Statistical problems in diffraction theory[1)]

Yu. A. Kravtsov, C. M. Rytov, and V. I. Tatarskiĭ

V. I. Lenin State Pedagogic Institute,
Radio Engineering Institute, USSR Academy of Sciences,
Institute of Atmospheric Physics, USSR Academy of Sciences
Usp. Fiz. Nauk **115**, 239–262 (February 1975)

Various formulations of problems in the statistical theory of diffraction and wave propagation are discussed: excitation of fields by random sources, diffraction of partially coherent waves, diffraction of waves by bodies having random shapes or positions, and diffraction and propagation of waves in a randomly inhomogeneous medium. For each of these types of problem, physical problems from acoustics, radio astronomy, radiophysics, optics, and other branches of physics are given as examples, and the methods (mostly approximate ones) most widely used for solving them are indicated. Among the problems discussed are those of the diffraction content of the radiation transport equation and the back scattering enhancement effect observed in the diffraction of waves by small bodies immersed in a randomly irregular medium. Examples of statistical problems of mixed type are also given.

CONTENTS

1. INTRODUCTION

In the theory of diffraction, as in other branches of physics, problems are frequently encountered that in principle require a statistical approach. In these problems one is interested rather in certain averaged quantities than in the actual values in particular instances. As in statistical physics, even if we could predict all the actual values ("microstates") of the diffraction field, we would still resort to an averaged description, if only because the actual values of the scattered field in individual instances are virtually never repeated and hence are of no interest in themselves. The statistical moments and probability distributions, on the other hand, are stable characteristics of the stochastic field, and it is they that enable us to obtain a natural and adequate description of stochastic processes.

The statistical problems of diffraction theory are extremely varied. They encompass a multitude of physical objects and are frequently closely associated with applications. On first encountering these problems, one may get the impression that he is dealing with a multitude of diverse problems. This impression is reinforced by the fact that many different methods and approximations are used in diffraction theory to solve both dynamical and statistical problems. Actually, however, the pattern becomes less confused if we classify the statistical diffraction problems according to their physical formulation, and not according to the mathematical methods used to solve them nor according to the traditional "branch" of physics (optics, acoustics, radiophysics, etc.) to which they belong. In this review we shall attempt to systematize statistical diffraction phenomena, limiting our discussion to linear and classical (nonquantum) problems of the statistical theory of waves.

2. CLASSIFICATION OF STATISTICAL DIFFRACTION PROBLEMS

It is natural to tie the classification of the statistical phenomena to the formulation of the dynamical problems of diffraction theory. A rather broad class of these problems are formulated as follows.

Let a body (or a system of bodies), bounded by the surface S, lie in a medium (homogeneous or inhomogeneous) in which waves of some definite physical type (electromagnetic, acoustic, elastic, spin, etc.) can propagate. We denote the corresponding linear wave operator (this is usually a differential operator—less frequently, an integro-differential operator) by L, so that in a region free of sources, the wave equation has the form

$$Lu = 0, \tag{1}$$

in which u is the field quantity, which may be a scalar or a vector (in the case of a vector field, L is a tensor operator).

The primary wave that strikes the body S (Fig. 1) is either produced by specified real sources q or is excited by virtual sources, in which case one assumes that the specified quantity is, for example, the field u_0 of the initial wave (which is most frequently a plane wave). The unknown quantity to be found is the scattered diffraction field. Of course, in addition to the sources, the wave equation, and the shape of the boundary surface S, one must also specify certain homogeneous boundary conditions on S as well as the conditions at infinity (the radiation conditions).

The same equation (1) and the same boundary conditions also serve for a statistical problem, but now

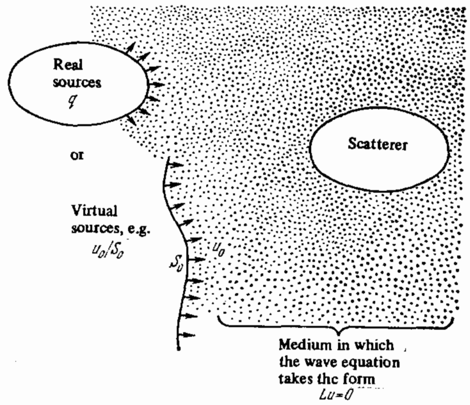


FIG. 1. Typical formulation of a problem in diffraction theory.

Eq. (1) is a stochastic equation, i.e., an equation that the individual realizations of the stochastic field must satisfy. In other words, the parameters, functions, and operators involved in the problem (all or some of them) are now random quantities, i.e., they are specified by their probability distributions. For example, if the primary field u_0 is specified on the surface S_0 (virtual sources) in the dynamical problem, it is the statistics of this field that is to be specified in the corresponding statistical problem. In particular, one may specify only the second moment of this field,

$$\Gamma^0_u(1, 2) = \langle u_0(1)\, u_0^*(2)\rangle, \tag{2}$$

which in optics is usually called the second order coherence function, or simply the coherence function (if u_0 is a vector field, the product $u_0(1)u_0^*(2)$ must be regarded as the outer (dyadic) product, so that in this case $\Gamma^0_u(1, 2)$ will be the coherence matrix). Higher moments (higher-order coherence functions) of the primary field u_0 may also be specified. Of course a complete statistical description of the field u_0 is given by the set of all n-dimensional (n = 1, 2, ...) probability densities. If we start with the dynamical problem formulated as above, we can treat any of the following as random variables:

a) the real sources q,
b) the virtual sources (say u_0/S_0),
c) the shape and position of the boundary surface S, and/or
d; the properties of the medium, i.e., the operator L itself.

In accordance with these different possible random variables, we may introduce four basic statistical schemes, which we shall arbitrarily call primary statistical schemes. Let us briefly sketch the mathematical "formulation" of these primary schemes.

a) Excitation of fields by random sources. The problems of this group are described by the inhomogeneous equation

$$L\tilde{u} = \tilde{q}.$$

(For the present we shall mark the symbols of stochastic quantities with a tilde; here these quantities are the sources $\tilde{q}$ and the field $\tilde{u}$ itself). For brevity we shall refrain from discussing the necessary homogeneous boundary conditions on S and the conditions at infinity: these conditions are mandatory for dynamical problems, as well as for statistical ones.

b) Diffraction of stochastic (partially coherent)[2] fields. The problems of this group are described by the homogeneous wave equation

$$\tilde{L}u = 0,$$

but with inhomogeneous stochastic boundary conditions of the type $\tilde{u}_0|_{S_0}$, which reflect the fact that the initial wave $\tilde{u}_0$ is a random quantity.

c) Diffraction by bodies with random shapes and/or positions. Here the boundary conditions are imposed on a random surface $\tilde{S}$, while the wave equation may be inhomogeneous ($L\tilde{u} = q$) in the case determinate real sources or homogeneous ($L\tilde{u} = 0$) but with inhomogeneous boundary conditions on S_0 in the case of determinate virtual sources.

d) Diffraction and propagation of waves in a randomly inhomogeneous medium. Here we are dealing with a stochastic operator $\tilde{L}$ for the propagation of waves in the medium. If $\tilde{L}$ is a linear differential operator the coefficients of the derivatives may be random variables, while if $\tilde{L}$ is an integro-differential operator, its kernel may be a random function. Thus, when the field is excited by real determinate sources, for example, the equation for the problem is $\tilde{L}\tilde{u} = q$.

It is actually found that the overwhelming majority of statistical problems in linear diffraction theory reduce to these four primary schemes. Of course problems of mixed type are also possible, but up to now there has been very little discussion of such problems. We shall discuss them briefly in Chap. 7.

Of course the mere formulation of the problem does not predetermine the methods (usually approximate ones) to be used for solving it. Actually, the fluctuations of various of the parameters and functions that occur in the conditions of the problem may be large or small (on some characteristic scale), smooth and slow or, on the contrary, sharp and fast, strongly or weakly correlated, and so on. These differences in the physical conditions require different approaches to an approximate treatment of the problem. That is why there are so many secondary statistical schemes that are not associated directly with the formulation of the problem, but with the various approximate methods that can be used for solving it. Moreover, it is just the fact that there are so many statistical schemes that makes it difficult to orient oneself among the problems of wave statistics.

In the subsequent exposition we shall first of all attempt to distinguish between primary and secondary statistical schemes. To do this we shall examine several important and interesting groups of statistical problems, adhering to the classification of the problems adopted above, based on their formulations. Incidentally, we shall touch upon the most widely used methods for solving them. Almost every one of the problems discussed below has generated an abundant literature, so here we shall merely cite summarizing monographs and review articles, citing the original papers only in special cases.

