

The pinch effect in a solid-state plasma

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This article reviews the experimental and theoretical studies of the pinch effect in a solid-state plasma in the period 1958-1975. Different methods of observing the pinch effect and ways of initiating and suppressing this effect are described. Special attention is paid to the time evolution of this phenomenon. The fundamental features of the pinch effect in semiconductors under lattice-heating conditions are pointed out. The problems are discussed in detail of plasma stability in the pinch effect, as well as the mechanisms of breakdown of a pinch in a longitudinal magnetic field. The form of the recombination-radiation spectra in the pinch effect in a degenerate electron-hole plasma is analyzed. The fundamental experimental results from studying a θ -pinch in InSb and Ge are given and analyzed. The applied aspects of studying the pinch effect in a solid-state plasma are pointed out, and unsolved problems are noted. Analogies are pointed out in a number of cases with the corresponding gas-discharge developments.

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1. INTRODUCTION

Very interesting discoveries were made in the field of studying plasma phenomena during 1958-1963. During these years, the pinch effect^[1,2] was observed, and helical instability,^[3-5] helicons,^[6-8] and the Gunn effect^[9,10] were discovered. This is just the period when the foundations were being laid for a new field in solid-state physics, which then vigorously developed and led to the creation of some very interesting scientific and technical developments. Yet the reader might object that the plasma-like behavior of the electron gas in metals was first indicated by the studies of Rutheman^[11] and Lang^[12] in 1948. Rutheman and Lang found that the energy losses upon passage of an electron beam through thin metal foils are discrete in nature. As Bohn and Pines^[13] soon showed, this effect stems from excitation of Langmuir oscillations (or plasmons^[14]) in the electron plasma of the metal being penetrated by the beam. While noting the importance of this discovery and of the subsequent studies along this line, I must note that these studies didn't lead to a further search for new plasma effects in solids, although certain treatments in the physics of a gaseous plasma were already known. Thus, in 1934 Bennett^[15] predicted the phenomenon of self-compression of an electron beam whose charge is neutralized by a background of slow ions owing to the intrinsic magnetic field of the beam current. Tonks^[16] termed this phenomenon the pinch effect. In 1956, Kurchatov^[17] reported at Harwell the results of the extensive program of Soviet studies of the pinch effect in a gaseous plasma in connection with the problem of controlled thermonuclear fusion (CTF).

In 1935, Eckersley^[18] determined the dispersion of

the so-called atmospheric whistlers,¹⁾ which had been discovered by Barkhausen.^[19] According to the data of^[20], Barkhausen made this discovery as early as the first World War while monitoring enemy telephone calls. We note that helicons in a solid-state plasma are a direct analog of the whistlers that arise in the plasma of the ionosphere.

In 1942, while analyzing the phenomena that occur in a sunspot region, Alfvén^[21] predicted a new type of electromagnetic waves that propagate in a highly conductive plasma in strong magnetic fields (Alfvén waves).

Even the cited studies in a gaseous plasma might have become a serious basis for seeking analogous effects in semiconductors and metals, since the notion of an electron-hole gas arose relatively early in the list of fundamental models of solids. However, plasma ideas began to penetrate broadly into solid-state physics around 1959. This had its objective reasons. In the postwar period, solid-state physics underwent an inner growth (no less vigorous than the development of the studies on CTF). However, the above-cited phenomena seemed so remote from solid-state physics that there could be no question of discovering them (or in jest, going from cosmic to solid phenomena). Moreover, extensive information on studies in plasma physics of gas discharges became available to the scientific community beginning in 1958, after the Geneva conference on peaceful uses of atomic energy. At the end of the fifties,

¹⁾The group velocity of these waves, which propagate along the magnetic field of the Earth, increases with frequency. Hence the waves of highest frequencies first reach the observer from the region of perturbation of the ionosphere. Accordingly, the tone falls with time^[20] (whistler).

plasma effects in solids asserted themselves in a most unexpected way. This seemed to be completely associated with the development of the technology of preparing superpure substances and of techniques of physical experimentation, including methods of generating a highly-concentrated non-equilibrium electron-hole plasma, synthesis of semiconducting compounds having high carrier mobility, the technique of low-temperature studies in strong magnetic fields, etc. In 1958, Ivanov and Ryvkin^[3] reported observation of low-frequency current oscillations in specimens of germanium (Ge) placed in a strong enough magnetic field. In the same year, at the Rochester conference on semiconductor physics, Glicksman and Steele^[1] reported the anomalous behavior of the volt-ampere characteristics (VAC) of specimens of indium antimonide (InSb) under impact ionization in the presence of a longitudinal magnetic field (H). The resistance of the specimens sharply rose at large currents when $H = 0$. The anomalous-resistance effect vanished in a longitudinal magnetic field comparable in magnitude with the intrinsic magnetic field of the current at the surface of the specimen. In 1959, Galt and his associates^[22] were studying absorption of electromagnetic waves in the centimeter range in superpure bismuth (Bi) in strong magnetic fields, and they discovered a linear dependence of the absorption coefficient on the magnetic field intensity. In 1961, Bowers and his associates^[8] measured the conductivity of sodium in strong magnetic fields at liquid-helium temperature ($T \approx 4^\circ \text{K}$) by the known method of determining the decay time of an electromagnetic signal applied to the specimen. The decay curve showed slowly damped oscillations whose frequency increased in direct proportion to the magnetic field intensity. I wish to emphasize that the design of the stated experiments involved no theoretical predictions, and we can only assume that a premonition of discovering a new direction in solid-state physics firmly dominated the minds of investigators in this period (the rather rapidly observed effects were moreover explained in a number of cases by the authors of the original experimental studies).

Glicksman^[5] was able to explain in 1961 the results of the experiments of Ivanov and Ryvkin^[3] on the basis of the theory of helical instability that Kadomtsev and Nedospasov^[4] had developed in application to the weakly ionized plasma of a gas discharge. In 1960, Kadomtsev and Nedospasov explained the phenomenon of anomalous diffusion in the plasma of the positive column of a gas discharge that had been found by Lehnert^[23] in 1958, practically simultaneously with the observations of Ivanov and Ryvkin.^[3]

As for the experiments of Glicksman and Steele,^[1] a year later^[2] these authors explained the anomalous behavior of the VAC of InSb specimens with the idea of a pinch effect. The electron-hole plasma is strongly compressed toward the axis of the specimen at large currents, the density of the plasma rises, and its resistance is increased by the enhanced electron-hole scattering and by quadratic bulk recombination. Here they showed that the criterion of strong compression that Bennett^[15] derived for a gas plasma was well satisfied under the experimental conditions of^[1]. The disappearance of the anomalous resistance in a strong enough magnetic field, as Ando and Glicksman showed considerably later,^[24] arises from breakdown of the pinch owing to onset of helical instability. The latter gives rise to an anomalous plasma diffusion current at the surface of the specimen

and to smoothing of the concentration profile. Thus, Glicksman and Steele's first experiments^[1] manifested very important plasma phenomena: the pinch effect and helical instability. Yet the experimental setup did not permit the authors of^[1] to observe directly the current and voltage oscillations associated with the growth of helical instability.

The results of the experiments of Galt and his associates^[22] were explained in 1961 by Buchsbaum and Galt,^[25] who showed that the incident wave is converted under the conditions of these experiments into an Alfvén wave whose phase velocity is directly proportional to the magnetic field intensity, which also governs the analogous variation of the absorption coefficient.

Bowers and his associates^[8] unambiguously explained the results of their measurements with the idea of helicons, which were rediscovered theoretically in 1960 as applied to a solid-state plasma by Aigrain,^[6] and independently by Konstantinov and Perel'.^[7]

Thus, as the brief historical statement given above implies, the existing theoretical and experimental store of material concerning a gaseous plasma facilitated in many ways the establishment of a plasma line of research in solid-state physics. The above-listed discoveries led to vigorous development of studies of plasma effects in semiconductors and metals. We can evaluate the development of the new branch of solid state physics not only by a quantitative index (hundreds of studies have been published on the different aspects of the phenomena cited above), but also qualitatively. A substantial fraction of these studies was constituted of those on plasma effects whose nature strongly depends on band-structure features, on the form of the carrier distribution functions, and on bulk and surface properties of solids. These studies culminated in 1963, when Gunn^[10] discovered the remarkable phenomenon of generation of microwave radiation by crystals of gallium arsenide and indium phosphide in strong electric fields. The Gunn effect is due to appearance of a negative differential conductivity in these crystals when the heating of the carriers by the electric field leads to a considerable intervalley redistribution.^[9a] Even before this effect had been discovered, Ridley^[9b] showed that a uniform carrier distribution becomes stratified under conditions of negative differential conductivity into regions of greater and lesser conductivity. It has since been shown^[26] that these regions (or domains) migrate through the crystal in the form of a solitary or shock wave of (field) density at a velocity equal to the drift velocity of the majority carriers. The Gunn effect had no analogy in a gaseous plasma. Yet the theory of collisionless shock waves that Sagdeev^[27] had developed facilitated in many ways the construction of a rigorous theory of this effect.

Extensive reviews^[28] have appeared in recent years that are concerned with detailed analysis of each of the above-cited phenomena in a solid-state plasma, apart from the pinch effect. In this review I would like to restore "justice" to one of the fundamental effects in a solid-state plasma, since the information of a review type^[29-32] on this phenomenon is extremely scanty and disconnected. I should note that a set of rather interesting and, above all, reliable information has been amassed in the studies on this topic, both on ways of initiating the pinch effect in semiconductors, and on methods of abolishing it. This information is extremely important in

designing high-current semiconductor lasers and in utilizing new narrow-band compounds having high carrier mobilities. We must also not forget the great conceptual importance of studies of the pinch effect in semiconductors for a gaseous plasma. I shall demonstrate this aspect below with a number of examples. The above-mentioned ideas also allow us to hope that this review, which has been written from the results of studies published during 1958–1975, may prove useful to a broad set of investigators. Let us recall the fundamental information on the pinch effect and the properties of an electron-hole plasma.

A pinch effect can arise only in a bipolar plasma when the specimen contains mobile carriers bearing opposite charges (electrons in the conduction band and holes in the valence band). In a monopolar plasma, space-charge forces prevent even a slight spatial redistribution of the carriers. One can create an electron-hole plasma of high concentration in a specimen by using interband breakdown in a strong electric field (impact ionization), by injection, or by two-photon absorption of laser radiation.^[30] One speaks in this case of a non-equilibrium plasma, since it is created by external sources. For example, in compounds such as $\text{Bi}_{1-x}\text{Sb}_x$ ($x = 0.065-0.23$, $T = 4^\circ\text{K}$), a compensated plasma is produced by impact ionization that has a rather high density^[33] ($n \approx 10^{15}-10^{16} \text{ cm}^{-3}$) that greatly exceeds the impurity concentration of electrons ($n_0 \approx 3 \times 10^{14} \text{ cm}^{-3}$), even in fields $\approx 2-3 \text{ V/cm}$. Such a low threshold for interband breakdown arises from the extremely narrow width of the forbidden band in these compounds ($0 < E_g < 0.24 \text{ eV}$). In n-InSb, impact ionization arises^[34-36] at an electric-field intensity $E \approx 200 \text{ V/cm}$ ($E_g = 0.2 \text{ eV}$ at a lattice temperature $T_c = 77^\circ\text{K}$). Upon further increase in the field, the conductivity of the plasma (σ) exceeds the impurity conductivity by more than one or two orders of magnitude. A highly effective method of producing a non-equilibrium plasma in semiconductors is the double-injection method,^[37] in which carriers of opposite signs are introduced into the crystal from opposing contacts.

An equilibrium neutral plasma exists in very pure semiconductors at high enough temperature, and in semimetals and metals having electron-hole conduction. Here the plasma concentration is fully determined by the lattice temperature and by the band-structure parameters (region of intrinsic conductivity). For example, in InSb the intrinsic plasma concentration at the lattice melting point^[38] ($\sim 808^\circ\text{K}$) is $n_p \approx 2 \times 10^{18} \text{ cm}^{-3}$. This greatly exceeds the non-equilibrium plasma density that can be produced in this compound by using the above-cited methods. At low temperatures the intrinsic conductivity of semiconductors is infinitesimally small. For example, the conduction of the overwhelming majority of semiconductor compounds at $T_c = 77^\circ\text{K}$ is of impurity type. In Bi (a semimetal), the concentration of electrons and holes at this temperature^[39] is $\approx 5 \times 10^{17} \text{ cm}^{-3}$. In tungsten^[31] (a metal having bipolar conduction), $n_p = 5 \times 10^{22} \text{ cm}^{-3}$.

I shall treat below two types of pinch effect in an electron-hole plasma: the Z and θ pinches, which are well known in a gaseous plasma.^[40] The Z pinch arises when a strong current passes through the specimen, and it is due to compression of the plasma by the intrinsic magnetic field of the current.^[15] The θ pinch amounts to compression of the plasma in a pulsed longitudinal

magnetic field that increases with time. Since the relaxation time of the carriers ($\tau_{e,h}$) with respect to the pulse is very short^[31] ($\sim 10^{-11}-10^{-12} \text{ sec}$) in a solid-state plasma, the redistribution of the plasma in the pinch effect arises from ambipolar drift of the electrons and holes toward the axis of the specimen (in contrast to a low-pressure gaseous plasma,^[41] the inertia effects are negligibly small). In the Z pinch, this drift arises in the longitudinal electric field and in the intrinsic magnetic field of the current transverse to it. In the θ pinch, it arises in the longitudinal magnetic field and in the aximuthal electric field induced by it. A pulsed regime is used in all studies of the Z pinch in a solid-state plasma so as to avoid thermal damage to the crystal or appreciable heating of the carriers. As a rule, the skin effect of the current is weakly marked (owing to the relatively low conductivity of the plasma and the long duration of the pulse), and the longitudinal electric field is constant over the cross section of the specimen. In this sense, we can classify the few experiments on the θ pinch in semiconductors into two groups. In one case, the skin effect is poorly marked, and the longitudinal magnetic field does not vary over the cross section of the specimen. In the other case, in which the dimension of the skin layer (δ) is less than the radius of the specimen, this field varies considerably in the radial direction.

The fundamental factors that hinder strong compression of the electron-hole plasma are ambipolar diffusion and bulk recombination of the carriers. Hence, the pinch effect can arise only in specimens having a high carrier mobility ($b_{e,h}$) and a relatively long bulk recombination time. This is just why studies of the pinch effect have as yet been conducted on a small number of semiconductor compounds:²⁾ InSb, Ge, and $\text{Bi}_{1-x}\text{Sb}_x$.

One of the fundamental criteria for appearance of a Z pinch in a plasma (Bennett's criterion^[15]) can be derived most easily by equating the pressure of the magnetic field of the current at the surface of a specimen having a uniformly distributed plasma to the pressure of the electron-hole gas:

$$\frac{H^2(r=R_0)}{8\pi} = P = nk(T_e + T_h), \quad (1)$$

We can easily determine from this the value of the critical current (the Bennett current) that must flow in the plasma in order to cause an appreciable spatial redistribution of the carriers:

$$I \geq I_B = \frac{2c^2k(T_e + T_h)}{ev_g}, \quad (2)$$

Here $v_g = (b_e + b_h)E$ is the drift velocity in the longitudinal electric field.

When Bennett's criterion (2) is satisfied, the drift component of the ambipolar velocity exceeds the velocity of ambipolar diffusion. As we see from (2), the Bennett current is large in specimens having a low mobility of carriers. We note that the Bennett criterion is a necessary condition for strong compression of a plasma in a Z pinch, but it is not sufficient. We shall discuss below this problem, as well as the approximations used in deriving (2).

