Boson-fermion symmetries and superfields

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This review provides an introduction to the theory of supersymmetry. The algebra of supersymmetries is developed, and its representations are given together with the ensuing surprising property of vanishing of the vacuum average of the energy-momentum tensor in supersymmetrical theories. Particular attention is paid to superfields—objects that combine fermion and boson fields with the aid of auxiliary anticommuting spinor coordinates. The Lagrangian formalism and the equations of motion for superfields are discussed. A detailed analysis is presented of the simplest supersymmetrical model of a chiral scalar superfield and its exceptional renormalizability. Gauge supersymmetrical theories are analyzed. The possibilities are discussed of a nontrivial unification of internal symmetries and supersymmetries. The article concludes with first attempts of spontaneous supersymmetry breaking, which is needed for the construction of future realistic supersymmetrical models.

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CONTENTS

"Nobody can foresee the next such law (of nature) that will be discovered. Nevertheless there is a structure in the laws of nature which we call the laws of invariance. This structure is so far-reaching in some cases that laws of nature were guessed on the basis of the postulate that they fit into the invariance structure"

E. P. Wigner in "The Role of Invariance Principles in Natural Philosophy" (Symmetries and Reflections, MIT Press, Cambridge, Mass., 1970)

1. INTRODUCTION

Symmetry principles have always played a very important role in physics. This role has grown particularly in recent times. An enormous number of new particles and resonances have been discovered, but the laws of nature which would describe quantitatively and exactly the interactions between the elementary particles, the spectrum of masses, etc., are not yet known. Only quantum electrodynamics can pretend to a quantitative description, of the purely electromagnetic interactions.

At the same time, a number of exact and approximate symmetry principles have been established, which enable us to find our bearings in the existing experimental data, suggest theoretical models and prompt the setting-up of new experiments facilitating a deeper understanding of nature. The isotopic invariance of the strong interactions and its generalization—the hypothesis of unitary SU(3) symmetry—have led to the hadron classification of Gell-Mann and Nishijima. During an investigation of the invariance with respect to the discrete symmetries (space reflection, time reversal and charge conjugation), one of the most beautiful effects in elementary-particle physics was discovered—the Gell-Mann-Pais-Piccioni effect for K mesons—and a phenomenological theory of the weak interaction was constructed. The recently discovered new narrow resonances, the ψ particles, are being discussed intensively on the basis of hypotheses of higher, charm symmetries, and the quark models, etc., associated with them.

Hypotheses concerning symmetry principles lead in certain situations to the construction of consistent theories. A combination of the idea of gauge symmetries and the idea of spontaneous symmetry-breaking has made it possible to construct a unified renormalizable theory of the weak and electromagnetic interactions.

Reigning supreme amongst the symmetries is the invariance under the Poincaré group, consisting of the Lorentz transformations, rotations and translations in space-time. Also of fundamental significance is the symmetry under permutations of identical particles, which leads to the classification of particles into fermions, possessing half-integer spin and obeying Fermi-Dirac statistics, and bosons, possessing integer spin and obeying Bose-Einstein statistics.

The transformations associated with all the traditional symmetries do not mix particles with different spins. Thus, the nucleons and the Λ, Σ and Θ particles appear in the same unitary-symmetry baryon octet and all have the same spin, equal to 1/2.

In the last few years, a new trend in the theory of elementary-particle symmetries—the study of supersymmetries—has appeared, and is developing rapidly. However paradoxical, it has turned out to be theoretically possible to connect the fields of particles with integer and half-integer spin, obeying different statistics, by supersymmetry transformations, and bring together bosons and fermions into generalized multiplets. The introduction of supersymmetries considerably extends the range of symmetry principles that are potentially applicable in relativistic quantum field theory and the theory of elementary particles. The first steps have already led to surprises: a new conserved spin-vector current is connected by a supersymmetry transformation with such a fundamental quantity as the energy-momentum tensor[36,102]; in the simplest supersymmetry model the number of divergences is sharply reduced in comparison with the usual field-theory models (only one renormalization constant is necessary, in place of the large number expected); all vacuum-loop diagrams and the vacuum-average of the energy-momentum tensor vanish, and so on.