3. EXCITATION OF FIELDS BY RANDOM SOURCES

Let $G = L^{-1}$ be the inverse of the operator L for the dynamical problem, i.e., the Green's function in terms of which the solution to Eq. (1) can be written in the form[3] u = Gq. This equation is a linear (operator) relation between the random sources q and the stochastic wave field u. The moments of the field u will therefore also be linearly related to the corresponding moments of q:

$$\langle u\rangle = G\langle q\rangle, \quad \langle u_1 u_2^*\rangle = G_1 G_2^* \langle q_1 q_2^*\rangle, \text{ etc.}$$

(the angle brackets denote statistical averages, i.e., averages over the statistical ensemble).

If we know (and use) the exact Green's function, we obtain exact expressions for the moments, but if we use various approximate Green's functions whose adequacy has not been tested by diffraction theory, we obviously obtain only approximate values for the moments. Thus, we have essentially a single method of solution for the problems of scheme (a) of Chap. 2. Here the great variety of problems is due mainly to the abundance of specific physical and technical problems that reduce to this scheme.

The best known group of applied problems of type (a) comprises those concerning the statistics of antennas, and in particular, problems concerned with the fluctuations of real currents. In these problems one is mainly interested in the statistical characteristics of the directional pattern $g(\theta, \varphi)$, so that the Fraunhofer approximation can be used for the Green's function. In this approximation $u \sim g e^{i\mathbf{k}\cdot\mathbf{R}}/R$, and the angular pattern g is related to the fluctuating currents in the antenna by a Fourier transformation. Here the principal results are well known and are described, for example, in Shifrin's book [1]. The distortion of the pattern is best seen in the case of antennas that are large (as compared with the wavelength) and therefore have narrow patterns (Fig. 2). As the antenna-current fluctuations increase, the radiation in the direction of the principal lobe falls off and the lobe itself broadens. In addition, the zeros (or minima) of the pattern become smoothed out and the side radiation (i.e., radiation in directions other than that of the principal lobe) increases.

The main difficulty in investigating the statistics of antennas does not seem to be in calculating the various integrals that arise (although we are far from overcoming all the difficulties even here), but in specifying the fluctuating currents along the antenna flare in a physically justified manner. The guiding role here must be assigned to experiment, since the current fluctuations are mainly due to errors in fabricating the antennas (spread of the parameters or uneven spacing of the radiating dipoles) and to random variations in the "feeding" of the dipoles.

Of the physical problems belonging to scheme (a) of Chap. 2 we first note the problem of the electromagnetic radiation from hot bodies. Here the statistics of the currents excited by thermal radiation is known: it is determined by the fluctuation-dissipation theorem as generalized to distributed electromagnetic systems (see [2]). These problems differ from those of the preceding group in that the greatest difficulty in solving them lies in finding the Green's function. Here we shall mention three problems whose solution requires calculation of the diffraction of thermal radiation.

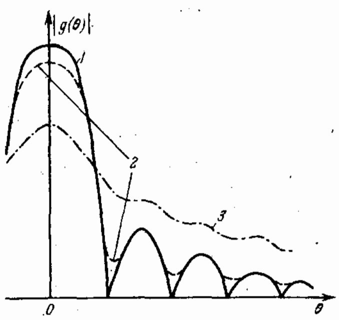


FIG. 2. Distortions of the directional pattern of an antenna due to the presence in the antenna of current fluctuations: 1—no current fluctuations; 2—weak current fluctuations; 3—strong current fluctuations.

a) Calculation of the intrinsic thermal radiation in antennas, waveguides, and other uhf electronic devices. The distinctive feature of this problem as compared with the problems of antenna statistics is that here one must also know the fields close to the antenna, and this naturally complicates the calculations [2].

b) Measurement of the parameters of large antennas (in particular, of radiotelescope reflectors) from the characteristics of the radiation received from hot bodies. A black disk is generally used as the standard body against which to calibrate the antenna system.

c) Determination of the properties of lunar and martian formations and of the surface of the earth (the ocean, arctic ice, deserts, etc.) from the intensity of the thermal radiation received from them at a distance. It is comparatively simple to calculate the thermal radiation for inverse problems of this type if the radiating medium is modeled as a uniform half-space with a plane boundary, but the calculations are much more difficult if the boundary is uneven or the radiating medium is inhomogeneous.

Another problem of general physical interest is the statistics of the field emitted by a large number of uncorrelated sources. Under certain assumptions (and sometimes quite rigorously) this problem encompasses the problems of the optical radiation of the atoms of a hot gas, the radiant emission from electrical discharges incident to snowfalls, the acoustic noise from rustling leaves on trees or from air bubbles bursting at the surface of the sea, and many others. Since the sources are independent, it is the intensities of the radiated fields that are additive. It would seem that there could be no question of diffraction phenomena under these conditions, since the fields of independent sources do not interfere with one another. Actually, however, interference still takes place, though it is not manifest in the spatial distribution of the intensity, but in the behavior of the coherence function of the radiated field.

Let the sources (for definiteness we shall assume them to be the atoms of a self-luminous body—a star, for example) occupy some finite volume of diameter a. We denote the intensity within the cloud of sources, reduced to the central plane z = 0, by $J(\boldsymbol{\rho}')$, where $\boldsymbol{\rho}' = (x'y')$ is a vector in the plane z = 0 (Fig. 3). Then the spatial coherence function of the radiated field at the plane z = const is given in terms of $J(\boldsymbol{\rho}')$ by the diffraction integral

$$\Gamma_u(\rho) \sim \int J(\rho')\, e^{ik\rho\rho'/z}\, d^2\rho', \qquad (3)$$

in which $\boldsymbol{\rho}(xy)$ is a vector in the plane z = const. In other words, the coherence function $\Gamma_u(\boldsymbol{\rho})$ is the Fourier transform of $J(\boldsymbol{\rho}')$, i.e., it behaves like the field of a wave that has passed through a variable-transparency screen whose transmission factor is proportional to $J(\boldsymbol{\rho}')$.

Equation (3), which expresses the so-called van Zittert-Zernike theorem [3], is widely used in optics and radioastronomy—we shall speak more of this later—but specialists in diffraction theory, who are used to dealing with coherent fields, know little about it. Nevertheless, the theorem can find interesting applications in diffraction problems too.

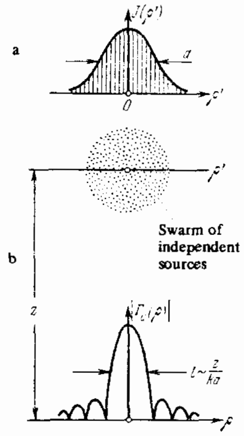


FIG. 3. The coherence function of a field excited by a swarm of independent sources. The coherence function $\Gamma_u(\boldsymbol{\rho})$ (b) is related to the reduced intensity distribution $J(\boldsymbol{\rho}')$ (a) by the Fourier transformation (3).

The theorem is valid for $z \gg a$, where a is the transverse size of the cloud of radiators. When a is large ($ka \gg 1$), therefore, the theorem is valid both in the distant zone ($z \gg ka^2$) and in the near zone ($z \ll ka^2$, but $z \gg a$). Use can be made of this circumstance to find the directional pattern in the distant zone from measurements in the near zone ($a \ll z \ll ka^2$) of the coherence function of the thermal radiation field from the appropriately heated antenna, for the coherence function and the directional pattern are expressed in terms of the same Fourier transformation. Another example concerns the acoustics of open concert halls (in closed rooms one would have to take the reflection from the walls and ceiling into account). The orchestra—say a group of violins—may be treated as a set of uncorrelated radiators (although the musicians play in unison, their violin strings vibrate with different, and indeed independent, phases). Let us ask how far from the stage the stereophonic effect will be perceptible. Following V. A. Zverev [4], we may assume that the stereophonic effect will be perceived when the range l of the correlations of the acoustic field is small compared with the distance y between the listener's ears ($l \ll y$), i.e., when the two ears receive uncorrelated vibrations. The van Zittert-Zernike theorem (3) gives the estimate $l \sim z/ka$ for the correlation (coherence) range l. It follows that the stereophonic effect disappears when $l \lesssim y$, i.e., when $z < kay$. This is not a very great distance: for a frequency of 3 kHz it amounts to ~ 30—60 m. If one wishes, one may regard this as a "diffraction explanation" for the lower price of tickets for the last rows.

The van Zittert-Zernike theorem can also be generalized to the case in which the sources are partially correlated. The coherence function $\Gamma_u(\boldsymbol{\rho})$ obviously "perceives" this correlation only when the correlation range of the sources exceeds the wavelength.

4. DIFFRACTION OF PARTIALLY COHERENT FIELDS

A typical formulation of a problem of type (b) would be as follows: The statistics of the primary field u_0, i.e., its moments (coherence functions), are specified on the plane $z = 0$, and it is required to find how these functions vary on moving away from that plane provided the field is subjected to certain transformations on the way (the wave passes through stops, lenses, etc.).

