Current-convective instability of a gas-discharge plasma

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Among the numerous instabilities of a plasma in a magnetic field, a prominent place is occupied by the instability that sets in when a constant electric field and a constant magnetic field are applied, and is manifest in the excitation of helical plasma-concentration waves. First discovered in semiconductor and gas-discharge plasma, it exists also in a fully ionized current-carrying plasma, and its modification exists in many plasma devices. The survey covers the main theoretical and experimental investigations of this instability, performed in gas discharges. The most complete data are available for a weakly-ionized positive column, in which the development of current-convective instability is investigated in detail in a wide range, from the limit of the onset of weak linear waves up to strong turbulence. Since it has been so thoroughly investigated, current-convective instability is used as an example to study various methods of stabilizing plasma in stabilities. These methods are the subject of Chap. 8 of the article.

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1. INTRODUCTION

In 1957, Ivanov and Ryvkin[1] discovered the presence of harmonic oscillations in the current flowing through thin germanium specimens placed in longitudinal electric and magnetic fields. The magnetic field had to be increased beyond a certain critical value before the oscillations could be excited.

In 1958, Lehnert[2] reported that, in the positive column of a gas discharge excited in a longitudinal magnetic field, the escape of charged particles to the wall was found to increase with increasing magnetic field under certain definite conditions.

By that time, it had become clear from the experimental data then available that transport processes in magnetized plasma could not be described in terms of classical ideas. Declassified information on controlled thermonuclear diffusion, which has now become available, shows that this fact had attracted very considerable interest. The publication of experimental papers on charged-particle diffusion in a magnetic field was therefore not surprising. These measurements were carried out on the weakly ionized plasma of a gas discharge even though the parameters of this plasma were quite different from those of the hot plasma in the hypothetic thermonuclear reactor.

The first experiments with the positive column of a helium discharge in a weak magnetic field showed that the discharge parameters were consistent with the well-known formula for transverse ambipolar diffusion[3]

$$D_{\perp}=\frac{D_{\parallel}}{1+(\omega_{He}\omega_{Hi}/\nu_e\nu_i)}; \qquad (1)$$

where $D_{\perp}$ and $D_{\parallel}$ are the ambipolar diffusion coefficients across and along the magnetic field, respectively, ω_{He} and ω_{Hi} are the electron and ion cyclotron frequencies, and ν_e and ν_i are the electron and ion collision frequencies with the neutral gas particles. It was found that there was a substantial reduction in $D_{\perp}$ when the field was increased up to 450 G.

The formula given by (1) was verified in [4] using the distribution of plasma density in the cathode region of a low-voltage arc in argon, where the plasma diffused simultaneously across and along the magnetic field. A reduction in $D_{\perp}$ by an order of magnitude was observed as a result of the small scattering cross sections at the electron temperature of 0.5 eV and high magnetic induction (up to 1000 G).

It was found in [2] that the state of the positive column was determined by (1) only up to a certain critical value of the longitudinal magnetic field (H_c) which was not reached in [3].

Lehnert's results were explained by Kadomtsev and Nedospasov,[5] who showed that an instability, subsequently called the current-convective instability, developed in the positive column. In foreign literature, this is frequently referred to as helical or screw instability.

Glicksman[6] applied the theory of current convective instability, developed in [5], to the electron-hole plasma in semiconductors, and used it to explain the results of Ivanov and Ryvkin. The four papers in [1,2,5,6] provided the foundation for extensive studies of the instability of plasmas in gas discharges and solids. In the present review, we shall be concerned with the gas-discharge plasma. Current convective instability in semiconductor plasmas has been reviewed by V. V. Vladimirov [Usp. Fiz. Nauk **115**, 73 (1975)] [Sov. Phys.-Usp. **18**, 37 (1975)].

2. THEORY OF CURRENT CONVECTIVE INSTABILITY IN THE POSITIVE COLUMN

A transverse concentration gradient, connected with the ionization of the gas and recombination at the tube walls, is established in the positive column under diffusion conditions. The physical mechanism responsible for the instability can be understood by inspection of Fig. 1, where, for simplicity, cylindric geometry is replaced by plane geometry, so that the x axis corresponds to the radial direction and the y axis to the azimuthal direction. Consider a positive fluctuation n′ in a layer inclined to the z axis. Since current must be conserved in this layer, the longitudinal electric field will fall, and electric charges will appear on the layer boundaries. These will produce a perturbation of the electric field in the y direction because the layer is inclined to the z axis. In crossed fields, the electron in the layer will drift in the x direction. The excess charge is compensated by the motion of unmagnetized ions, and the result of this is that the quasineutral layer moves along the x axis with a velocity determined both by electron drift and by ion mobility. The outflow of plasma into the region of lower concentration enhances the initial perturbation. For a negative density perturbation, an analogous instability picture is obtained by reversing the sign of the motion. Since the necessary conditions for the existence of this instability is the presence of a longitudinal current and a convective transverse motion in inhomogeneous plasma, this is often referred to as the current convective instability.

The development of the instability is impeded by a number of factors. These include, for example, enhanced longitudinal and transverse diffusion out of the perturbation region.

The radial electric field associated with ambipolar diffusion is directed toward the walls (along the x axis in Fig. 1), and produces electron drift along the y axis and a polarization of the layer with the opposite sign. This mechanism is more effective in stabilizing the higher modes, so that the m = 1 perturbation is the first to develop.

To obtain a correct determination of the stability limits, the authors of [5] considered small oscillations in the concentration and potential of the form f(r) exp (im Ψ + ikz − iωt).

Under the simplest diffusion conditions, the plasma in the positive column is described by the continuity equation

$$\frac{\partial n}{\partial t}+\operatorname{div} nv_e=\frac{\partial n}{\partial t}+\operatorname{div} nv_i=Zn \qquad (2)$$

the equation of motion of the electrons

$$\frac{T_e}{mn}\nabla n=-\frac{e}{mc}[\mathbf{v}_e\times\mathbf{H}]+\frac{e}{m}\nabla\varphi-\frac{\mathbf{v}_e}{\tau_e} \qquad (3)$$

and the equation of motion of the ions

$$v_i=-b_i\nabla\varphi; \qquad (4)$$

where n is the electron density (equal to the ion density), $Z(T_e)$ is the number of ionizations produced by one electron per unit time, T_e is the electron temperature determined from the balance conditions for particles in a stationary discharge (as usual, it is much greater than the temperature of ions and neutrals), φ is the electric potential, and b_i is the ion mobility.