We emphasize that, as yet, no realistic supersymmetry theory has been constructed. The equality of the masses of the fermions and bosons from the same irreducible supersymmetry representation forces us to seek a suitable supersymmetry-breaking. There are also other obscurities, e.g., associated with conservation of the number of fermions. We recall, however, that the Yang-Mills theory of gauge fields was also unrealistic (zero masses of the vector fields) at the time it was constructed. About fifteen years were needed to construct on its foundations, by invoking the idea of spontaneous symmetry breaking, a realistic unified-theory model of the weak and electromagnetic interactions, renormalizable and containing massive intermediate vector bosons. We can hope that a physical theory applying supersymmetries will be constructed in a shorter period. Already, supersymmetry is now of assistance in constructing field-theory models with asymptotically free massless[80,100] and (when the supersymmetry is broken) massive[21] particles. Supersymmetries are very unusual, distinctive and unfamiliar. In writing this article we have pursued the aim of giving as clear an introduction as possible to the theory of supersymmetries and assisting the reader to become accumtomed to them. We shall trace the characteristics of supersymmetry using the example of the simplest supermultiplet. It consists of a complex spinless field $A_1(x)$, a two-component spinor $\psi_\alpha(x)$ and an auxiliary complex spinless field $F_1(x)$. The infinitesimal supersymmetry transformations mixing bosons and fermions are written in the form

$$\begin{gathered}\delta_\zeta A_1(x) = \zeta^\alpha \psi_\alpha(x), \quad \delta_\zeta F_1(x) = -i\partial^\mu \psi^\alpha(x)(\sigma_\mu)_{\alpha\dot\beta}\bar\zeta^{\dot\beta}, \\ \delta_\zeta \psi_\alpha(x) = 2i(\sigma_\mu)_{\alpha\dot\beta}\bar\zeta^{\dot\beta}\partial^\mu A_1(x) + 2\zeta_\alpha F_1(x),\end{gathered} \tag{1.1}$$

where ζ is the spinor parameter of the transformation ($(\sigma_\mu)_{\alpha\dot\beta} = (1, \sigma_i)$; for the notation, see below). Since fermion fields anticommute and boson fields commute, there arises a very unusual situation in which the parameters of the transformation should anticommute ($\{\zeta_\alpha, \zeta_\beta\} = 0$), instead of being c-numbers as usual. The group property of these transformations is expressed in the fact that the commutator of two supersymmetry transformations is a translation. For example,

$$\begin{aligned}\delta_\eta\delta_\zeta A_1(x) &= 2i(\zeta\sigma_\mu\bar\eta)\,\partial^\mu A_1(x) + \zeta^\alpha\eta_\alpha F_1(x), \\ (\delta_\eta\delta_\zeta - \delta_\zeta\delta_\eta)\,A_1(x) &= 2i(\zeta\sigma_\mu\bar\eta - \eta\sigma_\mu\bar\zeta)\,\partial^\mu A_1(x).\end{aligned} \tag{1.2}$$

The translations commute with the transformations (1.1) and with them form a supergroup—a generalized group containing both commuting and anticommuting parameters. The mathematical theory of such generalized groups was developed quite recently, in 1970, by Berezin and Kats[7]. In order to obtain the corresponding generalized Lie algebra, we represent the transformation (1.1) in the form

$$\delta A_1(x) = i[\zeta^\alpha Q_\alpha + \bar Q_{\dot\alpha}\bar\zeta^{\dot\alpha}, A_1(x)], \tag{1.3}$$

where Q and $\bar Q$ are the spinor generators, the explicit form of which is defined in the appropriate field-theory model. In the commutator (1.2) of two such transformations, on account of the anticommutativity of the spinor parameters, anticommutators of the spinor generators arise, instead of commutators. The algebra

$$\{Q_\alpha, \bar Q_{\dot\beta}\} = 2(\sigma_\mu)_{\alpha\dot\beta}P^\mu, \quad \{Q_\alpha, Q_\beta\} = \{\bar Q_{\dot\alpha}, \bar Q_{\dot\beta}\} = 0, \quad [P_\mu, Q_\alpha] = [P_\mu, \bar Q_{\dot\alpha}] = 0, \tag{1.4}$$

arises, where $P_\mu = -i\partial_\mu$ is the generator of translations. The fact that the generators transforming fermions and bosons into each other should satisfy anticommutation relations is due, on the other hand, to their fermion character. In supersymmetry theories, the spin-vector current $J_{\mu,\alpha}(x)$, containing an odd number of fermion fields, is conserved:

$$\partial^\mu J_{\mu,\alpha} = 0, \quad Q_\alpha = \int d^3x J_{0,\alpha}(x). \tag{1.5}$$

