Vortex motion and resistivity of type-II superconductors in a magnetic field

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The review analyzes the physical mechanisms of energy dissipation in type-II superconducting alloys in the mixed state. A very simple microscopic theory is presented for the dissipative processes in alloys with paramagnetic impurities. The main premises of the microscopic theory are described and its results are presented for the resistivity in the case of ordinary alloys (without paramagnetic impurities) in weak magnetic fields smaller than the upper critical field, at the low temperatures and at temperatures close to critical, and also in the case of strong magnetic fields, on the order of the upper critical field, in the entire temperature interval. The theory is compared with experiment.

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CONTENTS

INTRODUCTION

Among the numerous singularities in the properties of the so-called type-II superconductors, which include superconducting alloys with sufficiently short mean free paths as well as pure niobium, the least trivial is the appearance of a finite conductivity in the presence of a magnetic field. This circumstance, which is of primary significance for practical applications of superconductivity, has been for the last decade the subject of numerous experimental and theoretical studies. It can now be regarded as established that this resistance is due to motion of the vortex structure first introduced by Abrikosov[1] to describe the unique behavior of type-II superconductors in a magnetic field. The vortex motion, as first noted in[2], is the result of the action exerted on the vortex by the superfluid component of the Lorentz-force current. In real superconductors the situation is greatly complicated by the action of the pinning forces, so that a properly uniform motion of a vortex lattice occurs only if the current through the sample is large enough to suppress the action of the pinning centers. We shall consider throughout just this case, and only then is the resistance a property inherent in the superconductor itself and does not depend on the details of the sample preparation. Figure 1 shows current-voltage characteristics obtained in the classical experiments of Kim, Hempstead, and Strand[3] for Nb-Ta alloys. We see that although the threshold current at which a finite potential difference appears is different for different samples, the slopes of all curves are the same at large currents.

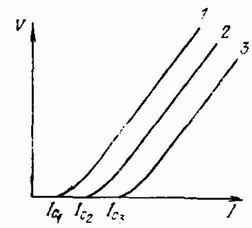

FIG. 1. Idealized current-voltage characteristics for samples with different degrees of structure inhomogeneity.

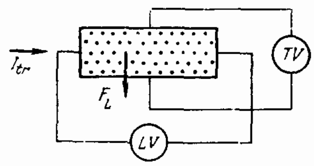

FIG. 2. Experimental setup for the measurement of ρ_f and of the Hall angle. The points mark the vortex filaments. LV and TV are the longitudinal and transverse voltages.

Figure 2 shows schematically the usual experimental setup. The magnetic field is perpendicular to the plane of the plate (film) of the investigated superconductor, and the points designate the vortex structure. A finite direction to the longitudinal current I_{tr} makes it possible in principle for a Hall potential difference to appear, and this difference was indeed observed in the experiments of Reed, Fawcett, and Kim.[4] The sample is usually a thin film. The use of such a setup is dictated by the desire to prevent bending of the vortex lines by the self-field of the current and by the action of the pinning forces, and also the desire to attain high homogeneity of the transport-current density. In fact, although at a finite resistance the average current density in a bulky sample would be different from zero, the Meissner effect on the surface would lead to a significant bending of the current line in the vicinity of a vortex filament if the filament density were not too large. From this point of view, the optimal films have dimensions smaller than the depth of penetration of the magnetic field.

A vortex filament carries a magnetic-field-flux quantum $\boldsymbol{\varphi}_0 = (hc/2e)\mathbf{n}_H$ (the vector $\mathbf{n}_H$ is directed along the field).[1)] Therefore the Lorentz force exerted by the flowing current on the system of currents in the

[1)]A list of symbols is given at the end of the article.

vortex (per unit vortex length)

$$F_L = \frac{1}{c}[j_{tr} \times \varphi_0],$$

is directed as shown in Fig. 2. If this force causes the vortex to acquire a velocity $v_\perp$, then the macroscopic electric field induced in the sample is

$$E = \frac{1}{c}[B \times v_L],$$

where the magnetic induction is $B = n_L\varphi_0$ (n_L is the density of the vortex filaments). Thus, the direction of the electric field is determined by the direction of vortex-filament motion. In a Galilean invariant system, such as helium, the vortex filaments at low temperatures move together at equilibrium with the superfluid component of the velocity. If the same were to occur in a superconductor, as proposed by de Gennes and Nozieres,[5] then according to Fig. 2, at least for a pure type-II superconductor (e.g., Nb), the largest quantity in the experiments would be the Hall potential difference (the transverse field E). The dragging of the vortices in the metal lattice can be hindered by their deceleration via some relaxation mechanism. If this deceleration is large, then the vortices move slowly under the influence of the force F_L in a direction transverse to the transport current—this is called viscous flow of the vortices. Experiment, even at low temperatures, offers decisive evidence in favor of predominance of the dissipation mechanisms whose nature will be discussed below. The Hall component of the velocity turns out to be small, and for the Hall angle α

$$\mathrm{tg}\,\alpha = \frac{v_\parallel}{v_\perp}$$

it is of the same order as in the normal metal.[4] At T = 0 the effective conductivity obtained in the experiments[3] turned out to be

$$\sigma_f = \frac{\sigma_n H_{c2}}{B}.$$

From the theoretical point of view, the fact described above should fit within the framework of the Bardeen-Cooper-Schrieffer (BCS) microscopic theory of superconductivity.[6] The fact that the vortex structure moves relative to the lattice calls for a generalization of the BCS theory to include the dependence of the parameters of the theory on the time. A microscopic theory of nonstationary processes in superconductors was constructed by Gor'kov and Éliashberg.[7-9] As is customary in superconductivity physics, a qualitative understanding of the phenomena in type-II superconductors, which occur when current flow, can be gained on the basis of the semiphenomenological considerations advanced in Chap. 1 of this review. In Chaps. 2 and 3 we discuss the results of the microscopic theory. In the final Chap. 4 we attempt to estimate the degree of quantitative correspondence between the microscopic theory and experiment.

1. SEMIPHENOMENOLOGICAL DESCRIPTION OF THE VISCOUS FLOW OF A VORTEX STRUCTURE

a) Thermodynamic properties of a type-II superconductor in a magnetic field. We summarize below the basic vortex data which we shall need subsequently. A more detailed exposition can be found in the book[10] or in the review of Schmidt and Mkrtchyan.[11]. As already mentioned, type-II superconductors include pure niobium and possibly vanadium, as well as superconducting alloys with small mean free paths.[12] From the physical point of view, the main property responsible for the qualitative change of the behavior of these objects in a magnetic field is the negative surface energy on the boundary between the normal and superconducting phases. In the Ginzburg-Landau theory[13] the surface energy is negative at $\kappa > 1/\sqrt{2}$, where the parameter

$$\varkappa = \frac{\sqrt{2}\, 2eH_{cm}\delta^2}{\hbar c}$$

characterizes, from the point of view of the BCS theory, the ratio of the depth of penetration δ of the field to the coherence radius ξ:

$$\varkappa = \frac{\delta}{\xi}.$$

Abrikosov[1] has shown, in the approximation of the Ginzburg-Landau theory, that the destruction of superconductivity in a magnetic field begins with a field $H_{c1} < H_{cm}$ and terminates at $H_{c2} > H_{cm}$, where H_{cm} is the thermodynamic field of the transition. At $\kappa \gg 1$ we have[1]

$$H_{c2} = \varkappa \cdot \sqrt{2} H_{cm}, \quad H_{c1} \approx H_{cm} \frac{\ln \varkappa}{\sqrt{2}\varkappa}. \tag{1.1}$$

In the field interval $H_{c1} < H \ll H_{c2}$ ($B \ll H_{c2}$) the magnetic field penetrates into the sample in the form of filaments, the structure of which is shown in Fig. 3. This figure shows schematically the distributions of the magnetic field H and of the order parameter Δ as functions of the distance ρ to the center of the filament. The magnetic flux connected with an individual filament is equal to hc/2e and is distributed over an area with dimensions on the order of the penetration depth. The field near the filament is therefore usually of the order of H_{c1}. The order parameter varies over distances $\xi = \delta/\kappa \ll \delta$ and tends rapidly to the equilibrium value Δ_∞ ($\Delta_\infty \equiv \Delta$ in the absence of a field). At the center of the core we have $\Delta(0) = 0$. In the field interval indicated above the filament cores do not overlap, and we can use the concept of individual filaments packed in the lattice and coupled with one another by a magnetic interaction whose characteristic effective radius is large in comparison with the dimension of the filament core. The macroscopic mean value of the magnetic field is the induction $B = n_L\varphi_0$.

When the fields increase to H_{c2}, the distances between filaments decreases to $d \sim \xi$. It is seen from Fig. 3 that the cores begin to overlap, and this leads to an appreciable decrease of the value of the order parameter averaged over the sample.

The microscopic theory of superconductivity had confirmed the general picture of the vortex state in a magnetic field, which was proposed in[1], in two respects. First, it has turned out[12] that in the vicinity of the transition theory the BCS theory goes over into the Ginzburg-Landau theory, and therefore the results of[1] are approximate in character in this region. The order parameter of the theory[1,3] is proportional to the energy gap Δ in the energy spectrum of the superconductivity in the BCS theory. Second, in the entire range of

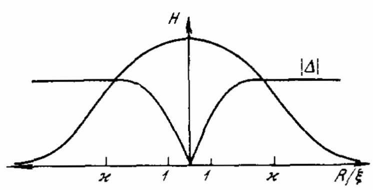

FIG. 3. Schematic plots of the order parameter and of the magnetic field for an isolated vortex filament.

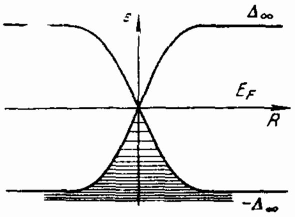

FIG. 4. "Band structure." The shaded area shows the occupied states.

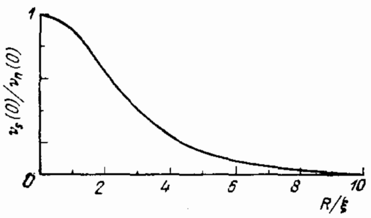

FIG. 5. Normalized state density on the Fermi surface for a "dirty" superconducting alloy. [16]

temperatures, the penetration of the magnetic field into a bulky sample begins at a certain weak field H_{c1} and proceeds in the manner described above in the form of a lattice of filaments. The fields H_{c1} and H_{c2}, accurate to temperature-dependent coefficients, are of the order of (1.1), where H_{cm} should be taken to be the thermodynamic critical field of the bulky superconductor at the given temperature.[14]

An important circumstance, which is to play an essential role in what follows, can be qualitatively deduced from the fact that the order parameter vanishes at the center of the filament, and also from the remark made above concerning the connection between the order parameter and the gap in the energy spectrum of the superconductor in the BCS theory. According to[6], in a homogeneous superconductor the energy spectrum of the excitation has a threshold, namely, the density of states $\nu_s(\epsilon)$ in the superconducting state is equal to

$$\nu_s(\varepsilon) = \nu_n \frac{\varepsilon}{\sqrt{\varepsilon^2 - \Delta^2}}.$$

It can thus be assumed that in a superconductor that is inhomogeneous in space the state density depends on the point, and the vanishing of the order parameter at the center of the filaments indicates that there is no excitation threshold near the center. This was first demonstrated by Caroli, de Gennes, and Matricon[15] within the framework of the microscopic theory for a pure type-II superconductor. The result of[15] is shown schematically in Fig. 4 in the form of the "band" structure of the spectrum of the superconductor. The shaded region shows the filled excitation states of energy $\epsilon < 0$ at absolute zero temperature. The distance between the two curves is the "local" threshold for the excitation of the ground state. As $\rho \to \infty$ this threshold is equal to $2\Delta_\infty$, but in the region $\rho \sim \xi$ there is no threshold. In this region we have $\nu_s(0)/\nu_n \sim 1$. Figure 5 shows the results of numerical calculations[1] of the function $\nu_s(\rho, 0)/\nu_n$ in the case of a so-called "dirty" alloy, when the mean free path is $l \ll \hbar v_F/T_c$. The value of ξ in Fig. 5 is chosen to be $\xi = \sqrt{D/2\Delta_\infty(0)} = \sqrt{l v_F/10.5 T_c}$.

Generally speaking, as first shown by Bardeen, Kummel, Jacobs, and Tewordt,[17], the structure of the energy spectrum of Fig. 4 at intermediate $\kappa \sim 1$ has a strong effect on the value of the critical field H_{c1} at low temperatures, since the energy of the vortex filament is determined in this case to a considerable degree by its core. The distribution of the magnetic field also has a different form. Compared with Fig. 3, the field in the vicinity of the center of the filament has a rather sharp maximum.[18] The origin of this effect follows most clearly from the results[47] for a pure type-II superconductor. In this case the Fermi excitations in the core of the filament have a conserved projection of the angular momentum m relative to the axis on the direction of the magnetic field, but in this case $\epsilon < 0$ (shown shaded in Fig. 4) corresponds to $m < 0$, whereas $\epsilon > 0$ always corresponds to $m > 0$. At zero temperature, as is indeed the case for Fig. 4, all the states with $m < 0$ are occupied and the states with $m > 0$ are empty. Thus, the vicinity of the vortex center is a solenoid of sorts. The effect of the concentration of the field in alloys with $\kappa \sim 1$[18] may turn out to be so strong that the field can reverse direction in order to preserve the total magnetic flux φ_0. We shall deal here in the main with the case $\kappa \gg 1$.

b) Physical nature of the **dissipative processes**. The just-described structure of the vortex suggests a certain simplified model in which the vortex consists of a perfectly normal core and of a region with dimension δ, in which a flux quantum is concentrated. On the boundary, the superconducting parameter changes jumpwise from its value in the homogeneous superconductor to zero. This rough picture makes it possible, however, to understand one of the mechanisms of energy dissipation in the vortex, viz., Joule heating of the normal excitations near the center of the filament. An estimate of the contribution of this mechanism was first made by Bardeen and Stephen[19] and was based on the following considerations. We write down the hydrodynamic equation for the velocity v_s of the "superfluid" electrons:

$$m \frac{d\mathbf{v}_s}{dt} = \nabla \mu, \qquad (1.2)$$

where μ is the chemical potential. If the vortex filament moves as a unit with constant velocity $\mathbf{v}_L$, then the total derivative $d\mathbf{v}_s/dt$ contains the term

$$(\mathbf{v}_L \nabla)\,\mathbf{v}_s = \nabla\,(\mathbf{v}_L \mathbf{v}_s), \qquad (1.3)$$

since curl $\mathbf{v}_s = 0$, here $v_L \ll v_s$, and $\mathbf{v}_s$ is the velocity field of the superfluid component outside the core of the vortex:

$$m\mathbf{v}_s = \mathbf{p}_s(\mathbf{r}) + \frac{e}{c}\mathbf{A}(\mathbf{r}). \qquad (1.4)$$

Inasmuch as in this model the order parameter is constant outside the vortex,

$$2\mathbf{p}_s = \hbar\nabla\theta,$$

i.e., it is proportional to the gradient of the phase of the Cooper-pair wave function (hence the coefficient 2!). The condition that the wave function be unique

$$2\oint \mathbf{p}_s\, dl = \hbar \cdot 2\pi$$

yields

$$(\mathbf{p}_s)_\varphi = \frac{\hbar}{2\rho}.$$

The potential $\mathbf{A}(\mathbf{r})$ in (14) is small (at $\kappa \gg 1$ the magnetic field inside the vortex is $\sim H_{c1}$), and we ultimately obtain

$$(\mathbf{v}_s)_\varphi = \frac{\hbar}{2m\rho}.$$

The additional term (1.3) in the expression for $d\mathbf{v}_s/dt$ is an additional acceleration that acts on the "superfluid" electrons because of the vortex motion. It is important that in the stationary picture of moving vortices the current line must inevitably enter into the vortex core, i.e., the superfluid current is converted on the vortex boundary into a normal electron current, and an electric field $E = -\nabla\Phi$ is produced in the normal region. According to (1.2), in the superconducting region there is added to the chemical potential a term $\mu_s \to \mu - (\mathbf{v}_L\cdot\mathbf{v}_s)$. In the normal region $\mu_n = \mu - e\Phi$. The equilibrium conditions calls for continuity of μ on the boundary. Since inside the vortex we have $\mathbf{j}_n = \sigma_n\mathbf{E}$ and div $\mathbf{j}_n = 0$, it follows that $\nabla^2\Phi = 0$. As a result we get

$$\mu_s = \mu - \frac{\hbar v_L y}{2\rho^2} \qquad (\rho > \xi),$$

$$\mu_n = \mu - \frac{\hbar v_L y}{2\xi^2} \qquad (\rho < \xi).$$

The electric field in the core of the vortex is $E = \hbar v_L/2\xi^2 e$, and we can write down for the energy dissipation per unit volume

$$W = \sigma_n E^2 n_L \pi\xi^2 = \frac{\sigma_n \hbar^2 v_L^2 \pi}{4\xi^2 e^2}\frac{B}{\varphi_0},$$

where n_L is the number of filaments per unit area. Expressing the velocity v_L in terms of the average field $\bar{E} = (v_L/c)B$, we obtain for the effective conductivity $W = \sigma_f\bar{E}^2$

$$\sigma_f = \frac{\sigma_n H_{c2}}{2B}. \qquad (1.5)$$

The crude model of a vortex with the core cut out does not make it possible, of course, to determine the numerical coefficient in this formula, but the order of magnitude turns out to be correct.

