Self-induced transparency effect

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Usp. Fiz. Nauk **114**, 97–131 (September 1974)

A review is given of the current status of the self-induced transparency effect. The McCall–Hahn theory (2π pulses) is considered and its generalization to the case of phase modulation of radiation pulses, degeneracy of transitions, and two-photon resonance is discussed. The relationship with the theory of nonlinear waves, obeying the Korteweg–de Vries equation, is pointed out. The possibility of similar coherent effects in direct interband transitions in semiconductors is discussed. A summary is given of the experimental results demonstrating a strong fall in the absorption and a considerable slowing down of light pulses, typical of the self-induced transparency. Possible physical applications of the effect, based on these properties, are discussed.

CONTENTS

INTRODUCTION

High-power coherent light pulses of duration shorter than the relaxation times of resonant transitions in many media (such as gases, alkali metal vapors, ruby, etc.) can now be generated by Q-switched or mode-locked lasers. The effects which accompany the propagation of such ultrashort pulses through resonantly absorbing media are no longer described by the laws based on the linear dispersion theory (low intensities) or on the rate equations for radiative transfer (incoherent interaction).

The dominant feature of the new effects is that the relaxation processes (collisions, spontaneous emission, etc.) have insufficient time to destroy the "phase memory" so that the polarization of the medium is a nonlinear function of the field amplitude and phase at all the preceding moments in time. Coherent effects of this kind include the photon echo[1] and optical nutation;[2] which are the optical analogs of the effects known from the nuclear magnetic resonance theory.

However, the consequences of the phase memory are manifested most clearly in the phenomenon of self-induced transparency discovered by McCall and Hahn.[3-5] It is found that above a certain intensity threshold the absorption of a pulse by resonant transitions falls strongly and a previously absorbing substance becomes almost completely transparent to the incident radiation. This is accompanied by considerable reduction in the velocity of a pulse compared with the phase velocity of light in the substance under consideration.[5-6]

The physics of the self-induced transparency can be understood by considering the dynamics of the interaction of a pulse with a medium. The leading edge of a pulse transfers absorbing particles from a lower energy state to a coherent superposition of the lower and upper states, so that some of the field energy is stored in the medium. If the pulse intensity is sufficiently high, at some moment the particles are all found in the upper state, i.e., the medium is completely inverted. The remainder of the pulse causes the particles to emit stimulated light and thus return the energy to the field. In this way, the particles are gradually transferred back to the lower state. Since the duration of a pulse is less than the relaxation time, this energy-transfer cycle should be completed before the relaxation processes can destroy the coherence of the interaction. Moreover, under certain conditions, all the energy acquired initially from the field returns to the pulse and restores its initial shape. In this way, a pulse may travel in an absorbing medium without losses, all the time expending and recovering its energy. Consequently, its velocity decreases. Here, it is convenient to separate this coherent bleaching of a medium from the bleaching effect due to the ordinary saturation of the absorption which occurs when the duration of high-power pulses is much longer than the relaxation time of the investigated medium. In the latter case, the relaxation processes play an important part in the interaction and, consequently, the ability of a medium to absorb or radiate energy is simply proportional to the difference between the populations of the upper and lower levels. Hence, it is clear that such a long pulse cannot transfer the medium to the inverted state and, at best, it can only equalize the probabilities of finding the particles

at the upper and lower energy levels. The energy necessary for this purpose is acquired from the leading edge of the pulse and this energy cannot be returned coherently. It is then emitted as spontaneous radiation or lost by nonradiative transitions, i.e., it is generally lost to the radiation pulse. Thus, a pulse of this kind traveling across a medium loses energy irreversibly in the bleaching process.

We shall consider the self-induced transparency effect in the wide sense of this term and discuss the coherence phenomena which accompany the propagation of ultrashort pulses across resonantly absorbing media.

However, we shall not deal with the photon echo or optical nutation. They can be regarded as independent effects without any necessary link with the self-induced transparency as such, and we shall direct the reader to appropriate reviews.[7,8] The relationship between the photon echo, optical nutation, and self-induced transparency is discussed in[9]. The coherence effects in the presence of a phase memory play equally important roles during the propagation of radiation across amplifying media (see also[10]). Some of the results obtained on this subject are given in reviews,[6,11,12] where the saturation of the absorption and amplification in the incoherent case is also discussed. It should be pointed out that such properties of the self-transparency effect as the strongly pronounced threshold nature of the transmission of light and the considerable reduction in the velocity of propagation are not only of general physical interest, but may find (and have already found) practical applications in measurements of short relaxation times, dipole moments of transitions, and other characteristics of resonances; there are also optoelectronic applications such as the reduction in the spatial size of light pulses or the use of self-transparency in discriminators, delay lines, and logic elements. The connection between the self-induced transparency effect and the general theory of solitons (Korteweg–de Vries equation[13]) is of considerable theoretical interest.

1. DERIVATION OF EQUATIONS DESCRIBING RESONANT INTERACTION OF PULSES WITH A MEDIUM

We shall describe the electromagnetic field of a pulse **E** using the Maxwell equations for a dielectric and we shall find the dipole moment **P**, induced in absorbing particles by the field **E**, from quantum-mechanical equations for the density matrix $\hat{\rho}$. The sum of all these dipoles determines the macroscopic polarization $\mathscr{P}$ of the medium. For the sake of self-consistency, this polarization should also occur in the equations which govern changes in the field. This semiclassical approach is generally acceptable in descriptions of the interaction between laser pulses and media (in the coherent and incoherent cases) and it works well if we can ignore the quantum fluctuations of the field (see, for example,[14]). The procedure is shown schematically in Fig. 1. We shall assume that a medium consists of two-level particles interacting with one another only via the radiation field. These particles may be atoms, ions, or molecules separated sufficiently far from one another and characterized by spectra with two levels ϵ_1 and ϵ_2 such that the transition between them ω_{21} is close to resonance with the carrier frequency ω of the incident light pulse. We shall assume that these particles are immobile but the results obtained can usually also be applied to moving atoms in gases.[12]

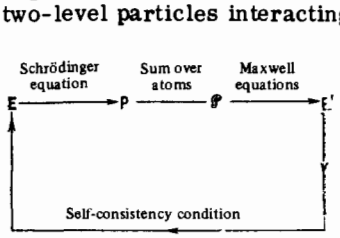

FIG. 1. Schematic diagram illustrating the procedure in obtaining a self-consistent system of equations for the field and medium.

The high-frequency dipole moment **P** induced by the field in such a two-level atom can be described by the formula

$$\mathbf{P} = \mathbf{e}P = \mathbf{Tr}(\hat{\mu}\hat{\rho}), \tag{1.1}$$

where **e** is the polarization vector and $\hat{\mu}$ is the dipole moment operator of the atom.

We shall assume that the field **E** represents a linearly polarized[1)] plane wave:

$$\mathbf{E} = \mathbf{e}E(z, t), \qquad e_z = 0. \tag{1.2}$$

The equation of motion for the density matrix is

$$i\hbar \frac{\partial}{\partial t}\rho = [\hat{\mathscr{H}}_0 + \hat{V}, \rho]_- - \hat{\Gamma}\hat{\rho}, \tag{1.3}$$

where $\mathscr{H}_0$ is the Hamiltonian of the atom under consideration in the absence of the field, and

$$\hat{V} = -\hat{\mu}E, \quad \hat{\mu} = (\hat{\mu}\mathbf{e}).$$

The term $\hat{\Gamma}\hat{\rho}$ describes the relaxation processes (collisions, spontaneous emission, etc.) which—on the whole—result in the decay of the dipole moment. We shall not give the intermediate steps but make a phenomenological allowance for the relaxation in the final equations.

In the representation in which $\hat{\mathscr{H}}_0$ is diagonal, the operators are of the form

$$\hat{\mathscr{H}}_0 = \begin{vmatrix} \varepsilon_1 & 0 \\ 0 & \varepsilon_2 \end{vmatrix}, \quad \hat{V} = -E\begin{vmatrix} 0 & \mu_{12} \\ \mu_{21} & 0 \end{vmatrix}, \quad \hat{\rho} = \begin{vmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{vmatrix}. \tag{1.4}$$

The dipole moment **P**(z, t) can be represented by

$$P = \rho_{12}\mu_{21} + \rho_{21}\mu_{12}, \tag{1.5}$$

and, going over from Eq. (3) to the equations for the quantities **P** and $n = \rho_{11} - \rho_{22}$, we obtain

$$\frac{\partial^2 P}{\partial t^2} + \omega_{21}^2 P = \frac{2\mu^2\omega_{21}}{\hbar} En, \tag{1.6}$$

$$\frac{\partial n}{\partial t} = \frac{2\hbar}{\omega_{21}} E \frac{\partial P}{\partial t}; \tag{1.7}$$

where $\mu = |\mu_{12}| = |\mu_{21}|$, $\omega_{21} = (\epsilon_2 - \epsilon_1)/\hbar$. The value of n governs the difference between the populations of the lower and upper energy states and, in the absence of the field, we have n = 1 (absorbing medium).

We shall assume that the following inequalities are satisfied:

$$\tau_p \gg \frac{1}{\omega}, \quad \mu E \ll \hbar\omega, \quad K \ll \varkappa, \tag{1.8}$$

where τ_p is the duration of the light pulse and **K** is the absorption coefficient in weak fields. Therefore, the field **E** can be represented in the form

$$E(z, t) = \mathscr{E}(z, t)\cos\Phi, \qquad \Phi = \omega t - \varkappa z + \varphi(z, t), \tag{1.9}$$

where $\mathscr{E}$ and φ are the "slow" amplitude and phase, and $\varkappa$ is the wave vector. The conditions of "slowness" are:

$$\frac{\partial\mathscr{E}}{\partial t} \ll \omega\mathscr{E}, \quad \frac{\partial\varphi}{\partial t} \ll \omega, \quad \frac{\partial\mathscr{E}}{\partial z} \ll \varkappa\mathscr{E}, \quad \frac{\partial\varphi}{\partial z} \ll \varkappa. \tag{1.10}$$

We shall assume that the field is coherent in the sense that the coherence length is greater than the length of the absorbing medium **L**. Similarly, the dipole moment **P**(z, t) can be separated into the fast and slow parts:

$$P(z, t) = P_1(z, t)\cos\Phi + P_2(z, t)\sin\Phi; \tag{1.11}$$

here P_1 is the reactive component and P_2 is the active component; we can also say that P_1 describes the contribution to the dispersion and P_2 the contribution to the absorption of light by the medium (in the weak-field case). It should be stressed that, in the coherent interaction of a high-power pulse with a medium, the field dependence of the polarization is strongly nonlinear and P_2 may contribute not to the absorption but to the slowing down of a pulse. The conditions of slowness for P_1 and P_2 are similar to those given in Eq. (1.10).

In terms of the slow variables,[5] the system (1.6) becomes (see also[23])

$$\left.\begin{aligned} \frac{\partial P_1}{\partial t} &= -\left(\Delta\omega+\frac{\partial\varphi}{\partial t}\right)P_2-\frac{P_1}{T_2},\\ \frac{\partial P_2}{\partial t} &= \left(\Delta\omega+\frac{\partial\varphi}{\partial t}\right)P_1+\frac{\mu^2}{\hbar}\mathcal{E}-n\frac{P_2}{T_2'},\\ \frac{\partial n}{\partial t} &= -\frac{\mathcal{E}}{\hbar}P_2-\frac{n-1}{T_1}; \end{aligned}\right\} \qquad (1.12)$$

here $\Delta\omega = \omega - \omega_{21}$ and $P_1(z, -\infty) = P_2(z, -\infty) = 0$.

We have introduced above the phenomenological decay constants T_1 and T_2', which describe the relaxation processes (this is a generalization of the well-known Weisskopf-Wigner method[15]). In particular, T_1 includes a contribution of only those processes which result in the transition of the excited atom to the lower state (these transitions usually give rise to the spontaneous radiation). On the other hand, T_2' also includes the contributions from the relaxation events such as collisions of excited atoms with one another in gases, with phonons in solids, and so on. Since, in this case, the particle energy does not change greatly but only the phase of its vibrations is disturbed, we usually have $T_2' \ll T_1$ (however, in the case of low gas pressures, we can have $T_2' \approx T_1$). The broadening of the transition line associated with T_2' is homogeneous. The criterion of the coherence of the interaction between a light pulse and a medium can be expressed in the form

$$\tau_p < T_2'. \qquad (1.13)$$

The relaxation terms on the right-hand side of Eq. (1.12) are small compared with the derivatives and we can seek a solution in the absence of relaxation and then find corrections in terms of the smaller parameter τ_p/T_2'.

In the derivation of the expression for the macroscopic polarization $\mathcal{P}$, we must allow for the fact that the resonant transition frequency ω_{21} is not the same for all the atoms but is distributed with a probability $g(\omega_{21} - \omega_{21}^0)$ in a range of frequencies around the average ω_{21}^0 (close to the pulse carrier frequency ω).

Then,

$$\mathcal{P} = eN\int g(\Delta\omega)\, P(\Delta\omega, z, t)\, d\Delta\omega \equiv eN\langle P\rangle, \qquad (1.14)$$

where $\Delta\omega = \omega - \omega_{21}$ is the integration variable and N is the number of particles (per unit volume) participating in the transitions under discussion. The line broadening associated with the distribution $g(\Delta\omega)$ is called inhomogeneous. Its width $1/T_2^* \sim 1/g(0)$ contributes to the total width of the transition line $1/T_2$.

The quantities P_1 and P_2 in the material equations (1.12) should now also be regarded as functions of the parameter $\Delta\omega$. The Maxwell equations for the field can easily be reduced to the wave equation

$$\frac{\partial^2 E}{\partial z^2} - \frac{\eta^2}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 \mathcal{P}}{\partial t^2}; \qquad (1.15)$$

here, η is the nonresonant component of the refractive index, so that the whole contribution of the resonance is included in $\mathcal{P}$. McCall and Hahn[4, 5] modified Eq. (1.15) to first-order shortened equations for the field amplitude $\mathcal{E}$ and phase φ. Although this approximation is not simply a consequence of the slowness conditions (1.10), it can be justified for the majority of experimental situations.[16] [Some of the results based on the use of the second-order equation (1.15) are discussed in[17-22]].