The supersymmetry algebra contains (1.4), the algebra of the Poincaré group and the commutators between Q_α, $\bar Q_{\dot\alpha}$ and the generators of the Lorentz group; these commutators are determined by the fact that Q_α and $\bar Q_{\dot\alpha}$ transform according to the representations (1/2, 0) and (0, 1/2) of the Lorentz group.

The irreducible representations of the supersymmetries, i.e., the supermultiplets, contain boson and fermion fields as components. We have become accustomed to the fact that the components of a given irreducible representation, i.e., of a multiplet, can be labeled by a certain index. For example, in isotopic symmetry, the "nucleon" N = (p/n) unifies the proton and neutron, so that N_1 describes the proton and N_2 the neutron, and so on. How can we provide the boson and fermion fields from a given supermultiplet, which possess different statistics, with some kind of indices and treat them in a unified manner? The answer to this question is connected with the realization of the spinor generators. Their anticommutator gives translations (cf. (2.3) in Sec. 2) and, therefore, supersymmetry transformations act on the space-time coordinates. However, it is impossible to construct a spinor generator from only the operations of differentiation with respect to x_μ and the coordinates themselves. We introduce additional anticommuting spinor "coordinates" θ^α and $\bar{\theta}^{\dot{\beta}}$:

$$\{\theta^\alpha, \theta^\beta\} = \{\theta^\alpha, \bar{\theta}^{\dot{\beta}}\} = \{\bar{\theta}^{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\} = 0, \tag{1.6}$$

and consider the eight-dimensional "superspace" x_μ, θ, $\bar{\theta}$. In the superspace it is now possible to realize the spinor generators in the form of translation operations with respect to the usual coordinates and spinor coordinates (cf. Sec. 3):

$$Q_\alpha = -i\frac{\partial}{\partial\theta^\alpha}, \quad \bar{Q}_{\dot{\alpha}} = i\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + 2(\theta\sigma_\mu)_{\dot{\alpha}}\partial^\mu, \tag{1.7}$$

and to consider superfields $\Phi_{\mu,\nu,\ldots,\alpha}(x, \theta, \bar{\theta})$. The simplest superfield is $\Phi(x, \theta)$ – a chiral scalar superfield (chiral because it depends on one chiral spinor θ, and scalar because it does not contain outer vector or spinor indices). We shall expand $\Phi(x, \theta)$ in a series in powers of θ. By virtue of (1.6), $(\theta^1)^2 = (\theta^2)^2 \equiv 0$, and the expansion is truncated at the second term (a product of three θ always contains two identical θ and is equal to zero):

$$\Phi(x, \theta) = A_1(x) + \theta^\alpha\psi_\alpha(x) + \theta^\alpha\theta_\alpha F_1(x). \tag{1.8}$$

The difference in the statistics of the fermion and boson fields is compensated by the introduction of the anticommuting spinor quantities θ.

Writing the rule for an infinitesimal supersymmetry transformation in the form

$$\delta\Phi(x, \theta) = i(\zeta^\alpha Q_\alpha + \bar{Q}_{\dot{\alpha}}\bar{\zeta}^{\dot{\alpha}})\Phi(x, \theta), \tag{1.9}$$

we obtain (1.1). Thus, it is possible to unite the boson and fermion components of a supermultiplet in a single object – a superfield. More complicated superfields depend on both θ and $\bar{\theta}$, have outer indices, and describe higher representations of the supersymmetry group. A transformation law of the type (1.9) is valid for all of them, and they are always equivalent to a certain finite set of fermion and boson fields in ordinary space, i.e., to a certain supermultiplet.

