The Hanle effect

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The history of the discovery, study, and explanation of the effect discovered fifty years ago by the German physicist Hanle and its development and generalization by the French school of physicists led by Kastler are described. The effect is observed optically in an increase in the degree of depolarization of the resonance fluorescence. It is studied in the range of magnetic fields in which the Zeeman splitting of the atomic magnetic sublevels does not exceed the level widths determined by thermal relaxation mechanisms and electromagnetic interaction. It is a direct optical manifestation of interference of degenerate atomic states and is intimately connected with the conservation law for the angular momentum of the photon-atom system. In the review, different applications of the effect to the measurement of mean lifetimes, linewidths, and Stark splitting constants of atoms in excited states are given. The resonance scattering of light by atoms in the ground state in a weak magnetic field is considered in detail—in particular, in the presence of radio-frequency magnetic fields oriented in different ways. The different possible types of parametric resonances in one or two radio-frequency fields for arbitrary orientation of the constant magnetic field are interpreted as the effect of crossing of the levels of an atom that is "dressed" by the radio-frequency field. Applications of the effect in the quantum magnetometry of ultraweak fields (down to 10^{-10} Oe) are examined.

CONTENTS

1. INTRODUCTION

Fifty years ago in Göttingen the German physicist Hanle published a paper[1] whose significance for the subsequent development of atomic physics and optics is difficult to overestimate. In this paper the phenomenon of strong depolarization of light of resonance frequency scattered by atoms in a weak external magnetic field was described. The character of the polarization of the scattered light depended in an essential way on the magnitude and direction of the field and on the direction of observation. Following[1], a series of papers[2] appeared in which, in particular, the character of the polarization of the 2537 Å resonance line of mercury and also of the resonance lines of vapors of other atoms, especially divalent metals (cadmium, calcium and zinc), were studied in detail. In magnetic fields less than or of the order of the earth's field, the polarization of the scattered light was observed to have a strong dependence on their intensity, and this dependence disappeared when the intensity was increased. Rayleigh, and also Wood and Ellett[3], who had discovered the effect of a magnetic field on the character of the polarization of resonantly-scattered light, came close to discovering the effect, which soon became known as the Hanle effect.

To explain the depolarization and rotation of the plane of polarization of the scattered light, Hanle made use of a model of a damped spatial oscillator situated in a magnetic field. More detailed calculations with this model were soon carried out by Breit and then by Soleillet[4]. The semi-classical theory explained the features of the observed phenomena for different orientations of the magnetic field with respect to the direction of propagation of the exciting light and for different types of polarization of the latter, for divalent metals in whose Zeeman effect a simple Lorentzian triplet appears. The first quantum-mechanical theory of the effect was developed in papers by Breit[5] and van Vleck[6]. In simple cases, its results coincided with those of the semi-classical theory.

The original variant of the theory of the Hanle effect is attractively simple and so instructive that we present it here, supplying only certain clarifications.

We shall consider the case of emission of light by an ensemble of atoms whose ground state is a singlet and has energy E_0 and whose excited state is a doublet with energy levels E_1 and E_2. If we associate a classical linear oscillator with the dipole transition, the intensity $\mathscr{E}$ of the field of the emitted wave can be written in the form

$$\mathscr{E} = \mathscr{E}_1 \exp\left[i\left(\frac{E_1-E_0}{\hbar}t+\varphi_1\right)\right]+\mathscr{E}_2\exp\left[i\left(\frac{E_2-E_0}{\hbar}t+\varphi_2\right)\right],$$

where the amplitudes E_1, E_2 and phases φ_1, φ_2 of each of the oscillations depend on the direction of observation, the polarization of the radiation, and the character of the transition. The intensity of the light emitted by an atom is proportional to $|\mathscr{E}|^2$ and equals

$$I(t) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\Omega t + \varphi).$$

Here it is assumed that $\varphi = \varphi_2 - \varphi_1$ is independent of time. In order that it be possible to observe these interference beats in the intensity of the radiation after averaging over the whole ensemble of atoms, it is necessary

that the time at which the atoms are excited can be determined with accuracy greater than $1/\Omega$, where $\Omega = |E_2 - E_1|/\hbar$ is the beat frequency. This is not the case for individual atoms under the conditions of the experiments of [1,2], since the individual atoms of the ensemble are excited at different moments of time. Nevertheless, the interference term can give a nonzero contribution to the experimentally observed mean value $\langle I \rangle \equiv I(\Omega\tau)$ of the intensity of the radiation, owing to the fact that the excited state of the atom has a finite mean lifetime τ and decays according to the law $(1/\tau)\exp(-t/\tau)$. Assuming that $\tau_1 = \tau_2$, we obtain

$$I(\Omega\tau) = \int_0^\infty I(t)\frac{\exp(-t/\tau)}{\tau}dt = \\ = \cos\varphi\int_0^\infty \exp\left(-\frac{\xi}{\Omega\tau}\right)\cos\xi\,d\xi - \sin\varphi\int_0^\infty \exp\left(-\frac{\xi}{\Omega\tau}\right)\sin\xi\,d\xi,$$

or

$$I(\Omega\tau) = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\frac{\cos\varphi - \Omega\tau\sin\varphi}{1+\Omega^2\tau^2}.$$

It is instructive to trace how, from the periodic time dependence $\cos(\Omega t + \varphi)$ of the beats, a quasi-resonant dependence of the mean intensity $I(\Omega\tau)$ on the frequency Ω arises on averaging over the moments of excitation of the atoms. The dependence $I(\Omega\tau)$ is determined by integrals containing a product of a decaying exponential factor with a harmonic function. In weak fields ($\Omega\tau \ll 1$) the exponential decreases rapidly with increasing ξ and the harmonic function does not have time to vary. On the other hand, in strong fields ($\Omega\tau \gg 1$) the exponential decreases slowly and the harmonic factors are averaged to zero.

Thus, there appears an effect with an apparently resonant (pseudo-resonant) dependence of the intensity of the resonance luminescence on Ω—the beat frequency between the sublevels.

Depending on the value of φ ($0 \le \varphi \le \pi/2$), the contour of the luminescence line as a function of frequency can have the form (superimposed on $I_1 + I_2$) of a Lorentzian curve ($1/(1 + \Omega^2\tau^2)$) with width $2/\tau$ at half height (the case $\varphi = 0$) or of a dispersion curve with maxima at the points $\pm 1/\tau$ (the case $\varphi = \pi/2$). Since the energy levels are not parallel and diverge with increasing magnetic field, the linewidth as a function of the magnetic field depends not only on τ but also on the Landé factor, and is inversely proportional to the latter. The values of the phase can be varied using the observational scheme containing light polarizers that was used by Hanle himself (Fig. 1). In this case, the vapor of an even-even isotope of mercury, e.g., Hg^{200}, whose level scheme is depicted in Fig. 2, is a very suitable real object for observing the effect. The magnitude of the effect for $\varphi = 0$ is a maximum when $\Omega = 0$, i.e., when the levels are degenerate and completely overlap, and for $\varphi = \pi/2$ is a maximum when $\Omega\tau \sim 1$, i.e., when the levels of the doublet have a splitting of the order of their width, and decreases rapidly with increasing Ω. If the doublet under consideration is a Zeeman doublet, as is the case in the example with mercury, then $\Omega = g(e/mc)H_0$, where g is the Landé factor for the sublevels considered. The condition $\Omega\tau \sim 1$ is fulfilled in fields $H_0 \sim (mc/e)/g\tau$. For $\tau = 10^{-8}$ sec, H_0 is of the order of a few Oersted. For metastable levels, H_0 may turn out to be very small. Thus, the intensity of the resonance luminescence varies appreciably in weak magnetic fields and ceases to depend on the field when $\Omega\tau \gg 1$. The Hanle effect was

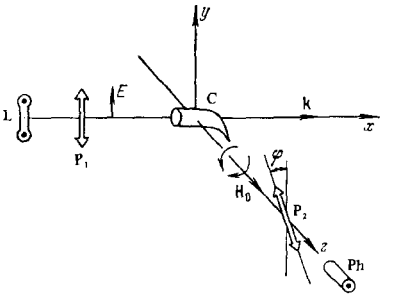

FIG. 1

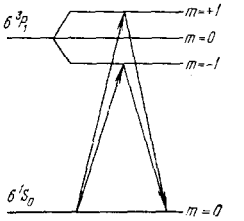

FIG. 2

FIG. 1. Scheme of observation of the Hanle effect in the excited state. (L—pumping lamp, C—resonance cell, P_1 and P_2—polarizer and analyzer, Ph—photon collector, k—direction of the beam of light rays of resonance frequency).

FIG. 2. Scheme of the Zeeman levels of the isotope Hg^{200} in the ground (6^1S_0) and excited (6^3P_1) states.

observed in the resonance scattering of light by atoms in an excited state. The possibility of observing the effect is associated with the fact that the mean lifetime τ in the excited state is considerably greater than the time during which the excitation of the atom occurs. The latter is of the order of the correlation time of the pumping resonance light field and thus depends on the extent to which it is monochromatic.

Nils Bohr, having learned of Hanle's experiments before his article was published, immediately turned his attention to the fact that the observed degree of depolarization in the presence of the magnetic field turns out to be considerably greater than the expected mean value that should be found for arbitrary orientations of the atoms with respect to the field. He had the opportunity to discuss this question with Heisenberg, who remarked that an external magnetic field exerts no influence on the polarization of the scattered radiation only in the case when its direction coincides with that of the electric vector of the linearly polarized exciting light. The observed effect was unexpected and appeared at first to be incompatible with the existing ideas on the structure of atoms. In Bohr's paper "On the polarization of fluorescent light"[7], which was published soon after Hanle's paper, it was asserted that the state of the polarization of the light emitted by an atom does not vary on adiabatic decrease to zero of some auxiliary external magnetic field oriented with respect to the quantum system in such a way that the symmetry of the latter remains unchanged. This statement was based on work carried out in 1916 by P. Ehrenfest, who showed that during an infinitesimally slow change of external or internal parameters of a mechanical system (including quantum systems), the system goes over continuously into a new state, and certain combinations of these parameters—the adiabatic invariants of the system—are conserved. The quantum numbers (adiabatic invariants of the system) are Ehrenfest's adiabatic invariants[8]. In the paper by Heisenberg[9] which followed these papers, the postulate that was later named the principle of spectroscopic stability was put forward. Starting from the fact that a field that does not affect the symmetry of a system does not alter the polarization of its radiation, Heisenberg came to the conclusion that for linearly polarized light the auxiliary magnetic field should be oriented parallel to the electric vector of the light wave. But in the case of circular polarization, the symmetry axis is the direction of propagation of the light, and the

auxiliary magnetic field should be oriented along it. The practical use of this principle is that it permits one to regard resonance scattering in zero field as the limit of scattering in the case of the Zeeman effect when the auxiliary field tends to zero. It was considered for a long time that the principle of spectroscopic stability did not follow from the basic propositions of quantum mechanics and was an additional postulate, analogous to the "law of the parallelogram of forces" in classical mechanics.

The discovery of the polarization of resonance radiation and the study of the effect of both strong and weak external fields on its character were made in a period in which atomic structure was being actively studied and in which quantum mechanics was established, and this explains the great interest shown in this effect. Even earlier it had become clear that the photon is not only characterized by a definite energy and momentum, but, depending on the state of polarization, also possesses a definite angular momentum[10]. The conservation of this angular momentum in elementary acts of interaction with atoms was found to be intimately related to the selection rules for the magnetic quantum numbers[11].

