Electrons and holes in an inertial-force field

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1. INTRODUCTION

It may seem far fetched to raise the question of how carriers in a solid behave in inertial or gravitational forces[1)]. Indeed, the progress in solid state theory is so great that it is difficult to visualize that in such seemingly settled problems as the dynamics of the carriers, particularly their motion in the field of inertial forces, there is still something unexplained. After all, the first electron-inertial experiments were performed by Tolman and Stewart[1] back in 1916 (!).

Still, in spite of the fact that a correct interpretation of the experiments of Tolman and co-workers[1,2] was offered long ago (Darwin[3], Ginzburg[4]), they are not clearly explained in a number of articles and monograms, and this leads frequently to misunderstandings. Thus, for example, one can encounter the statement that under the conditions of electron-inertial experiments the action of the crystal potential is not equivalent to replacing the mass of the free electron m_0 by the effective mass m (i.e., that the effective-mass approximation is incorrect); that holes cannot behave under the influence of inertial forces like quasiparticles with positive charge and with positive effective mass. The electron-inertial experiments of Tolman and Barnett (the reverse Tolman experiment) are frequently classified in accordance with whether the electric circuit is closed or open. This does not agree with the real conditions of the Tolman and Barnett experiments, who always measured the current in a closed circuit and not the electric field in the absence of current.

Whereas a correct explanation of the Tolman experiments has been in existence since 1936[3], and the matter reduces only to the errors in the analysis and to inaccurate formulations of the later studies which to be sure frequently led to an incorrect understanding of the gist of the phenomenon, the situation is different with respect to the reverse Tolman experiment which was first performed by Barnett in 1931[5]. This effect has not been correctly explained to this day. An attempt was even made by Brown and Barnett[6] in 1952, in connection with the results of the reverse Tolman experiment, to cast doubts on the main premises of the band theory. Starting with the fact that the obtained mass/charge ratio m/e of the carriers in the metals with positive Hall effect (Mo, Zn[6], and Cd[7]) turn out to differ in sign and in magnitude (with accuracy on the order of a percent) to m_0/e for free electrons, they stated that "the assumption that the positive Hall effect can be explained by assuming a positive value of m/e has been refuted, since experiment shows this ratio to be negative also for metals with positive Hall coefficients." This statement was criticized in articles by Rostoker[8], Shockley[9], and Ginzburg[4]. We encountered, however, no satisfactory explanation of Barnett's experiments in either these articles or in other papers known to us.

Thus, in spite of the fact that the electron-inertial effects discovered about 60 years ago have been the subject of many studies and are described in textbooks and monographs (see[10–14]), the question of their correct interpretation calls, in our opinion, for an additional analysis. It is necessary to examine thoroughly the behavior of electrons and holes of a solid in the field of inertial forces. In particular, an answer must be found to the question whether it is possible to describe the motion of carriers quasiclassically (by Newton's equation) for such nonelectromagnetic forces. It is also necessary to explain why the expression for the Tolman and Barnett electron-inertial effects contains the free-electron mass m_0, whereas the dynamic properties of the electrons and holes in a solid can be described by a quasiclassical equation that contains the effective mass m. We shall attempt below to clarify these questions and present a simple explanation of Barnett's experiments.

The idea of Tolman's experiments, advanced already by Maxwell in his treatis of electricity and magnetism, reduces to the following. Imagine a ring made of metallic wire and rotating uniformly about its axis. If the ring is stopped, then the electrons will move by inertia for a certain time relative to the ions of the crystal lattice, and this will result in a current I, and a certain amount of electricity Q will be transported along the periphery of the ring. It is clear that if the electrons were rigidly bound to the ions, i.e., if they were to move with the same velocity as the ions, then there would be no current in the circuit. To the contrary, if there were no interaction whatever between the electrons and the lattice, the average electron velocity v relative to the lattice would at each instant of time be equal and opposite to the linear velocity[2)] u of the points of the ring, and the density of the current due to the motion of the ions relative to the electrons would be $j(t) = enu(t) = -env(t)$, where $-e$ is the electron charge and n is the electron concentration. In real conductors we encounter the intermediate case: owing to the interaction between the electrons and the lattice, they are frequently dragged by the nonuniform motion of the latter. In this case the same current $-env(t)$ is produced as in the case when there is no interaction; the binding of the electrons affects the average velocity v.