The introduction of superfields makes it possible to construct, algorithmically, economically and conveniently, a Lagrangian theory of superfields, which turns out in the end to be equivalent to an ordinary local theory of boson and fermion fields, with specific relationships between the interaction constants. A characteristic feature is the presence of auxiliary fields (of the type $F_1(x)$ in (1.8)), which appear without derivatives in the Lagrangian and can be eliminated. After their elimination the supersymmetry remains in implicit form, and is manifested in the conservation of the spin-vector current.

The idea of a possible unification of fermions and bosons into one family was discussed by Lipkin[56] [1)] in 1964 on a level of semi-fantasy (the "barbaryon classification"). He noted that the SU(3) classification of the baryon, pseudoscalar, vector and antibaryon octets is "invariant" under simultaneous interchange of the isotopic spin with the ordinary spin and of the

	$S=1/2,\ B=1$	$S=0,\ B=0$	$S=1,\ B=0$	$S=1/2,\ B=-1$
$T=1/2,\ Y=1$	N	K	K^*	$\bar{\Xi}$
$T=0,\ Y=0$	Λ	η	$\omega\ (\varphi)$	$\bar{\Lambda}$
$T=1,\ Y=0$	Σ	π	ρ	$\bar{\Sigma}$
$T=1/2,\ Y=-1$	Ξ	$\bar{K}$	$\bar{K}^*$	$\bar{N}$

hypercharge with the baryon number (see the table).

In the usual SU(3) classification, particles with the same spin S and baryon number B are combined in octets whose components are characterized by the values of the isospin T and hypercharge Y (the columns of the table).

In the barbaryon classification, particles with the same isospin and hypercharge are combined in one family, and the components of each family have different ordinary spin and baryon number. It is suggested that we consider the direct product of the usual SU(3) symmetry and the barbaryon symmetry. We note also that, in connection with generalized current algebras, in the sixties attempts were undertaken to introduce fermion currents that change baryon number[38,62,110], and certain field-theory models with fermion generators were also studied[91,92].

Supersymmetries acquired a firm basis after the pioneering papers[26,27,57] of Gol'fand and Likhtman, who proposed and investigated spinor extensions of the Poincaré group, supersymmetry algebras, and their representations. In connection with the possible interpretation of the neutrino as a Goldstone particle[104], a significant contribution to the development and understanding of supersymmetries was made by Volkov and Akulov[16,18,20], who considered their nonlinear realizations. Interest in supersymmetries increased sharply as a result of the appearance of the constructive articles of Wess and Zumino[12,13], which displayed an exceptional property – the renormalizability of the model they proposed. Wess and Zumino did not know about the previous papers, and generalized the so-called supergauge symmetries that arise in dual models[3,25,34,64]. An important step was made by Salam and Strathdee[77], who introduced anticommuting spinor coordinates, a superspace and the concept of a superfield. We remark that the possible spinor structure of space-time was discussed by Smrz[88] and Araki and Okubo[4] in connection with the desire to unify the internal and space-time symmetries, and is being publicized by Penrose[70] in connection with the prospects for quantizing the theory of gravitation. A possible interrelation between the supersymmetries and the theory of gravitation has been stressed by D. Volkov and Soroka[19]. The fact, established by Ferrara and Zumino[102], that the conserved spin-vector current and the energy-momentum tensor appear in the same superfield-supercurrent impels us to think seriously in this direction.

We shall summarize the content of the review. In the second section the uniqueness of the minimal extension of the algebra of the Poincaré group is demonstrated and the irreducible representations are described. For nonzero rest mass an irreducible representation with given superspin Y contains fields with spin Y + 1/2, Y, Y and Y − 1/2, and for zero rest mass contains only two fields, with neighboring superspin values. From the algebra itself, it is already simple to obtain the important consequence that in any supersym-

[1)] It is not without interest that Lipkin's article develops his humorous remark from "Jocular Physics for Pedestrians" (1962) (unpublished).