It was first demonstrated clearly in[1,2] that the process of resonance scattering of polarized light can be controlled by means of an external magnetic field interacting with the scattering atoms. Thus, another aspect (closely related to the selection rules) of the conservation of angular momentum in elementary processes of excitation of atoms by resonance photons was elucidated: the excited state of the atoms becomes polarized and the atom acquires a definite angular momentum. An extensive literature is devoted to all these questions, and they have invariably been given their due weight in monographs on the resonance scattering of light, luminescence and the theory of atomic spectra[12]. By the end of the 1930's it seemed that these phenomena had been studied sufficiently and interest in them gradually declined.

In 1949, twenty-five years after the appearance of Hanle's paper, thanks to the work of Brossel and Kastler the attention of physicists was again drawn to the elementary acts of interaction of quantum systems with an electromagnetic field of resonance frequency. It was shown that in the process of resonance scattering the angular momentum of the photon can be transferred to the atoms, as a result of which, considerable polarization of the atoms can be attained not only in the excited state but also in the ground state. Thus, the possibility of optical orientation of atoms[13] by polarized light of resonance frequency was discovered, and is an effective method of obtaining a high degree of magnetic polarization of atoms and of producing a different kind of rearrangement of populations between the magnetic and hyperfine sublevels of the ground state. This method of orienting the angular momenta of atoms, which has become known as the method of "optical pumping," was later the subject of numerous experimental investigations and lay at the basis of some extremely important scientific and technical applications, a survey of which can be found in[14]. The effectiveness of the method of optical pumping of atoms by light of resonance frequency can be seen by comparing the degree of orientation that can be attained by the "brute-force method" due to relaxation to a configuration of thermal equilibrium in a strong external magnetic field H_0. Inasmuch as the degree of orientation R in the latter method is determined by the relation $R = \tanh(\mu H_0/kT) \approx \mu H_0/kT$, in order to obtain $R \gtrsim 0.1$ at room temperatures fields of intensity $H_0 \sim 3 \times 10^5$ Oe and higher are required.

Alongside the development of optical-pumping methods, methods for the optical detection of the states of atomic systems have been successfully developed and refined. They began to play a special role after the discovery of the effects of crossing and anti-crossing of atomic energy levels[15] in a magnetic or electric field, these effects being one of the manifestations of interference of atomic states. In particular, by that time it had been elucidated that the Hanle effect, already known for many years, could be interpreted as the result of interference of degenerate Zeeman levels of the excited state of the atom in zero magnetic field.

The interference aspect of the Hanle effect was completely incomprehensible in the first years after its discovery. The degeneracy of the Zeeman sublevels of the excited states in zero magnetic field (when the interference effect is a maximum) was perceived as a difficulty in the theoretical description of the effect, and the above-mentioned principle of spectroscopic stability was specially put forward to partially overcome this difficulty.

Interest in the observable optical manifestations of the interference of atomic states, at the basis of which lies the well-known superposition principle of quantum mechanics, has been revived comparatively recently as a result of a series of experimental[15] and theoretical papers[16]. In[16,17a] the universality of the phenomenon of interference of atomic states as one of the basic propositions of quantum mechanics was demonstrated. In these papers, not only quantum systems containing electrons, but also atomic nuclei, nucleons and mesons are considered in the form of examples. The states of a quantum system can interfere independently of the nature of its levels. Therefore, interference of states can be manifested not only in the case of splitting of levels of individual atoms in external fields, but also in cases when the appearance of additional levels is due to collective interactions. This can give the possibility of obtaining important information on internal molecular crystal fields, fields of impurity atoms and color centers in crystals, etc. These interesting problems, however, still require detailed study.

In complete analogy with the Hanle effect for atoms in an excited state, the character of the absorption of light by the atoms should also experience a change when the Zeeman sublevels of the ground state of the atomic system cross in zero field. Nowadays, we can only be surprised that this simple conclusion was drawn many years after the first experiments of Hanle and the discovery of the phenomenon of optical pumping. Attention was first drawn to the existence of very narrow resonance lines near zero magnetic field in[31]. Somewhat later[33a], it was proposed that the phenomenon of crossing of the Zeeman levels of the ground state could be applied to measure very weak magnetic fields. This suggestion was developed further in papers by French physicists of Kastler's group[33b,c], who carried out a detailed theoretical and experimental investigation of the phenomenon of crossing of ground-state levels. In these papers, special attention was directed to the practical application of the phenomenon for the development of a magnetometric apparatus possessing a record sensitivity for optical methods (down to 10^{-10} Oe), comparable only with the sensitivity of modern magnetometers with

superconducting detectors, in which the phenomenon of quantization of the magnetic flux in doubly-connected conductors (the Josephson effect) is utilized[34].

Below we shall examine the new results obtained in the study of the crossing of levels of both the excited and the ground states in magnetic and electric fields, and also in weak and ultraweak (insufficient to resolve the levels) magnetic fields. Then we shall discuss certain possibilities for applying these results in quantum magnetometry[33d].

2. CROSSING OF ZEEMAN LEVELS IN ZERO FIELD AND THE PRINCIPLE OF SPECTROSCOPIC STABILITY

While developing the Bohr-Heisenberg principle of spectroscopic stability, Kastler in his dissertation[35] formulated a postulate according to which, for atoms absorbing light in the absence of an external magnetic field, the "symmetry axis of the incident electromagnetic vibrations" serves as the quantization axis. Somewhat later[13], he noted that optical excitation leads to selection of the angular state of the atoms with respect to the polarization of the beam of light rays, and, in agreement with[10,11], connected this phenomenon with the law of conservation of angular momentum in the photon-atom system. The role, noted in these papers, of the polarization of the light ray defining the direction of the axis of quantization of the angular momentum of the atomic systems in the absence of other singled-out directions has made the interpretation of all the effects observed near the value $\mathbf{H}_0 = 0$ much easier, and has thereby turned out to be "convenient." Of course, as the quantization axis we could take the direction of the field $\mathbf{H}_0$ irrespective of the direction and magnitude of the latter, but the theoretical analysis of the level-crossing effect, while remaining as rigorous as for any other choice of quantization axis, then loses its lucidity and, most important of all, its simplicity. In the case when the directions of the light ray and of the magnetic field do not coincide, the precession of the spins about $\mathbf{H}_0$ masks the action of the optical pumping, since the symmetry of the magnetic field becomes the symmetry of the problem under consideration and the angular aspect of the strictly optical pumping becomes secondary.

The results of a consistent quantum theory of optical pumping do not depend, of course, on the choice of quantization axis, since the calculation can be carried out in any basis. However, in a very weak or zero magnetic field it is considerably simpler and more convenient to use as the basis the eigenstates of some Hermitian operator describing the system of atoms interacting with the light ray[36].

It is possible to indicate a simple way, needing no extra postulates, of finding a physically distinct quantization-axis direction that is "convenient" for solving quantum-mechanical problems[37].

We shall assume that the complete operator $\hat{\mathscr{H}}$ of the energy of a rigid system of particles (atoms) interacting with each other and with an external magnetic or electric field is known. Then the physically distinguished direction or directions, if they exist, necessarily satisfy the condition

$$\{\hat{J}_\alpha, \hat{\mathscr{H}}\} = 0, \tag{1}$$

where the curly brackets denote the quantum Poisson bracket (or commutator), and $\hat{J}_e$ denotes the projection of the total angular momentum of the system on this direction. We shall assume that this direction is specified by the unit vector $\mathbf{e}$ in an arbitrarily oriented laboratory coordinate frame. Then, according to the definition, $\hat{J}_e = (\mathbf{e} \cdot \mathbf{J}) = e_\alpha J_\alpha$ and (1) takes the form

$$e_\alpha \{\hat{J}_\alpha, \hat{\mathscr{H}}\} = 0. \tag{2}$$

The orientation of the vector $\mathbf{e}$ in space is determined by the symmetry group of the complete operator of the energy of the particles in the field. We shall assume that the operator $\hat{\mathscr{H}}$ does not depend explicitly on time. The component along this direction of the total moment of the external forces acting on the system of particles under consideration equals zero, and the corresponding component $\hat{J}_e$ of the angular-momentum operator does not depend on time. The angular-momentum components in the plane perpendicular to $\mathbf{e}$ do not have well-defined values, and their mean value precesses about the direction of the quantization axis $\mathbf{e}$.

In order to find the required direction, we go over from the operators to the observable (mean) quantities; for this, we perform the contraction (trace) of the expression (2) with the Gibbs density operator $\hat{\rho}$. As a result we obtain two linearly independent equations (the direct equation and its conjugate):

$$e_\alpha \mathrm{Sp}\,\langle \hat{\rho} \{\hat{J}_\alpha, \hat{\mathscr{H}}\}\rangle = 0, \quad e_\alpha \mathrm{Sp}\,\langle \{\hat{J}_\alpha, \hat{\mathscr{H}}\}\hat{\rho}^+\rangle = 0. \tag{3}$$

In order to solve this system of equations, we introduce vectors $\mathbf{A}$ and $\mathbf{A}^+$ with components

$$A_\alpha = \mathrm{Sp}\,\langle \hat{\rho} \{\hat{J}_\alpha, \hat{\mathscr{H}}\}\rangle, \quad A_\alpha^+ = \mathrm{Sp}\,\langle \{\hat{J}_\alpha, \hat{\mathscr{H}}\}^+ \hat{\rho}^+\rangle;$$

as can be seen from (3), these vectors are orthogonal to $\mathbf{e}$. Therefore, the vector $\mathbf{\Pi} = [\mathbf{A} \times \mathbf{A}^+]$ is parallel to $\mathbf{e}$, since $[\mathbf{e} \times \mathbf{\Pi}] = 0$, or

$$\frac{e_x}{\Pi_x} = \frac{e_y}{\Pi_y} = \frac{e_z}{\Pi_z}.$$

Thus, the orientation of the vector $\mathbf{e}$ in space is determined by the components of the vector $\mathbf{\Pi}$, which are proportional to the direction cosines e_α:

$$\mathbf{\Pi} = [\mathrm{Sp}\,\langle \hat{\rho} \{\hat{\mathbf{J}}, \hat{\mathscr{H}}\}\rangle, \mathrm{Sp}\,\langle \{\hat{\mathbf{J}}, \hat{\mathscr{H}}\}^+, \hat{\rho}^+\rangle]. \tag{4}$$

To find the components of the vector $\mathbf{\Pi}/\Pi = \mathbf{e}$ it is necessary, knowing the Hamiltonian of the problem, to calculate the Poisson bracket (commutator) and average the result over the Gibbs ensemble.

We shall assume that the total Hamiltonian of the system can be represented in the form $\hat{\mathscr{H}} = \hat{\mathscr{H}}_0 + \hat{\mathscr{H}}_0$ is the Hamiltonian of the system in the absence of the field and $\hat{\mathscr{H}}'$ describes the interaction with the field. If $\hat{\mathscr{H}}_0$ turns out to be an isotropic operator, the symmetry of the states of the quantum system (i.e., of the eigenstates of the operator $\hat{\mathscr{H}}_0 + \hat{\mathscr{H}}'$) is entirely determined by the form of the interaction operator.