Formally, this problem is easy to solve. if one knows knows how a determinate (fully coherent) wave is transformed, one need only average the solution to the determinate problem over the ensemble representing the statistics of the "incoming" field u_0. However, this natural approach usually leads to integrals that it is difficult to evaluate. For example, to calculate the relative intensity fluctuations $\beta = \langle (I - \langle I \rangle)^2 \rangle / \langle I \rangle^2$ (the "flicker index") one must evaluate an eightfold integral. One cannot evaluate this integral rigorously even for the simplest possible model problem of the intensity fluctuations behind a random phase screen[4] (Fig. 4).

Fortunately, qualitative physical considerations frequently suggest approximate methods for calculating the complicated multiple integrals. In the phase screen problem, the calculation of the flicker index β can be carried through to the end for the case of weak phase fluctuations ($\langle \psi^2 \rangle \ll 1$). For medium and strong phase fluctuations ($\langle \psi^2 \rangle \gg 1$), on the other hand, one can use perturbation theory to calculate β at small distances z (the intensity fluctuations directly behind the screen are small), while at large distances one can simply use a normal field distribution. The field distribution becomes normal at large distances z because at such distances many uncorrelated waves from different parts of the screen arrive at the same observation point. Finally, in the intermediate focusing region one can find the asymptotic behavior of the field for large phase fluctuations ($\langle \psi^2 \rangle \gg 1$). In this way we obtained the approximate $\beta(z)$ curves shown in Fig. 5 (see [5,6] and the references cited in [7]).

Calculations of this type are used in radioastronomy. A random phase screen serves as a (not entirely satisfactory) model of the ionosphere or the interplanetary plasma with random irregularities. On traversing the randomly inhomogeneous ionospheric plasma or solar corona, the wave becomes randomly phase modulated, and this leads to amplitude fluctuations at the Earth. From an analysis of these fluctuations one can derive information about the parameters describing the irregularities in the ionosphere or the solar corona.

However, the principal "user" of the theory of the diffraction of partially coherent fields is of course optics, in which the concept of coherence arose about a century ago. For a long time this remained a qualitative concept of the "yes-no" type, and only in the last 15—20

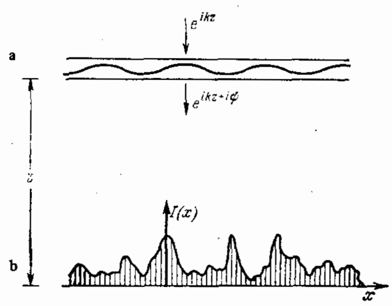


FIG. 4. Intensity fluctuations I(x) (b) in a wave that has traversed a phase screen (a).

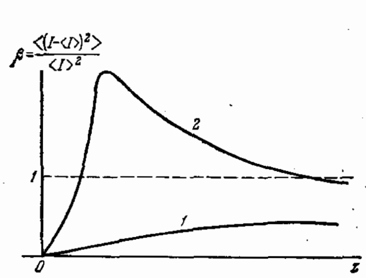


FIG. 5. Approximate behavior of the flicker index behind a random phase screen; curves 1 and 2 are for weak ($\langle \psi^2 \rangle \ll 1$) and strong ($\langle \psi^2 \rangle \gg 1$) phase fluctuations, respectively.

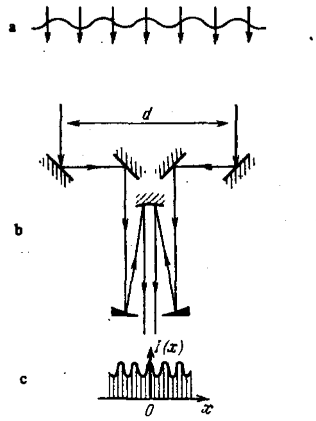


FIG. 6. Schematic diagram of the Michelson stellar interferometer.

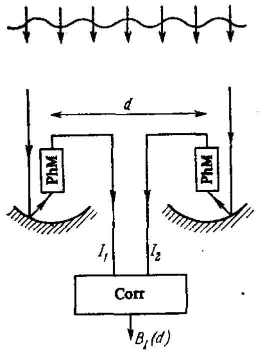


FIG. 7. Schematic diagram of the intensity interferometer of Brown and Twiss: PhM—photomultiplier; Corr—correlator.

years has the necessity of a quantitative measure of coherence been recognized and the corresponding theory (including the theory of partial polarization) been constructed. This theory reduces essentially to the calculation of the statistical moments of the wave field, i.e., the coherence functions of various orders, and mainly the second order coherence function. These coherence functions describe the statistical relation (correlation) between the field values at different points in space and at different instants in time. Now let us consider a few typical problems selected from those that arise and are solved in statistical optics.

a) Optical interferometry. Interferometry not only enables us to measure the coherence function (spatial or temporal) of the optical field, but in a number of cases also directly to see it. The temporal coherence function is related to the spectrum of the radiation by a Fourier transformation, and this makes it possible to measure the line shape. Such line shape measurements are actually made with interferometers intended for spectral analysis. As regards the spatial coherence function, we note that for self-luminous objects it is related by a Fourier transformation to the brightness distribution $J(\rho')$ on the surface of the radiating body (the van Zittert-Zernike theorem). Hence by measurement of the spatial coherence function we can (in principle) recover the surface brightness distribution $J(\rho')$, i.e., we can recover the shape of the luminous object or can at least determine its angular dimensions. This possibility is realized in the Michelson stellar interferometer. In this instrument (Fig. 6) one observes the interference between two light beams "cut" by two mirrors from the field of the partially coherent wave (a) emitted by a star. The visibility of the interference pattern (c) is determined by the degree of coherence of the field at points separated by a distance d equal to the interferometer base length (b). If the distance d between the mirrors is smaller than the coherence range l of the incident wave, the two light beams will be coherent, and when they are brought together at the center one will observe a contrasty interference pattern, whereas if $d \gtrsim l$, the interference pattern will be "washed out." From Eq. (3) we see that $l \sim 1/k\theta$, where $\theta = a/z$ is the angular diameter of the star. Hence by determining the base length d_0 at which the interference pattern disappears one can estimate the angular diameter of the star: $\theta \sim 1/k\theta \sim 1/kd_2$. Further, by analyzing the behavior of the visibility of the interference pattern as a function of the base length d we can (in principle) measure the coherence function $\Gamma_u(\rho)$, and from it we can determine the brightness distribution $J(\rho)$ over the investigated object.

b) The intensity interferometer. The use of the Michelson stellar interferometer to measure the spatial coherence function involves great difficulties associated with the high sensitivity of the instrument to the phases of the interfering waves. Even slight fluctuations of these phases and of the directions of the wave front at the two interferometer mirrors such as might be caused by atmospheric effects smear out the pattern of interference bands (just as would vibration of the widely separated mirrors) and thereby make it difficult to measure d_0. These difficulties do not arise when working with the intensity interferometer proposed by Brown and Twiss[8] (Fig. 7). With this instrument one does not measure the spatial correlation function of the wave field u itself, but rather the intensity fluctuations of this field:

$$B_I(\rho) = \langle I_1 I_2 \rangle - \langle I_1 \rangle \langle I_2 \rangle.$$

Moreover, this quantity is not measured by optical (interference) methods, but electrically, using a correlator to which are fed electrical signals proportional to the intensities I_1 and I_2 measured at points separated by the distance d. From this point of view the term "intensity interferometer" may not be entirely suitable: the device is essentially an intensity correlator, i.e., a device for measuring the correlation of the intensity fluctuations of the light field.

The light field of a star is a superposition of fields from a great number of independent sources and therefore conforms to Gaussian statistics. For a Gaussian field u with zero mean, however, the intensity correlation function is related to $\Gamma_u(\rho)$ by the simple equation $B_I(\rho) = |\Gamma_u(\rho)|^2$. Hence the measured values permit us to draw virtually the same conclusions concerning the brightness distribution over the visible surface of the star as would the values of $\Gamma_u(\rho)$. Thus, not only the coherence function for the field itself, but also the intensity correlation function, carries diffraction information concerning the light field of the star. Moreover, what we measure here is the modulus of $\Gamma_u(\rho)$, which is not highly sensitive to fluctuations.

c) Optical image formation. It seems obvious that the degree of temporal and spatial coherence of the field illuminating the object must affect the quality of the image formed in cameras, microscopes, and other optical instruments, since the field at each point of the image plane is a superposition of waves emitted (or reemitted) by the object. Actually, however, in most cases the image quality depends little on the character of the light that illuminates the object. This is due to the fact the field at a point A' in the image plane (Fig. 8) is made up of contributions from only a small neighbor-

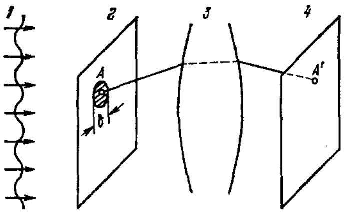


FIG. 8. Diagram illustrating the formation of an optical image under partially coherent illumination: 1—partially coherent light; 2—object plane; 3—optical system; 4—image plane.