metry theory the vacuum-average of the energy-momentum tensor vanishes. In Sec. 3 we discuss the superspace and superfield concepts, and the properties of superfields under space reflection, time reversal, etc. We attempt to show that a superfield theory is not much more complicated to handle than a field theory, and we emphasize some far-reaching analogies. The generalized mathematical analysis developed by Berezin[6] in a Grassmann algebra turns out to be adequate for the problem and simplifies the situation. In Sec. 4 we discuss a simple form of the Lagrangian theory of superfields. As in the case of ordinary fields (e.g., the vector field A_μ), general superfields are burdened with spare components, which are either made to vanish by additional conditions following from the equations of motion (for nonzero rest mass) or are made harmless by the corresponding gauge invariance. We then describe the construction of the supercurrent-superfield containing the conserved spin-vector current and energy-momentum tensor. Section 5 is devoted to invariant perturbation theory for superfields, based on the simple example of the model of Wess and Zumino. The use of formal integration over the anticommuting variables[6], the simple properties of the delta-function of the variables, and the Ward identities (cf.[107] and our article[61]) make it possible to trace, simply and in detail, the concellation of divergences and other surprising features of this model. The reader who is not interested in the technique of the calculations can pass over subsections (c) and (d) of this section. The 1974 investigations on the supersymmetry generalization of the Yang-Mills theory and of electrodynamics are described briefly in Sec. 6 [14,80,87,100]. The possibility of even more profound gauge theories of supersymmetries, in which the gauge fields have spin 1 and 3/2, is contemplated. In Sec. 7, the nontrivial unification of the internal symmetries and supersymmetries is briefly described[20,33,78,82,83,109]. In its time, the unitary SU(6) symmetry was proved, in the framework of ordinary Lie groups, to be incapable of being "relativized" without violation of the locality of the theory. Supersymmetries make it possible to approach this problem anew: it is possible to combine internal symmetries and supersymmetries in a local theory, but the supermultiplets obtained should be excessively large and the masses of all the particles belonging to them should coincide[51a,109]. The key problem of the breaking of the supersymmetries, without whose solution it is impossible to construct a physical supersymmetry model, is discussed very briefly in the last section. The brevity is due to the fact that there have been only a few attempts as yet, and the search for a suitable realistic supersymmetry-breaking is a matter for the future.

In the article we use two-component spinors with indices with and without a dot (cf., below, the subsection "Notation" in this section); this is technically convenient for discussing the model of Wess and Zumino. The reader can dig useful information about the equivalent formalism with Majorana spinors out of the very full article by Salam and Strathdee[82].

Notation. The signature of the metric tensor $\eta_{\mu\nu}$ is $(+---)$. Two-component spinors with dotted and undotted indices are used[9,11,67]:

$$\theta_{\dot{\alpha}} = \bar{\theta}_{\dot{\alpha}}, \quad \theta^{\alpha} = \varepsilon^{\alpha\beta}\theta_{\beta}, \quad \theta_{\alpha} = \varepsilon_{\alpha\beta}\theta^{\beta},$$

$$\varepsilon^{\alpha\beta} = -\varepsilon^{\beta\alpha}, \quad \varepsilon^{\dot{\alpha}\dot{\beta}} = -\varepsilon^{\dot{\beta}\dot{\alpha}}, \quad \varepsilon^{12} = -\varepsilon_{12} = \varepsilon^{\dot{1}\dot{2}} = -\varepsilon_{\dot{1}\dot{2}} = 1,$$

$$\theta Q = \theta^{\alpha}Q_{\alpha} = -\theta_{\alpha}Q^{\alpha}, \quad \bar{\theta}\bar{Q} = \bar{\theta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}} = -\bar{\theta}^{\dot{\alpha}}\bar{Q}_{\dot{\alpha}},$$

$$(\sigma_{\mu})_{\alpha\dot{\beta}} = (1, \sigma)_{\alpha\dot{\beta}}, \; (\tilde{\sigma}^{\mu})^{\alpha\dot{\beta}} = (1, -\sigma)^{\dot{\alpha}\beta}$$