We shall now give a self-consistent system of equations for the coherent interaction:

$$\frac{\partial P}{\partial t} = -\left(\Delta\omega+\frac{\partial\varphi}{\partial t}\right)P_2, \qquad (1.16)$$

$$\frac{\partial P_2}{\partial t} = \left(\Delta\omega+\frac{\partial\varphi}{\partial t}\right)P_1+\frac{\mu^2}{\hbar}\mathcal{E}n, \qquad (1.17)$$

$$\frac{\partial n}{\partial t} = -\frac{\mathcal{E}}{\hbar}P_2, \qquad (1.18)$$

$$\frac{\partial\mathcal{E}}{\partial z}+\frac{\eta}{c}\frac{\partial\mathcal{E}}{\partial t} = -\frac{2\pi\omega N}{\eta c}\langle P_2\rangle, \qquad (1.19)$$

$$\left(\varkappa-\frac{\omega\eta}{c}\right)\mathcal{E}+\left(\frac{\partial\varphi}{\partial z}+\frac{\eta}{c}\frac{\partial\varphi}{\partial t}\right)\mathcal{E} = -\frac{2\pi\omega N}{\eta c}\langle P_1\rangle. \qquad (1.20)$$

The system (1.16)–(1.20) can be studied by numerical methods. However, we shall first consider some important analytic results which can be obtained subject to additional assumptions.

2. McCALL-HAHN SOLUTIONS

We shall consider the special case[5] when there is no phase modulation:

$$\varphi(z, t) = \text{const}. \qquad (2.1)$$

In this case, the field $\mathcal{E}$ is given just by Eq. (1.19), whereas Eq. (1.20) reduces to the condition which must be obeyed by the dispersion law $\varkappa(\omega)$. Thus, in this case, there is no reactive nonlinearity because $\langle P_1\rangle \propto E$

We can easily see that Eqs. (1.16)–(1.18) can be reduced to an equation for the rotation of a virtual vector **R** with the components $(P_1/\mu, P_2/\mu, n)$:[22, 24]

$$\frac{\partial\mathbf{R}}{\partial t} = [\mathbf{\Omega}, \mathbf{R}],\ \mathbf{R}(\Delta\omega, z, -\infty) = (0, 0, 1), \qquad (2.2)$$

where the angular velocity "vector" $\mathbf{\Omega}$ is

$$\Omega = \left(-\frac{\mu\mathcal{E}}{\hbar},\ 0,\ \Delta\omega\right). \qquad (2.3)$$

Hence, it follows that

$$R^2 = \frac{P_1^2+P_2^2}{\mu^2}+n^2 = 1. \qquad (2.4)$$

In the absence of the field, the atom is in its lower state, i.e., **R** = (0, 0, 1) and it begins to rotate during a pulse. If the field transfers the atom to a state with equal probabilities for both energy levels, the corresponding rotation is through the angle $\pi/2$, whereas complete population inversion corresponds to the rotation through π. If we ignore relaxation, the rotation of the vector **R** under the action of a pulse $\mathcal{E}$ is completed before the relaxation effects (T_1, T_2') can have any significant effect.

In the case of particles in exact resonance, $\Delta\omega = 0$, the rotation equation (2.2) is easily integrated:

$$P_1 = 0,\ P_2 = \mu\sin\psi,\ n = \cos\psi, \qquad (2.5)$$

where the angle of rotation is

$$\psi = \frac{\mu}{\hbar}\int_{-\infty}^{t}\mathcal{E}(z, t_1)\, dt_1. \qquad (2.6)$$

The complete rotation is

$$\vartheta(z) = \psi(z, \infty) = \frac{\mu}{\hbar}\int_{-\infty}^{\infty}\mathcal{E}(z, t)\, dt \qquad (2.7)$$

and it is an important characteristic of the interaction. The quantity $\vartheta(z)$ is also called the area of a pulse. It follows from Eqs. (2.5) and (2.6) that if $X = 2\pi n$ (n = 1,

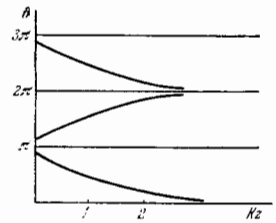

FIG. 2. Area theorem for a non-degenerate one-photon resonance.

2, . . .), such a pulse returns the resonant atoms exactly to the lower state so that all the energy first stored in a medium reverts to the radiation field. We shall show later that this is true not only for resonant atoms but for atoms with arbitrary detuning $\Delta\omega$ (subject to the condition that $\Delta\omega \ll \omega$). Consequently, such a $2\pi m$ pulse moves across a resonantly absorbing medium without losing energy.

We shall now consider the behavior of a pulse whose initial area is $\vartheta_0 = \vartheta_{(0)}$. Using Eqs. (1.19), (1.16), and (1.17), we find that

$$\frac{d\vartheta}{dz} = -\frac{2\pi\omega\mu^2 N}{\eta c\hbar^2}\int_{-\infty}^{\infty}\mathscr{E}(z, t_1)\left\langle n(\Delta\omega, z, t_1)\lim_{t\to\infty}\frac{\sin\Delta\omega(t-t_1)}{\Delta\omega}\right\rangle dt_1. \quad (2.8)$$

Going to the limit,[25]

$$\lim_{t\to\infty}\frac{\cos xt}{x} = \pi\delta(x)\lim_{\varepsilon\to 0}\frac{\varepsilon}{x}, \quad \lim_{t\to\infty}\frac{\sin xt}{x} = \pi\delta(x), \quad (2.9)$$

we obtain the "area theorem"[5]

$$\frac{d\vartheta}{dz} = -\frac{K}{2}\sin\vartheta, \quad (2.10)$$

where

$$K = K(\omega) = \frac{4\pi^2\omega\mu^2 N g(\Delta\omega_0)}{\eta c\hbar}, \quad \Delta\omega_0 = \omega - \omega_{21}^0. \quad (2.11)$$

The quantity $K(\omega)$, which occurs in the area theorem given by Eq. (2.10), is simply the absorption coefficient of a weak monochromatic field of frequency ω. We shall consider the specific case when the spectrum of a light pulse is narrower than an inhomogeneously broadened line, i.e., we shall assume that

$$\tau_p \gg T_2^*. \quad (2.12)$$

In this case, the value of K varies only a little within the spectrum of the field.

If the pulse being considered is sufficiently weak, i.e., if $\vartheta_0 \ll 1$, we find from Eq. (2.10) that

$$\vartheta(z) = \vartheta_0\exp\left(-\frac{Kz}{2}\right). \quad (2.13)$$

Applying Eq. (2.12), we obtain the same exponential decay law of the pulse energy $W = (\eta c/8\pi)\int\mathscr{E}^2(z, t)\,dt$:

$$W(z) = W_0\exp(-Kz). \quad (2.14)$$

Thus, in the case of a weak pulse, $\vartheta_0 \ll 1$, the usual Beer absorption law (2.14) is also valid in the coherent case if the spectrum of the field is much narrower than the total line width.

The general solution of Eq. (2.10) is

$$\operatorname{tg}\frac{\vartheta}{2} = \operatorname{tg}\frac{\vartheta_0}{2}\cdot\exp\left(-\frac{Kz}{2}\right). \quad (2.15)$$

This solution is shown graphically in Fig. 2. The area of a pulse tends to the nearest stable value $\vartheta = 2\pi n$. In particular, there is a steep threshold at $\vartheta_0 = \pi$, i.e., pulses with smaller areas decay in a distance of several reciprocals of the absorption coefficient, whereas pulses with a larger area are converted to a form with a stable area.

Although allowance for other factors results in some broadening of the threshold, the discriminatory nature of the transmission of the pulses in the self-transparency effect is not in doubt and can be utilized (see also[26]). It should be pointed out that the area ϑ is related in a simple manner[26] to the spectrum of a pulse $\mathscr{L}(z, \nu) = \left|\int_{-\infty}^{\infty}\mathscr{E}\exp[-i(\omega-\nu)t]dt\right|^2$:

$$\vartheta^2(z) = \frac{\mu^2}{\hbar^2}\mathscr{L}(z, \nu=\omega). \quad (2.16)$$

We shall seek the steady-state solution of Eqs. (1.16)–(1.20) for the field and medium when all the quantities depend only on the variable $\tau = t-(z/v)$. (It should be noted that the velocity is $v < c/\eta$.) This solution can be found subject to the additional condition, which is in agreement with the results to be discussed later, that the active component of the polarization can be represented in the factorized form:

$$P_2(\Delta\omega, \tau) = P_2(0, \tau)\chi(\Delta\omega). \quad (2.17)$$

Using Eq. (2.17), we find from the system (1.16)–(1.20) that the steady-state value of the quantity $\psi(\tau)$ is described by the equation

$$\frac{d^2\psi}{d\tau^2} = \frac{1}{\tau_p^2}\sin\psi, \quad (2.18)$$

where

$$\tau_p^2 = \frac{\eta c\hbar\,(1/v)-(\eta/c)}{2\pi\omega\mu^2 N\langle\chi\rangle_i}. \quad (2.19)$$

This is analogous to the equation of motion of a physical pendulum, which is disturbed from its upper unstable equilibrium position. All the solutions are periodic, except for

$$\operatorname{tg}\frac{\psi}{4} = \exp\frac{\tau}{\tau_p}, \quad (2.20)$$

which corresponds to the motion of the pendulum with zero initial velocity. Naturally, only Eq. (2.20) corresponds to a pulse limited in time and characterized by an energy $W < \infty$. Hence, we obtain the following expression for the field $\mathscr{E}$:[5]

$$\mathscr{E} = \frac{2\hbar}{\mu\tau_p}\sin\frac{\psi}{2} = \frac{2\hbar}{\mu\tau_p}\operatorname{sech}\frac{\tau}{\tau_p}. \quad (2.21)$$

We can easily see that the area is $\vartheta + 2\pi$ so that the steady-state solution obtained here is a 2π pulse. Hence, it follows directly that other solutions with constant areas $\vartheta = 2\pi n$ $(n \neq 1)$ are not steady-state pulses.

3. PROPERTIES OF 2π PULSES

We shall first consider the behavior of the polarization and inversion during the passage of a 2π pulse through a medium. Using Eq. (2.21), we readily find from Eqs. (1.16)–(1.18) that

$$P_1(\Delta\omega, \tau) = -2\mu\Delta\omega\tau_p\chi(\Delta\omega)\sin\frac{\psi}{2}, \quad P_2(\Delta\omega, \tau) = \mu\chi(\Delta\omega)\sin\psi, \quad (3.1)$$

$$n(\Delta\omega, \tau) = 1 - 2\chi(\Delta\omega)\sin^2\frac{\psi}{2}. \quad (3.2)$$

It follows from Eq. (2.4) that the function χ has the Lorentzian form $(1 + \Delta\omega^2\tau_p^2)^{-1}$.

A remarkable property of the system (3.1)–(3.2) is that the "vectors" $R(\Delta\omega, \tau)$ are rotated through 2π in the case of atoms with an arbitrary detuning from resonance $\Delta\omega$, so that the absorption of light in a resonant transition is identically zero (in the absence of relaxation), irrespective of the profile of the inhomogeneously broadened lines representing this transition. However, the velocity of the incident pulse v decreases. We readily find from Eqs. (1.20) and (2.19) that

$$\frac{1}{v} - \frac{\eta}{c} = \frac{K_\tau(\omega)\tau_p}{2}, \quad \varkappa(\omega) = \frac{\omega}{c}\Pi_\tau(\omega), \quad (3.3)$$

where

$$K_\tau = \frac{4\pi\mu^2\omega N}{\eta c\hbar\tau_p}\left\langle\frac{1}{\Delta\omega^2+\tau_p^{-2}}\right\rangle, \quad \Pi_\tau(\omega) = \eta - \frac{2\pi\mu^2 N}{\eta\hbar}\left\langle\frac{\Delta\omega}{\Delta\omega^2+\tau_p^{-2}}\right\rangle. \quad (3.4)$$

The quantities $K_\tau(\omega)$ and $\Pi_\tau(\omega)$ differ from the

ordinary absorption coefficients K and the ordinary refractive index Π only by the fact that the relaxation rate $\Gamma = 1/T_2'$ in Eq. (3.4) is now represented by the spectral width of the incident pulse $1/\tau_p$. If $T_2' \ll \tau_p$, we find that $K_\tau \approx K$ and $\Pi_\tau \approx \Pi$.

In the case of considerably delays, when $K\tau_p/2 \gg \eta/c$, we have

$$v \approx \frac{2}{K\tau_p}. \tag{3.5}$$

Equation (3.5) demonstrates the unusual role of the absorption coefficient K. It no longer describes the absorption of light but it occurs in the expression for the velocity of the pulses and the high values of K correspond to slow pulses. A different expression for the velocity,[28] which has a simple physical meaning, is given by

$$\frac{1}{v} = \frac{\eta}{c}\left(1 + \frac{U_m}{U_f}\right), \tag{3.6}$$

where U_m and U_f are the average energies stored in the medium (m) and in the radiation field (f). The reduction in the velocity of a 2π pulse then means that a considerable proportion of its energy is the excitation energy of the medium. The velocity v deduced from Eq. (3.3) is the generalization of the concept of the group velocity to the nonlinear coherent interaction case. If the carrier frequency ω is sufficiently far from the central frequency of the line ω_{21}^0, the nonlinear effects disappear and v becomes the ordinary group velocity v_g in the dispersion theory.[29,30]

4. ALLOWANCE FOR TRANSIENT CONDITIONS

Although our steady-state model of a 2π pulse in the absence of relaxation gives a qualitative description of the coherent interaction between light and matter, any comparison with real experiments demands allowance for various factors which are ignored in Chaps. 2 and 3.

a) **Transient stage.** This stage is described by the complete system (1.16)–(1.20), which can only be solved numerically. However, some qualitative results follow directly from the area theorem (2.10). As mentioned earlier, light pulses whose initial areas lie in the range $\tau < \vartheta_0 < 3\pi$ are converted into 2π pulses.