At low temperatures and high concentrations of electrons and holes, at which the Fermi energy (ϵ_F) is considerably greater than the thermal energy:^[45]

²⁾The information on studying the pinch effect in indium arsenide (InAs)^[42-44] is extremely meager.

$$\varepsilon_F = \frac{(3\pi^2)^{2/3} \hbar^2 n^{2/3}}{2 m_{e,h}} \gg kT_{e,h}, \quad (3)$$

the plasma is described by Fermi-Dirac statistics, and here one speaks of a degenerate electron-hole gas. For example, in Bi the plasma is partially degenerate ($\varepsilon_{e,hF} \approx kT_c$) even at room temperature ($T_c = 300^\circ \text{K}$). In InSb, strong degeneracy of the electron component at liquid-helium temperature arises at $n \approx 10^{15} \text{ cm}^{-3}$ because of the small effective mass of the electrons ($m_e \approx 0.013m_0$, where m_0 is the mass of a free electron). Even when one of the components of the plasma is degenerate, the gas pressure of the carriers will be determined by precisely this component, and the ambipolar diffusion coefficient becomes a function of the concentration ($D \sim n^{2/3}$). Hence, when the latter is large, pinching of the degenerate plasma is hindered. For just this reason, and also because of the small drift velocities ($v_g \leq 10^3 \text{ cm/sec}$ when $j \leq 10^7 \text{ A/cm}^2$, $n = 5 \times 10^{22} \text{ cm}^{-3}$), the pinch effect is practically impossible in metals having electron-hole conduction. We can determine the critical Bennett current in the case of strongly degenerate electrons and holes by taking account of the fact that the plasma pressure in this case is^[45]

$$P = \frac{2}{5} n (\varepsilon_{eF} + \varepsilon_{hF}) = \alpha n^{5/3}, \quad (4)$$

where

$$\alpha = \frac{(3\pi^2)^{2/3}}{5} \hbar^2 \left(\frac{1}{m_e} + \frac{1}{m_h} \right).$$

We can show by using (1) and (4) that

$$I_B = \frac{2c^2 \alpha n^{2/3}}{ev_g}. \quad (5)$$

Thus, under conditions of a strongly degenerate plasma, the Bennett current increases with increasing concentration, since the effective temperature of the carriers (the Fermi energy) rises.

Study of a non-equilibrium degenerate plasma in semiconductors is of considerable scientific and technical interest, since a population inversion arises in such a plasma for optical transitions between the conduction band and the valence band. For direct transitions (that do not involve phonons), quanta of the energy^[46]

$$\hbar\nu - E_g < \varepsilon_{eF} + \varepsilon_{hF} \quad (6)$$

are not absorbed, since they can't raise an electron from the valence band to the conduction band. Yet such quanta can induce a backward transition and lead to the growth of a photon avalanche. Then one speaks of a negative absorption coefficient for quanta whose energy satisfies the condition (6). For direct transitions, this coefficient is determined by the product of the ordinary absorption coefficient (positive) times the population difference of the corresponding levels in the conduction and valence bands.^[47] The negative absorption is maximal when both types of carriers are strongly degenerate (the population difference is unity), and this fact is taken into account in designing semiconductor lasers. A number of studies^[48, 49] have noted that the pinch effect that arises at high injection currents is an undesirable phenomenon in those semiconductor lasers in which the radiation output emerges perpendicular to the current, owing to strong absorption of the photon avalanche in the rest of the crystal (outside the plasma filament). Yet if the radiation emerges in the axial direction,^[50] then a pinch effect can enhance the laser effect by increasing the plasma density near the axis of the specimen. Thus, the applied aspects of studying the pinch effect in semicon-

ductors consist not only in inventing ways of initiating it, but also of abolishing it.

In closing the Introduction, I would like to emphasize that a number of narrow-band compounds have already been synthesized having a high carrier mobility, and this number will grow like a photon avalanche in line with the growing demands of semiconductor electronics. In these compounds the pinch effect will be a common phenomenon, even at very low fields ($\sim 1 \text{ V/cm}$), and this will alter the fundamental electrical characteristics of the crystals. Hence we can assuredly say that the knowledge gathered in studying the pinch effect in "selected" compounds will help in hastening the applied use of narrow-band semiconductors.

2. DISCOVERY OF THE PINCH EFFECT IN AN ELECTRON-HOLE PLASMA

The history of the discovery of the Z pinch in semiconductors is rather interesting and instructive. In 1958, Glicksman and Steele,^[34] and also independently, Prior^[35] and Kanai^[36] discovered the effect of impact ionization in single crystals of n-InSb ($n_0 \approx 2 \times 10^{14} \text{ cm}^{-3}$, $T_c = 77^\circ \text{K}$). The current through the crystal at $E \approx 180\text{--}200 \text{ V/cm}$ ($I \approx 5 \text{ A}$)^[34] increased sharply upon a small increase in the electric field. As measurements of the Hall coefficient^[34] showed, this feature of the VAC was caused by generation of electron-hole pairs upon interband breakdown. The pattern of the phenomenon seemed quite clear until Glicksman and Steele^[1] studied the form of the VAC of these specimens in a longitudinal magnetic field ($\mathbf{H} \parallel \mathbf{E}$). These measurements (Fig. 1) showed that the current rises with increasing voltage much more rapidly after the onset of avalanche breakdown in the presence of the longitudinal magnetic field (curves 2 and 3) than when $H = 0$ (curve 1). All of the VAC's practically coincide up to the onset of impact ionization (Fig. 1). The impression was created that the longitudinal magnetic field destroys the anomalous resistance that was inherent in the electron-hole plasma in these experiments.³⁾ As we see from Fig. 1, the "destructive" action of the magnetic field depends on both its magnitude and on the current passing through the crystal. If $H = 350 \text{ Oe}$, then the effect of the magnetic field diminishes when $I > 13 \text{ A}$. When $I > 20 \text{ A}$, at which the intrinsic magnetic field of the current $H_\varphi(R_0) \approx H$, it vanishes completely (curves 1 and 2 merge). If $H = 3500 \text{ Oe}$, then the effect of abolishing anomalous resistance is strongly marked up to the currents $I \approx 50 \text{ A}$ that were attained in these experiments. The voltage applied to the crystal (U) was measured between two contacts lying 0.25 cm apart (the crystal dimensions $V = 0.039 \times 0.06 \times 0.8 \text{ cm}^3$). In order to avoid lattice heating, square current pulses were used of duration $\tau_p = 10^{-6} \text{ sec}$, and rise (or decay) time $\approx 10^{-7} \text{ sec}$. At low voltage, the current obeys Ohm's law (Fig. 1); when $U > 2\text{--}3 \text{ V}$ ($E > 8\text{--}10 \text{ V/cm}$), the current increases more slowly with increasing field. This is due to heating of the electrons and decrease in their mobility. The current begins to rise more rapidly when $U > 20 \text{ V}$ (pair generation sets in). A sharp break in the VAC is observed at $U \approx 45\text{--}50 \text{ V}$ ($E \approx 180\text{--}200 \text{ V/cm}$). At this field value, the rate of impact ionization, and correspondingly, the concentration of pairs are great enough

³⁾A magnetic field $H = 350 \text{ Oe}$ cannot substantially change the rate of impact ionization $g(E)$. This fact has been confirmed by later measurements of $g(E)$ in a longitudinal magnetic field.^[51,73]

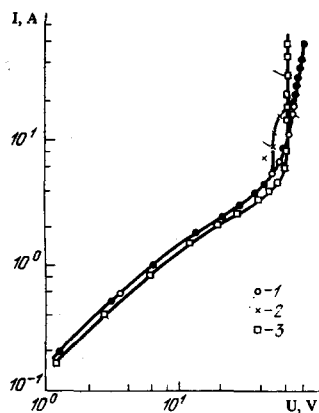


FIG. 1. Volt-ampere characteristic of n-InSb at $T_c = 77^\circ\text{K}$. ^[1,2] H (in Oe): 0 (1), 350 (2), and 3500 (3). $E = 4$ V/cm.

to change appreciably the resistance of the crystal ($\Delta n \approx n_0$). Curve 3 ($H = 3500$ Oe) is somewhat displaced with respect to the others. Apparently this involves ^[2] distortion of the current lines near the end contacts (even a weak non-parallelism of the current and the magnetic field in this region can affect the shape of the VAC). Moreover, the rate of impact ionization in such a strong magnetic field can be somewhat diminished, ^[51] and the impact breakdown will shift toward higher E values. A deviation of curves 1 and 2 arises at $I \approx 5$ A (i.e., practically at the instant of avalanche breakdown), and it increases with increasing current up to $I \approx 13$ A. We can easily calculate by using the graph of Fig. 1 that, when $I = 10$ A, the resistance of the non-equilibrium plasma (apart from the impurity component) is approximately twice as large when $H = 0$ as when $H = 350$ Oe.

Glicksman and Steele ^[2] soon proposed an explanation of the observed VAC anomalies based on the rather unexpected hypothesis of appearance of a pinch effect in the above-described experiments, which in turn could be abolished by a longitudinal magnetic field. Actually, the plasma concentration sharply increases under pinch-effect conditions, electron-hole scattering is enhanced, ^[44, 52, 53] and the mobility of the electrons is correspondingly diminished ($b_e \sim 1/n$). The increase in the resistance of the plasma in the pinch effect can also arise from the nonlinear nature of bulk recombination, ^[54-56] whose rate increases in the region of strong compression of the plasma. For just these reasons, it takes a stronger electric field (or higher rate of impact ionization) to maintain a given current under pinch-effect conditions. We note that the magnetoresistance effects caused by interaction of the radial ambipolar current of particles with the intrinsic magnetic field of the current in the experiments of Glicksman and Steele, ^[1] as well as in all of the subsequent studies of the pinch effect in InSb, are negligibly small ($(b_e b_h / c^2) H_\phi^2 \ll 1$).

When the pinch is abolished in a longitudinal magnetic field, the anomalous resistance naturally must vanish. To a certain degree, Glicksman and Steele ^[2] were prepared for such an interpretation of their experiments. In 1956, the Soviet physicists Bezbatchenko, Golovin, et al. ^[57] published the results of studying a Z pinch in a deuterium plasma ($I \lesssim 700$ kA, $\tau_{\text{pulse}} \approx 10^{-5}$ sec). It turned out that the compression of the plasma appreciably declined upon increasing the longitudinal magnetic field up to 6×10^3 Oe. Apparently the results of this very study aided Glicksman and Steele ^[2] in interpreting resistance anomalies of an electron-hole plasma on the basis of the idea of a pinch effect. Yet we must note that

the stabilizing action of a longitudinal magnetic field on the pinch effect that had been observed in ^[57] involved partial trapping of the magnetic field by the plasma, whereby the effective pressure of the plasma rose. In these experiments, the Maxwell diffusion time of the magnetic field ($\tau_m = 4\pi\sigma R^2/c^2$) was comparable with the duration of the pulse ($\sigma \approx 4 \times 10^{14}$ sec⁻¹, $R \approx 2$ cm). Yet in the case of the experiments of Glicksman and Steele ^[1,2], $\tau_m < 10^{-9}$ sec, and the concept of a "frozen-in" magnetic field, which was also adopted as an argument in this study, is absolutely false. And only in 1967 did Ando and Glicksman ^[24] firmly prove that the breakdown of the pinch in a longitudinal magnetic field is caused by excitation of helical instability. ^[3-5] However, at the dawn of these studies, it was hard to foresee such a nontrivial mechanism of suppression of the pinch effect in a semiconductor plasma.

The authors of ^[2] gave the following estimate to favor the appearance of a pinch effect in the electron-hole plasma of InSb. If the current $I \approx 5$ A at which one observes branching of the VAC curves in an external magnetic field (Fig. 1) corresponds to the Bennett current of (2), then we can estimate from this relationship the overall temperature of the electrons and holes if we know the drift velocity. With the value $v_g \approx 5 \times 10^7$ cm/sec, as obtained by measuring the Hall constant R_H ($v_g = R_H j$, where j is the current density), we can show that $k(T_e + T_h) \approx 0.13$ eV. In the opinion of the authors of ^[2], this estimate is quite reasonable, since the temperature of the electrons and holes cannot appreciably exceed the energy of the optical phonons (≈ 0.025 eV in InSb) if the lattice is not heated during the pulse. In this case ($kT_e > 0.025$ eV), the electrons intensively emit optical phonons (in a time $\sim 10^{-12}$ sec ^[50]), and this process prevents further heating of the plasma. Hence the compression regime of the plasma in the pinch effect in semiconductors is close to isothermal, ^[50, 58, 59] in which the temperatures of the electrons and holes match the energy of the optical phonons. We can make more precise the estimate of Glicksman and Steele ^[2] by accounting for the fact that the current passing through the plasma (apart from the impurity component) figures in the Bennett criterion (2), while the value of the drift velocity chosen by the authors of ^[2] is evidently too high (when $v_g = 5 \times 10^{17}$ cm/sec, the impurity current > 4 A). According to the data of ^[36, 60], $v_g \approx 3 \times 10^7$ cm/sec at the onset of impact breakdown in n-InSb. We can get this same value by assuming that the concentrations of the plasma and of the impurity electrons are approximately equal at the point of branching of the VAC in a magnetic field. After introducing the appropriate corrections, we can find that $k(T_e + T_h) \approx 0.05$ eV. This agrees far better with the ideas presented above and with the results of later studies. ^[58, 60-62]

Thus, the estimates made show that one of the fundamental criteria for strong compression of a plasma in the pinch effect (the Bennett criterion) can be fulfilled under the conditions of Glicksman and Steele's experiments. ^[1] This criterion has played an extremely important role in "identifying" the pinch effect in a solid-state plasma.

The existence of a critical current passing through the plasma (I_p) at which a pinch effect arises in an impact-ionized plasma in n-InSb has been proved more firmly in the experiments of Chynoweth and Murray, ^[60] who made measurements analogous to those of ^[1] on

specimens having a higher resistance (owing to decrease in the cross-sectional area S). Hence they were able to distinguish between the current for onset of impact ionization ($I \approx 1.5$ A) and for the pinch effect ($I \approx 4$ A) (Fig. 2). It is an essential point that the results of the measurements of [60] on single-crystal and polycrystalline specimens agreed, while the value of the critical current depended weakly on the dimensions of the specimen, in agreement with Eq. (2).

Interesting measurements that confirm Glicksman and Steele's [1,2] fundamental conclusions have been performed on p-InSb ($T_c \approx 77^\circ\text{K}$) in an injection regime. [62] In order to observe a pinch effect in p-InSb specimens from the shape of the VAC, it is quite necessary to inject into the crystal a plasma whose concentration is comparable with the impurity concentration, since the mobility of electrons is much greater than that of holes ($b_e/b_h > 50$ [62]), and a non-equilibrium plasma contributes appreciably to the conductivity of the specimen, even when $n \ll p_0$. The concentration of impurities should not be very large: $p_0 \lesssim 10^{15}-10^{16}$ cm^{-3} . The mobility of the electrons is appreciably diminished in the opposite case. [62] Thus, in the experiments of [62], an anomaly in the VAC stemming from the pinch effect was observed at $n/p_0 \gtrsim 0.06$.

In closing the description of the first studies on the pinch effect in semiconductors, I should note that all of the conclusions on the nature of the observed anomalies in the VAC were to some extent hypothetical, and the investigators were faced with the problem of inventing more direct methods for observing the strong compression of the plasma in the pinch effect. The next chapter treats this problem.