The second mechanism, which was pointed out by Tinkham, is essentially connected with the inhomogeneity of the order parameter in the vortex. In an immobile reference frame, at a given point of the metal through which a vortex filament passes, the order parameter varies with time from an equilibrium value $|\Psi_0|$ far from the vortex to zero, and then increases again as the vortex filaments moves farther from the point. The characteristic time of passage of the filament is $t_0 \sim \xi/v_L$, whereas obviously there is also another time τ_0 during which an equilibrium distribution of the order parameter is established. If F is the value of the free-energy density, then in the case of slow motion $\tau_0 \ll t_0$ the fraction of the energy

$$\frac{\tau_0}{t_0}\bar{F}$$

is dissipated in irreversible fashion. The dissipation per unit volume in a unit time is

$$W \sim \frac{\tau_0}{t_0^2}\bar{F}\xi^2 n_L \sim \frac{v_L^2\tau_0 H_{cm}^2 B}{\varphi_0}.$$

For the effective conductivity we obtain

$$\sigma_f = \frac{\tau_0 H_{cm}^2 c^2}{B\varphi_0} \approx \tau_0\frac{c^2}{\delta^2}\frac{H_{cm}^2}{H_{c1}B} \approx \frac{\tau_0 c^2}{\delta^2}\frac{H_{c2}}{B}.$$

As the characteristic relaxation time it is natural to choose $\tau_0 \sim \hbar/\Delta(T)$. Inasmuch as in the limit of a small mean free path ($\kappa \gg 1$) we have[21]

$$\delta \sim c\sqrt{\frac{\hbar}{\Delta\sigma_n}},$$

we again obtain expression (15) as an estimate of this mechanism. Actually, as we shall show below, in the vicinity of the critical temperature the mechanism wherein the order parameter relaxes prevails.

c) **Dissipation function.** The dissipation mechanisms discussed above for the motion of vortex filaments were brilliantly unified by Schmid[22] in a phenomenological theory constituting an attempt to generalize the Ginzburg-Landau theory to the nonstationary case. The simplest approach, in accord with the foregoing, would be to include in the Ginzburg-Landau equation for the wave function a term with the first derivative with respect to time, thus ensuring relaxation of the order parameter, whereas in the expression for the current one adds a term in the form $\sigma_n\mathbf{E}$ corresponding to dissipation of the energy of the normal component. With the aid of the resultant equations it becomes possible to write down a dissipation function that describes the irreversible losses.

The standard method that permits a description of slow relaxation of any parameter η is to connect the rate of change of η with the system energy by the relation

$$\gamma\dot{\eta} = -\frac{\delta F}{\delta\eta}. \qquad (1.6)$$

The free energy F of the superconductor consists of three parts. First, the energy F_n of the metal in the normal state, from which other energies are reckoned usually in superconductivity theory. The second term is the energy F_{em} of the electromagnetic field

$$F_{em} = \frac{1}{8\pi}\int(\mathbf{H}^2 + \mathbf{E}^2)\,dV. \qquad (1.7)$$

The third and final contribution F_{sn} is given by the expression of the Ginzburg-Landau theory, with an addition to the free energy to account for the transition to the superconducting state and for the interaction of the superfluid currents with the electromagnetic field:

$$F_{sn} = \int\left[C_1|\Psi|^2 + \frac{C_2}{2}|\Psi|^4 + \frac{1}{2m}\left|\left(-i\hbar\nabla - \frac{2e}{c}\mathbf{A}\right)\Psi\right|^2\right]dV. \qquad (1.8)$$

We define the electric field $\mathbf{E}$ in the following manner:

$$\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t} - \nabla\Phi.$$

Before we write down the equation for Ψ, we note that the left-hand side of (1.6) should in this case also be gauge-invariant because of the complex character of the order parameter in the Ginzburg-Landau theory. Therefore, in any case, differentiation with respect to time always accompanies the electrostatic potential

$$\hbar\frac{\partial\Psi}{\partial t} \to \left(\hbar\frac{\partial}{\partial t} + 2ie\Phi\right)\Psi.$$

However, as already mentioned, the macroscopic theory makes it possible to establish the physical meaning of the function as a quantity proportional to the wave function of the Cooper pair. The dependence of the phase of the latter on the time is determined at equilibrium by the value of the chemical potential[23]:

$$\Psi(\mathbf{r}, t) = \Psi(\mathbf{r})\,e^{-2i\mu t/\hbar}.$$

Thus, the left-hand side of Eq. (1.6) for Ψ contains the time derivative in the combination

$$\hbar\frac{\partial}{\partial t} + 2ie\Phi + 2i\mu = \hbar\frac{\partial}{\partial t} + 2ie\tilde{\Phi},$$

where

$$\tilde{\Phi} = \Phi + \frac{\mu}{e}$$

is the electrochemical potential, and it is most natural to reckon chemical potential from its value for the normal metal in the absence of a field.

In the isotropic model

$$\delta N = \frac{3N_0}{mv_F^2}\delta\mu = \frac{3N_0 e}{mv_F^2}(\tilde{\Phi} - \Phi).$$

The electroneutrality condition, which is formulated as div $\mathbf{j} = 0$ by virtue of the continuity equation

$$\operatorname{div} \mathbf{j}+\frac{\partial (Ne)}{\partial t}=0,$$

means constancy of the charge density. Therefore the vanishing of δN means that we can neglect the difference between $\tilde{\Phi}$ and Φ.[2)] We have

$$\frac{\delta F_{sn}}{\delta \Psi^*(\mathbf{r})}=C_1\Psi+C_2|\Psi|^2\Psi+\frac{1}{2m}\left(-i\hbar\nabla-\frac{2e}{c}\mathbf{A}\right)^2\Psi, \tag{1.9}$$

$$\frac{\delta F_{sn}}{\delta \mathbf{A}(\mathbf{r})}=-\frac{\mathbf{j}_s}{c}=-\frac{1}{c}\left[-\frac{ie\hbar}{m}(\Psi^*\nabla\Psi-\nabla\Psi^*\cdot\Psi)-\frac{4e^2}{mc}|\Psi|^2\mathbf{A}\right]. \tag{1.10}$$

In accordance with (1.6), we write down a generalized equation that describes the relaxation of the order parameter Ψ of the Ginzburg-Landau theory, in the form

$$\gamma\left(\hbar\frac{\partial\Psi}{\partial t}+2ie\Phi\Psi\right)=-\frac{\delta F_{sn}}{\delta\Psi^*} \tag{1.11}$$

and a complex-conjugate equation for Ψ^*.

Equation (1.11), jointly with Maxwell's equations

$$\operatorname{rot}\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}, \tag{1.12}$$

$$\operatorname{rot}\mathbf{H}=\frac{4\pi}{c}\mathbf{j}+\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} \tag{1.13}$$

and expressions (1.7), (1.8), (1.9), and (1.10) makes it possible to formulate the energy conservation law. Indeed, let us write down the time derivative of the total free energy

$$\frac{dF}{dt}=\frac{dF_{em}}{dt}+\frac{dF_{sn}}{dt}.$$

The first term in this expression is transformed with the aid of (1.12) and (1.13) into

$$\frac{dF_{em}}{dt}=\int(-\operatorname{div}\mathbf{S}-\mathbf{jE})\,dV, \tag{1.14}$$

where S is the Poynting vector:

$$\mathbf{S}=\frac{c}{4\pi}[\mathbf{E}\times\mathbf{H}].$$

In the second term it is necessary to differentiate Ψ, Ψ^*, and $\mathbf{A}$ with respect to time. We obtain directly

$$\begin{aligned}\frac{dF_{sn}}{dt}=\int\left[\dot{\Psi}(\mathbf{r})\frac{\delta F_{sn}}{\delta\Psi(\mathbf{r})}+\dot{\Psi}^*(\mathbf{r})\frac{\delta F_{sn}}{\delta\Psi^*(\mathbf{r})}+\dot{\mathbf{A}}\frac{\delta F_{sn}}{\delta\mathbf{A}(\mathbf{r})}\right]d^3\mathbf{r}\\+\int\frac{\hbar}{2m}\operatorname{div}\left[\dot{\Psi}\left(\hbar\nabla+\frac{2ie}{c}\mathbf{A}\right)\Psi^*+\text{c.c.}\right]d^3\mathbf{r}.\end{aligned} \tag{1.15}$$

The integrand in the first term of the right-hand side of (1.15) can be written, according to (1.9), (1.10), and (1.11), in the following manner:

$$\begin{aligned}&-\frac{\mathbf{j}_s}{c}\frac{\partial\mathbf{A}}{\partial t}-\frac{2\gamma}{\hbar}\left|\hbar\frac{\partial\Psi}{\partial t}+2ie\Phi\Psi\right|^2\\&\quad-\frac{2ie\overline{\Phi}}{\hbar}\Psi\frac{\partial F_{sn}}{\partial\Psi}+\frac{2ie\Phi}{\hbar}\Psi^*\frac{\delta F_{sn}}{\delta\Psi^*}\\&\quad=-\mathbf{j}_s\left(\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}\right)-\frac{2\gamma}{\hbar}\left|\hbar\frac{\partial\Psi}{\partial t}+2ie\Phi\Psi\right|^2+\Phi\operatorname{div}\mathbf{j}_s.\end{aligned}$$

Combining the so-transformed expression for (1.5) with (1.14) and recalling that the total current density includes a dissipative term

$$\mathbf{j}=\mathbf{j}_s-\sigma_n\left(\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}+\nabla\Phi\right),$$

we obtain

$$\frac{dF}{dt}=-\int W(\mathbf{r})\,d^3\mathbf{r}-\int\operatorname{div}\mathbf{j}_E\,d^3\mathbf{r},$$

where $\mathbf{j}_E$ is the density of the energy flux

$$\begin{aligned}\mathbf{j}_E&=-\frac{\hbar}{2m}\left[\dot{\Psi}\left(\hbar\nabla+\frac{2ie}{c}\mathbf{A}\right)\Psi^*+\text{c.c.}\right]-\Phi\mathbf{j}_s+\mathbf{S},\\W(\mathbf{r})&=\frac{2\gamma}{\hbar}\left|\hbar\frac{\partial\Psi}{\partial t}+2ie\Phi\Psi\right|^2+\sigma_n\left|\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}+\nabla\Phi\right|^2.\end{aligned} \tag{1.16}$$

It should be noted with respect to (1.7) that the electric field is produced in the present situation as a result of an induction mechanism. Therefore, as is always the case in the theory of quasistationary currents in a metal (see, e.g.,[24]), the electric field E is small. Estimating it from (1.12), we see that $E\sim(\omega d/c)H \ll H$, where d is the characteristic dimension of the problem. Therefore, properly speaking, we could neglect E^2 in comparison with H^2 in expression (1.7) for the electromagnetic-energy density. Similarly, we can omit from the right-hand side of (1.3) the term with $(1/c)\partial E/\partial t$, since the characteristic frequencies ω are small:

$$\omega\ll\sigma_n.$$

It is then easy to verify that this Maxwell's equation can be represented in the form

$$\frac{1}{c}\mathbf{j}_n=\frac{1}{4\pi}\operatorname{rot}\mathbf{H}-\frac{1}{c}\mathbf{j}_s,$$

i.e.,

$$-\frac{\sigma_n}{c}\left(\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}+\nabla\Phi\right)=\frac{\delta F}{\delta\mathbf{A}(\mathbf{r})}. \tag{1.17}$$

Thus, Eqs. (1.11) and (1.17) determine the generalized friction forces corresponding to the variables Ψ, Ψ^*, and $\mathbf{A}$, while W is by the same token the dissipation function.

Introducing the phase of $\Psi=|\Psi|e^{i\theta}$, we rewrite (1.6) as follows:

$$W(\mathbf{r})=\frac{2\gamma}{\hbar}\left[\left(\hbar\frac{\partial|\Psi|}{\partial t}\right)^2+|\Psi|^2(\hbar\dot{\theta}+2e\Phi)^2\right]+\sigma_n\left(\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}+\nabla\Phi\right)^2. \tag{1.18}$$

Subtracting from (1.11) its complex conjugate, we obtain

$$2\gamma(\hbar\dot{\theta}+2e\Phi)|\Psi|^2=\frac{\hbar}{2e}\operatorname{div}\mathbf{j}_s,$$

and since $\operatorname{div}\mathbf{j}_s=-\operatorname{div}\mathbf{j}_n$, the last relation is transformed into an equation from which it is necessary to determine the scalar potential Φ:

$$2\gamma(\hbar\dot{\theta}+2e\Phi)|\Psi|^2=\frac{\hbar\sigma_n}{2e}\left(\nabla^2\Phi+\frac{1}{c}\operatorname{div}\frac{\partial\mathbf{A}}{\partial t}\right). \tag{1.19}$$

By the same token, the problem of finding the resistance of the vortex structure at low velocities of its motion has in principle been solved. Indeed, all the quantities characterizing the vortex system can be written in first order in the form

$$f(\mathbf{r},t)=f_0(\mathbf{r}-\mathbf{v}_Lt)+f_1(\mathbf{r}-\mathbf{v}_Lt),$$

where the corrections f_1 to the distributions of $|\Psi|$, θ, and $\mathbf{A}$ in the moving reference frame are themselves linearly small relative to the velocity of motion $\mathbf{v}_L$. The time derivatives in (1.18) and (1.19) can therefore be written in the form

$$\frac{\partial|\Psi|}{\partial t}=-(\mathbf{v}_L\nabla)|\Psi_0|,\quad\frac{\partial\theta}{\partial t}=-(\mathbf{v}_L\nabla)\theta_0,$$

i.e., they are determined by the solution of the static problem. The potential Φ is proportional to the velocity and should be determined from the inhomogeneous linear equation (1.19), which also contains known quantities. The dissipation per unit volume is proportional to

[2)]The electroneutrality condition in the form $\operatorname{div}\mathbf{j}=0$ is equivalent, accurate to terms $(\omega/\omega_p)^2$ (where ω_p is the plasma frequency) to the approach with the Poisson equation for a self-consistent potential. From the condition $\operatorname{div}\mathbf{j}=0$, generally speaking, it does not follow that $\operatorname{div}\mathbf{E}=0$. The longitudinal electric field is the result of Coulomb forces exerted by the ions on the electrons which shifts relative to the latter under the influence of the external action, and is necessary to satisfy the electroneutrality condition in first-order approximation. In particular, in the problem in question, where the parameters of the superconductor are indetermined in space, an alternating charge density $\delta(Ne)$ arises, and is generally speaking extremely small.

v_L^2. The average electric field $\overline{E}$ is determined from the condition

$$\overline{\mathbf{E}}=\frac{1}{c}[\mathbf{B}, \mathbf{v}_L]. \qquad (1.20)$$

Thus, expressing the velocity in terms of $\overline{E}$, we obtain an energy dissipation proportional to the square of the macroscopic electric field, and this yields directly the specific conductivity σ_f in the superconducting phase:

$$\overline{W}=\sigma_f\overline{E}^2.$$

If the vortex lattice is rarefied enough, as is the case either when the filament concentration is very small, or when $\kappa \gg 1$ in the field interval $B \ll H_{c2}$, then $\rho_f = \sigma_f^{-1}$ is obviously proportional to the number of filaments. It is therefore convenient to write ρ_f in the form

$$\rho_f=\beta^{-1}(T, B)\frac{B}{H_{c2}(T)}, \qquad (1.21)$$

where the dimensionless coefficient $\beta(T, B)$ is to be determined, and $H_{c2}(T)$ is the upper critical field at the given temperature.

