PHYSICS OF OUR DAYS

Superfluidity in pulsars

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GENERAL PROPERTIES OF PULSARS

The statement of the problem of the properties of the nuclear matter of which stars are made is connected with the formulation of the ideas of quantum mechanics. Notice must be taken of the pioneering work done in this direction by Landau[1] and published back in 1932. Beade and Zwicky[2] have shown in 1934 that the formation of superdense neutron stars can be the result of supernova explosions. In 1959, Migdal[3] has proposed that nuclear matter in neutron stars, whose density reaches $\sim 10^{14}$ cm^3, constitutes a liquid in superfluid state, in spite of the fact that its temperature immediately after formation of the star can reach 10^9 degrees[4].

Let us recall in this connection the main premises of the theory of superfluidity of liquid helium, developed by Landau[5], Bogolyubov[6], and Feynman[7]. At 0°K, all the liquid helium is in the ground state and is superfluid in this state, at which it has zero viscosity. However, the presence of a superfluid ground state does not mean that all the helium atoms are concentrated in momentum space at the zero level. To the contrary, later experiments have shown that only 7.4% of the material (and perhaps even less) is in the "condensate" state, even at temperatures very close to 0°K. The remaining atoms of the liquid helium, owing to the interaction between them, are "pushed out" of the condensate and have on the average rather appreciable momenta. At temperatures different from zero, there appear in liquid helium thermal excitations, the number of which is insufficient to set the entire liquid in thermal motion, so that in the temperature interval from zero to 2.17°K one can speak both of a superfluid liquid that has not yet been set in thermal motion, and a normal liquid that is already in thermal motion. At a temperature above 2.17°K, i.e., at the second-order phase-transition point, the amount of heat is already sufficient to cause thermal motion of all of the liquid, so that the liquid loses by the same token its superfluid properties and is converted into matter having the most ordinary properties. The existence of two components in helium II was experimentally demonstrated by Andronikashvili[9], viz. the normal component was set to oscillate, whereas the superfluid component remained immobile.

It is easy to visualize an analogous situation also in the case of the nuclear matter of a star, which consists mainly of neutrons with a small admixture of protons ($\sim$1%) and electrons (also $\sim$1%) and is in the superfluid state[10]. The difference between the liquid helium and nuclear matter inside a star is that the neutrons, protons, and electrons, being fermions, cannot "condense" directly in momentum space, and in order for such a phenomenon to be able to take place it is necessary that they combine pairwise into bosons. The fermion pairing mechanism was observed in superconductors and, as is well known, is named after Cooper[11]. The phase-transition point above which nuclear matter loses its superfluidity is in the region $(10^{10}-10^{12})$°K (see, e.g.,[12]), whereas the pulsar temperature is $T \sim 10^{8}$°K and is quite uniform over the entire star. At temperatures of about 100 million degrees, the bulk of the superfluid nuclear matter has only a small admixture of the normal electronic part.

In 1964, Ginzburg and Kirzhnits[13] have proposed that rotating nuclear matter in the superfluid state should form quantized vortices, in analogy with the superfluid phase of liquid helium at transcritical velocities, when the normal and superfluid parts can no longer move independently of each other.

The appearance of quantized vortices in the superfluid component of liquid helium moving with transcritical velocity at temperatures below the phase-transition point was predicted by Onsager in 1949[14]. Onsager postulated, in addition, that the angular momentum of the helium atom around the core of such a vortex should be equal to Planck's constant or its multiple. It follows therefore that the vortex has a quantized circulation.

Vinen[15] has subsequently confirmed experimentally the validity of Onsager's hypothesis.

The theory of quantized vortices was developed by Feynman[7], who has shown that at sufficiently high velocities the liquid helium should be permeated with quantized vortices. In a rotating vessel, the number of vortices per unit area of the liquid cross section should be proportional to the angular velocity, and is given by

$$N = \frac{m\omega_0}{\pi\hbar}, \qquad (1)$$

where ω_0 is the angular velocity and m is the mass of the helium atom.

As to the critical velocity ω_{c_1} at which the first vortex appears, Arkhipov[16] and Vinen[17] have obtained

$$\omega_{c_1} = \frac{\hbar}{mR^2} \ln \frac{R}{a_0}, \qquad (2)$$

where R is the radius of the vessel and a_0 is the radius of the core of the vortex ($a_0 \sim 3 \times 10^{-8}$ cm).

It should be noted that until Onsager predicted the need for quantizing the orbit of the helium atom around the vortex, it was assumed that quantization scales were limited to atomic dimensions (i.e., to a value on the order of 10^{-8} cm), in which, according to Bohr's well known postulate, the electron orbit is quantized. Under laboratory conditions, on the other hand, atoms located several centimeters away from the vortex axis are set in vortical motion. Thus, the Onsager hypothesis has increased the quantization scales by eight orders of magnitude. In the case of a star, the quantization scales should increase even more.

The hypotheses of Migdal, Ginzburg, and others remained unverified until recently. Only the discovery, in 1967, of the first pulsar has afforded real possibilities of verifying these theoretical guesses.

As is well known, a pulsar is a neutron star of relatively small dimensions with radius $R \sim 10^6$ cm and mass equal approximately to that of the sun. It is called "pulsar" because its brightness increases periodically.

According to present-day concepts, the pulsar is produced as a result of supernova explosion. Thus, for example, at the present day about 30 radio pulsars out of the 105 observed ones are attributed to remnants of various supernovas[18,19].