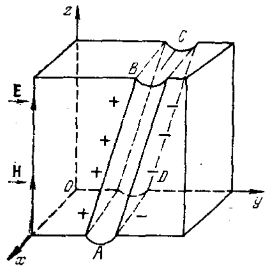


FIG. 1

In accordance with the conditions under which Lehnert established the presence of the anomalous phenomena, it is assumed here that the collision frequency between ions and neutrals is much greater than both the cyclotron frequency and the frequency of the above oscillations. For electrons, on the other hand,

$$\omega_{He}\tau_e=\frac{eH\tau_e}{mc}\gg 1.$$

Having taken the radial perturbation profile in the form $f(r)\sim J_1(\alpha_1 r/a)$, where α_1 is the first root of the Bessel function J_1, the authors of [5] derived the dispersion relation for ω and the condition for loss of stability (a is the tube radius):

$$Kx^4+Fx^2+G<mx\,\frac{eE_0a}{T_e},\quad x=\frac{k(\omega_{He}\tau_e)a}{\alpha_1}. \qquad (5)$$

The positive coefficients K, F, and G are not very dependent on the mobility ratio for ions and electrons, and decrease with increasing $\omega_{He}\tau_e$.

In a long tube, the quantity X can have arbitrary values. Hence, the instability arises when the left-hand side of (5) is equal to the right-hand side at a single point. The derivatives of the two are then also equal.

Since all the coefficients in (5) are roughly of the same order of magnitude, x turns out to be of the order of unity, i.e., the instability appears for long-wave perturbations: $ka=\alpha_0 x/\omega_{He}\tau_e\ll 1$.

The equations describing diffusion processes in plasmas have the feature that the length, time, and magnetic field are present only in combination with pressure, i.e., in the form ap, tp, and H/p. This follows from the fact that Z is proportional, and b, D, and τ_e are inversely proportional, to the pressure. The equations also contain an implicit dependence on the electron and ion temperatures which are determined by the energy balance, i.e., in the final analysis, by the ratio E/p.

All this leads to the conclusion that if there are two geometrically similar systems with equal values of the parameters ap, E/p, and H/p, then the processes occurring in them differ only by the time scale. This result is valid even when the effect of the magnetic field on the motion of the ions is taken into account together with their diffusion and inertia.

Since (2) is linear in the density n, the absolute magnitudes of n and of the electric current do not affect processes in the column.

We may therefore conclude that, if the degree of ionization is so low that collisions between ions and electrons can be neglected, then we have the following similarity relation: E/p and ω/p are functions of only ap and H/p, and do not depend on the current.

To calculate the critical electric and magnetic fields, the instability wavelengths, and the oscillation frequencies, we must know the ratio Ea/T_e as a function of ap and H/p. The values of E and T_e can, at least in principle, be determined theoretically from the balance equations for the particles and energy. However, this leads to certain difficulties in the determination of the energy distribution of fast electrons and energy losses by radiation. Theoretical values of the critical parameters for the instability of the column in a magnetic field are therefore better determined on the basis of reliable experimental data on $E/p = f_1(ap)$ and $T_e = f_2(ap)$, obtained without the magnetic field.

The similarity laws for the positive column are, of course, determined by the diffusion lifetime of the plasma $\tau \sim a^2/D_a$. Since (1) is valid in the subcritical region, the electron temperature and the electric field in a longitudinal magnetic field will be the same as in the discharge in zero magnetic field (with the same pressure), but with radius greater by a factor $\sqrt{1 + (\omega_{He}\omega_{Hi}/\nu_e\nu_i)}$:

$$\frac{E}{p} = f_1\left(ap\sqrt{1+\frac{\omega_{He}\omega_{Hi}}{\nu_e\nu_i}}\right), \qquad (6)$$

$$T_e = f_2\left(ap\sqrt{1+\frac{\omega_{He}\omega_{Hi}}{\nu_e\nu_i}}\right). \qquad (7)$$

Instead of (6), we can use the dependence of the mean fraction κ of the energy of an electron lost upon collision on E/p, namely $Ea/T_e \sim ap\sqrt{\kappa}$.

This procedure was, in fact, used in [5,7] to calculate the values of the critical parameters.

Johnson and Jerde[8] obtained a more rigorous solution of the stability problem in the form of a Bessel-function series, and showed that the approximation adopted in [5] was satisfactory for a quantitative comparison with experimental data. The density perturbation profile used by these workers is then the same as in [5], but the potential differs by the factor $1/J_0(\beta_0 r)$.

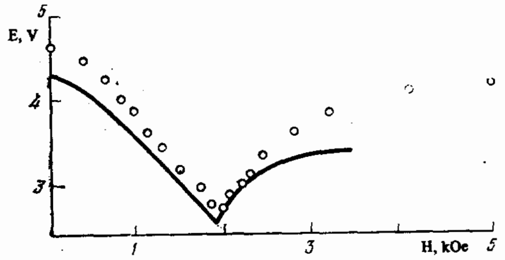


FIG. 2. Plot of the function E(H) for the helium column: points—experiments [9] with a = 1 cm, p = 0.89 Torr, curve—theoretical. [5]

3. EXPERIMENTAL DATA ON CURRENT CONVECTIVE INSTABILITY OF THE POSITIVE COLUMN

There are now extensive and mutually consistent data on positive-column instability in a longitudinal magnetic field and, especially, on the magnitude of the critical field H_c. In early papers,[2,9-12] H_c was determined from the position of the minimum on the E(H) curve. This method is valid, for example, in the case of helium discharges for which this curve exhibits a break at the critical point (Fig. 2). However, in many cases, for example, in argon and mercury, this break is not present. At the present time, the critical point is usually determined from the onset of oscillations.

a) The values of H_c. Figure 3 shows that the similarity law $H_c/p = f(ap)$ is valid for helium. The theoretical values are found to be in good quantitative agreement with experimental results. In accordance with the theoretical predictions, H_c is a slowly-varying function of the discharge current.

The dependence of H_c on ap in hydrogen is shown in Fig. 4. Comparison between Figs. 3 and 4 will show that, for equal values of ap, the critical field in hydrogen is lower than in helium by a factor of roughly 4. In hydrogen, electron energy losses are substantially greater because of inelastic collisions with the molecules. For equal ap, therefore, the ratio E/T_e in this gas is several times greater than that in helium. According to (5), this leads to the corresponding reduction in H_c.

The electron mobility in argon is known to be very dependent on the energy of the electrons and, consequently, on E/p. The longitudinal field in the column is, therefore, a nonmonotonic function of gas pressure.[13] This, in turn, leads to a nonmonotonic dependence of H_c on pressure (Fig. 5).

Data on the critical values in neon and mercury are listed below (see Figs. 11 and 12).

Molecular additions to inert gases result in a substantial reduction in H_c.[14-16] As in the case of pure hydrogen, this reduction is due to the increase in the

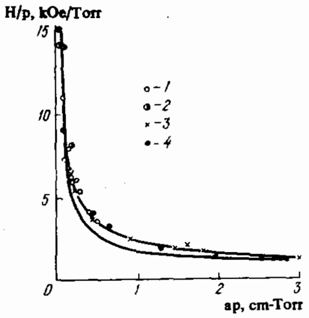


FIG. 3

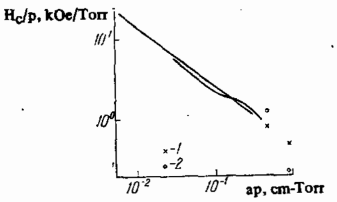


FIG. 4

FIG. 3. H_c/p as a function of ap for helium: lower curve—theoretical; [7] experimental points taken from [12] (1), [11] (2), [9] (3), and [10] (4).