Let $\kappa \gg 1$, so that in fields $B \ll H_{c2}$ it suffices to confine oneself to the motion of an individual filament, a motion due to the local (i.e., in the vicinity of the given filament) superfluid component of the current j_{tr}. For unaccelerated motion, the Lorentz force $\mathbf{F}_L = (1/c)[\mathbf{j}_{tr} \times \varphi_0]$ per unit volume of the vortex is offset by the viscous-friction force $-\eta \mathbf{v}_L$. If $j_{tr} = \sigma_f \overline{E} = (\sigma_f/c)v_L B$, then the viscosity coefficient can be expressed from this in terms of the conductivity

$$\eta=\frac{\sigma_f\varphi_0 B}{c^2}=\beta\,\frac{\sigma_n\varphi_0 H_{c2}}{c^2}. \qquad (1.22)$$

No special assumptions were made above. At $\kappa \gg 1$, Eq. (1.19) and the expression (1.18) for W can be simplified. Indeed, in this case, besides the fact that we can choose $\operatorname{div} A = 0$ for an individual filament, the magnitude of the vector potential $\mathbf{A}$ at distances $\rho \sim \xi$ is small in comparison with $(c/2e)\nabla\theta$:

$$\hbar\nabla\theta \sim \frac{1}{\rho} \sim \frac{1}{\xi},\quad \frac{e}{c}A \sim \frac{e}{c}H(0)\rho \sim \frac{H(0)\xi}{\varphi_0} \sim \frac{\xi}{\delta^2}.$$

The second term in the curly brackets (1.18) and the term $\sigma_n(\nabla\Phi^2)$ are both transformed with the aid of (1.19) into

$$\frac{1}{2e}(\hbar\dot{\theta}+2e\Phi)\,\sigma_n\nabla^2\Phi+\sigma_n(\nabla\Phi)^2=\frac{\hbar}{2e}\dot{\theta}\sigma_n\nabla^2\Phi+\sigma_n\operatorname{div}(\Phi\nabla\Phi)$$

Discarding the surface contribution, we obtain for the energy dissipation per unit length of the vortex

$$\overline{W}=\int\left[2\gamma\hbar\,(\mathbf{v}_L\nabla\,|\,\Psi\,|)^2+\frac{1}{2e}\sigma_n\dot{\theta}\nabla^2\Phi\right]d^2\mathbf{r}. \qquad (1.23)$$

The two terms in (1.23) represent respectively the contribution made to the energy dissipation by the relaxation of the order parameter and by the ohmic dissipation by the normal excitations in the core of the vortex.

Qualitatively, formulas (1.22) and (1.23) correspond to the general experimental situation. Since, however, the equations (1.11) and (1.17) follow quantitatively from the microscopic superconductivity theory only in alloys with paramagnetic impurities, we shall defer the calculation and the discussion of (1.23) to the appropriate section.

2. RESISTANCE OF TYPE II SUPERCONDUCTORS IN SIMPLE MICROSCOPIC THEORY

Simple equations such as (1.11) and (1.17) appear in the microscopic theory only in two cases, alloys with large concentration of paramagnetic impurities, and alloys in a strong magnetic field ($H_{c2} - H \ll H_{c2}$). In Chap. 3 below we shall explain qualitatively the formalism of the BCS microscopic theory becomes so complicated when it comes to the study of kinetic phenomena in superconductors. Here we note only briefly that this circumstance stems from the threshold character of the electronic spectrum of the superconductor. The paramagnetic impurities and the magnetic field have one common property that alters in principle the indicated feature of the spectrum, namely, they violate the invariance of the electron Hamiltonian to time reversal. As is well known, in Cooper pairing the wave functions of two electrons are obtained from each other by the time-reversal operation. Violation of this invariance by the action of the magnetic field, or as a result of the interaction of the electron with the proper angular momentum of the impurity, suppresses the pair production. In a definite range of concentrations this suppression manifests itself in the so-called "gapless superconductivity"[25] where there is no energy threshold for the excitation. In the limit when the density of states of the superconductor is $\nu_s(\epsilon) \approx \nu_n(0)$, the kinetic equations become much simpler and take the form of the diffusion equations of the preceding section.[7]. A magnetic field acts similarly, as proved in the thermodynamic case by Maki[14] and by Caroli, Cyrot, and de Gennes.[26]. The nonlinear kinetic equations of the diffusion type (1.11), as it turns out, can be written down also in this case[8], but, as we shall see, the situation in the region $|H - H_{c2}| \ll H_{c2}$ is much simpler everywhere except in the immediate vicinity of T_c. The point is that the strong magnetic field suppresses the superconductivity in the entire volume. Therefore, instead of the picture of localized vortex filaments, the state of the superconductor corresponds in this limiting case to a strong overlap of the filaments, and the order parameter is uniformly small throughout the entire volume.

a) **Superconducting alloys with paramagnetic impurities. Weak and intermediate fields** ($B \ll H_{c2}$). The behavior of these alloys in an alternating field in the limiting case of high concentrations ($T_c \ll T_{c0}$ or $\tau_s T_{c0} \ll 1$) is described by the equations

$$\left.\begin{aligned}&\frac{\partial|\Delta|}{\partial t}+\frac{\tau_s}{3}\left[-\pi^2(T_c^2-T^2)+\frac{|\Delta|^2}{2}\right]|\Delta|\\&\qquad-D\nabla^2|\Delta|+\frac{4e^2}{c^2}DQ^2|\Delta|=0,\\&\frac{c}{2e}|\Delta|^2\tilde{\mu}+D\operatorname{div}(|\Delta|^2\mathbf{Q})=0,\\&\mathbf{j}=\sigma_n\mathbf{E}-\frac{2\sigma_n\tau_s}{c}|\Delta|^2\mathbf{Q};\end{aligned}\right\} \qquad (2.1)$$

here $D = lv_F/3$ is the diffusion coefficient and τ_s is the average time between the collisions of the electrons with the impurity, accompanied by spin flip. Equations (2.1) were written for the order parameter of the microscopic theory of superconductivity, and the following notation was introduced:

$$\mathbf{Q}=\mathbf{A}-\frac{c}{2e}\nabla\theta,\quad \tilde{\mu}=\dot{\theta}+2e\Phi \qquad (2.2)$$

(θ is the phase of the order parameter; throughout this section we use $\hbar = 1$).

The correspondence between Eqs. (2.1), (2.2) and (1.11), (1.17) is complete if we put $\Psi = \Delta\sqrt{N_0\tau\tau_s/2}$ and $\gamma = \hbar/2mD$. Therefore, in particular, the viscosity η could have been determined by using expression (1.23) for the dissipation function. We use here a different approach[27] with an aim, in particular, of demonstrating the concept of the equivalence of the Lorentz force and the Magnus force, meaning to show that the motion of the vortex is caused by the field of the superfluid velocity in its vicinity. More important is the fact that the developed approach is applicable also in the case when equations such as (1.11) and (1.17) do not hold.

We shall assume below that the constant of the Ginzburg-Landau theory is $\kappa = c(48\pi D\sigma_n)^{-1/2} \gg 1$. Therefore, as already mentioned, in the calculations all the quantities will be suitable over distances $\rho \ll \delta$ in which the magnetic field (and consequently its contribution to the vector potential A) can be neglected up to fields $H \ll H_{c2}$. Therefore even if the distances d between the filaments are small in comparison with the penetration depth, but still large relative to the coherence radius, the expression for Q in (2.2) reduces to the form $\mathbf{Q} = -(c/2e)\nabla\theta$.

Putting $2e\Phi = \mu^*$ and denoting by $\mathbf{j}_s = \mathbf{j}/e$ the electron momentum flux density, we can rewrite the system (2.1) in such a way that the electron charge is completely eliminated from it (we shall henceforth omit the symbol for the absolute value of Δ):

$$\dot{\Delta} + \frac{\tau_s}{3}\left[-\pi^2(T_c^2 - T) + \frac{\Delta^2}{2}\right]\Delta - D\nabla^2\Delta + D(\nabla\theta)^2\Delta = 0, \quad (2.3)$$

$$\Delta^2(\dot{\theta} + \mu^*) - D\,\mathrm{div}\,(\Delta^2\nabla\theta) = 0, \quad (2.4)$$

$$\mathbf{j}_s = -\frac{\sigma_n}{2e^2}(\nabla\mu^* - 2\tau_s\Delta^2\nabla\theta). \quad (2.5)$$

Equations (2.4) and (2.5), jointly with div $\mathbf{j} = 0$, yield

$$D\nabla^2\mu^* - 2\tau_s\Delta^2(\dot{\theta} + \mu^*) = 0. \quad (2.6)$$

We shall seek a solution of these equations for a slowly moving vortex in the form

$$\Delta = \Delta_0(\mathbf{r} - \mathbf{v}_L t) + \Delta_1(\mathbf{r} - \mathbf{v}_L t), \quad \theta = \theta_0(\mathbf{r} - \mathbf{v}_L t) + \theta_1(\mathbf{r} - \mathbf{v}_L t). \quad (2.7)$$

We substitute (2.7) in (2.3) and (2.5), confining ourselves to terms linear in v_L:

$$-(\mathbf{v}_L\nabla)\Delta_0 + \frac{\tau_s}{3}\left[-\pi^2(T_c^2 - T^2) + \frac{3\Delta_0^2}{2}\right]\Delta_1 - D\nabla^2\Delta_1 + D(\nabla\theta_0^2)\Delta_1 + 2D(\nabla\theta_0\nabla\theta_1)\Delta_0 = 0, \quad (2.8)$$

$$\mathbf{j}_{s1} = -\frac{\sigma_n}{2e}(\nabla\mu^* - 2\tau_s\Delta^2\nabla\theta_1 - 4\tau_s\Delta_0\Delta_1\nabla\theta_0). \quad (2.9)$$

By virtue of the spatial homogeneity, the origin in the static solutions Δ_0, θ_0, and $\mathbf{j}_{s0}$ of (2.1) can be shifted by an arbitrary vector d: $f(\mathbf{r}) \to f(\mathbf{r} + \mathbf{d})$. It is therefore obvious that $\Delta_d = (\mathbf{d}\cdot\nabla)\Delta_0$ and $\theta_d = (\mathbf{d}\cdot\nabla)\theta_0$ satisfy Eq. (2.8) without the inhomogeneous term $(\mathbf{v}_L\cdot\nabla)\Delta_0$ and the linearized equation div $\mathbf{j}_d = 0$ with expression (2.9) (without the term $\nabla\mu^*$) in place of $\mathbf{j}_d$, if we replace throughout Δ_1 by θ_1 and Δ_d by θ_d. Bearing this in mind, we multiply (2.8) by Δ_d and integrate over the volume of a cylinder with radius larger than ξ. Denoting analogously $\Delta_v = (\mathbf{v}_L\cdot\nabla)\Delta_0$ and using the fact that Δ_1 attenuates over distances of the order of ξ, we can integrate the terms $D\Delta_d\nabla^2\Delta_1$ by parts, discarding the surface integrals. We obtain

$$-\int\Delta_v\Delta_d d^3\mathbf{r} + 2\int D\left[(\nabla\theta_0\nabla\theta_1)\Delta_0\Delta_d - (\nabla\theta_0\nabla\theta_d)\Delta_0\Delta_1\right]d^3\mathbf{r} = 0.$$

Combining the second integral with the aid of the expression for $\mathbf{j}_{s1}$ and $\mathbf{j}_d$, we get

$$-\int\Delta_v\Delta_d d^3\mathbf{r} + \frac{De^2}{\tau_s\sigma_n}\int(\mathbf{j}_{sd}\nabla\theta_1 - \mathbf{j}_{s1}\nabla\theta_d)\,d^3\mathbf{r} - \frac{D}{2\tau_s}\int(\nabla\mu^*\nabla\theta_d)\,d^3\mathbf{r} = 0.$$

Integrating by parts, we obtain (div $\mathbf{j}_{d,1} = 0$)

$$\frac{De^2}{\tau_s\sigma_n}\int(\mathbf{j}_{sd}\theta_1 - \mathbf{j}_{s1}\theta_d)\,dS = \int(\Delta_v\Delta_d)\,d^3\mathbf{r} + \int\Delta_0^2\tilde{\mu}\theta_d\,d^3\mathbf{r},$$

where, according to (2.6), $\mu = \mu^* - (\mathbf{v}_L\cdot\nabla)\theta_0$ satisfies the equation

$$D\nabla^2\tilde{\mu} - 2\tau\Delta_0^2\tilde{\mu} = 0$$

with a boundary condition that calls for finite μ^* at $\rho = 0$.[28,29] The surface integral can be easily expressed in terms of $\mathbf{j}_{s1\infty} = \mathbf{j}_{s\,tr}$. Indeed, at $\rho \gg \xi$ we have

$$\mathbf{j}_{s1\infty} = -\frac{\sigma_n\tau_s\Delta_\infty^2}{e^2}\nabla\theta_1, \quad (\mathbf{j}_{sd})_\rho = -\frac{\sigma_n\tau_s\Delta_\infty^2}{e^2\rho^2}[\mathbf{n}_H, \mathbf{d}]_\rho.$$

Whence $\theta_1 = \frac{e^2}{\sigma_n\tau_s\Delta_\infty^2}(\mathbf{j}_{s1\infty}\rho)$,

$$([\mathbf{n}_H\times\mathbf{d}]\,\mathbf{j}_{1\infty}) = \frac{\sigma_n\tau_s}{2\pi eD}(\mathbf{v}_L\mathbf{d})\left[\int 2\pi\rho\left(\frac{d\Delta_0}{d\rho}\right)^2 d\rho + \int 2\pi\Delta_0^2\tilde{\mu}_1\,d\rho\right], \quad (2.10)$$

where $\tilde{\mu} = v_L\tilde{\mu}_1\sin\varphi$ and $\tilde{\mu}_1 = -1/\rho$ as $\rho \to 0$. If $v_L = \bar{E}c/B$ and $\mathbf{j}_{1\infty} = \mathbf{j}_{tr}$, then we obtain for the conductivity

$$\sigma_f = \frac{6\alpha\sigma_n H_{c2}}{B}, \quad (2.11)$$

where we have used the expression for the upper critical field $H_{c2} = c\tau_s\Delta_\infty^2/12cD$ in the given concrete problem, and the numerical coefficient α is equal to

$$\alpha = \int_0^\infty \rho\left(\frac{df}{d\rho}\right)^2 d\rho + \int_0^\infty \tilde{\mu}f^2\,d\rho = \alpha_1 + \alpha_2, \quad (2.12)$$

where the function f is the solution of the stationary problem for a single vortex in terms of the dimensionless variables[1,27] (in units of $\xi = \sqrt{6D/\tau_s\Delta_\infty^2}$)

$$\frac{d}{d\rho}\left[\frac{1}{\rho}\frac{d}{d\rho}(\rho f)\right] + (1 - f^2)f = 0,$$

and $\tilde{\mu}_1$ in the same units satisfies the equation

$$\frac{d}{d\rho}\left(\frac{1}{\rho}\frac{d}{d\rho}(\rho\tilde{\mu}_1)\right) - 12f^2\tilde{\mu}_1 = 0, \quad \tilde{\mu}_1 = -\frac{1}{\rho} \quad \text{at} \quad \rho \to 0. \quad (2.13)$$

The quantity α_1 was calculated in[28] and found to be $\alpha_1 = 0.279$.[3)] Equation (2.13) was solved numerically by Likharev and Kupriyanov[28] and by Hu (see[29,30]); α_2 was found equal to 0.159. Thus, $\alpha = 0.438$. The two terms of (2.12), in accord with (1.23), describe the contribution to the energy dissipation by the order-parameter relaxation mechanism (α_1) and by the ohmic losses (α_2).

b) Equations of motion and deformations in a vortex lattice in the presence of a transport current. In the preceding section we considered one filament in the field of the superfluid current flowing around it. The transport current is produced in the volume of the superconductor, of course, only at a finite vortex-filament density. In weak fields $B \ll H_{c2}$, a very important role is played by edge effects, since the Meissner effect causes the current to flow in the main near the surface of the superconductor and to go into the interior only in the vicinity of the filaments. This is the reason for one of the mentioned difficulties in the measurement of the vortex resistance in superconductors in the case of weak fields. The second factor, which plays an essential role, is the current's own field, which leads to the appearance of a magnetic-field component perpendicular to the applied external field. This component bends the vortex lattice. Thus, in a bulky sample the current near an individual filament does not coincide in general with the average density of the transport current.[4)]

When considering a vortex lattice, we shall assume that $B \ll H_{c2}$, i.e., the distances d between the vortices are large in comparison with ξ ($\kappa \gg 1$). We locate the

[3)]In [27] we cited a less accurate value, $\alpha_1 = 0.247$.