3. METHODS OF DETECTING THE PINCH EFFECT IN A SOLID-STATE PLASMA

In [58,63], they used an optical method of detecting the pinch effect in an impact-ionized plasma in n-InSb. Osipov and Khvoshchev [58] studied the intensity distribution of the recombination radiation of the plasma ($\lambda \approx 4-6$ μm) over the cross-section of the specimen ($V = 0.26 \times 0.14 \times 1.5$ cm^3 , $T_c = 77^\circ\text{K}$). They used a scanning system with a moving mirror, which made it possible to verify graphically the existence of the compressed plasma filament, and to observe the change of shape of the glowing region when acted on by an external

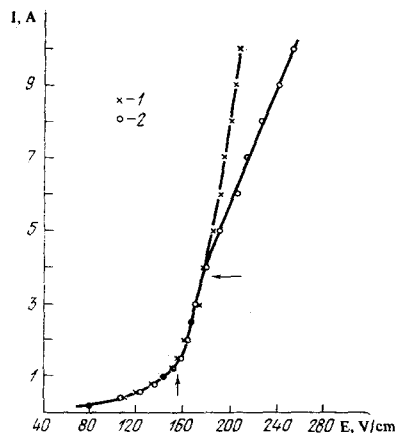


FIG. 2. Volt-ampere characteristic of n-InSb at $T_c = 77^\circ\text{K}$. [60] H (in Oe) = 350 (1) and 0 (2). The vertical arrow indicates the onset of impact ionization, and the horizontal arrow the pinch-effect.

magnetic field. Such measurements permit one to determine the density distribution of the non-equilibrium plasma. Figure 3 shows the change in the nature of the distribution of infrared (IR) emission along the 0.26 cm face when acted on by a longitudinal magnetic field. The current through the specimen ($I_{\text{pinch}} \approx 50$ A, $\tau_{\text{pulse}} = 4 \times 10^{-6}$ sec) was held constant as the magnetic field was varied. When $H = 0$, the glowing region was localized in the center of the specimen in the shape of a narrow filament (curve 1 of Fig. 3). An estimate of the dimensions of the glowing region with account taken of the resolving power of the scanning system gave ≈ 0.02 cm (the radius of the filament $r_p \approx 0.01$ cm). Here the mean plasma concentration in the filament was $\bar{n} \approx 3 \times 10^{16}$ cm^{-3} ($v_g \approx 3 \times 10^7$ cm/sec). The glowing region expands with increasing magnetic field, while the emission intensity increases. These data (Fig. 3, curves 2-4) unequivocally indicate abolition of the pinch in the longitudinal magnetic field, since the absorbing region between the filament and the surface of the crystal diminishes as the spatial distribution of the plasma expands. Accordingly, the intensity of the radiation should increase, especially in the short-wave region of the spectrum. The absorption coefficient in the region of the studied wavelength range [58,64] is $\approx 10^3$ cm^{-1} , and it increases with the frequency as $\sqrt{h\nu - E_g}$. [64]

An appreciable instability in the emission distribution arises in a magnetic field of $H = 400$ Oe (curve 3 in Fig. 3). At $H = 10^3$ Oe, it acquires a two-humped shape, which seems to indicate a tubular type of intensity distribution. In order to observe the true spectrum of the recombination radiation of the plasma in the pinch effect, Osipov and Khvoshchev [58] "drew" the glowing filament to the surface of the specimen with a transverse magnetic field [65] (H_{\perp}). As H_{\perp} increased, the intensity of emission grew while the spectrum was shifted toward shorter wavelengths. In the range 2.5×10^3 Oe $\leq H_{\perp} < 7 \times 10^3$ Oe, the observed spectrum ceases to change, and the authors of [58] think that it is the true recombination-radiation spectrum of the plasma filament. Comparison of the short-wavelength region of the obtained spectrum with that of a black body showed that the overall temperature of the electrons and holes was approximately twice the energy of an optical phonon (≈ 0.05 eV). It is pleasant to note that Osipov and Khvoshchev's experiments have played a decisive role in "confirming" the pinch effect in a solid-state plasma.

In [66], the spatial distribution of a plasma in the pinch effect (n-InSb, $T_c = 220^\circ\text{K}$, $\tau_{\text{pulse}} \approx 3.3 \times 10^{-7}$ sec) was determined by using a high-frequency probe. The re-



FIG. 3. Intensity distribution of the recombination radiation over the cross-section of an n-InSb specimen: [58] H (Oe) = 0 (1), $H = 200$ (2), 400 (3), and 1000 (4). The current I_p in the plasma = 50 A, and the current-pulse duration $\tau_{\text{pulse}} = 4$ μsec .

flected signal was measured, being a function of the plasma concentration at the site of the probe on the surface of the crystal. The specimens were plates ($V = 0.07 \times 0.01 \times 0.14 \text{ cm}^3$). They were scanned along the 0.07-cm dimension. The resolving power of this system was $\approx 0.01 \text{ cm}$. Narrowing of the spatial distribution of the plasma was seen with increasing current, and at $I \approx 20 \text{ A}$, the half-width of this distribution was at the level of the resolving power of the measuring system. These experiments also indicate strong pinching of the plasma. The pinch was abolished in a longitudinal magnetic field, and oscillations appeared on oscillograms of the reflected signal that indicated onset of instability in the plasma.

Chen and Ancker-Johnson^[67] applied a laser-probe method ($\lambda = 10.6 \text{ }\mu\text{m}$, CO_2 laser) for determining the spatial distribution of the plasma in p-InSb (injection regime) in the pinch effect. The theoretical side of this method in which one studies the IR absorption of free carriers, was developed by Harrick^[68] in 1956. These experiments^[67] showed that a pinch effect arises only at high enough currents passing through the plasma. Figure 4 shows some individual results of these measurements.

Morisaki^[59,69] has applied an interesting method for determining the dimensions of the plasma filament in experiments with n-InSb ($n_0 = 10^{14} \text{ cm}^{-3}$, $T_e = 77^\circ \text{K}$, $\tau_{\text{pulse}} = 10^{-6} \text{ sec}$, $V = 0.1 \times 0.1 \times 0.5 \text{ cm}^3$). By using a resonance method, these studies measured the inductivity of the plasma filament in the frequency range 800–1400 MHz. Since the inductivity of the plasma filament increases with decreasing radius, then in principle one can determine the current-dependence of the radius of the filament from the shift in the resonance frequency as the current is increased. Figure 4 shows the results of these measurements. For comparison, the same diagram shows the data of analogous measurements using other methods. As we see from this diagram, the radius (r_p) of the filament decreases as the current varies from 13 to 20 A (the resonance is poorly marked for $I < 13 \text{ A}$). Then it increases, for some reason that is not fully understood. In Morisaki's opinion^[59], this might involve the onset of a population inversion in the plasma at high currents when the plasma concentration is high enough. The arising photon avalanche will restrict the growth of density in the filament, and when it emerges from its boundaries, it will produce non-equilibrium pairs and will expand the pinch channel. According to the calculations of^[50], the criterion (6) for transitions $h\nu = E_g$ is fulfilled at $n \approx 2 \times 10^{17} \text{ cm}^{-3}$ ($T_{e,h} = 250^\circ \text{K}$). According to Morisaki's estimates,^[59] the mean plasma concentration reaches just this value at a current of $\approx 20 \text{ A}$, at which the radius of the filament begins to increase. However, as I see it, this explanation is highly problem-

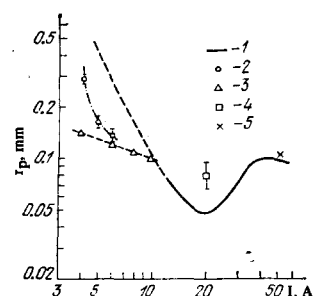


FIG. 4. Current-dependence of the radius of the plasma filament in InSb as measured by various methods. [59] Solid curve 1—data of Morisaki [59] (by measuring the inductivity of the specimen); 2—Ancker-Johnson and Chen [67] (laser probe); 3—Shotov and his associates [117] (from recombination-radiation spectrum); 4—Toda [66] (by measuring a reflected UHF signal); 5—Osipov and Khvoshchev [58] (from the width of the glowing region).

atical. Actually, at such a density, in spite of the rather high degeneracy of the electron gas ($T_e = 250^\circ \text{K}$), the holes still obey Boltzmann statistics, and the negative absorption coefficient will be very small, even if the condition (6) is satisfied.^[47] Hence it is hardly possible to explain the expansion of the plasma filament by the onset of a photon avalanche. Perhaps there is a second version of the observed filament expansion that involves lattice heating. Yet, by Morisaki's estimates,^[59] this heating was insignificant in his experiments. If the concentration of thermally ionized plasma sharply increases in the pinch process, then plasma can be ejected^[70] from the near-axial (hottest) region in a radial direction. This would also expand the pinch channel. The true reason for this effect is not clear. By using data^[71] on the $v_g(E)$ relationship in the pinch regime (obtained by measuring the Hall e.m.f. in the intrinsic magnetic field of the current), the shape of the VAC,^[71] and the results of measuring the radius $r_p(I)$ of the plasma filament^[59], Morisaki determined the relation of the mean density in the pinch channel to the current (the solid curve in Fig. 5). In constructing the "theoretical" relationship (dotted in Fig. 5), he used only the experimental $v_g(I)$ relationship, and the plasma density was determined from the condition that the magnetic and gas-kinetic pressures should be equal at the surface of the filament. Here he assumed that the temperature of the electrons and holes during plasma compression is equal to the energy of an optical phonon (the isothermal approximation). He also corrected for degeneracy of the electron component of the plasma. The rather good agreement (at $I < 20 \text{ A}$) of the relationships shown in Fig. 5 favors an isothermal type of plasma compression in the pinch effect.

One of the most interesting methods of observing the pinch effect in semiconductors involves time scans of the voltage on the specimen in a controlled-current regime. This problem will be taken up in the next section.

4. DYNAMICS OF THE PINCH EFFECT IN A NON-EQUILIBRIUM ELECTRON-HOLE PLASMA

Just as in a gaseous plasma,^[40,41] the pinch effect in semiconductors is essentially dynamic in nature. That is, it is determined by a rather complicated time course. This nature is manifested with special force in the pulsed methods of studying the pinch effect. Under conditions of impact ionization (or injection), one can observe a pinch effect if a non-equilibrium plasma of high enough concentration can fill the crystal. Thus, the time for impact ionization $g^{-1}(E) = (d \ln n/dt)^{-1}$ should be less than or comparable with the pulse duration. In this sense, the

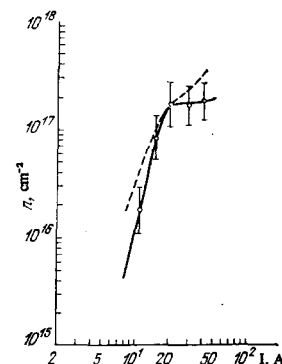


FIG. 5. Current-dependence of the mean concentration in the pinch channel. [59] Solid line—experimental data; dotted line—calculated.

concept of a critical field for impact breakdown ($E \approx 180-200$ V/cm in the experiments of [1,34-36]) has a relative meaning, since an appreciable filling of the specimen with plasma strongly depends on the relationship between the pulse duration and the impact-ionization time. Thus, the size of the breakdown field shifts to higher values for shorter current pulses. At the same time, the characteristic time for pinching of the plasma (τ_{pinch}) should also be shorter than the pulse duration. Since bulk recombination hinders plasma accumulation processes, the characteristic times of the corresponding processes (g^{-1} , τ_{pulse}) must be less than the bulk-recombination time (τ_r). Thus, fulfillment of the Bennett criterion alone does not suffice for observing a pinch effect.

In order to study the dynamics of the pinch effect in semiconductors, Glicksman and Powlus [61] measured the voltage across n-InSb specimens ($n_0 = 0.1-2 \times 10^{14}$ cm $^{-3}$, $V = 0.042 \times 0.040 \times 0.25$ cm 3 , $T_c = 77^\circ$ K) as a function of the time in a regime of square current pulses (of varying amplitude) or duration $(0.25-1.4) \times 10^{-6}$ sec. Figure 6 shows the corresponding time scans of the current and voltage ($\tau_{\text{pulse}} \approx 3 \times 10^{-7}$ sec, $n_0 = 2 \times 10^{14}$ cm $^{-3}$). As we see from Fig. 6a, the field and current oscillograms have the same shape at low current (or voltage) values, and no signs of impact ionization (and naturally, also of pinching) are seen. At large currents (Fig. 6b, $E \approx 250$ V/cm), the time for impact ionization is short enough⁴⁾, and a plasma of high enough concentration is created in the crystal within the pulse duration (the voltage declines). Within a certain time t_2 (see Fig. 6) after the onset of impact ionization, whose size strongly depends on the current, the voltage across the specimen rises again. This effect is due to the increase in the resistance of the plasma owing to pinching. Quite evidently (Fig. 6b-d), the characteristic time for compression of the plasma ($\tau_{\text{pinch}} \approx t_2$) decreases with increasing current. For relatively small currents ($I \approx 5$ A, Fig. 6b), noticeable signs of a pinch (anomalous resistance) arise at the end of the pulse, in spite of the fact that a dense enough plasma had been produced far earlier in the specimen. Hence a pinch effect could not develop at $I \approx 5$ A for shorter pulse durations ($\tau_{\text{pulse}} < \tau_{\text{pinch}}$).

Figure 7 shows the experimental and theoretical relationships of the pinching time [31,61] to the current in the plasma (I_p). The data of the experiments are given for specimens having different impurity concentrations, while the theoretical curves are drawn for different values of the hole mobility, which decreases with increasing content of impurities in the crystal. [74] As we see from this diagram, the theoretical values of τ_{pinch} agree well with experiment. Hence I shall briefly take up the derivation of the theoretical pinch $\tau_{\text{pinch}}(I_p)$ relationship, while restricting the treatment to simpler estimates than those given in [50,75]. As I have noted, the compression of the plasma in the pinch effect in semiconductors arises from ambipolar drift of the electrons and holes in the longitudinal electric field and the azimuthal magnetic field of the current (H_ϕ). One can

⁴⁾Detailed data on the $g(E)$ relationship obtained in a regime of nanosecond voltage pulses are given in [51,72,73] In Morisaki's opinion, [59] the values of $g(E)$ should be higher under pinch-effect conditions than those measured by the authors of [51,72] since the distribution function of the electrons is shifted here toward higher energies.

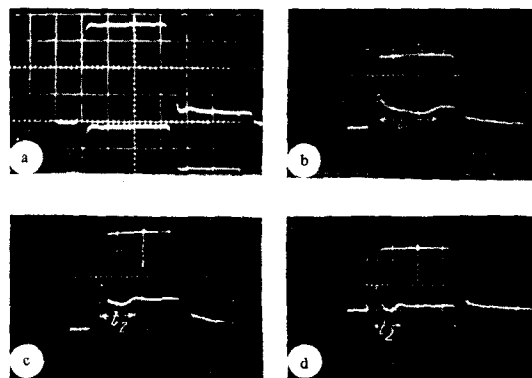


FIG. 6. Time scans of the current (upper beam) and electric field intensity (lower beam) in n-InSb specimens ($T_c = 77^\circ$ K). [61] Current (in A): 1.4 (a), 5.5 (b), 7.3 (c), and 9.7 (d). Field: a) 39 V/cm is the value of a large division, b-d) 70 V/cm per division. Time scale—0.1 μ sec/division. The field is measured from the lower mark.

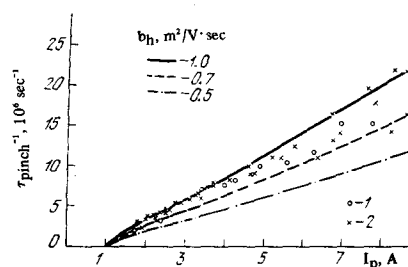


FIG. 7. Relationship of the pinching time ($\tau_{\text{pinch}} = t_2$) to the current in the plasma. [31,61,95] Experimental points: n_0 (cm $^{-3}$) = 2×10^{14} (1) and $(1-4) \times 10^{13}$ (2). The curves are the calculated data for various values of the mobility of holes.

derive an expression for the velocity of ambipolar drift ($v_{er} = v_{hr} = v_r$)

$$v_r = -\frac{b_e b_h}{4\pi e (b_e + b_h)} \frac{H_\phi}{rn} \frac{\partial r H_\phi}{\partial r} - \frac{D}{n} \frac{\partial n}{\partial r} \quad (7)$$

by using the equation of motion for electrons and holes and the Maxwell equation ($\text{curl } H = (4\pi/c)j$). Inertia effects were not taken into account in deriving (7) ($\tau_{\text{pinch}} \gg \tau_{e,h}$). It was assumed that the mean free path of the carriers and the Debye screening radius are considerably smaller than the characteristic inhomogeneity dimension of the plasma $(d \ln n/dr)^{-1}$, and the relaxation time of the carriers with respect to the pulse depends weakly on the energy,⁵⁾ while the plasma concentration is large ($n \gg n_0, p_0$).