FIG. 4. H_c/p as a function of ap for hydrogen: curves—experiment according to [10]: 1—experiment, [9] 2—calculation. [7]

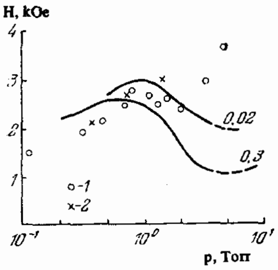


FIG. 5

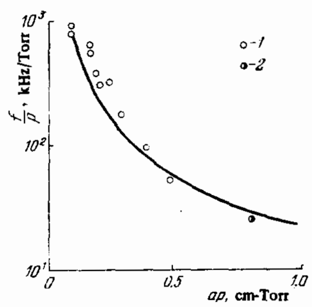


FIG. 6

FIG. 5. H_c as a function of p for argon (a = 1.25): curves—calculation in [7] for a current of 25 and 300 mA. Experimental points taken from [9] (1) and [10] (2) (recalculated in accordance with the similarity rule).

FIG. 6. F/p as a function of ap for helium: experimental points taken from [13] (1) and [18] (2): curve—theoretical.

ratio Ea/T_e which characterizes the level of inelastic losses.

b) Oscillation frequency. For magnetic fields that are not much greater than H_c, oscillations of frequency

$$f = \frac{\omega}{2\pi} \approx \frac{3D_e\beta_1^2}{2\pi\omega_{He}\tau_e}(0.6 + x^2) \approx \frac{10b_iD_e}{\omega_{He}\tau_e b_e a^2}. \qquad (8)$$

are found to appear in the column.

Like striations,[17] these are "diffusion"-type oscillations whose frequency is inversely proportional to the diffusion lifetime of the charged particles in the plasma of the column. The rotation of the helical perturbation is largely due to its longitudinal and azimuthal (Hall) drift.

Figure 6 shows theoretical and experimental results of f/p as a function of ap in the case of helium. The experimental results for neon are shown in Fig. 7. The agreement with the theoretical similarity law is found to hold to within 10–30% when the argument is varied by an order of magnitude and the ratio f/p by more than two orders of magnitude.

Molecular additions to the gas increase the frequency at the critical point.[16] Thus, for example, the addition of H_2 to Ne will increase this frequency by a factor of 4–7. This is due to a reduction in H_c since, according to (8), we have $f \sim H_c^{-1}$.

c) Perturbation wavelength. Helical distortion of the column was observed in [18,19] for $H \geqslant H_c$, in accordance with the predictions in [5]. The wavelength of this deformation has been measured by Paulikas and Pyle in a helium discharge for different p and a, and the results are shown in Figs. 8 and 9. The theoretical curves are also reproduced.

It is important to note that a number of workers have reported theoretical curves which differ somewhat from those in [5,7]. This is due to the fact that the values of Ea/T_e were different from those employed in (5). They are found from the particle and energy balance equations,[18] or are taken directly from experiment.[20] In the latter case, difficulties in determining T_e by the probe method in a strong magnetic field were probably responsible for substantial uncertainties.

A set of probes was used in [21] to verify that the oscillations corresponded to m = 1 in helium near H_c. Subsequent studies showed that m = 1 was the correct mode for the helical perturbations in other inert gases as well. However, at low pressures and currents, it was found that m = 2 in nitrogen[22] and in hydrogen.[23] In [22,23], instability began to develop for $\omega_{He}\tau_e \approx 1$. If we abandon the condition $\omega_{He}\tau_e \gg 1$, the stabilizing role of azimuthal electron drift is reduced, so that helical perturbations can begin with m = 2.

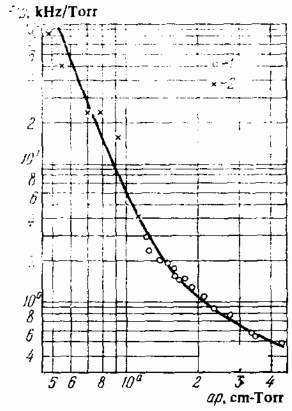


FIG. 7. F/p as a function of ap for neon, taken from [18] (1) and [16] (2).

It is clear from the foregoing examples that the theory of current convective instability given in [5] provides an adequate description of the experimental facts within its range of applicability. Further experimental data on the positive column in a longitudinal magnetic field can also be found in [24–27].

4. CURRENT CONVECTIVE INSTABILITY OF THE POSITIVE COLUMN OUTSIDE THE RANGE OF VALIDITY OF THE THEORY [5]

The validity of the theory may be restricted by finite column length and the fact that some of the assumptions underlying the theory may not themselves be valid (strong magnetization of electrons, diffusion-type discharge, ionization proportional to electron density, and so on).

Timofeev[28] has generalized the theory of current convective instability to the case of finite column length, and took into account the magnetization of ions. He found the stabilization conditions for the first instability modes in a short column, the length of which imposed a lower limit on possible wave numbers. Under these conditions, there are two values of the magnetic field for each mode: instability sets in when the lower of the two is reached, and stabilization is achieved when the larger value is exceeded.

When $k^2 > 1.6E_0/aT_e$, the column is stable for all harmonics, independently of the magnetic field. The excitation and subsequent stabilization of helical oscillations in short positive columns as H increases has been observed by a number of authors.[24,29–31] Akhmedov and Zaĭtsev[32] have investigated the reduction in H_c as the length L of the column inside a solenoid was reduced. They showed that H_c did not vary until L reached a critical value L_c and thereafter there was a rapid reduction in H_c (Fig. 10).

The current convective instability will not develop for any value of H when the solenoid is sufficiently short.[28,31]

The instability of the positive column in low-pressure mercury vapor, when the mean free path is greater than the radius of the tube, is investigated in [29]. Figure 11

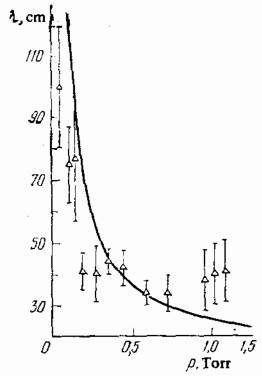


FIG. 8

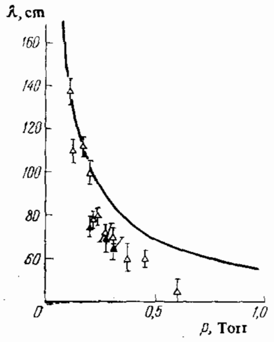


FIG. 9

FIG. 8. Wavelength as a function of pressure [18] for helium (a = 0.9 cm).

FIG. 9. Wavelength as a function of pressure for helium (a = 2.75 cm).