Starting from the concept of the conduction electrons as a gas of free particles, Tolman calculated in the following manner the amount of electricity passing through the circuit. Let the linear velocity of the points of the ring prior to the start of the deceleration be u_0, and let the acceleration produced by the stopping be w $(w < 0)$[2)]. Under the influence of the inertial forces, the electrons acquire an acceleration $-w$ relative to the lattice. The motion of the electrons is such as if they were acted upon by an electric field of intensity E_{TS} defined by the relation

$$-eE_{TS} = -mw, \tag{1}$$

where m is the electron mass.

As a result of the displacement of the electrons relative to the ions, a current I is produced in the ring and satisfies the equation

$$\frac{L}{c^2}\frac{dI}{dt} + RI = \oint E_{TS}\,ds, \tag{2}$$

where L is the self-inductance coefficient, R is the resistance of the ring, and s is the circumference of the ring. Integrating this equation with respect to time from the start of the deceleration t_1 to the stopping of the ring t_2, we obtain

$$R\int_{t_1}^{t_2} I\,dt = -\frac{m}{e}su_0, \tag{3}$$

since the current vanishes at the instants of time t_1 and t_2, and the self-induction emf makes no contribution to the effect; $u(t_1) = u_0$, $u(t_2) = 0$.

Tolman used expression (3) to determine the ratio m/e. From the known values of the ring resistance R, its rotational speed u_0 prior to the start of the deceleration, and the measured amount of electricity Q flowing through the circuit during the deceleration time $t_2 - t_1$, he obtained the ratio m/e. And from the sign of the resultant potential difference he determined the sign of the charges making up the current. Tolman's experiments have shown that the charges are negative sign and the ratio m/e is numerically close to $m_0/(-e)$.

This result may contradict the main conclusions of solid-state theory, as was indeed suggested by Brown and Barnett[3]. According to the theory, the electric conductivity and other kinetic characteristics of metals and semiconductors are determined by the dynamic properties of the electrons and holes and by their dispersion law, which usually is more complicated than the dispersion law for free electrons $\epsilon = \hbar^2k^2/2m_0$. But in the case when the dispersion law is isotropic and quadratic, the relation $\epsilon(k) = \hbar_2k_2/2m$ contains the effective mass m rather than the mass m_0. Let us attempt to cope with this apparent contradiction, but let us first describe briefly how the electron-inertial experiments were actually performed. These experiments were performed in two different versions.

1. In the Tolman-Stewart experiments[1] a coil of metallic wire was rotated rapidly about its own axis, and was stopped within a fraction of a second. A ballistic galvanometer connected with flexible wires to the ends of the coil was used to measure the amount of electricity Q flowing through the coil during the stopping time. In the second version of the experiments of the same type, performed by Tolman et al.[2], a hollow cylinder of the investigated material executed torsional oscillations. Owing to the acceleration of the cylinder during the oscillations, periodic currents were produced in it and were measured with a vibration galvanometer. The value of m/e was determined from the known frequency and amplitude of the oscillations and from the measured value of the current.

2. In experiments of a different type, performed by Barnett et al.[5,6], they measured the mechanical momentum produced in the coil when the current flowing through the coil was changed (the reverse Tolman experiment). The scheme of the experiment is the following. Alternating current was made to flow through a coil vertically suspended on a filament. This current caused oscillations of the coil about its axis. To increase the measurement accuracy, a compensating device was used (the alternating magnetic field of the compensation coil acted on magnets fastened to the investigated coil), making it possible to keep the coil immobile. Knowing the moment Fr of the couple of forces that maintain the coil at rest, (r is the radius of the coil), and the rate of change dI/dt of the current flowing in the coil, Barnett calculated the ratio m/e. A detailed description of the procedure of the electron-inertial experiments is contained in the 1935 review by Barnett[15].