We can easily estimate by using (7) the characteristic time for pinching if we neglect the diffusion component of the velocity ($I > I_B$) and the magnetoresistance ($j_z = env_g$):

$$\tau_{\text{pinch}} \approx \frac{c^2 R_0^2}{4b_n v_g I_p} \quad (8)$$

Here R_0 is the initial dimension of the plasma. We assume that $b_e \gg b_h$.

Equation (8) agrees well with the experimental data (Fig. 7) in the high-current region ($I_p > 3$ A, $R_0 = 0.02$ cm, $v_g \approx 3 \times 10^7$ cm/sec).

Let us try by using (7) to justify [29] the expression

⁵⁾If we consider the abundance of various mechanisms of scattering (by phonons, or impurities, or electron-hole scattering), whose roles under pinch-effect conditions have not been determined too exactly, this assumption is to a certain degree justified. [75]

for the critical Bennett current of (2). If we use the condition of equilibrium $v_r = 0$ (which it is if we neglect processes of generation and recombination of carriers), and integrate the corresponding expression over the radius, then we can derive the relationship

$$I^2 = 2c^2 N k (T_e + T_h),$$

where N is the number of particles per unit length of the plasma filament. In deriving this equation, from which we can easily find the value of I_B in (2), we have assumed that $n = 0$ at the surface of the filament (or of the specimen). We note that the most rigid assumption used in this calculation involves neglecting the generation and recombination processes.⁶⁾ Evidently, strictly speaking, it is justified to use the Bennett criterion in identifying the pinch effect in a series of experimental studies if the critical currents at which the VAC anomalies arise correspond to the condition $\tau_{\text{pinch}} \ll g^{-1}(E)$, τ_p , R_0/s , where s is the rate of surface recombination. These criteria can be satisfied for a given current if the radius (R_0) of the specimen is not too large.

One can show^[50, 75] that when $I > I_B$, the plasma filament collapses to a point ($r_p \rightarrow 0$) within the time τ_{pinch} of (8) in the approximation in which the number of particles is conserved in isothermal compression. This evidently absurd result, which does not fit the experimental data (as we see from Figs. 6c and 6d, onset of a steady state is observed after a non-stationary phase of pinching), stems from using the corresponding approximations, which fail upon strong compression of the plasma. First, the rate of quadratic bulk recombination sharply rises, and the number of particles is not conserved during compression.^[54-56] Second, upon strong compression, the plasma becomes degenerate, and the effective temperature of the carriers ($\epsilon_F \sim n^{2/3}$) rises.^[50, 76] This factor, which stabilizes further compression, is enhanced^[50] under conditions of transverse breakdown^[77] in the Hall field

$$E_r = \frac{v_g H_\phi}{2c}.$$

Interestingly, as early as 1932, in the paper "On the Theory of the Stars," Landau pointed out the need of taking account of the strong degeneracy of the electron gas in treating the problem of gravitational collapse.^[78] To a certain degree, accounting for the degeneracy of the electron-hole gas under strong compression is equivalent to the adiabatic approximation that Glicksman^[75] used to rule out collapse in the theory of the pinch effect. However, this approximation is not always justified in a solid because a considerable heating of the electron-hole plasma cannot happen if the lattice is not heated.

Moreover, with strong compression of the plasma, when $b_e b_h H_\phi^2 / c^2 \gg 1$, we must take account of the magnetoresistance and the decrease in the velocity of radial drift. In a fixed-current regime, the increase in resistance of the plasma leads to a considerable redistribution of the electric field and to a change in rate of generation of non-equilibrium carriers.

The above-cited factors become essential long before the approximation used in deriving (7) breaks down. Accordingly, a number of theoretical studies^[56, 79-84] have been concerned with calculating the steady-state and quasi-steady-state characteristics of a plasma under

⁶⁾In this approximation under equilibrium conditions ($\partial n / \partial t = 0$), the magnetoresistance is zero ($v_r = 0$).

pinch-effect conditions with account taken of various processes that stabilize against further compression of the plasma.

Let us turn to describing the very interesting results obtained by Glicksman and Powlus.^[61] In the authors' words, a steady state of a plasma becomes unstable in the presence of a longitudinal magnetic field, and as the field (H) increases, the main sign of a pinch in these experiments (anomalous resistance of the plasma) vanishes. Unfortunately, the pertinent oscillograms have not been published. Glicksman and Powlus very cautiously advanced the hypothesis that the observed instability corresponds to helical instability,^[3-5] but they gave no proof. We note that a voltage instability was observed at high enough currents ($I > 7$ A), even in the absence of a longitudinal magnetic field.^[61] Measurements analogous to the above-described^[61] have also been performed on specimens of p-InSb in an injection regime.^[62] This study gave some interesting data on the $\tau_{\text{pinch}}(I)$ relationship for different injection levels (n/p_0), as well as results of studying the plasma instability that arises both in the pinch regime ($H = 0$) and during breakdown of plasma compression in a longitudinal magnetic field.

Thus far in discussing the experimental results, we have restricted the treatment to the purely magnetic stage of the pinch effect in semiconductors, and have practically not treated the phenomena caused by lattice heating in the region of localization of the plasma filament.

Let us proceed to discuss this interesting problem.

5. THE MAGNETOTHERMAL PINCH IN SEMICONDUCTORS

When the current passing through the plasma is high enough, the magnetic compression (Bennett pinch) gives rise to a thin plasma filament into which practically all the current is shunted. Accordingly, all of the power is liberated within the channel of this filament. If the duration of the current pulse is long enough, then the lattice temperature in the pinch channel increases, and the equilibrium plasma concentration increases. Upon strong lattice heating, the equilibrium plasma generated by thermal ionization plays a substantial role in the overall balance of numbers of carriers, and its density can appreciably exceed the concentration of the non-equilibrium plasma. In Ancker-Johnson's terminology,^[30] this stage of the pinch effect is called the magnetothermal stage. At very high powers, the magnetothermal pinch is converted into a thermal pinch,^[38, 85] which has the nature of an explosion, and is accompanied by melting of the lattice in the region where the plasma filament lies.

All of the listed stages in the pinching of a plasma have been studied experimentally and theoretically in the classical studies of Ancker-Johnson and Drummond^[38, 70] for InSb. As I have emphasized, the magnetic pinch plays the role of a detonator to give rise to the magnetothermal and thermal pinch effects. At large currents, the originally produced plasma is compressed in a relatively short time ($\tau_{\text{pinch}} \ll \tau_{\text{pulse}}$) into a thin filament whose radius $r_p \ll R_0$ (Fig. 4). We can estimate the change in the lattice temperature within the filament by the relationship^[75]

$$\Delta T_c = \frac{(P_L - P'_L) \Delta t}{\pi r_p^2 c_p} \quad (9)$$

Here P_L is the power generated per unit length of the specimen (of the filament), P'_L is the losses involving heat conduction, c_p is the heat capacity of the crystal, and $\Delta t = \tau_{\text{pulse}} - \tau_{\text{pinch}}$.

When the power and the pulse duration are large enough, the lattice heating can be very substantial if the radius of the filament is small (Eq. (9)). We recall that the equilibrium plasma concentration at $T_c = 300^\circ\text{K}$ in InSb^[53] $n_p \approx 2 \times 10^{16} \text{ cm}^{-3}$, while the mean density in the pinch channel in the experiments of Osipov and Khvoshchev^[58] was $\bar{n} \approx 3 \times 10^{16} \text{ cm}^{-3}$. Let us estimate, by using (9), the role of lattice heating in the previously discussed studies.^[58, 59] If we assume that $P'_L \approx 0.4 \text{ kW/cm}$,^[38] $c_p = 1 \text{ W} \cdot \text{sec} \cdot \text{cm}^{-3} \cdot ^\circ\text{K}^{-1}$,^[86] then in Osipov and Khvoshchev's experiments ($I_{\text{pinch}} = 50 \text{ A}$, $E \approx 300 \text{ V/cm}$, $\tau_{\text{pulse}} = 4 \times 10^{-6} \text{ sec}$, $r_p = 0.01 \text{ cm}$, $T_c = 77^\circ\text{K}$), $\Delta T_c \approx 200^\circ$. Thus the heating at the end of the pulse is rather large. In the experiments of Morisaki^[59] ($I = 20 \text{ A}$, $E \approx 300 \text{ V/cm}$, $\tau_{\text{pulse}} = 10^{-6} \text{ sec}$, $r_p = 5 \times 10^{-3} \text{ cm}$, $T_c = 77^\circ\text{K}$), $\Delta T_c \approx 60^\circ$, and we can consider the lattice heating not to affect substantially the balance of numbers of carriers. The reliability of these estimates corresponds to the degree of accuracy with which the radius of the plasma filament is determined in the cited studies.

If thermal plasma generation plays an essential role, then in a fixed-current regime, the electric field intensity can decrease during lattice heating. Hence the heating ceases at a certain stage of the compression, since the generated power diminishes. Correspondingly the plasma density falls, and the voltage across the specimen increases. Figure 8 shows the calculated time-dependences⁷⁾ of the lattice temperature (curve 2) and of the electric field intensity at the specimen (3) in a magnetothermal pinch regime. For comparison, the same diagram (in the inset) gives a characteristic oscillogram of $E(t)$ obtained in the experiments on p-InSb. The qualitative courses of the calculated and observed $E(t)$ relationships are practically the same. Hence, in the opinion of the authors of^[30, 70] the anomalous resistance of the plasma that has been observed in many experiments can arise partially from the discussed specific lattice heating under controlled-current conditions. The calculations of^[70] were performed within the framework of the following model. The plasma filament that had arisen from magnetic compression was divided into two regions: in the outer one, the temperature of the carriers was assumed equal to the energy of an optical phonon and the effect of heat generation was not considered; in the inner (hot) region, intensive lattice heating occurs, and the plasma concentration is determined by the equilibrium⁸⁾ value:^[87]

$$n(r, t) = n_p = 4.2 \cdot 10^{14} \text{ cm}^{-3} \left(\frac{T_c}{K} \right)^{3/2} \exp \left(-0.125 \frac{eV}{kT_c} \right) \text{ for InSb.} \quad (10)$$

Curve 1 in Fig. 8 describes the variation in the plasma density $n(t)$ in the outer region, for which the inner region supplies plasma. According to the calculations of^[70], when the power is large enough, the plasma concentration in the outer region can be even greater than in the inner region, and the spatial distribution of the plasma is

⁷⁾The time is measured in Fig. 8 from the instant $t = \tau_{\text{pinch}}$ in arbitrary units.

⁸⁾A more general expression for n_p that accounts for possible degeneracy of the electron gas and an $E_g(T_c)$ dependence has been given in^[88].

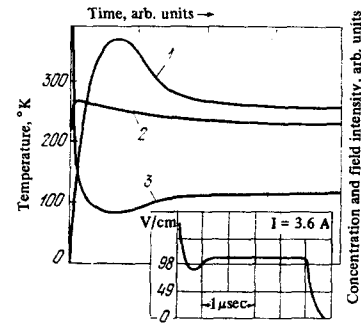


FIG. 8. A magnetothermal pinch.^[30, 70] Calculated time-dependences: plasma concentration in the outer region of the pinch channel (1), lattice temperature (2)—inner region, electric field intensity (3). Inset: results of measuring $E(t)$ in an injection regime (p-InSb).

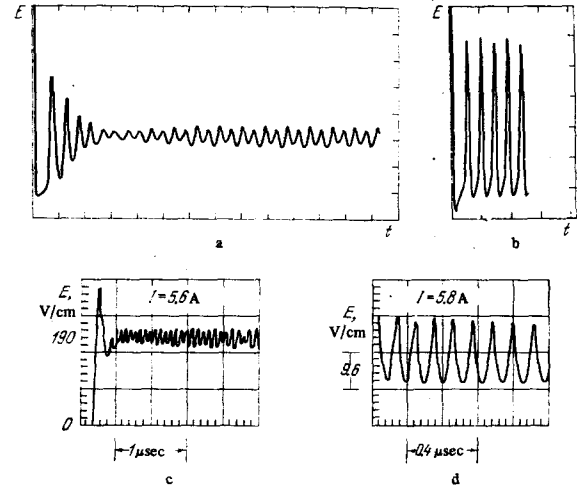


FIG. 9. A magnetothermal pinch.^[70] Calculated (a, b) and experimental (c, d) time-dependences of the electric field intensity.

tubular in nature. This feature of the solutions arises from the large temperature gradient in the region near the axis ($\nabla T < 0$), so that the Lorentz forces and the plasma pressures ($\nabla n T$) are balanced even when $\nabla n > 0$ near the axis of the specimen. We note that the tubular plasma distribution observed in Osipov and Khvoshchev's^[58] experiments in the presence of a longitudinal magnetic field is apparently due to onset of helical instability.

At large enough currents, the establishment of a steady state of the plasma in a magnetothermal pinch is oscillatory in nature.^[70] This is caused by relaxation phenomena in the fixed-current regime (periodic heating and cooling of the lattice). Figure 9 shows the calculated (a, b) and experimental⁹⁾ (c, d) $E(t)$ relationships. These relationships agree well in the shape of the oscillations. It is hard to say anything about the quantitative agreement, since the calculated results are given in arbitrary units. The frequency of the relaxation oscillations of the voltage increases with increasing current^[70], in line with the experimental data.

Let us proceed to discuss the thermal stage of the pinch effect. Burgess^[85] has proposed a model of a thermal pinch that consists in the following. If a semiconductor crystal is put into a cryostat, then when a strong enough current passes through it, the heating is a maximum in the region near the axis. Accordingly, the

⁹⁾The experiments were performed on p-InSb in an injection regime.^[70]

conductivity increases, and the current is concentrated in this region. This process at high currents is explosive in nature, and it ends with lattice melting in the region near the axis. The conditions for onset of a thermal pinch are considerably eased if the region of local lattice heating is "preconditioned" by magnetic compression of a non-equilibrium plasma. Let us relate briefly the corresponding experiments.^[38] The studies were performed on InSb specimens ($T_c = 77^\circ \text{K}$), both in an injection regime (p-type crystals) and with impact ionization (n-type) in the power range $2 \text{ kW/cm} \leq P_L \leq 18 \text{ kW/cm}$ ($1.4 \times 10^{-6} \text{ sec} \leq \tau_{\text{pulse}} \leq 7.2 \times 10^{-6} \text{ sec}$). Table I gives the parameters of the specimens, the pulse, and the plasma for certain of the studied crystals. Specimen No. 5 corresponds to an n-type crystal.