However, the 2π pulses obtained in this way differ from one another in respect of their duration and intensity. This is due to the fact that the parameter τ_p in Eq. (2.21) is equal to the initial pulse duration τ_{in} only if on entry into the medium under consideration it is of the form described by Eq. (2.21). If $\pi < \vartheta_0 < 2\pi$, the area under the pulse increases and its energy decreases during the transient stage because the medium still absorbs significantly. Consequently, the duration of the pulse increases so that, finally, we have $\tau_p > \tau_{in}$. However, if $2\pi < \vartheta_0 < 3\pi$, the area under the pulse decreases, whereas its energy remains practically constant. Consequently, the pulse duration decreases and we have $\tau_p < \tau_{in}$.[2)] A remarkable property of the self-transparency effect is that the pulse which emerges from an absorbing medium is characterized by the maximum value of the field exceeding the initial value. These features of the transient stage are responsible for the appearance of a delay which depends strongly on the initial area under a pulse ϑ_0. In particular, calculations[27] show that if $\vartheta_0 \approx \pi$, the delay during the transient stage (due to the increase in the pulse duration) may be considerably greater than the steady-state delay given by Eq. (3.12) (see also[33]).

b). **Relaxation processes.** The results for a steady-state 2π pulse, given by Eqs. (2.21) and (3.1)–(3.5), are obtained on the assumption that there is no relaxation. If we allow for the fact that T_1 and T_2' are finite, we find that a resonant transition absorbs the pulse energy but to a much smaller extent that predicted by the Beer law (2.14).

We can show[5] that, in the first order with respect to τ_p/T_2', the relaxation has the following consequences.

1) The area of a 2π pulse changes little:

$$\vartheta = 2\pi\left(1 - \frac{\tau_p}{T_2'}\right). \tag{4.1}$$

2) The energy of a pulse W decreases in accordance with the law

$$\frac{dW}{dz} \approx -n\hbar\omega\tau_p\left[\frac{1}{T_2'}\left\langle\frac{1}{1+\Delta\omega^2\tau_p^2}\right\rangle + \frac{2}{3}\left(\frac{1}{T_1} - \frac{1}{T_2'}\right)\left\langle\frac{1}{(1+\Delta\omega^2\tau_p^2)^2}\right\rangle\right], \tag{4.2}$$

3) The total delay, described earlier by the formula

$$t_d = \frac{K\tau_p}{2}L,$$

is now of the form

$$t_d = -T_e \ln\left(1 - \frac{K_\tau\tau_{in}L}{2T_e}\right), \quad T_e = \frac{3}{8}\left(\frac{2}{T_2'} + \frac{1}{T_1}\right)^{-1}. \tag{4.3}$$

The reduction in the energy in the case of a constant-area 2π pulse increases the pulse duration τ_p until[33,34] the pulse which has traveled a distance z_c approaches the duration of the relaxation time ($\tau_p \approx T_2'$); subsequently, the pulse is absorbed very rapidly in accordance with the incoherent interactions laws.

It should be noted that the gradual absorption of a pulse due to the relaxation processes described by Eqs. (4.1)–(4.31) can be compensated, at least to some extent, by weak focusing of a light beam in a medium.[5,16,32]

c) **Transverse structure of optical field.** According to the model considered in Secs. 2 and 3, the field is a plane wave $\mathcal{E}(z, t)$. In a real situation, there is also an additional transverse structure in the field (for example, that associated with the transverse modes of the laser pulse). We shall assume[5] that the transverse structure can be described by the dependence $\mathcal{E}(r)$, where r is the distance from the beam axis; for the sake of brevity, we shall ignore the variables z and t. If $\mathcal{E}(r)$ falls sufficiently slowly, we can still use our results (2.21) for a plane wave and introduce a parametric dependence on r. We shall assume that the transverse structure of the field at the entry to a medium is such that at the center of the beam we have $\vartheta_0(r = 0) > \pi$, with $\vartheta_0(r_c) = \pi$.

Then, in the central part of the beam ($r < r_c$), we find that a short transient stage is followed by the appearance of 2π pulses with $\vartheta(r) = 2\pi$, whereas the peripheral part of the beam ($r > r_c$) is absorbed rapidly (the process is known as pulse stripping). However, the central part of the beam is no longer homogeneous because the initial distribution $\mathcal{E}_0(r)$ gives rise to a range of pulse durations $\tau_p(r)$ and amplitudes $E(r) \propto 1/\tau_p$ of the 2π pulses. The region of the highest intensity in a pulse ($r \approx 0$) leaves behind the peripheral parts ($r \approx r_c$) so that the field profile becomes elongated.

We shall assume that the recording of a pulse involves averaging over the detector aperture r_a; then,

$$\mathcal{E}^2(t) \sim \int_0^{r_a} \frac{1}{\tau_p^2(r)}\,\mathrm{sech}^2\left(\frac{t}{\tau_p(r)} - \frac{K_\tau L}{2}\right) r\,dr. \tag{4.4}$$

If the condition $r_a \ll r_c$ is not satisfied, the recorded

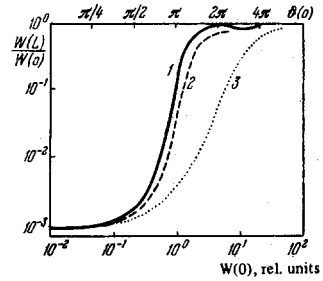

FIG. 3

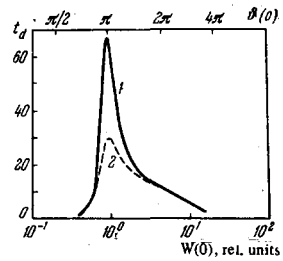

FIG. 4

FIG. 3. Transmission W(L)/W(0): 1) $T_2^* \ll \tau_p \ll T_2'$; 2) $T_2' = 2.5\,\tau_p$; 3) $T_2' \ll \tau_p$.

FIG. 4. Delay time t_d: 1) $T_2^* \ll \tau_p \ll T_2'$; 2) $T_2' = 2.5\,\tau_p$.

intensity may differ considerably from the shape of a 2π pulse.

Other deviations appear because of the dependence of the refractive index $\Pi_\tau(\omega)$ on the pulse duration $\tau_p(r)$ as given by Eq. (3.4), unless the exact resonance condition $\omega = \omega_{21}^0$ is satisfied. In this case, self-focusing or self-defocusing of the beam may occur,[5,16] depending on the sign of the difference $\Delta\omega_0 = \omega - \omega_{21}^0$.

The estimates given in Sec. 4b and the points 1–3 are confirmed by numerical integration of the system (1.16)–(1.19) making allowance for the finite values of the relaxation times T_1 and T_2', detuning from the resonance $\Delta\omega$, and presence of nonresonance losses.[16,27] This solution allows us to obtain the true transmission curve W(L)/W(O) and the delay t_d of a pulse interacting with a specified transition (Figs. 3 and 4) (see also[16,35,36]).

The influence of the relaxation is manifested by the change of the transmission curve to a monotonically rising form and by its downward shift from the total transparency level W(L)/W(O) = 1. Although the delay of a 2π pulse changes only slightly, it can decrease strongly during the transient stage (for $\vartheta_0 \gtrsim \pi$). The disappearance of oscillations in the transmission curve and of a sharp maximum in the delay curve[3)] for $\tau_p \approx T_2'$ may be used to estimate the value of T_2'. Methods suitable for more accurate measurements of the relaxation times are discussed in[26].

The results in Chaps. 2–4 apply to an inhomogeneously broadened line ($T_2^* \ll T_2'$). If a line is homogeneously broadened, the area theorem given in the form (2.10) ceases to be valid. However, calculations show[32,34,37] that, even in this case, the main properties of the self-transparency effect are retained.

5. $2\pi n$ PULSES (SPLITTING INTO SOLUTION)

It follows from the area theorem (2.10) that there are pulses which can move across a medium without a change in their area. However, we have seen in Chap. 2 that only the 2π pulses represent a steady-state solution which does not change in shape. Therefore, we have to return to the system (1.16)–(1.20) of the transient equations in order to study other solutions.

We shall first consider a simplified system,[38-42] in which there is no homogeneous broadening and the pulse frequency is equal to the transition frequency:

$$g(\Delta\omega) = \delta(\omega_{21} - \omega_{21}^0),\ \omega_{21}^0 = \omega. \tag{5.1}$$

This approximation is valid if the pulse spectrum is much wider than an inhomogeneously broadened line,

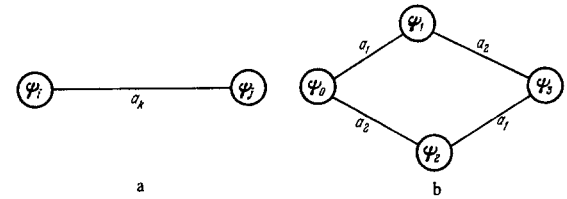

FIG. 5. a) Schematic representation of the Bäcklund transformation. b) Relationship between four particular solutions.

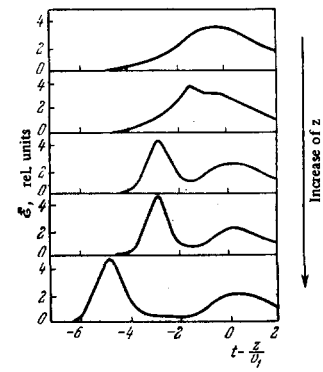

FIG. 6. Splitting of a 4π pulse into two solitons.

i.e., if $T_2^* \gg \tau_p$ (or if $T_2^* \gg \hbar/\mu\mathscr{E}$). However, in many cases, the conclusions reached employing this model differ only slightly from the results in the opposite case when a line exhibits a strong inhomogeneous broadening ($T_2^* \ll \tau_p < T_2'$). As a rule, this is due to the factorized dependence of the polarization on the frequency detuning $\Delta\omega$ given by Eq. (2.17). If the condition (5.1) is satisfied, the transient system of equations simplifies considerably and it can be reduced to a single equation for ψ [see Eq. (2.6)]

$$\frac{\partial^2\psi}{\partial\xi\,\partial\tau} = -\sin\psi, \tag{5.2}$$

where we have introduced the dimensionless variables

$$\xi = \alpha z,\ \tau = c\alpha\left(t - \frac{\eta z}{c}\right),\ \alpha^2 = \frac{2\pi\omega\mu^2 N}{\eta\hbar c^2}.$$

The nonlinear differential equation (5.2) is encountered in a differential geometry, theory of dislocations, field theory, superconductivity, physics of elementary particles, and so on.[42] There is a regular method for obtaining particular solutions, which are very useful for the understanding of the process of obtaining isolated results by numerical calculations. This method is based on the Bäcklund transformation equations, which are

$$\begin{aligned}\frac{\partial}{\partial\tau}\frac{\psi_1-\psi_0}{2} &= a\sin\frac{\psi_1+\psi_0}{2},\\ \frac{\partial}{\partial\zeta}\frac{\psi_1+\psi_0}{2} &= -\frac{1}{a}\sin\frac{\psi_1-\psi_0}{2}.\end{aligned} \tag{5.3}$$

We can show that ψ_0 and ψ_1 in the system (5.3) also satisfy Eq. (5.2). Consequently, if only the solution ψ_0 is known, we can obtain a new solution ψ_1 by the method of quadratures. In practical calculations, it is convenient to use the schematic representation shown in Fig. 5a. We can demonstrate that the four particular solutions linked as shown in Fig. 5b satisfy the relationship

$$\operatorname{tg}\frac{\psi_3-\psi_0}{4} = \frac{a_1+a_2}{a_1-a_2}\operatorname{tg}\frac{\psi_1-\psi_2}{4}. \tag{5.4}$$

A new solution ψ_3 can be obtained from Eq. (5.4) without integration provided we know the three particular solutions ψ_0, ψ_1, and ψ_2.

Since $\psi = 0$ is a solution of Eq. (5.2), it follows that if we use it as ψ_0 in Eq. (5.3), we obtain the following solution for the field

$$\mathscr{E} = \frac{2\hbar}{\mu\tau_p}\operatorname{sech}\frac{t-(z/v)}{\tau_p}, \tag{5.5}$$

where

$$\tau_p=\frac{1}{ac\alpha},\quad \frac{1}{v}-\frac{\eta}{c}=\alpha^2 c\tau_p^2.$$

In this way, we can find a 2π pulse in the absence of inhomogeneous broadening. However, a special analysis is needed[42,43] to find the stability of this solution. In order to obtain a 4π pulse, we can use Eq. (5.4) and select the three particular solutions

$$\psi_0=0,\ \operatorname{tg}\frac{\psi_i}{4}=\exp\left(a_i\tau-\frac{\zeta}{a_i}\right)\qquad (i=1,2). \tag{5.6}$$

The field $\mathcal{E}$ now becomes

$$\frac{\mathcal{E}}{A}=\left(\frac{\operatorname{sech} X_1}{\tau_1}+\frac{\operatorname{sech} X_2}{\tau_2}\right)[1-B(\operatorname{th} X_1\cdot\operatorname{th} X_2-\operatorname{sech} X_1\cdot\operatorname{sech} X_2)]^{-1}, \tag{5.7}$$

where

$$X_{1,2}=\frac{t-(z/v_{1,2})}{\tau_{1,2}},\quad A=\frac{2\hbar}{\mu}\frac{\tau_1^2-\tau_2^2}{\tau_1^2+\tau_2^2},\quad B=\frac{2\tau_1\tau_2}{\tau_1^2+\tau_2^2}; \tag{5.8}$$

the velocities v_1 and v_2 are given by the formulas

$$\frac{1}{v_{1,2}}-\frac{\eta}{c}=\alpha^2 c\tau_{1,2}^2. \tag{5.9}$$

It follows from Eq. (5.7) that over sufficiently long distances a 4π pulse splits into two 2π pulses (Fig. 6).