2. TOLMAN'S EXPERIMENTS

Let us consider now what information concerning the carriers in a conductor can be extracted from Tolman and Barnett's electron-inertial experiments. We present first Darwin's explanation of Tolman's experiments[3]. To simplify the analysis somewhat, we assume that the metallic rod moves in translation with constant velocity u_0 along its length (say, along x axis) and is stopped at the instant of time t_1 with a constant acceleration w ($w < 0$) until it is completely stopped at the instant of time t_2.

The change of the state of the electron in an immobile or uniformly moving conductor is described by the time-dependent Schrödinger equation

$$\left[-\frac{\hbar^2}{2m_0}\nabla^2 - eU(x, y, z)\right]\psi = i\hbar\frac{\partial\psi}{\partial t}, \tag{4}$$

where u(x, y, z) is the periodic potential of the crystal lattice.

If we reckon time from the instant t_1, then the Schrödinger equation for the electron in a conductor moving with acceleration takes the form

$$\left[-\frac{\hbar^2}{2m_0}\nabla^2 - eU\left(x - \frac{1}{2}wt^2, y, z\right)\right]\psi = i\hbar\frac{\partial\psi}{\partial t}. \tag{5}$$

It is convenient to change over to a non-inertial coordinate system, i.e., to introduce the new variables

$$x' = x - \frac{1}{2}wt^2, \quad y' = y, \quad z' = z, \quad t' = t.$$

We can then show that the function

$$\psi(\mathbf{r}, t) = \psi'(\mathbf{r}', t)\exp\left[\frac{i}{\hbar}\left(m_0wt'x' + \frac{1}{6}m_0w^2t'^3\right)\right] \tag{6}$$

transforms (5) into

$$\left[-\frac{\hbar^2}{2m_0}\nabla'^2 - eU(x', y', z') + m_0wx'\right]\psi' = i\hbar\frac{\partial\psi'}{\partial t'}. \tag{7}$$

V. L. Ginzburg[4] derived expression (7) (more accurately, the term m_0wx) without a transformation to the accelerated coordinate system, but using the principle of the equivalence of gravitational and inertial forces. According to this principle, the motion of an electron in a reference frame having an acceleration w is the same as in a system at rest in the presence of a homogeneous gravitational field of intensity $-w$ and potential wx (we assume that the acceleration w is directed along the x axis). The potential energy of the electron in the gravitational field is then m_0wx, and this quantity should be added to the potential energy $-eU$ in the Schrödinger equation (4), in which no account was taken of the influence of the gravitational field.

Comparing (7) with the Schrödinger equation for an electron moving under the influence of a homogeneous electric field with intensity $E = (E, 0, 0)$ in an immobile conductor

$$\left[-\frac{\hbar}{2m_0}\nabla^2 - eU(x,\ y,\ z) + eEx\right]\psi = i\hbar\frac{\partial\psi}{\partial t}, \tag{8}$$

we note that the action of the force $-m_0w$ is equivalent to the action of the force $-eE$. Thus, the inertial force $-m_0w$ accelerates the electron by exactly the same amount as the electric field

$$E_{TS} = \frac{m_0}{e}w. \tag{9}$$

A similar result is obtained, of course, for the real versions of Tolman's experiments, in which a nonuniformly rotating coil was used, rather than a linearly moving rod. A distinction should be made, however, between two possible situations: 1) the electric circuit of which the accelerated conductor is a part, or the circuit consisting only of the accelerated conductor (for example a rotating coil), is closed; 2) the electric circuit is open. In the first case, the action of the inertial force gives rise to a flow of electrons in a circuit, i.e., produces a current, which can be measured, say with a galvanometer. On the other hand, if the circuit is open, then when the electrons are moved by the inertial force $-m_0w$, an electric field E_{TS} is established in the conductor and balances the action of the force $-m_0w$. The field E_{TS} can in principle be measured by some contactless method, say with a capacitor. In practice, however, it is more convenient to measure the current produced by the electric field. To this end the conductor can be abruptly stopped and the circuit can be closed through an immobile part. After the stopping of the conductor, the field E_{TS} is no longer balanced by the force $-m_0w$ and produces a current.