It may happen that the operator $\mathscr{H}'$ contains terms that oscillate rapidly with time. Then the expression (4) defining the vector $\mathbf{\Pi}$ must be averaged over a sufficiently long time interval (compared with the period of these oscillations). The correctly averaged vector $\langle \mathbf{\Pi} \rangle$ defines the direction of the experimentally observed quantization axis (e.g., the direction of the mean magnetization of an ensemble of atoms that is established in the field of a circularly polarized light wave of resonance frequency).

We shall consider a few simple examples of the interaction of systems of particles of various multiplicity with an external field.

For an electric dipole with moment ! xternal electric field **E** we find, carrying out the operations indicated above,

$$\Pi_\alpha = \langle \varepsilon_{\alpha\beta\gamma}\varepsilon_{\beta\sigma\delta}\varepsilon_{\gamma\nu\mu}E_\delta E^*_\mu p_\sigma p^*_\nu\rangle \tag{5a}$$

and, analogously, for a magnetic dipole **m** in a magnetic field,

$$\Pi_\alpha = \langle \varepsilon_{\alpha\beta\gamma}\varepsilon_{\beta\sigma\rho}\varepsilon_{\gamma\nu\mu}H_\rho H^*_\mu m_\sigma m^*_\nu\rangle. \tag{5b}$$

It follows directly from (5) that in constant fields the quantization axis is always directed along the field.

More interesting is the case when the electric or magnetic field depends on time. For example, let

$$\mathbf{H}(t) \equiv \{h_0,\ i\eta h_0,\ 0\}\exp(-i\omega t);$$

then, as one can easily verify,

$$\langle\Pi\rangle = h_0^2\left\{\begin{array}{l}\frac{i}{2}(1-\eta^2)(m_y m_z^* - m_z m_y^*),\\ \frac{1}{2}(1-\eta^2)\,\eta\,(m_y^* m_z - m_z^* m_y),\\ \eta m_z^* m_z\end{array}\right\}. \tag{6}$$

A completely analogous expression is also obtained for an electric dipole in an oscillating electric field.

It can be seen from (6) that in the case of circular polarization of the field of the wave, the direction defined by the vector $\langle\Pi\rangle$ coincides with the direction of the wave vector, i.e., with the direction of propagation of the wave, and changes on change of sign of the polarization. For linear polarization of the incident wave, the direction singled out for a magnetic dipole, as can be seen easily from (6), coincides with the direction of the oscillations of the magnetic vector of the wave. For an electric dipole this direction coincides with the direction of the electric vector.

These simple conclusions constitute the content of the Bohr-Heisenberg principle of spectroscopic stability.

If a magnetic dipole is situated simultaneously in constant and oscillating magnetic fields, $\mathbf{H}_0$ and $h_0\exp(-i\omega t)$,

$$\mathbf{H}(t) = (0,\ 0,\ H_0) + (h_0,\ 0,\ 0)\exp(-i\omega t),$$

then

$$\Pi = \left\{\begin{array}{l}h_0^2(\mu_y^+\mu_z - \mu_z^+\mu_y),\\ 0,\\ H^2(\mu_x^+\mu_y - \mu_y^+\mu_x)\end{array}\right\}.$$

Thus, the action of the oscillating field leads to a declination of the quantization axis, determined by the ratio of the squares of the amplitudes.

Other more complicated problems can also be solved in an analogous way; in particular, the case of elliptical polarization of the pumping field and also the case of mixed polarization specified by Stokes parameters have been considered.

Such an approach to the determination of the form of the symmetry of an open quantum system is completely general. It permits one to find the direction of the natural axis of quantization for a Hamiltonian of arbitrary form, using only the standard formalism of quantum mechanics. The principle of spectroscopic stability, as we see, is a consequence of quantum mechanics. We remark, however, that the direction of the symmetry axis can also be found, by means of Poisson brackets, in the framework of classical analytical mechanics.

3. THE HANLE EFFECT IN EXCITED STATES OF ATOMS

Expensive equipment is not required to observe the signals from the interference of atomic states. The necessary minimum is available to any laboratory, and the success of the work is determined, as usual, by the capabilities of the investigators.

An important feature of the interference of atomic states in sufficiently diffuse gases is the absence of Doppler broadening of the observed signals. Since, in this case, states of one and the same atom interfere, the width of the resonance curves is determined by the widths of the interfering levels. This feature of the Hanle effect essentially distinguishes the methods based on observations of it from the usual methods of atomic spectroscopy and makes it highly effective in the study of many features of the fine and hyperfine structures of atomic levels.

The method is not in competition with other known methods in the determination of the lifetimes and widths of resonance levels[17,18], in the study of the effect of different kinds of broadening interactions and shifts[19], and in measurements of the g-factors of excited states[17c, 20, 38]. The range of applications of the methods encompasses equally both the excited and the ground states of atoms, for an arbitrary time dependence of the excitation[21].

The phenomenon of interference of atomic states opens up new possibilities for measuring the Lamb shift by measuring the lifetimes of the excited states of atoms[22].

We note an interesting dynamical method of obtaining polarized atoms or ions of deuterium[23], which also permits one to measure "in passing" the Lamb shift of its levels. Deuterium atoms in the metastable state $2S_{1/2}$ are formed from deuterons by charge exchange. If we select a magnetic field such that in it the Zeeman levels of the $2S_{1/2}$ and $2P_{1/2}$ states cross, the metastable atoms are given the possibility of a transition to the ground state. Their number then rapidly decreases with time, as is observed experimentally. Knowing the intensity of the field (570 Oe) in which the crossing of the levels occurs, we can calculate the Lamb shift of the $2P_{1/2}$ level. On rapid reversal of the field direction, the process of decay of the metastable state can be repeated, the levels having "changed places." At the end of the decay process there remain only deuterium atoms in the ($F = 3/2$, $m_F = \pm 1/2$)-states. If at this time we apply a sufficiently strong saturating radio-frequency field of resonance frequency, then, owing to the hyperfine interaction, the electronic spin of the shell of each atom is transferred to the nucleus. Thus, it is possible to obtain a high degree of polarization of the nuclei of deuterium atoms. By ionizing them, we can obtain deuterons or negative deuterium ions with a high degree of polarization.

The Stark effect can be studied by the method of interference of levels[24,25]. In this case the measurements are usually performed in both an electric field and a constant magnetic field simultaneously. Crossing of two of the terms under consideration, e.g., the fine structure, is realized by means of the magnetic field, and then the electric field is switched on, displacing each of these terms differently (the quadratic Stark effect). By restoring the crossing position on the fre-

quency scale of the transition by varying the magnetic field, it is possible to determine the Stark constant. If the hyperfine levels are well resolved (Rb^{85}, Rb^{87}), the measurements can be performed in only one electric field.

On passage of linearly polarized light of non-resonance frequency through a gas consisting of atoms oriented by resonance light and possessing magnetic gyrotropy, either the Faraday effect (the rotation of the plane of polarization) or birefringence is observed, depending on the direction of the light[26]. The effect also occurs for pumping light, but, because of the strong absorption, it can be observed only in thin layers. The phenomenon occurs in the absence of an external magnetic field. If a magnetic field is switched on and varied within limits such that crossing of the corresponding levels occurs, an appreciable depolarization of the signal occurs and the phenomenon of rotation or birefringence disappears. The half-width of the region of depolarization when the resonance light is observed along the field (forward scattering in a thin layer) corresponds approximately in this case to the Doppler half-width[27].

The Hanle effect has also been detected in reflected light[28]. The authors raised the pressure of mercury vapor in a flask and at a certain pressure observed a selective (metallic) reflection of part of the resonance light at its surface. The effect appeared when the mean distance between the atoms became less than a wavelength; it is due to the phase correlations which then arise between groups of atoms and lead to interference of the σ^+ and σ^- emissions[40,41] of different atoms. The half-width of the observed signal in this case also turns out to be equal to the Doppler half-width[29].

To study the crossings of the levels, any excitation mechanism that leads to a statistically-nonequilibrium occupation of the magnetic sublevels of the excited states can be used. This could be, e.g., collisions with electrons[40] or ions[41].

A number of interesting effects are observed on optical pumping of atoms by polarized laser radiation[63]. Because of the high spectral density of the radiation, induced resonance optical transitions are the most probable, and this leads to polarization of the levels associated with these optical transitions. Observation of the optical fluorescence lines emitted in transitions from these levels gives information about the atomic constants and relaxation processes. In particular, in[63] the effect of an external magnetic field on the transverse alignment was observed. The case of high laser-radiation intensity, when nonlinear effects appear, was considered in[64-66]. In particular, a broadening of the Hanle resonance lines with increasing intensity of the resonance radiation was observed; the lineshapes remained close to Lorentzian. A theoretical analysis of these phenomena can be found in[67].

Recently, the Hanle effect, in combination with electronic excitation, has been used to study gas discharge, in particular, in a neon-helium mixture[30], and to analyze highly-excited states, including the paradoxical cases of observation of the phenomenon in laser transitions with isotropic excitation.

Certain examples, not considered here, of the application of the Hanle effect in atoms in excited states are discussed in detail in the review[17c]. An extensive bibliography on the questions considered here can be found in[38].

4. THE HANLE EFFECT IN THE GROUND STATE OF ATOMS

Interference between sublevels of the ground state of atoms can be detected from the appearance of modulation in the absorption of light of resonance frequency passing through a cell filled with vapor of these atoms. It is possible to create the necessary correlation between the random values of the phases of the wavefunctions of the atomic electrons by placing the cell in a weak constant field $\mathbf{H}_0$ and an oscillating radio-frequency (RF) field of resonance frequency γH_0. The intensity of the light passing through the cell is related in a simple way (linearly) to the components of the magnetization $\mathbf{M}$ arising during the optical orientation of the atoms[32].

To illustrate the phenomenon we shall consider the simple case when the atoms have pairs of Zeeman sublevels in both the ground and the excited state. The optical cell containing the atoms is illuminated by circularly polarized σ^+-light of resonance frequency, propagating along the axis Ox, which is the quantization axis. In the absorption of the circularly polarized light the angular momentum of the photons is transferred to the atomic system, which thus acquires a macroscopic magnetization. This circumstance is reflected in the selection rules, which can always be regarded as a consequence of the angular-momentum conservation law. Since optical transitions from the state $m_f = 1/2$ under the action of σ^+-light are forbidden, and the spontaneous inverse transitions from the excited state can occur to both sublevels $m_f = \pm 1/2$, as a result of the large number of acts of absorption and emission of photons by the atomic ensemble there arises in the latter a nonequilibrium stationary population distribution over the sublevels of the ground state. Then the intensity I of the absorbed light is proportional to the number of atoms able to absorb σ^+-photons, i.e., to the number of atoms in the sublevel $m_f = -1/2$. It can be shown that, to within a constant, the quantity I is proportional to the x-component of the magnetization M of the atomic ensemble[14b, 32].

The equation determining the rate of change of the magnetization of a two-level system in an arbitrarily oriented uniform magnetic field $\mathbf{H}$ has, in the case of isotropic relaxation, the form[14b]

$$\frac{d\mathbf{M}}{dt} = \frac{\mathbf{M}_0 - \mathbf{M}}{T_p} - \frac{\mathbf{M}}{T} + \gamma[\mathbf{M}, \times \mathbf{H}]; \qquad (7a)$$

here, γ is the gyromagnetic ratio of the atoms in the ground state, $1/T_p$ is a constant characterizing the rate of pumping under the action of the orienting light and is proportional to the light intensity (T_p is the pumping time), and $1/T$ is the thermal-relaxation rate, which is principally determined by collisions between atoms and with the cell walls.