Results of this kind were obtained in numerical calculations based on the complete system of equations[3] and in experiments.[16,32] The physical cause of the conversion of a 4π pulse into two 2π pulses or solitons can be understood by recalling that each 2π cycle causes the vector R to rotate completely once. Therefore, the central part of a pulse is continuously interacting with atoms which have returned to the lower state and it is gradually "eaten away." The remainder of the pulse is, on the whole, amplified because of the stimulated emission until the area $\vartheta = 2\pi$ is reached. A similar analysis yields expressions for the 6π, 8π, . . . pulses.[42,44,45] In all cases, the $\vartheta = 2\pi n$ area splits into n independent 2π pulses, each moving at its own velocity. This property is a unique characteristic of the self-transparency effect, in sharp distinction to the incoherent saturation of the absorption when only the leading edge of the pulse becomes steeper at sufficiently high intensities.

The existence of steady-state 2π pulses and the splitting of these pulses into separate solitons demonstrate a relationship with the theory of nonlinear waves of different physical origin such as hydrodynamic, plasma, and other waves.[13] The agreement between many results obtained in our discussion and in these theories is due to the similarity of the equation of motion of a light pulse(5.2) and of the Korteweg-de Vries equation in the soliton theory. Under steady-state conditions, this equation is identical with the equation for the intensity $\mathcal{E}^2$ which follows from Eq. (2.18). Moreover,

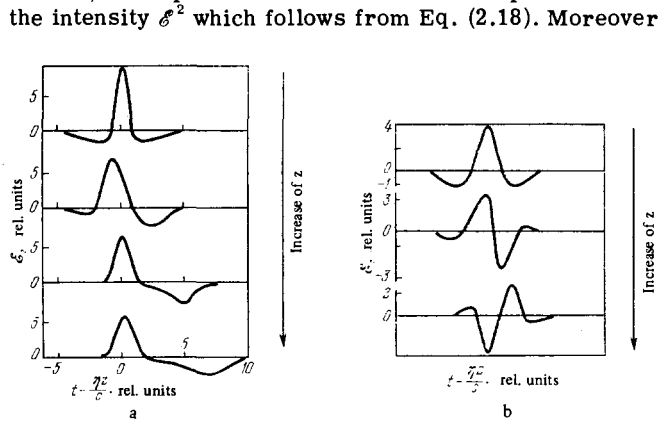

FIG. 7. Motion of splittable (a) and unsplittable (b) 0π pulses.

the optical equation (5.2) and the Korteweg-de Vries equation obey certain conservation laws,[42] which can be used to determine the amplitudes and durations of the final solitons if we know the parameters of the initial pulse, and this can be done without numerical calculations. An analysis shows that this similarity is also observed in the case of an arbitrary inhomogeneous line broadening.[42,46] The relationship between the solutions in the form of $2\pi n$ pulses and the problems in the scattering theory is discussed in[45,47].

6. 0π PULSES

Nontrivial pulses with $\vartheta = 0$, within which the field changes sign, are special among the constant-area solutions. These 0π pulses are a special case of the solution with a phase modulation. Although a discontinuity of the phase at the point where E = 0 is in formal conflict with the slowness conditions (1.10), a more detailed analysis allowing for the phase modulation[48] gives similar results. Thus, if a pulse entering a medium is characterized by a phase shift, we find that–even if $\vartheta_0 < \pi$–its energy need not be rapidly absorbed. In this case, the prediction of the area theorem ($\vartheta \to 0$) becomes ambiguous and we have to investigate the transient process for a specific initial shape of a pulse.

We shall first consider the analytic form of some of the 0π pulses in the absence of inhomogeneous broadening.[42] One of the solutions of this kind can be obtained from Eqs. (5.6)–(5.8):

$$\frac{\mathcal{E}}{A}=\left(\frac{\operatorname{sech} X_1}{\tau_1}-\frac{\operatorname{sech} X_2}{\tau_2}\right)[1-B(\operatorname{th} X_1\cdot\operatorname{th} X_2+\operatorname{sech} X_1\cdot\operatorname{sech} X_2)]^{-1}. \tag{6.1}$$

Over long distances we again obtain two independent 2π pulses but with their phases shifted by 180° relative to one another (Fig. 7a). A completely different type of 0π pulse can be obtained if we assume that the parameters a_1 and a_2 in Eq. (5.6) are complex and that $a_1 = a_2^* = a$. Then,

$$\mathcal{E}=\frac{4\hbar}{\mu\tau_p}\operatorname{sech} X_+\left(\cos X_- -\frac{\operatorname{Re} a}{\operatorname{Im} a}\sin X_-\operatorname{th} X_+\right)\times\left[1+\left(\frac{\operatorname{Re} a}{\operatorname{Im} a}\right)^2\sin^2 X_-\cdot\operatorname{sech}^2 X_+\right]^{-1}, \tag{6.2}$$

where

$$X_+=\frac{t-(z/v_+)}{\tau_p},\quad X_-=\frac{\operatorname{Im} a}{\operatorname{Re} a}\frac{t-(z/v_-)}{\tau_p},\quad \frac{1}{v_\pm}=\frac{1}{c}\left(\eta\pm\frac{1}{|a|^2}\right). \tag{6.3}$$

In contrast to the preceding case, this pulse does not split: it moves as a compact entity of duration τ_p and is subject to a considerable delay (Fig. 7b).

The results in Chaps. 5 and 6 can be generalized to the case of an arbitrary inhomogeneous broadening[46] and we can find an analytic form of the $2\pi n$ pulses (n = 0, 1, 2, . . .) which move across a resonantly absorbing medium without change in their energy.

Numerical calculations[49] which allow for the finite values of the relaxation times T_1 and T_2' demonstrate a gradual fall in the field energy. The relaxation also slows down the splitting of pulses but has hardly any effect on their delay.

Since the 0π pulses considered above are only the special case of the solutions with constant phase shifts, they do not give information on the motion of an arbitrary initial field with $\vartheta_0 = 0$. However, a numerical solution of the system (1.16)–(1.19) in the $T_2^* \ll \tau_p$ case reveals the following relationships.[29]

a) If the initial pulse consists of two parts of areas $\vartheta_1 \approx -\vartheta_2 \approx 2\pi$, the solution is a 0π pulse of the kind

that can split and is characterized by very low energy losses [see Eq. (6.1)].

b) If $\vartheta_1 \approx -\vartheta_2 = \pi$, we obtain a 0π pulse of Eq. (7.2) which cannot split and which moves as a whole subject to a considerably delay.

c) If $\vartheta_1\chi - \vartheta_2 \ll \pi$, then (in spite of the phase shift) the absorption obeys the usual exponential Beer law (2.14) because the interaction of such a pulse with a medium is linear and its spectrum is much narrower than the line width.

d) The behavior of the pulses with $\vartheta_0 \neq 0$ may depend strongly on the presence of the initial phase shifts.

We shall close this section by considering a third type of 0π pulse,[50-52] which appears when the width of the field spectrum is much greater than the line width ($\tau_p \ll T_2^* \leq T_2'$). In this case, we may expect an anomalously weak absorption even in the case of weak pulses ($\vartheta_0 \ll 1$), accompanied by the "eating away" ("burning") of a hole in a spectrum of width $\sim 1/T_2^*$. The solution for the field $\mathcal{E}$ is of the form[50]

$$\mathcal{E}(z, t)\exp(i\varphi) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \varepsilon(0, \nu)\exp\left[-i\nu\left(t - \frac{\eta}{c}z - A_\nu z\right)\right]d\nu, \quad (6.4)$$

where

$$\varepsilon(0, \nu) = \int_{-\infty}^{\infty} \mathcal{E}(0, t)\exp[i\varphi(0, t) + i\nu t]\,dt, \quad A_\nu = \alpha\alpha^2\left\langle \frac{1}{(1/T_2') + i(\Delta\omega - \nu)}\right\rangle. \quad (6.5)$$

Figure 8a gives the results of a numerical calculation based on Eq. (6.4) for an initial Gaussian field profile. We can see that the pulse becomes oscillatory. The absorption of energy (Fig. 8b) deviates strongly from the Beer law, even for $\tau_p \gtrsim T_2$.[50]

7. PHASE MODULATION EFFECTS

We shall now discuss a more general case than the phase discontinuity in a 0π pulse. It is convenient to discuss separately the transient stage and the steady-state motion of a pulse.

a) It follows from the general equations (1.16)–(1.20) that there is no phase modulation during the transient stage only if the initial pulse is unmodulated [$\varphi(0, t)$ = const] and its frequency ω is identical with the central frequency ω_{21}^0 of a symmetric inhomogeneously broadened line. On the other hand, the results for a 2π pulse without phase modulation [Eqs. (2.21) and (3.1)–(3.4)] are applicable to any frequency ω (and not just $\omega = \omega_{21}^0$). We may therefore expect that, when the carrier frequency ω of a pulse entering a medium is not in exact resonance with the transition, the field is still

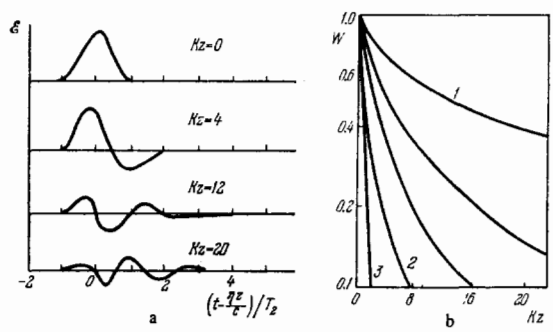

FIG. 8. Propagation of a weak 0π pulse with a wide spectrum (a) and absorption of the energy of such a pulse (b): 1) $\tau_p/T_2 = 0.4$; 2) $\tau_p/T_2 = 2$; 3) Beer law.

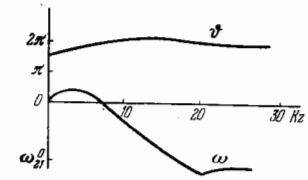

FIG. 9. Changes in the area ϑ and in the average frequency of a pulse ω with distance in the case of pulling of the pulse frequency to the central frequency of a line.

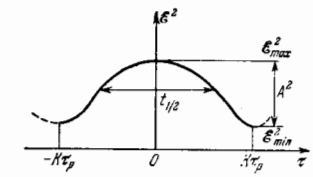

FIG. 10. One period of a steady-state solution (7.3).

transformed into a steady-state 2π pulse. However, because of the phase modulation, the carrier frequency during the transient stage may differ from its initial value ω_{in}.[53] In order to determine the resultant shift of the spectrum, we must integrate numerically the system of equations describing the transient stage. It is found that the sign and magnitude of the shift $\omega - \omega_{in}$ depend on the shape of the initial pulse, inhomogeneous line width, and so on.[53,54] Under certain conditions, the pulse frequency may be pulled toward the center of the transition line: $\omega \to \omega_{21}^0$.[53] Figure 9 shows the change in the pulse area ϑ and in the average frequency ω resulting from such pulling. If the phase modulation exists already in the initial pulse, we find that the frequency may be shifted even when it is in exact resonance with a transition ($\omega_{in} = \omega_{21}^0$).[54]

It should be pointed out that although the phase modulation can alter the spectrum of a pulse, it can only affect weakly such characteristics as the transmission and delay curves.[12,27]

b) The steady-state solution considered so far, i.e., a 2π pulse, is not phase-modulated. We shall now seek different types of steady-state solution which are also characterized by $\varphi(\tau) \neq$ const. We shall show later that they all represent not single pulses but infinite trains of pulses.[48,55-63]

Under steady-state conditions the system (1.16)–(1.20) can be used to obtain a general expression for the dispersion and phase modulation law [subject to the condition (2.17)]:

$$\varkappa - \frac{\eta\omega}{c} = -\frac{\langle\Delta\omega\chi\rangle}{\langle\chi\rangle}\left(\frac{1}{v} - \frac{\eta}{c}\right), \quad \frac{d\varphi}{d\tau} = \frac{C_1}{\mathcal{E}^2}. \quad (7.1)$$

The phase modulation constant C_1 may clearly differ from zero only for waves which are infinite along the time axis and, if $C_1 = 0$, we again have a 2π pulse of Eq. (2.21). The general equation for the field is

$$\left(\mathcal{E}\frac{d\mathcal{E}}{d\tau}\right)^2 = \frac{\mu^2}{4\hbar^2}(-\mathcal{E}^4 + M\mathcal{E}^4 + N\mathcal{E}^2 + Q). \quad (7.2)$$

The solution of Eq. (7.2) with arbitrary constants M, N, and Q can be expressed in terms of the elliptic Jacobi functions

$$\mathcal{E}(\tau) = \frac{2\hbar m}{\mu\tau_p l}\sqrt{1 - l^2\,\mathrm{sn}^2\left(\frac{\tau}{\tau_p}, m\right)}, \quad C_1^2 = \frac{16m^4\hbar^2}{\mu^2\tau_p^4 l^6}(1 - l^2)(l^2 - m^2); \quad (7.3)$$

here, sn is the elliptic sine,[64] whose period is $2K(m, \pi/2)$,[4)] where $m^2 \leq l^2 \leq 1$.

The field (7.3) is an infinite periodic pulse train. There is no phase modulation only if $l^2 = 1$ or $l^2 = m^2$.[56-58] One period of such a field is shown in Fig. 10. We can easily demonstrate that the solution is single-valued if we specify three physical parameters, which are the depth of the intensity modulation A^2,

maximum field value $\mathscr{E}^2_{max}$, and "duration" τ_p, all of which satisfy the same inequality $A \leq \mathscr{E}_{max} \leq 2\hbar/\mu\tau_p$.

Consequently, it is sufficient to control only the envelope of $\mathscr{E}$ at the entry[48] to excite this type of wave in a medium. A wave without phase modulation can also form from a step-like initial pulse.[65]

We must stress that all the steady-state solutions discussed so far are obtained on the assumption that there are no relaxation processes ($T_1 = T_2' = \infty$). A necessary condition for the existence of these solutions is, therefore, $\tau_p \ll T_2'$, i.e. each period in a train should be sufficiently short. The influence of the finite value T_2' on the evolution of such a wave has not yet been investigated and no experimental results have yet been obtained.