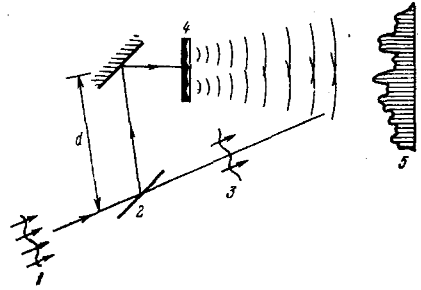


FIG. 9. Simplified diagram illustrating the formation of a hologram: 1—partially coherent light; 2—semitransparent mirror; 3—reference beam; 4—semitransparent object; 5—interference pattern (hologram).

hood of the optically conjugate point A of the object plane, the diameter δ of this "formation region" being, in the case of good optical instruments, of the order of the Rayleigh limit of resolution. The path difference for rays from the center and periphery of this region is about a half wavelength, and this is less than the length Δ of a coherent wave train for any illumination since even for illumination with daylight we have $\Delta \sim 3\lambda_0$, where λ_0 is the mean wavelength of visible light. That is why a photographic image formed with daylight illumination is almost indistinguishable from an image formed with strictly monochromatic illumination.

The problem of the effect on image formation of the spatial coherence of the illuminating light is solved in a similar way. It is clear that the spatial incoherence can have an effect only if the coherence range l is shorter than the radius δ of the image forming region, but since the instrument does not resolve details smaller than δ, the image quality will be virtually the same for $l \gg \delta$ (only the illuminance at A' will be different). Nevertheless, there are definite differences between coherent and incoherent illumination. These differences are manifest, for exampke, in the resolving power, and also in the fact that with incoherent illumination, the degree of coherence of the field after passing through the optical system is independent of the aberrations. From this, for example, we can draw the practical conclusion that it is entirely unnecessary to use a good aberration-free condenser to illuminate the field of a microscope.

When the image is recorded photographically, the quality of the final image will depend not only on the illumination, but also on the structural irregularity (graininess) of the photographic emulsion. The fluctuations due to graininess can also be described by statistical methods. Various models of the grain structure have been proposed; with the aid of these models one can estimate the maximum information that can be carried by a photographic image[9].

d) Holographic image formation. We recall that a hologram is an interference pattern formed by bringing together into a single plane a reference light beam from a laser and a second beam from the same source that has been diffracted by the object (Fig. 9 is a simplified diagram illustrating hologram formation). The contrast of the interference pattern is determined by the degree of coherence of the reference and diffracted light. The theory of the diffraction of partially coherent fields made it possible to establish the requirements on the degree of monochromaticity of the light for the successful formation and reconstruction of holograms. It was found that under certain conditions holograms can be obtained with incoherent illumination, i.e., by using "ordinary" light sources (rather than lasers), which, however, must be very nearly monochromatic if a high-contrast hologram is to be formed. The quality of a holographic image, like that of an ordinary photograph, still depends on the graininess of the photographic film on which it is recorded. Here, too, the statistical diffraction theory enables us to estimate the optimal achievable characteristics of a hologram. What was just said applies equally to diffraction estimates of the random errors introduced by irregularities in the refractive index and by other defects of lenses, photographic films, and other elements of optical systems[10].

The results of the theory of the diffraction of partially coherent fields are widely applied not only in optics, but also in other branches of physics.

1) As is well known, the primary limitation on the possibilities of x-ray structure analysis are due to the low degree of coherence (monochromaticity) of the x radiation. Because of the short coherence length of the x-rays one can observe the interference only of waves scattered from atoms lying close together in a crystal lattice, but not the interference of waves scattered from large crystals. This reduces the accuracy in determining interatomic distances and other characteristics of the crystal. If one could obtain sufficiently monochromatic x-ray beams, one would obviously be able sharply to improve the results obtained by x-ray structure analysis. In this connection we recall that when Gabor proposed holography he was pursuing this same goal—to enhance the information borne by x-ray photographs. Of course the difficulties in producing sources of sufficiently coherent x radiation (x-ray lasers) are rather of experimental than of theoretical type: the problem is to find materials with suitable properties and to achieve sufficiently powerful and at the same time sufficiently monochromatic pumping.

2) The use of the theory of the diffraction of partially coherent fields in radioastronomy is not limited to the calculations of fluctuations beyond a phase screen that we discussed above. This theory finds considerably more important applications in the development of radiointerferometers, which can be used, in particular, to measure the angular dimensions of extraterrestrial radio sources. The radiointerferometer operates on the same principle as the Michelson stellar interferometer, but the actual device is based on an entirely different technology. First, large radio antennas (Fig. 10) are employed instead of spaced mirrors. Further, the base d for the measurements is not just a few meters long, but may amount to several thousand kilometers. This makes it possible to equal, and even to exceed, the angular resolution of optical interferometers, despite the enormously longer wavelength. The radio signals received by the antennas are recorded on magnetic tape, the recordings being synchronized with standard atomic clocks. The recordings are subsequently processed together to determine the degree of coherence of the two

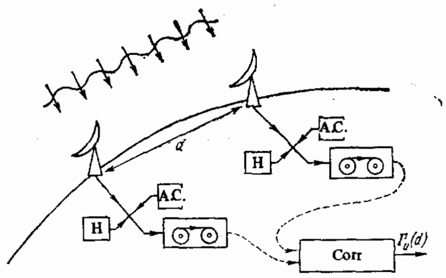


FIG. 10. Diagram illustrating the measurement of the spatial coherence function of the radio emission from extraterrestrial radio sources, using a long-base radiointerferometer: H—heterodyne, A.C.—atomic clock, Corr—correlator.

signals, and from this one can judge the angular dimensions of the radio sources [11]. Of course the technical differences should not mask the essence of the interference method of measuring angular dimensions, which, in the last analysis, is based on the van Zittert-Zernike theorem.

Having constructed interferometers with bases comparable with the radius R_E of the earth, the radioastronomers are now discussing an even more impressive project: to join all the large radiotelescopes on the earth earth together into a single system. This amounts essentially to constructing a peculiar radio reflector with dimensions of the order of R_E that consists of separate "fragments"—the individual radiotelescopes. The principal difficulty in producing such a reflector is encountered in attempting to "phase" the individual "fragments." In optics this problem is solved by properly choosing the positions and orientations of the reflectors, whereas here it can be solved by introducing appropriate time delays in the tape recordings. In this case the inevitable errors in synchronizing the recordings will play a part similar to that played by random aberrations in optical systems.

3) The principle of holography finds application in radiophysics in the so-called synthetic aperture method [12,13]: In this method one undertakes coherent processing, i.e., one records not only the amplitude, but also the phase, of the radar signals emitted from an airplane or artificial earth satellite and reflected from the earth's surface. The motion of the receiver replaces the long and, in general, wide-aperture antenna. The recording that preserves the coherence data (radiohologram) is analogous to the interference pattern in optical holography (here the emitted signal plays the part of the reference beam in the optical case). On "reconstituting" the hologram, one can resolve details with angular dimensions of the order of λ/vT, whereas the usual limit of resolution is λ/a (a is the size of the radio antenna, v is the velocity of the aircraft, and T is the coherent processing time); the product vT is the effective size of the synthetic aperture.

Without dwelling on the details of radioholography, we note that the limiting accuracy with which a region can be mapped is mainly determined by the phase errors in the transmitting and receiving channels: the permissible coherent processing time T is limited to the time during which the random phase excursions remain small as compared with π. Here, too, the phase fluctuations are analogous to random aberrations in optics. The difficulties associated with preserving the phase are also the limiting factor in another application of radioholography to radar astronomy: the mapping of lunar and planetary surfaces using the motion of the earth as the basis for the synthetic aperture method [14].

Coherence theory also provides an adequate statistical description of partially polarized waves. Here the basic statistical characteristic is not the coherence function, as in the scalar theory, but the coherence matrix (polarization matrix) $\Gamma_{ik}(1,2) = \langle E_i(1)E_k^*(2)\rangle$, where E_i and E_k are components of the electric field vector. This matrix provides a complete description of a vector field in the context of correlation theory. It is better, however, to expand the stochastic wave field in plane waves and deal with the latter. Such quantitative characteristics of a partially polarized wave as the degree of polarization and the Stokes parameters, which are unambiguously related to the elements of the polarization matrix, can be introduced for a plane wave.

The theory of partial polarization, like scalar coherence theory, finds an extremely wide range of applications: in astrophysics it serves as a basis for choosing between various mechanisms for optical, radio, or x-ray emission (thermal emission, synchrotron radiation, etc.) that might explain the observed polarization data; in atmospheric optics the degree of polarization makes it possible to judge the characteristics of the scatterers; and so on. In order fully to solve vector problems of types a), b), c), and d) one must, in general, evaluate the elements of a coherence matrix.