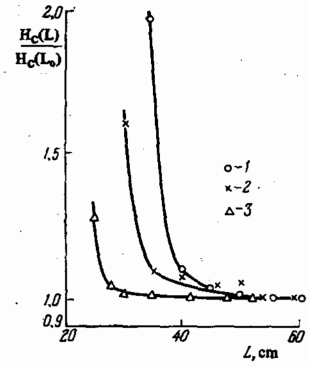


FIG. 10

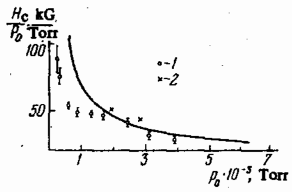


FIG. 11

FIG. 10. H_c as a function of the length of the positive column: [32] helium, p = 0.2 Torr; a = 1.5 cm (1), 1.25 cm (2), and 0.9 cm (3).

FIG. 11. H_c/p_0 as a function of pressure for mercury: a = 1.6 cm, line–theoretical; [7] 1–experiments, [29] 2–recalculated data from [38].

shows H_c/p_0 as a function of pressure, reported in [29]. When $p_0 > 1.5 \times 10^{-3}$ Torr, the experimental points lie near the curve calculated from the formulas in Chap. 2.[7] At lower pressures, the theoretical curve is in clear disagreement with experimental data (the calculated H_c tends to infinity). The inertia of the ions is then appreciable, and the ion-acoustic drift instability (also called drift-dissipative instability) sets in.[33-36] This was discussed in connection with the mercury discharge experiments in [29,34] and, subsequently, the connection with other types of column instability at low pressures was considered in [37].

Repkova and Spivak[38] were the first to investigate the positive column in a longitudinal magnetic field in the case of mercury. They found that, as the field was increased, there was a nonmonotonic variation in the discharge parameters. Analysis of these data and comparison with the results reported in [29] leave no doubt that Repkova and Spivak were dealing with an unstable discharge. Figure 11 shows some of the critical points for plasma parameters in a magnetic field (taken from [38]) recalculated in accordance with the similarity rules. It is clear that the appearance of instability can indeed explain the complicated form of these functions. However, interest in plasma instabilities in a magnetic field had not yet arisen and all this work became undeservedly forgotten.

Akhmedov and Zaĭtsev[39] discovered that the onset of column contraction when the pressure was increased was accompanied by a reductionin H_c. This is particularly clearly defined in neon (Fig. 12). The results of Akhmedov and Zaĭtsev were explained in [40], where the dependence of the ionization frequency on electron density was incorporated into the theory of current convective instability. Under the conditions corresponding to Fig. 12, this dependence arises as a result of the deficit of fast electrons as compared with the Maxwellian energy distribution. Since the redistribution of energy among the electrons occurs as a result of collisions between them, both the size of the deficit and the quantity Z are very dependent on n.[41] When the term $\partial Z/\partial n$ is taken into account in the perturbations, the function $H_c(p)$ is found to be a decreasing one (curve 2 in Fig. 12). In this pressure range, H_c is, of course, very dependent on the discharge current.[39] The reduction in H_c reduces the stabilizing role of the azimuthal electron drift when $\omega_{He}\tau_e$ is less than unity. It is possible that the m = 2 instability in nitrogen and hydrogen,[22,23] and the dependence of H_c on the current, observed in the case of nitrogen,[42] are connected with the strong dependence of Z on n in molecular gases.

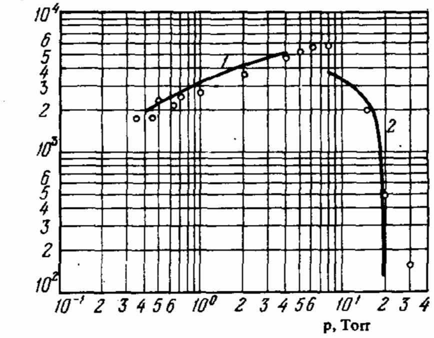


FIG. 12. H_c as a function of pressure in neon: theoretical curves from [7] (1) and [40] (2), experimental points from [10], [39].

At still higher pressures, when the discharge column contracts to a thin filament, current convective instability results in the winding of this filament into a helix. When E is parallel to H, the right-handed twist of the filament is further enhanced by the additional force $[\mathbf{j} \times \mathbf{H}]/c$ observed by Elenbaas[43] as far back as 1951. When E and H are antiparallel, the spiral is left-handed.[39]

5. FINITE-AMPLITUDE WAVES

The finite amplitude of the oscillations near the critical point was taken into account in [5] within the framework of the quasilinear approximation. The additional flow of plasma which appears as a result of current convection is proportional to the square of the oscillation amplitude and is a linear function of the difference $\Delta H = H - H_c$. The function E(H), calculated in [5] with allowance for convection, is shown in Fig. 2. It is in agreement with experimental data up to H = 3 kG, where the average particle flux to the wall is greater than the diffusion flux by a factor of four. As this flux increases, there is an increase in ionization and, consequently, in T_e and E as well.

Holter and Johnson[44,45] have carried out detailed calculations of the characteristics of a column with a nonlinear m = 1 spiral mode, the critical parameters for the excitation of waves with m = 2, and the dependence of the longitudinal electric field, the wavelength, and the oscillation frequency on ΔH. The result was the correct, i.e., increasing, dependence of the oscillation frequency on ΔH, previously observed in [12,21]. In the $Ne + H_2$ mixture containing a high relative amount of hydrogen, this function was found to be increasing.

The radial profile of plasma in the positive column in the postcritical region was examined experimentally in [46-49]. It was found that there was good agreement with the results reported in [44]. In particular, the fluctuation part of the density was found to be proportional to the Bessel function of order one.[47]

A rigorous nonlinear theory of helical oscillations of small amplitude has been developed by Simon and Shiau.[50] They took into account second harmonics of the same order of small quantities as the quadratic (in amplitude) terms, and the presence of ions of different mass. As in [5], these calculations were based on the classical experimental data reported by Garfield.[13] The electric field and the electron temperature were calculated. The results for the helium column were found to be in good agreement with the experiment of Hoh and Lehnert.[9] Calculations of the amplitude of the second harmonic of the oscillations, and allowance for the finite rate of recombination of the plasma at the walls of the tube, were carried out by Holter.[51] Daugherty and Ventrice[52] calculated the density profile for finite-amplitude perturbations with m = 1 and m = 2.

The excitation of current convective instability in a variable magnetic field was found to be subject to a hysteresis effect in the theoretical paper.[44] This was connected with the fact that helical perturbation of the discharge current produced an additional magnetic field parallel to the external field. Hence, when the external field is reduced, the oscillations may stop for external fields less than H_c. Zaïtsev and Shvilkin[53] and Robertson[54] have observed this type of hysteresis. However, the paramagnetic effect of the instability is too small, and the observed hysteresis was probably due to nonlinear processes connected with the ionization balance.