If we introduce the notation

$$\Gamma_1 = \frac{1}{\tau} = \frac{1}{T_p} + \frac{1}{T}, \quad M_0' = M_0 \frac{\tau}{T_p},$$

then Eq. (7a) takes the form

$$\frac{d\mathbf{M}}{dt} = \frac{\mathbf{M}_0' - \mathbf{M}}{\tau} + \gamma[\mathbf{M}, \times \mathbf{H}], \qquad (7b)$$

which coincides with the well-known Bloch equations for the case when the longitudinal (T_1) and transverse (T_2)

relaxation times are equal. Analysis of the more realistic case when $T_1 > T_2$ does not lead to substantial complications in the solution of the problem. The magnitude and direction of the vector M_0' are determined in the case under consideration not by the field H but by the intensity and direction of propagation of the resonance light. Equation (7) for spins $S = 1/2$ can be obtained completely rigorously by means of the density-matrix formalism. For many-level systems $S > 1/2$, it should be replaced by a system of equations which, however, is equivalent to the vector equation (7) in the first approximation.

First we shall consider the case when the constant magnetic field H_0 lies perpendicular to the orienting light ray and is varied near the value $H_0 = 0$. If, e.g., the field H_0 is parallel to Oz, the stationary solution of Eq. (7) has the form

$$M_z = 0, \quad M_\pm = \frac{M_0'}{1 \pm i\omega_0\tau},$$

where $\omega_0 = -\gamma H_0$ and $M_\pm = M_x \pm iM_y$. Hence it follows that

$$M_x = \frac{M_0'}{1+\omega_0^2\tau^2}, \quad M_y = \frac{M_0'\omega_0\tau}{1+\omega_0^2\tau^2}, \quad M_z = 0. \tag{8}$$

The magnetization component M_z remains equal to zero, and the components M_x and M_y have, respectively, the shape of absorption and dispersion curves centered at the point $H_0 = 0$, with width $\Delta\omega_0\tau = 2$ at half-height. Thus, the quantity

$$\Delta H_0 = \frac{2}{\gamma\tau} \tag{9}$$

is directly proportional to the width of the Zeeman sublevels and determines the linewidth of the crossing signal in units of the magnetic-field intensity. The components M_x and M_y of the stationary magnetization satisfy the equation

$$\left(M_x - \frac{M_0'}{2}\right)^2 + M_y^2 = \left(\frac{M_0'}{2}\right)^2.$$

When the field is varied, the stationary-magnetization vector M describes a semicircle in the (x, y) plane; the field value $H_0 = 0$ corresponds to $M = (M_0', 0)$. When the field H_0 is increased, the angle $\varphi = \arctan \omega_0\tau$ also increases.

It can be seen from Fig. 3 that for $\omega_0 = 0$ $M_x = M_0'$, whereas in a strong field ($\omega_0\tau \gg 1$) all three components of M vanish. This means that transverse pumping in a strong field cannot lead to orientation in an atomic system; this had been discovered experimentally by Wood and Ellett [3] for an excited state.

The results obtained have a simple geometrical interpretation. Pumping by light of resonance frequency orients the atomic moments (spins) along the axis Ox. The oriented spins precess in the (x, y) plane with frequency ω_0 about the direction of the field H_0. Owing to relaxation processes, with the passage of time the orientation of the spins is destroyed, with time constant τ. Therefore, at the moment of time t, only spins oriented in the time interval from $t - t'$ to t, where $t - t' \sim \tau$, make an appreciable contribution to the magnetization.

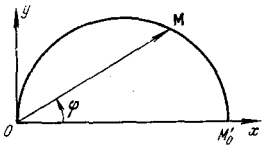

FIG. 3. Hodograph of the stationary magnetization in a magnetic field transverse to the pumping axis.

In a weak magnetic field ($\omega_0\tau \ll 1$), the oriented spins remain parallel to each other, $M_y = 0$, and M_x is a maximum and equal to M_0'. In a strong magnetic field ($\omega_0\tau \gg 1$) the spins have time to perform many rotations during the relaxation time and their distribution in the (x, y) plane becomes isotropic, as a result of which the magnetization M vanishes. In fields intermediate in magnitude ($\omega_0\tau \sim 1$), both M_x and M_y are nonzero. The condition that the pumping ray be transverse with respect to H_0 is essential for observation of the effect under consideration: for purely longitudinal pumping (with the orienting ray parallel to H_0) precession of the spins does not occur.

It is also not difficult to find the stationary solutions of Eq. (7) in the case when the field H_0 is oriented arbitrarily in space. Introducing the notation $\omega_q = -\gamma H_q$, q = x, y, z, we obtain

$$M_x = M_0' \frac{\Gamma_f^2 + \omega_x^2}{\Gamma_f^2 + \omega_x^2 + \omega_y^2 + \omega_z^2}, \quad M_y = M_0' \frac{\Gamma_f\omega_z + \omega_x\omega_y}{\Gamma_f^2 + \omega_x^2 + \omega_y^2 + \omega_z^2},$$
$$M_z = M_0' \frac{\omega_x\omega_z - \Gamma_f\omega_y}{\Gamma_f^2 + \omega_x^2 + \omega_y^2 + \omega_z^2}. \tag{10}$$

The expressions (10) enable us to determine the character of the deformation of the curve of the level-crossing signal in the case when the angle between H_0 and the axis F of the beam of light rays differs from $\pi/2$.

In order to record the above-described signal experimentally, one can use either the change in the intensity I_1 of the pumping light that has passed through the cell, or the change in the intensity of the fluorescence light. The quantity I_1 is connected with M_x by the following relation:

$$I_1(M_x) = I_1(0) + k_0 I_0 M_x, \tag{11}$$

where I_0 is the intensity of the incident light, and k_0 is a coefficient depending on the density of atoms in the cell.

As can be seen from formula (9), the signal from crossing of levels in the ground state has a very small width. In fact, in the case of nuclear paramagnets that can be oriented optically (Hg^{199}, Hg^{201}, Cd^{111}, Cd^{113}, He^3), for relaxation times $\tau \sim 1$ sec we have $\Delta H_0 \sim 10^{-3}$ Oe, since $\gamma/2\pi \sim 10^3$ Hz/Oe. For electronic paramagnets, such as Na^{23}, K^{39}, Rb^{87}, for the same values of τ the crossing linewidth turns out to be smaller by several orders ($\Delta H_0 \sim 10^{-6}$ Oe). It is precisely this fact that establishes the possibility of using the effect described as an extremely sensitive method of measuring ultra-weak magnetic fields.

5. CROSSING OF LEVELS IN THE PRESENCE OF RADIO-FREQUENCY FIELDS

Up to now, in placing the atoms in an external field we have assumed that the field is constant in time. The influence of modulation of the external electric or magnetic fields on the Stark and Zeeman effects has been treated theoretically in, e.g., [42]. The appearance in the luminescence spectrum of a set of additional lines was predicted, for the observation of which, however, it is necessary that the modulation frequency be greater than the Doppler linewidth. Therefore, to observe the effects predicted by the theory, ultra-strong constant magnetic fields and oscillating fields in the UHF range are necessary. Of course, no interference effects were taken into account in the treatment. For this reason it seemed that there were no grounds to expect any noticeable effects in the behavior of atoms in weak oscillating fields.

The possibility of the existence of interference effects in weak fields was first pointed out by Podgoretskiĭ and Khrustalev[16b, 17a]. In particular, they treated qualitatively the very important particular case when the oscillating radio-frequency field $H_1 \cos \omega t$ is oriented parallel to the constant field H_0, and predicted the appearance of interference beats and changes in the mean emission intensity. Owing to its simplicity, this case, which they called the parametric-resonance case, has found the most important practical applications.

The phenomenon of parametric resonance admits an intuitive semi-classical and quantum interpretation[16c, 17b]. The observed spectrum can be explained qualitatively in several ways: in the language of the theory of oscillations and of the theory of multi-quantum transitions, and, as will be shown below, as anti-crossing of levels of an atom "dressed" by the HF field.

The observed frequency spectrum can be explained most simply on the basis of well-known propositions from the theory of oscillations. The oscillating magnetic field $H_1 \cos \omega t$ parallel to the constant field modulates the spacing between the energy sublevels of each atom, and this affects the resonance luminescence of the atoms, in which beats appear with frequencies that are equal to or multiples of the frequency of modulation of the sublevels. The greatest depth of modulation occurs when the transition frequencies or multiples of them coincide with the modulation frequency. Then the mean emission intensity in the direction of observation also changes.

The quantum theory of parametric resonance, based on the use of the density-matrix method in first order of perturbation theory for the excited states of the atom, and a simple interpretation of the observed effects were developed in[16c].

When the atoms are in the ground state, their behavior can be described on the basis of equations of the Bloch type. Parametric resonance as a phenomenon of coherent multi-photon absorption was first treated quantitatively in[45,46,49]. Such a treatment makes it possible to predict qualitatively the basic features of this interesting phenomenon in weak oscillating fields.

An idea of the general quantum theory of interference phenomena as applied to the cases of the excited and ground states can be gained from the review[39]. The interference of the sublevels of the ground state can be recorded from the beats in the absorption of the atoms. In essence, the first such experiment was set up by Bell and Bloom[48], but, in terms of the interference of states, it was interpreted by Aleksandrov[44]. Parametric resonance in the ground state was first observed in cesium[44] and mercury[47] vapors.

An elegant method for describing the interaction of an atom with oscillating fields of any nature has been proposed and developed in papers by Kastler's school. It makes it possible to describe rigorously, from a single standpoint, all the coherent observable features of the resonance luminescence of atoms, both in the excited state and in the ground state. In contrast to the methods that had been applied earlier to solve problems of this type, in this method a consistent quantum-mechanical description is given not only of the behavior of the electrons in the atom but also of the electromagnetic field, both optical and radio-frequency. It is assumed that the Hamiltonian describing the behavior of the atom (e.g., in a weak pumping field of optical frequency and a strong oscillating radio-frequency field) can be represented in the form of two parts: $\hat{\mathscr{H}} = \hat{\mathscr{H}}_0 + \hat{\mathscr{H}}'$. The first part $\hat{\mathscr{H}}_0$ describes the atom and its interaction with the external radio-frequency magnetic field. The interaction of the atom in the radio-frequency field with the field of optical frequency is treated as a perturbation $\hat{\mathscr{H}}'$. The electromagnetic field is treated in the second-quantization representation, and the atom is described in terms of a density matrix. In this case, the eigenstates of the energy operator are states with energies $\gamma H_0 \hbar m + n\hbar\omega$, where m is the magnetic quantum number, n is the number of photons of the RF field, and ω is the frequency of this field. Concrete problems are solved either by means of perturbation theory or by using a diagram technique[49b,50].

The Hamiltonian $\hat{\mathscr{H}}_0$ describes the evolution of the atom and RF field as that of a single system; because of this, the method has come to be called the method of the atom "dressed" by a field, or simply the "dressed-atom" method. It is applicable for solving problems in which the oscillating RF field and constant field H_0 form any angles with the optical-pumping direction[51a]. With its help it is especially simple to describe the case of parametric resonance[51b], which can be considered as ordinary paramagnetic resonance with anti-crossing of the levels of the atom dressed by the RF field[39,45].