[4)]We shall show, however, that expression (2.11) for the conductivity does not change when account is taken of the latter circumstance, so long as the current's own field is small compared with H.

origin of the coordinates on one of the filaments. The coordinates of the points along the filament are characterized by a two-dimensional deformation vector $\mathbf{u}(z, t)$, and the coordinates along the remaining filaments are $\rho_{0i} + \mathbf{u}_i(z, t)$, where ρ_i is the equilibrium position of the i-th filament. The bending of the filaments changes the picture of the motion near the selected vortex ($\rho \ll d$). Although the velocity and deformation are by definition small, the zero-order approximation for an individual filament corresponds now not only to a displacement but also to a rotation of the equilibrium solutions:

$$\Delta = \Delta_0(\rho - \mathbf{u}(z, t)) + \Delta_1, \quad \mathbf{Q} = \mathbf{Q}_0(\rho - \mathbf{u}(z, t)) + [\delta\varphi, \mathbf{Q}_0) + \mathbf{Q}_1, \quad \delta\varphi = \left[\mathbf{n}_H \times \frac{\partial \mathbf{u}}{\partial z}\right] \quad (2.14)$$

(cf. (2.7)!). As a result of calculations that are perfectly analogous to those that have led us to (2.10), we obtain[27]

$$([\mathbf{n}_H, \mathbf{d}]\,\mathbf{j}_{1\infty} = \frac{mp_F\tau_s e}{2\pi^3}\int [\Delta_d(\dot{\mathbf{u}}\nabla)\Delta_0 + \tilde{\mu}\Delta_0^2\theta_d]\,d^2\mathbf{r} - \frac{mp_F\tau_s eD}{2\pi^3}\int\left[\Delta_d\left(\frac{\partial^2\mathbf{u}}{\partial z^2}\nabla\right)\Delta_0 + \theta_d\Delta_0^2\left(\frac{\partial^2\mathbf{u}}{\partial z^2}\nabla\right)\theta_0\right]d^2\mathbf{r}. \quad (2.15)$$

The second integral in (2.15) reduces to the form

$$-\frac{mp_F\tau_s De\Delta_\infty^2}{2\pi^2}\left\{\varkappa^2\frac{\varepsilon}{2\pi} + \int_\rho^\infty \frac{d}{d\rho}\left[\rho\left(\frac{2e}{c}Q_0\right)^2\right]\rho\,d\rho\right\}\frac{\partial^2\mathbf{u}}{\partial z^2}\,\mathbf{d},$$

where $\epsilon = 2\pi\kappa^{-2}(\ln\kappa + 0.18)$ is the energy per unit length of the vortex.[1] Ultimately,

$$\mathbf{j}_{1\infty}(\rho) = 6\alpha\frac{\sigma_n H_{c2}}{c}[\mathbf{n}_H, \dot{\mathbf{u}}] - \frac{c\varphi_0}{16\pi^2\delta^2}\left[\mathbf{n}_H, \frac{\partial^2\mathbf{u}}{\partial z^2}\right]\left\{\varkappa^2\frac{\varepsilon}{2\pi} + \int_\rho^\infty \frac{d}{d\rho}\left[\rho\left(\frac{2e}{c}Q_0\right)^2\right]\rho\,d\rho\right\} \quad (2.16)$$

(the last term contains the dependence on ρ).

This relation enables us to establish a connection between the filament velocity and the deformation of the vortex lattice. To this end we proceed in the following manner: The current $j_{1\infty}(\rho)$ at not large distances from the core of the selected filament can be obtained directly from the Londons' equation

$$\delta^2\,\mathrm{rot}\,\mathrm{rot}\,\mathbf{h} + \mathbf{h} = \varphi_0\sum_j\int d\mathbf{S}_j\,\delta(\mathbf{R} - \mathbf{R}_j);$$

where $\mathbf{h}$ is the microscopic magnetic field, $d\mathbf{S}_j$ is the length element along the j-th filament, $\delta(\mathbf{R} - \mathbf{R}_j)$ is a three-dimensional δ function, and $\mathbf{R}_j$ is a three-dimensional vector drawn to a point on the j-th filament. From this we easily obtain an expression for the current $\mathbf{j} = (c/4\pi)\,\mathrm{curl}\,\mathbf{h}$:

$$\mathbf{j}(\mathbf{R}) = -\frac{c\varphi_0}{16\pi^2\delta^4}\sum_j\int\left\{d\mathbf{S}_j,\ \mathrm{grad}\left[\frac{\exp(-|\mathbf{R} - \mathbf{R}_j|/\delta)}{|\mathbf{R} - \mathbf{R}_j|}\right]\right\}. \quad (2.17)$$

Assuming that $\mathbf{u}(\mathbf{R})$ varies slowly over distances on the order of δ and d, we expand (2.17) in powers of the small lattice deformations up to second-order terms in the derivatives $\nabla_i^2\mathbf{u}$:

$$\mathbf{j}_{1\infty} = \frac{c}{B}\left\{(c_{11} - c_{66})[\mathbf{n}_H \times \nabla\,\mathrm{div}\,\mathbf{u}] + c_{66}[\mathbf{n}_H \times \nabla^2\mathbf{u}] + c_{44}\left[\mathbf{n}_H \times \frac{\partial^2\mathbf{u}}{\partial z^2}\right] + (c_{12} - c_{66})\frac{\partial}{\partial z}\,\mathrm{rot}\,\mathbf{u}\right\} - \frac{c\varphi_0}{16\pi^2\xi^2}\frac{\varepsilon}{2\pi}\left[\mathbf{n}_H \times \frac{\partial^2\mathbf{u}}{\partial z^2}\right] - \frac{c\varphi_0}{16\pi^2\delta^2}\left[\mathbf{n}_H \times \frac{\partial^2\mathbf{u}}{\partial z^2}\right]\left[K_0\left(\frac{\rho}{\delta}\right) + \frac{\rho}{2\delta}K_1\left(\frac{\rho}{\delta}\right)\right], \quad (2.18)$$

where c_{ij} are the elastic moduli of the triangular lattice of vortex filaments, calculated by Labusch.[31] Comparing expressions (2.16) and (2.18) at $\xi \ll \rho \ll d$, we obtain an equation that describes the motion of the vortex lattice

$$6\alpha\frac{H_{c2}\sigma_n}{c^2}B[\mathbf{n}_H \times \dot{\mathbf{u}}] = (c_{11} - c_{66})[\mathbf{n}_H \times \nabla\,\mathrm{div}\,\mathbf{u}] + c_{66}[\mathbf{n}_H \times \nabla^2\mathbf{u}] + c_{44}\left[\mathbf{n}_H \times \frac{\partial^2\mathbf{u}}{\partial z^2}\right]. \quad (2.19)$$

The lattice deformation and its velocity are determined by the applied macroscopic field $\mathbf{E}$ and magnetic field $\mathbf{H}$ or by the transport current $\mathbf{j}_{tr}$. The problem thus consists of expressing the lattice deformation in terms of $\mathbf{j}_{tr}$. To this end we write down the thermodynamic Gibbs potential[24] for a system of vortices in a given external field $\mathbf{H}$[32]:

$$\mathscr{F} = \mathscr{F}_0 + \int\left\{\left[\frac{1}{2}(c_{11} - c_{66})\left(\frac{\partial u_i}{\partial x_i}\right)^2 + c_{66}\left(\frac{\partial u_k}{\partial x_i}\right)^2 + c_{44}\left(\frac{\partial u_i}{\partial z}\right)^2\right] - \frac{\mathbf{H} - \mathbf{H}_0}{4\pi}\mathbf{B}\right\}d^3\mathbf{r}; \quad (2.20)$$

here $\mathscr{F}_0$ and $\mathbf{H}_0$ correspond to a superconductor with an undeformed lattice of vortex filaments, so that at $\mathbf{H} = \mathbf{H}_0$ and $\partial u/\partial x_i = 0$ the potential is $\mathscr{F} = \mathscr{F}_0$.

Its variation with respect to the deformation vectors $\mathbf{u}_i$ at a constant $\mathbf{H}$ determines the forces acting on the vortex lattice. The induction $\mathbf{B}$ changes in this case by an amount

$$\delta\mathbf{B} = B_0\frac{\partial\mathbf{u}}{\partial z} - B_0\mathbf{n}_H\frac{\partial u_i}{\partial x_i}. \quad (2.21)$$

In steady-state uniform motion, the lattice is at equilibrium. Therefore the condition that the potential (2.20) be a minimum yields

$$(c_{11} - c_{66})\nabla\,\mathrm{div}\,\mathbf{u} + c_{66}\nabla^2\mathbf{u} + c_{44}\frac{\partial^2\mathbf{u}}{\partial z^2} = \frac{B_0}{c}\mathbf{j}_{tr}, \quad (2.22)$$

where, by definition,

$$\frac{4\pi}{c}\mathbf{j}_{tr} = \mathrm{rot}\,\mathbf{H} = \left[\mathbf{n}_H \times \frac{\partial\mathbf{H}}{\partial z}\right] - [\mathbf{n}_H, \nabla(H_z - H_0)].$$

We see thus that the right-hand side of (2.19) is directly equal to the macroscopic transport current

$$\mathbf{i}_{tr} = 6\alpha\sigma_n\frac{H_{c2}}{c}[\mathbf{n}_H \times \dot{\mathbf{u}}] \quad (2.23)$$

in accordance with the statement made above.

The average electric field $\overline{\mathbf{E}}$ can be obtained from Maxwell's equation

$$\mathrm{rot}\,\overline{\mathbf{E}} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t},$$

where $\mathbf{B}$ is given by (2.21):

$$\overline{\mathbf{E}} = \frac{B_0}{c}[\mathbf{n}_H \times \dot{\mathbf{u}}]. \quad (2.24)$$

Substituting (2.24) in (2.23) we see that the superconductor has a finite conductivity

$$\sigma_f = \sigma_n 6\alpha\frac{H_{c2}}{B_0}.$$

We note once more that Eq. (2.22) has the same form as the conditions for the equilibrium of an elastically deformed lattice

$$\frac{\partial\sigma_{ik}}{\partial x_k} = F_i;$$

σ_{ik} is the stress tensor, where the role of the external force is played by the Lorentz force.

$$\mathbf{F}_L = \frac{1}{c}[\mathbf{j}_{tr} \times \mathbf{B}_0].$$

Taking the force acting on one vortex we find that $\mathbf{F}_L = \eta\mathbf{v}_L$ ($\mathbf{v}_L = \dot{\mathbf{u}}$ is the vortex velocity), where the viscosity coefficient (per unit length of the vortex) is

$$\eta = \frac{6\alpha\sigma_n\varphi_0 H_{c2}}{c^2}.$$

Substituting here $\varphi_0 = \pi\hbar c/e$ and $H_{c2} = \hbar c/2e\xi^2$ we see that

$$\eta = \frac{3\alpha\pi N_0\hbar^2\tau}{m\xi^2},$$

i.e., the electron charge drops out from the expression for the viscosity coefficient, in full accord with the fact that Eqs. (2.1) are the generalized equations of electrodynamics for a superfluid Fermi liquid. Therefore the Lorentz force is equivalent to the Magnus force exerted by the superfluid component on a unit length of the vortex. As to the kinematics of vortex motion, the predominant role is played not by the dragging of the vortex by the superfluid component of the liquid, but by the action of the viscous forces, due to the invariance of the electron system in the alloy to Galilean transformations.

We point out also that the results of this section, which concern the connection between the transport current and the lattice deformations, are based on the Londons' equations far from the vortex, and have therefore a macroscopic character. The details of the kinetic dissipation processes determined by the microscopic model enter at $\kappa \gg 1$ only in the value of the constant in expressions (2.19) and (2.23) for one filament.

c) Superconducting alloys with paramagnetic impurities. Strong fields ($B \approx H_{c2}$). Let us examine first the equilibrium structure of the vortex lattice in a strong field. In this case, as already mentioned, the superconductivity is strongly suppressed in the entire volume of the superconductor. It follows from the microscopic theory[14] that in fields close to H_{c2} the superconductor is described by equations of the Ginzburg-Landau type in the entire temperature interval $0 < T < T_c$. Confining ourselves for the purpose of illustration in this section to the simplest case of alloys with paramagnetic impurities, we write down the static equation (2.1)

$$\frac{\tau_s}{3\hbar^3}\left[-\pi^2(T_c^2-T^2)+\frac{|\Delta|^2}{2}\right]\Delta-D\left(\nabla-\frac{2ie}{\hbar c}\mathbf{A}\right)^2\Delta=0.$$

In order for this picture to be more general and more lucid, it is convenient to introduce the scale of length in this equation

$$\xi(T)=\sqrt{\frac{6D\hbar^2}{\tau_s\Delta_\infty^2}},\ \text{where}\ \Delta_\infty=\sqrt{2\pi^2(T_c^2-T^2)}$$

is the equilibrium value of the order parameter Δ in the absence of a magnetic field. We then have

$$\xi^{-2}\left(1-\frac{|\Delta|^2}{\Delta_\infty^2}\right)\Delta+\left(\nabla-\frac{2ie}{\hbar c}\mathbf{A}\right)^2\Delta=0. \quad (2.25)$$

In the considered range of fields, Δ/Δ_∞ is small and the gauge usually chosen for **A** is $\mathbf{A} = (0, Hx, 0)$ (we recall that the magnetic field is directed along the z axis). At $H = H_{c2}$ the solution (2.25), which is periodic in x and y, as shown by Abrikosov[1], takes the form

$$\Delta=\sum_{n=-\infty}^{\infty}C_n\exp(inqy)\exp\left[-\frac{e}{\hbar c}H\left(x-\frac{n\hbar cq}{2eH}\right)^2\right]. \quad (2.26)$$

The coefficients C_n satisfy the periodicity conditions $C_{n+\nu} = C_n$, where ν is an integer; the parameters q and ν are determined by the concrete geometric structure of the lattice. For a triangular lattice we have $\nu = 2$, with $C_1 = iC_0$. Each term in (2.26) describes the nuclei of the superconducting phase in the form of a strip in (2.26) describes the nuclei of the superconducting phase in the form of a strip of width $\xi(T)$ (we recall that $H_{c2} = \hbar c/2e\xi^2$, and therefore at $H = H_{c2}$ the argument of the exponential contains simply $-(1/2\xi^2)(x - x_0)^2$, $x_0 = -n\hbar cq/2eH$). The lines $|\Delta|^2 = \text{const}$ for a triangular lattice are shown in Fig. 6.[33]

If the field H is slightly smaller than H_{c2}, then we have $B \neq H_{c2}$ and the connection between the induction B and the magnetic field H can be established from the equation

$$\frac{c}{4\pi}\operatorname{rot}\mathbf{h}=\mathbf{j}=\frac{\hbar c^2}{16\pi\delta^2 e}\left[\Delta^*\left(-i\nabla-\frac{2e}{\hbar c}\mathbf{A}\right)\Delta-\Delta\left(-i\Delta+\frac{2e}{\hbar c}\mathbf{A}\right)\Delta^*\right]\Delta_\infty^{-2},$$