The youngest of the pulsars discovered to date is NP 0532 (or PSR 0532) in the Crab nebula (Fig. 1). The age of this most interesting formation is not yet 1000 years. It was produced upon appearance of the "celestial guest" in 1054, as recorded by the Japanese and Chinese astronomers. Located at the center of the Crab nebula is the brightest pulsar PSR 0532, the period of revolution of which is $P \approx 0.033$ sec[20] (the value of P is presently measurable to 11 significant figures). Observation of this pulsar with a stroboscope has shown that if the stroboscope and the pulsar rotate in phase, then the pulsar can be seen. In the case of a phase deviation, the pulsar "vanishes" (Fig. 2). Pulsars emit in the main electromagnetic waves in the meter and decimeter bands. It appears that only 3 out of 115 pulsars (including x-ray pulsars) emit visible light. About 10 pulsars emit also x rays, and the Crab-nebula pulsar PSR 0532 emits in addition γ rays[21].

A fortunate concurrence of circumstances enables an earth-based observer to determine the angular velocity of pulsars. The reason is that the magnetic axes of the pulsars are inclined to the axes of their proper rotation, so that the luminosity of the pulsar is seen by the terrestrial observer as a periodic function of the time. It is necessary to accept here the point of view that the pulsar radiates particularly strongly in the direction of its own magnetic axis[22].

According to contemporary concepts[12], the pulsar structure is the following: it has an outer solid iron crust of thickness $d \approx 3$ km. Inside the solid crust is located the superfluid neutron liquid.

The sudden jumps in the angular velocity that occur in the pulsar PSR 0833-45[23], which belongs to the remnants of the supernova Vela X and of the pulsar PSR 0532 of the Crab nebula[24] have given rise to numerous hypotheses concerning the nature of this phenomenon. The most probable of these is at present the Ruderman starquake hypothesis[25].

FIG. 1. Photograph of Crab nebula.

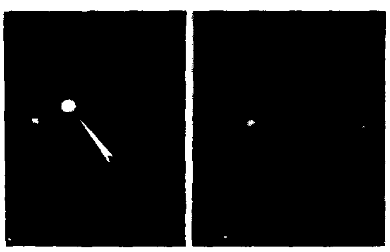


FIG. 2. Photograph of central region of the Crab nebula, obtained with a stroboscope. On the left-hand photograph, the stroboscope and the pulsar PSR 0532 rotate in phase.

A few words concerning this hypothesis. As a result of the rapid rotation, the pulsar should be slightly oblate at the poles, and the gravitational forces are balanced by centrifugal forces; the star is at equilibrium. In the course of time the angular velocity of the pulsar decreases (to be sure, slowly). This should upset gradually the equilibrium between the gravitational and the centrifugal forces. Mechanical stresses can appear in the solid crust of the pulsar and can become reinforced, and ultimately case destruction of the crust. The iron armor of the star cracks under these conditions, sinking at the equator and rising at the poles. Naturally, this should cause a jumplike increase in the angular velocity of the star. Measurements made in 1969 have shown for the first time that the jump in the angular velocity of the pulsars PSR 0833-45 (Vela X) and PSR 0532 (Crab nebula) corresponded to decreases of 0.1 and 10^{-3} cm in their radii, respectively, equivalent to relative increases in their angular velocities $\Delta\omega_0/\omega_0 = 2 \times 10^{-6}$ and 3×10^{-9}, respectively.

It should be noted that besides the starquake mechanism, the pulsar acceleration can be caused, e.g., by planetary perturbation[27], and also by volcanic activity[28]. In our opinion, however, the most probable together with starquake is also the pulsar-acceleration mechanism proposed by Packard[29]. According to Packard, a neutron star is accelerated as a result of the decay of a metastable excess of vortices; when these vortices are destroyed, they should give up their angular momentum to the liquid, and through it to the solid crust (this will be discussed in detail later on).

Let us examine the plot of $\omega_0 = F(t)$ for pulsars. (Fig. 3). The ordinates represent the angular velocity in a logarithmic scale, and the abscissas the time. The plot shows the times characterizing the behavior of the pulsars and connected with their internal nature.

One of them (t_0) pertains to the change of the damping law: prior to the velocity jump, the deceleration is given by $\omega_0 = f(t)$. After the velocity jump, the deceleration takes the form $\omega_0 = \varphi(t)$. The law $\omega_0 = \varphi(t)$ is much stronger than the $f(t)$ law. The period during which the $\varphi(t)$ law goes over into $f(t)$ will be called the relaxation period designated t_0. For the pulsars PSR 0833—45 and PSR 0532 we have $t_1 = 1.2$ year $= 3.7 \times 10^7$ and $t_2 = (7 \pm 3)$ days $= 6 \times 10^5$ sec, respectively[12].

Another characteristic time is the time that separates two successive velocity jumps. We designate it by τ. We have $\tau_1 = 2$ years for PSR 0833-45 and $\tau_2 = 3$ months for PSR 0532, respectively[30].

The third characteristic time is connected with the periodic revolution-frequency variation that occurs after the velocity jump. This characteristic time, which we denote by θ, is $\theta_1 = 7$ months[31] for PSR 0833-45 and $\theta_2 = 4$ months[32] for PSR 0532, respectively.

A comparison of the plots of $\omega_0 = f(t)$ and $\omega_0 = \varphi(t)$ shows quite unequivocally the interior of the neutron star is not a solid. On the other hand, if the interior of the pulsar were a classical liquid, then the relaxation time, under the most varied assumptions, should not exceed a value on the order of 10^{-7}–10^{-17} sec.[12] It is required to find a hydrodynamic system such as to satisfy all three characteristic times defined above.