As a rule, the field intensity E declines considerably during the pulse in a fixed-current regime (to 45%, Table I, specimen No. 4). This indicates a considerable increase in the plasma concentration in the pinch channel owing to lattice heating. We recall that the overall mobility of electrons and holes in InSb declines rather strongly with increasing temperature:^[89]

$$b_e + b_h \approx 7 \cdot 10^9 \left(\frac{T_c}{\text{K}} \right)^{-1.6} \text{ cm}^2 \text{ V}^{-1} \text{ sec}^{-1} (T_c > 200^\circ \text{K}) \quad (11)$$

Nevertheless, the conductivity of the plasma rises owing to the more substantial increase in the concentration Eq. (10)). At high powers ($P_L > 6 \text{ kW/cm}$), the specimens spontaneously cleave at the end of the pulse in the longitudinal direction, and a recrystallized channel is seen on the cleavage surface, as in Fig. 10 (specimen No. 2, Table I). At lower powers, $2 \text{ kW/cm} < P_L < 6 \text{ kW/cm}$, such channels have been found after cleavage by ultrasound. Table I gives the dimensions of these channels. A lava eruption of the molten lattice material was observed in the region of the contacts in a number of cases (a volcano under laboratory conditions!) We can naturally assume that the dimensions of the found channels match the site of the plasma filament that arises from magnetic compression of the plasma. This idea is confirmed by the calculations of Akranov,^[76] who

TABLE I

$S \times 10^3, \text{ cm}^2$ (area of square cross-section)	$I, \text{ A}$	$I_p, \text{ A}$	$P_0 \times 10^{-14}, \text{ cm}^{-3}$	$E(t=0), \text{ V/cm}$	$-\Delta E(t = \tau_{\text{pulse}}), \text{ V/cm}$	$\tau_{\text{pulse}}, \mu\text{sec}$	$r_p \times 10^3, \text{ cm}$ (channel radius)
6.40	42	40	5.3	450	98	2.5	4
7.75	19	17.3	6.3	360	20	1.5	2.8
2.50	15.5	9.4	92	400	130	6.4	2.5
2.50	22	15	90	460	210	7.2	2.5
2.50	18	12.7	$0.7_{(\tau_{90})}$	400	40	3.8	3



FIG. 10. A thermal pinch.^[29,38] A photograph of a recrystallized channel formed at the site of the plasma filament (see Table I, specimen No. 2).

determined the current-dependence of the radius of the plasma filament from a condition of the type of (1), with account taken of degeneracy of the electron component of the plasma at the high lattice temperature. The calculated relationship agrees well with the data given in Table I.

I emphasize that a thermal pinch must arise at much higher currents and powers in the absence of a magnetic stage of compression, while its radius will exceed by an order of magnitude the dimensions of the observed channels. This conclusion has been drawn^[29] after performing the appropriate calculations on a computer for crystals having the parameters indicated in Table I. Hence we can affirm that the studies of Ancker-Johnson and Drummond^[38,70] have permitted a firm proof of the appearance of a magnetic pinch effect in the electron-hole plasma of semiconductors.

I also note the extremely interesting study of all of the stages of the pinch effect that Dobrovolskiĭ and Vinoslavskii^[90] have carried out on crystals of n-Ge in an injection regime with power supplies up to 200 kW/cm in currents of $\approx 200 \text{ A}$ ($\tau_{\text{pulse}} \leq 5 \times 10^{-5} \text{ sec}$). In the opinion of Brandt and his associates,^[33] lattice heating substantially influences the dynamics of the pinch effect in the $\text{Bi}_{1-x}\text{Sb}_x$ compounds.

The analog of the magnetothermal pinch in a gas-discharge plasma is to a certain extent the overheat instability^[91] that develops in a weakly ionized plasma at high enough powers: $\sigma E^2/P > \tau_{\text{pulse}}^{-1}$, where P is the gas pressure. The reasons why this instability arises are the following. If an increase in the temperature of the neutral gas has arisen in any region of the discharge, then this will enhance the Joule heating in this region, since the density of the gas decreases ($P = \text{const.}$). The heating leads to further temperature increase and decreased gas density. The degree of ionization sharply rises in the "overheated" region, and the current of the discharge is shunted into this region. Owing to filament formation by the current, the uniform glow of the discharge ceases. Pinching of the filament can hasten the development of overheat instability. The studies of Velikhov and his associates^[92] have shown that overheat instability is the fundamental factor that disrupts the generation of coherent radiation in combination-pumped gas lasers. People use through-pumping of neutral gas in these systems at rather high velocity in order to prevent the onset of overheat instability.^[92] I shall discuss below also another possibility of suppressing this instability in connection with studies^[24,93] of breakdown of the magnetothermal pinch in an electron-hole plasma in the presence of a longitudinal magnetic field. Let us proceed to discuss the problems of stability of the pinch effect in a solid-state plasma.

6. STABILITY OF AN ELECTRON-HOLE PLASMA UNDER PINCH-EFFECT CONDITIONS

Even the first studies^[61, 62, 70, 94] on the pinch effect in semiconductors under anomalous-resistance conditions noted an oscillation of the voltage across the specimen. These oscillations were quickly damped at low currents, but they existed throughout the pulse upon increasing the current. As a rule, the frequency and amplitude of these oscillations increase^[70, 95] with increasing I_p . Figure 11 shows a typical oscillogram of the voltage (a) across the specimen under pinch-effect conditions ($I = 15 \text{ A}$) for

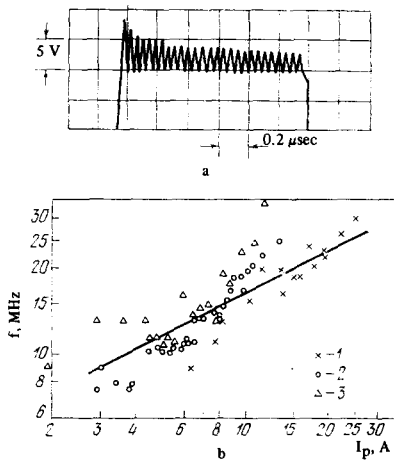


FIG. 11. Stability of the pinch in an impact-ionization regime (n-InSb). [61,95] a) Voltage oscillogram, $I = 15$ A, $n_0 = 2 \times 10^{14}$ cm $^{-3}$; b) current-dependence of the frequency of pinch oscillations; [95] 1— $n_0 = 2 \times 10^{14}$ cm $^{-3}$; 2— 5×10^{13} cm $^{-3}$, L (length of the specimen) = 0.8 cm; 3— $n_0 = 5 \times 10^{13}$ cm $^{-3}$, $L = 0.4$ cm.

n-InSb, and the dependence of the frequency of the observed oscillations on the current (b). As we see from comparing the results shown in Figs. 7 and 11b, the frequency of the oscillations agrees in order of magnitude with τ_{pinch}^{-1} .

The literature contains varied opinions on the nature of these oscillations, which arise even when $H = 0$. On the one hand, these oscillations can involve relaxation processes that accompany the establishment of a steady-state (the final form of the spatial distribution). [61,82] As the calculations of [82] have shown, the relaxation oscillations that arise in the pinch effect under impact-ionization conditions may not involve lattice heating, [70] and can correspond to oscillations in the mean plasma concentration with account taken of quadratic bulk recombination. The frequency of these oscillations should be comparable with τ_{pinch}^{-1} , while the decay time is of the order of the ambipolar diffusion time ($\tau_D \approx R_0^2/D$) or the bulk recombination time.

Steele and Hattori [43] have advanced the hypothesis that the observed oscillations stem from excitation of sound waves under conditions in which the velocity of ambipolar drift exceeds the velocity of sound. In this case, standing waves should be excited in the specimen at the frequencies: $f_n = nv_s/2b$ ($n = 1, 2, \dots$; Fabry-Perot resonances), where b is the dimension of the specimen in the direction of compression of the plasma, and v_s is the velocity of sound. Comparison with the experimental data on the threshold for excitation of oscillations shows good agreement, both for n-InSb [61] and for InAs [43] for the mode $n = 2$. In the opinion of the authors of [43], the excitation of precisely this mode arises from the need of satisfying the criterion of weak damping of the oscillations ($\omega\tau_p > 1$). The mechanism of excitation of sound waves that Steele and Hattori proposed has actually been observed in InSb. [96] Nevertheless, the strong dependence of the frequency of the pinch oscillations on the current (Fig. 11b) does not allow us to draw an unequivocal conclusion favoring this mechanism. The hypothesis of excitation of Alfvén waves [62,95] that propagate azimuthally along the intrinsic magnetic field of the current under the conditions of the experiments of [61,62] is ill-grounded, since these waves should be strongly damped ($\omega\tau_{e,h} \ll 1$). The most detailed

study of pinch oscillations in the absence of an external magnetic field has been performed by Ancker-Johnson and Chen [93,97] on p-InSb specimens in an injection regime. In these studies, they used a system of lateral and longitudinal probes in order to determine the spatial structure of perturbations of the type of $\exp i(kz + m\varphi + \omega t)$. It turned out that the spontaneous oscillations that arise in the pinch regime are due to excitation of the so-called sausage modes ($m = 0$) that are well known to investigators in the controlled thermonuclear fusion field. There is an opinion [40] that it is precisely the sausage modes of the plasma filament that facilitate the appearance of neutron radiation of accelerator type in powerful gas discharges. Here, as a rule, along with the mode $m = 0$, the helical mode ($m = 1$) was also excited at the same frequency, but with a far smaller amplitude. [93] The modulation of the current and voltage by $m = 0$ perturbations attained the value of $\approx 38\%$ ($T_c = 77^\circ\text{K}$) (Fig. 12), while at a lower lattice temperature it was as much as 50% ($T_c = 4^\circ\text{K}$). The amplitude of the spontaneous oscillations considerably increased in the direction of drift of the minority carriers (electrons) (Fig. 12). They showed that the relative value of the modulation and the frequency of the oscillations (Fig. 13) vary in time. Figure 14 shows the relationship of the wavelength ($\lambda = 2\pi/k$) to the current for the modes $m = 0$ and 1. As we see from this diagram, the sausage modes are of a long-wave type ($\lambda = 2L$) that depends weakly on the current. The wavelength of the helical mode declines rather strongly with increasing I . At values of I below the threshold for exciting spontaneous oscillations, but larger than the critical pinching current, spatial amplification was shown to be possible for a signal that corresponds to the mode $m = 0$ at the frequencies of spontaneous excitation. An amplification was attained of

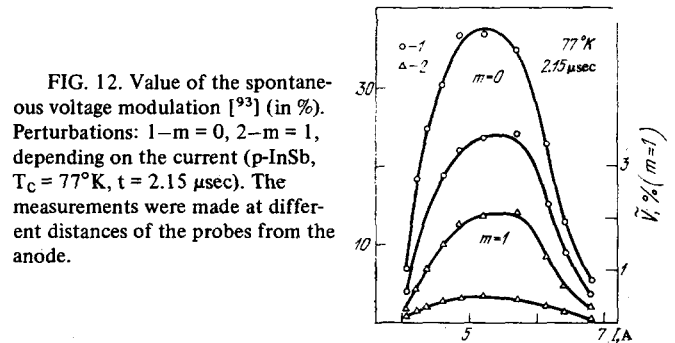


FIG. 12. Value of the spontaneous voltage modulation [93] (in %). Perturbations: 1— $m = 0$, 2— $m = 1$, depending on the current (p-InSb, $T_c = 77^\circ\text{K}$, $t = 2.15$ μsec). The measurements were made at different distances of the probes from the anode.

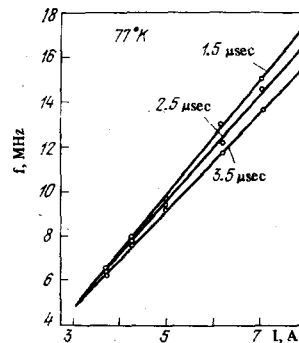


FIG. 13

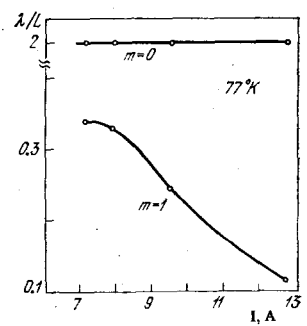


FIG. 14

FIG. 13. Current-dependence of the oscillation frequency at different instants of time. [93]

FIG. 14. Current-dependence of the wavelength ($\lambda = 2\pi/k$) of the modes with $m = 0$ and 1. [93]

≈ 19 dB/cm ($f = 6$ MHz). A number of the experimental results obtained in these studies^[93, 97] can be explained within the framework of the magnetothermal-pinch theory.^[70] A dispersion relationship was derived in^[98] for perturbations $\sim e^{Dt}$ by using a previously proposed model.^[70] A set of parameters enters into this relationship that cannot be calculated or directly measured (e.g., the current in the "hot" region of the pinch channel, the dimensions of this region, etc.) However, one can determine these parameters by fitting the experimental current- and time-dependences of the oscillation frequency to the theoretical ones. After this rather complicated procedure, one can calculate the current- and time-dependences of the oscillation amplitude, which actually agree well with the experimental results. Perhaps the "magnetothermal" interpretation of the pinch oscillations is true as applied to experiments under conditions of a prolonged current pulse,^[94] since the fundamental parameters of the plasma vary strongly in time at precisely this stage of the pinch. Yet the pertinent calculations^[98] are far from perfect. One cannot explain on their basis the strong spatial dependence of the amplitude of spontaneous oscillations (see Fig. 12). These calculations do not consider the spatial structure of the perturbations (in particular, the appearance of the mode $m = 1$ cannot be understood within the framework of this theory¹⁰). In this regard, the results of experiments in a regime of spatial amplification of the mode $m = 0$ cannot be explained. Hence the problem of the reason for excitation of sausage modes under pinch-effect conditions in semiconductors remained open even after these studies. This was all the more true, since the theoretical studies of Igitkhanov and Kadomtsev^[101] had shown that excitation of long-wavelength sausage modes in the pinch-effect in semiconductors may not involve lattice heating; it suffices^[101] to take correct account of the drift currents in the intrinsic magnetic field of the current.

I should note that no study has been made of the spatial structure of the perturbations responsible for the pinch oscillations in specimens of n-InSb (impact-ionization regime). To a certain extent, this gap has been filled by the studies of Brandt and his associates,^[33] who studied pinch oscillations in the compound $\text{Bi}_{0.912}\text{Sb}_{0.088}$ under conditions of interband breakdown. The authors of^[33] assume that the observed oscillations ($f = 1-30$ MHz) are due to the magnetothermal nature of the pinch, but the measurements of the spatial structure of the perturbations have shown that the mode $m = 0$ did not arise with an appreciable amplitude. As they say, comment is superfluous. Hence the problem of the nature of pinch oscillations, as I see it, needs further study.

The fullest experimental study of pinch stability in InSb in the presence of a longitudinal magnetic field has been performed in^[24, 93].

It was shown in^[93] that the amplitude of the oscillations corresponding to the mode $m = 0$ decreases sharply in a longitudinal magnetic field comparable in magnitude with the intrinsic magnetic field produced by the current at the surface of the plasma filament. Upon further increase in the field, the percent composition of the modes $m = 0$ and 1 in the spectrum of the oscillations is comparable in amount (Fig. 15). In weaker magnetic fields:

¹⁰⁾The appearance of helical structures in a gas-discharge plasma in the absence of an external magnetic field has been found in a hollow mercury discharge.^[99] The possible appearance of such structures in weakly conductive media has been discussed in^[100].

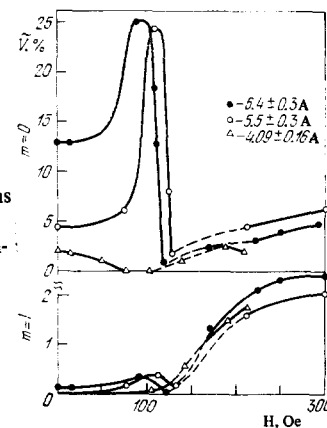


FIG. 15. The value of the voltage modulation by the perturbations $m = 0$ (upper part of the diagram) and $m = 1$, as a function of the longitudinal magnetic field intensity.^[93]

$H < H_\phi(r_p)$, the amplitude of the pinch oscillations ($m = 0$) sharply increases. The reason for this effect is not clear. The authors of^[93] think that the sharp attenuation of the mode $m = 0$ in strong fields is due to abolition of the magnetothermal pinch owing to onset of helical instability.