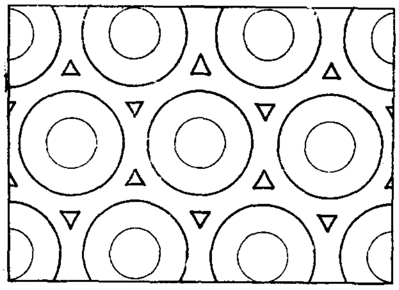

FIG. 6. Lines of constant density of the superconducting electrons for a triangular lattice of vortex filaments. [33]

$\delta = \hbar c\,(8\pi\sigma_n\tau_s\Delta_\infty^2)^{-1/2}$ is the depth of penetration of the magnetic field. Using (2.26), we can show that

$$j_x=-\frac{c^2\hbar}{16\pi\delta^2 e}\frac{\partial}{\partial y}\frac{|\Delta|^2}{\Delta_\infty^2},\quad j_y=\frac{c^2\hbar}{16\pi\delta^2 e}\frac{\partial}{\partial x}\frac{|\Delta|^2}{\Delta_\infty^2}, \quad (2.27)$$

whence

$$h_z=H-\frac{\hbar c}{4e\delta^2}\frac{|\Delta|^2}{\Delta_\infty^2}.$$

Averaging h_z over the volume of the sample we obtain

$$\bar{h}_z=B=H-\frac{\hbar c}{4e\delta^2}\frac{\overline{|\Delta|^2}}{\Delta_\infty^2}. \quad (2.28)$$

To find $\overline{|\Delta|^2}$ we multiply (2.25) by Δ^* and integrate over the volume of the superconductor. Then, taking (2.26) and (2.28) into account, we obtain

$$\frac{\overline{|\Delta|^2}}{\Delta_\infty^2}=\frac{1-(H/H_{c2})}{\beta_L[1-(2\varkappa)^{-2}]}, \quad (2.29)$$

where $\beta_L = \overline{|\Delta|^4}/(\overline{|\Delta|^2})^2$. For a triangular lattice we have[33] $\beta_L = 1.16$. We shall find it more convenient to express $\overline{|\Delta|^2}$ not in terms of H but in terms of B, since it is precisely the induction B which is equal to the field H in the vacuum at the sample surface. Substituting (2.28) in (2.29), we find

$$\frac{\overline{|\Delta|^2}}{\Delta_\infty^2}=\frac{2\varkappa^2[1-(B/H_{c2})]}{\beta_L(2\varkappa^2-1)+1}. \quad (2.30)$$

In the concrete case of a superconductor with paramagnetic impurities in strong fields, it is easiest to find the resistance[34] by starting from expression (1.16) given in Chap. 1 for the dissipation function, but putting there $\gamma = \hbar/2mD$ and $\Psi = \Delta\sqrt{N_0\tau\tau_s/2}$:

$$W=\frac{N_0\tau\tau_s}{2mD}\left|\hbar\frac{\partial\Delta}{\partial t}+2ie\Phi\Delta\right|^2+\sigma_n\left|\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}+\nabla\Phi\right|^2.$$

It is convenient to choose the gauge for the potential such that the electric field, which is homogeneous in zeroth order in $|\Delta|^2$, is described by the scalar potential

$$\Phi=-E_x x,\quad \text{a}\quad \mathbf{A}=(0,\ H_{c2}x,\ 0).$$

In this case the second term in W yields, after averaging over the sample, simply $\sigma_n\bar{E}^2$ accurate to $|\Delta|^2$, since the deviations of the field from the mean value, $\delta E = E - \bar{E}$, drop out: $\overline{\delta E} = 0$ and $\overline{(\delta E)^2} \sim |\Delta|^4$. To calculate the first term, we note that in first order in the velocity we can assume $\partial\Delta/\partial t = -(\mathbf{v}_L\cdot\nabla)\Delta_0$, where Δ_0 is the equilibrium value of Δ, given by (2.26), and the average electric field is $\bar{\mathbf{E}} = -(1/c)\mathbf{v}_L\times\mathbf{B} = -(1/c)\mathbf{v}_L\times\mathbf{H}_{c2}$, so that in the chosen gauge of Φ the velocity $\mathbf{v}_L$ is directed downward along the y axis. Using this, we can write

$$\left|\hbar\frac{\partial\Delta}{\partial t}+2ie\Phi\Delta\right|^2=v_{Ly}^2\hbar^2\left[\left(\frac{\partial}{\partial y}+\frac{2ie}{\hbar c}A_y\right)\Delta^*\left(\frac{\partial}{\partial y}-\frac{2ie}{\hbar c}A_y\right)\Delta\right].$$

To simplify the calculation of these expressions, we introduce the operators

$$\Pi_{\pm}=\left(\nabla-\frac{2ie}{\hbar c}\mathbf{A}\right)_x \pm i\left(\nabla-\frac{2ie}{\hbar c}\mathbf{A}\right)_y,$$
$$\Pi^*_{\pm}=\left(\nabla+\frac{2ie}{\hbar c}\mathbf{A}\right)_x \pm i\left(\nabla+\frac{2ie}{\hbar c}\mathbf{A}\right)_y,$$

which act on Δ and Δ^*, respectively.

With the aid of (2.26) we can easily verify that $\Pi_+\Delta_0 = 0$ and $[\Pi_+, \Pi_-]_- = (4e/\hbar c)H_{c2}$. We therefore have

$$\overline{\left|\hbar\frac{\partial\Delta}{\partial t}+2ie\Phi\Delta\right|^2}=\frac{v_{Ly}^2\hbar^2}{4}\overline{[(\Pi_+^*\Delta_0^*)(\Pi_-\Delta_0)]}=\frac{v_{Ly}^2\hbar^2}{4}\overline{(\Delta_0^*\Pi_+\Pi_-\Delta_0)}$$
$$=\frac{v_{Ly}^2\hbar^2}{4}\overline{(\Delta_0^*[\Pi_+,\Pi_-]\Delta_0)}=\frac{ev_{Ly}^2\hbar^2}{\hbar c}H_{c2}\overline{|\Delta|^2}=2e^2\xi^2(\bar{E})^2\overline{|\Delta|^2}.$$

Thus

$$\overline{W}=\sigma_n\bar{E}^2\left(1+6\frac{\overline{|\Delta|^2}}{\Delta_\infty^2}\right).$$

Equating $\overline{W} = \sigma_f\bar{E}^2$, we obtain[34]

$$\sigma_f/\sigma_n=1+6\overline{|\Delta|^2}/\Delta_\infty^2=1+\frac{12\varkappa^2}{\beta_L(2\varkappa^2-1)+1}\left(1-\frac{B}{H_{c2}}\right). \qquad (2.31)$$

This result is customarily represented as the slope of the plot of the resistivity ρ_f against B:

$$S=\left[\frac{H_{c2}}{\rho_n}\frac{d\rho_f}{dB}\right]_{B=H_{c2}}=\frac{12\varkappa^2}{\beta_L(2\varkappa^2-1)+1}.$$

In dirty alloys with $l \ll \xi$ the Ginzburg-Landau parameter κ is large, therefore

$$S\approx\frac{6}{\beta_L}\approx 5.17.$$

We note that the conductivity in strong fields can be obtained also by direct calculation of the current in Eqs. (2.1). The equation for the order parameter has then an exact solution.

Let us write down again the time-dependent equations (2.1) in the form (2.25)[7]:

$$-D^{-1}\left(\frac{\partial}{\partial t}+\frac{2ie}{\hbar}\Phi\right)\Delta+\xi^{-2}\left(1-\frac{|\Delta|^2}{\Delta_\infty^2}\right)\Delta+\left(\nabla-\frac{2ie}{\hbar c}\mathbf{A}\right)^2\Delta=0, \qquad (2.32)$$

$$\mathbf{j}=\sigma_n\left(-\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}-\nabla\Phi\right) \qquad (2.33)$$
$$+\frac{\hbar c^2}{16\pi\delta^2 e\Delta_\infty^2}\left[\Delta^*\left(-i\nabla-\Delta\frac{2e}{\hbar c}\mathbf{A}\right)\Delta-\Delta\left(-i\nabla+\frac{2e}{\hbar c}\mathbf{A}\right)\Delta^*\right].$$

As before, we choose the gauges for the potentials in the form $\Phi = -\bar{E}_x x$ and $A = (0, H_{c2}x, 0)$. Since $\bar{E} = -(1/c)v_L \times B$, it follows that $v_{Lx} = 0$ and $v_{Ly} = -c\bar{E}_x/H_{c2}$. It is easy to verify that the solution of (2.32) with these potentials is the function[22]

$$\Delta=\sum_{n=-\infty}^{\infty}C_n\exp[inq(y-v_{Ly}t)]\exp\left[-\frac{eH_{c2}}{\hbar c}\left(x-\frac{n\hbar cq}{2eH_{c2}}+\frac{iv_{Ly}\hbar c}{4DeH_{c2}}\right)^2\right], \qquad (2.34)$$

which, obviously, describes uniform motion of the initial equilibrium structure (2.26) with velocity v_{Ly}. Using the obtained expression for Δ, we easily calculate the current (2.33). By the same procedure as in the derivation of (2.27) we get

$$j_x=\sigma_n\left(-\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}-\nabla\Phi\right)_x-\frac{\hbar c^2}{16\pi\delta^2 e\Delta_\infty^2}\frac{\partial|\Delta|^2}{\partial y}-\frac{\hbar c^2 v_{Ly}}{16\pi\delta^2 eD}\frac{|\Delta|^2}{\Delta_\infty^2},$$
$$j_y=\sigma_n\left(-\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}-\nabla\Phi\right)_y+\frac{\hbar c^2}{16\pi\delta^2 e\Delta_\infty^2}\frac{\partial|\Delta|^2}{\partial x}.$$

After averaging over the sample, we have[22,34]

$$\bar{j}_y=0,\quad \bar{j}_x=\sigma_n\bar{E}_x-\frac{\hbar c^2 v_{Ly}}{16\pi\delta^2 eD}\frac{\overline{|\Delta|^2}}{\Delta_\infty^2}=\sigma_n\bar{E}_x\left(1+6\frac{\overline{|\Delta|^2}}{\Delta_\infty^2}\right).$$

We thus obtain again for the conductivity σ_f the expression (2.31). Formulas (2.31) if taken literally (the expression for κ!) pertain the case of paramagnetic alloys. The idea of the calculation, however, is close in the general case (which will be discussed later on) to that of the scheme described above.

3. RESULTS FOR GENERAL CASE OF SUPERCONDUCTING ALLOYS

a) Viscous flow of vortices in superconducting alloys. The case $B \ll H_{c2}$. The dissipation-function method described in Sec. c of Chap. 1 cannot be used directly to calculate the viscosity for the motion of vorte filaments in ordinary superconducting alloys. The nonequilibrium processes in superconductors should be described by the kinetics of the excitations in the superconductors. The corresponding system of nonequilibrium kinetic equation, which generalizes the ordinary kinetic equation in a normal metal, was derived by Eliashberg from the microscopic theory.[35] It is too complicated to present in the present review. As to the qualitative aspect of the matter, it can be understood by considering the singularities of viscous flow of vortices near the critical temperature. In this region, our subsequent reasoning, as shown by the detailed microscopic theory,[36] is also qualitative in character.

Near T_c, the vortex dimension increases, $\xi(T) \propto (T_c - T)^{-1/2}$. The motion of the vortices leads to a deviation of the excitation distribution function from the equilibrium Fermi function $n_0(\epsilon) = (e^{\epsilon/T} + 1)^{-1}$. Since the times of electron energy relaxation due to collisions with one another and with phonons are large, the steady state sets in via outflow of the nonequilibrium excitations to infinity. The corresponding times are determined by the diffusion relation $Dt_{diff} \sim \xi^2 \propto (T_c - T)^{-1}$.

We write down the BCS-theory equation that defines $\Delta(\mathbf{r}, t)$, assuming that the dependence of the latter on $\mathbf{r}$ and t is slow enough:

$$\Delta(\mathbf{r},t)=\lambda\int_0^\infty\frac{\nu_s(\varepsilon,\mathbf{r})}{\nu_n}\Delta(\mathbf{r},t)[1-2n(\varepsilon_{\mathbf{p}},\mathbf{r},t)]\frac{d\varepsilon}{\varepsilon}\frac{dO_{\mathbf{p}}}{4\pi}. \qquad (3.1)$$

With respect to the level density ν_s/ν_n it is assumed that it is determined by the usual relation $\nu_s = 0$ at $\epsilon < \Delta$ and $\nu_s = \nu_n\epsilon\sqrt{\epsilon^2-\Delta^2}$, but with a local value $|\Delta(\mathbf{r}, t)|$. In Eq. (3.1), $n(\epsilon_p, \mathbf{r}, t)$ is the distribution function of the normal excitations, which differs little from the equilibrium $n = n_0(\epsilon) + n_1(\epsilon_p, \mathbf{r})$. In the vicinity of the critical temperature at thermodynamic equilibrium, Eq. (3) can again be expanded in terms of Δ/T_c and $\nabla\Delta/T_c$, and we get the known equations of the Ginzburg-Landau theory. Therefore, with an aim at repeating the derivation of Sec. a of Chap. 2, which enabled us to find the connection between the superfluid component of the current with the vortex velocity, we write down, in analogy with (2.8), the linearized equation for the correction Δ_1 to the gap on account of the motion of the vortex:

$$\frac{\pi}{8T_c}\frac{\partial\Delta_0}{\partial t}+\frac{\pi}{8T_c}D\left[\nabla^2\Delta_1-\frac{4e^2}{c^2}Q_0^2\Delta_1-\frac{8e^2}{c^2}(Q_0Q_1)\Delta_0\right] \qquad (3.2)$$
$$+\left[\frac{T_c-T}{T_c}-\frac{7\zeta(3)}{8(\pi T_c)^2}3\Delta_0^2\right]\Delta_1=-\Delta_1^{(a)},$$

where the left-hand side (without $\partial\Delta/\partial t$) is the linearization of the Ginzburg-Landau equation for alloys (Gor'kov[12]), and the right-hand side, according to (3.1), is

$$\Delta_1^{(a)}(\mathbf{r}_1,t)=-2\int_\Delta^\infty\frac{\Delta_0}{\sqrt{\varepsilon^2-\Delta_0^2}}n_1(\varepsilon_{\mathbf{p}},\mathbf{r})\,d\varepsilon.$$

For the function n_1 we write down the usual kinetic equation

$$\frac{\partial n_0(\varepsilon)}{\partial t}+\frac{\partial\varepsilon}{\partial\mathbf{p}}\frac{\partial n_1}{\partial\mathbf{r}}-\frac{\partial\varepsilon}{\partial\mathbf{r}}\frac{\partial n_1}{\partial\mathbf{p}}=-\frac{1}{\tau_\varepsilon}(n_1-\bar{n}_1), \qquad (3.3)$$

where

$$\bar{n}_1 = \int n_1(\varepsilon_p) \frac{dO_p}{4\pi}, \quad \varepsilon_p = \sqrt{v_F^2 (p - p_F)^2 + \Delta^2 (\mathbf{r} - \mathbf{v}_L t)},$$

$$\mathbf{v}_\varepsilon = \frac{\partial\varepsilon}{\partial\mathbf{p}} = \mathbf{v}_F \frac{\sqrt{\varepsilon^2 - \Delta^2}}{\varepsilon}, \quad \tau_\varepsilon v_\varepsilon = l.$$

It is possible to prove rigorously the local equation (3.3) only with the aid of the microscopic theory,[36] but it can be used in the vicinity of T_c because Δ is small in comparison with T. Therefore almost all the electrons are excited and the contribution from the paired electrons has an additional smallness Δ/T_c. If there are many impurities, i.e., $l \ll \xi$, then (3.3) reduces in the usual manner to the diffusion equation, since the second term of the expansion of the dependence of $n_1(\epsilon_p)$ on the direction $\mathbf{p}$ in Legendre polynomials

$$n_1(\varepsilon_p) = \bar{n}_1 + (\mathbf{v}_F \mathbf{n}_1) + \ldots$$

is small to the extent that $l/\xi \ll 1$. Indeed, taking the zeroth and first harmonics of Eq. (3.3), we obtain

$$\mathbf{n}_1 = -\frac{v_\varepsilon \tau_\varepsilon}{v_F} \nabla \bar{n}_1 = -\frac{l}{v_F} \nabla \bar{n}_1, \tag{3.4}$$

$$D\nabla^2 \bar{n}_1 = \frac{\partial n_0}{\partial \varepsilon} \frac{\Delta}{\sqrt{\varepsilon^2 - \Delta^2}} \frac{\partial\Delta}{\partial t}. \tag{3.5}$$