Baym, Pethick, Pines, and Ruderman[33] start from the premise that the cavity inside the pulsar solid crust of Fe^{56} is filled with a two-component liquid constituting a mixture of superfluid and normal nuclear matter in which 98% of the particles are neutrons, ~1% are protons, and ~1% are electrons. The PSR 0833-45 pulsar can furthermore contain at its center a solid core[30] consisting of hadrons or of a neutron crystal lattice. The core, as well as the electrons and protons, are rigidly bound to the crust by the ultrastrong magnetic field of the star. The interaction between the normal and superfluid parts, or between the superfluid part and the crust or the core, is effected by quantized vortices, the presence of which in neutron stars was postulated by Ginzburg and Kirzhnits[13].

DESCRIPTION OF INSTALLATION

To ascertain the hydrodynamic features of pulsar rotation, experiments were performed at the low-temperature division of our Institute[34-36], on liquid-helium-filled vessels of cylindrical or spherical shape, freely suspended on a magnetic suspension in such a way that they could execute sufficiently uniform rotation. When desired, an abrupt jolt was applied to these vessels by an electric pulse. This increased the angular velocity by various amounts. This imitated the results of a starquake in a pulsar.

A schematic diagram of the instrument is shown in Fig. 4. A hollow glass sphere 1 (or a thin-wall Plexiglas beaker) together with a brass disk 2 having a relatively large moment of inertia were securely mounted with an aligned shaft 3 constituting a stainless tube of 3 mm diameter. The sphere (or beaker) served as a vessel for the liquid helium and could be filled through

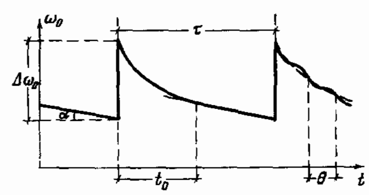


FIG. 3. Schematic representation of the change of the pulsar angular velocity with time. The plot shows the most typical times connected with the internal structure of the neutron star (the ordinates are in logarithmic scale).

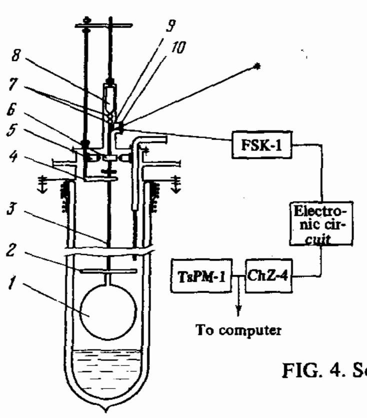


FIG. 4. Schematic diagram of instrument.

a narrow opening of 0.5 mm diameter in its upper part. The small diameter of the opening ensured a negligible loss of helium II film from the vessel during the experiment. The vessel was refilled with the liquid before each experiment.

A steel sphere 7 of 2 mm diameter was soldered to the second end of the tube 3. A similar sphere was fastened to the end piece of electromagnet 8 mounted in the upper part of the cover of the dewar vessel. When the electromagnet winding was connected to a dc line fed from storage batteries, the spheres were attracted to each other and the vessel, being suspended, could rotate freely. The small contact area of the well-polished sphere surfaces made for negligible friction between the moving and stationary parts of the instrument: the vessel, spun to angular velocities ~4 sec^{-1}, rotated for 70–80 minutes before it stopped. The small value of the friction at the point of contact of the spheres contributed also to the flow of minimum current through the electromagnet coil; this current was regulated in such a way that the lifting force of the electromagnet exceeded only slightly the weight of the rotating part of the instrument. The shaft 3 also carried the securely fastened rotor of a small induction motor, the stator of which was made up of coils 5 that produced a rotating magnetic field (there were six coils). The initially immobile instrument was accelerated to $\omega_0 \sim 7$ sec^{-1} within a time ~10 sec. By turning on the stator coils for a short time it was possible to increase the angular velocity of the vessel abruptly. There were of course no vertical oscillations of the rotating vessel. The amplitude of the radial oscillations of the adjusted instrument, on the other hand, did not exceed ~1 mm (amounting to $\sim 1.6 \times 10^{-3}$ rad). A brake was provided to stop the rotating system.

A mirror 9 was mounted on the upper part of the tube 3 and reflected a focused light beam to a type FSK-1 photoresistor connected in a special electronic circuit that measured (with the aid of a ChZ-4 frequency meter) and automatically recorded (with the aid of a TsPM-1 digital printing unit the time between to succeeding pulses produced when the light spot illuminated the photoresistor. The period of revolution was measured accurate to 10^{-5} sec. The experimental data were reduced with the M-220-M computer.

The moment of inertial of the liquid was 15–20% of the total moment of inertia of the rotating part of the instrument.

We used also another magnetic-suspension variant. Unlike the first, the second employed no bearings, so that the rotating system could be made as light as possible and the relative moment of inertia of the superfluid liquid could be increased (to 70% of the total moment of inertia of the rotating part) (Fig. 5). In this variant of the instrument, the rotation and the torsion (or acceleration) of the rotating part was recorded in the same manner as in the first, but there were some differences, viz., the shaft of the instrument terminated in this case in a special core of Armco steel, 1. When the Armco end piece approached the electromagnet 2, the inductance of the coil 5 changed, and this caused an immediate response of the electronic control system 4, which decreased the lifting force of the magnet 3 by acting on the magnet power supply. When the end piece 1 was moved away from the inductance 5, the current in the electromagnet coils increased. The suspension

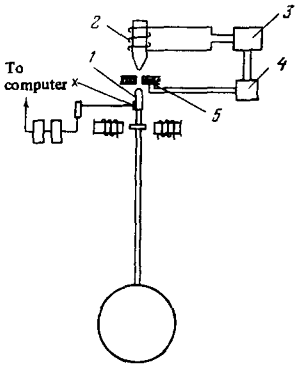


FIG. 5. Schematic diagram of bearing-free magnetic suspension.