(where the σ are the Pauli matrices), $\sigma_{\mu\nu} = (\sigma_{\mu}\tilde{\sigma} -$ spinor $\Psi = \begin{pmatrix} \psi_{\alpha} \\ \bar{\kappa}^{\dot{\alpha}} \end{pmatrix}$ and the Majorana bispinor $\Psi = \begin{pmatrix} \psi_{\alpha} \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}$. A parametrization of the γ matrices and charge-conjugation matrices is assumed in which

$$\gamma_{\mu} = \begin{pmatrix} 0 & \sigma_{\mu} \\ \tilde{\sigma}_{\mu} & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} \varepsilon_{\alpha\beta} & 0 \\ 0 & \varepsilon^{\dot{\alpha}\dot{\beta}} \end{pmatrix}, \quad \{\gamma_{\mu}, \gamma_{\nu}\} = 2\eta_{\mu\nu},$$

$$\partial^{\mu} = \frac{\partial}{\partial x_{\mu}}, \quad \hat{\partial} = \sigma_{\mu}\partial^{\mu}, \quad \tilde{\partial} = \gamma_{\mu}\partial^{\mu}, \quad \varepsilon_{0123} = 1.$$

2. SUPERSYMMETRY ALGEBRA

a) Minimal Spinor Extension of the Poincaré Group

The simplest and most popular group of supersymmetries corresponds to the minimal spinor extension of the Poincaré group. Besides the generators of 4-rotations ($L_{\mu\nu}$) and translations (P_μ), with the commutation relations

$$\begin{aligned} [L_{\mu\nu}, L_{\lambda\rho}] &= -i(\eta_{\mu\lambda}L_{\nu\rho} + \eta_{\nu\rho}L_{\mu\lambda} - \eta_{\mu\rho}L_{\nu\lambda} - \eta_{\nu\lambda}L_{\mu\rho}) \\ [L_{\mu\nu}, P_{\lambda}] &= -i(\eta_{\mu\lambda}P_{\nu} - \eta_{\nu\lambda}P_{\mu}), \quad [P_{\mu}, P_{\nu}] = 0, \end{aligned} \tag{2.1}$$

only one spinor generator Q_α and its conjugate $\bar{Q}_{\dot{\alpha}}$ are included in its algebra. By definition of a spinor,

$$[L_{\mu\nu}, Q_{\alpha}] = \frac{1}{2}(\sigma_{\mu\nu})_{\alpha}{}^{\beta}Q_{\beta}, \quad [L_{\mu\nu}, \bar{Q}_{\dot{\alpha}}] = \frac{1}{2}(\tilde{\sigma}_{\mu\nu})_{\dot{\alpha}}{}^{\dot{\beta}}\bar{Q}_{\dot{\beta}}. \tag{2.2}$$

In order to close the algebra, as was discussed in the Introduction we must specify the anticommutators between the spinor generators and the commutators between these and the translations. The only possible algebra that does not require extra generators is found to be the algebra[2)]

$$\{Q_{\alpha}, \bar{Q}_{\dot{\beta}}\} = 2(\sigma_{\mu})_{\alpha\dot{\beta}}P^{\mu}, \tag{2.3a}$$

$$\{Q_{\alpha}, Q_{\beta}\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0, \tag{2.3b}$$

$$[Q_{\alpha}, P_{\mu}] = [\bar{Q}_{\dot{\alpha}}, P_{\mu}] = 0 \tag{2.3c}$$

(the definition of the matrices σ_μ, $\sigma_{\mu\nu}$ and $\tilde{\sigma}_{\mu\nu}$ was given in the subsection "Notation" in Sec. 1).

We shall outline the proof. The generators are described by the following representations of the Lorentz group: $P_\mu \to (1/2, 1/2)$, $L_{\mu\nu} \to (1, 0) + (0.1)$, $Q_\alpha \to (1/2, 0)$, $\bar{Q}_{\dot{\alpha}} \to (0, 1/2)$. It follows from this that the most general form of the relations (2.3b) and (2.3c) consistent with relativistic invariance is

$$\{Q_{\alpha}, Q_{\beta}\} = c_1(\sigma_{\mu}\tilde{\sigma}_{\nu})_{\alpha\beta}L^{\mu\nu}, \tag{2.3b'}$$

$$[Q_{\alpha}, P_{\mu}] = c_2(\sigma_{\mu}\bar{Q})_{\alpha} \tag{2.3c'}$$

and the conjugate relations (c_1 and c_2 are constants). Commuting (2.3c′) with P_μ and applying the Jacobi identities, we find that $c_2 = 0$. By next commuting (2.3b′) with P^μ, we see that $c_1 = 0$ too. The most general form of the relation (2.3a) reduces to the replacement of the coefficient 2 in the right-hand side by a positive real constant A, whose value is determined by the normalization of Q. The positive sign of this con-