Ando and Glicksman^[24] first showed that abolition of a pinch in a longitudinal magnetic field is due to onset of helical instability.^[3-5] As I have noted, this effect has been observed by using various methods^[58, 59, 61, 66] from the time of discovery of the pinch effect in semiconductors.^[1, 2] The reason for abolition of the pinch remained unclear for a long time, since estimates have shown that one cannot use such well-known concepts as "freezing-in" of the magnetic field in the plasma and "magnetization" of the radial current to explain this effect. After some extremely important experiments^[61, 62] studying time scans of the voltage across the specimen, the picture became somewhat clearer. It turned out that the disappearance in a longitudinal magnetic field of the anomalous resistance that is inherent in a plasma under pinch-effect conditions is accompanied by instability. Here the fundamental symptoms of a pinch are restored if one passes through the specimen a large current^[1, 2, 60] whose magnitude increases^[60] in direct proportion to H (Fig. 16). The hypothesis was advanced in^[61, 62] that the abolition of the pinch might involve helical instability. What might the grounds be for such a conclusion? In order to answer this question, let us examine the mechanism of excitation of this instability.^[4, 102, 103] In a d.c. electric field, the helical perturbations ($m = 1, k \neq 0$) of electron and hole density become spatially separated. This gives rise to transverse fluctuational fields of ambipolar origin that oppose charge accumulation. Drift currents arise in these fields and in the d.c. magnetic field ($H \parallel E$) that are directed toward the surface of the specimen. These currents can cause growth of the original perturbations of density and potential if the velocity of transverse drift exceeds the velocity of ambipolar diffusion. Hence helical instability arises only at large enough intensities of the electric and magnetic fields.^[4, 103] Under conditions of growing helical instability, the resistance of the specimen increases sharply.^[103, 104] This is due to the anomalously large plasma current at the surface of the crystal,^[105] where it disappears owing to surface recombination.

Actually the pinch effect is also a plasma instability that is inverted with respect to helical instability. Indeed, the strong compression of the plasma arises whenever the drift current in the fields E and H_ϕ directed

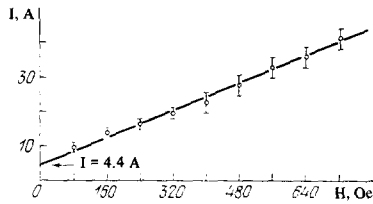


FIG. 16. Relationship of the critical current at which a pinch effect arises to the longitudinal magnetic field intensity. [60] n-InSb, $T_c = 77^\circ\text{K}$.

toward the axis of the specimen exceeds the diffusional current. This instability is also accompanied [1, 2, 61, 62] by onset of anomalous resistance, but it arises from other causes, namely the enhanced electron-hole scattering and quadratic bulk recombination. Naturally the pinch effect will be suppressed under conditions of growing helical instability, since the drift currents responsible for growth of these instabilities travel in opposite directions. In just the same way, helical instability can be suppressed at high enough currents at which a pinch effect arises. Here one observes an extremely interesting phenomenon in which the anomalous resistance caused by the one instability is suppressed by onset of the other one, although each of them separately increases the resistance of the specimen. Isn't this indeed an excellent illustration of the philosophical principle of unity and conflict of opposites?

We can naturally assume that a pinch will be abolished whenever the characteristic time of development of helical instability is smaller than the pinching time (in essence one is comparing the increments of two instabilities). Let us try to estimate by using this criterion the value of the longitudinal magnetic field at which the pinch effect should be abolished. Let us consider a plasma filament (of radius r_p) in which the plasma is distributed uniformly across the cross-section. With such a density distribution a superficial helical wave can be excited. [108] If we neglect the magnetic field of the current and diffusion, the increment of this wave in the approximation where $(H/c)^2 b_{e,h}^2 \ll 1$ has the form [108] $\gamma_B \approx 2(b_e b_h / cr_p) EH (k \sim 1/r_p)$. If we compare this expression with the pinching increment $\gamma_{\text{pinch}} = \tau_{\text{pinch}}^{-1}$ (8), we find that the above-mentioned criterion is fulfilled if $H > H_\phi(R_0) R_0 / r_p$, where R_0 is the radius of the specimen. This very crude estimate nevertheless agrees well with the experimental results, [24, 93] and it can be used for pinch diagnostics. Thus, for example, if one has determined experimentally the smallest field H at which the fundamental symptoms of a pinch disappear, then one can estimate the radius of the plasma filament. More precise calculations that take account of the spatial distribution of the plasma in the pinch effect and the intrinsic magnetic fields of the currents have been performed in [106]. We note also [107], where they calculated the threshold for excitation of helical instability in a weak pinch effect (the intrinsic magnetic field of the current was taken into account as a perturbation).

We can pose the inverse problem and treat the situation in which the specimen before the onset of the current pulse lies in a longitudinal magnetic field, so that helical instability is excited in the plasma at relatively low currents before the onset of the pinch. Correspondingly the problem amounts to how the critical pinching current will depend on the magnetic field intensity. Comparison of the increments $\gamma(R_0)$ shows that the pinch effect will "open up" at currents such that $H_\phi(R_0) > H$.

Thus, $I_{\text{crit}} \sim H$. This relationship agrees well with the experimental data [60] (see Fig. 16).

And now I shall relate some very interesting results obtained by Ando and Glicksman. [24] The experiments were performed on crystals of n-InSb ($T_c = 77^\circ\text{K}$, $n_0 = 5 \times 10^{13} \text{ cm}^{-3}$) by the method described in [61]. Time scans were taken on a specimen in a fixed-current regime in the presence of a d.c. magnetic field that could be turned on either before the onset of the current pulse or at definite instants of time when the pinch had already been able to develop. Figure 17 shows one of the characteristic oscillograms. The magnetic field (250 Oe) was turned on in these measurements 1.5 μsec after the onset of the current pulse (10 A). As we see from this diagram, a sharp peak appears first on the voltage scan that corresponds to onset of impact breakdown. During the time t_1 from the switching-on of the magnetic field, no changes are yet observed on the U scan, but then, after the time t_2 , the voltage falls to its pre-pinch value. In this state, the U scan exhibits oscillations ($f \approx 10 \text{ MHz}$). This indicates the onset of instability in the plasma. They determined the spatial structure of the perturbations by using a system of probes. It turned out that the instability is due to excitation of the helical mode ($m = 1$). As a rule, perturbations with $m = 2$ were also observed along with this mode. We note that abolition of the pinch [33] in $\text{Bi}_{1-x}\text{Sb}_x$ compounds is also accompanied by excitation of waves with $m = 1$ and 2.

The time $\tau = t_1 + t_2$ is one of the fundamental decay characteristics, and it corresponds to the time of onset of instability. Figure 18 shows the $\tau(B, I)$ relationship. We can see well that the decay time declines with increasing magnetic field (the increment γ_B increases), and it increases with the current (diffusion appreciably reduces the value of γ_B upon strong compression of the plasma). The problem of the excitation of the mode $m = 2$ that accompanies the helical wave under pinch-effect conditions has not been studied theoretically. In the absence of a pinch, the threshold for exciting this wave is always higher [103] than for the mode $m = 1$ (under the condition $(H/c)b_{e,h} < 1$). It is not ruled out that this threshold is lowered in the pinch effect, in which the role of the intrinsic magnetic field of the currents is increased. The spatial redistribution of the plasma arising from onset of helical instability can also lead to the same effect. These problems in a gas plasma have been discussed in [108].

I should note another extremely important result that Ando and Glicksman [24] obtained. At large currents ($\tau_{\text{pulse}} = 5 \mu\text{sec}$) in these experiments, a magnetothermal pinch arose, and the voltage across the specimen diminished in time ($H = 0$). When a longitudinal magnetic field was turned on and then off, the voltage across the crystal was appreciably higher than the corresponding value at the same instant of time in the total absence of a magnetic field. This fact indicates a considerable lattice "cooling" in the presence of the magnetic field that destroys the magnetothermal pinch channel owing to development of helical instability.

Upon taking account of the direct analogy between the overheating instability in a weakly-ionized gas-discharge plasma and the magnetothermal pinch in semiconductors, a theoretical foundation has been found in [109] for possible suppression of overheating instability in gas lasers with combined pumping in the presence of a longitudinal magnetic field ($H \parallel E$). The conditions were deter-

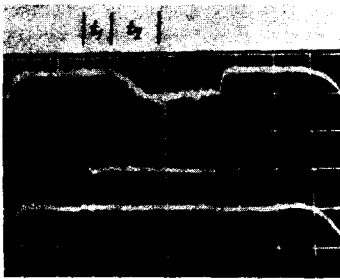


FIG. 17. Time scans of the voltage (upper), longitudinal magnetic field (middle), and current (lower) in an n-InSb specimen. $T_c = 77^\circ\text{K}$, $H = 250\text{ Oe}$, $I = 10\text{ A}$.^[24] The value of a large division is 25 V (in the vertical direction), and 0.5 μsec (in the horizontal direction). $L = 0.52\text{ cm}$.

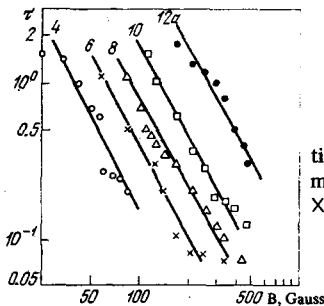


FIG. 18. Relationship of the decay time of the pinch to the longitudinal magnetic field. ^[24] $V = 0.051 \times 0.051 \times 0.52\text{ cm}^3$.

mined^[109] under which the increment of helical instability in the plasma of a non-autonomous gas discharge exceeds the increment of overheating instability.

The extremely effective studies of Ancker-Johnson and her associates^[110] on suppression of sausage modes ($m = 0$) in a solid-state pinch by using a feedback system (or automatic regulation) have also a certain conceptual significance for physicists concerned with problems of stabilizing instabilities in thermonuclear apparatus. In these experiments, which were performed on p-InSb specimens, the regulation system consisted of two pairs of probes lying at different distances from the cathode. The instability signal, as detected by the central pair of probes, was amplified, phase-shifted, and applied to the other pair, which lay near the cathode. With an appropriate phase shift, they observed suppression ($\approx 25\text{ dB}$) of the amplitude of the oscillations, which was detected by a third (independent) pair of probes lying near the anode.

We have thus far restricted the discussion of results to studies of the pinch effect in semiconductors. At large currents, VAC anomalies analogous to the above-described arise also in Bi.^[54,55] Yet the authors of^[29,31,55] deem the interpretation of these experiments based on the idea of a pinch effect to be not fully unequivocal. We shall discuss this problem in the next section in the light of the earlier study of Borovik^[111] (which was apparently poorly known to the authors of^[29,31,55]), who studied the variation of the resistance of thin plates of Bi at high current densities.

7. ON THE PINCH EFFECT IN BISMUTH

Borovik^[111] discovered in 1953 a considerable increase ($\approx 30\text{--}60\%$) in the resistance of Bi specimens at current densities $j \approx (6\text{--}12) \times 10^5\text{ A/cm}^2$, which indicated a strong violation of Ohm's law. The experiments were performed on plates of thickness $b = 2$ or $7.5\text{ }\mu\text{m}$ at temperatures $T_c = 20.4^\circ$ or 77°K in a pulsed regime ($\tau_{\text{pulse}} = 1.5\text{ }\mu\text{sec}$, with insignificant lattice heating). In principle, the observed effect could be associated with the appearance of magnetoresistance in the rather strong intrinsic magnetic field of the current. Along this line, Borovik performed additional experiments to determine

TABLE II

No. 1 ($T_c = 77^\circ\text{K}$, $b = 7.5\text{ }\mu\text{m}$, $\Omega_0 = 2.9\text{ ohm}$)				No. 2 ($T_c = 20.4^\circ\text{K}$, $b = 7.5\text{ }\mu\text{m}$, $\Omega_0 = 1.39\text{ ohm}$)				No. 3 ($T_c = 20.4^\circ\text{K}$, $b = 2\text{ }\mu\text{m}$, $\Omega_0 = 3.6\text{ ohm}$)			
$j \cdot 10^{-5}$, A/cm ²	$\frac{\Delta\Omega}{\Omega}$	$\left(\frac{\Delta\Omega}{\Omega}\right)_H$	$\left(\frac{\Delta\Omega}{\Omega}\right)_j$	$j \cdot 10^{-5}$, A/cm ²	$\frac{\Delta\Omega}{\Omega}$	$\left(\frac{\Delta\Omega}{\Omega}\right)_H$	$\left(\frac{\Delta\Omega}{\Omega}\right)_j$	$j \cdot 10^{-5}$, A/cm ²	$\frac{\Delta\Omega}{\Omega}$	$\left(\frac{\Delta\Omega}{\Omega}\right)_H$	$\left(\frac{\Delta\Omega}{\Omega}\right)_j$
4.4	0.13	0.027	0.1	6.8	0.45	0.13	0.32	7.7	0.35	0.02	0.33
4.9	0.2	0.035	0.17	12	0.63	0.30	0.33	10.3	0.42	0.05	0.37

the magnetoresistance $(\Delta\Omega/\Omega)_H$ of the studied specimens at small current densities. The appropriate measurements were performed in transverse magnetic fields equal to the mean value in the specimen of the intrinsic magnetic fields of the currents at which the resistance anomalies were observed ($H_\perp = H_j/2$, where $H_j = 2\pi j b/c$). It turned out that the observed increase in the resistance $(\Delta\Omega/\Omega)$ cannot be explained by the effect of the magnetoresistance alone, since in a number of cases this effect was small ($\approx 10\text{--}20\%$ of the main value). The quantity $(\Delta\Omega/\Omega)_j = \Delta\Omega/\Omega - (\Delta\Omega/\Omega)_H$, which defines the effect of the "pure" change in the resistance, proved to be rather significant ($\approx 30\%$) at current densities $\approx 10^6\text{ A/cm}^2$ ($T_c = 20.4^\circ\text{K}$). Table II gives the results of these measurements (here Ω_0 is the resistance of the specimen at low current density). Quite naturally, the measured magnetoresistance at a fixed current density (Table II) falls with increasing temperature of the crystal (the mobility of electrons and holes decreases) and with decreasing thickness of the specimen (H_j becomes smaller).

We note that the effect of the "pure" variation of the resistance at large current densities is manifested most clearly at low temperatures and at low crystal thicknesses (Table II, No. 3). Borovik was not able to explain the observed resistance anomalies, and this study has been to a certain extent forgotten.

After the results of the series of experiments^[1,58,60-62] on InSb had been reliably interpreted within the framework of the pinch effect, Hattori and Steele^[54] recalled Borovik's studies.^[111] These authors, and then also Schillinger^[55] repeated part of the experiments that Borovik had described, and also discovered deviations of the VAC of Bi ($T_c = 77^\circ\text{K}$, $\tau_{\text{pulse}} = 0.25\text{--}1\text{ }\mu\text{sec}$) from Ohm's law at large current densities ($j \geq 10^5\text{ A/cm}^2$). In^[54], the experiments were performed on rectangular plates of dimensions $0.0212 \times 0.0076 \times 0.326\text{ cm}$. These dimensions corresponded to the bisector, to the threefold, and the twofold crystallographic axes of the specimen. The current was passed in the direction of the twofold axis. Schillinger measured the VAC of filamentous specimens (polycrystals) of diameter 0.02 cm . We should note that the dimensions of the crystals were unfortunately chosen in these experiments.^[54,55] With such large thicknesses (0.0076 cm ; 0.02 cm), the magnetoresistance could be considerable, and the authors of^[54,55,112] had to carry out a complicated calculational effort in order to try to distinguish the change in resistance involving the pinch effect from magnetoresistance. What changes in the resistance might be due to pinching of an equilibrium plasma?^[54]

On the one hand, the pinch effect should decrease the magnetoresistance of the specimen (Ω_H) , since the diffusional component of the ambipolar current increases upon compression of the plasma, while the surface of the

specimen becomes a source of carriers to the bulk^[54, 55, 112] (surface generation). The effect of surface generation, which arises when $n < n_p$ near the surface of the specimen, will be manifested appreciably^[112] if $s\tau_p/b \gg 1$ ($\tau_p \ll \tau_{\text{pulse}}$). On the other hand, the effects of quadratic bulk recombination and of decreased carrier mobility in the region of elevated concentration ($n > n_p$) will increase Ω_H in the pinch effect. It becomes clear from the foregoing how difficult it is to interpret unambiguously the VAC anomalies of Bi whenever the effects of pinching and of magnetoresistance play an appreciable role. The difficulty of the VAC calculation is aggravated by the strong anisotropy of the mobility tensor for electrons and holes in Bi,^[11] and by the need of taking account of intervalley transitions.