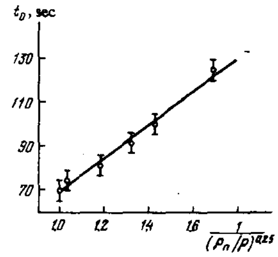


FIG. 6

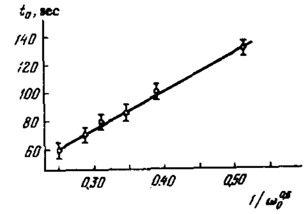


FIG. 7

FIG. 6. Plot of t_0 against ρ_n/ρ.
FIG. 7. Plot of t_0 against ω_0.

system was subject to practically no vertical oscillations.

Copper screens in contact with a helium bath shielded the rotating system reliably against thermal radiation.

THE TIME t_0

It is natural to assume that the relaxation time t_0, after the lapse of which the angular velocity of the instrument is altered only because of ordinary damping, is effected by the following parameters: the radius of the beaker (or sphere), the temperature of the liquid (or the ratio ρ_n/ρ, where ρ_n is the density of the normal component and ρ is the total density of the liquid), the angular velocity ω_0 (the number of vortices), and the magnitude of the velocity jump $\Delta\omega_0$.

We have studied the influence of each of these parameters on the time t_0.

To perform experiments with a spherical vessel, we chose a sphere with minimal difference between diameters measured in different directions in the equatorial plane. The experiments were performed with a sphere having $R = 3.4 \pm 0.95$ cm.

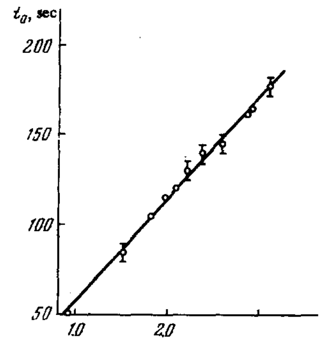
We investigated initially the temperature dependence of the relaxation time in a smooth-surface sphere filled with liquid helium. These experiments were performed at constant $\omega_0 = 5.02\ \text{sec}^{-1}$ and at identical $\Delta\omega_0 = 1.50\ \text{sec}^{-1}$. As seen from Fig. 6, the relaxation time increases with decreasing temperature and has a power-law dependence on the ratio ρ_n/ρ. This dependence will be discussed in greater detail later on.

In the next series of experiments we investigated the dependence of t_0 on the angular velocity at the fixed values $T = 1.46°$K and $\Delta\omega_0 = 1\ \text{sec}^{-1}$. The results shown in Fig. 7 indicate that the relaxation time increases with decreasing ω_0, corresponding to a decrease in the number of vortices that couple the solid shell to the superfluid liquid.

To investigate the dependence of the relaxation time on the velocity jump $\Delta\omega_0$, the experiments were performed at fixed $T = 1.46°$K and $\omega_0 = 5\ \text{sec}^{-1}$. The results shown in Fig. 8 indicate that the relaxation time increases logarithmically with increasing jump.

Similar results were obtained also in the case when the vessel used for the helium II was a Plexiglas beaker, 70.0 mm in diameter and h = 40 mm high. The beaker had a very thin wall, 0.5 mm thick. The cover of the beaker was also of Plexiglas and was glued to the beaker. Introduction of a coaxial cylinder into the beaker, which decreased the linear dimension by a

FIG. 8. Plot of t_0 against $\Delta\omega_0$.



factor of two, decreased the relaxation time also by a factor of two. Thus, $t_0 \sim R$, where R is the linear dimension of the volume occupied by the liquid.

Taking into account the arguments presented in this section and the results of the measurements of the relaxation time, shown in Figs. 6–8, and making furthermore use of dimensionality theory, we have derived a semi-empirical formula[1)]. We started from the assumption that the dimensionless quantity $\omega_0 t$ can be expressed in the following form (with dimensionless arguments):

$$\ln\left(1+c\frac{\Delta\omega_0}{\omega_{c1}}\right) \quad \omega_0 t = f_1\left(\frac{m\omega_0 R^2}{\hbar}\right) f_2\left(\frac{\rho_n}{\rho}\right) f_3\left(\frac{\Delta\omega_0}{\omega_0'}\right), \tag{3}$$

where ω_0' is a certain quantity with dimensionality ω_0' m is the mass of the helium atom, and $\hbar$ is Planck's constant. A reduction of the data obtained at constant ρ_n/ρ, $\Delta\omega_0$, and R has shown that

$$\omega_0 t = (3 \pm 0.3)\left(\frac{m\omega_0 R^2}{\hbar}\right)^{0.4 \pm 0.05}, \tag{4}$$

where the dimensionless numerical parameters were obtained by least squares. Thus, $t \sim \omega_0^{0.6}$ and $t \sim R^{0.8}$ (instead of $t \sim R\omega_0^{-1}$ obtained in a preliminary estimate[34,35]). The temperature dependence of t_0, with the remaining variables constant, is given by

$$t_0 = \frac{70 \pm 5}{(\rho_n/\rho)^{0.25 \pm 0.01} + B}. \tag{5}$$

The dependence of t_0 on the velocity jump $\Delta\omega_0$, the other variables being constant, is described by a formula in the form

$$t_0 = (57 \pm 5)\ \ln\ (1 + c'\Delta\omega_0). \tag{6}$$

Combining the relations obtained in this manner, we have

$$t_0 = A\left(\frac{mR^2}{\hbar}\right)^{\beta} \omega_0^{\beta-1} \left(\frac{\rho_n}{\rho}\right)^{-\alpha} \ln(1 + c'\Delta\omega_0). \tag{7}$$