The concept of an atom "dressed" by a field is undoubtedly richer and more rigorous than the ideas developed in[16c,43,44], and makes it possible to describe by a single method all the varieties of coherent effects, both in the excited state and in the ground state of the atom. In the treatment of atoms in the ground state, the optical field is represented not in the form of photons but in the form of the "holes" that can be left by optical excitation in the ground state. This corresponds to the observation of parametric resonance in the ground state from the absorption.

Using the idea of an atom "dressed" by a field, we can next determine its magnetic moment $\mu = -\langle \partial \hat{\mathscr{H}} / \partial H \rangle$ and g-factor in the usual way. These "dynamical" quantities determined in this way do not coincide with the moment and g-factor of the free atom, but depend, naturally, on the amplitude of the RF field[52] and turn out to be proportional to the Bessel function $J_0(\gamma H_1/\omega)$. Using the theoretically determined dependence of the magnetic moment and g-factor on the amplitude H_1 of the RF field, it has turned out to be possible to make them vanish. In this case, the atom ceases to interact with the constant field H_0, the intensity of the transitions that do not depend on the field increases, and the lifetime pertaining to these transitions is increased[53]. Experiments with vapor of Hg^{199} in the ground state have fully confirmed these conclusions.

By establishing a suitable value of the RF-field amplitude H_1, it is possible to equalize the Larmor frequencies of two atoms with different g-factors. In this case, as was predicted and detected experimentally, in exchange collisions a transfer of coherence between atoms with substantially different static values of the g-factor should occur. Thus, in spin-exchange collisions, transfer of orientation from atoms of one sort to atoms of another occur[54].

Starting from the concept of an atom "dressed" by a field it is possible to perform a natural classification of the types of crossing of levels. In[55] it was shown

that all crossings of levels can be divided into two essentially different types. Crossing of the first type occurs between levels of different symmetry, and crossing of the second type occurs between levels with the same symmetry or with no definite symmetry. These two types are distinguished by characteristics of the crossing signals. The existence of different types of crossing of levels is conveniently illustrated by means of the energy diagram of an atom "dressed" by RF photons of different polarization. All forms of crossing of the first type and certain crossings of the second type can be found by studying the symmetry of the Hamiltonian $\hat{\mathscr{H}}_0$ by a method analogous to that given above for finding the quantization axis. Using a quantum formulation of the Majorana theorem, it is also possible to identify all the remaining crossings with crossings, of all orders, of the second type (Majorana crossings). In[55b] the results are given of experiments on the observation of the resonance phenomena near the points of these crossings. A particular case of a crossing of the second type is the crossing in zero field of the levels of an atom "dressed" by RF photons of circular polarization[56]. Other examples of crossings of the second type can be found in[55b], where results of experiments on the observation of resonance phenomena near the points of these crossings are also given.

We proceed now to consider the Hanle effect in the ground state in the presence of RF fields. The solution of the problem of the crossing of levels of an atom in a strong radio-frequency field becomes substantially simpler if we make use of the "dressed-atom" theory. Without giving the detailed calculations, we shall cite certain results of this theory[50].

As earlier, we shall assume that atoms situated in a weak field H_0 are subjected to optical pumping by a beam of rays of circularly polarized light, parallel to the axis Ox, and to the action of a radio-frequency field $H_1 \cos \omega t$ oriented along the axis Oz. We shall assume that Γ_f, $\omega_0 \ll \omega$, i.e., that the evolution of the atomic system as a result of relaxation and interaction with the field H_0 is insignificant over the period of the radio-frequency field. Such a situation is, in essence, the Hanle effect for an atom dressed by a field[49b,c]. In zero field the energy spectrum of a "dressed" atom with total moment F consists of an infinite number of $(2F + 1)$-fold degenerate levels, separated by an interval $\hbar\omega$ and corresponding to a complete system consisting of the atom in the presence of 0, 1, ..., n, ... photons with frequency ω. The field H_0 lifts the degeneracy in each of the multiplets, and, consequently, crossing of an infinite set of levels of the joint system occurs at the point $H_0 = 0$.

Quantitatively, the problem of finding the magnetization vector M, which is proportional within the given multiplet F to the mean value of the orientation vector $\langle \mathbf{S} \rangle$ ($\mathbf{M} = (ge\hbar/2mc)\langle \mathbf{S} \rangle$), is solved by reducing it to the equivalent problem for a new observable quantity—the mean value $\langle \tilde{\mathbf{S}} \rangle$ of the orientation of the "dressed" atom. As shown in[49c], the equation of motion for $\langle \tilde{\mathbf{S}} \rangle$, like that for $\langle \mathbf{S} \rangle$ of the "bare" atom, has the form of the Bloch equations:

$$\frac{d}{dt}\langle \tilde{\mathbf{S}} \rangle = -\Gamma_f \langle \tilde{\mathbf{S}} \rangle + \gamma [\langle \tilde{\mathbf{S}} \rangle \times \tilde{\mathbf{H}}] + \tilde{\mathbf{S}}_0' \Gamma_f, \tag{12}$$

but the quantities $\tilde{\mathbf{H}}$ and $\tilde{\mathbf{S}}_0'$ are defined by the formulas

$$\left.\begin{aligned} \tilde{H}_{x,y} &= H_{x,y} J_0\left(\frac{\omega_1}{\omega}\right), \\ \tilde{H}_z &= H_z, \\ \tilde{\mathbf{S}}_0' &= \mathbf{M}_0' J_0\left(\frac{\omega_1}{\omega}\right). \end{aligned}\right\} \tag{13}$$

In these expressions, $J_0(\omega_1/\omega)$ is the Bessel function of order zero, and $\omega_1 = -\gamma H_1$. The stationary solutions of Eq. (12) are written analogously to the expressions (10):

$$\left.\begin{aligned} \langle \tilde{S}_x \rangle &= \tilde{S}_0' \frac{\Gamma_f^2 + \tilde{\omega}_x^2}{\Gamma_f^2 + \tilde{\omega}_x^2 + \tilde{\omega}_y^2 + \tilde{\omega}_z^2}, \\ \langle \tilde{S}_y \rangle &= \tilde{S}_0' \frac{\Gamma_f \tilde{\omega}_z + \tilde{\omega}_x \tilde{\omega}_y}{\Gamma_f^2 + \tilde{\omega}_x^2 + \tilde{\omega}_y^2 + \tilde{\omega}_z^2}, \\ \langle \tilde{S}_z \rangle &= \tilde{S}_0' \frac{\tilde{\omega}_x \tilde{\omega}_z - \Gamma_f \tilde{\omega}_y}{\Gamma_f^2 + \tilde{\omega}_x^2 + \tilde{\omega}_y^2 + \tilde{\omega}_z^2}. \end{aligned}\right\} \tag{14}$$

The quantities M_x, M_y and M_z, recorded in the laboratory coordinate frame, are related to the components of the vector $\langle \tilde{\mathbf{S}} \rangle$ as follows:

$$\mathbf{M} = R(\mathbf{H}_1)\langle \tilde{\mathbf{S}} \rangle. \tag{15}$$

where $R(H_1)$ is the matrix of the rotation through an angle $(\omega_1/\omega)\sin \omega t$ about the axis of the oscillations of the field H_1, i.e., about the axis Oz.

Using the expressions (11), (14) and (15), we find

$$I_1 = I_1(0) + k_0 I_0 \left\{ \langle \tilde{S}_x \rangle \left[J_0\left(\frac{\omega_1}{\omega}\right) + \sum_{p>0} 2J_{2p}\left(\frac{\omega_1}{\omega}\right) \cos 2p\omega t \right] - \langle \tilde{S}_y \rangle \sum_{p \geqslant 0} 2J_{2p+1}\left(\frac{\omega_1}{\omega}\right) \sin (2p+1)\,\omega t \right\}, \tag{16}$$

where $J_q(\omega_1/\omega)$ are Bessel functions of order q.

It can be seen from this formula that, in the light that has passed through the cell, intensity modulation is displayed in all the harmonics of the frequency ω whose amplitude varies in a resonance manner near the value $H_0 = 0$. In particular, at frequency ω we have

$$I_1(\omega) = k_0 I_0 \left[-2J_1\left(\frac{\omega_1}{\omega}\right) \right] \langle \tilde{S}_y \rangle \sin \omega t. \tag{17}$$

It should be noted that in the particular case when the field H_0 is parallel to the field H_1 (parametric resonance) and perpendicular to the beam of the pumping light rays, the problem can also be solved exactly without assuming that the field H_0 is small (i.e., for an arbitrary value of ω_0).

The solution of Eqs. (12), (15) for M_Z with the initial condition $M_Z(0) = 0$ is, as before, $M_Z = 0$. But the solution for M_+ becomes time-dependent and can be represented in the form

$$M_+(t) = M_0' [A_0 + \sum_{p>0} (A_p e^{ip\omega t} + A_{-p} e^{-ip\omega t})], \tag{18}$$

where

$$A_0 = \sum_{n=-\infty}^{\infty} \frac{J_n^2(\omega_1/\omega)}{1 - i(\omega_0 + n\omega)\tau}, \quad A_{\pm p} = \sum_{n=-\infty}^{\infty} \frac{J_n(\omega_1/\omega) J_{n \pm p}(\omega_1/\omega)}{1 - i(\omega_0 + n\omega)\tau}.$$

The expressions (18) for the case of parametric resonance can be obtained by solving equations of the Bloch type without invoking the "dressed-atom" formalism[16c]. The value of the approach described above lies in its universality. It enables one to solve problems in which it is possible to separate out strong and weak perturbations of the system by a field, as is the case, e.g., for resonance in two radio-frequency fields.

It can be seen from (18) that adding an oscillating magnetic field $H_1 \cos \omega t$ to H_0 leads to the appearance of two important features of the "dressed-atom" Hanle effect.

First, the transverse magnetization M_+ becomes a periodic function of time and contains harmonics with frequencies $p\omega$.

Secondly, a macroscopic transverse magnetization appears not only in the vicinity of the field $H_0 = 0$ but also for other values of the field H_0 satisfying the resonance condition: $\omega_0 = n\omega$, where n is an integer.

The width of these resonances does not depend on ω_1 and is easily determined from the expressions (14). The fact that M_+ depends on time is fairly obvious, since there is a periodic external perturbation $H_1 \cos \omega t$. The appearance of resonances for nonzero values of the constant field H_0 admits the following intuitive interpretation. The field $H_1 \cos \omega t$ modulates the frequency of the Larmor precession about the axis Oz, and so the spins are not distributed isotropically in the (x, y)-plane (as was the case in the Hanle effect for $\omega_0\tau \ll 1$), but are grouped in packets. The time-averaged angular velocity of precession remains equal to ω_0, and therefore in one period of the field H_1 the angle of rotation of the spins amounts to $\omega_0(2\pi/\omega)$. In order that the grouping effect build up, the relation $\omega_0 \cdot 2\pi/\omega = 2\pi n$, i.e., the condition $\omega_0 = n\omega$ already obtained (parametric resonance), should be fulfilled.

The most important feature of parametric resonance is the fact that the linewidth is independent of the amplitude H_1 of the modulating field. The reason for this lies in the fact that an oscillating field oriented strictly parallel to Oz cannot induce magnetic-dipole transitions between eigenstates of an atomic system in the field H_0, but leads only to modulation of a parameter—the frequency of the Larmor precession in the magnetic field. Therefore, the Zeeman levels are not broadened, and the resonance line remains very narrow. A rigorous approach to the problem using quantization of the field[46, 49a, c] leads to the same conclusion. The interaction of the atom with the radio-frequency field can be described as the virtual absorption and emission of radio-frequency quanta[46, 49a]. The absence of real transitions leads to the absence of broadening of the lines by the field $H_1(t)$. We note also that parametric resonances arise at the points corresponding to the crossing of energy levels of the total system "atom plus radio-frequency field," as is completely obvious for resonance in zero field.