We shall end this section by considering the cases when the phase modulation may also appear for isolated 2π pulses.[48,52,61,66-68] This occurs, for example, if a nonresonant refractive index η depends on the field $\mathscr{E}$. For the sake of simplicity, we shall consider a Kerr nonlinearity[48] $\eta = \eta_0 + \eta_2\mathscr{E}^2$, where $\eta_2\mathscr{E}^2 \ll 1$. (A more general nonlinearity is considered in[66]). Then, if $v \ll c/\eta_0$, we obtain

$$\tau_p \frac{d\varphi}{d\tau} = \frac{3}{2}\frac{\varkappa}{K_\tau}\frac{\eta_2\mathscr{E}^2}{\eta_0}, \tag{7.4}$$

and even in the case of a weak nonlinearity ($\eta_2\mathscr{E}^2 \ll \eta_0$), we can have $\tau_p d\varphi/d\tau \gtrsim 1$, since, usually, $\varkappa \gg K_\tau$.

An additional phase modulation may also result from the influence of other levels on the interaction of a pulse with a resonant transition.[67,69]

8. DEGENERACY OF TRANSITIONS

We shall now consider the interaction between a pulse and a degenerate transition when several two-level transitions of different kinds are in resonance (within the limits of the line width) with the pulse field. It is convenient to distinguish the following two cases.

a) In the first case, the field frequency ω is in resonance with transitions between a large number of quite different levels characterized by a wide distribution of the dipole moments of the transition μ. This "accidental" degeneracy may occur if the absorption spectrum of a medium is locally very rich in lines and is effectively continuous.

b) In the second case, the upper and lower levels are degenerate in respect of the projections of the total momentum j_2 and j_1. In this case, we may be dealing with an atomic transition or a vibration-rotational transition in a molecule.[33,69] In the latter case, an additional k degeneracy may occur.[70]

We shall confine ourselves to case B; the results for case A are basically identical with the conclusions reached for B in the case of strong degeneracy $j \gg 1$.[33,71]

We shall thus assume that the incident field is in resonance with the transition whose upper and lower levels can be represented by the quantum numbers (j_2, m_2) and (j_1, m_1), where $|m_1| \leq j_1$ and $|m_2| \leq j_2$.

The following selection rules apply here:

$$j_2 = j_1 - 1, \quad j_2 = j_1, \quad j_2 = j_1 + 1, \tag{8.1}$$

which corresponds to $P(j_1)$, $Q(j_1)$, and $R(j_1)$ types of transition.

In the case of a linearly polarized pulse field eE, the projection of the total momentum along the direction e is quantized and only transitions with $m_1 = m_2 = m$ are allowed. Thus, we can consider separate subtransitions $(j_1, m) \to (j_2, m)$. If the relaxation time T_3 between the separate sublevels is sufficiently long,

$$T_3 > \tau_p, \tag{8.2}$$

these subtransitions interact completely with the field and each is characterized by its own dipole moment μ_m.

Exactly as in Chaps. 5 and 6, it is convenient to consider first a model (5.1) postulating the exact resonance and the absence of homogeneous broadening.[69] This allows us to understand qualitatively the characteristic features of a degenerate transition.

A system analogous to (1.16)–(1.19) is now of the form

$$\frac{\partial P_{2m}}{\partial t} = \frac{\mu_m^2}{\hbar}\mathscr{E}n_m, \tag{8.3}$$

$$\frac{\partial n_m}{\partial t} = -\frac{\mathscr{E}}{\hbar}P_{2m}, \tag{8.4}$$

$$\frac{\partial\mathscr{E}}{\partial z} - \frac{\eta}{c}\frac{\partial\mathscr{E}}{\partial t} = -\frac{2\pi\omega N_j}{\eta c}\sum_m P_{2m}; \tag{8.5}$$

here, the index m has $2j + 1$ values and indicates that the quantity in question refers to a subtransition with the projections $m_1 = m_2 = m$. We shall consider the specific case when $j = j_1 \leq j_2$ and the initial differences between the populations are such that $N_j = N(2j + 1)^{-1}$.

Equations (8.3)–(8.5) yield expressions which describe the changes in the area under a pulse $\vartheta_j(z)$ and in the total energy $W(z)$:

$$\frac{d\vartheta_j}{dz} = -\frac{K_j}{2}\sum_m c_m \sin(c_m\vartheta_j), \tag{8.6}$$

$$\frac{dW}{dz} = -\frac{N_j\hbar\omega}{2}\sum_m [1 - \cos(c_m\vartheta_j)]; \tag{8.7}$$

here,

$$\vartheta_j = \frac{\mu_j}{\hbar}\int_{-\infty}^{\infty}\mathscr{E}(z, t)\,dt, \quad K_j = \frac{4\pi\omega\mu_j^2 N_j T_2'}{\eta c\hbar},$$
$$\mu_j = \max_m \mu_m, \quad c_m = \frac{\mu_m}{\mu_j}, \quad 0 \leqslant c_m \leqslant 1. \tag{8.8}$$

The quantities c_m which describe the relative weight of each subtransition have the following explicit form:[70]

$$\left.\begin{aligned} &\frac{(j^2 - m^2)^{1/2}}{j} - \text{type } P(j), \\ &\frac{|m|}{j} - \text{type } Q(j), \\ &\frac{[(j+1)^2 - m^2]^{1/2}}{j+1} - \text{type } R(j). \end{aligned}\right\} \tag{8.9}$$

The right-hand sides of Eqs. (8.6)–(8.7) describe, respectively, the macroscopic polarization and the energy which remains in the medium after the passage of a pulse $\mathscr{E}$.

We first note that, for several exceptional transitions, which we shall denote by $0 \rightleftarrows 1$, $1/2 \rightleftarrows 1/2$, $1/2 \rightleftarrows 3/2$, and $1 \rightleftarrows 1$, the degeneracy has no influence on the propagation of an ultrashort pulse because only the sublevels with identical values of μ are optically coupled. In the case of these special transitions, the sums over m in Eqs. (8.6) and (8.7) reduce to a single term, as in the nondegenerate case and this leads–in particular–to the existence of 2π pulses which propagate without absorption.[5)] It follows from the properties of the solution (2.21) that this result also applies to an arbitrary inhomogeneously broadened line $g(\Delta\omega)$. The transitions which are not exceptional in the sense described above are characterized by sums over m which reduce to several terms and we can show that for all the P(j) and R(j) transitions the right-hand sides of Eqs. (8.6) and (8.7) do not have common zeros for any value $\vartheta_j \neq 0$, so that the propagation of a steady-state

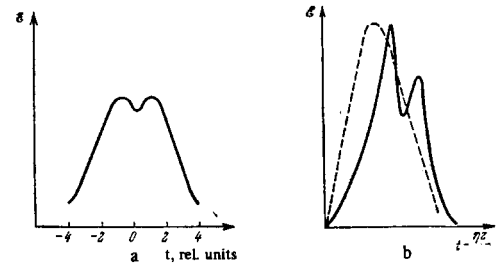

FIG. 11. Shape of a steady-state pulse for the Q(2) transition in the absence of inhomogeneous broadening (a) and the transient stage in the case of inhomogeneous broadening of the Q(2) transition (b).

pulse without resonance losses is impossible.[69]

The situation is different for the Q(j) transitions. Since the ratio of the dipole moments μ_m in Eqs. (8.8) and (8.9) is made up of integers, the right-hand sides vanish simultaneously not only for $\vartheta_j = 0$ but also for $\vartheta_j = 2\pi j$. In this case the vectors $\mathbf{R}_m$ of all the subtransitions return exactly to the initial position after the passage of a pulse.

Under steady-state conditions, the equation for a pulse becomes

$$\left(\frac{\mu_1 \mathcal{E}}{\hbar}\right)^2 = \left(\frac{d\psi}{d\tau}\right)^2 = \frac{2}{\tau_p^2} \sum_m [1 - \cos(c_m \psi)]. \qquad (8.10)$$

A pulse corresponding to j = 2 is shown in Fig. 11a. In general, if $j \gtrsim 1$, the field profile is symmetric and has j maxima (j + 1/2 in the case of half-integral values of j).[69] The nature of the oscillations can be easily understood by considering the example of j = 2 when a steady-state pulse is a 4π pulse for two subtransitions ($m = \pm 2$, $c_m = 1$). In the nondegenerate case it would seem that the pulse should split into two parts. However, the splitting process is not completed because of an "excess" pair of subtransitions ($m = \pm 1$, $c_m = 1/2$) for which the field is simply a 2π pulse.

If $\vartheta_j = 0$, Eqs. (8.6) and (8.7) can be solved for all three types of transition P(j), Q(j), and R(j). We have seen in Sec. 6 that, apart from the normal absorption, such solutions may also correspond to the formation of 0π pulses ($\vartheta_j = 0$, $W > 0$).

Since the steady-state equation (8.10) has no such solutions, these 0π pulses should then travel, keeping the area under the pulse and its energy constant but not conserving the pulse shape. These 0π pulses are naturally not identical with the analogous pulses in the case of nondegenerate transitions [see Eqs. (6.1) and (6.2)]. In fact, calculations[49] show that, for example, a 0π pulse which can undergo splitting [Eq. (6.1)] interacts with a degenerate transition, assuming rapidly the same shape as a pulse which cannot split [Eq. (6.2)] and moves subsequently, gradually losing its energy.

9. TRANSIENT PROCESSES AND ROLE OF INHOMOGENEOUS BROADENING OF DEGENERATE TRANSITIONS

We must now consider the evolution of a pulse toward its steady-state shape, given by Eq. (8.10) and discuss the influence of the inhomogeneous broadening ignored in Chap. 8.

It is convenient to consider separately the cases of small values of j and of strong degeneracy ($j \gg 1$).

a) Numerical calculations for a wide inhomogeneously broadened line indicate[33] that, in the case of small values of j, the initial pulse assumes (after the transient stage) an oscillatory form with j maxima and this form is close to the steady-state solution of Eq. (8.10). This evolution is shown in Fig. 11b for the Q(2) transition in the case when the energy of a weak pulse obeying the Beer law decreases by a factor of $\sim 10^3$ (at z = L).

b) The sibuation is quite different in the case of strongly degenerate transitions ($j \gg 1$). Here, the decisive factor is the wide range of values of the dipole moment of the transition μ. We shall consider the specific case of the Q(j) transitions but our conclusions also apply qualitatively to the P(j) and R(j) transitions and to the "accidental" degeneracy when a large number of transitions is active simultaneously (at least, in the case when $\vartheta_0 \gg 1$[33,71]). In the case of large values of j, the summation in Eqs. (8.6) and (8.7) should be replaced with integration. In the Q(j) case the equations for the area and energy of a pulse become

$$\frac{d\vartheta}{dz} = -\frac{K}{2} \frac{\sin\vartheta - \vartheta\cos\vartheta}{\vartheta^2}, \qquad (9.1)$$

$$\frac{dW}{dz} = -\frac{N\hbar\omega}{2}\left(1 - \frac{\sin\vartheta}{\vartheta}\right). \qquad (9.2)$$

Calculations indicate[33,71] that the behavior of a strongly degenerate transition varies greatly with the initial area ϑ_0 under a pulse.

1. If $\vartheta_0 \ll 1$, we have the normal linear absorption and the influence of degeneracy reduces to the appearance of an extra factor of the order of unity in the absorption coefficient.

2. If $\pi \leq \vartheta_0 \leq 1.4\pi$, i.e., in the range which contains the first unstable root of the area equation for nondegenerate transitions, the corresponding pulse travels, conserving its area but losing its energy. Since $W \sim \vartheta^2/\tau_p$, the duration of the pulse τ_p increases and this results in a delay of the field maximum.

This situation resembles that encountered in the transient stage under nondegenerate conditions, when $\vartheta_0 \approx \pi$ (Chap. 4). The similarity becomes greater if the length of the absorbing medium is not large, $KL \sim 1$, so that the energy losses do not result in a significant absorption of a pulse.[71]

3. If $\vartheta_0 \gg 1$, a strongly degenerate transition behaves quite differently from a nondegenerate transition.

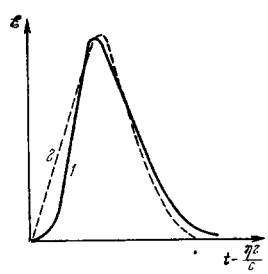

FIG. 12. Transient stage in the case of the Q(10) transition: 1) z = 0; 2) z = L.

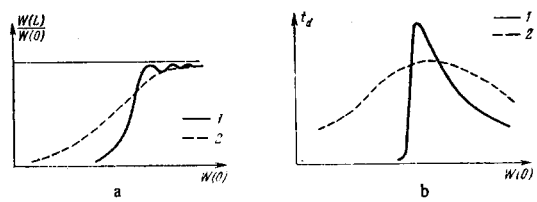

FIG. 13. Transmission (a) and delay (b) curves for nondegenerate (1) and strongly degenerate (2) transitions.

Physically, this means that after the passage of a strong pulse a medium may be in a state with zero macroscopic polarization and an energy density equal to $N\hbar\omega/2$, as in the ordinary noncoherent saturation of the absorption. This is due to the fact that, in the strongly degenerate case, there is an interference between the contributions made to the macroscopic polarization by the separate subtransitions so that, in the case of large values of ϑ, these contributions "quench" one another.[71]

Calculations show that, under these conditions, an initial pulse does not assume its steady-state shape with j maxima. Instead the leading edge becomes steeper but the rest of the pulse is practically unaffected (Fig. 12).[33] These conclusions are also qualitatively valid for any strongly degenerate transition.

The transmission and delay curves can be calculated by numerical integration of the appropriate equations, allowing for the inhomogeneous broadening and the finite nature of the relaxation time[33] (Fig. 13).

In the strongly degenerate case, the transparent state is achieved much more smoothly without oscillations at $\vartheta_0 = 2\pi, 4\pi, \ldots$, and the dealy curve is much broader. On the other hand, in both cases, the region of maximum delay ($\vartheta_0 \approx \pi$) coincides with an inflection in the transmission curve.

It should be stressed that the delay time t_d for a strongly degenerate transition is entirely due to an increase in the pulse duration and not a reduction in its velocity.