The least-squares fit yields $A = 1.0 \pm 0.1$, $\beta = 0.40 \pm 0.05$, $\alpha = 0.25 \pm 0.01$, $c' = 5.1 \pm 0.2$. The quantity B does not differ from zero within the limits of errors (although in principle $B \neq 0$, because $t \neq \infty$ at $\rho_n/\rho = 0$). The expression $c'\Delta\omega_0$ in the last term is preferable to $c''\Delta\omega_0/\omega_0$, i.e., c' in (6) is a constant. We have therefore expressed this term in the form $c\Delta\omega_0/\omega_{c1}$, where ω_{c1} is the critical velocity of the vortex formation, and $c = c'\omega_{c1} = (6.6 \pm 0.2)\cdot 10^{-4}$ (in the reduction of the results obtained with our instrument, it is simply an identity transformation; the replacement of $c\Delta\omega_0$ by $\Delta\omega_0$ $\Delta\omega_0/\omega_{c1}$, however, becomes essential and ensures similarity when dealing with pulsars, for which $\Delta\omega_0$ and ω_{c1} differ greatly from the corresponding values for helium II).

If we assume now that the processes in an accelerated vessel containing a neutron superfluid liquid are the same as in helium II under similar conditions, then we must assume that our formula (7) is valid also for a neutron superfluid liquid and can be used to calculate the relaxation times observed after pulsar acceleration. A similar calculation yields for the relaxation processes in pulsars

$$t_{\text{calc}}^{(\text{PSR } 0833\text{-}45)} \approx 0.6\cdot 10^7 \text{ sec},$$
$$t_{\text{calc}}^{(\text{PSR } 0532)} \approx 2\cdot 10^6 \text{ sec}$$

as against the observed values $t_1 = 3.7 \times 10^7$ and $t_2 = 6 \times 10^5$ sec, respectively. We used in the calculations the following presently accepted pulsar parameters: $R = 10^6$ cm, $\rho_n/\rho = 1\%$, ω_0 (PSR 0532) = 190 sec^{-1}, ω_0 (PSR 0833-45) = 70 sec^{-1}, $\Delta\omega_0$ (PSR 0532) = $1.3 \cdot 10^{-6}$ sec^{-1}, $\Delta\omega_0$ (PSR 0833-45) = $1.6 \cdot 10^{-4}$ sec^{-1}, and for the constants that enter in (7) we used the values obtained from the helium experiments.

THE TIME θ

There are several reasons for the oscillation of the angular velocity of a freely rotating superfluid liquid. One of the most important is the action exerted on the walls of the rotating vessel by the lattice of quantized Onsager-Feynman vortices oscillating in a direction perpendicular to the rotation axis. These oscillations were investigated theoretically by Tkachenko[37].

Naturally, an experimental observation of this effect calls for a very sensitive instrument with a small moment of inertia and small damping of the angular velocity. We used the second type of instrument. The vessel for the helium was: 1) a Plexiglas beaker of 64 ± 0.05 mm diameter, 50 mm height, and wall thickness 0.2 mm, with smooth internal surfaces, 2) the same beaker with a rough bottom and cover (the roughness was produced by small sand grains with linear dimensions 0.01 mm), 3) a glass sphere with smooth internal surface and diameter 68 ± 1.5 mm. The ratio of the moments of inertia of the superfluid component of the helium II (at $T \approx 1.46°$K) and of the vessels were 2.38, 3.01, and 0.89 for vessels 1, 2, and 3, respectively.

Just as in the earlier experiments, the procedure was the following: the instrument was set to rotate and, after reaching a definite angular velocity, was left to itself. We measured the dependence of the angular velocity on the time. The setup was connected on line to a computer and the experimental results were automatically reduced.

Figure 9 shows the results obtained in experiments with a vessel with smooth surfaces. From an examination of curve 1 it is clear that immediately after the acceleration the velocity of the initially immobile instrument decreases abruptly, and then varies periodically with a period of ~30 sec. The rotation becomes more and more uniform with time, although there are always traces of definite perturbations of the rotation. The same figure shows the time dependence of the angular velocity at 12 minutes after the start of the rotation (curve 2, which is a continuation of curve 1). If the rotating instrument is set to oscillate radially with amplitude $\varphi \gtrsim 10^{-2}$ rad (the period of the radial oscillations of the instrument is ~1.7 sec), then the angular velocity of the instrument begins to oscillate as the amplitude of the radial oscillations increases (curve 3).

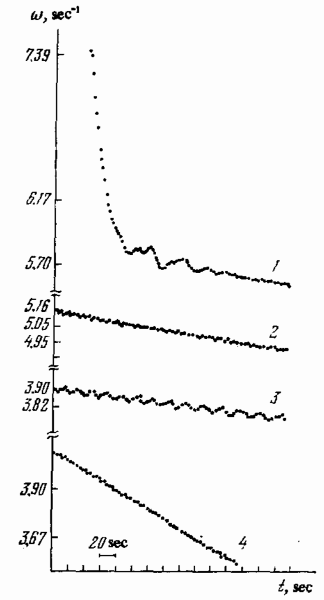


FIG. 9. The variation of the angular dependence of a freely rotating vessel with liquid helium [36]. Curves 1–3 pertain to He II; curve 2 is continuation of 1 and 3 is a continuation of 2; 1—start of rotation, 3—rotation when the instrument oscillates radially, 2—relatively smooth damping approximately 12 min after the start of the rotation, 4—He I with the instrument oscillating (ω_0 is shown in logarithmic scale).