6. TURBULENT POSITIVE COLUMN

When $H \gg H_c$, a broad oscillation spectrum is excited in the positive column and the plasma becomes turbulent. A striking feature, discovered in early work, is the relatively rapid saturation of the E(H) curve when convective transport of plasma to the tube walls is independent of the magnetic field. Kadomtsev[55] used the analogy with the Prandtl theory of a submerged turbulent jet to introduce the idea of the mixing length into the description of the turbulent positive column. During the convective motion, plasma reaches the wall in the form of individual helical tubes and is, in turn, penetrated by "bubbles" free of plasma, which arrive from outside. The gradient $\nabla n/n$ is large on the surface of each such bubble and, therefore, current convective instability ensures that they are rapidly mixed into the plasma. The entire situation thus assumes a turbulent character. Since the density perturbation is produced by convective motion, the average density pulsation can be represented by $n' = l\, dn/dx$, where l is the effective mixing length traversed by the plasma tubes during convection prior to disintegration during interaction with other perturbations. The velocity pulsation v' is related to the density pulsation by the formula $\gamma n' \sim v'\, dn/dx$, where γ is the maximum instability growth rate.

Kadomtsev neglected ionization in the continuity equation, assumed that $\omega_{Hi}\tau_i \gg 1$, and showed for $H \gg H_c$ that

$$\gamma = U\frac{d\ln n}{dx}, \quad U = \frac{1}{2}b_i E\sqrt{\frac{b_i}{b_e}}, \tag{9}$$

from which the radial diffusion current can be shown to be

$$q = \langle n'v'\rangle = -Ul^2\frac{|\nabla n|}{n}\nabla n. \tag{10}$$

Assuming, as in the Prandtl theory, that l is constant over a cross section and is proportional to the tube radius a, Kadomtsev calculated the radial density profile and used the experimental data on plasma diffusion to show that $l \sim 0.15a$. The calculated dependence of $\theta_s = E_s/E_0$ on the magnetic field, where E_s is the electric field in the turbulent column and E_0 is the field for H = 0, was found to be in good agreement with the experimental data reported by various authors (Fig. 13).

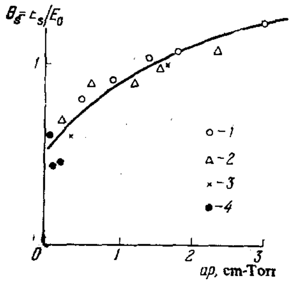


FIG. 13

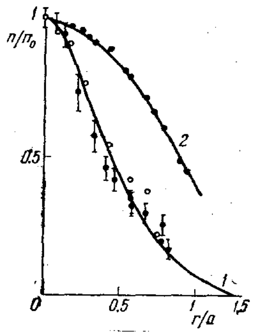


FIG. 14

FIG. 13. 1—a = 1 cm, 2—a = 0.7 cm, 3—a = 0.535 cm (according to the data from [9]), and 4—a = 1 cm (according to the data from [11]).

FIG. 14. The function n(r) for the positive column in helium: [56] 1—H = 1500 Oe, 2—H = 0; p = 0.02 Torr.

A mobile electric probe was used in [56] to determine the n(r) provile in the helium positive column for different gas pressures and $H > H_c$. For $\omega_{Hi}\tau_i < 1$ and $H \approx H_c - 3H_c$, the density profile was found to be nearly the same as $J_0(\alpha_0 r/a)$ although for $H \approx 3H_c$ the longitudinal electric field was close to the saturation value characteristic for the turbulent state. For $H = 8H_c$ and $\omega_{Hi}\tau_i \approx 5$, the discharge was found to be concentrated on the tube axis (Fig. 14) as predicted in [55]. This was connected with the fact that the turbulent diffusion coefficient was proportional to $|\nabla n|/n$ and increased in the direction of the wall. The theoretical curve in Fig. 14 is corrected for the fact that the density n_s near the wall should be of the order of the density pulsation and $q = n_s U$. This ensures that n vanishes for an extrapolated radial distance $R' = a + 2l \approx 1.3a$.

It is important to recall the well-known methodologic difficulties encountered in experiments on discharges in strong longitudinal magnetic fields, which have not been adequately taken into account in a number of papers. They are connected with the magnetic focusing of the electrons by the nonuniform magnetic field at the end of the solenoid facing the cathode.[57] This focusing may distort the radial distribution of the plasma at large distances from the region in which the field is nonuniform, especially when the cathode lies near the solenoid. It occurs because the electron mobility is highly anisotropic. Effects of this kind have frequently been observed, for example, in [58–60]. The second factor associated with this diffusion is the change in sign of the radial electric field, since the magnetic field collects the electrons on the axis. More recent measurements have shown that this factor has a drastic effect on some of the parameters of the turbulent column.[61]

The side effects of focusing can be successfully suppressed by increasing the tube diameter outside the solenoid. This was used in [56], where an increase in the tube radius outside the solenoid by a factor of two had practically no effect on the n(r) profile measured at a distance of eight diameters from the edge of the solenoid. Turbulent transverse diffusion examined in these experiments exceeded classical laminar diffusion by roughly three orders of magnitude.

The theory of the turbulent positive column was thus confirmed experimentally during the first stage of these investigations.

The radial density distribution in the post-critical region, measured in [48,62,63], was found to be uncontracted. In these experiments, the ions were still only weakly magnetized, and the results were not very different from those obtained in [56] for $\omega_{Hi}\tau_i < 1$. They can be compared with laminar convection[44] rather than with the theory of the turbulent column.[55]

As in the case of hydrodynamic turbulence, measurements of average quantities (wall currents and density profiles) were followed by correlation measurements of the pulsation structure in the turbulent column.

Woehler[64] was the first to determine the correlation between signals from two probes at different points in the positive column, using the Lissajous figures.

The oscillation spectra, wave velocities, and correlations between oscillations in longitudinal and transverse directions were measured in [61] for a broad range of experimental conditions. The strong effect of magnetic focusing mentioned above was observed and was found to be accompanied by a change in the sign of the radial field in a substantial part of the tube and by the appearance of radial waves. These phenomena were suppressed by increasing the radius of the tube outside the solenoid and by placing the axis of the tube at a small angle (of the order of one degree) to the field H. The effects of this angle on the distribution of the oscillations were also reported in [65]. As the magnetic field was increased, the small-scale oscillations associated with the development of current convective instability under the action of the longitudinal current did not occur,[61] but new types of oscillation were observed. Figure 15 shows the polar correlation function[66] $R(\tau) = (1/T)\int_0^T \operatorname{sgn} u(t)\, \operatorname{sgn} v(t+\tau)\,dt$ obtained for $H \approx 10H_c$ for signals from probes at the wall and on the axis for three different cases. If the random quantities u(t) and v(t) are normally distributed, then

$$R(\tau) = \frac{2}{\pi}\arcsin F(\tau),$$

where

$$F(\tau) = \frac{\int\limits_0^T u(t)\, v(t+\tau)\, dt}{T\sqrt{\overline{u^2}}\sqrt{\overline{v^2}}}$$

is the usual correlation function. Figure 15 shows that the oscillations exhibit strong correlation over the entire cross section of the column. Measurements performed by Sheffield[63] have also confirmed the strong correlation between the oscillations. For $H_c < H < 4H_c$, a nonlinear spiral was seen [the function $F(\tau)$ was calculated theoretically in [67] near H_c], and for $4H_c < H < 8H_c$, unstable helical perturbations were observed.