In the presence of an external electric field E_{TS}, the kinetic equation for the electrons can be written in the form

$$-\left(\frac{\partial f_0}{\partial \varepsilon}\right)e\mathbf{v}\mathbf{E}_{TS} + \left(\frac{\partial f}{\partial t}\right)_{\text{cr}} = 0, \tag{10}$$

and the current density is

$$\mathbf{j} = \sigma\mathbf{E}_{TS}, \tag{11}$$

where f_0 is the equilibrium (Fermi) distribution function, $(\partial f/\partial t)_{\text{col}}$ is the collision integral, and σ is the conductivity.

By measuring the quantity of electricity Q passing through the circuit during the deceleration time $t_2 - t_1$ we can, knowing the values of σ and u_0, determine $m_0/(-e)$ from (9) and (11):

$$Q = \frac{m_0}{(-e)}\sigma u_0. \tag{12}$$

We have thus verified that the Tolman-Stewart extraneous field E_{TS}, which is produced in an accelerated conductor, depends on the mass of the free electron and not on the effective mass.

To determine the value of $m_0/(-e)$ from the Tolman experiments it is necessary in essence to know, besides σ, also the current density j and the acceleration of the conductor w.

In all the possible attainable accelerations w, the field E_{TS} is quite weak and cannot give rise to interband transitions. In such a field, the Bloch wave function $\psi_{\mathbf{k}}(\mathbf{r}) = u_{\mathbf{k}}(\mathbf{r})\exp(i\mathbf{kr})$, as first demonstrated by Houston[15], assumes after a time interval dt the form

$$u_{\mathbf{k}-\frac{e}{\hbar}\mathbf{E}_{TS}dt} \times \exp\left[i\left(\mathbf{k} - \frac{e}{\hbar}\mathbf{E}_{TS}\,dt,\ \mathbf{r}\right)\right],$$

i.e., the wave function changes in accordance with the relation

$$\frac{d\mathbf{p}}{dt} \equiv \dot{\mathbf{p}} = -e\mathbf{E}_{TS} \qquad (\mathbf{p} = \hbar\mathbf{k}). \tag{13}$$

In the case of an isotropic quadratic dispersion, this equation takes the form

$$m\dot{\mathbf{v}} = -e\mathbf{E}_{TS}. \tag{13a}$$

Thus, the motion of a conduction electron in a Tolman-Stewart field can be described by a quasiclassical equation.

Similar results are obtained for the field E_{TS} (9) and for the current j (11) in the case of hole conductivity. This follows from the fact that the field of the inertial forces acts on each individual electron separately (its energy changes by m_0wx), regardless of the degree of filling of the energy band. Consequently, the results are valid in the case of hole conductivity. This can be verified by considering holes directly.

To this end we recall how the hole concept is introduced. Assume that the energy band contains n states, of which p are not occupied by electrons. Heisenberg has first shown in 1931[17] that p hypothetical positively charged particles, holes, produce exactly the same current as n − p electrons that fill partly the energy band. To explain the equivalence of a partially filled band and a hole gas, let us compare two aggregates of particles: 1) n − p electrons, which leave p states out of the n possible states of the band unoccupied; 2) n electrons, which fill completely the band, and p hypothetical positively charged holes.