Similar results were obtained also when the rotating vessel was a beaker with rough end surfaces.

Control experiments performed in helium I as well as in an empty instrument (without liquid) have shown that the radial oscillations of a rotating system, with amplitudes $\varphi \gtrsim 10^{-2}$ rad, cause no oscillations of its rotation (curve 4). As to experiments with a spherical glass vessel, in this case there were similar velocity oscillations, but they were weaker than in the case of the cylindrical vessel.

As is well known, the angular velocity of the pulsar PSR 0532, after acquiring acceleration, began to oscillate with an approximate period of four months[32]. The amplitude of the oscillations was approximately one-tenth of the velocity jump produced when the rotation was accelerated. According to Ruderman[31] this phenomenon can be attributed to perturbations of the vortex lattice of the superfluid neutron liquid, in which Tkachenko oscillations can propagate in analogy with the vortex lattice of helium II.

A rather rough estimate of the period of the θ oscillation of the angular velocity can be obtained by using a relation derived by Ruderman for the fundamental modes of the oscillations of the vortex lattice in a cylindrical vessel.

According to Ruderman[31], if we disregard the moment of inertia of a cylindrical vessel of infinite length, the period of the fundamental mode of the Takchenko oscillations of a vortex lattice can be calculated from the formula

$$\theta \sim \frac{4\pi}{5}\sqrt{\frac{M}{\hbar\omega_0}}R, \qquad (8)$$

where M is the boson mass, ω_0 is the angular velocity, and R is the radius of the vessel.

In view of the roughness of the foregoing estimates, it can be assumed that the velocity oscillations have the same nature in both cases, in spite of the fact the finite dimensions of the instrument and the presence of a vessel with a relatively large moment of inertia can change the parameters of the fundamental mode of the vortex-lattice oscillations. In any case, it is clear that the oscillations of the angular velocity after acceleration of the instrument is the result of the presence of a vortex lattice, since helium I reveals no such oscillations either at the start of the rotation or when the instrument executes radial oscillations.

THE TIME τ

In 1971, approximately two years after the observation of the first acceleration, the pulsar PSR 083-45 (Vela X) was accelerated once more and again with $\Delta\omega_0/\omega_0 \sim 2 \times 10^{-6}$.[38] As to the PSR 0532 pulsar of Crab nebula, it underwent in 1971 several accelerations; in the spring, summer (August 1) and fall (October 25) (i.e., almost every three months), each time with $\Delta\omega_0/\omega_0 \sim 10^{-9}$.[30] A calculation made by Pines and Shaham[39] has shown that whereas the frequency of the acceleration of the pulsar PSR 0532 in the Crab nebula can still be explained by means of the starquake mechanism, the two-years' interval between the accelerations of the pulsar PSR 0833-45 is too short. Calculations performed for this pulsar have shown that the period between its accelerations should not be less than several dozen years or several centuries, and that the acceleration of the other "older" pulsar is a most rare phenomenon, occurring once in several milennia. The authors have therefore introduced the concept of "corequake", According to this new hypothesis, the neutron solid crystal lattice making up the core of the PSR 0833-45 pulsar, just like the oblate solid iron crust, approaches a spherical shape from time to time, viz., its radius decreases at the equator and increases at the poles, and at the same time a jump occurs in the angular velocity of the star. To explain the two-years' interval between the accelerations, the authors had to assume that the neutron solid is more brittle than the iron crust of the pulsar. Let us however, make use of the two-component model of the pulsar to explain the frequency of its acceleration, and let us consider the acceleration mechanism considered by Packard[29], based on observations of the metastable state of rotating helium II, a mechanism capable of explaining the observed time intervals.

Back in 1958, Andronikashvili and Tsakadze[40] have observed that the vortex effects produced when rotating helium II is heated are preserved for a long time in rotating helium I. In 1964—1966, a large group of experimental studies was made in Tbilisi, aimed at studying the metastable regime of the rotation of helium II. Using a procedure involving the measurement of the damping of the axial-torsional oscillations of a disk immersed in rotating helium II, Andronikashvili, Mesoed, and Tsakadze[41] and Andronikashvili, Gudzhabidze, and Tsakadze[42] have shown that the metastable state with vortices can be preserved in rotating liquid helium for a rather long time, about 30 minutes (Fig. 10).

The reverse of the phenomenon, namely prolonged rotation of helium II without a vortex lattice, was demonstrated by Andronikashvili, Babalidze, and Tsakadze[43]. This experiment was carried out using second sound propagating in a radial resonator along its radius. The second sound in the resonator was first tuned at a fixed temperature in the helium II. The liquid was then heated above T_λ and set to rotate until the entire system rotated in equilibrium. Next, without stopping the rotation, the liquid was cooled to the initial temperature. The time of establishment of the equilibrium amplitude (for the given velocity and temperature) of the second sound was measured. These times turned out to be appreciable and reached 1000 seconds and more under certain conditions (Fig. 11).

Analogous metastable states of rotating liquid helium II were subsequently observed, with slow and smooth variation of the angular velocity, by Packard and Sanders[44]. They investigated the capture of negative ions by vortices. The acceleration of the instrument in their experiment did not exceed 2 rad/sec in 10 hours. It turned out that at negative acceleration the vortices disappear not in a smooth manner that follows the change of the velocity, but in jumps, after a definite excess number of vortices is accumulated. The vortex production likewise did not follow the variation of the angular velocity, namely, the increase in the number of vortices was likewise not smooth but in jumps (Fig. 12).