Assuming as usual that $\partial\Delta/\partial t = -(\mathbf{v}_L \cdot \nabla)\Delta_0$ and putting $\bar{n}_1 = -\tilde{w}_\epsilon v_L \cos\varphi$, we obtain from (3.5) the following equation for $\tilde{w}$:

$$D \frac{d}{d\rho}\left[\frac{1}{\rho} \frac{d}{d\rho} (\rho \tilde{w}_\varepsilon)\right] = \frac{1}{4T} \mathrm{ch}^{-2} \frac{\varepsilon}{2T} \frac{d}{d\rho} \sqrt{\varepsilon^2 - \Delta^2}, \tag{3.6}$$

which can be easily solved[36] with the boundary conditions

$$\tilde{w}_\varepsilon(\rho) \to 0 \quad \text{as} \quad \rho \to \infty \qquad (\varepsilon > \Delta_\infty),$$
$$\tilde{w}_\varepsilon(\rho_0) = 0 \quad \text{at} \quad \Delta_0(\rho_0) = \varepsilon \qquad (\varepsilon < \Delta_\infty).$$

repeating the derivation of relation (2.10), we get

$$([\mathbf{n}_H \times \mathbf{d}], \mathbf{j}_{tr}) = \frac{m p_F e}{\pi^3} \int \Delta_1^{(a)} (\mathbf{d}\nabla) \Delta_0 \rho \, d\rho \, d\varphi = \frac{m p_F e}{\pi^2} (\mathbf{v}_L \mathbf{d}) \int_0^\infty \rho \, d\rho \frac{d\Delta^2}{d\rho} \int_\Delta^\infty \frac{\tilde{w}_\varepsilon(\rho)\, d\varepsilon}{\sqrt{\varepsilon^2 - \Delta^2}}. \tag{3.7}$$

In this expression we have retained only the principal term due to the nonequilibrium character of the distribution function. All the remaining terms, particularly the Joule dissipation, make smaller contributions. The reason is clear from the forms of the right-hand sides of (3.2) and (3.5). From (3.5) we estimate

$$n_1 \sim \frac{v_L \Delta_0^2 \xi}{D\sqrt{\varepsilon^2 - \Delta^2}} \frac{1}{2T} \mathrm{ch}^{-2} \frac{\varepsilon}{2T}.$$

The integral in (3.7) with respect to ϵ makes the main contribution in the region $\epsilon \sim \Delta$. The quantity $\Delta_1^{(a)}$ is of the order of $v_L \Delta_0^2 \xi / DT \propto v_L \sqrt{T_c - T}$, and can be compared with the contribution, say, due to the diffusion derivative $(\pi/8T_c)\partial\Delta/\partial t \sim v_L \Delta_0 / T\xi \propto v_L (T_c - T)$ in the left-hand side of (3.2). It can be similarly verified that the contribution made to the ohmic current from n_1, $\mathbf{j} \propto -e\int v_F n_1 d\epsilon\, dO_p$ is due to electrons with $\epsilon \sim T$, and therefore the energy dissipation due to the electrons is determined unjust as in Sec. b of Chap. 1 by the estimate[19] of the fraction of the superconductor volume occupied by the normal core of the vortex. This mechanism always makes a contribution on the order of unity to the coefficient of σ_f, whereas calculation of (3.7)[37] yields for the conductivity of the alloys near T_c the expression[5)]

$$\sigma_f = 1.1 \left(1 - \frac{T}{T_0}\right)^{-1/2} \sigma_n \frac{H_{c2}(T)}{B_0}.$$

From the point of view of Eqs. (2.11), this result means an additional factor $(1 - T/T_c)^{-1/2}$ in the coefficient α. In other words, as $T \to T_c$, owing to the large dimensions of the vortex, the relaxation of the order parameter is slower than proposed in[20]. We call attention also to the fact that, just as above, the charge of the electron drops out of (3.7), since $\mathbf{j}/e$ has the meaning of the electron momentum flux density.

We have thus seen that the square root singularity in the density of states of the BCS superconductivity theory leads in (3.1) to a strong dependence of the order parameter on the nonequilibrium increments to the excitation distribution function in the region $|T - T_c| \ll T_c$. Evidence of this singularity can appear even in the case when $\nu_s(\epsilon)$ does not become infinite, i.e., the singularity is only smoothed out because of effects that violate the Cooper pairing. For example, if the concentration of the paramagnetic impurity is not too high, so that the transition temperature T_c changes little, $|T_c - T_{c0}| \ll T_{c0}$ ($\tau_s T_{c0} \gg \hbar$), then, as shown by Éliashberg,[38], in the temperature region $\tau_s \Delta \ll \hbar$ there is likewise an effective increase of the order-parameter relaxation time. The conductivity of the vortex lattice ($H \ll H_{c2}$) is[27]

$$\sigma_f = 2.15 \frac{\tau_s T_c}{\hbar} \sigma_n \frac{H_{c2}}{B_0}.$$

The case when an analogous role is played by the magnetic field for an ordinary superconducting alloy was considered by Larkin and Ovchinnikov[39,40] (see the next section).

We have discussed in rather great detail the calculation of the conductivity (or viscosity) of the vortex structure in the vicinity of the critical temperature. The noted singularities of this limiting case make it possible to obtain relatively simple formulas. The general case of arbitrary temperatures is exceedingly laborious if for no other reason that the spectrum itself, the state density, and the magnitude of the gap change significantly over small distances. General equations that make it possible in principle to obtain by means of numerical calculations the conductivity of a lattice of vortex filaments were obtained for dirty alloys from the complete system of kinetic equations by Gor'kov and Kopnin[41]. However, a solution for these equations was obtained only for the case of low temperatures. The conductivity at $T = 0$ is[41]

$$\sigma_f = \frac{0.9\sigma_n H_{c2}(0)}{B_0}.$$

b) **Superconducting alloys. Strong fields** ($B \approx H_{c2}$). The microscopic theory of the motion of vortex filaments in alloys in the region of strong fields $H_{c2} - H \ll H_{c2}$ is much simpler than in weak fields since, as indicated in Sec. 2 of Chap. 2, the order parameter $\Delta/\Delta_\infty \sim \sqrt{1 - (H/H_{c2})}$ is small in the entire volume of the superconductor. Therefore in the thermodynamic (static) case there exists a generalization of the Ginzburg-Landau theory,[14] where the expansion of the free energy is in terms of the parameter $\Delta/T \sim \sqrt{H_{c2} - H}$. To summarize briefly the results, we indicate that, accurate to linear terms, the equation for Δ takes in this theory the form

$$\left[\ln \frac{T}{T_{c0}} + \psi\left(\frac{1}{2} + \hat{\rho}\right) - \psi\left(\frac{1}{2}\right)\right] \Delta = 0; \tag{3.8}$$

here $\psi(z)$ is the logarithmic derivative of the gamma function, and $\hat{\rho}$ is an operator

5)The coefficient 1.1 in this expression is obtained by using the interpolation curve for $\Delta_0(\rho)$. At the same time, a computer calculation [37] of the integral in (3.7) yielded $\beta = 2.85[1 - (T/T_c)]^{-1/2}$. Such a discrepancy is strange, but no other calculations of β have been published to date.

$$\hat{\rho} = \frac{\hbar D}{4\pi T}\left(\nabla - \frac{2ie}{\hbar c}\mathbf{A}\right)^2.$$

Since the function (2.26)

$$\Delta = \sum_{n=-\infty}^{\infty} C_n \exp(inqy) \exp\left[-\frac{eH_{c2}}{\hbar c}\left(x - \frac{n\hbar cq}{2eH_{c2}}\right)^2\right]$$

is an eigenfunction of the operator $\hat{\rho}$:

$$\hat{\rho}\Delta = \rho_0\Delta,$$

where $\rho_0 = 2DeH_{c2}/4\pi T_c$, Eq. (3.8) reduces to an algebraic equation, from which we determine the upper critical field $H_{c2}(T)$:

$$\ln\frac{T}{T_{c0}} + \psi\left(\frac{1}{2}+\rho_0\right) - \psi\left(\frac{1}{2}\right) = 0.$$

The current density in the mixed state is

$$\mathbf{j} = \frac{\sigma_n}{4\pi Te}\psi'\left(\frac{1}{2}+\rho\right)\left[\Delta^*\left(-i\nabla - \frac{2e}{\hbar c}\mathbf{A}\right)\Delta - \Delta\left(-i\nabla + \frac{2e}{\hbar c}\mathbf{A}\right)\Delta^*\right].$$

The connection between the induction and the field is given by a formula similar to (2.28):

$$-4\pi M = H - B = \frac{\sigma_n}{eTc}\overline{|\Delta|^2}\psi'\left(\frac{1}{2}+\rho_0\right). \tag{3.9}$$

For M, in turn, we have

$$M = -\frac{H_{c2}-H}{4\pi}\frac{1}{(2\varkappa_2^2(T)-1)\beta_L}, \tag{3.10}$$

where we have introduced a new parameter $\kappa_2(T)$.[14,26] As $T \to T_c$ we have $\rho \to 0$ and all the foregoing formulas go over into the ordinary expressions of the Ginzburg-Landau theory, while

$$\varkappa_2(T) \to \varkappa.$$

A plot of $\kappa_2(T)$ is shown in Fig. 7.

In the description of the kinetic phenomena in superconducting alloys at $H \approx H_{c2}$, the distribution functions of the nonequilibrium excitation can also be expanded in the order parameter, since the singularity of the state density $\nu_s(\epsilon)$ is completely smoothed out. It is clear that in this case the state density differs little from its value in the normal metal to the degree that $|\Delta|$ is small. The expansion parameter, however, is in this case the ratio Δ/ϵ_0, where $\epsilon_0 = 2DeH_{c2}(T)/c$. As shown by the microscopic theory[9], the contribution of the nonequilibrium excitations to the order-parameter relaxation processes has an additional smallness $(\Delta/\epsilon_0)^2$ relative to the relaxation processes described in Sec. c of Chap. 1. Therefore in the case when Δ/ϵ_0 is small a simple generalization of the static equation for Δ to the nonstationary case consists, roughly speaking of replacing[42,43], just as in the case of paramagnetic impurities (2.32), the operator $D[\nabla - (2ie/\hbar c)\mathbf{A}]^2$ in the static equation (3.8) by

$$-\left(\frac{\partial}{\partial t} + \frac{2ie}{\hbar}\Phi\right) + D\left(\nabla - \frac{2ie}{\hbar c}\mathbf{A}\right)^2.$$

Therefore the solution in the presence of an electric field is, as before, an expression of the type (2.34), which describes a vortex lattice moving as a unit. As to the electric current, the contribution of the nonequilibrium excitations turns out to be comparable with the ordinary ohmic current even in the case when their contribution can be neglected in the equation for Δ.[44]

It is of interest to note that the inequality $\Delta/\epsilon_0 \ll 1$ at a given field H can be violated if T is close enough to T_c, since $\Delta \sim \sqrt{T_c - T}$ and $\epsilon_0 \sim (T_c - T)$. This occurs at $1 \gg 1 - (H/H_{c2}) \gg 1 - T/T_c$. In this case the problem of finding the conductivity becomes much more complicated.

In that simple case when the inequality $\Delta/\epsilon_0 \ll 1$ is satisfied, the calculation of the conductivity in the mixed state is analogous to the scheme described in Sec. c of Chap. 2, and consists of finding the corrections, proportional to $\overline{|\Delta|^2}$, for the current $\sigma_n\overline{E}$ in the normal state.

In this review we cannot dwell on the details of the rather cumbersome calculations of the conductivity in this case. We confine ourselves only to the results.

The first attempt at applying the microscopic theory to the problem of the resistive state of type-II superconducting alloys was made by Caroli and Maki.[49] Their result is

$$\sigma_f = \sigma_n\left[1 + \frac{c}{4\pi TeDH_{c2}}\psi'\left(\frac{1}{2}+\rho_0\right)\overline{|\Delta|^2}\right].$$

Using (3.9) and (3.10), we can represent this result in the form

$$\frac{\sigma_f}{\sigma_n} = 1 + \frac{4\varkappa^2 a^2}{[2\varkappa_2^2(T)-1]\beta_L + 1}\left(1 - \frac{B}{H_{c2}}\right),$$

where $a = \pi^2/2\sqrt{14\zeta(3)} = 1.20$.

However, no account was taken in[42] of the contribution of the nonequilibrium excitations to the electric current. This circumstance was first pointed out by Thompson.[44] The expression obtained by him is

$$\frac{\sigma_f}{\sigma_n} = 1 + \frac{4\varkappa^2 a^2 L_D(T)}{[2\varkappa_2^2(T)-1]\beta_L + 1}\left(1 - \frac{B}{H_{c2}}\right), \tag{3.11}$$

where $L_D(T) = 2 + [\rho_0\psi''((1/2)+\rho_0)/\psi'((1/2)+\rho_0)]$. A similar result was obtained later by others[43,45,46]. As $T \to T_c$, the conductivity is exactly double the value calculated in[42]. A plot of $S = [(H_{c2}/\rho_n)d\rho_f/dB]_{B=H_{c2}}$ against T is shown in Fig. 8.

Formula (3.11) no longer holds in if T is in the immediate vicinity of T_c. As already noted above, at $1 - (H/H_{c2}) \gg 1 - (T/T_c)$ an important role is assumed by the nonequilibrium excitations upon relaxation of the order parameter. The phenomena that evolve in this temperature region have the same physical nature as in weak fields, and were discussed in the preceding section. The conductivity of superconductors in this region of temperatures was calculated by Larkin and Ovchinnikov.[39,40] They have shown that for superconducting alloys

$$\frac{\sigma_f}{\sigma_n} = 1 + 0.18\left(1 - \frac{T}{T_c}\right)^{-1/2}\left[\frac{2\varkappa^2}{\beta_L(2\varkappa^2-1)+1}\left(1 - \frac{B}{H_{c2}}\right)\right]^{3/2}.$$

On the other hand, in the case when the Cooper pair-

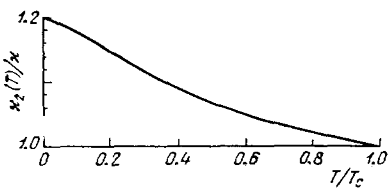

FIG. 7. Dependence of the generalized Ginzburg-Landau parameter κ_2 on the temperature [26] (κ is the ordinary Ginzburg-Landau parameter [13]).

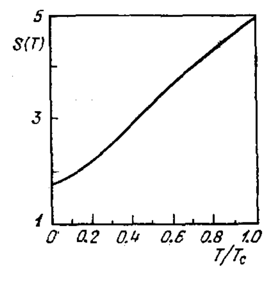

FIG. 8. Temperature dependence of the slope $S(T) = [(H/\rho_n)\,d\rho_f/dH]_{H_{c2}}$ in strong fields. [44]

ing is affected also by a small concentration of paramagnetic impurities, viz., $\tau_s T_{c0} \gg \hbar$, $\Delta^2\tau_s \gg \hbar\epsilon_0$, i.e., in the field range

$$\frac{\hbar}{\tau_s T_c} \ll X \ll \frac{\hbar^2}{\tau_s^2 (T_c - T) T_c},$$

where

$$X = \frac{2\varkappa^2}{\beta_L (2\varkappa^2 - 1) + 1}\left(1 - \frac{B}{H_{c2}}\right),$$

the conductivity is[39]

$$\frac{\sigma_f}{\sigma_n} = 1 + 0.32 \frac{\tau_s T_c}{\hbar} X^2.$$

Near the critical field H_{c2} it turns out to be possible to investigate dissipative processes in superconductors at an arbitrary ratio of the mean free path l and the correlation radius ξ_0 (the dimension of the Cooper pairs).[47] Because the formulas are cumbersome, we refer the reader to the original paper.

c) **Surface impedance.** As already mentioned in the Introduction, the observation of the picture of the viscous flow of vortices in the experiment is made difficult by the action of the pinning forces, so that the procedure frequently used to determine the resistivity of a superconductor in the mixed state is to measure the energy dissipated when a magnetic wave is reflected from the superconductor surface (usually in the centimeter-wavelength band). This dissipation, as is well known, is determined by the real part of the impedance (the surface resistance)

$$Z(\omega) = -\frac{4\pi i\omega}{c^2}\lambda(\omega),$$

where $\lambda(\omega)$ is the "skin" depth of penetration of the electromagnetic field.