The currents produced by these two aggregates of particles are the same, if the electric field E and the crystal-lattice field E_i (the field produced by all the charges of the crystal except that considered) cause p holes to move in the same manner as those p electrons of the filled band (second aggregate), which occupy p states that are free in the real band (first aggregate)[4)]. This identical motion of p electrons and p holes can be obtained if the holes are assigned a mass $-m_0$. Indeed, the quasiclassical equation of motion of the electron $m\dot{v} = -eE$ can be rewritten in the form[5)]

$$m_0\dot{\mathbf{v}} = -e\mathbf{E} - e\mathbf{E}_i. \tag{14}$$

In order for a hole with charge +e to move with the same acceleration as the electron, the equation of motion must obviously be

$$-m_0\dot{\mathbf{v}} = e\mathbf{E} + e\mathbf{E}_i. \tag{15}$$

This means that the hole must be assigned a mass $-m_0$. Thus, since the signs of the charge and the mass of the holes in the electrons are opposite, they are accelerated in the same manner. We see that if, instead of p vacant states of the band, we consider p electrons $(m_0 - e)$ and p holes $(-m_0 + e)$,[6)] we can reduce the motion of n − p electrons to the motion of p holes. Heisenberg[17] obtained this result on the basis of a quantum-mechanical analysis. He has shown that the time-dependent Schrödinger equation for a hole, in contrast to Eq. (4) for an electron, takes the form

$$\left[\frac{\hbar^2}{2m_0}\nabla^2 + eU(x,\ y,\ z)\right]\psi = i\hbar\frac{\partial\psi}{\partial t}. \tag{16}$$

The wave functions ψ_e and ψ_h of the electron and of the hole are complex conjugate, so that the electrons and holes have equal probability distributions $\psi_e\psi_e^*$ and $\psi_h\psi_h^*$ in space and in time. This means in turn that the hole continuously accompanies the electron.

It is possible to go over from the holes $(m_0, +e)$, introduced above for the crystal and subject to the action of the crystal potential, to ordinary holes, which are quasiparticles with effective mass $m > 0$ and $e > 0$, in analogy with the procedure used for the electrons. To this end it is necessary, assuming the impurity potential $V(r)$ to be smooth enough, to eliminate from the Schrödinger equation the periodic potential $U(r)$ and replace the mass $-m_0$ by the effective mass m, thus taking into account the action of the lattice forces.

We now return to the conditions of Tolman's experiment, i.e., we consider a conductor having hole-type conductivity moving with acceleration. If the same procedure is carried out as for the electron i.e., if we change from the Schrödinger equation in the immobile coordinate system (16) to the equation in the accelerated coordinate system, then we obtain

$$\left[\frac{\hbar^2}{2m_0}\nabla'^2+eU(x',\ y',\ z')-m_0wx'\right]\psi'=i\hbar\frac{\partial\psi'}{\partial t'}. \qquad (17)$$

It follows therefore that a positively charged hole with mass $-m_0$ is acted upon by an inertial force $+m_0w$ whose sign is opposite that of the inertial force acting on the electron[7]. The inertial force depends on the mass m_0 of the free electron (or $-m_0$ of the hole), and not on the effective mass m, since the crystal potential, as we have seen, does not influence the inertial force.

A comparison of (17) with the Schrödinger equation for a hole in an electric field E, i.e., with Eq. (8), in which the electron hole $-e$ should be replaced by the hole charge $+e$, and the mass m_0 by the mass $-m_0$, we get

$$E_{TS}=\frac{m_0}{e}w. \qquad (9a)$$

It is seen from (9a) that in the case of hole conductivity ($-m_0+e$) the Tolman-Stewart field (and the current $j=\sigma E_{TS}$) has the same sign as in the case of electron conductivity ($+m_0$, $-e$), since E_{TS} is determined by the mass to charge ratio of the particle.