5. DIFFRACTION BY BODIES HAVING RANDOM SHAPES OR POSITIONS

The problems of this type can be separated into two subgroups: diffraction from bodies with random shape, and from bodies with random position. Let us consider these subgroups one at a time.

If there are many random irregularities on a surface, we say that the surface is rough or statistically uneven. All real surfaces are rough, some being rougher than others. From the point of view of diffraction, even the "ideally flat" surface of a liquid at rest is rough, for it actually fluctuates as a result of the thermal motion of the molecules. Random irregularities are also present on ideally polished telescope mirrors and lenses, as well as radiotelescope reflectors, not to mention such uneven surfaces as the agitated surface of the sea, the lunar landscape, asphalt, and paper. The principal measure of the degree of roughness is the ratio of the height ζ of the irregularities to the wavelength λ. The same surface can obviously be very rough for short waves (e.g., ripples on the water for light waves) and practically smooth for long waves (e.g., those same ripples for long radio waves).

Problems of the scattering of waves from rough surfaces can be solved only by approximate methods. Two methods, developed originally for dynamical problems, have been most widely used: perturbation theory, and Kirchoff's method. Perturbation theory is suitable for irregularities whose height ζ is small compared with the wavelength λ (Fig. 11, a), while Kirchoff's method is applicable not only to the case of low irregularities, but also to that of high ones ($\zeta \gg \lambda$), which, however, must have a large radius of curvature ($R \gg \lambda$) (Fig. 11, b). Both methods require still another condition to be met: the irregularities must be mildly sloping, i.e., if l is a characteristic length of an irregularity, we must have $\zeta \ll l$. When this condition is violated it proves to be

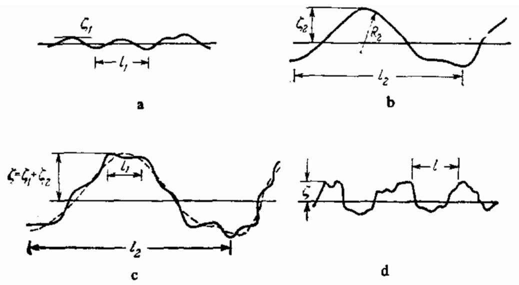


FIG. 11. Types of surface irregularity for which approximate calculations of the scattered field can be made by different methods: a) perturbation theory ($\zeta_1 \ll \lambda$, $\zeta_1 \ll l_1$); b) Kirchoff's method ($\zeta_2 \ll l_2$, $R_2 \gg \lambda$); c) a combined method (the surface roughness consists of "small ripples on a large wave," and in addition to the inequalities required for cases (a) and (b), here we must also have $l_1 \ll l_2$); d) the Green's function method ($\zeta \ll \lambda$, $l \ll \lambda$).

virtually impossible to carry through the required statistical averaging of the formulas of the dynamical theory.

The overwhelming majority of the results obtained up to now have been obtained by the two methods mentioned above. A unified exposition of scattering theory including the new approaches to the problem is given in the book by Bass and Fuchs[15] and in the brief but capacious review article[16]. The main application of the theory has been to the scattering of radio waves, light, and sound by the statistically uneven surface of the sea. The radiation scattered from the agitated sea surface bears useful information on the spectrum and height of the waves, on the vertical and horizontal velocities of the oscillatory motion of the particles in the wave, on wind directions, etc. Relatively little attention has been given to other applications, although the theory does not suffer from a lack of objects for study.

The large reflecting antenna of a radiotelescope is one of these objects. The irregularities of such a dish are especially noticeable at short wavelengths. They lead to essentially the same effects as current fluctuations in real antennas. In open resonators, irregularities of the mirrors tend to reduce the Q factor (since part of the radiation gets "splashed" out of the resonator on account of scattering from irregularities) and to shift the eigenfrequencies (because of an effective change in the distance between the mirrors[17]). In optical systems, surface irregularities of the lenses, as well as bulk irregularities in the refractive index of the glass, lead to certain image defects, and in particular, to broadening of the diffraction spot. In microwave waveguides, irregularities in the walls are responsible for the transformation of some types of waves into other types[18]. Phenomena that lead to similar results also arise in quasioptical microwave transmission lines on account of defects in the lenses and reflectors. We note further that in the propagation of light in thin films, irregularities in the interfaces result in strong damping of the waves on account of radiation losses.

The statistical theory of scattering from rough surfaces has been employed to analyze the reflection of radio waves from the moon and planets. The main difficulty here was to choose an appropriate model correlation function for the irregularities. Attempts to introduce an effective impedance of the earth to describe the propagation of surface waves as well as the propagation of ultralong radio waves in the waveguide bounded by the ionosphere and the earth's surface have been more comforting. The theory has also been applied to a number of phenomena in solid state physics (the interaction of phonons with walls) and seismology (the literature on these topics is summarized in[15]).

In addition to the two basic methods discussed above one also sometimes uses a combined approach (perturbation theory together with Kirchoff's method), which is suitable for analyzing scattering in the case of two-component roughness of the "small ripples on a large wave" type (Fig. 11, c). In particular, with this approach one can successfully account for certain features of the large angle scattering of radio waves from the agitated surface of the sea. These features are not in accordance with a theory that takes into account only fine or only coarse irregularities of the sea swell.

Multiple scattering (or multiple rereflection) of the waves, which would greatly complicate the problem, is neglected in all three methods: perturbation theory, Kirchoff's method, and the combined method. Attempts have recently been made to overcome this difficulty within the context of the integral-equation and Green's-function methods. In the first of these attempts, proposed by U. P. Lysanov, Green's integral formula is treated as an integral equation for the field on the rough surface. This equation can be solved under the assumption that the surface slopes even less steeply than is permitted for the perturbation-theory or Kirchoff's-method calculations. But then the results of both these methods are obtained from the integral equation as particular cases[16].

Another approach, the Green's function method, was taken from quantum electrodynamics and is based on the approximate selective summation of infinite perturbation-theory series. In this method we escape from the requirement that the irregularities be gently sloping, but at the price of submitting to a new limitation: the scale of the irregularities must be small ($l \ll \lambda$; Fig. 11, d).

The Green's function method proved to be very effective for analyzing the transformations of waves of various types in rough-wall waveguides, and also for describing the propagation of long radio waves above the uneven surface of the earth[15].

Diffraction from irregularities that are neither small nor gently sloping is, as before, a bottleneck of the theory. Correlation theory, which relates the statistics of the scattered field to the statistics of the irregularities as specified by their correlation function alone, proves to be ineffective here. In this direction we have still not gotten beyond the use of model ideas[19] (e.g., hemispheres, semicylinders, or semiellipsoids randomly distributed on a plane (Fig. 12, a) or a set of plane areas with random dimensions and inclinations (Fig. 12, b)). Lambert's law (the cosine law), which describes what is called diffuse scattering, is frequently used for approximate calculations of the scattering from a strongly broken up surface, although the conditions under which the scattering of radiation would rigorously obey Lambert's law are still unclear (from both the theoretical and the experimental points of view).

Now let us consider the other subgroup of the problems of group b of Chap. 2—scattering from bodies having random positions in space. If we are dealing with just one body, then obviously no fundamentally new problems arise. Scattering from a multitude of bodies, how-

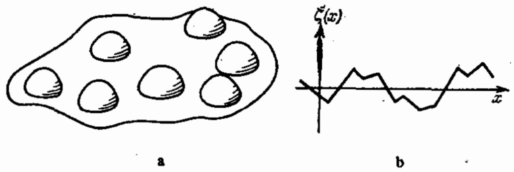
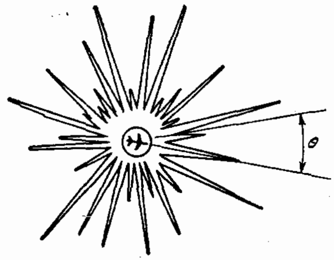


FIG. 12. Models of highly uneven surfaces used in wave scattering calculations: a) hemispheres randomly distributed on a half plane; b) an aggregate of plane areas with random inclinations.

FIG. 13. Example of the irregular diffraction patterns observed in the scattering of short waves from bodies having complex but determinate shapes.



ever, is an extremely general and important problem that has a wide range of applications and involves the problem of allowing for multiple scattering. In radiometeorology we deal with the scattering of radio waves from raindrops, snowflakes, hailstones, clouds, and mist; in atmospheric optics, with scattering from aerosols; in radar, with the reflection of radio waves from clouds of metallic needles; in sonar, with the scattering of sound from air bubbles and plankton; in astrophysics, with scattering from interplanetary and interstellar dust clouds; and so on.

Related problems arise in the treatment of molecular scattering in liquids and gases, in the analysis of incoherent scattering of electromagnetic waves by free electrons in laboratory and ionospheric plasmas, etc. Under these conditions the diffraction (boundary) problem itself simplifies greatly since here the problem is that of the diffraction of waves by point scatterers.