Recently Nadirashvili and Tsakadze[45], using the oscillating-disk procedure, measured the relaxation times in helium II for a rapid (within 1 sec) decrease of the vessel angular velocity. They found that whereas the decay of the vortices begins instantaneously, immediately after rotating vessel with helium II is stopped, the vortex structure remains unchanged for 500 seconds and more in the case of a rapid decrease of the angular velocity (Fig. 13).

Thus, these studies have demonstrated convincingly that both the onset and the decay of the vortex lattice in

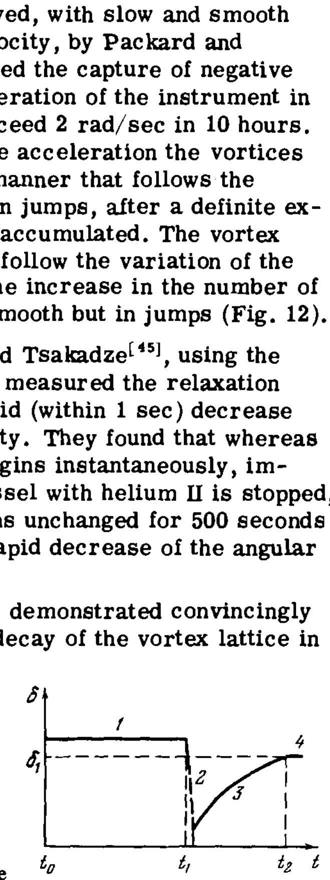


FIG. 10. Dependence of the logarithmic damping decrement of the disk oscillations on the time [42]. The oscillations take place in rotating helium I obtained by heating rotating HE II. At the instant t_0 the liquid is heated above the λ point, at the instant t_0 the damping decreases abruptly, and at t_2 the damping reaches its equilibrium values. Lengths of sections: 1) ~18 min, 2) ~1 min, 3) 12 min.

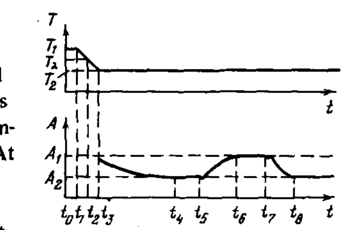


FIG. 11. Time variation of the second-sound amplitude in a radial resonator in He II under conditions when rotating He I is cooled to temperatures below the λ point [43]. At the instant t_1 the cooling of He I begins, the liquid is cooled below the λ point at the instant t_2, and at the instant t_3 the cooling stops and second sound appears. At the instant t_4 the second-sound amplitude reaches its equilibrium value (the interval (t_3, t_4) can reach 1000 sec, the rotation of the instrument is stopped at the instant t_5, at t_6 the amplitude of the second sound reaches its equilibrium value (for $\omega_0 = 0$), rotation of the instrument begins at t_7, and the amplitude of the second sound reaches an equilibrium value (for $\omega_0 \neq 0$) at the instant t_8.

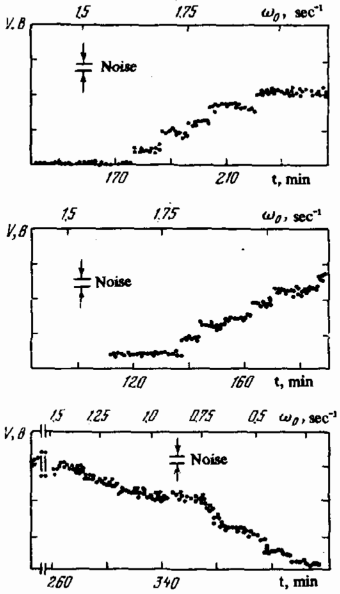


FIG. 12. Illustration of the ability of helium II to execute metastable rotation [44]. Ordinates—potential difference proportional to the number of vortices.

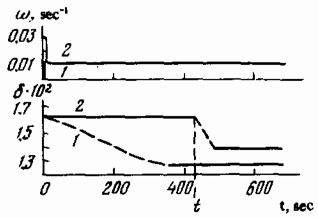


FIG. 13. Time dependence of the logarithmic damping decrement of the oscillations of a disk in rotating He II under conditions when its angular velocity is varied [45]: 1—after stopping the vessel, 2—after changing the angular velocity of the vessel.

liquid helium II proceed with large relaxation times. In other words, helium II can rotate for a long time without the appearance of vortices. At the same time, helium II can rotate also with an excess number of vortices that does not correspond to the given angular velocity (metastable rotation that does not correspond to the minimum free energy of the liquid).

Let us examine the interaction of the vessel with the superfluid liquid. Let the vessel and the liquid be initially at rest, and let us then raise the vessel velocity to a definite value and then leave it alone. The first to take place is a redistribution of the angular momentum between the vessel and the liquid, resulting in an abrupt decrease in the angular velocity of the shell. After the liquid and vessel begin to rotate at equal velocity, the rotation damping coefficient γ should become constant. Its value depends not only on the external decelerating force, but also on the intensity of the interaction of the vessel with the liquid. Indeed, the liquid "overtakes" the decelerating vessel, and its interaction with the shell has an accelerating character, i.e., it decreases the damping. On the other hand, by transfering the angular momentum to the vessel, the liquid itself gradually loses velocity, and consequently the number of vortices should decrease. Actually, however, the number of vortices remains constant for a certain time, making the vortex lattice metastable. When a definite degree of metastability is reached, some of the vortices decay again and transfer the angular momentum to the vessel or to the liquid. In the former case the vessel, becoming itself accelerated, accelerates also the liquid. The superfluid liquid is now in a metastable state relative to the lattice with a (small) number of vortices that does not correspond to the lattice angular velocity.