To determine the acceleration of the hole it is necessary, obviously, just as in the case of the electron, to take into account not only the inertial forces but also the lattice forces, i.e., to use Eq. (15). For a quadratic isotropic dispersion law, Eq. (15) reduces to the quasiclassical equation of motion of a hole in a crystal with effective mass $m > 0$:

$$m\dot{\mathbf{v}} = e\mathbf{E}_{TS}.$$

3. BARNET'S EXPERIMENTS

In an external electric field, the electrons and ions of a conductor are accelerated. If the electric field is constant, then the average velocities of the electrons and ions are also constant, and consequently the momenta of the systems of electrons and holes remain unchanged. If alternating current flows in the conductor, especially when an electric circuit is closed or open, the momentum of the electron system, as well as of the ion system changes. If the conductor is not clamped, say it is suspended on an elastic filament, then a change in the current is accompanied by a change of the momentum **P** (see (22)) of the conductor as a unit (under Barnett's conditions, the angular momentum is changed). Of course, if the investigated conductor is clamped, then the changes of the electron and ion momenta are transferred to the clamping body via the conductor boundary (constraints).

To determine the momentum acquired by a conductor in a unit time as a result of the acceleration of the electrons and ions by an external electric field E, we consider in place of the Schrödinger equation (8) the quasiclassical equation of motion (13) (which is valid for quantum-mechanical mean values)

$$\dot{\mathbf{p}} = -e\mathbf{E},$$

which takes the form (13a) for a quadratic isotropic dispersion law. Equation (13a) can be represented in a different form, if we write down explicitly all the electron interactions that are implicit in the effective mass m:

$$m_0\dot{\mathbf{v}}_s=-e\mathbf{E}-e\mathbf{E}_s^{ie}-e\mathbf{E}_s^{ee} \qquad (s=1,\ 2,\ \ldots,\ n), \qquad (18)$$

where E_s^{ie} is the effective electric field produced by all the ions and acting on the electron, and E_s^{ee} is the effective field characterizing the averaged action of all the remaining electrons on the s-th electron.

The motion of the t-th ion is described by the equation

$$M\dot{\mathbf{V}}_t=e\mathbf{E}+e\mathbf{E}_t^{ie}+e\mathbf{E}_t^{ii} \qquad (t=1,\ 2,\ \ldots,\ n), \qquad (19)$$

where E_t^{ii} is the effective electric field characterizing the averaged action exerted on the t-th ion by all the others, and M is the mass of the ion (the masses of all ions are assumed to be equal).

The terms eE_s^{ee} drop out of the sum of the forces acting on all the n electrons, since the total momentum of the electron system cannot be altered by the electron-electron interactions[8]. Analogously, all the terms eE_t^{ii} drop out from the sum of the forces acting on the n ions. Thus, the sum of the forces acting on the electrons and ions is

$$\sum_{s=1}^{n} m_0\dot{\mathbf{v}}_s+\sum_{t=1}^{n} M\dot{\mathbf{V}}_t=0. \qquad (20)$$

This means that the sum of the forces acting on the electrons is equal and opposite to the sum of the forces acting on the ions or, equivalently, the total momentum of the system of electrons and ions remains unchanged:

$$\sum_{s=1}^{n} m_0\mathbf{v}_s+\sum_{t=1}^{n} M\mathbf{V}_t=\text{const}.$$

It follows from (20) that

$$m_0\left(\sum_{s=1}^{n}\dot{\mathbf{v}}_s-\sum_{t=1}^{n}\dot{\mathbf{V}}_t\right)=-\sum_{t=1}^{n}(M+m_0)\dot{\mathbf{V}}_t, \qquad (21)$$

where

$$\sum_{t=1}^{n}(M+m_0)\dot{\mathbf{V}}_t=\dot{\mathbf{P}} \qquad (22)$$

is the change of the momentum of the conductor as a unit as a result of the accelerated motion of the electrons and ions in the conductor. The current density produced by the electrons moving relative to the ions is

$$\mathbf{j}=-e\left(\sum_{s=1}^{n}\mathbf{v}_s-\sum_{t=1}^{n}\mathbf{V}_t\right). \qquad (23)$$