[2)]The supersymmetry algebra (2.1)–(2.3), like the algebra of the Poincaré group, is not a simple superalgebra. We note that Kats [50a] has given a classification of all the simple superalgebras.

stant is connected with the remarkable fact that the Hamiltonian $\mathcal{H} = P^0$ can be expressed in terms of the spinor generators[37]. From (2.3a) it follows that

$$P^0 = \frac{1}{4}\{Q_\alpha, (\sigma^0)^{\alpha\dot\beta}\bar{Q}_{\dot\beta}\} = \frac{1}{4}\{Q_\alpha, (Q_\alpha)^+\}. \quad (2.4)$$

Under spatial reflection,

$$Q_\alpha \to i(\sigma_0\bar{Q})_\alpha, \quad P^0 \to P^0, \quad P^i \to -P^i, \quad (2.5)$$

and this operation is an automorphism of the algebra.

In the literature, the superalgebra (2.1)–(2.3) is often used in the bispinor formalism rather than in the spinor formalism of van der Waerden. In view of this we shall give the formulas for comparison. For the Majorana bispinor generator

$$S = \frac{1}{\sqrt{2}}\begin{pmatrix} Q_\alpha \\ \bar{Q}^{\dot\alpha} \end{pmatrix}, \quad \bar{S} = S^+\gamma_0 \quad S_c = C\bar{S}^T = S \quad (2.6)$$

the superalgebra relations (2.2), (2.3) take the form

$$\{S_\rho, \bar{S}_\tau\} = -(\gamma_\mu C)_{\rho\tau} P^\mu, \quad [P^\mu, S] = 0, \quad [L_{\mu\nu}, S] = \frac{1}{2}\sigma_{\mu\nu} S. \quad (2.3')$$

This form is used in the papers of Salam and Strathdee[77–84] and of certain other authors.

We emphasize that the superalgebra (2.1)–(2.3) was first proposed and investigated by Gol'fand and Likhtman[26,27] in a bispinor formalism with the generators

$$W = \frac{1}{\sqrt{2}}\begin{pmatrix} Q_\alpha \\ 0 \end{pmatrix}, \quad \bar{W} = \frac{1}{\sqrt{2}}(0, \bar{Q}_{\dot\alpha}), \quad (2.7)$$

for which it follows from (2.1)–(2.3) that

$$\{W, \bar{W}\} = \frac{1}{2}(1+\gamma_5)\gamma_\mu P^\mu, \quad [L_{\mu\nu}, W] = \frac{1}{2}\sigma_{\mu\nu} W,$$
$$\{W, W\} = \{\bar{W}, \bar{W}\} = [P_\mu, W] = [P_\mu, \bar{W}] = 0. \quad (2.8)$$

Gol'fand and Likhtman[26,27] also discussed other algebras in the bispinor formalism. There are also wider algebras—in particular, the spinor extension of the conformal group[97,33], first realized by Wess and Zumino[12], and also supersymmetry algebras with internal symmetries included. We shall direct our attention mainly to the simplest supersymmetry, based on the algebra (2.1)–(2.3).