All the results derived in Chaps. 8 and 9 are subject to the following comment. Our conclusions are based essentially on the condition (8.2), which allows us to consider separate subtransitions as independent of one another. However, we can have a situation[72] when the opposite inequality is true, i.e., when the relaxation between degenerate sublevels is capable of maintaining the same populations of these levels. Under these conditions, the coherence of the interaction of each separate transition with the field breaks down. However, we can show that the transition as a whole interacts coherently with the field (provided $\tau_p < T_2'$) and the dipole moment of the transition is[73] $\mu\sqrt{\sum_m \mu_m^2}$. Then, irrespective of the values of j_2 and j_1, the transition behaves as "effectively nondegenerate" and the results obtained for a two-level transition are still valid. In particular, steady-state 2π pulses should form, the strong field should split into solitons, and so on.

In the "accidental" degeneracy case, there are no such equal populations of the levels and the transition always behaves as strongly degenerate. This can sometimes be used to determine the nature of the degeneracy of a given transition.[72]

10. INFLUENCE OF STATIC FIELDS

We shall now consider the influence of static (electric or magnetic) fields on the self-induced transparency in the case of a degenerate transition. In general, a Stark or Zeeman shift of the sublevels is approximately equivalent to an inhomogeneous broadening of a transition and, consequently, it simply introduces additional losses. However, an important exception to this rule is represented by the $1/2 \rightleftarrows 1/2$, $1/2 \rightleftarrows 3/2$, $0 \rightleftarrows 1$, and $1 \rightarrow 1$ transitions, which behave as nondegenerate in the absence of an additional field. We shall show later that the application of a static field

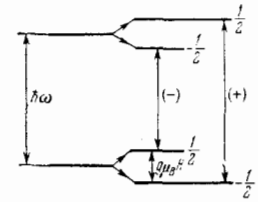

FIG. 14. Scheme of the $1/2 \rightarrow 1/2$ transition.

makes the characteristics of steady-state pulses interacting with such transitions dependent on the field and on the Stark or Landé constants of the levels involved.

a) **Magnetic fields.** If a static magnetic field H_c is aligned longitudinally with respect to the direction of propagation of light, we find from the theory of the Zeeman effect[70] that a line splits into a series of σ components corresponding to the subtransitions with $\Delta m = \pm 1$ and all these components should be circularly polarized. If the initial pulse is linearly polarized, it can be represented by two superimposed circularly polarized (+) and (−) pulses.

However, it should be noted that these pulses now interact with transitions of different frequencies because, as expected the laws of dispersion and velocity may be different for such pulses. The difference between the laws of dispersion corresponds to the well-known Faraday rotation of the plane of polarization and the difference between the velocities gives rise to a gradual spatial separation of the (+) and (−) pulses.

In the specific case of the $1/2 \rightarrow 1/2$ transition, one of the pulses (−) interacts with the subtransition $\Delta m = -1$ and the other (+) interacts with the subtransition $\Delta m = 1$. It is assumed that $\omega = \omega_{21}^0$ so that the shift away from the resonance condition is symmetric (Fig. 14). When both pulses become 2π pulses, their dispersion laws are described by

$$\varkappa_\pm(\omega) = \frac{\omega}{c}\Pi_\tau^\pm(\omega) = \frac{\omega}{c}\left(\eta - \frac{2\pi\mu^2 N}{\eta\hbar}\left\langle\frac{\Delta\omega}{\Delta\omega^2+\tau_p^{-2}}\right\rangle_\pm\right). \qquad (10.1)$$

The averaging $\langle \ldots \rangle_\pm$ denotes integration over inhomogeneously broadened lines $g_\pm(\Delta\omega)$ and the centers of these lines $\omega_{21}^\pm$ are shifted relative to one another by geH_c/mc (g is the Landé factor).

Expressed differently, Eq. (10.1) represents the rotation of the plane of polarization through an angle $A = (\varkappa_+ - \varkappa_-)z$ and, in the first order with respect to H_c, we obtain

$$\frac{dA}{dz} = V(H_c), \qquad (10.2)$$

where the Verdet constant is of the form[28]

$$V = 4ge\frac{\pi\omega\mu^2\tau_p^2 N}{\eta mc^2}\left\langle\frac{1-\Delta\omega^2\tau_p^2}{(1+\Delta\omega^2\tau_p^2)^2}\right\rangle; \qquad (10.3)$$

the averaging $\langle \ldots \rangle$ is carried out over an "unperturbed" inhomogeneously broadened line $g(\Delta\omega)$ with its center at $\omega_{21}^0 = \omega$. An estimate obtained in[28] on the basis of Eq. (10.3) for the parameters typical of a resonant transition in potassium vapor gives an anomalously large Verdet constant (1.9 rad.cm^{-1}.Oe^{-1}).

This "giant" rotation is exactly equal[74] to the normal Faraday rotation calculated using the linear dispersion theory for a spectral line whose decay constant is $\Gamma = 1/\tau_p$ because the dispersion law (10.1) is the same as in the case of a weak field.

It is important to note that only under the conditions corresponding to the self-transparency effect, when a 2π pulse can move steadily across a medium characterized by strong dispersion and absorption, does this classical formula have its full meaning (see also[75]).

If frequency pulling takes place, the angle of rotation of the plane of polarization A does not remain constant but "moves"[74] at a phase velocity c/η:

$$A=\frac{\eta}{c}(\omega_{21}^{+}-\omega_{21}^{-})\left(z-\frac{c}{\eta}t\right). \qquad (10.4)$$

The results obtained for the $0 \rightleftarrows 1$ and $1 \rightarrow 1$ transitions differ little from the case $1/2 \rightarrow 1/2$ just considered (see also[76]).

Strictly speaking, all these results are valid only as long as the (+) and (−) pulses are spatially coincident. Over long distances, the nature of the polarization varies along the field profile in accordance with the following sequence:[5] circular (+), elliptic, linear, elliptic, linear, elliptic, circular (−). Later, two spatially separate 2π pulses, (+) and (−), form and move at different velocities $v_{\pm}$.[5,75]

b) **Electric field.** We shall consider the case when a static field E_c is oriented parallel to the polarization vector of the wave field $\mathbf{e}$.[77] In this case, a pulse interacts with the subtransitions $\Delta m = 0$, whose frequencies depend on the value of E_c. The shift of the energy levels is described by the well-known formula for the quadratic Stark effect[70]

$$\Delta\varepsilon_n=-\frac{E_c^2}{2}\left\{\alpha_n+2\beta_n\left[m^2-\frac{j(j+1)}{3}\right]\right\}. \qquad (10.5)$$

Since the electric field does not lift the degeneracy of the sign of m, it follows that the transitions $0 \rightleftarrows 1$, $1/2 \rightleftarrows 1/2$, $1/2 \rightleftarrows 3/2$, and $1 \rightarrow 1$ can still be described by the results for the nondegenerate case but the velocity v and nature of the dispersion law $\kappa(\omega)$ of a 2π pulse now depend on the detuning $\Delta\omega_0(E_c) = \omega_{21} - \omega_{21}^0$. For the sake of simplicity, we shall give the formulas only for the case when, in the absence of the field E_c, the condition (5.1) is satisfied:

$$\begin{aligned}&\frac{1}{v}-\frac{\eta}{c}=\frac{2\pi\mu^2\omega\tau_p^0 N\Gamma_{j_1j_2}}{\eta c\hbar(1+\Delta\omega_0^2\tau_p^2)},\\ &\varkappa-\frac{\eta\omega}{c}=-\Delta\omega_0\left(\frac{1}{v}-\frac{\eta}{c}\right),\end{aligned} \qquad (10.6)$$

where $\Delta\omega_0 = (\mathscr{E}^2/2\hbar)r_{j_1j_2}$, and the values of the parameters $r_{j_1j_2}$ and $\Gamma_{j_1j_2}$ are listed in the table below

$j_1 \rightarrow j_2$	$\frac{1}{2}\rightarrow\frac{1}{2}$	$1\rightarrow 1$	$1\rightarrow 0$	$0\rightarrow 1$	$\frac{1}{2}\rightarrow\frac{3}{2}$	$\frac{3}{2}\rightarrow\frac{1}{2}$
$r_{j_1j_2}$	$\alpha_2-\alpha_1$	$(\alpha_2-\alpha_1)+\frac{2}{3}(\beta_2-\beta_1)$	$(\alpha_2-\alpha_1)-\frac{2}{3}\beta_1$	$(\alpha_2-\alpha_1)+\frac{2}{3}\beta_2$	$(\alpha_2-\alpha_1)-2\beta_2$	$(\alpha_2-\alpha_1)+2\beta_1$
$\Gamma_{j_1j_2}$	1	$\frac{2}{3}$	$\frac{1}{3}$	1	1	$\frac{1}{2}$

11. POSSIBLE SELF-INDUCED TRANSPARENCY IN SEMICONDUCTORS

Several coherent effects may appear when ultrashort light pulses interact with semiconductor crystals. The nature of these effects depends on the energy band structure, nature of optical transitions, degree of doping of the semiconductor crystal, and other parameters.

We shall consider particularly the case when the photon energy is less than the forbidden band width $\hbar\omega < \Delta$. In the case of weak pulses, a strong plasma reflection[78] is expected in the range

$$\omega < \Omega \qquad (11.1)$$

where $\Omega = (4\pi e^2 N_c/\eta^2 m_c)^{1/2}$ is the plasma frequency, N_c is the density of electrons in the conduction band, and m_c is their effective mass. However, in the case of semiconductors with a nonparabolic dispersion law, the value of Ω may depend strongly on the incident radiation intensity.

It is shown in[79] that a high-power light pulse of a shape close to that given in Eq. (2.2) may cross a semiconductor such as PbTe or InSb in the plasma reflection region defined by Eq. (11.1).

A close analog of the self-transparency occurs[80] in the case of direct interband transitions when $\hbar\omega > \Delta$. The absorption of monochromatic radiation in such transitions can be described formally as in the case of an inhomogeneously broadened two-level transition. In particular, the absorption coefficient in weak fields is of the form[81]

$$K_{\Gamma}(\omega)=\frac{4\pi\omega\Gamma}{\eta c\hbar}\sum_{\mathbf{k}}\frac{\mu_{\mathbf{k}}^2 n_{\mathbf{k}}}{\Delta\omega_{\mathbf{k}}^2+\Gamma^2}; \qquad (11.2)$$

here,

$$\Delta\omega_{\mathbf{k}}=\omega-\frac{\Delta+\varepsilon_c(\mathbf{k})+\varepsilon_v(\mathbf{k})}{\hbar}, \quad \varkappa \ll k, \qquad (11.3)$$

and, by analogy with the two-level transitions, we have introduced $\mu_{\mathbf{k}}$, which is the dipole moment of a transition between Bloch states in the valence band $\psi_v(\mathbf{k})$ and in the conduction band $\psi_c(\mathbf{k})$; $n_{\mathbf{k}}$ is the difference between the populations of these states; $\epsilon_c(\mathbf{k})$ and $\epsilon_v(\mathbf{k})$ are the dispersion laws of the bands; Γ is the rate of decay of the interband polarization due to electron-electron and electron-phonon collisions. The law of conservation of the quasi-momentum $\mathbf{k}$ couples optically the pairs of states $\psi_c(\mathbf{k})$ and $\psi_v(\mathbf{k})$ in the conduction and valence bands. The expression (11.2) differs from the formula for the absorption coefficient in the case of a two-level transition because the averaging over the profile of an inhomogeneously broadened line is now replaced with the summation over the quasicontinuous spectrum in the energy bands. However, there is one fundamental difference: the two-level atoms which absorb light are basically localized, whereas the electron states in pure semiconductors are smeared over the whole crystal because the probability of finding an electron at a given point ($|\psi_c(\mathbf{k})|^2$, $|\psi_v(\mathbf{k})|^2$) is a periodic function of the coordinates and the period corresponds to the crystal lattice parameter.

An ultrashort light pulse interacts coherently with a semiconductor if

a) the pulse duration τ_p is less than any of the relaxation times, which include the spontaneous recombination time T_1, polarization decay time T_2' electron-electron collision time T_{ee}, and electron-phonon collision time T_{ep};

b) the carrier mobility is incapable of disturbing the coherent coupling between the field and the polarization produced by it.

The first condition is clearly analogous to the criterion $\tau_p < T_2'$ in the case of a two-level transition and the second is entirely due to the presence of the long-range order in a semiconductor crystal.