From (21) and (23) we get the sought relation

$$\frac{\dot{\mathbf{P}}}{\dot{\mathbf{j}}}=\frac{m_0}{e}. \qquad (24)$$

The foregoing analysis of the Barnett experiment shows that a change $\dot{\mathbf{P}}$ in the conductor momentum occurs only if alternating current flows, i.e., when $\dot{\mathbf{j}} \neq 0$. It can be easily verified that a definite contribution to this momentum change $\dot{\mathbf{P}}$ is made by the inertial force acting on the electrons. Assume that in the laboratory frame the electron and ion accelerations in an external electric field are $\dot{\mathbf{v}}$ and $\dot{\mathbf{V}}$, respectively. We assume that all the ions have the same acceleration. In the coordinate system connected with the ions, the relative acceleration of the electron is $\dot{\mathbf{v}}_r = \mathbf{v} - \mathbf{V}$. The equation of motion of the electrons in the laboratory frame is $m_0\dot{\mathbf{v}} = \mathbf{F}$ ($\mathbf{F}$ is the resultant of all the forces), and in the coordinate system connected with the conductor, the transport inertial force $-m_0\dot{\mathbf{V}}$ enters in the equation of motion, namely $m_0\dot{\mathbf{v}}_r = \mathbf{F} - m_0\dot{\mathbf{V}}$. Thus, the conductor-momentum change $\dot{\mathbf{P}} = -m_0 \sum_{s=1}^{n} (\dot{\mathbf{v}}_r)_s$ is due not only to the force $n\mathbf{F}$, but also to the inertial force $\mathbf{F}_{in} = -nm_0\dot{\mathbf{V}}$.

Thus, by measuring in Barnett's experiment the changes in the densities of the current and of the momentum[9)] it is possible to determine the ratio m_0/e from (24). As regards the conductivity σ, it depends, of course, on the effective mass m and not on m_0.

It is easily seen that for holes $(-m_0 + e)$ we get the same result (24) as for electrons $(+m_0, -e)$, since the ratio $\dot{P}/\dot{j}$ is determined by the ratio of the mass to the charge of the particle.

As already noted in the Introduction, Barnett's experiments were sometimes incorrectly interpreted (following Rostoker[8]). Although Rostoker's criticism of the already-cited conclusion by Brown and Barnett[6], concerning the error in the conclusions of solid-state theory, is in general correct, his particular analysis of Barnett's experiments raises objections. Rostoker's explanation reduces to the fact that since the current density is $j = -e\sum_{\mathbf{k}} v(\mathbf{k})$ and the momentum of the aggregate of the electrons is $P' = m_0 \sum_{\mathbf{k}} v(\mathbf{k})$, it follows that

$$\frac{P'}{j} = -\frac{m_0}{e}. \qquad (25)$$

A similar interpretation of Barnett's experiments is given in other papers[4,12,14].

It must first be emphasized that this explanation has no bearing on Barnett's experiment in which he measured the change $\Delta P/\Delta t$ of the conductor momentum, i.e., the change produced in the momentum of the aggregate of the electrons and ions when the current in the conductor changes, and not the momentum P' of the electrons alone (incidentally, it is generally incomprehensible how P' can be determined experimentally).

As to (25), all that it states is that the momentum of the aggregate of electrons is proportional to the electric current carried by the electrons. But to explain Barnett's experiments it is important to determine the momentum that is transferred to the conductor as a result of the acceleration of the conduction electrons. Here, too, there is one more weak point in Rostoker's arguments. He assumes that when a current $j = -e\sum_{\mathbf{k}} v(\mathbf{k})$ flows the conductor acquires from the ele electrons a momentum $P' = m_0 \sum_{\mathbf{k}} v(\mathbf{k})$. Yet it is not at all clear beforehand the conduction electrons transfer to the crystal lattice a momentum P' rather than a quasimomentum $P'' = m\sum_{\mathbf{k}} v(\mathbf{k})$, and this is precisely what had to be demonstrated.