Following Packard[29], we now use the facts described above to discuss the behavior of pulsars. In particular, since the starquake mechanism cannot explain the observed short period between the accelerations of the neutron stars, let us see whether the decay of the metastable vortices can ensure more frequent acceleration of the vortices. Assume that following a sudden vanishing of a certain metastable number of vortices the angular momentum of the liquid L_{liq} changes by an amount ΔL_{liq}. The solid shell of the pulsar should then accelerate in accordance with the obvious relation

$$\frac{\Delta\omega_0}{\omega_0} = \frac{J_{liq}}{J_{sh}}\frac{\Delta L_{liq}}{L_{liq}}, \tag{9}$$

where J_{sh} is the moment of inertia of the shell. For a neutron star, the ratio J_{sh}/J_{liq} can in principle range from 10 to zero. According to Packard, the observed pulsar acceleration can be explained using reasonable values $J_{sh}/J_{liq} \sim 10^{-1}$–$10^{-2}$. We use the expression

$$N = 4\pi R^2\omega_0\frac{M}{h},$$

which describes the equilibrium number of vortices. Here N is the number of vortices, R is the radius of the vessel, ω_0 is the angular velocity of the liquid, and M is the mass of a single boson for a star of the mass of the helium atom for laboratory experiments. Knowing the rate at which the rotation slows down and using the expression

$$\dot{N} = 4\pi R^2\dot{\omega}_0\frac{M}{h},$$

we can calculate also the excess number of vortices "accumulated" per unit time (say in one day). Using these formulas for the pulsar PSR 0532, we obtain

$$N = 6\cdot 10^{1\cdot},$$
$$\dot{N} = 7\cdot 10^{11}\ \text{day}^{-1}.$$

Pulsar acceleration in the Crab nebula takes place every 90 days, and during that time 10^{13} vortices accumulate, amounting to $\sim 10^{-2}$ per cent of their total number. We take this excess of vortices to be the critical value for any pulsar. Then under this assumption, we can calculate for the pulsar PSR 0833-45 the time interval τ between the accelerations of this pulsar. The obtained value $\tau = 2$ years coincides exactly with the observed intervals between the acceleration of this pulsar. It is easy to verify that the total angular momentum of the decayed vortices is more than enough to explain the observed jump in the angular velocity.

Using similar calculations for the fastest pulsars, we can compile the presented table (the pulsar parameters were taken from[46]).

It is seen from the table that if the transition of the pulsar from the metastable state to a state with minimum free energy for the particular instant of time can actually cause acceleration of the shell of the neutron

Pulsar	Period of revolution, P, sec	Acceleration $\dot{P}$, nsec/day	Total number of vortices N	$\dot{N}$, day^{-1}	Time τ: calculated on the basis of our arguments	Time τ: observed
PSR 0532 (crab)	0.033099324	36.518	$6\cdot10^{17}$	$7\cdot10^{11}$		90 days
PSR 0833-45 (vela X)	0.089209298	10,823	$2\cdot10^{17}$	$3\cdot10^{10}$	2 years	2 years
PSR 0611-22	0.334911898	4.84	$6\cdot10^{16}$	10^{9}	15 years	
PSR 0736	0.374918324	1.73	$5\cdot10^{16}$	$2.5\cdot10^{8}$	30 years	
PSR 2045-16	1.961565351	0.963	10^{16}	$1.5\cdot10^{6}$	$3\cdot10^{4}$ years	

star, then the pulsar acceleration is not so rare a phenomenon and can be observed also for relatively "old" slowly rotating stars.

To verify Packard's hypothesis directly, we observed the time variation of the angular velocity of a duraluminum vessel with helium II, in which the superfluid component had a moment of inertia amounting to 0.75 of the total moment of inertia of the rotating system. It is known that free rotation with small damping follows the law $\omega_0 = \exp(-\gamma/Jt)$, where J is the moment of inertia of the system and γ is the damping coefficient of the rotation. According to preliminary data, after the lapse of a sufficiently long time (~15 min) after the start of the rotation, velocity jumps appear on the hitherto smooth plot of $\omega_0 = f(t)$; these jumps are possibly due to the vanishing of metastable vortices. The experiments in this direction should be continued.

CONCLUSION

We thus arrive at the conclusion that accelerations of at least two types can be realized in pulsars: I—acceleration of the solid part of the pulsar (due, for example to the Ruderman starquake mechanics), followed by dragging of the superfluid liquid into rotation, and II—acceleration of the superfluid liquid (due, for example, to its metastable rotation) with subsequent acceleration of the solid shell of the pulsar. As reported by Pines[47], a detailed reduction of the data, carried out by Lohsen[48], shows that the acceleration of the PSR 0532 pulsar in October 1971 differed from the acceleration of 1969, namely: there was no prolonged relaxation process after the jump in the velocity. It can therefore be concluded that this is connected with realization of acceleration of the second type in pulsars.

We can conclude on the basis of our research that the two-component model of the neutron star is in a position to explain the existence of all three characteristic times t_0, θ, and τ, whereas other pulsar models can account only for the existence of any one o- these times taken separately.

This work was stimulated by a lecture delivered by Professor D. Pines of the University of Illinois, delivered at the Physics Institute of the Georgian Academy of Sciences, and by the discussions that followed this paper.

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