The problem of the diffraction of waves by a multitude of scatterers is most often (more accurately, "almost always") solved in the single-scattering approximation[20]. In this approximation one assumes that each scattering object scatters the incident wave as though the other objects were not there. The randomness of the field reemitted by an individual scatterer is due to the scatterer's random position (and sometimes orientation), which may be correlated with the positions and orientations of the nearest neighbors. Formally, the single-scattering approximation fits into scheme (a), since it is a problem of the radiation from many discrete random sources whose statistics are fixed by the properties of the primary field and the statistics of the positions (orientations) of the scatterers.

In many cases the single-scattering approximation proves to be entirely satisfactory. As the number of scatterers increases, however, the single-scattering theory finally ceases to be justified and multiple scattering must be taken into account. The simplest way to do this is to introduce the extinction of the primary wave, i.e., to take into account the attenuation of the primary field as a result of scattering. Neither this method, however, nor a consistent calculation of the double or tripple scattering can be effective enough: if the contribution from double scattering is appreciable, then as a rule the contribution from higher-multiplicity scattering cannot be neglected. Under these conditions all we can do is to describe the multiple scattering by means of a radiation transport equation, which is usually introduced phenomenologically (from energy balance considerations) and in recent years has received support from statistical wave theory (we shall discuss this later).

We can also assign the problem of the diffraction of short waves by bodies of complex shape, e.g., by airplanes, to scheme c), even though this assignment is somewhat arbitrary. Although in this case the surface of the scatterer is determinate, the scattering pattern $g(\theta)$ is so complex and irregular a function of θ that it is expedient to describe it in terms of probability concepts (Fig. 13).

The random character of the behavior of the scattering pattern $g(\theta)$ is to be attributed to the fact that many "highlights" ("bright spots") form on the surface of a body of complex shape when it is irradiated, and reflections from these highlights break up the interference pattern. Various probability models of the "highlights" are discussed, for example, in[21].

We note still another nonstandard problem of type (c), which lies somewhat to one side of the traditional path: the diffraction of waves by plane screens with statistically uneven edges. Such a screen can serve as a model for a mountain ridge in calculating the diffraction of uhf radio waves[22] or for the edge of the lunar disk, which diffracts light waves from stars and radio waves from extraterrestrial radio sources.

6. DIFFRACTION AND PROPAGATION OF WAVES IN A RANDOMLY INHOMOGENEOUS MEDIUM

The problem of the fluctuations of waves in randomly inhomogeneous media (scheme (d) of Chap. 2) has been comprehensively treated in monographs[23–25] and in a number of review articles (see, e.g., [7,26]). Here we shall therefore merely recall the approaches that have been adopted and give an account of some of the practical problems.

As a rule, the operator $\tilde{L}$ describing the propagation of waves in a randomly irregular medium will contain a "large" regular part L and a small perturbing part V, so that the field will satisfy the equation

$$\tilde{L}u = (L + V)\,u = 0. \qquad (4)$$

The simplest thing one can do with this equation is to solve it by perturbation theory, taking V as the perturbing part of the operator. Then in the first approximation we obtain the single-scattering approximation. The singly scattered field u_1 satisfies the equation

$$Lu_1 = -Vu_0,$$

where u_0 is the primary field. This is obviously another variant of scheme (a) of Chap. 2—the excitation of fields by distributed random sources.

The single-scattering approximation is usually called the first Born approximation, or simple the Born approximation, since Born successfully employed it to solve the quantum mechanical problem of the scattering of an electron by a nonuniform potential[5]. The Born approximation gives an entirely adequate account of a wide range of phenomena associated mainly with light scattering. The spectral analysis of light scattered by liquids or transparent solids can provide information concerning certain parameters of the material that it would be difficult or quite impossible to measure by other methods. As examples we may mention the study of scattering from the thermal fluctuations of the dielectric constant (Rayleigh scattering of light), the study of the critical opalescence, etc. The Born approximation gives a good account of the scattering of uhf radio waves in the ionosphere and of many important features of the far tropospheric propagation of these waves. The scattering of microwaves has recently come to be used to investigate the characteristics of laboratory plasmas.

As the distances become longer or the fluctuations V become greater, the first Born approximation eventually becomes inadequate and multiple scattering has to be taken into account. In the case of large and smooth irregularities with dimensions $l \gg \lambda$, one can take multiple scattering into account by the geometric optics method, Rytov's method of smooth perturbations, or the parabolic equation method. Unlike the geometric optics method, the last two methods describe the diffraction phenomena and involve the so called parabolic equation, i.e., the wave equation for the complex amplitude U ($u = Ue^{ikz}$, where z is the wave propagation direction) from which the second derivative $\partial^2 U/\partial z^2$ has been dropped, only the first derivative $\partial U/\partial z$ being retained. This approximation is valid precisely when the irregularities are large. The difference between the smooth-perturbation and parabolic-equation methods is that in the first of these methods the parabolic equation is written for the complex phase (i.e., essentially for the logarithm of the amplitude U), while in the second, it is written for the complex amplitude itself.

Despite the fact that the initial (unaveraged) equations of the geometric-optics and smooth-perturbation methods describe multiple scattering, one actually solves them by perturbation theory, taking the deviations of the dielectric constant of the medium from the average value as the small perturbing factor. Of course the perturbation series obtained by these methods, unlike the Born approximation series, are not expansions of the field itself, but of the phase (in the geometric-optics method) or the complex phase (in the smooth-perturbation method). Even though one has to limit the calculations to the first approximation, the geometric-optics and smooth-perturbation methods are still fairly effective in accounting for multiple scattering at not very great distances, provided the relative intensity fluctuations are not large. By now a great many specific problems of atmospheric optics, sonar, radar, and radioastronomy have been solved by these methods[7].

The study of light intensity fluctuations on paths near the earth's surface has now become an effective method of investigating the microstructure of turbulent streams in the atmosphere. A similar method, but using uhf radio waves, has received general recognition in research on plasma turbulence.

The intensity fluctuations of starlight (twinkling) also can provide a source of valuable information about tropospheric turbulence. Similar information on the electron concentration fluctuations in the ionosphere and interplanetary plasma can be extracted from radioastronomical observations of radio waves from extraterrestrial radio sources. Similar studies of the fluctuations in the interstellar plasma became possible with the discovery of quasars and pulsars.

The study of fluctuations of light in the atmosphere has recently received considerable stimulus from laser technology. Broadening of laser beams and phase, propagation-direction, and field-intensity fluctuations in the beam are of interest in connection with laser communication and ranging systems. Related problems also arise in radar and sonar.

Unlike the geometric-optics and smooth-perturbation methods, the parabolic-equation method enables us to go beyond the limits of perturbation theory and derive equations for the coherence functions of arbitrary order, which are valid not only in the region of small intensity fluctuations, but also in the region of strong fluctuations. Moreover, the equation for the second order coherence function can be solved exactly. The principal efforts of investigators working in this field are now being directed toward the solution of the equation for the fourth order coherence function, which is just the one that describes the intensity fluctuations (see, e.g.,[26-28]).

The general theory of multiple scattering does not make use of the parabolic equation, but employs the complete wave equation and is therefore competent to deal with problems involving not only large irregularities, but also fine bulk irregularities[6]. Approximate equations in closed form for the moments of the field have now been obtained within the limitations of the general theory of multiple scattering. The derivation of these equations was actually based on the selective summation of perturbation series, i.e., on the Green's function method developed in quantum electrodynamics, very diverse methods being used to sum the series (diagram techniques, perturbation theory for statistical operators, etc.—see[25,26,29]).

Multiple scattering theory has not yet had any very great specific successes. Essentially new results have been obtained, unfortunately, only in the problem of the propagation and scattering of waves in a medium with strongly fluctuating parameters (when the perturbation operator V in Eq. (4) is "not small"). Of course here, as in the problem of scattering from a rough surface, the irregularities must be on a small scale ($l \ll \lambda$). Nevertheless, multiple scattering theory has yielded a number of results of some theoretical importance.

First, within the context of the general theory of multiple scattering one can derive all the equations obtained by approximate methods and, what is even more important, one can determine the limits of applicability of these approximate methods. In other words, multiple scattering theory enables us to mark out the places occupied by the various approximate methods in the general scheme.

Second, it has been possible, by the aid of multiple scattering theory, to provide a "statistical-wave" basis for the radiation transport equation and to determine the "diffraction" content of that equation.

