" for the instability.

There are two other effects that are analogous to the anomalous Doppler effect: the induced normal Doppler effect, and the induced Vavilov-Čerenkov effect.[2,4,18,19] The latter may be called inverse Landau damping from the name of an effect that plays an exceptional part in plasma physics.[20,21]

Now we must add the following supplement of a fundamental nature to all that has been said above. In discussing beam instabilities we always considered the case of a so-called "monoenergetic" beam, in which the velocity spread of the electrons was so small that all the beam electrons were moving faster than the wave they were exciting (Fig. 3a). Such a beam is said to be in the hydrodynamic regime[22]; in this regime $u > \omega/k_z$, and the anomalous Doppler effect takes place. However, the following question arises: What will happen if the velocity spread of the beam electrons is not "small enough," so that, as illustrated in Fig. 3b, the condition $u > \omega/k_z$ is not satisfied for all the beam electrons? In answering this question we limit ourselves to the case of one-dimensional motion ($\mathbf{k} = \mathbf{k}_z$). The situation turns out to be radically different in a beam having a large velocity spread (such a beam is said to be in the kinetic regime[22]): as the velocity spread increases, the anomalous Doppler effect disappears and, simultaneously therewith, the negative-energy beam wave ceases to be

[4)]For $\omega_1 \gg \omega_H$ one obtains the space-charge beam waves previously described: $\omega - k_z u = \pm\omega_1$.

excited—it damps out.[5)] This coincidence—the simultaneous disappearance of the anomalous Doppler effect and the instability (buildup) of the negative-energy wave—is naturally not accidental, but provides further evidence of the deep physical analogy between these two phenomena.

The boundary between the hydrodynamic and kinetic regimes of a beam corresponds to the (limiting) velocity spread[6)]

$$\frac{\Delta u}{u} \approx \frac{|\omega - k_z u_0|}{k_z u_0}. \tag{4.3}$$

This means, according to (1.8), (2.2), and (2.8), that

$$\frac{\Delta u}{u_0} = \left(\frac{n_1}{n_e}\right)^{1/3} \tag{4.3'}$$

for the case of a beam in a collisionless plasma,

$$\frac{\Delta u}{u_0} = \left(\frac{m}{M}\right)^{1/3} \tag{4.3''}$$

in the case of a one-dimensional beam moving through a background of mobile ions, and

$$\frac{\Delta u}{u_0} \approx \frac{\omega_1}{k_z u_0} \approx \frac{\omega_1}{\omega} \tag{4.3'''}$$

for the case of the beam in a traveling-wave tube, a resistive-wall amplifier, and similar microwave devices.

When the velocity spread Δu of the beam rises above the indicated limit, the instability of the beam in plasma media does not entirely disappear, but another (kinetic) mechanism, the induced Vavilov-Čerenkov effect,[2,4,22] comes into play, and according to Fig. 3, b, this just corresponds to the Čerenkov condition $\omega/k_z u = 1$. Thus, the Vavilov-Čerenkov effect begins to come into play just when the conditions for instability (buildup) of negative-energy waves cease to be satisfied. In particular, therefore, the Vavilov-Čerenkov effect can have nothing to do with the mechanism responsible for the operation of the traveling wave tube and similar microwave devices, for the operation of these devices, as was shown above, depends on the instability of the negative-energy waves, i.e., on the induced anomalous Doppler effect.

Let us give another example illustrating the analogy we are pursuing. It is known that when a beam in the kinetic regime passes through a plasma, the collisions of the plasma electrons tend to damp out the beam instability.[4,5,10b] This alone is evidence that the waves generated in the beam-plasma system in this regime have positive energy. In fact, if there were negative-energy waves in the system, the collisions of the plasma electrons would not tend to damp them out, but to build them up, as was shown in Chap. 3 by the example of the dissipative instability of a beam in a plasma and in a resistive-wall amplifier. Here, too, there is a regular coincidence of two circumstances: first, the waves have positive energy; and second, according to Fig. 3 the conditions for the anomalous Doppler effect are not satisfied.

The existence of a direct relationship between the induced anomalous Doppler effect and the instability of negative-energy waves provides the basis for the very fine idea[24] of collective acceleration of ions to relativistic energies in powerful high-voltage electron beams. According to this idea, the accelerating agent is the slow cyclotron wave (4.2), whose amplitude increases with time on account of the energy losses to the ions being accelerated (in accordance with the effect of dissipation on a negative-energy wave). The acceleration takes place in a continually falling magnetic field, and on account of this the phase velocity ω/k_z of the slow wave rises, tending toward the velocity of the (relativistic) beam electrons, i.e., toward the velocity of light.

Finally, we note in the same context that the coincidence discussed above is also found in very interesting experiments[23] on the generation of ultrahigh-power microwave oscillations by intense relativistic beams in a spatially modulated magnetic field: first, there is no anomalous Doppler effect (since $\omega > k_z u$); and second, the space-charge waves that build up in the beam (which is twisted into a helix), and it is these waves that determine the beam's coherent radiation, turn out to have positive energy.[23]

Thus, all the examples examined here further illustrate and confirm the conclusion reached above that there is a deep physical analogy between the induced anomalous Doppler effect and the instability of negative-energy waves in "beamlike" dispersive media.

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[5)]We shall not discuss the kinetic regime of a beam since such a discussion would not conform to the general style of this article. A clear and rigorous exposition of such topics will be found in[5]. The kinetic instability of a beam under the condition $\omega - k_z u = -n\omega_H$ for the anomalous Doppler effect has been investigated theoretically in[25] for the case of three-dimensional motion.

[6)]As an approximate value of Δu one may take the full width at half maximum of the electron velocity distribution function $f(u)$ (see Fig. 3).

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