The calculations of^[54] reduced the effect of pinching on the shape of the VAC merely to taking account of the diffusional component of the ambipolar current (the balance of numbers of carriers was not treated). The choice of such a simplified model involved the fact that the calculated value of Ω_H when $\tau_p \rightarrow 0$ (a pinch does not arise) proved to be greater than the change in resistance measured in the experiments of^[54]. One can get agreement of the calculated and experimental data^[54] by choosing $\tau_p \approx 10^{-8}$ sec. Then the criterion for appearance of a pinch $\tau_{\text{pinch}} < \tau_p$ is satisfied when $I > 22$ A (the VAC anomalies arose at precisely this current^[54]). Bennett's criterion ($\tau_{\text{pinch}} < \tau_D = b^2/4D$) is satisfied at $I > 15$ A. The authors of^[54] consider these estimates to indicate that the pinch effect influences the observed resistance of the Bi plates. In this regard, Hattori and Steele^[54] made VAC measurements in a longitudinal magnetic field. It turned out that the VAC anomalies disappear when $H > H_j$ (the resistance of the specimen rises in weaker fields). Yet it remained unclear whether these effects involve a decrease in the ambipolar drift velocity ($b_e b_h / c^2 H^2 > 1$) or the onset of helical instability in the plasma of Bi.

Schillinger's calculations^[55] eliminated a number of the restrictions that Hattori and Steele^[54] had used. Figure 19 shows the fundamental results of these calculations,^[12] which took account of quadratic bulk recombination and surface generation. The values of the current and of the electric field in this diagram are given in units of I_p and E_p , which correspond to the values of these quantities as defined by the Bennett criterion (5). Let us discuss these results. If the diffusion length $\tilde{L} = \sqrt{D\tau_p} = 0$, then the corresponding VAC describes only the effect of magnetoresistance (a pinch does not arise in this case). The VAC for the value $\tilde{L} = a$ (a is the radius of the specimen) describes the variation of the resistance of the specimen under pinch-effect conditions. In the case where $\tilde{L} = a$, $s = 0$, in which surface genera-

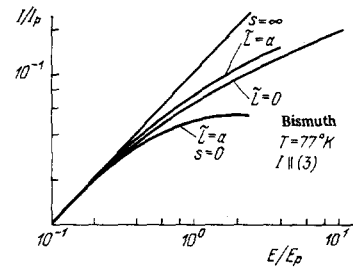


FIG. 19. Calculated volt-ampere characteristics of bismuth.^[55] The current is parallel to the threefold axis of the specimen; a is the radius of the specimen, \tilde{L} is the diffusion length, and s is the rate of surface recombination.

tion plays no role, the deviation of the VAC from Ohm's law becomes more marked than when $\tilde{L} = 0$. This involves the enhancement of quadratic bulk recombination in the pinch effect, which diminishes the number of carriers in the bulk of the specimen. When $s = \infty$ (surface generation is substantial), the VAC is "smoothed," and it even passes somewhat above the VAC that describes the magnetoresistance ($\tilde{L} = 0$). However, as Schillinger^[55] notes, it is hard to distinguish the effects of magnetoresistance and of pinching from the observed $I(E)$ relationships. Hence the problem of a pinch effect in Bi has remained open after the experiments performed in^[54, 55].

And in this sense, as we see it, the results of Borovik's^[111] experiments admit a more unequivocal treatment of the observed VAC anomalies. The effects of magnetoresistance were considerably smaller in these experiments, even at high current densities, owing to the small thickness of the plates. And what is most important, Borovik was able to detach these effects from the main one. Hence, the "pure" change in resistance of the specimens in these experiments could be associated with the onset of the pinch effect. However, this effect should arise at far higher current densities in plates of small thickness. Let us try to estimate the value j_{crit} at which a pinch effect should arise under the conditions of Borovik's experiments. For a plate,

$$\tau_{\text{pinch}}^{-1} = 2 \frac{b_H H \mu_g}{cb} = \frac{4\pi}{c^2} b_H j v_g.$$

The criterion $\tau_{\text{pinch}} < \tau_D$ is of the form

$$j > j_{\text{crit}} = \frac{c}{b} \sqrt{\frac{k(T_e + T_h) n_p}{\pi}}. \quad (12)$$

When the electron-hole gas is degenerate, we should take the Fermi energy ($\epsilon_{e,hF}$) in place of the quantity $kT_{e,h}$.

For the experiments of^[54] ($T_c = 77^\circ\text{K}$, $n_p = 4.6 \times 10^{17} \text{ cm}^{-3}$,^[39] $\epsilon_{eF} = 0.026 \text{ eV}$, $\epsilon_{hF} = 0.015 \text{ eV}$, $b = 7.6 \times 10^{-3} \text{ cm}$), the value $j_{\text{crit}} = 1.2 \times 10^5 \text{ A/cm}^2$ calculated by using (12) agrees well with the more precise estimate given in^[54]. In estimating j_{crit} from Borovik's data, we take account of the fact that the plasma concentration^[113] in Bi at $T_c = 20.4^\circ\text{K}$ is about half as much as at $T_c = 77^\circ\text{K}$, while the mobility of the electrons and holes is increased by a factor of three.^[113] The appropriate calculations by (12) show that $j_{\text{crit}} \approx 1.2 \times 10^6 \text{ A/cm}^2$ ($T_c = 77^\circ\text{K}$; $b = 7.5 \times 10^{-4} \text{ cm}$); or $7 \times 10^5 \text{ A/cm}^2$ ($T_c = 20.4^\circ\text{K}$; $b = 7.5 \times 10^{-4} \text{ cm}$); or $2.3 \times 10^6 \text{ A/cm}^2$ ($T_c = 20.4^\circ\text{K}$; $b = 2 \times 10^{-4} \text{ cm}$).

Thus the data given in the second column of Table II correspond best to the pinch hypothesis of the origin of the "pure" resistance of Bi specimens at high j values.

¹¹The equal-energy surfaces of Bi for electrons are described by three ellipsoids^[39] lying symmetrically with respect to the trigonal axis. The isoelectronic surface for holes forms one ellipsoid of revolution (the axis of revolution is parallel to the threefold axis). The concentrations of electrons and holes are equal. Data on the concentration and mobilities of the carriers in the directions of the corresponding axes are given in^[39].

¹²A not completely correct account was taken of ambipolar diffusion in deriving the initial equation for the plasma density.^[55] The author neglected the concentration-dependence of the ambipolar diffusion coefficient (the Fermi energy). This inaccuracy does not affect the qualitative course of the calculated VAC's.

We note that such estimates are relative in nature. Unconditionally, a weaker redistribution of the plasma $(n - n_p)/n_p \ll 1$ will arise even when $j < j_{crit}$. The problem is whether such a redistribution can lead to an appreciable change in the resistance of the Bi specimens. The answer is as yet unclear. As I see it, the elucidation of the real reasons for the appearance of the considerable VAC anomalies in thin Bi specimens at high current densities deserves further attention. The solution of this problem would seem to require using more direct methods of determining the spatial redistribution of the plasma, e.g., microwave diagnostics. Studies of the VAC of Bi in a longitudinal magnetic field must be continued. According to Hattori and Steele's estimates,^[54] helical instability should be excited under the conditions of their experiments at frequencies ≈ 500 MHz. The frequency of these waves might be considerably higher in thinner plates. The experimental information^[114] on excitation of helical instability in a Bi plasma is extremely meager and indefinite.

We note that the studies^[54,55] are essentially the first attempt to investigate the pinch effect in a degenerate electron-hole plasma. We shall relate in greater detail the development of this line of study in the next section.

8. THE PINCH EFFECT UNDER CONDITIONS OF STRONG PLASMA DEGENERACY

The recombination-radiation spectra should apparently undergo the greatest changes in the pinch effect in a non-equilibrium Fermi plasma in semiconductors, since as the plasma concentration in the filament (and correspondingly also ϵ_F) is increased, an ever greater number of energy states will participate in emission processes. We recall that as early as 1954, Burstein^[115] discovered a remarkable effect of shift of the long-wavelength absorption edge in n-InSb toward shorter wavelengths upon strong doping ($n > 10^{18} \text{ cm}^{-3}$, $T_C = 300^\circ \text{K}$). This effect is due to the strong degeneracy of the electrons in the conduction band, and it will be manifested in the recombination radiation of the degenerate plasma as a shift of the spectral emission-intensity peak toward shorter wavelengths as the plasma concentration is increased in the pinch channel (Fig. 20). This feature of the spectra will be enhanced when a population inversion for interband transitions arises. Schmidt^[50] and Steele^[116] have discussed this problem as applied to the pinch effect in InSb. The criterion of population inversion (6) for the transitions $h\nu \approx E_g$ for InSb ($m_e = 0.013m_0$, $m_h = 0.3m_0$) is satisfied^[50] at $n \geq 2 \times 10^{17} \text{ cm}^{-3}$ if the temperature of the carriers is $\approx 250^\circ \text{K}$. At a lower plasma temperature ($T_e = T_h = 10^\circ \text{K}$), this condition (6) is satisfied at relatively low densities ($n > 10^{15} \text{ cm}^{-3}$). Emission will be amplified at the appropriate frequencies under inversion conditions (negative absorption).

The recombination-radiation spectra in the pinch effect in a degenerate InSb plasma have been studied experimentally in the effective investigations of Shotov and his associates.^[49,117] In these experiments, a high-density electron-plasma was created by double carrier injection into pure p-InSb crystals ($p_0 \approx 10^{13} \text{ cm}^{-3}$, $V = 0.05 \times 0.05 \times (0.015 - 0.045) \text{ cm}^3$) by using n'ip' contact structures. Plasma injection was carried out with rectangular current pulses ($\tau_{pulse} = 10^{-5} - 10^{-6} \text{ sec}$) at a crystal temperature of $T_C = 4.2^\circ \text{K}$. The recombination-radiation spectrum was broadened with increasing cur-

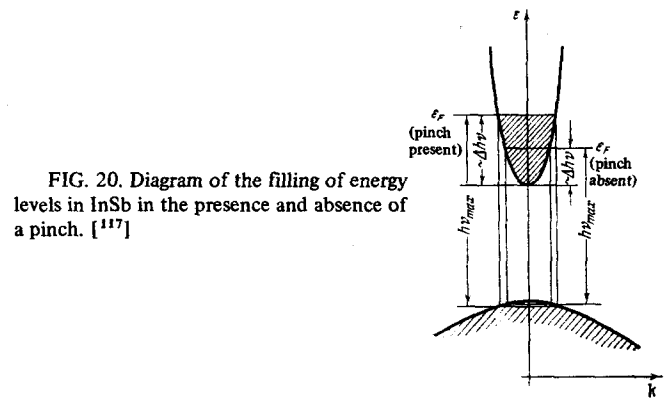


FIG. 20. Diagram of the filling of energy levels in InSb in the presence and absence of a pinch. [117]

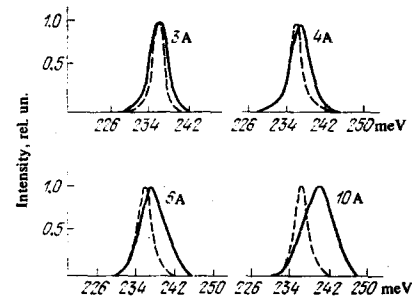


FIG. 21. Emission spectra of the plasma (p-InSb) at different currents. [49,117] Solid lines are for $H = 0$, dotted for $H = 350$ Oe.

rent ($\lambda \approx 5 \mu\text{m}$), while the spectral intensity peak shifted toward shorter wavelengths (Fig. 21), starting at currents ≥ 4 A. By comparing the half-width of the emission spectrum with the Fermi energy of the electrons in the conduction band, we can estimate that the mean plasma concentration is $\approx 0.5 \times 10^{16} \text{ cm}^{-3}$ when $I \approx 7$ A ($\epsilon_{eF} \approx 6.5 \text{ meV}$). At an electron temperature of $\approx 14^\circ \text{K}$ ($I = 7$ A), which was determined from the short-wavelength part of the emission spectrum^[118] which varies like $\exp[-(h - E_g)/kT_e]$, the degree of degeneracy of the electron component of the plasma is rather high ($\epsilon_{eF}/kT_e > 4$). A direct experimental proof of the degeneracy of the plasma in the experiments of Shotov and his associates is that they got coherent emission under conditions such as were realized in^[119]. The shift of the emission intensity peak to shorter wavelengths can naturally be attributed to the onset of a pinch effect (see Fig. 20). We recall that, for a nondegenerate plasma, the spectral emission peak with a pinched plasma is shifted to longer wavelengths with increasing current.^[58,120] In a longitudinal magnetic field ($H = 350$ Oe), one does not observe any appreciable changes in the emission spectrum with increasing I (Fig. 21). This is apparently due to abolition of the pinch by onset of helical instability. We should note that the critical currents for onset of a pinch, as determined from the form of the VAC,^[49,117] prove to be two or three times higher than the corresponding values obtained by using the spectral method. What are the reasons for this disagreement? The anomalous resistance in the pinch effect in a degenerate plasma is due to quadratic bulk recombination^[55,80,81] and to the decreased mobility of the electrons with increased concentration caused by scattering by acoustic phonons^[81] ($b_e \sim n^{-1/3}$).^[13] In a fixed-current

¹³⁾Electron-hole scattering does not depend on the plasma density [44,81] at high degeneracy ($\tau_{h-e}^{5/2} \sim T_e^{3/2}/n$, $T_{e,eff} \sim n^{2/3}$).

regime, as the resistance of the plasma is increased by the pinch effect, the rate of double injection will also increase (the voltage across the specimen increases). At relatively low currents, it will be difficult to determine the critical pinch current from the form of the VAC. Thus, for example, under double-injection conditions, the calculated VAC in the pinch effect with account taken of quadratic bulk recombination will be ohmic in nature.^[121] Hence, in the experiments of Shotov and his associates, the spectral method of observing the pinch effect is apparently the most sensitive. By using data from measurements analogous to those shown in Fig. 21, Shotov and his associates^[117] determined the radius of the plasma filament by assuming that the number of carriers remains invariant upon compression of the plasma as the longitudinal magnetic field is switched off. These data are given in Fig. 4. We should note that these estimates of r_p that assume that $h\nu_m - E_g \approx \epsilon_e F$ demand a certain caution. In rigorous calculations of the spectral distribution of the radiation emerging at the surface of the crystal, one must take account of both negative-absorption processes in the filament of quanta whose frequency satisfies the condition (6), and of positive-absorption processes in the region between the surface of the crystal and the pinch channel (one must also bear in mind the actual spatial distribution of the plasma or the Fermi energy). The competition of the stated processes will determine all the spectral features of the radiation that emerges at the surface of the crystal. When the plasma filament is rather close to the surface of the specimen, negative absorption plays the predominant role, and this shifts the spectral distribution toward shorter wavelengths with increasing current. At large values of I at which the filament is far from the surface of the crystal, positive absorption plays the major role, and it suppresses the short-wavelength radiation from emerging at the surface of the crystal. These problems have been treated in part in^[122], whose results are applicable only when the electrons and holes are strongly degenerate (the negative-absorption coefficient is a maximum^[47]). However, under the conditions of the experiments of^[49, 117], the hole gas is not degenerate ($\epsilon_{hF}/kT_h < 1$), and the negative-absorption coefficient will be small.^[47] Hence theoretical exploration must be continued along the line of more reliable quantitative interpretation of the spectral characteristics of the emission observed in the experiments of Shotov and his associates.^[49, 117] It has been noted in^[48, 49] that a pinch effect is undesirable in semiconductor lasers where the radiation output passes perpendicular to the current, owing to the strong absorption in the rest of the crystal. One can use a longitudinal magnetic field^[44] to suppress the pinch (Fig. 21), or crystals having a short distance between the contacts. The current at which a pinch effect arises is considerably higher in short crystals than in long ones, as has been pictorially demonstrated in^[48, 49]. Thus, a pinch effect did not arise even at $I = 20$ A in a crystal of length $L = 0.015$ cm. This circumstance permitted them to observe coherent emission from such specimens in the absence of an external magnetic field.^[48] In longer crystals ($L = 0.04$ cm) at high currents ($I \approx 12$ A), they observed a dumbbell-shaped plasma distribution along the length of the specimen (the

plasma lay nearer to the surface of the crystal at the contacts). They drew this conclusion from measuring the integral intensity distribution of the emission along the length of the specimen (the emission is brighter near the contacts). The mentioned facts can be explained rather simply.^[49, 79] If the front of the injected plasma traverses the distance between the contacts in a time shorter than the pinching time of (8), then plasma compression cannot take place. The corresponding condition has the form $L \ll h = v_0 \tau_{\text{pinch}} = c^2 R_0^2 v_0 / 4b_h v_g I$, where v_0 is the velocity of the front. Hence a pinch effect does not arise in short specimens. Since injection occurs uniformly over the cross section of the contacts, an appreciable compression of the plasma occurs at the distance h from the ends of the specimen, which is determined by the path length that the plasma front traverses in the time τ_{pinch} . This situation gives rise to the dumbbell distribution of the plasma in the pinch effect under double-injection conditions. If we assume that the velocity of the front is determined by the drift velocity of holes^[49] ($v_0 = b_h E$), as the slower carrier, then the pertinent estimates of $I_{\text{crit}}(L)$ and h agree well with the experimental data.^[49]

With this, we shall close the discussion of the studies on the Z-pinch in a solid state plasma, and shall proceed to treating the pulsed method of compressing a plasma in a longitudinal magnetic field that increases with time.