In a weak field $B \ll H_{c2}$ the problem of reflection of an electromagnetic wave from the surface of a superconductor in the mixed state can be solved, in first order in ω, with the aid of Eqs. (2.21), (2.22), and (2.24), and also the relations

$$\mathbf{j}_{tr} = \sigma_f \mathbf{E} \times \mathbf{j}_{tr} = \frac{c}{4\pi}\operatorname{rot}\mathbf{H} = \left[\mathbf{n}_H \times \frac{\partial \mathbf{H}}{\partial z}\right] - [\mathbf{n}_H \times \nabla (H_z - H_0)].$$

We note that all these relations are quite general in character and do not depend on the concrete model of the superconductor.

It is easy to show[27] that following the reflection of the electromagnetic a superconductor in the mixed state behaves like an anisotropic "medium" of sorts, described by Maxwell's equations

$$\operatorname{rot}\mathbf{E}_\sim = -\frac{1}{c}\frac{\partial \mathbf{B}_\sim}{\partial t}, \quad \operatorname{rot}\mathbf{H}_\sim = \frac{4\pi}{c}\mathbf{j}_\sim$$

and by the material equations

$$\mathbf{j}_\sim = \sigma_f \mathbf{E}_\sim, \quad \mathbf{B}_\sim = \mu \mathbf{H}_\sim$$

(the tilde labels the alternating quantities), and the magnetic permeability μ depends on the direction of the magnetic field of the wave relative to $\mathbf{H}_0$. If $\mathbf{H}_\sim$ is parallel to $\mathbf{H}_0$, then μ is given by

$$\mu_\parallel = \frac{B_0^2}{4\pi c_{11}},$$

and if $\mathbf{H}_\sim$ is perpendicular to $\mathbf{H}_0$, then

$$\mu_\perp = \frac{B_0^2}{4\pi c_{44}}.$$

The moduli c_{11} and c_{44} are equal to[31]

$$c_{11} = \frac{B^2}{4\pi}\frac{\partial H}{\partial B} + \frac{1}{8\pi}\int_0^B B'^2 \frac{\partial^2 H(B')}{\partial B'^2} dB', \quad c_{44} = \frac{BH}{4\pi}.$$

At $\kappa \gg 1$ and in the field interval $H_{c1} \ll H \ll H_{c2}$ we have

$$c_{11} \approx \frac{B^2}{4\pi} - \frac{BH_{c1}}{16\pi \ln \varkappa}.$$

Proceeding in the usual manner,[24] we can obtain the "macroscopic skin" depth of penetration of the electromagnetic field

$$\lambda_{\parallel,\perp} = \frac{(1+i)c}{\sqrt{8\pi\mu_{\parallel,\perp}\sigma_f\omega}}.$$

The surface resistance is

$$R_f = \operatorname{Re} Z(\omega) = \sqrt{\frac{2\pi\omega}{\mu\sigma_f c^2}},$$

where it is necessary to use for σ_f the corresponding expressions obtained in Chaps. 2 and 3 (see also (4.1a), (4.2a), and (4.3)).

This approach is valid when the "skin" depth $\lambda(\omega)$ is large in comparison with the depth of penetration of the constant field:

$$\delta = \frac{c}{2\pi}\sqrt{\frac{\hbar}{\Delta\sigma_n \operatorname{th}(\Delta/2T)}},$$

so that the effects on the surface itself can be neglected. Unfortunately, the inequality $\lambda \gg \delta$ is satisfied only in the meter wavelength band. Thus, even at temperatures lower than T_c we have

$$\frac{\lambda}{\delta} \sim \sqrt{\frac{T_c}{\hbar\omega}\frac{H_0}{H_{c2}}},$$

where the characteristic values of $T_c/\hbar$ are of the order of 10^{11} sec^{-1}.

In the case of strong fields ($H_0 \approx H_{c2}$) the frequency dependence of the impedance can be calculated more fully. We put $\mathbf{j} = -Q(\omega)\mathbf{A}$. The skin depth λ is expressed in terms of $Q(\omega)$ by means of $\lambda^{-2} = 4\pi Q(\omega)/c$. The kernel $Q(\omega)$ can be represented in the form

$$Q(\omega) = Q_n(\omega) + Q'(\omega),$$

where

$$Q_n = -\frac{i\omega\sigma_n}{c}$$

corresponds to the normal metal and Q' is a small correction necessitated by the incomplete suppression of the superconductivity. We obtain for the surface resistance

$$\frac{R_f - R_n}{R_n} = \frac{\operatorname{Re} Z(\omega) - \operatorname{Re} Z_n(\omega)}{\operatorname{Re} Z_n(\omega)} = \frac{c}{2\omega\sigma_n}[\operatorname{Im} Q'(\omega) - \operatorname{Re} Q'(\omega)]. \quad (3.12)$$

In the limit as $\omega \to 0$ this relation yields

$$\frac{R_f - R_n}{R_n} = \frac{\rho_f - \rho_n}{2\rho_n} \quad (3.13)$$

($R_n = \sqrt{2\pi\omega/\sigma_n c^2}$ is the surface resistance of the normal metal). We note that the derivative $\partial R_f/\partial B|_{H_{c2}}$ is expressed directly in terms of the derivative $d\rho_f/dB|_{H_{c2}}$:

$$\left(\frac{H_{c2}}{R_n}\frac{dR_f}{dB}\right)_{H_{c2}} = \frac{1}{2}\left(\frac{H_{c2}}{\rho_n}\frac{d\rho_f}{dB}\right)_{H_{c2}} \equiv \frac{1}{2}S(T).$$

The kernel $Q'(\omega)$ was calculated by Thompson.[44] At low frequencies $\hbar\omega \ll \pi(T_c - T_c)$ we have

$$\frac{\operatorname{Re} Q'}{\omega} = 0, \qquad \frac{\operatorname{Im} Q'}{\omega} = \frac{4eML_D(T)}{8\pi T\rho_0}, \quad (3.14)$$

where the magnetic moment is given by (3.9). From this we readily obtain with the aid of (3.12) and (3.13) the expression (3.11) for the conductivity. In the region near T_c, where the frequency ω becomes comparable with $\pi(T_c - T)/\hbar$ for $\hbar\omega \ll \pi T_c$ and $T_c - T \ll T_c$, we have

$$\begin{aligned} \operatorname{Re} Q' &= -4eM\left[\frac{1}{2}\frac{\omega^2}{\omega^2+\varepsilon_0^2} + \frac{\omega^2\{1 - 2\rho[\psi''(1/2)/\psi'(1/2)]\}}{\omega^2+4\varepsilon_0^2}\right], \\ \operatorname{Im} Q' &= 2eM\omega\left[\frac{\varepsilon_0}{\omega^2+\varepsilon_0^2} + \frac{4\varepsilon_0\{1 - 2\rho_0[\psi''(1/2)/\psi'(1/2)]\}}{4\varepsilon_0^2+\omega^2} + \frac{3\psi''(1/2)\hbar}{4\pi T\psi'(1/2)}\right]. \end{aligned} \quad (3.15)$$

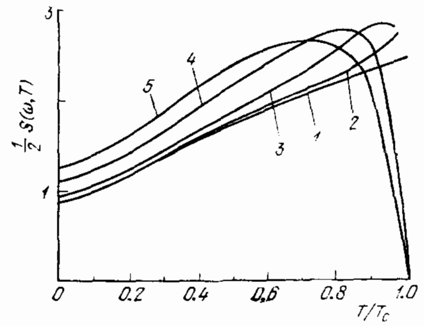

FIG. 9. Family of curves for the slope (1/2) S (ω, T) = {[H/$R_n(\omega)$] d$R_f(\omega)$/dH}$_{H_{c2}}$, expressed in terms of the surface impedance. [48] 1—ν/Tc = 0; 2—ν/Tc = 0.4, $\omega/\epsilon_0(0)$ = 0.0109; 3—ν/Tc = 2, $\omega/\epsilon_0(0)$ = 0.0544; 4—ν/Tc = 6, $\omega/\epsilon_0(0)$ = 0.1633; 5—ν/Tc = 10, $\omega/\epsilon_0(0)$ = 0.2771. S (0, T) (curve 1) coincides with the slope S (T) = [(H/ρ_n) dρ_f/dH]$_{H_{c2}}$ (see Fig. 8) of the resistance in a constant field.

Figure 9, which is taken from[48], shows the numerically calculated slopes

$$\frac{1}{2}S(\omega,T)=\left(\frac{H_{c2}}{R_n}\frac{dR_f}{dB}\right)_{H_{c2}}$$

for different values of the parameter $\omega/\epsilon_0(0)$.

4. STATUS OF EXPERIMENTAL RESEARCH

a) Summary of principal theoretical formulas. Before we proceed to discuss the experimental data, we summarize the formulas describing the conductivity of various superconducting alloys in weak and strong fields at different temperatures.

1) Alloys with large concentration of paramagnetic impurities.

a) Weak fields $H \ll H_{c2}$, $\kappa \gg 1$[27-30]:

$$\frac{\sigma_f}{\sigma_n}=\frac{2.63H_{c2}(T)}{B},\quad \frac{\rho_f}{\rho_n}=\frac{0.381B}{H_{c2}(T)}. \tag{4.1a}$$

b) Strong fields $H \approx H_{c2}$[34]

$$\frac{\sigma_f}{\sigma_n}=1+\frac{12\kappa^2}{\beta_L(2\kappa^2-1)+1}\left(1-\frac{B}{H_{c2}}\right),\quad \frac{\rho_f}{\rho_n}=1-\frac{12\kappa^2}{\beta_L(2\kappa^2-1)+1}\left(1-\frac{B}{H_{c2}}\right) \tag{4.1b}$$

(in the limit $\kappa \gg 1$) we have in practice $\rho_f/\rho_n = 1 - 5.17\,[1-(B/H_{c2})]$).

2) Alloys with low paramagnetic-impurity concentrations:

a) $H\ll H_{c2}$, $\kappa\gg 1$, $\tau_s T_c\sqrt{[1-(T/T_c)]}\ll 1$[27]:

$$\frac{\sigma_f}{\sigma_n}=2.15\frac{\tau_s T_c}{\hbar}\frac{H_{c2}(T)}{B}. \tag{4.2a}$$

b) $H\approx H_{c2}$, $\hbar/\tau_s T_c\ll 1-(B/H_{c2})\ll \hbar^2/\tau_s^2 T_c(T_c-T)$[39]:

$$\frac{\sigma_f}{\sigma_n}=1+0.32\frac{\tau_s T_c}{\hbar}\left[\frac{2\kappa^2}{\beta_L(2\kappa^2-1)+1}\left(1-\frac{B}{H_{c2}}\right)\right]^2. \tag{4.2b}$$

At $\kappa \gg 1$ we have

$$\frac{\sigma_f}{\sigma_n}=1+0.24\frac{\tau_s T_c}{\hbar}\left(1-\frac{B}{H_{c2}}\right)^2.$$

3) Ordinary superconducting alloys (without paramagnetic impurities). Weak fields $H \ll H_{c2}$ ($\kappa \gg 1$).

a) Low temperatures $T \ll T_c$[41]:

$$\frac{\sigma_f}{\sigma_n}=\frac{0.9H_{c2}(0)}{B},\quad \frac{\rho_f}{\rho_n}=\frac{1.1B}{H_{c2}(0)}. \tag{4.3a}$$

b) High temperatures $T \to T_c$[36,40] 6)

$$\frac{\sigma_f}{\sigma_n}=1.1\frac{1}{\sqrt{1-(T/T_c)}}\frac{H_{c2}(T)}{B},$$

6)See footnote 5.

$$\frac{\rho_f}{\rho_n}=0.9\sqrt{1-\frac{T}{T_c}}\frac{B}{H_{c2}(T)}. \tag{4.3b}$$

4) Alloys without paramagnetic impurities. Strong fields.

a)
$$\frac{\sigma_f}{\sigma_n}=1+\frac{5.76\kappa^2L_D(T)}{[2\kappa_2^2(T)-1]\beta_L+1}\left(1-\frac{B}{H_{c2}}\right). \tag{4.4a}$$

At $\kappa \gg 1$ we have

$$\frac{\sigma_f}{\sigma_n}=1+2.49L_D(T)\left[\frac{\kappa}{\kappa_2(T)}\right]^2\left(1-\frac{B}{H_{c2}}\right),$$
$$\frac{\rho_f}{\rho_n}=1-2.49L_D(T)\left[\frac{\kappa}{\kappa^2(T)}\right]^2\left(1-\frac{B}{H_{c2}}\right)$$

(see Fig. 8).

b) $1-(H/H_{c2})\gg 1-(T/T_c)$[39]:

$$\frac{\sigma_f}{\sigma_n}=1+0.18\left(1-\frac{T}{T_c}\right)^{-1/2}\left[\frac{2\kappa^2}{\beta_L(2\kappa^2-1)+1}\left(1-\frac{B}{H_{c2}}\right)\right]^{3/2}. \tag{4.4b}$$

At $\kappa \gg 1$ we have

$$\frac{\sigma_f}{\sigma_n}=1+0.14\left(1-\frac{T}{T_c}\right)^{-1/2}\left(1-\frac{B}{H_{c2}}\right)^{3/2},$$
$$\frac{\rho_f}{\rho_n}=1-0.14\left(1-\frac{T}{T_c}\right)^{-1/2}\left(1-\frac{B}{H_{c2}}\right)^{3/2}.$$

It should be noted that so far (with the exception of the interpolation curve obtained by Danilov, Kuprianov, and Likharev[37]) no calculations have been made of the dependence of ρ_f on H in arbitrary fields.7)

b) Results of comparison of theory and experiment. Let us now dwell briefly on the experimental measurements of ρ_f in the mixed state. At the present time two methods are being used for this purpose: measurements with direct current and the method of surface resistance.

In the first method the sample is placed in an external magnetic field $H_{c1} < H < H_{c2}$ and an electric current j_{tr} perpendicular to the magnetic field is made to flow through the sample. The experimental setup is shown in Fig. 2. The resistance ρ_f is identified with the slope of the linear section of the current-voltage characteristic. Already, as already mentioned many times above, such a "viscous" flow of vortices is impeded by the pinning of the vortices by the inhomogeneities of the sample structure. This causes the linear sections of the current-voltage characteristics to be located in regions of currents large enough to exceed on the average the so-called critical current j_c at which the vortices become detached from the inhomogeneities; in addition, these sections turn out to be relatively short. All this makes the reduction of the experimental data difficult and decreases the accuracy of the experiment. To obtain reliable results it is therefore necessary to subject the samples to a special chemical treatment that eliminates the defects of the structure.

There are also other difficulties connected with the bending of the vortex filaments, which is due not only to the pinning forces but also to the Meissner effect and to the need to take into account the field produced by the transport current itself. If the transport-current density is j_{tr}, then the field on the surface of a sample of thickness D is

$$H\approx\frac{2\pi}{c}j_{tr}D.$$

7)Figure 10 shows the plot obtained in [37] for ρ_f against the ratio B/H_{c2} for alloys with large concentration of paramagnetic impurities. The slope of the $\rho_f(B)$ curve in strong fields is equal to 5.17, in agreement with the result of [34], but the slope obtained in [37] for weaker fields is 0.318 and does not agree with the results of [28] and [30]. In our opinion this discrepancy is due to an error made in [37].