As we mentioned above, up to now the radiation transport equation has been introduced phenomenologically on the basis of the concept of ray tubes (i.e., within the limitations of geometric optics) and the energy balance condition. In the simplest case (the stationary scalar problem) the transport equation has the form

$$\mathbf{n}\,\frac{\partial I(\mathbf{n},\,\mathbf{R})}{\partial \mathbf{R}} = -\alpha I(\mathbf{n},\,\mathbf{R}) + \oint \sigma(\mathbf{n},\,\mathbf{n}')\,I(\mathbf{n}',\,\mathbf{R})\,d^2\mathbf{n}'. \tag{5}$$

The left-hand side represents the change in the "beam intensity" I(n, R) in the direction n, while the first term on the right describes the extinction and attenuation, i.e., the decrease in energy as a result of scattering and absorption (α represents the sum of the extinction and absorption coefficients), and the second term represents the increase in the energy in the direction n as a result of scattering from directions n′ (σ(n, n′) is the scattering cross section per unit volume). In the phenomenological derivation, the "beam intensity" I(n, R) is treated as a photometric quantity without considering the microscopic meanings of α and σ.

A new view of the radiation transport equation was reached in recent years when it was shown in several ways that the radiation transport equation could be derived, under certain assumptions, from the equation for the coherence function, i.e., in essence, directly from the stochastic wave equation (see[30], where the pertinent literature is cited). In such a derivation the phenomenological parameters α and σ can be related to the statistical characteristics of the medium, and in particular, to the spectral density of the fluctuations of the parameters describing the medium, while the beam intensity I(n, R), i.e., the energy characteristic of the field, can be related to the second order coherence function $\Gamma_u(\mathbf{r}, \mathbf{R})$ (here $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ is the distance between the observation points $\mathbf{r}_1$ and $\mathbf{r}_2$, and $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$ is the coordinate vector of the "center of gravity").

The relation between I(n, R) and $\Gamma_u(\mathbf{r}, \mathbf{R})$ is simple:

$$\Gamma_u(\mathbf{r}, \mathbf{R}) = \oint I(\mathbf{n}', \mathbf{R})\, e^{ik\mathbf{n}'\mathbf{r}} d^2\mathbf{n}' \tag{6}$$

(the integration here, as in Eq. (5), is taken over the unit sphere). This relation was first noted by Dolin[31] in the small-angle approximation, i.e., in the case in which the wave is scattered at small angles and the beam intensity I(n, R) differs appreciably from zero only in a narrow cone about the primary wave propagation direction.

Since the coherence function $\Gamma_u(\mathbf{r}, \mathbf{R}) = \langle u(1)u^*(2)\rangle$ satisfies a set of two wave equations corresponding to the equations for u(1) and u(2), and thus describes the diffraction of waves, it follows from Eq. (6) that even the beam intensity I(n, R) also bears diffraction information. In particular, in the small angle approximation the radiation transport equation is equivalent to the parabolic equation for the coherence function. In other words, solving the radiation transport equation in the small angle approximation is equivalent to solving the diffraction (parabolic) equation for the coherence function. Still another example illustrating the diffraction content of the radiation transport equation was given by Watson[32], who showed that if we solve the transport equation by perturbation theory (this is possible if α and σ are small enough or if the dimensions of the scattering volume are small), then in the first approximation for the coherence function as calculated with Eq. (6) we obtain the result of single-scattering theory, i.e., the result of pure diffraction theory. A few more subtle questions concerning the relation between the radiation transport equation and the theory of coherence (the conditions for unambiguous correspondence of these theories to one another, differences in formulating the boundary conditions for I and Γ_u, etc.) are discussed in[33].

In the statistical wave derivation of the radiation transport equation one also obtains other important and useful results. First, the transport equation can be derive with allowance for the transformation of the coherent component $\langle u\rangle$ of the field into an incoherent part (the corresponding equations have been obtained not only for the scalar problem[30], but also for the electromagnetic problem[34]). In calculating the coherent field one simultaneously determines the refractive index of the stochastic medium[35]. The attenuation (extinction) of the coherent field is to be attributed precisely to the fact that it "feeds" the incoherent component. Such a refinement of the radiation transport equation is meaningful for problems of coherent optics and radiophysics, but not for the traditional problems of the transport of radiant energy, in which there is no coherent field.

Second, the wave derivation of the transport equation reveals the role of a specific interference effect that may be called "back scattering enhancement". This effect manifests itself, in particular, in the fact that the back scattering cross section of a body that is small (compared with λ) and is immersed in a medium with large random irregularities increases with increasing fluctuations of the dielectric constant of the medium, the angular dependence of the scattering cross section assuming the form shown in Fig. 14[36]. As the figure shows, the enhancement of the back scattering ($\theta \sim \pi$) is accompanied by a weakening of the scattering in directions close to π, the total cross section remaining the same as in the absence of fluctuations. It turns out that when the back scattering enhancement effect is taken into account the radiation transport equation becomes ineffective in certain parts of space, e.g., in a small neighborhood of a point source[37] or in a narrow sector about the backward direction, provided the scattering volume is irradiated by a plane wave[31]. This effect does not appear when radiant energy is being transported in stellar and planetary atmospheres, however, because there the sources are spread throughout the entire scattering volume.

Within the limitations of multiple scattering theory one can obtain approximate equations in closed form for the moments of a wave field not only in the case of a continuous randomly inhomogeneous medium (bulk irregularities), but also for the moments of a wave field scattered by a multitude of discrete disseminated centers (diffraction from bodies or point particles occupying random positions–statistical scheme c)). Here, too, one can derive the radiation transport equation from the general equations for multiple scattering theory (see, e.g.,[39,40]) and establish the microscopic meanings of the phenomenological parameters α and σ. It turns out that σ, the scattering cross section per unit volume, is not in general equal to the product $\sigma_{sp}N$ of the scattering cross section σ_{sp} for a single particle by the particle concentration N: the equality $\sigma = \sigma_{sp}N$ is valid only for small N; when N is large the so-called cooperative effects predicted by G. V. Rozenberg[41,42] come into play. In the papers just cited, and also in[43,44], Rozenberg discusses a vector form of the radiation transport equation that describes the behavior of the Stokes-parameter vector. Thus, the radiation transport equation can provide a complete description of the coherence characteristics of the field, including the polarization characteristics.

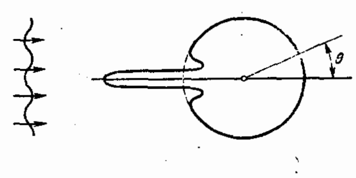


FIG. 14. Angular dependence of the scattered intensity from a small body immersed in a medium having large random irregularities, showing the enhanced scattering in the backward direction ($\theta \sim \pi$).

7. MIXED PROBLEMS

Up to now we have been discussing what we may call "pure" statistical schemes. Actually, various combinations of these schemes may be encountered, say of type (ac) or (bd) of Chap. 2. Since the number of mixed-type problems that have been solved is still not very great, we shall merely mention a few particular problems of this type.

a) Thermal radiation from a randomly irregular layer. Such a problem has been treated (in the Born approximation) in[45] (also see the bibliography in that paper) in connection with the analysis of experimental data on the thermal radiation from antarctic ice.

b) Thermal radiation from rough surfaces has been discussed in[46] as applied to good conductors (e.g., the reflectors of radio antennas) and in[47,48] in connection with the statistically uneven surface of the sea. The analysis of the thermal radiation from the disturbed sea is much complicated by the appearance of a third statistical factor–discrete scatterers consisting of foam or air bubbles.

c) The diffraction of waves in a randomly inhomogeneous medium in the presence of discrete randomly distributed scatterers has been investigated in[49], where the radiation transport equation for this case is derived. The range of application of this theory includes the transport of radiation in a turbulent atmosphere in the presence of an aerosol. One result of[49] is (at least at first glance) somewhat unexpected: the cross sections for scattering from bulk turbulent irregularities and from the aerosol are not additive; on the contrary, the expression for the scattering cross section contains a term due to the correlation between the bulk irregularities and the aerosol concentration.

d) The diffraction of partially coherent fields in a randomly inhomogeneous medium is of interest in connection with many applications: the passage through the earth's ionosphere or the interplanetary medium of radio waves from a radio source of finite angular dimensions[5,50], the broadening of a partially coherent (in cross section) laser beam in a turbulent atmosphere[51], etc.

e) The scattering of waves from a rough surface surrounded by a randomly inhomogeneous medium is distinguished by a number of special features, some of which have been analyzed in[52,53].

f) The diffraction of partially coherent light by a screen with a statistically uneven edge is of interest in connection with the determination of the angular diameters of stars by the lunar occultation method, since the unevenness of the edge of the lunar disk may limit the scope of this method.

Of course the above list is far from complete, but it may give some idea of current trends in linear statistical wave theory. The statistical phenomena incident to the propagation of waves in nonlinear media and quantum effects incident to the diffraction of electromagnetic waves require separate treatment.

In concluding, the authors wish to thank G. V. Rozenberg for valuable remarks.

---

1) This review is based on a report at the Sixth All-Union Symposium on the Diffraction and Propagation of Waves, Erevan, October 1971.