In deriving equations for the coherent interaction between a light pulse and a semiconductor crystal, we can conveniently replace the Bloch functions with the localized Wannier functions.[82] The two-band Schrödinger equation can then be reduced to

$$i\hbar\frac{\partial}{\partial t}f_{ck}(\mathbf{R},t)=\varepsilon_c(-i\nabla)f_{ck}-\mu_{\mathbf{k}}Ef_{vk}, \qquad (11.4)$$

$$i\hbar\frac{\partial}{\partial t}f_{vk}(\mathbf{R},t)=\varepsilon_v(-i\nabla)f_{vk}-\mu_{\mathbf{k}}Ef_{ck}; \qquad (11.5)$$

here f_{ck} and f_{vk} are the amplitudes of the probability that at a moment t an electron is in the Wannier state

ψ_{cR}, ψ_{vR} if this electron is in the Bloch state $\psi_v(\mathbf{k})$ before the application of the field. The initial conditions for Eqs. (11.4) and (11.5) are

$$f_{ck}(\mathbf{R}, -\infty) = 0, \quad f_{vk}(\mathbf{R}, -\infty) = e^{-i\mathbf{kR}}. \tag{11.6}$$

We can show that the second of the above conditions can be satisfied if the "displacement" of a carrier during a pulse τ_p is small compared with the "length" of the pulse $l_p = v\tau_p$. In the parabolic band case, the latter condition becomes

$$\frac{\hbar k_0}{m_{cv}} \ll v, \tag{11.7}$$

where

$$\frac{1}{m_{cv}} = \frac{1}{m_c} + \frac{1}{m_v}, \quad \frac{\hbar^2 k_0^2}{2m_{cv}} = \hbar\omega - \Delta. \tag{11.8}$$

If the condition (11.7) is satisfied, the differential operators in Eqs. (11.4) and (11.5) can be replaced with the c number $\epsilon_c(\mathbf{k}) - \Delta - \epsilon_v(\mathbf{k})$, and the equations for the field and medium are then easily reduced to the form

$$\left.\begin{aligned}
\frac{\partial P_{1\mathbf{k}}}{\partial t} &= -\left(\Delta\omega_{\mathbf{k}} + \frac{\partial\varphi}{\partial t}\right) P_{2\mathbf{k}}, \\
\frac{\partial P_{2\mathbf{k}}}{\partial t} &= \left(\Delta\omega_{\mathbf{k}} + \frac{\partial\varphi}{\partial t}\right) P_{1\mathbf{k}} + \frac{\mu_{\mathbf{k}}^2}{\hbar}\mathcal{E} n_{\mathbf{k}}, \\
\frac{\partial n_{\mathbf{k}}}{\partial t} &= -\frac{\mathcal{E}}{\hbar} P_{2\mathbf{k}}, \\
\frac{\partial\mathcal{E}}{\partial z} + \frac{\eta}{c}\frac{\partial\mathcal{E}}{\partial t} &= -\frac{2\pi\omega}{\eta c}\sum_{\mathbf{k}} P_{2\mathbf{k}}, \\
-\left(\varkappa - \frac{\eta\omega}{c}\right)\mathcal{E} + \left(\frac{\partial\varphi}{\partial z} + \frac{\eta}{c}\frac{\partial\varphi}{\partial t}\right)\mathcal{E} &= -\frac{2\pi\omega}{\eta c}\sum_{\mathbf{k}} P_{1\mathbf{k}};
\end{aligned}\right\} \tag{11.9}$$

here, P_{1k} and P_{2k} are the reactive and active components of the polarization for the $ck \rightarrow vk$ transition; η is the nonresonant part of the refractive index of the semiconductor (considered ignoring the contributions of the conduction and valence bands). The interband dipole moment of an allowed transition can be approximated satisfactorily by assuming that $\mu_k = \mu$.

In this case, the system (11.9) is of the same form as the system for a two-level transition (1.16)–(1.20), except that the averaging over an inhomogeneously broadened line is replaced by the summation of the values of the wave vector $\mathbf{k}$ in the first Brillouin zone.

In particular, the "area theorem" is valid:

$$\frac{d\vartheta}{dz} = -\frac{K_s}{2}\sin\vartheta, \tag{11.10}$$

where

$$K_s = \frac{\mu^2\omega}{4\pi\eta c\hbar}\int n_{c\mathbf{k}}\delta(\Delta\omega_{\mathbf{k}})\,d^3\mathbf{k}. \tag{11.11}$$

If $h\omega - \Delta \gg \hbar\Gamma$, we find that $K_S(\omega) \approx K_\Gamma(\omega)$ [see Eq. (11.2)].

Since the absorption coefficient of direct interband transitions can be very large ($\sim 10^4$ cm^{-1}), the transient stage extends over much shorter distances than in media with two-level transitions. Moreover, since the density of states varies very slowly over the width of the pulse spectrum, effects resembling the weak 0π pulses of Chap. 6 cannot occur in a semiconductor.

A 2π pulse in a semiconductor has its previous form (2.21). The velocity of this pulse and the dispersion law are

$$\left.\begin{aligned}
\frac{1}{v} - \frac{\Pi}{c} &= \frac{2\pi\mu^2\omega^2\tau_p^2}{\eta c\hbar}\sum_{\mathbf{k}}(1 + \Delta\omega_{\mathbf{k}}^2\tau_p^2)^{-1} \approx \frac{K_\Gamma\tau_p}{2}, \\
\varkappa &= \frac{\omega}{c}\Pi(\omega),
\end{aligned}\right\} \tag{11.12}$$

where $\Pi(\omega)$ is identical (if $\hbar\omega - \Delta \ll \hbar/\tau_p$) with the ordinary refractive index of a semiconductor.

Estimates based on the system (11.12) show that, if $K_\Gamma = 10^4$ cm^{-1} and $\tau_p = 5 \times 10^{-12}$ sec, the velocity of

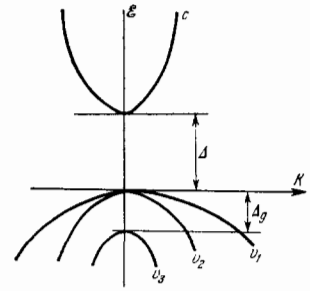

FIG. 15. Energy band structure of a semiconductor.

a 2π pulse is three orders of magnitude lower than the velocity of light and the length of such a pulse is three orders of magnitude shorter than the length of a pulse in air. This "miniaturization" effect of the self-induced transparency may find applications in optoelectronics.

If the dipole matrix element near a band edge can be described by $\mu_k \propto \mathbf{k}\cdot\mathbf{e}$, a light pulse interacts with a semiconductor in exactly the same way as with a strongly degenerate two-level transition (Chap. 9).

The results obtained for a two-band model of a semiconductor are easily modified to allow for the real band structure such as that of the III–V compounds (Fig. 15). If $\Delta < \hbar\omega < \Delta + \Delta_g$, transitions take place between the conduction band and the light-hole (v_2) and heavy-hole (v_1) bands.[78] The laws of conservation of energy and quasimomentum "burn" two inhomogeneously broadened transitions in the energy spectrum and these transitions are separated by

$$\delta = (\hbar\omega - \Delta)\frac{m_{cv_1} - m_{cv_2}}{m_c}.$$

If the condition $|\sigma| \gg \hbar/\tau_p$ is obeyed, a 2π pulse has the same form as in the case of two-level transitions but the expressions (11.11) and (11.12) for the velocity of a pulse and for the absorption coefficient should be summed over v_2 and v_1. The contribution of the heavy-hole band is m_{cv_2}/m_{cv_1} times greater than that of the light-hole band.

We shall now consider the influence of the doping of a crystal. The energy states in a doped semiconductor containing N_i donor or acceptor impurities are characterized by a dimensionless parameter $s = a_B N_i^{1/3}$, where a_B is the Bohr radius of an impurity center. In the case of low impurity concentrations ($s \ll 1$), the energy band structure is practically the same as that of an undoped semiconductor but an additional impurity scattering of carriers with a characteristic time T_i has to be allowed for. If the duration of a light pulse satisfies the condition $\tau_p < T_i$, such scattering does not have sufficient time to disturb the coherence of the interaction with the field.

In the heavy doping case ($s > 1$), the very concept of the dispersion law loses its meaning.[83] The density of states changes considerably and tails appear in the forbidden band. The presence of a large number of randomly distributed impurity centers can be described by introducing a fluctuation potential U which acts on electrons in the semiconductor. It is usual to employ the bent-band approximation,[83] assuming that the bottom of the conduction band and the top of the valence band simply shift in accordance with the form of U(R).

The equations for the polarization and population difference can be written formally in their previous form (11.9) except for the substitution $\Delta\omega_k \rightarrow \Delta\omega_k(U)$,

whereas the field equation has to be averaged over the impurities:

$$\frac{\partial \mathcal{E}}{\partial z}+\frac{\eta}{c}\frac{\partial \mathcal{E}}{\partial t}=-\frac{2\pi\omega}{\eta c}\int B(U)\sum_{k}P_{2k}(U)\,dU; \qquad (11.13)$$

here B(U) is the distribution of the fluctuation potential; $\Delta\omega_k(U) = \Delta\omega_k + \Delta\epsilon_{vk}(U) + \Delta\epsilon_{ck}(U)$; $\Delta\epsilon_{ck}$ and $\Delta\epsilon_{vk}$ are the displacements of the energy states.

It is important to note that, in general, $\Delta\epsilon_{ck}(U) \neq \Delta\epsilon_{vk}(U)$ because of the localization effect[84] the result of which is that a potential well U of width Γ_0 does not contain bound states if $U < \hbar^2/m\Gamma_0^2$. If the mass of a hole m_v is much greater than the mass of an electron m_c, we can use the approximation

$$\Delta\omega_k(U)=\begin{cases}\Delta\omega_k+U, & U_v<U<U_c,\\ \Delta\omega_k, & U<U_v,\ U>U_c,\end{cases}\qquad U_{c,v}=\frac{\hbar^2}{m_{c,v}\Gamma_0^2}. \qquad (11.14)$$

This approximation works well if $U_v < \gamma < U_c$ (γ is the rms value of the potential U and Γ_0 is the Debye screening radius), which is true of several heavily doped semiconductors.[85]

We can now use Eqs. (11.9), (11.13), and (11.14) to obtain the area theorem

$$\frac{d\vartheta}{dz}=-\frac{K_i}{2}\sin\vartheta, \qquad (11.15)$$

where

$$K_i=\frac{2\pi^2\mu^2\omega}{\eta c}\sum_{k}n_{0k}\int_{U_v}^{U_c}B(U)\,\delta(\Delta\omega_k+U)\,dU,$$

and n_{0k} is the initial Fermi difference between the populations in the band. We can easily show that K_i is identical with the usual absorption coefficient of transitions between density-of-states tails of a heavily doped semiconductor.[86]

12. MEDIA WITH TWO-PHOTON RESONANCE TRANSITIONS

Some coherent effects may also occur during the propagation of an ultrashort light pulse through a medium exhibiting the two-photon resonance.[87-89] It is assumed[90] that the atomic spectrum includes levels ϵ_1 and ϵ_2 such that $\omega_{21} = (\epsilon_2 - \epsilon_1)/\hbar \approx 2\omega$.

The equations for the difference between the level populations and the polarization in the case of an ultrashort pulse ($\tau_p < T_2'$) can be reduced to

$$\left.\begin{aligned}\frac{\partial P_1}{\partial t}&=-\left(\Delta\omega+2\frac{\partial\varphi}{\partial t}+\frac{r_{22}-r_{11}}{4\hbar}\mathcal{E}^2\right)P_2,\\ \frac{\partial P_2}{\partial t}&=\left(\Delta\omega+2\frac{\partial\varphi}{\partial t}+\frac{r_{22}-r_{11}}{4\hbar}\mathcal{E}^2\right)P_1+\frac{|r_{21}|^2}{2\hbar}\mathcal{E}n,\\ \frac{\partial n}{\partial t}&=-\frac{\mathcal{E}^2}{2\hbar}P_2,\end{aligned}\right\} \qquad (12.1)$$

where

$$r_{21}=r_{12}^*=\sum_{m\geq 3}\frac{\mu_{1m}\mu_{m2}}{\hbar(\omega_{m2}+\omega)},\qquad r_{11}=\frac{2}{\hbar}\sum_{m\geq 3}\frac{|\mu_{1m}|^2\omega_{m1}}{\omega_{m1}^2-\omega^2}$$

Bearing in mind that the resonance frequencies ω_{21} of atoms are distributed in accordance with the profile of an inhomogeneously broadened line $g(\Delta\omega)$ ($\Delta\omega = 2\omega - \omega_{21}$), we find that the slow field amplitude and phase are given by

$$\frac{\partial \mathcal{E}}{\partial z}+\frac{\eta}{c}\frac{\partial \mathcal{E}}{\partial t}=-\frac{2\pi\omega N}{\eta c}\mathcal{E}\langle P_2\rangle, \qquad (12.2)$$

$$-\left(\varkappa-\frac{\omega\eta}{c}\right)+\left(\frac{\partial\varphi}{\partial z}+\frac{\eta}{c}\frac{\partial\varphi}{\partial t}\right)=-\frac{2\pi\omega N}{\eta c}\left\langle P_1-\frac{r_{22}-r_{11}}{2}(n-1)\right\rangle. \qquad (12.3)$$

Under exact resonance conditions, $[g(\Delta\omega) = \delta(\Delta\omega)]$, Eqs. (12.1)–(12.3) readily yield an expression which describes the evolution of a pulse:

$$\mathcal{E}^2=\frac{\mathcal{E}^2(0,\ t-(\eta z/c))}{1+2K_2z\{\sin\psi(0,\ t-(\eta z/c))+K_2z[1-\cos\chi(0,\ t-(\eta z/c))]\}}, \qquad (12.4)$$

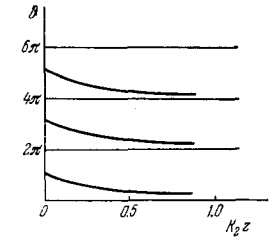

FIG. 16. Energy theorem under two-photon resonance conditions.

where

$$\psi=\frac{|\tilde{r}_{21}|^2}{2\hbar}\int_{-\infty}^{t}\mathcal{E}^2(z,\ t_1)\,dt_1,\quad K_2=\frac{2\pi\omega N|r_{21}|^2}{\eta c(\tilde{r}_{21})},\quad |\tilde{r}_{21}|^2=|r_{21}|^2+\frac{(r_{22}-r_{11})^2}{4}.$$

The quantity $\vartheta(z) = \psi(z, \infty)$ is directly proportional to the total energy of a pulse. We can show that

$$\operatorname{ctg}\frac{\vartheta}{2}=\operatorname{ctg}\frac{\vartheta_0}{2}+2K_2z,\quad \vartheta_0=\vartheta(0). \qquad (12.5)$$

The solution of Eq. (12.5), which we can call the energy theorem by analogy with Eqs. (2.10) and (2.15), shows that the initial energy ϑ of a pulse is absorbed, tending to a stable value $\vartheta = 2\pi n$ (n = 0, 1, . . .), as shown in Fig. 16. A steady-state 2π pulse has the Lorentzian profile:

$$\mathcal{E}^2=\frac{4\hbar}{|\tilde{r}_{21}|\tau_p}\left(1+\frac{\tau^2}{\tau_p^2}\right)^{-1},\quad \tau=t-\frac{z}{v}. \qquad (12.6)$$

The pulse velocity v is related to its duration τ_p by

$$\frac{1}{v}-\frac{\eta}{c}=2K_2\tau_p. \qquad (12.7)$$

Other steady-state solutions represent an infinite train of 2π pulses:

$$\mathcal{E}^2=\frac{2\hbar}{|\tilde{r}_{21}|\tau_p}(C^2-1)\left(C+\sin\frac{\tau\sqrt{C^2-1}}{\tau_p}\right)^{-1},\quad C>1. \qquad (12.8)$$

In the case of detuning from resonance, ($\Delta\omega_0 = 2\omega_{in} - \omega_{21}^0 \neq 0$), the phase modulation alters the initial frequency ω_{in}:[88]

$$2\frac{\partial\varphi}{\partial t}+\Delta\omega_0=\left[2\frac{\partial}{\partial t}\varphi\left(0,\ t-\frac{\eta z}{c}\right)+\Delta\omega_0\right]\frac{\mathcal{E}^2(0,\ t-(\eta z/c))}{\mathcal{E}^2(z,\ t)}. \qquad (12.9)$$

As long as the detuning $\Delta\omega_0$ is not too large, such a distortion of the spectrum has little influence on the evolution of a pulse. Therefore, if $4\pi > \vartheta_0 > 2\pi$, the transient stage, whose duration now depends on $\Delta\omega_0$, is followed by the formation of a quasisteady 2π pulse which is described approximately by Eqs. (12.6) and (12.7). A delay ($v < c/\eta$) results in an effective pulling of the pulse frequency toward the central frequency ω_{21}^0 (this happens near the maximum of the pulse) and the spectrum becomes asymmetric:

$$2\omega=2\left(\omega_{in}+\frac{\partial\varphi}{\partial t}\right)\to\omega_{21}^0. \qquad (12.10)$$

Exact solutions of Eqs. (12.1)–(12.3) are not known for the inhomogeneous broadening case. However, we may expect the broadening to result in additional losses of the pulse energy, even in the case when $2\omega_{in} = \omega_{21}^2$.