Although, up to now, no limitations have been imposed on the frequency of the field $H_1(t)$, to improve the conditions for observation of an optical crossing signal it is convenient to choose $\omega \gg 1/\tau$, i.e., nonadiabatic modulation. With such a choice, the amplitude of the measured signal at frequency ω is directly proportional to the magnitude of the static external field H_0, and this is very convenient for measurements of weak magnetic fields.

For practical use in magnetometry, the effect considered possesses a whole series of advantages as compared with the Hanle effect for a "bare" atom in the ground state. We shall list them:

1) Outside the resonance region (for $\omega_0\tau \gg 1$), i.e., when the condition for crossing of sublevels is violated, the amplitude of the modulation of the magnetization M_x tends to zero. In experiments on the Hanle effect in an unmodulated field, small changes in the intensity of the ray passing through are measured, amounting in the neighborhood of the resonance to only a few percent of the intensity outside the resonance region. Therefore, the problem arises of separating these from the slow variations in the intensity of the initial ray. The usual technique of adiabatically slow modulation of the magnetic field in the vicinity of each value of H_0 is not very effective, since the frequency Ω of such modulation should satisfy the condition $\Omega < 1/\tau$, i.e., falls in a region of high spectral density of noise.

2) The resonance linewidth does not depend on the amplitude H_1 of the modulating field. It varies according to the relationship between the components of the constant field H_0, and for $\gamma H_{x,y} \ll \Gamma_f$ is equal to $2\Gamma_f$.

3) To measure the components of the field H_0 it is always possible to use a resonance line in the form of a dispersion curve. In this case, the amplitude of the optical signal recorded is, within the limits of the linewidth, proportional to the field component being measured.

The above-mentioned features of parametric resonance in zero field enable us to use it as an effective method for measuring static or slowly varying ultra-weak magnetic fields with a record sensitivity, for optical magnetometers, of 10^{-9}–10^{-10} Oe[33]. However, the practical use of the effect in its pure form (H_1 strictly parallel to H_0) is frequently difficult, inasmuch as the direction of the very small field being measured needs to be specified previously in order to orient the experimental setup spatially.

An interesting feature of the expression (17) is the dependence, given by formula (13), of the frequency of the Larmor precession in the constant field H_0 on the frequency and amplitude of the modulating field $H_1(t)$. It disappears if $H_1 \parallel H_0$ and is a maximum when $H_1 \perp H_0$. In the latter case, the precession frequency is decreased by the factor $J_0(\omega_1/\omega)$. As a result, broadening of the crossing curves corresponding to $H_x = 0$ and $H_y = 0$ arises. In particular, the linewidth tends to infinity at $J_0(\omega_1/\omega) = 0$.

The characteristics of the signal M_x recorded at frequency ω were subjected in[33c] to a thorough experimental check in a vapor of oriented Rb^{87} atoms, for which the relaxation time amounted to 1 sec in a cell with a paraffin coating. To eliminate the effect of the earth's magnetic field and magnetic disturbances, magnetic screening was used. A magnetic field of arbitrary direction was produced inside the screen by means of three pairs of Helmholtz coils. Although the ground state of Rb^{87} contains two hyperfine levels, this fact does not essentially change the result (17). The dependence of the amplitude of the signal M_x on the constant field is determined by the factor

$$\frac{\Gamma_f\tilde{\omega}_z + \tilde{\omega}_x\tilde{\omega}_y}{\Gamma_f^2 + \tilde{\omega}_x^2 + \tilde{\omega}_y^2 + \tilde{\omega}_z^2}.$$

If the field H_0 is small, i.e., $\gamma H_0 \ll \Gamma_f$, then from (17) it is easy to obtain

$$M_x(\omega) = -2M_0' J_0\left(\frac{\omega_1}{\omega}\right) J_1\left(\frac{\omega_1}{\omega}\right)\left[\frac{\omega_z}{\Gamma_f} + J_0^2\left(\frac{\omega_1}{\omega}\right)\frac{\omega_x\omega_y}{\Gamma_f^2}\right]\sin\omega t. \quad (19)$$

Thus, in the first approximation, the measured signal depends only on the component H_z of the constant weak field.

We shall discuss the shape of the detected signal for different relationships between the components of the static field H_0:

a) $H_x = H_y = 0$. $M_x(\omega)$ has the shape of a dispersion curve with its center at the field $H_z = 0$ and with width

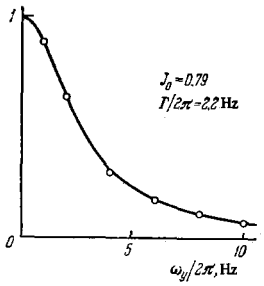

FIG. 4. Dependence on $\omega_y/2\pi$ of the sensitivity in the measurement of the z-component of the field.

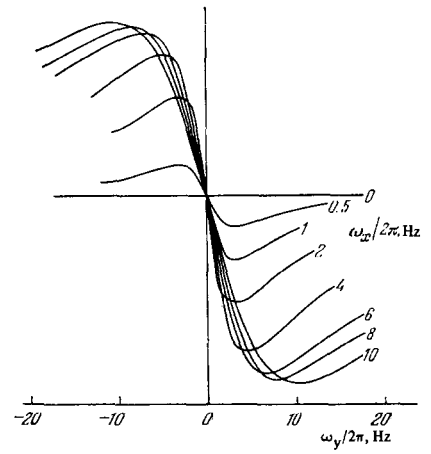

FIG. 5. Resonance curves for values of the field H_X for $\omega_Z = 0$ ($\Gamma_f/2\pi$ = 2.2 Hz, $J_0(\omega_1/\omega_0) = 0.79$).

$2\Gamma_f$. The variation of $M_X(\omega)$ for small H_Z is linear in the field.

b) One of the transverse components is nonzero. For definiteness, let $H_X = 0$ and $H_Y \neq 0$. The resonance curve is a dispersion curve, $\Gamma_f\tilde{\omega}_Z/(\Gamma_f^2 + \tilde{\omega}_Y^2 + \tilde{\omega}_Z^2)$, but its width is increased:

$$\Delta\omega_z = 2\left[\Gamma_f^2 + J_0^2\left(\frac{\omega_1}{\omega}\right)\omega_y^2\right]^{1/2},$$

and the slope of the resonance curve at the point $\omega_Z = 0$ is decreased, as a result of which the accuracy of a measurement of H_Z is lowered in the presence of $H_Y \neq 0$.

c) Both transverse components are nonzero: $H_X \neq 0$, $H_Y \neq 0$. The resonance curve does not have a pure dispersion shape. It is broadened, and its center is displaced and is found at the point $\tilde{\omega}_Z = -\tilde{\omega}_X\tilde{\omega}_Y/\Gamma_f$, i.e., $M_X(\omega)$ vanishes for $H_Z \neq 0$.

d) The longitudinal field-component $H_Z = 0$. If we now vary H_Y, a signal appears only under the condition $H_X \neq 0$, and disappears for $H_X = 0$. The resonance curve has the shape of a dispersion curve, $\tilde{\omega}_X\tilde{\omega}_Y/(\Gamma_f^2 + \tilde{\omega}_X^2 + \tilde{\omega}_Y^2)$, with width $\Delta\tilde{\omega}_Y = 2(\Gamma_f^2 + \tilde{\omega}_X^2)^{1/2}$. The greatest sensitivity to variations of H_Y, corresponding to the maximum slope of the curve, is attained when $\tilde{\omega}_X = \Gamma_f$. It is of the same order as in the determination of H_Z in case (a).

e) $H_X \neq 0$, $H_Z \neq 0$. The detected signal $M_X(\omega)$ vanishes under the condition $\Gamma_f\tilde{\omega}_Z + \tilde{\omega}_X\tilde{\omega}_Y = 0$, i.e., when $\omega_Y = -\omega_Z/J_0(\omega_1/\omega)$, if the sensitivity with respect to variations of H_Y is chosen to be optimal: $\tilde{\omega}_X = \Gamma_f$. Although a number of approximations were made in calculating the results described above, the experiment of[33c] gives excellent agreement with the theory (Figs. 4–6).

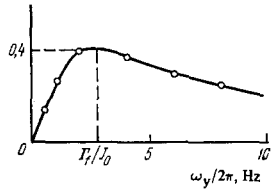

FIG. 6. Dependence on $\omega_X/2\pi$ of the sensitivity in the measurement of the y-component of the field.

In the conditions of the experiment described above, let the vapor of the atoms in the cell be subjected to the action of an additional oscillating magnetic field $\mathbf{H}_2 \cos\omega t$, parallel to the axis Oy. We shall assume that the following inequalities are satisfied:

$$\omega \gg \Gamma_f,\ \Omega,\ \omega_0,\ \Omega \gtrsim \Gamma_f,\ \omega_0.$$

The exact solution of the analogous problem, i.e., the determination of the stationary values of the components of M(t), which are now functions of time, encounters considerable difficulties. However, using the "dressed-atom" formalism, in this case too it is possible to simplify the calculation considerably and obtain a result correct to terms of first order inclusive in Γ_f/Ω. In this case, an atom "dressed" by the field $\mathbf{H}_1\cos\omega t$ is considered, and the above-mentioned transition is effected to the equivalent problem for the reduced mean value $\langle\tilde{\mathbf{S}}\rangle$, the evolution of which is determined in this case by the field $\tilde{\mathbf{H}}_0 + \tilde{\mathbf{H}}_2 \cos\Omega t$, where the wavy lines over the field values indicate that these quantities are defined by means of the transformation (13). Thus, everything reduces to calculating the characteristics of the parametric resonance excited by the field $\tilde{\mathbf{H}}_2 \cos\Omega t$, where the wavy lines over the field values indicate that these quantities are defined by means of the transformation (13). Thus, everything reduces to calculating the characteristics of the parametric resonance excited by the field $\tilde{\mathbf{H}}_2 \cos\Omega t$ in a system of atoms "dressed" by the field $\mathbf{H}_1 \cos\omega t$. Formally, the expressions obtained in this case should coincide with those found above, with the sole difference that quantities that are "twice-dressed" by two radio-frequency fields will appear in them:

$$\tilde{\tilde{\omega}}_x = \omega_x J_0\left(\frac{\omega_1}{\omega}\right) J_0\left(\frac{\tilde{\Omega}_2}{\Omega}\right),\quad \tilde{\tilde{\omega}}_y = \omega_y J_0\left(\frac{\omega_1}{\omega}\right),\quad \tilde{\tilde{\omega}}_z = \omega_z J_0\left(\frac{\tilde{\Omega}_2}{\Omega}\right),$$

$$\tilde{\Omega}_2 = -\gamma H_2 J_0\left(\frac{\omega_1}{\omega}\right),\quad \tilde{\tilde{S}}_0' = M_0' J_0\left(\frac{\omega_1}{\omega}\right) J_0\left(\frac{\tilde{\Omega}_2}{\Omega}\right),$$

and the final expressions for the quantities actually recorded will take the form, with the assumption $\omega_0 \ll \Gamma_f$,

$$\left.\begin{aligned} M_x(\omega) &= -2\tilde{\tilde{S}}_0' J_1\left(\frac{\omega_1}{\omega}\right)\left(\frac{\tilde{\tilde{\omega}}_z}{\Gamma_f} + \frac{\tilde{\tilde{\omega}}_x\tilde{\tilde{\omega}}_y}{\Gamma_f^3}\right)\sin\omega t,\\ M_x(\Omega) &= -2\tilde{\tilde{S}}_0' J_0\left(\frac{\omega_1}{\omega}\right) J_1\left(\frac{\tilde{\Omega}_2}{\Omega}\right)\left(\frac{\tilde{\tilde{\omega}}_y}{\Gamma_f} - \frac{\tilde{\tilde{\omega}}_x\tilde{\tilde{\omega}}_z}{\Gamma_f^3}\right)\sin\Omega t,\\ M_x(\omega, \Omega) &= 4\tilde{\tilde{S}}_0' J_1\left(\frac{\omega_1}{\omega}\right)\frac{J_1(\tilde{\Omega}_2/\Omega)}{J_0(\tilde{\Omega}_2/\Omega)}\frac{\Gamma_f}{\Omega}\left(\frac{\tilde{\tilde{\omega}}_x}{\Gamma_f} - \frac{\tilde{\tilde{\omega}}_y\tilde{\tilde{\omega}}_z}{\Gamma_f^3}\right)\sin\omega t\cos\Omega t, \end{aligned}\right\} \quad (20)$$

where $M_X(\omega)$, $M_X(\Omega)$ and $M_X(\omega, \Omega)$ are respectively the components of the macroscopic magnetization along the axis Ox varying with frequencies ω and Ω and with a combination of the frequencies ω and Ω.