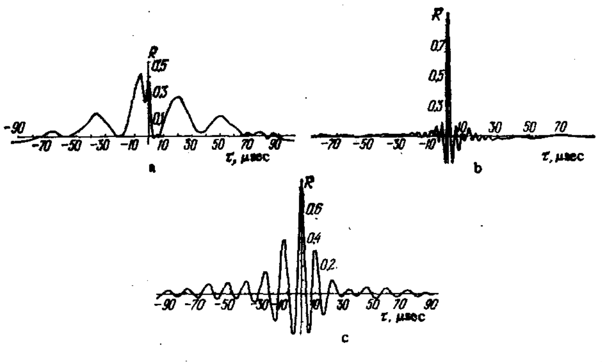


FIG. 15. Polar correlation function for signals from wall and axial probes: helium, p = 0.1 Torr, a = 1.6 cm; [61] (a) H = 5400 Oe, angle between tube axis and field $\alpha = 0$; (b) ditto with $\alpha = 1.2°$; (c) H = 5200 Oe, tube expanded at ends.

These measurements are in conflict with the turbulent column theory since, for correlations as strong as this, $l \approx a$ and the plasma current to the wall should, according to (10), be greater than the observed current by an order of magnitude.

An attempt to remove this contradiction by taking into account fluctuations in the electron temperature and volume ionization was made in [40]. In the case of perturbations with transverse size comparable with the tube radius, we may suppose that the current density in unaltered. Positive density fluctuations then correspond to negative fluctuations in the Joule heat release, and this leads to a reduction in $Z(T_e)$ and to an additional stabilization of the instability. The final result is that the maximum growth rate of the current convective instability is given by

$$\gamma = U\frac{d\ln n}{dx} - 2Z_0, \tag{11}$$

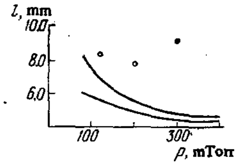
where Z_0 is the ionization frequency in strong fields. The formula given by (10) remains valid when $l \sim a$ if γ is sufficiently small, and this leads to the result Z = const (ap) · U/a obtained by Kadomtsev, which is in agreement with the experimental results for helium (Fig. 13).

Halsteth and Pyle[68] have extended this work to the region of strong magnetic fields, and have shown that small-scale turbulence begins to predominate for $H > 15H_c$, in agreement with the theoretical prediction reported by Sheffield.[63] When $H = 20H_c$, measurements of spatial correlation between pulsations yield $l \sim 0.19a$ (Fig. 16). The radial density profile n(r) is also in good agreement with the turbulent-column theory. These data were obtained for H = 12 kG in a helium discharge at pressures of 0.02–0.4 Torr and tube radius of 2.75 cm.

Details of the transition to fully developed turbulence are still not clear, and further experiments are necessary to elucidate this situation. Since the degree of magnetization of the ions is quite substantial, the ratio H/H_c is not sufficient to characterize the turbulent column. Further experiments with different gases are desirable. For small-scale pulsations, highly elongated in the direction of the field, the conditions j = const which, as indicated above, acts as a stabilizing factor is no longer valid. On the contrary, one would expect an enhancement of the perturbations by ionization and overheating, and this may be of the same order as the growth rate of current convective instability.

It is quite likely that effects associated with other instabilities of inhomogeneous plasma will appear in strong magnetic fields for $\omega_{Hi}\tau_i > 1$. At sufficiently low gas pressures, longitudinal ion-acoustic waves[7,69–72] are excited in the positive column, and

FIG. 16. Mixing length as a function of pressure: [68] helium, a = 2.75 cm, ○—experimental points, curves—calculated using [55].



the spectrum of these oscillations lies at higher frequencies.

Striations are another, more common, form of oscillations of the positive column. The longitudinal field has a very complicated effect on the parameters of these striations and their boundaries. In particular, it has been found that the striations are suppressed when the helical perturbations appear.[73] However, later work shows that both types of oscillation are present simultaneously, and the amplitude of the striations begins to fall with increasing H until the current convective instability appears.[74,75]

Anomalously strong scattering of microwaves by the positive column at 34 and 30 GHz was reported in [76,77]. Some data on the mercury column with H up to 80 kG are reported in [78].

The gas-discharge column in a strong magnetic field is very convenient for the investigation of interactions between different types of wave, which is one of the most complicated problems in the physics of turbulent plasma. The possibilities of this approach have not been exhausted by far.

7. CURRENT CONVECTIVE INSTABILITY IN DIFFERENT PLASMA SYSTEMS

So far, we have been considering the instability of a cylindric column of weakly ionized plasma with a longitudinal current. The current convective instability can also appear in many other situations encountered in different plasma configurations.

It occurs in fully ionized plasma carrying a longitudinal current when there is a finite transverse temperature gradient dT_0/dr and, consequently, an analogous gradient in the conductivity.[79] The only difference in the mechanism responsible for the development of instability in this case is that, when $\omega_{Hi}\tau_i \gg 1$, the layer with enhanced conductivity (Fig. 1) drifts as a whole with velocity cEH/H^2.

This instability was the first dissipative instability of inhomogeneous plasma due to departures from ideal properties (in the present case, finite electrical conductivity σ_0) to be investigated theoretically. Following the work reported in [79], the possibility of current convective instability in the Tokamak, Stellarator, and Zeta installations was discussed in [80,81]. Current convective instability in fully ionized plasma was observed experimentally in [82].

Particle and energy losses in the Tokamak system due to current convection are of particular interest. Analyses carried out by Kadomtsev and Pogutse[83] and by Artsimovich[84] have shown that these losses present no particular danger to the establishment of conditions necessary for thermonuclear reactions. Thermal conductivity in the direction of the magnetic field, which stabilizes temperature perturbations, ensures that only fluctuations with small transverse dimensions (determined by the crossing of the magnetic lines of force) can develop, and their contribution to the transverse heat flux is small.

Vladimirov[85] has discussed the effect of current convective instability in combination with "ambipolar sound" in plasma in which ions escape to the walls without collisions. Vladimirov's results are in agreement with experimental data on mercury discharges in short solenoids.[29]

Current convective instability in the positive column of noncylindric geometry and in the presence of diaphragms was investigated experimentally in [86]. The radial distribution of density and electric-field fluctuations during developed instability in a column with conducting walls were obtained in [87]. The azimuthal component of the field fluctuation decreases near the wall, and this leads to a weakening of convection. There is a corresponding increase in plasma density and in the radial component of the electric field.

Comparison of current convective instability in coaxial and ordinary cylinders was carried out experimentally in [88,89].