9. THE θ -PINCH IN SEMICONDUCTORS

While the discovery of the Z-pinch in semiconductors happened somewhat unexpectedly, and it was not directly associated with the analogous studies in a gaseous plasma, the experimental study of the θ -pinch in an electron-hole plasma, which began^[124] in 1967, was entirely initiated by the gas-discharge developments,^[40] The experiments of Hübner and his associates^[124-126] were performed on n-InSb ($n_0 = 2 \times 10^{14}$ cm⁻³), both in the region of intrinsic conductivity^[124, 125] ($T_c = 250-300^\circ$ K, $n_p \approx 0.7-2 \times 10^{16}$ cm⁻³), and in a transverse-breakdown regime^[126] ($T_c = 77^\circ$ K) produced by an induced electric field, and they amount to a skin-effect-influenced θ -pinch regime (in the experiments of^[125], $\beta = \delta/R_0 \approx 1/3$).

The first experiments^[124] determined the variation in the magnetic flux penetrating the specimen during a magnetic-field pulse ($\dot{B}_{z, \text{max}} \approx 1.7 \times 10^{11}$ Oe/sec, rise time $\approx 2 \times 10^{-8}$ sec, $R_0 = 2.6 \times 10^{-1}$ cm, $T_c = 250^\circ$ K). An increase in the magnetic induction flux in the specimen was observed at the instant of time that corresponded to the maximum azimuthal electric field intensity. The authors of^[124] believe that this effect is due to onset of a θ -pinch, since the magnetic field penetrates more freely into the specimen toward the axis upon decompression of the plasma. Strictly speaking, the results of these experiments do not allow an unequivocal treatment, and in^[125, 126], they tried to determine more directly the plasma distribution by a laser-probe method. The experiments of^[125, 126] measured the infrared absorption ($\lambda = 10.6$ μ m, CO₂ laser) of the plasma of the specimen ($R_0 = 4.5 \times 10^{-1}$ cm, $L = 0.1$ cm) during a magnetic field pulse. One can reconstruct from these data the form of $n(r, t)$ (the laser beam was focused on suitable points of the end), since the absorption is directly proportional to the plasma concentration^[127] if $\kappa L \ll 1$ ($\kappa \sim n$ is the absorption coefficient of free carriers for the probe radiation). Figure 22 shows the results of these measurements for the case of an intrinsic plasma^[125]

¹⁴⁾Besides the function of suppressing the pinch, a strong magnetic field ($h\nu_{ce} \gg kT_e$) will increase the density of states in the conduction band, owing to Landau quantization, and will correspondingly decrease the threshold current for generating coherent radiation.^[119, 123]

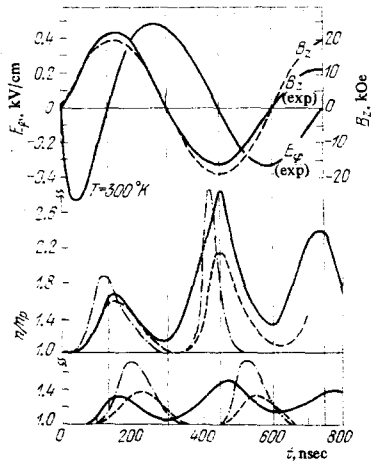


FIG. 22. A θ -pinch in InSb. $^{[125]}$ $T_c = 300^\circ\text{K}$, $R_0 = 0.45$ cm, $L = 0.1$ cm. Time-dependence of the plasma concentration at the distance 3.2 mm from the axis of the specimen (middle part of the diagram) and 0.7 mm (lower part). Solid lines—measured data, dotted—calculated. Upper part of the diagram—vacuum oscillograms (specimen absent) of the magnetic and electric fields. B_z (dotted line) in the pulse shape that was used in the calculations; n_p is the equilibrium concentration.

($T_c = 300^\circ\text{K}$). They unequivocally indicate a θ -pinch. Naturally, the compression of the plasma toward the axis of the specimen arises in the odd quarters (Fig. 22) of the harmonic magnetic-field (B_z) pulse, since in this case, $B_z \dot{B}_z > 0$, and the drift of the plasma is directed toward the center. Interestingly, the maximum compression of the plasma was attained in these experiments at some distance from the axis of the crystal (Fig. 22). This feature is characteristic $^{[128]}$ of a skin-effect-influenced compression regime, since here the magnetic field is small in the region near the axis. The compression is greatest in the third quarter of the period. The point is $^{[128]}$ that the magnetic field in the plasma near the end of the second quarter of the period ($t = T/2$) under skin-effect conditions differs from zero, since the plasma traps the field, and the spatial distribution of electrons and holes does not return near this instant of time to the original state ($n = n_p$). At the beginning of the third quarter of the period, the external magnetic field has a sign opposite to that of the field in the plasma. On the one hand, this gives rise to a neutral layer in the plasma ($B_z = 0$) and to a counter movement of the particles in this layer. On the other hand, the magnetic-field front that penetrates into the plasma becomes considerably steeper, owing to compensation of fields. These effects favor a stronger compression of the plasma. In principle, the θ -pinch should be even more strongly marked in the subsequent stages of compression. However, with a long duration of the magnetic pulse, bulk recombination quenches the effect, $^{[128]}$ and the redistribution of the plasma is less pronounced (Fig. 22, fifth quarter of the period, $\tau_p/T \approx 1/3$ $^{[125]}$). The dotted curves (Fig. 22) apparently correspond to the results of numerical integration of the equations for the skin-effect-influenced θ -pinch, which Bruhns and Hübner have cited in a later study $^{[129]}$ (unfortunately, Hübner and his associates $^{[125]}$ gave no explanation of how they got the calculated data). The parameters of the plasma that were used in these calculations were chosen as follows: $n_p = 2 \times 10^{16}$ cm^{-3} , $\tau_p = 1.6 \times 10^{-7}$ sec, $b_e = 2.1 \times 10^7$ CGSE units, $b_h = 0.01 b_e$ (dotted curves), or $0.02 b_e$ (dot-dash curves).

An experimental study of a θ -pinch without the skin-

effect ($\beta \gg 1$) in cylindrical specimens of n-Ge that had a conductivity close to the intrinsic value ($T_c = 300^\circ\text{K}$, $n_p \approx 2.5 \times 10^{13}$ cm^{-3} , $b_e = 10^8$ CGS esu units, $b_h = b_e/2$, $R_0 = 0.15$ cm, $L = 0.4$ cm) is given in $^{[130]}$. These experiments were stimulated by theoretical studies $^{[131]}$ that had shown that, in a skin-effect-free θ -pinch regime, the maximum plasma density is attained at the axis of the specimen, with

$$n_{\max} \approx h n_p \quad (h \gg 1), \quad (13)$$

Here $h = \sqrt{b_e b_h / c H}$.

Equation (13) holds if the duration of the magnetic-field pulse is considerably shorter than the bulk-recombination time ($\tau_D = R_0^2/D \approx 2 \times 10^{-4}$ sec in the experiments of $^{[130]}$). Otherwise the plasma compression is less marked. Thus, if $\tau_{\text{pulse}} > \tau_p$, then the concentration effect is strongly damped $\sim \exp(-t/\tau_p)$. We can justify Eq. (13) by using a rather simple estimate. If we neglect diffusion and bulk recombination, then the spatial distribution of the plasma should have the form of a plateau (if $\beta \gg 1$). The movement of the boundary (r) of this distribution under ambipolar-drift conditions is determined by the equation $^{[131]}$ $v_r = \dot{r} = -[h\dot{h}/(1+h^2)]r/2$ ($E_\phi = -(r/2c)\dot{H}$, $\dot{H} = dH/dt$). Hence $r = R_0(1+h^2)^{-1/4}$. If we assume that the number of particles is conserved in compression ($n_p R_0^2 = nr^2$), then $n = n_p \sqrt{1+h^2} \approx n_p h$ in the plateau region. Thus, strong compression of the plasma in a θ -pinch in semiconductors will be attained only when $h \gg 1$. This condition is the fundamental feature of pulsed plasma compression in a skin-effect-free regime. When $h \ll 1$, the particles cannot migrate into the region near the axis within the pulse duration. The results of integrating the equations for a skin-effect-free θ -pinch on a computer $^{[132]}$ for different forms of magnetic-field pulses agree well with Eq. (13) in its region of applicability.

Figure 23 shows a block diagram of the experiments. $^{[130]}$ A pulsed magnetic field having an amplitude of 500 kOe ($h_m \approx 12$) and a rise time to the maximum value of $\tau_{\text{pulse}} \approx 2$ μsec ($\beta \geq 100$) was created by discharging a bank of condensers into a massive one-turn solenoid made of brass, with an inner diameter of 0.45 cm and length 0.6 cm. Just as in $^{[125]}$, the variation of the plasma density during the θ -pinch was recorded from the absorption of a probe radiation ($\lambda = 10.6$ μm) by free carriers. The laser radiation was focused by a system of mirrors onto the end of the specimen. The diameter of the spot of light did not exceed 0.03 cm, which allowed scanning across the cross-section of the crystal. The experiments measured the relative variation of the radiation intensity transmitted through the specimen that was caused by the variation in plasma

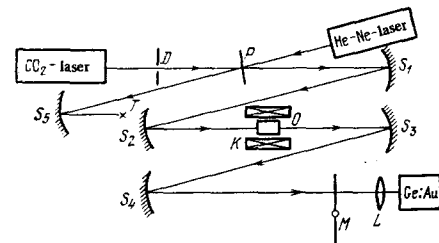


FIG. 23. The θ -pinch in Ge (diagram of the experiments $^{[130]}$). D—diaphragm, P—beam splitter made of BaF_2 , S_{1-5} —mirrors, K—solenoid, O—specimen, T—thermocouple, M—mechanical modulator, L—field lens.

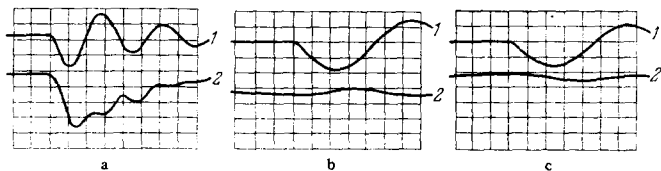


FIG. 24. Oscillograms of the magnetic field (1) and the infrared absorption signals (2). a) $r = 0$, time scale $2.5 \mu\text{sec/div.}$; b) $r = 0.75 R_0$, $1 \mu\text{sec/div.}$; c) $r = 0.5 R_0$, $1 \mu\text{sec/div.}$ In all cases the voltage scale is 25 mV/div. The amplitudes of the calibration current signals Φ_0 were obtained by using a mechanical interruptor on the probing infrared radiation (100% modulation), and for the cases of a-c), they were equal, respectively, to 1, 0.8, and 0.7 V. [130]

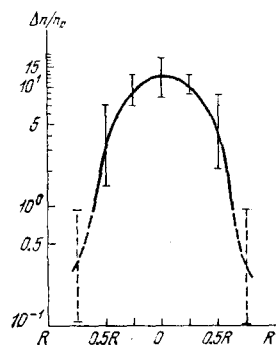


FIG. 25. A θ -pinch in Ge. The spatial distribution of the plasma density at maximum compression in the experiments of [130].

density along the axis of probing: [127] $\eta = \Delta\Phi/\Phi_0 = 1 - e^{-\Delta n q L} \approx \Delta n \cdot q L$ ($\eta \ll 1$). Here Φ_0 is the intensity of the transmitted radiation in the unperturbed plasma, $\Delta\Phi = \Phi_0 - \Phi(t)$, Δn is the variation in the plasma concentration during compression, and q is the absorption cross-section of free carriers for the radiation ($q \approx 7 \times 10^{-16} \text{ cm}^2$ [133] in Ge when $T_c = 300^\circ \text{K}$, $\lambda = 10.6 \mu\text{m}$). This relationship for η has been derived without taking account of multiple reflection of the radiation from the ends of the specimen, and it is valid [127] when $\tilde{R}^2 \ll 1$, where \tilde{R} is the reflection coefficient ($\tilde{R} = 0.36$ in Ge).

Figure 24 shows oscillograms of the signals from the radiation detector (2), together with oscillograms of the magnetic field (1). These measurements were performed at the axis of the specimen and at the points $r = 0.75 R_0$; $0.5 R_0$. An analysis of the data given in Fig. 24 indicates that the greatest infrared absorption, and hence also the maximum increase in plasma density, arises in the region near the axis (Fig. 24a). At the same time, the peripheral region (Fig. 24b) is depleted of carriers (the polarity of the transmitted infrared signal is changed). The value $\eta_{\text{max}} \approx 0.08$ calculated from the oscillograms at $r = 0$ corresponds to a maximum increase in the plasma density at the axis of the specimen of $\Delta n \approx 10 n_p$. This agrees well with the calculation from (13). Figure 25 shows the spatial distribution of the plasma density as measured at maximum compression in these experiments.

An analogous type of plasma redistribution in a θ -pinch can also arise in the weakly ionized plasma of a gas discharge. This problem has been treated in detail in the theoretical paper [134]. A skin-effect-free θ -pinch in a gas-discharge plasma can serve as one of the methods of pumping an optically active medium. Hence model experiments on semiconductors have a certain practical importance. Since the θ -pinch in semiconductors is a rather effective contactless method of producing a non-equilibrium plasma, the analogous developments in semiconductor lasers are also not ruled

out. The radiation in this case can be extracted through the ends. In this sense, the compounds $^{33}\text{Bi}_{1-x}\text{Sb}_x$ are of great interest. In these compounds, the θ -pinch will lead to a gigantic increase in plasma concentration owing to transverse breakdown. The recombination radiation in these crystals can have a rather varied frequency spectrum that depends both on the width of the forbidden band and on the value of the Fermi energy of the carriers.

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