It can be seen from these expressions that each of the oscillating components M_X is proportional to the first power of each of the components of the external constant field and to the second power of its other components, and this makes it possible to effect a simultaneous measurement of all three components of a weak magnetic field.

6. APPLICATION OF THE EFFECT OF CROSSING OF GROUND-STATE LEVELS IN QUANTUM MAGNETOMETRY

Because of the exceptional narrowness of the crossing lines, any of the described variants of the signal from the crossing of ground-state levels in zero field can, in principle, be used to measure ultra-weak magnetic fields. A magnetometer of such a type was first described in[33]. The block scheme of the experimental setup is depicted in Fig. 7. The resonance optical cell, containing Rb^{87} vapor, has paraffin-coated walls, ensuring a relaxation time $\tau \sim 1$ sec. Optical orientation of the Rb^{87} atoms in the ground state is produced by means of the D_1-line ($\lambda = 7947$ Å) of circularly polarized light from a Rb^{87} spectral lamp. The cell is placed in the center of a multi-layer magnetic screen with screening coefficient $\sim 10^5$. A system of Helmholtz coils is used to produce cancellation of the residual magnetic field, component by component, and also to produce the constant field $\mathbf{H}_0$ and the oscillating field $\mathbf{H}_1 \cos \omega t$ ($\omega/2\pi = 420$ Hz). The x-component of the magnetization of the Rb^{87} vapor in the cell is detected by the intensity of the pumping light passing through it, and this light is led out of the magnetic screen by means of a light-guide and recorded by a photomultiplier. Amplification and synchronous detection of the signal are performed at the frequency of the first harmonic of the parametric resonance ($\omega/2\pi = 420$ Hz).

Figure 8 shows the experimental curve of the amplitude of this harmonic, obtained by slow scanning of the field $\mathbf{H}_0$ in the vicinity of the zero value. The signal was measured under conditions of low pumping-light intensity, when the resonance linewidth is a minimum and is determined principally by thermal relaxation. The experiment shows good agreement with the theory, despite the numerous assumptions associated with the derivation of the expression (19).

We note that for the coated optical cells used in the experiment, resonance-line broadening associated with inhomogeneity of the magnetic field is absent, since it is averaged as a result of the rapid motion of the oriented atoms (this is not true for cells with a buffer gas). However, too low an intensity of the pumping light does not ensure maximum sensitivity to magnetic-field variations, inasmuch as the magnitude of the detected optical signal is then decreased.

We shall consider in more detail the question of the dependence of the sensitivity of the magnetometer on the different parameters of the apparatus, with the purpose

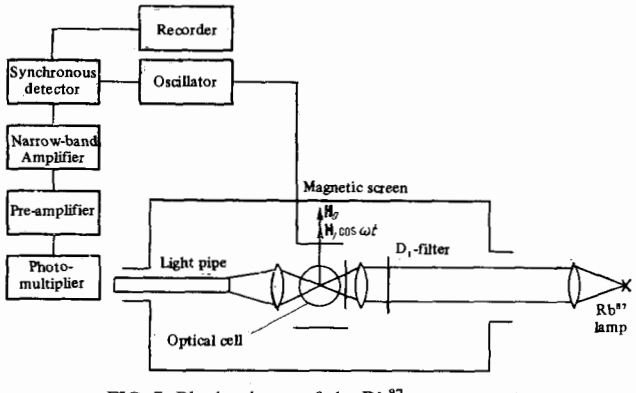

FIG. 7. Block scheme of the Rb^{87} magnetometer.

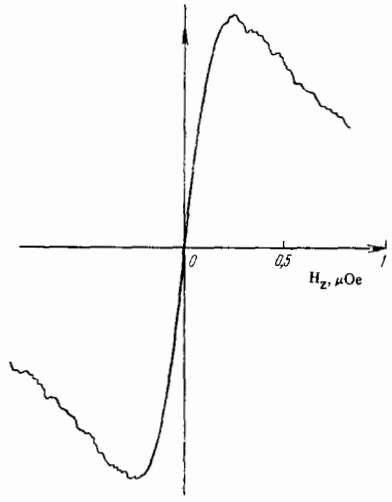

FIG. 8. Curve of the amplitude of the first harmonic of $M_x(\omega)$.

of optimizing these parameters. The sensitivity σ_i of the magnetometer in the measurement of the i-th component of the field $\mathbf{H}$ can be defined as $\sigma_i = 1/h_i$, where h_i is the smallest variation of the given magnetic-field component that can be measured, i.e., that can be distinguished from the noise. We shall assume that h_i is that variation of the external-field component H_i for which the change δV_i in the signal at the output of the synchronous detector coincides with the noise level B_i:

$$\delta V_i = \frac{\partial V}{\partial H_i} h_i = B_i,$$

whence we obtain

$$\sigma_i = \frac{1}{B_i} \frac{\partial V}{\partial H_i}.$$

If I_0 is the intensity of the light effecting the pumping, the amplitude $I_1(\omega)$ of the first harmonic of the intensity of the light passing through has the form

$$I_1(\omega) = 2k_0 I_0 M_0' J_0\left(\frac{\omega_1}{\omega}\right) J_1\left(\frac{\omega_1}{\omega}\right) \frac{\omega_0 \tau}{1 + \omega_0^2 \tau^2}, \qquad (21)$$

where k_0 is a coefficient proportional to the vapor density in the cell. Since the signal V at the output of the synchronous detector is proportional to $I_1(\omega)$ ($V = k_1 I_1(\omega)$), for the sensitivity σ_i in zero field we obtain the expression

$$\sigma_i = \frac{1}{B_i} 2k_0 k_1 I_0 M_0' J_0\left(\frac{\omega_1}{\omega}\right) J_1\left(\frac{\omega_1}{\omega}\right) \gamma\tau. \qquad (22)$$

The sources of the noise characterized by the quantity B_i can be divided into two groups: magnetic noise, associated with random variations of the field inside the screen, and noise of different origin (the intrinsic noise of the instruments, shot noise, and so on). The problem of separating the signal of the field being measured from the noise of a magnetic nature can be solved only if certain properties of the signal are known beforehand (e.g., that the field is periodic in time or in space). In such a case, use of the appropriate measurement technique may make it possible to lower the general level of uncontrollable random noise and to obtain high sensitivity. But if there are no prior data on the character of the variation of the field being measured, it cannot be separated from the magnetic noise—in particular, from random variations of the earth's magnetic field and field variations due to the gradual magnetization of the screen, and this imposes certain bounds on the limiting sensitivity of the apparatus.

Under the conditions of the experiment of[33b], the magnetic noise, including the noise associated with the instability of the source supplying the compensating coils, was of low level (the noise levels in the region of maximum sensitivity, i.e., for $H_0 = 0$, and for $H_0 \gg \Gamma_f$ were found to be the same, indicating the dominant role of noise of a nonmagnetic nature).

If we assume that the origin of the noise is associated mainly with the shot effect in the photomultiplier, we can obtain the following expression for the sensitivity of the magnetometer[36]:

$$\sigma_z = 2k_0 M_0 J_0\left(\frac{\omega_1}{\omega}\right) J_1\left(\frac{\omega_1}{\omega}\right)\left(\frac{\pi\tau_D I_0 \eta}{h\nu}\right)^{1/2} \frac{\gamma\tau^2}{T_p}, \qquad (23)$$

where we have used the expression (22) for σ_i and the formula for the shot effect:

$$B_i = k_1\left(\frac{h\nu I_0}{\eta\pi\tau_D}\right)^{1/2} \qquad (24)$$

with the condition $I_1 \approx I_0$; here η is the quantum yield of the photocathode, ν is the pumping-light frequency, and τ_D is the integration constant in the synchronous detection.

According to (23), to achieve the maximum sensitivity in measuring a field we must choose a working substance for which the gyromagnetic ratio of the atoms in the ground state is as large as possible, and also ensure a long relaxation time τ. The amplitude and frequency of the modulating field $H_1 \cos \omega t$ appear in the expression for the sensitivity through the factor $J_0(\omega_1/\omega)J_1(\omega_1/\omega)$, which attains its maximum value, equal to 0.34, when $\omega_1/\omega = 1.1$. Since the pumping rate is proportional to the light intensity: $1/T_p = \alpha I_0$, and $1/\tau = (1/T) + \alpha I_0$, the sensitivity σ_z is proportional to the quantity $I_0^{3/2}/(\alpha I_0 + T^{-1})^2$, and this gives the optimal value $I_0 = 3/\alpha T$ (the resonance linewidth $\Gamma_f = \tau^{-1}$ is four times greater than the relaxation width). Experiment gives a lower optimal value of I_0, so that $\Gamma_f = 2.5/T$.

To determine experimentally the sensitivity attained, rectangular current pulses corresponding to known variations of the field H_0 were fed into the Helmholtz coils producing the fields H_0 and $H_1 \cos \omega t$. Figure 9 shows the signal at the output of the synchronous detector when the amplitude of the field variations was 2×10^{-9} Oe ($\tau_D = 3$ sec). It can be seen that field variations $\sim 10^{-9}$ Oe are easily registered. In the measurement of periodic variations of the field, the use of accumulation of the signal over a large number of periods makes it possible to raise the sensitivity of the apparatus. Figure 10 shows the oscillogram of the signal at the output of the multi-channel analyzer for amplitude $\delta H_0 = 3 \times 10^{-10}$ Oe with accumulation over 3000 periods ($\tau_D = 0.1$ sec).

Up to now it has been assumed that the field H_0 being measured is oriented strictly along the axis Oz. To estimate the effect of the transverse components of the field H_0 on the magnitude of the detected signal, we can make use of the expression (19), which gives, in the first nonvanishing approximation in the small transverse components of the field H_0, the corrections to M_x due to the nonparallel directions of the fields H_0 and $H_1 \cos \omega t$. They are of second order in the transverse components of the field H_0. Thus, in a weak field a magnetometer of this type selectively detects the z-component of the field in the first approximation.