4. CONCLUSION

We can now answer the questions raised at the start of these remarks. We have explained that the quantities measured in the experiments of Tolman and Barnett (the charge Q connected with the field E_{TS}, and the ratio $\dot{P}/\dot{j}$), depend on the ratio $m_0/(-e)$ of the free electron. As to behavior of the carriers in a crystal in the presence of inertial forces under the conditions of Tolman's experiments, their dynamics can be described by the quasiclassical equation of motion

$$\dot{\mathbf{p}} = e\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}\times\mathbf{H}] + \mathbf{E}_{TS}\right) \qquad (e>0 \text{ or } e<0), \qquad (26)$$

while the kinetic coefficients (the Hall constant, the magnetoresistance, the thermoelectric power, etc.) can be obtained in the usual manner from the kinetic equation that contains the field E_{TS} along with other fields.

In the effective-mass approximation, all the dynamic and kinetic characteristics of the electrons and holes in the presence of inertial forces described by the field E_{TS} are expressed in terms of the effective mass m, whereas the field E_{TS} itself is determined by the mass m_0. Thus, the fact that the action of the inertial forces reduces to the action of an extraneous field E_{TS} enables us to describe the motion of the carriers in the conductor in exactly the same manner as in the case of an electric field E.

Arguments analogous to those presented above with respect to the electron-inertial effects can be applied also to gyromagnetic phenomena. This leads us to the conclusion that, for example, in the Einstein-de Haas experiments, one determines the g-factor of the electrons without allowance for their interaction with the crystal lattice, i.e., $1 \le g \le 2$, and not the effective g-factor g^* obtained in paramagnetic or ferromagnetic resonance. The cause of the difference between the values of g and g^* is the same as the cause of the difference between the value of m_0 obtained in electron-inertial experiments and the value m obtained from cyclotron resonance, namely, the internal interactions of the electrons with the lattice cannot change the angular momentum of the conductor as a whole.

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It was assumed for a long time that the electron-inertial experiments provide direct proof that the carriers in a conductor are electrons with mass m_0 and negative elementary charge $-e$. Solid-state theory, however, shows rigorously, as we well know, that the carriers of the dynamic and kinetic properties are quasiparticles—electrons and holes, that can differ both in mass and in the sign of the charge from the corresponding values for the free electrons. The advantage of electron-inertial experiments is that under those conditions when the collective properties of the quasiparticles do not manifest themselves it is possible to determine directly the characteristics (mass and charge) of the structure units of the solid, namely the electrons, and thus verify once more that our concepts concerning the solid state are correct.

1)The for sake of brevity, we shall refer henceforth to inertial forces only, since, as is well known, inertial forces and gravitational forces are equivalent.

2)The average velocity of the electrons relative to the laboratory frame is zero, since they are not dragged by the lattice. It is assumed that the wire is thin enough, so that the dependence of the velocity u on the distance of the given point of the ring to its axis can be neglected.

[3] Here w is the tangential compoment of the acceleration of the points of the ring.

[4] We recall that the electrons of a completely filled energy band make no contribution to the current.

[5] We note that the quasiclassical approximation for the periodic field E_i of a crystal lattice is generally speaking incorrect, since the electron wavelength is $\lambda \gtrsim a$, where a is the lattice period. However, this approximation is frequently used, since it provides greater clarity, albeit at the expense of rigor. In this case, the use of the quasiclassical approximation can be justified by the fact that the results of such an analysis coincide with the solution of the exact quantum-mechanical problem. [[17]].

[6] Electroneutrality is of course, conserved here.

[7] In particular, the earth's gravitational force exerts a force m_0g, on an electron in an immobile conductor, and a force $-m_0g$, on a hole where g is the acceleration due to gravity. When the electrons are displaced downward or the holes are displaced upward, the Tolman-Stewart field $E_{TS} = (m_0/e)g$ is produced and balances the gravitational force. This field, which is present in all conductors situated in the earth's gravitational field, is quite small, $E_{TS} \approx 5.5 \times 10^{-13}$ V/cm.

[8] This result is obtained automatically by summing all the terms.

$$eE_s^{ee} = \sum_{u \neq s} \frac{e^2(\mathbf{r}_u - \mathbf{r}_s)}{|\mathbf{r}_u - \mathbf{r}_s|^3}$$

over s from 1 to n.

[9] The momentum density is defined by us as the momentum per unit volume

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