Guest and Simon[90] used the analogy with the screw instability of the positive column to put forward an explanation of the instability of plasma in a container with conducting walls. It is well known[91] that the diffusion of charged particles in a plasma of this kind is not ambipolar; an electron current which can be the origin of instability appears in the direction of the magnetic field. Vdovin has shown[92] that plasma inhomogeneity in the direction of the magnetic field can lead to analogous instability.

It is shown in Chap. 2 that drift in a radial electric field, which takes place toward the wall at the critical point, will stabilize the discharge. The change in the sign of the field changes the direction of the electron drift and leads to instability. This is, essentially, the current convective instability due to the current perpendicular to H for which purely azimuthal modes are stable. The possibility of current convective instability in the case of positively charged walls was noted by Simon[93] and in the case of the Penning discharge by Hoh[94] (see also [95]). Penning discharge instabilities have frequently been observed experimentally, including the excitation of low-frequency oscillations and screw perturbations accompanied by anomalous diffusion.[96-100] In addition to purely drift instabilities, the effects discussed in [93,94] can also appear in such discharges. In particular, the Hoh theory was confirmed in [100] for discharges in molecular gases.

The results reported in [93] were used by Garrison and Hassan in an analysis of plasma stability in the MHD accelerator[101] and by Kim in connection with a special case of deceleration of a plasma current by a transverse magnetic field.[102] The modification of current convective instability discussed in [93,94] was used in [103] in connection with self-sustaining discharges which appear under certain definite conditions as a result of the motion of a gas in a transverse magnetic field. High density gradients are present in such discharges at the entrance to and exit from the MHD channel. On the boundary located under the current, an instability with a growth rate

$$\gamma = (\omega_{Hi}\tau_i)(\omega_{He}\tau_e)^3 u\varkappa,$$

can occur, $\kappa = d \ln n/dx$ and u is the gas velocity. These effects have not been investigated experimentally although they may have been present in models of the MHD generator with hot electrons.[104,105]

Mihailovskiĭ and, independently, Rukhadze et al. have investigated the instabilities of magnetized plasmas in which an electron beam which was inhomogeneous in the transverse direction lay along the magnetic field.[106, 107] These instabilities are the limiting case of the instability of inhomogeneous current-carrying plasma when the motion of electrons is unaffected by collisions between electrons and heavy particles. The current convective instability discussed in [5] is the other limiting case of frequent collisions. It is shown in [106] that there are low-frequency and high-frequency ($\omega \gg \omega_{Hi}$) collisionless current convective instabilities. Under the simplest assumptions, involving potential oscillations and electron velocity U_0 in the beam independent of the coördinates, the mechanism responsible for the first of these instabilities is as follows. Suppose that an electric-field perturbation with components E_x and E_z appears in plasma containing a beam which is inhomogeneous in the y direction. The change in the current in a small plasma volume during a time interval δt is:

$$\delta j_z = e\left(n_0 \frac{e}{m} E_z \delta t + U_0 \delta n\right).$$

The perturbation δn is connected with drift in the y direction under the action of E_x:

$$\delta n = \frac{cE_x}{H_0} \frac{\partial n_0}{\partial y} \delta t.$$

Hence,

$$\frac{\delta j_z}{\delta t} = \frac{e^2 n_0}{m} E_z + \frac{cE_x}{H_0} \frac{\partial j_z}{\partial y}.$$

The change in the current reacts on the electric field which produces it. Differentiating the equation $\partial E/\partial t = -4\pi j$ with respect to time, we obtain

$$\frac{\partial^2 E_z}{\partial t^2} = -4\pi \left(\frac{e^2 n_0}{m} E_z + \frac{cE_x}{H_0} \frac{\partial j_z}{\partial y}\right). \tag{12}$$

An aperiodic instability occurs at $-(E_x/E_z)(c/H_0) \cdot \partial j_0/\partial y > e^2 n_0/m$ and, since $E_x/E_z = k_x/k_z$, the instability condition can be written in the form

$$\frac{[\mathbf{k} \times \nabla j_0]}{e n_0 \omega_{He} k_z} > 1. \tag{13}$$

Further interesting developments in this direction are discussed in the reviews of Nezlina and Bogdankevich and of Rukhadze.[108, 109]

Current convective instability during turbulent current heating of plasma is investigated both theoretically and experimentally in [110]. In this case, the current convective instability has various features connected with the anomalous electrical resistance during the excitation of ion-acoustic oscillations by the current. It gave rise to the appearance of a discontinuity in the potential in the plasma, a sharp change in the distribution of the current over the cross section, and low-frequency fluctuations in the longitudinal and azimuthal magnetic field. Figure 17 shows the dependence of the time for development of the instability as a function of the magnetic field, as reported in [110]. The theoretical and experimental curves are in good agreement.

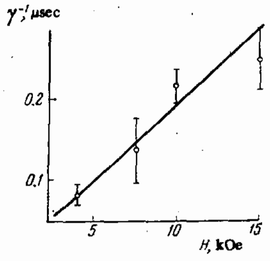


FIG. 17. Time for the development of current-convective instability as a function of H: [110] straight line—theoretical plot of the reciprocal of the damping rate.

8. STABILIZATION OF CURRENT CONVECTIVE INSTABILITY

Once the mechanism responsible for the current convective instability was reliably established, it became possible to investigate different methods of stabilizing it. Methods of stabilizing instabilities have been very extensively discussed in the physics of high-temperature plasma. The positive column is a convenient means of investigating these methods.

The first serious investigation in this area was reported by Gierke and Wohler.[20] They tried to obtain convincing evidence for the mechanism responsible for current convective instability and have substantially extended the experimental conditions used by Lehnert. Thus, they investigated the effect of high-frequency electric and azimuthal magnetic fields on the discharge. They showed that the imposition of an additional high-frequency field of sufficient amplitude gave rise to an increase in H_c. At the same time, the energy released by the high-frequency field ensured that a sufficient level of electron temperature was maintained for lower values of the constant electric field. According to (5) this should lead to an increase in H_c. After Gierke and Wohler, the effect of the high-frequency field on current convective instability in the positive column was investigated in detail by Akhmedov and Zaĭtsev,[32] Rugge and Pyle,[111] and Powers.[112] In particular, Rugge and Pyle investigated the dependence of H_c on the frequency of the applied field in the high-frequency discharge right up to the complete suppression of screw perturbations at high frequencies. Powers[112] obtained the similarity law $H_c/p = f(ap)$ for the high-frequency discharge, which was analogous to that shown in Fig. 3.

In addition to the increase in H_c, Akhmedov and Zaĭtsev observed the opposite effect, i.e., a reduction in the critical field when a low-amplitude high-frequency field was applied to part of the column. They explained this by suggesting that, in a weak high-frequency field which reduced the longitudinal electric field by only a few percent, the diffusion current to the tube wall fell as a result of the presence of the force

$$F = -\frac{e^2}{4m(\omega^2 + \nu^2)} \frac{dE^2}{dx}$$

where E and ω are, respectively, the amplitude and frequency of the high-frequency field, and ν and m are, respectively, the collision frequency and mass of the charged particles.