Two-photon coherent effects can also occur in the presence of two pulses[89] of frequencies $\omega_1 + \omega_2 \approx \omega_{21}^0$, in which case one of the pulses can change with time because of a change in the intensity of the other pulse. Moreover, a resonant two-photon transition may compete with a nonresonant one-photon transition and, under these conditions, the second harmonic may be generated in a centrosymmetric crystal.[91]

The interaction between an ultrashort light pulse and a semiconductor under two-photon resonance conditions $\Delta/2 < \hbar\omega < \Delta$ may, under certain conditions, result in coherent saturation of the absorption at much lower values of the field than in the conventional saturation case.[92,93]

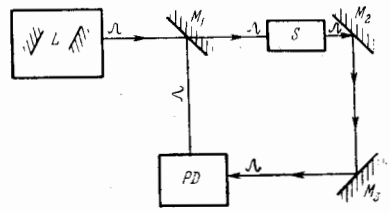

FIG. 17. Simplified schematic diagram of the apparatus used in studies of the self-induced transparency: L is a laser system; M_1-M_3 are mirrors; S is an absorbing medium; PD is a photodetector.

13. SUMMARY OF EXPERIMENTAL RESULTS

Over twenty experiments have been carried out on the self-transparency effect. On the whole, the results obtained indicate that the remarkable properties of the coherent interaction between light pulses and matter, described above, do indeed occur in real materials but a quantitative agreement with the experimental results can be obtained only if we allow for the relaxation processes, degeneracy of the transitions, transverse structure of the field, time profiles of the pulses entering a medium and their phase modulation, profile of an inhomogeneously broadened line, etc. However, the majority of the experimental results can be understood qualitatively even on the basis of the simple McCall-Hahn model (Secs. 1 and 2). These include a very strong drop in the absorption when the pulse intensity exceeds the threshold value and a considerable delay in a medium which depends strongly on the initial field intensity. The condition $\vartheta_0 = \pi$ of Eq. (2.15) easily yields the threshold value of the optical flux necessary for the observation of the self-transparency effect:

$$Q_{th} \sim \frac{\pi \eta c}{4} \left(\frac{\hbar}{\mu T_2'} \right)^2. \qquad (13.1)$$

Hence, it follows that the effect should be observed more easily in a medium with a large dipole moment of the transition μ and a long relaxation time T_2'.

The experimentally determined thresholds Q_{th} are in good agreement with the estimate represented by Eq. (13.1) and lie in a wide range from ~1 W/cm^2 for SF_6 to ~10^2 kW/cm^2 for ruby and ~10^2 MW/cm^2 for semiconductors. A common feature of all the investigations is the use of Q-switched lasers or tunable dye lasers. The duration of laser pulses is usually in the range 1–300 nsec (~1 nsec in the case of semiconductors).

A simplified schematic diagram of the apparatus used is shown in Fig. 17.

The media in which the self-transparency effect has been observed so far can be divided into four main categories: solid dielectrics, alkali metal vapors, molecular gases, and semiconductors.

a) **Solid dielectrics**. The self-transparency effect was first observed experimentally in a ruby crystal in the fundamental papers of McCall and Hahn.[4,5] Since then, the details of the effect in ruby have been studied in detail.[35,36,94] The radiation used in these experiments was generated by a Q-switched laser operating at 77°K. The pulses were of 10–30 nsec duration and passed through a resonant absorber in the form of a ruby crystal cooled with liquid helium to T = 4–60°K. The laser transition frequency was close to the frequency of the corresponding transition in the cooled ruby crystal.

The relaxation time at 4°K was fairly long (T_2 ~ ~ 50–80 nsec)[94] and the pulses could interact coherently with the investigated ruby crystal. The inhomogeneously broadened line was considerably wider than the pulse spectrum ($T_2^* \sim 0.03$ nsec). The self-induced transparency was observed when the optical flux reached the threshold $Q_{th} \sim 100$ kW/cm^2 and at this threshold the absorption fell by a factor of up to ~10^5. The delay of the pulses in the ruby crystal represented a reduction in the velocity down to $10^{-2}c/\eta$. A systematic study of the transmission and delay curves and of the influence of temperature on these curves demonstrated good agreement with the McCall-Hahn theory subject to allowance for the transverse structure of the field (Sec. 4). The high threshold flux and the need to employ liquid helium were attributed to the small dipole moment of the transition ($\mu \sim 5 \times 10^{-21}$ cgs esu) and the relatively fast relaxation processes in ruby.

b) **Alkali metal vapors**. At low concentrations ($N \sim 10^{11}$–10^{12} cm^{-3}) of atoms, a pulse of $\tau_p \sim 1$–10 nsec duration can interact coherently with such vapors. The large dipole moments of the transitions in alkali metal atoms ($\mu \sim 10^{-18}$ cgs esu) should make it possible to observe the self-transparency effect at low threshold fluxes $Q_{th} \sim 1$–10 W/cm^2.

The most systematic studies were those reported in[16,32]. In these studies, use was made of an Hg II laser and a Pockels cell to produce pulses of $\tau_p \sim 10$ nsec duration. A static magnetic field ($H_c \sim 75$ kOe) produced a resonance between the laser frequency and the frequency of a two-level transition (5s-5p) in a rubidium atom. When the self-transparency appeared, the absorption fell by a factor of ~100 and the delay corresponded to an extremely large reduction in the pulse velocity by a factor of ~1500. Special measures were taken to reduce the phase modulation (single-mode laser radiation, control of the spectrum) and to avoid deviations from the plane-wave conditions (screens with small apertures were used). Consequently, the results of these experiments (transmission and delay curves, splitting of 2π pulses, and so on) were found to be in excellent agreement with the theory. A reduction in the pulse duration by an order of magnitude was observed for the first time in the self-transparency effect. This was achieved by reducing the width of 3π pulses by conversion into 2π pulses and a simultaneous slight focusing, which balanced out the energy losses.

The self-transparency in potassium and sodium vapors was also investigated.[30,95] Tunable-frequency dye lasers were used for the first time in experiments of this kind and this made it easier to achieve resonance in the investigated substance. The range of dispersion delays in the case when the laser frequency was far from resonance was studied in[30]; the transmission and delay curves were also obtained.

The use of a magnetic field $H_c = 5$ kOe in[95] for the separation of a pure two-level transition and the employment of screens with small apertures made it possible to observe the majority of the self-transparency effects in their pure form.

c) **Molecular gases**. The most thoroughly investigated substance among the molecular gases is SF_6.[71,72,96-99] Pulses of 20–300 nsec duration were generated using Q-switched CO_2 lasers ($\lambda = 10.6$ μ). At low SF_6 pressures, the relaxation time was $T_2' \lesssim 1$

msec, which ensured a low threshold $Q_{th} \sim 0.1$–5 W/cm^2. Coherent effects were observed for the CO_2 laser lines in the 10.6 μ range: P(20), P(18), P(16), and P(14). In most of these investigations, there was a considerable deviation from the nondegenerate transition case: the absorption and delay curves were smoother than expected, splitting of 2πm pulses was not observed, and so on.

The exact nature of the vibration-rotational transitions in SF_6 responsible for the self-transparency effect is not known but the experimental results are in good agreement with the calculations[33,71] carried out for the strongly degenerate case. It was found that the leading edges of the pulses became steeper in the case of interaction with the P(18) transition, which was evidence of the degeneracy of the transition in the $\tau_p < T_2'$ case. On the other hand, the results[72] obtained for the P(14) line were in excellent agreement for the theory of nondegenerate transitions, as manifested by the steep transmission and delay curves and by the splitting of 2πm pulses. However, we should bear in mind that the results reported in[72] could also be explained by assuming an effective lifting of the degeneracy (Chap. 9).

The self-transparency effect was also observed in other molecular gases such as ammonia NH_3[51,100] and boron trichloride.[101] Other experimental investigations were concerned with effects similar to those discussed above. They included the observation of a delay in neon[102] experienced by pulses generated by a mode-locked He-Ne laser, as well as some fall in absorption and a delay in the case of transition to a free state of Cl_2,[103] etc.

In their first main paper,[5] McCall and Hahn pointed out that an effect similar to the self-induced transparency should also occur in fields other than optical. This was confirmed experimentally[104] for an ultrasonic wave interacting with an MgO crystal containing paramagnetic impurity centers. In this case, a strong fall in absorption, delay, and splitting into separate 2π pulses were observed. Ultrasonic experiments differed in some details from the optical experiments and were much easier to perform. In particular, the carrier frequency of the ultrasonic pulse and the transition frequency in a crystal could both be varied easily. This made it possible to demonstrate experimentally for the first time[75] the Faraday rotation of the plane of polarization of a 2π pulse as well as the splitting into (+) and (–) pulses. Experiments were also carried out recently under two-photon resonance conditions. Thus, a two-photon self-transparency in potassium vapor was reported in[89]. The sum of the energy of photons produced by a ruby laser and of the first Stokes component of these pulses in nitro-benzene was close to the transition energy. The large matrix elements of the transition made it possible to observe the effect at low thresholds $Q_{th} \sim 2$ kW/cm^2.

We shall now consider the experiments on the coherent interaction between ultrashort light pulses and semiconductors. We pointed out in Sec. 11 that the self-transparency effect could occur in the interband transitions in several semiconductors and, because of the large linear absorption coefficient, one could expect a very large drop in the absorption and a considerable delay even in small crystals ($L \lesssim 1$ cm). However, these effects were difficult to achieve because of the shortness of the relaxation times. The polarization decay time T_2' of semiconductors is governed by fast electron-electron and electron-phonon collisions and, under ordinary conditions, these collisions thermalize the carriers in the allowed bands.

One obviously has to use picosecond pulses and work at the lowest possible temperatures. It is also desirable that the excess of the photon energy over the forbidden band width be less than the optical phonon energy $\hbar\omega - \Delta \lesssim \hbar\omega_{opt}$ so as to avoid activating the fast relaxation via the optical phonons.

Equation (13.1) predicts that the threshold flux should be $Q_{th} \sim 10^2$ MW/cm^2 in the case of pulses of $\tau_p \sim 10^{-12}$ sec duration ($\mu = 10^{-17}$ cgs esu).

Although it was difficult to satisfy all the requirements mentioned above, some of the coherent effects had been observed in semiconductors. They included the observation of the one-photon self-transparency in an elegant experimental investigation reported in[105]. The use of picosecond pulses of the second harmonic of a neodymium laser and of a specially grown $CdS_{0.6}Se_{0.4}$ crystal revealed a strong drop in the absorption and a reduction in the pulse velocity. Earlier experiments[106] revealed that the propagation of Nd laser pulses across a GaAs crystal reduced the two-photon interband absorption by several orders of magnitude and could produce some delay.

CONCLUSIONS

Investigations of the self-induced transparency effect have now grown into a fairly independent branch of nonlinear optics which is developing rapidly in the experimental and theoretical aspects.[6] Although the basic physical principles are now clear, many important aspects are still awaiting investigation. These aspects include:

1) the relationship between the self-transparency effect and the general theory of solitons;

2) the spectral changes associated with the phase modulation;

3) the special features of the self-transparency observed for picosecond pulses generated by mode-locked lasers;

4) the search for new absorbing media in which the self-transparency or its acoustic and other analogs can be observed;

5) the generation of ultrashort pulses on the basis of the self-transparency effect;

6) the measurement of the relaxation times, cross sections, and dipole moments of various transitions, and the determination of the Stark and Landé factors from the results of studies of the characteristics of 2π pulses;

7) the possibility of long-range coupling in the strong-absorption regions;

8) the construction of optoelectronic elements such as discriminators, delay lines, logic elements, etc.

The authors are grateful to B. Ya. Zel'dovich and I. I. Sobel;man for numerous discussions.

[1)] The main results are applicable also to circularly polarized light. [4,5]

[2)] This consequence can be used in practice in the shaping of ultrashort pulses. [16,31,32]

[3)]We are assuming that the plane-wave condition is still satisfied (for example, with the aid of an aperture).
[4)]K(x, p) is a complete elliptic integral of the first kind. [64]
[5)]In the case of circularly polarized light, only the transitions $0 \rightleftharpoons 1$, $1/2 \rightarrow 1/2$, and $1 \rightarrow 1$ behave as nondegenerate. [5]
[6)]The papers cited as [107] appeared after the completion of this review.

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Translated by A. Tybulewicz