It follows from the results of Chap. 2 that the expressions for the conductivity in weak fields have been obtained under the assumption that the self-field of the current is small in comparison with the applied field, which at a low vortex concentration is of the order of H_{c1}. On the other hand, the transport current should exceed j_c, and we therefore obtain the estimate

$$j_c \ll j_{tr} \ll \frac{cH_{c1}}{2\pi D}.$$

Satisfaction of this inequality is possible only for a sufficiently thin sample. Thus, for example, for a sample of a well-annealed alloy of Nb + 45% Ta, the critical field H_{c1} is of the order of 200 Oe, and the density of the critical current is of the order of 5×10^3 A/cm².[49] This yields

$$5 \cdot 10^3 \text{ A/cm}^2 \ll j_{tr} \ll \frac{3 \cdot 10^2}{D} \text{ A/cm}^2$$

Thus, in this case the sample thickness should be less than 10^{-2}–10^{-3} cm, and the corresponding current density will exceed 10^4 A/cm². In a thin sample this leads to a large Joule heating of the sample itself as well as the conduction contacts. All this calls for effective cooling.

These difficulties can be overcome to a certain extent by using the method based on the measurement of the absorption coefficient in the reflection of an electromagnetic wave from the surface of the superconductor in the mixed state (the surface-resistance method). The expressions for the impedance of the superconductor and its connection with the resistance in the mixed state are given in Sec. c of Chap. 3. It was also indicated there that this method is suitable, in the main, in the case of an extremely large vortex-filament concentration. This method also suffers from the same shortcoming that it cannot be used in practice to measure ρ_f near T_c. The point is that as $T \to T_c$, at any given frequency of the electromagnetic wave, the ratio ω/ϵ_0 or ω/Δ ceases to be small, thus making it difficult to determine the value of ρ_f from the results of the experiment (see Fig. 9).

To study the dissipation of the energy in the mixed state one can use in principle also a method based on generation of the oscillations of the vortex filaments under torsional vibrations of the sample in a magnetic field perpendicular to the rotation axis.[50] This method, however, is at present still in the development stage, and no concrete results on ρ_f have been obtained as yet.

The first to study viscous flow of the vortices in type-II superconductor were Kim et al.[3] They obtained for the alloy $Nb_{0.5}Ta_{0.5}$ the plot of ρ_f vs. H shown in Fig. 11. Qualitatively, the $\rho_f(H)$ retain the same form also for other superconductors at arbitrary temperatures and fields. Kim et al. have proposed the empirical formula

$$\frac{\rho_f}{\rho_n} = \beta^{-1}(T)\frac{B}{H_{c2}(T)},$$

where $\beta(T) = H_{c2}(0)/H_{c2}(T)$. As $T \to 0$ this expression for β agrees well with the theoretical value $\beta \approx 0.9$ (4.3a), but at finite temperatures this formula does not describe satisfactorily the temperature dependence of $\beta(T)$ (see (4.3b)).

The measurement of ρ_f in various fields has been the subject of a large number of experiments. Let us dwell first on the results of measurements of ρ_f of alloys in weak fields. Figure 12 shows the data of of[3,51-54], which show the temperature dependence of the

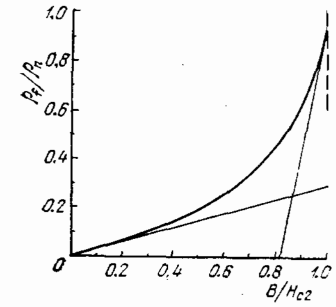

FIG. 10. Interpolation curve for alloys with large concentration of paramagnetic impurities. [37] The straight lines give the slopes of the curves for weak and strong fields.

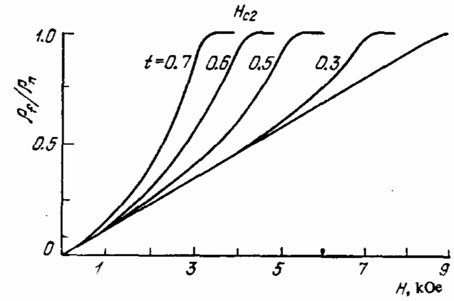

FIG. 11. Dependence of ρ_f on the magnetic field for the alloy $Nb_{0.5}Ta_{0.5}$. $T_c = 6.15°K$, $t = T/T_c$.

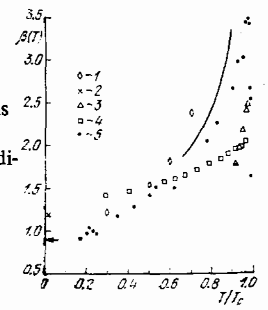

FIG. 12. The function $\beta(T)$ (see formulas (1.21) and (4.3b)). Solid line–plot of the function $1.1[1 - (T/T_c)]^{-1/2}$. The arrow indicates the theoretical value $\beta(0) = 0.9$. 1–results of [3], 2–[51], 3–[52], 4–[53], 5–[54].

function $\beta(T)$. The solid line is a plot of the function $1.1[1 - (T/T_c)]^{-1/2}$, and the arrow marks the value $\beta(0) = 0.9$ (see (4.3a) and (4.3b)). The most important fact here is that the experiment confirms the theoretically predicted[36] growth of $\beta(T)$ near the critical temperature, a reflection of the slowness of the relaxation processes near T_c.

Because of its relative simplicity, the viscous flow of vortices in fields close to H_{c2} was studied theoretically much earlier[42-44] than in the weak-field region[27-30,36,39,41]. This explains why for almost five years most experiments were made only in strong fields. The values of ρ_f were measured there in practically the entire range of temperatures and for a variety of alloys with a wide range of parameters.[48,55-61] The experimental data revealed here an unusually large scatter. Some of the causes of this scatter are indicated in Kim's review.[62] In addition the $\rho_f(H)$ curves, especially those obtained by the dc method, frequently have near H_{c2} a dip that may be due to the presence of pinning. Some investigated used for the slope $d\rho_f/dH$ H_{c2} the value of $d\rho_f/dH$ ahead of the dip, and others the value past the dip.

By way of example we present here data on the measurement of ρ_f in strong fields, obtained by Axt and Joiner[56] by the dc current method. The shaded region in Fig. 13 contains the experimental points corresponding to the alloys $Nb_{0.5}Ta_{0.5}$, $Nb_{0.1}Ta_{0.9}$,

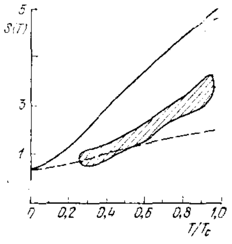

FIG. 13. Data of [56] on the measurement of ρ_f near H_{c2} for NbTa, InBi, and PbTl of varying purity. The shaded region corresponds to the experimental points. The solid line is the theoretical plot [41] (see Fig. 8; formula (4.4a)), and the dashed line is the result of Caroli and Maki. [42]

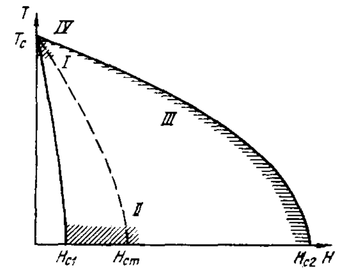

FIG. 16. The shaded regions correspond to those ranges of the parameters H/H_{c2} and T/T_c where there are analytic expressions for σ_f. Region I corresponds to formulas (4.2a) and (4.3b), II to (4.3a), III to (4.2b) and (4.4a), and IV to (4.4b).

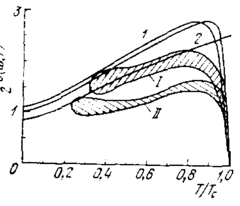

FIG. 14

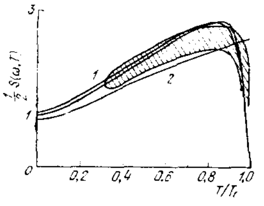

FIG. 15

FIG. 14. Data of [48] on the measurement of R_f by the surface-resistance method for NbTa alloys. I—region of experimental points for $Nb_{0.5}Ta_{0.5}$, $Nb_{0.75}Ta_{0.25}$, II—$Nb_{0.9}Ta_{0.1}$, $Nb_{0.95}Ta_{0.05}$. Curves 1 were calculated respectively for the critical points 5.8 and 8.5°K and for the frequency $\nu = 3.14 \times 10^{10}$ Hz. Curves corresponds to the slope of S(T) in a direct current.

FIG. 15. Data of [48] on the measurement of R_f by the surface-resistance method for the alloys $Pb_{0.5}In_{0.5}$, $Pb_{0.83}In_{0.17}$ and $Pb_{0.9}In_{0.1}$. Curves 1 were calculated for the critical temperatures 6.2°K and 7.0°K. $\nu = 3.14 \times 10^{10}$ Hz.

$In_{0.98}Bi_{0.02}$, $In_{0.96}Bi_{0.04}$, $Pb_{0.6}Tl_{0.4}$, $Pb_{0.39}Tl_{0.61}$, and $Pb_{0.95}Tl_{0.05}$. The solid line corresponds to the theoretical dependence (4.4a), and the dashed line to the results of the theory of Caroli and Maki, with which all the experimental data prior to the publications of Thompson's paper were obtained.[44]

The most reliable at present are the data of Pedersen, Kim, and Thompson,[48] obtained by the surface-resistance method using a differential technique that permits direct measurement of $dR_f(\omega)/dH$. An advantage of this method is that it admits of a simple extrapolation of the slope dR/dH to fields H_{c2}. Figure 14 shows data[48] for NbaTa alloys of varying purity. The solid curves 1 correspond to numerical calculations in accord with the theoretical formulas (3.14) and (3.15) (frequency 3.14×10^{10} Hz and critical temperatures 5.8 and 8.5°K). Curve 2, calculated from (3.14), is the low-frequency limit ($\omega \to 0$). Figure 15 shows the analogous data for PbIn alloys. Curves 1 were calculated for the critical temperatures 6.2 and 7.0°K. It is curious to note a circumstance pointed out in[48], that many experimental data on the slope of the plot of the resistance against the field near H_{c2} agree better with Maki's earlier formula[42] than with the undoubtedly more correct results of Thompson (3.11). It is difficult to say as yet whether this fact is due to purely psychological causes or has a more substantial cause.

CONCLUSION

It follows from this brief survey of the experimental data that the theory agrees in the main with experiment, in spite of the large difficulty of obtaining final results in either direction. The simplest and most well-developed theoretical conclusions, such as for example the formulas for alloys with paramagnetic impurities or the behavior of the conductivity near T_c are precisely the ones that raise additional difficulties when it comes to verify them in the experimental situation. Modern microscopic theory of superconductivity makes it possible in principle to calculate the conductivity of superconductors in the mixed state at arbitrary temperature in the entire range of magnetic fields. However, relatively simple analytic expressions for the conductivity can be obtained only in rather narrow ranges of the parameters H/H_{c2} and T/T_c. The shaded regions in Fig. 16 are those in which there are analytic expressions for σ_f (see the summary of the formulas); only for alloys with large concentration of paramagnetic impurities has the conductivity σ_f been calculated in the entire range of temperatures for both small and large fields. The calculation of the conductivity in regions where the parameters H/H_{c2} and T/T_c are not close to zero or unity entails great difficulties and is apparently possibly only by using numerical methods. On the whole, we can state that the microscopic theory provides not only a qualitative but also a quantitative description of the chosen unique circle of phenomena.

It was our aim in this review to report on the physical mechanisms that govern the viscous flow of vortices, and present an idea of the methods used to resolve these problems in the theoretical papers. The number of these papers is at present quite large, but nevertheless a large number of phenomena related to those touched upon in this review have still no substantial microscopic description. This pertains, in our opinion, to most thermomagnetic phenomena (at least in the field interval $H \ll H_{c2}$), and also to the Hall effect. Starting with[19], where it was noted that the Hall effect in a superconductor should be of the same order as in a normal metal, numerous attempts were made to obtain this effect quantitatively from the microscopic theory (see, e.g.,[63,64]). For various reasons, not all these attempts are presently sufficient. It should be added that the experimental data on the Hall effect are likewise quite contradictory.

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LIST OF SYMBOLS

- $\mathbf{A}$ — vector potential of electromagnetic field,
- a = 1.20,
- $\mathbf{B}$ — magnetic induction,
- B_0 — the same for undeformed vortex-filament lattice,
- c_{ij} — elastic moduli of triangular vortex-filament lattice,
- D — diffusion coefficient,
- d — distance between vortex filaments,
- $\mathbf{d}$ — arbitrary translation vector of vortex structure,
- $\bar{\mathbf{E}}$ — electric field intensity,
- $\mathcal{F}$ — Gibbs' thermodynamic potential in a given external field H,

F — free-energy density,
F_L — Lorentz force acting on an individual vortex of unit length
$f = \Delta/\Delta_\infty$,
H_{cm} — thermodynamic critical magnetic field,
H_{c1} — lower critical magnetic field,
H_{c2} — upper critical magnetic field,
H — macroscopic intensity of magnetic field,
H_0 — the same for undeformed lattice of vortex filaments,
h — microscopic magnetic field,
j — electric current density,
j_{tr} — density of average (macroscopic) electric current (transport current),
$j_{1\infty}$ — current density produced when an individual filament moves at large distances for, its core,
j_c — critical pinning current,
K_0, K_1 — Bessel functions of imaginary argument,
$L_D(T) = 2 + \rho_0\psi''((1/2)+\rho_0)/\psi'((1/2)+\rho_0)$,
l — mean free path,
M — magnetic moment per unit volume of the superconductor,
N — electron-number density,
n_L — vortex-filament density,
$n_0(\epsilon)$ — Fermi distribution function,
$n(\epsilon, r, t)$ — distribution function,
n_H — unit vector in the magnetic-field direction,
p_F — electron momentum on the Fermi surface,
$Q = A - (c/2e)\nabla\theta$,
Q_0 — value of Q for immobile vortex,
$Q_d = (d\nabla)Q_0$,
$R_f = \mathrm{Re}\, Z(\omega)$—surface resistance in mixed state,
$R_n = \mathrm{Re}\, Z_n(\omega) = \sqrt{2\pi\omega/\sigma_n c^2}$—the same in the normal state,
$S = [(H_{c2}/\rho_n)(d\rho_f/dB)]$—slope of dependence of resistivity on B,
T — temperature,
T_c — critical temperature,
T_{c0} — critical temperature in the absence of a magnetic field and paramagnetic impurities,
$u_i(z, t)$ — deformation vector of i-th vortex filament,
v_L — velocity of motion of vortex filament,
v_F — velocity on the Fermi surface,
W — dissipation function,
$Z(\omega)$ — impedance,
$\beta_L = 1.16$—constant of triangular vortex-filament lattice,
Δ — order parameter (energy gap),
Δ_0 — the same for an immobile vortex,
Δ_∞ — value of Δ in the absence of a magnetic field,
ϵ — energy of excitations, reckoned from the Fermi surface,
$\epsilon_0 = 2DeH_{c2}/c$,
η — viscosity coefficient for vortex,
θ — phase of order parameter,
$\kappa = \delta/\xi$ — parameter of the Ginzburg-Landau theory,
$\lambda(\omega), \lambda_{\parallel,\perp}$ — skin depth of penetration of alternating electromagnetic field,
λ — constant of Cooper interaction of electrons
μ — chemical potential,
$\tilde{\mu} = \dot{\theta} + 2e\Phi$,
$\mu_{\parallel,\perp}$ — magnetic permeability,
ν_n — density of states in the normal metal on the Fermi surface,
$\nu_s(\epsilon)$ — density of states in superconductor,
ξ — coherence radius,
ρ — distance from center of vortex,
ρ_{0i} — equilibrium position of i-th vortex filament
ρ_f — resistivity in mixed state,
$\rho_0 = \epsilon_0/4\pi T$,
σ_n — conductivity of normal metal,
$\sigma_f = \rho_f^{-1}$ — conductivity of superconductor in mixed state,
τ — free path time of electrons with respect to collisions with impurity atoms,
τ_s — free path time of electron with respect to spin flip,
Φ — scalar potential,
φ — azimuthal angle in cylindrical coordinate system,
$\varphi_0 = hc/2e = 2\cdot10^{-7}$ G-cm^2,
Ψ — order parameter in Ginzburg-Landau theory,
ψ — derivative of gamma function (psi function)
ψ', ψ'' — derivatives of psi function.

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