Physical considerations suggest that current convective instability should be impeded, or suppressed altogether, when the electrons can move across the magnetic field and can neutralize the polarization of the layer due to fluctuation in conductivity. This possibility can be associated, for example, with gradient or centrifugal drifts in an inhomogeneous field. Fowler[113] has considered the possible stabilization of screw modes by a "magnetic well." He has established a stabilization criterion for the first mode of current convective instability in the positive column, which can be written in the form

$$\frac{\Delta H}{H} > \left|\frac{k_z(\omega_{He}\tau_e)}{k_\perp}\right| \frac{aE_0}{T_e} \sim 1, \tag{14}$$

i.e., the change in the field over the cross section of the column should be large and of the order of the field itself. Condition (14) has a simple physical interpretation. In the time $t \approx (b_e k_z E_0)^{-1}$ during which the electrons are

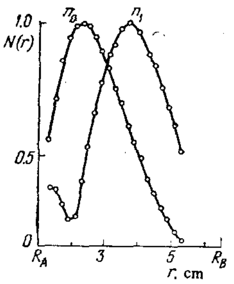


FIG. 18. Profiles of stationary distribution of plasma (n_0) and helical perturbation (n_1) in a coaxial discharge [114].

displaced along the tube to a distance of the order of a wavelength, they are also subject to drift in the inhomogeneous magnetic field through a distance

$$k_\perp^{-1} \approx v_{\mathrm{dr}} t = \frac{T_e}{m_e \omega_{He}} \frac{\Delta H}{Ha} (b_e k_z E_0)^{-1}.$$

The effect of the azimuthal magnetic field can be observed in tubes of coaxial geometry when an isolated metal rod carrying a current is placed along the axis of the tube.[20] When the additional force $[j_z \times H_\varphi]$ points in the outward direction, the screw perturbation occurs for lower H_c, and vice versa.

Reynolds and Holt[114] have investigated the current convective instability of a hollow plasma column in a helical magnetic field with shear. They showed both experimentally and theoretically that, in addition to the crossing of the magnetic lines of force and the azimuthal field H_φ, there is a substantial change in the profile of the stationary distribution of the plasma which, in turn, affects its stability.

This change corresponds to attraction or repulsion of currents flowing in the plasma and in the rod, depending on whether they are parallel or antiparallel. In the hollow column, the helical perturbation develops only in the region of the negative plasma density gradient (Fig. 18). The azimuthal field H_φ modifies not only H_c but also the rotational frequency of the helix, and destabilization facilitates the relatively rapid emergence of the second mode. The effect of the magnetic field due to the plasma column itself on its stability in a longitudinal field is discussed in [115].

Rutscher and Rüzicka have observed the motion of the helical wave in a coaxial tube at moderate gas pressures when the discharge was highly contracted.[116] When the force $[j \times H]$ presses the current filament against the inner wall of the tube, the waves move in the direction of the anode, and when it presses the current against the outer wall the waves move in the direction of the cathode.

A time-independent additional magnetic field is considered in the papers cited above. Johnson and Jerde have determined theoretically the effect of a longitudinal field which increases linearly in time, and showed that a sufficiently strong, azimuthal, induced electric field will stabilize the discharge.[117] The stabilization is connected both with the effect of j_φ, considered in Chap. 1, and additional Joule heating of the electrons. They have observed experimentally the stabilization of helical instability in a rapidly growing magnetic field and its destabilizationin a decreasing field.[118]

A fundamentally different method of stabilization by a time-dependent field has been proposed by Ivanov et al.[119] If H_φ is a periodic function of time, and the magnetic lines of force are twisted so that the plasma succeeds in moving along these lines to a distance equal to the characteristic transverse size of the perturbations, the instability cannot develop. For the current convective instability, this stabilization condition has the form[120, 121]

$$\frac{k_\perp^2 D_a}{\Omega} \left(\frac{H_\varphi}{H_0}\right)^2 > 1, \tag{15}$$

where Ω is the frequency of H_φ and D_a is the ambipolar diffusion coefficient in the direction of the field. Since Ω can be greater than the frequency of the helical perturbations (8), the condition given by (15) leads to the relatively stringent requirement that $H_\varphi \sim H_0$. The idea of dynamic stabilization of current convective instability by a time-dependent H_φ has been successfully tried in the experiment described in [122]. An alternating current of 75 A and frequency $\Omega/2\pi = 100$ kHz was passed through the central rod in a tube of 1.4 cm radius filled with helium at $p = 0.1$ Torr. When the longitudinal magnetic field reached the value $H_0 = 750$ G, the introduction of the alternating current reduced the amplitude of the helical oscillations by a factor of two.

Helical waves can be excited artificially for $H \leqslant H_c$ by applying a local periodic disturbance to the positive column. Thus, for example, the spatial enhancement of helical waves in He, Ne, and Hg is determined in [123]. When this is incorporated in a negative feedback system, the result may be an effective means of stabilizing current convective instabilities.

It is well known that the idea of using negative feedback to suppress various instabilities has been very effective in the physics of high-temperature plasmas. Arsenin and Chuyanov[124] and Arsenin[125] have investigated theoretically the various possible methods of stabilizing current convective instabilities in the gas discharge column. A continuous distribution of additional electron sources in the positive column, the intensity of which was controlled by a negative feedback system, was investigated in [125] and a condition for the suppression of instability was found. Because of the large-scale nature of the helical waves, it is evidently sufficient to control the plasma density by electric probes located at a number of points within a wavelength.

9. CONCLUSIONS

It is evident from the above review that the instability of magnetized current-carrying plasma is now a relatively well understood phenomenon. Owing to the relative simplicity of the experimental systems, and the availability of adequate theoretical models, it has been possible to achieve good quantitative agreement between theory and experiment in all the major areas. This refers to the linear theory and small-amplitude oscillations, and to nonlinear processes.

In many cases, current convective instability has an appreciable effect on the characteristics of various technologic systems incorporating a magnetic field, for example, plasmatrons and gas lasers (the literature in this field is not included in the present review).

Current convective instability has played an important role in filling the gap between the physics of low temperature plasmas in gas discharges and the physics of high-temperature plasmas. Analyses of this instability have shown that many of the phenomena occurring in hot plasmas, which are independent of the absolute temperature level, can be effectively investigated under much simpler conditions. This has facilitated the discovery of a large class of dissipative plasma instabilities and the development of a general theory of turbulent processes in plasmas as a result of the data obtained from studies of current convective instability.

The idea of current convective instability has been extensively developed in solid state physics. Studies of this instability have always been international in character and this has been facilitated by the fact that they do not require complicated and expensive installations. Many universities are therefore participating in the investigation of the current convective instability.

Currently available data on turbulent phenomena in the positive column and on methods of suppressing plasma instabilities have not by far exhausted all the possibilities. New results can be confidently expected in this field.

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