2) As applied to random wave fields, the terms "stochastic" and "partially coherent" are synonymous.

3) We no longer mark the symbols for stochastic quantities with a tilde.

4) That is what a layer that alters the phase of a wave crossing it is called: if the plane wave e^{ikz} falls on the screen, the field directly behind the screen will be given by $u_0 = e^{ikz + i\psi}$, where $\psi(x, y)$ is a random phase function. Specifying the statistics of the field at the screen obviously also fixes the statistics of the field at the plane $z = 0$. In this case the "system" that "transforms" the field after it crosses the screen is simply free space. Because of diffraction, the wave exhibits fluctuations after passing through the random phase screen (Fig. 4, b), even though it has a constant intensity on the phase screen itself (Fig. 4, a).

5) From a historical point of view this might have been called the Rayleigh approximation, since Rayleigh was the first to use perturbation theory to calculate the scattering of light by small particles; even now in optics one speaks of "Rayleigh scattering" and not of "Born scattering."

6) See below concerning multiple scattering from a multitude of discrete scatterers.

---

1. Ya. S. Shifrin, Voprosy statisticheskoĭ teorii antenn (Problems of the Statistical Theory of Antennas), Sov. radio, Moscow, 1970.
2. M. L. Levin and S. M. Rytov, Teoriya ravnovesnykh teplovykh fluktuatsiĭ v élektrodinamike (Theory of Equilibrium Thermal Fluctuations in Electrodynamics), Nauka, Moscow, 1967.
3. Max Born and Emil Wolf, Principles of Optics, Pergamon Press, Oxford, New York, 1959, 1964 (Russ. Transl. 1970).
4. V. A. Zverev, in "Materialy V Vsesoyuznoĭ shkoly po golografii", Novosibirsk, 1973 (Materials of the Fifth All-Union School of Holography, Novosibirsk, 1973), Izd. LIYaF AN SSSR, Leningrad, 1974, p. 472.
5. Ya. I. Al'ber, L. M. Erukhimov, Yu. A. Ryzhov, and V. P. Uryadov, Izv. vuzov (Radiofizika) 11, 1371 (1968).
6. V. I. Shishov, ibid. 14, 85 (1971).
7. Yu. N. Barabanenkov, Yu. A. Kravtsov, C. M. Rytov, and V. I. Tatarskiĭ, Usp. Fiz. Nauk 102, 3 (1970) [Sov. Phys.-Usp. 13, 551 (1971)].
8. R. Hanbury Brown, Contemp. Phys. 12, 357 (1971).
9. Edward L. O'Neill, Introduction to Statistical Optics, Addison-Wesley, Reading, Mass., 1963 (Russ. Transl., Mir, M. 1966).
10. R. J. Collier et al., Optical Holography, Academic, 1971.
11. B. F. Burke, Phys. Today 22, (10) 65 (1969).
12. R. O. Harger, Synthetic Aperture Radar Systems: Theory and Design, N. Y.–L., Academic Press, 1970.
13. N. I. Burenin, Radiolokatsionye stantsii s sintezirovannoĭ aperturoĭ (Synthetic Aperture Radar Stations), Sov. radio, M., 1972.
14. Radar Astronomy. Ed. J. V. Evans, T. Hagfors (MIT Lincoln Lab.), N. Y., McGraw-Hill Book Co., 1968.
15. F. G. Bass and I. M. Fuks, Rasseyani voln na statisticheski nerovnoĭ poverkhnosti (Scattering of Waves from a Statistically uneven Surface), Nauka, M., 1972.
16. A. B. Shmelev, Usp. Fiz. Nauk 106, 459 (1972) [Sov. Phys.-Usp. 15, 173 (1972)].
17. S. M. Kozel and G. R. Lokshin, Radiotekh. Élektron. 19, 1142 (1974).
18. R. B. Vaganov, R. F. Matveev, and A. S. Meriakri, Mnogovolnovye volnovody so sluchainymi neodnorodnostyami (Multiwave Waveguides with Random Irregularities), Sov. radio, M., 1972.
19. R. B. Patterson, J. Accoust. Soc. Amer. 36, 1150 (1960); J. E. Burke, V. Twersky, ibid. 40, 883 (1966).
20. H. C. Van de Hulst, Light Scattering by Small Particles (Russ. Transl. IL, M., 1961).

[21]E. A. Shtager, Izv. vuzov (Radiofizika) 16, 962 (1973).
[22]Yu. M. Polishchuk, Radiotekh. Élektron. 16, 2056 (1971).
[23]L. A. Chernov, Rasprostranenie voln v srede co sluchaĭnymi neodnorodnostyami (Propagation of Waves in a Medium with Random Irregularities), AN SSSR, M., 1958.
[24]S. M. Rytov, Vvendenie v statisticheskuyu radiofiziku (Introduction to Statistical Radiophysics), Nauka, M., 1966.
[25]V. I. Tatarskiĭ, Rasprostranenie voln v turbulentnoĭ atmosfere (Propagation of Waves in a Turbulent Atmosphere), Nauka, M., 1967.
[26]V. I. Klyatskin and V. I. Tartarskiĭ, Izv. vuzov (Radiofizika), 15, 1433 (1972); V. I. Klyatskin, ibid. 16, 1629 (1974).
[27]V. I. Tatarskiĭ, Rasprostranenie korotkikh voln v srede so sluchaĭnymi neodnorodnostyami v priblizhenii markovskogo sluchaĭnogo protsessa (Propagation of Short Waves in a Medium having Random Irregularities in the Markov-Random-Process Approximation). Preprint AN SSSR (Otdelenie okeanologii, fiziki atmosfery i geografii), Moscow, 1970.
[28]V. I. Shishov, Zh. Eksp. Teor. Fiz. 61, 1399 (1971) [Sov. Phys.-JETP 34, 744 (1972)].
[29]L. A. Apresyan, Izv. vuzov (Radiofizika) 17, 165 (1974).
[30]Yu. N. Barabanenkov, A. G. Vinogradov, Yu. A. Kravtsov, and V. I. Tatarskiĭ, ibid. 15, 1852 (1972).
[31]L. S. Dolin, ibid. 7, 669 (1964); 11, 840 (1968).
[32]K. M. Watson, J. Math. Phys. 10, 688 (1969).
[33]G. I. Ovchinnikov and V. I. Tatarskiĭ, Izv. vuzov (Radiofizika) 15, 1419 (1972).
[34]L. A. Apresyan, ibid. 16, 461 (1973).
[35]Yu. A. Ryzhov and V. V. Tamoĭkin, ibid. 13, 356 (1970).
[36]A. G. Vinogradov, Yu. A. Kravtsov, and V. I. Tatarskiĭ, ibid. 16, 1064 (1973).
[37]A. G. Vinogradov and Yu. A. Kravtsov, ibid. 16, 1055 (1973).
[38]Yu. N. Barabanenkov, ibid., p. 88.
[39]V. M. Finkel'berg, Zh. Eksp. Teor. Fiz. 53, 401 (1967) [Sov. Phys.-JETP 26, 268 (1968)].
[40]Yu. N. Barabanenkov and V. M. Finkel'berg, Zh. Eksp. Teor. Fiz. 53, 978 (1967) [Sov. Phys.-JETP 26, 587 (1968)].
[41]G. V. Rozenberg, Usp. Fiz. Nauk 56, 77 (1955).
[42]G. V. Rozenberg, Usp. Fiz. Nauk 69, 57 (1959) [Sov. Phys.-Usp. 2, 666 (1960)].
[43]G. V. Rosenberg, Appl. Opt. 18, 2855 (1973).
[44]G. V. Rozenberg, Opt. Spektrosk. 28, 392 (1970), [Opt.-Spectrosc. 28, 210 (1970)].
[45]A. S. Gurvich, V. I. Kalinin, and D. T. Matveev, Izv. Akad. Nauk SSSR (Fizika atmosfery i okeana) 9, 1247 (1973).
[46]V. V. Karavaev, Izv. vuzov (Radiofizika) 10, 658 (1967).
[47]A. G. Pavel'ev, Radiotekh. Elektron. 12, 1178 (1967).
[48]S. T. Wu, A. K. Fung, J. Geophys. Res. 77, 4231 (1972).
[49]G. I. Ovchinnikov, Izv. Akad. Nauk SSSR (Fizika atmosfery i okeana) 10, 88 (1974).
[50]V. I. Shishov, Izv. vuzov (Radiofizika) 15, 1279 (1972).
[51]A. I. Kon and V. I. Tatarskiĭ, ibid. 15, 1547 (1972).
[52]A. G. Vinogradov, ibid. 17, 1584 (1974).
[53]A. G. Vinogradov and A. B. Shmelev, in Voprosy izlucheniya i rasprostraneniya voln (Problems of the Emission and Propagation of Waves) (Trudy RTI AN SSSR, No. 8), M., 1974, p. 74.

Translated by E. Brunner