As already noted, the use of resonances of this type makes it possible to perform a systematic measurement of all the components of a constant field. The physics of the phenomenon differs little, proportionally, from ordinary parametric resonance, and the realization of the corresponding measuring apparatus reduces only to certain technical alterations of the apparatus described above. The sensitivity with respect to measurements of the transverse components of the field H_0 turns out to be lower than in the measurement of H_z. The measurement procedure is the same. Suppose that the external field is previously compensated, and a weak field δH_0 is being measured. According to formulas (14) and (17), the change in the detected signal is proportional to δH_z in the first approximation, and this enables us to measure this component. By "decompensating" H_y, we can make the signal sensitive to δH_x in the first approximation. For optimal decompensation ($\tilde{\omega}_y = \Gamma_f$) the signal is proportional to the sum $\delta H_z + J_0(\omega_1/\omega)\delta H_x$. Deducting from this the contribution of the variation δH_z, we obtain a signal proportional to δH_x. δH_y is determined analogously. From the expressions (14) and (17) it is easy to obtain the relative sensitivity of the measurements of the different components: $\sigma_{x,y} = \frac{1}{2}J_0(\omega_1/\omega)\sigma_z$. The greatest sensitivity in the measurement of variations of the components H_x and H_y is attained when the expression $J_0^2(\omega_1/\omega)J_1(\omega_1/\omega)$ equals its maximum value 0.27, which gives the optimal value of the field amplitude H_1: $\omega_1/\omega = 0.85$. This differs from the optimal value for measurement of H_z ($\omega_1/\omega = 1.1$), so that in practice some intermediate value is taken.

Experimentally[36], field variations $h_z \approx 6 \times 10^{-10}$ Oe and $h_{x,y} \approx 1.5 \times 10^{-9}$ Oe have been detected. An instrument making it possible to compensate the three field components automatically by means of the procedure outlined above is described in[57].

Application of an additional magnetic field $H_2 \cos \Omega t$ directed along the axis Oy makes it possible to register all the field components simultaneously. As shown above, the intensity of the light passing through the cell contains harmonics proportional to the different components of the field H_0 (20).

In[36] oscillating magnetic fields with frequencies $\omega/2\pi = 420$ Hz and $\Omega/2\pi = 15$ Hz were used. Harmonics of the frequencies ω and Ω, with amplitudes proportional to the field components H_z and H_y respectively, are detected in phase by two synchronous detectors. The signal proportional to H_x is the product of two harmonics, and it can be regarded as the amplitude modulation, with

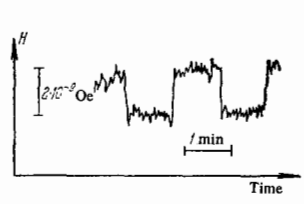

FIG. 9. Signal at the output of the synchronous detector when step-like magnetic-field pulses are applied to the cell.

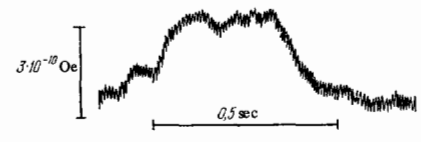

FIG. 10. The signal obtained at the output of the synchronous detector for accumulation over 3000 periods.

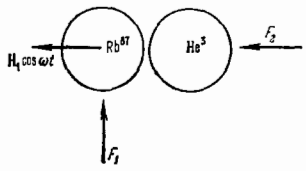

FIG. 11. Scheme of the experiment for observing the magnetization of He^3 vapor.

frequency Ω, of a harmonic of the frequency ω. To distinguish it a third synchronous detector is used, processing the signal from the output of the synchronous detector tuned to the frequency ω. The sensitivity is optimized with respect to the fundamental parameters of the experiment in the same manner as described above. New optimal values of the amplitudes of the oscillating fields are then obtained. According to (20), the sensitivities in the measurements of the different components do not coincide but are proportional to different combinations of Bessel functions, and this gives the following theoretical optimal-parameter values: $\omega_1/\omega \approx 0.84$, $\widetilde{\Omega}_2/\Omega \approx 1.1$. Then $\sigma_z = 0.51\sigma_z^0$, $\sigma_y = 0.56\sigma_z^0$, and $\sigma_x = 0.52\,(\Gamma_f/\Omega)\sigma_z^0$, where σ_z^0, defined by the expression (23), is the optimal sensitivity in the presence of one modulating oscillating field $\mathbf{H}_1$. The choice of the frequency of the modulating field $\mathbf{H}_2 \cos \Omega t$ is determined by a compromise between the need to ensure maximum Γ_f/Ω for the greatest sensitivity in measuring the x-component of the field, and the necessity of fulfilling the condition $\Omega > \Gamma_f$ in order that it be possible to take into account only the resonance terms in zero field (n = 0 in the expression (18)).

The simultaneous recording of the signals in the measurement of the three components of the field $\mathbf{H}_0$, obtained in [36], shows that the method makes it possible to detect field variations $h_z \approx 10^{-9}$ Oe, $h_y = 1.5 \times 10^{-9}$ Oe, $h_x \approx 3 \times 10^{-8}$ Oe. We note that the experimentally obtained optimal values of the amplitudes of the oscillating fields are given by $\omega_1/\omega \approx 0.8$, $\widetilde{\Omega}_2/\Omega \approx 0.95$.

A beautiful illustration of the possibilities of practical application of magnetometers of the type described is the experiment of [58], in which the ultra-weak static magnetic field of optically oriented He^3 vapor was detected. The scheme of the experiment is depicted in Fig. 11. An optical cell containing He^3 nuclei that can be oriented by means of an F_2 ray (λ = 10,830 Å, circular polarization) is placed next to a cell with Rb^{87} atoms, which serves as the detecting element of the magnetometer. The static magnetic field of the polarized He^3 nuclei is modulated by a field $\mathbf{H}_1 \cos \omega t$ parallel to it, and transverse pumping of the Rb^{87} is effected by a circularly polarized F_1 ray (the D_1 line), so that the conditions for the level-crossing parametric resonance described in Sec. 5 are fulfilled. The detected signal is the first harmonic (of frequency ω) of the modulation of the intensity of the ray F_1 passing through the Rb^{87} cell. The signal is proportional to the static field averaged over the volume of this cell, owing to the averaging of the rate of precession of the magnetization in the motion of the oriented Rb^{87} atoms.

The optical pumping time for He^3 is very long ($\sim$ 10 min), as is well illustrated by the experimental curve (Fig. 12) of the transitional process of establishing the stationary polarization of the He^3 nuclei after the ray F_2 is switched on. The time for relaxation of the orientation of the ground state of He^3 after F_2 is switched off is still longer (several hours). To eliminate the distorting influence of the gradual magnetization of the

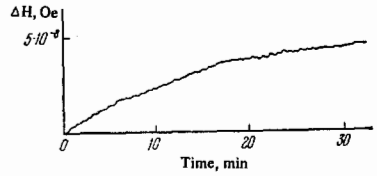

FIG. 12. Transitional process establishing the stationary polarization of He^3 nuclei under the action of optical pumping.

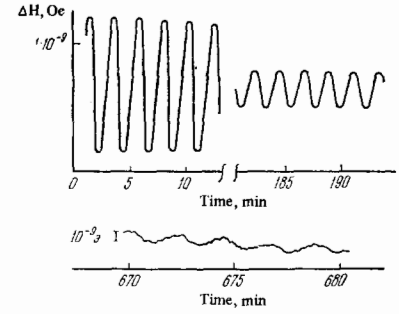

FIG. 13. Modulation of the magnetization of He^3 in the ground state after the optical pumping is switched off.

magnetic screen, the following technique is used to separate out the signal associated with the polarization of the He^3 nuclei. A very weak constant magnetic field h (2 μOe) is applied perpendicular to the pumping ray F_2, as a result of which the He^3 spins precess with a very low Larmor frequency ($\nu \approx 6 \times 10^{-3}$ Hz), and the field in the Rb^{87} cell is modulated with this same frequency. Figure 13 shows a recording of this modulation, which was registered with certainty even eleven hours after the F_2 ray was switched off. The method of magnetostatic measurement of the polarization not only demonstrates the unique possibilities for detecting ultraweak fields on the basis of the parametric resonance of the crossing of levels in the ground state of alkali-metal atoms, but is also of interest in its own right, since, in contrast to other existing methods, it does not introduce perturbations into the system of nuclei. The long relaxation time of He^3 nuclei in the ground state makes it possible to construct a sensitive position quantum gyroscope [59]. It can be used for an elegant demonstration of the Earth's rotation [60]. If the He^3 nuclei in the cell are oriented in a direction perpendicular to the Earth's axis, then after six hours the nuclei, maintaining their direction, will be parallel to the Earth's surface. After a further six hours their direction with respect to the Earth will be opposite to the initial direction. This experiment is an unusual variant of Foucault's experiment.

In a paper by Slocum [61] an analogous setup was described in which He^4 atoms in the metastable state 2^3S_1 were used as the working substance. It should be noted that, despite the broader line ($\sim 10^{-3}$ Oe) obtained in this experiment, the magnitude of the signal-to-noise ratio was exceptionally high and reached 35×10^3. This made it possible to ensure a sensitivity of the apparatus of about 10^{-8} Oe in the range of fields from zero to 10^{-3} Oe for a synchronous-detection time constant of 2 sec.

We note that the sensitivity attained with this apparatus is a record for helium magnetometers. This work demonstrates well the flexibility of the Hanle effect as a method for studying different physical quantities, in particular, thermal-relaxation times and the lifetime of the metastable 2^3S_1 state of He^4.

The possibilities for practical application of magnetometers of the type described to very precise measurement of field components whose absolute values are not small seem very limited, inasmuch as the magnitude of the fields being measured is limited by the difference between the longitudinal and transverse relaxation times, which reaches two orders of magnitude, and by the resonance linewidth, of order 10^{-6} Oe for the alkali metals and $\sim 10^{-3}$ Oe for He^4. However, there is a theoretical possibility of using also resonances that appear in nonzero fields ($n \neq 0$ in expression (18)) for magnetic-field measurements. The widths of these lines are the same as in the case $n = 0$, and so, in principle, it is possible to obtain the same sensitivity. If we vary the frequency of the modulating field $H_1 \cos \omega t$, the use of resonances of this type can make it possible to encompass measurements of a fairly broad interval of values of the field H_0. Yet another possibility of measuring fields that are not small by crossing methods is provided by the method of precision component-by-component compensation of the external field and subsequent measurement of the residual field[62].

The sensitivity of the realized magnetometers whose principle of operation is based on the phenomenon of crossing of sublevels in the ground states of atoms does not, as yet, reach the sensitivity values that are theoretically possible for devices of this type[33a,d]. To raise the sensitivity it is necessary to increase the signal-to-noise ratio. In addition, to increase the precision of measurements of the absolute values of a field, and not only of its variations, it is necessary to take careful account of the systematic errors, i.e., errors that are uncontrollable in the measurement of the effect and can displace the center of the resonance line from the point corresponding exactly to the zero value of the field being measured. One of the sources of such errors can be light of nonresonance frequency, which is always contained in the pumping ray and induces "virtual" transitions which lead to a shift of the levels and to displacement of the resonance. A detailed study of the energy levels of atoms perturbed by nonresonance light is carried through in[36]. It may be thought that the advance of interference methods into the region of nonzero fields will, as follows from the aforesaid, have no essential advantages as compared with magnetic-resonance methods[33d].

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