The superalgebra (2.1)–(2.3) is a particular case of a graded algebra[7,46]. (The generators Q, $\bar{Q}$ are assigned the grade 1, and $L_{\mu\nu}$ and P_μ the grade 0. The commutation relations for operators X and Y with grades k_X and k_Y are given in the form $XY - (-1)^{k_X k_Y} \times YX$.) Generalized Lie groups (we shall call them supergroups) correspond to graded algebras. The transformation parameters associated with grade-1 generators anticommute, i.e., are elements of a Grassmann algebra[6,7]. Thus, in our case we must introduce the spinor parameters ζ, $\bar\zeta^{\dot\alpha}$:

$$\{\zeta^\alpha, \bar\zeta^{\dot\alpha}\} = \{\zeta^\alpha, \zeta^\beta\} = \{\bar\zeta^{\dot\alpha}, \bar\zeta^{\dot\beta}\} = 0, \quad (2.9)$$

and also

$$\{\zeta^\alpha, Q_\beta\} = \{\zeta^\alpha, \bar{Q}_{\dot\beta}\} = \{\bar\zeta^{\dot\alpha}, Q_\alpha\} = \{\bar\zeta^{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0. \quad (2.10)$$

By invoking the spinor parameters we can represent the anticommutators (2.3a), (2.3b) in the form of commutators of spinor transformations[77]:

$$[\zeta_1 Q, \overline{Q\zeta_2}] = 2\zeta_1\sigma_\mu\bar\zeta_2 P^\mu, \quad [\bar\zeta_1\bar{Q}, \bar\zeta_2\bar{Q}] = 0, \quad [\zeta_1 Q, \zeta_2 Q] = 0. \quad (2.11)$$

A finite transformation of the supergroup is written as (c_μ are the parameters of translations)

$$G(\zeta, c) = \exp[-i(\zeta Q + \bar{Q}\bar\zeta + c_\mu P^\mu)]. \quad (2.12)$$

The law of combination and the group properties can be seen from the multiplication formula[3)]

$$G(\zeta_1, c_1)G(\zeta_2, c_2) = G(\zeta_1 + \zeta_2, c_{1\mu} + c_{2\mu} - i\zeta_1\sigma_\mu\bar\zeta_2 + i\zeta_2\sigma_\mu\bar\zeta_1). \quad (2.13)$$

The reader can find a rigorous mathematical definition of the operation of supergroups in the articles[7,9a]. A specific feature of a supergroup is the fact that the group law

$$c_\mu = c_{1\mu} + c_{2\mu} - i\zeta_1\sigma_\mu\bar\zeta_2 + i\zeta_2\sigma_\mu\bar\zeta_1,$$
$$\zeta = \zeta_1 + \zeta_2, \quad (2.14)$$

is a mapping on to a superalgebra with parameters c_μ, ζ, $\bar\zeta$ of a superalgebra with twice the number of parameters ($c_{1\mu}$, $c_{2\mu}$, ζ_1, $\bar\zeta_1$, ζ_2, $\bar\zeta_2$).

b) Energy-Momentum Tensor and Spin-Vector Current

In a supersymmetry field theory there arises a conserved spin-vector current $J^\mu_\alpha(x)$ ($\partial_\mu J^\mu_\alpha(x) = 0$), in terms of which the spinor generators—the "supercharges"—are written in the form

$$Q_\alpha = \int d^3x J^0_\alpha(x), \quad \bar{Q}_{\dot\alpha} = \int d^3x \bar{J}^0_{\dot\alpha}(x). \quad (2.15)$$

Zumino[36] noticed that the basic commutation relation (2.3a) can be regarded as the result of integrating the local relation

$$\{J^\mu_\alpha(x), \bar{Q}_{\dot\beta}\} = 2(\sigma_\nu)_{\alpha\dot\beta} T^{\mu\nu}(x) + \text{o.S.t.} \quad (2.16)$$

where o.S.t. denote the operator Schwinger terms[4)], and $T_{\mu\nu}$ is the energy-momentum tensor (we recall that $P^\nu = \int d^3x T^{0\nu}(x)$). It follows from (3.21) that in the limit of exact supersymmetry, when the supercharges are well-defined and give zero when they act on the vaccum, the vacuum-average of the energy-momentum tensor should vanish:

$$\langle T^{\mu\nu}(x)\rangle = 0. \quad (2.17)$$

Here we encounter a very unusual situation: up to the present, no field models have been known in which the vacuum average of the energy-momentum tensor, associated with the induced cosmological term in the Einstein equations, vanishes identically. We note that the identity (2.17) loses its validity on spontaneous symmetry breaking[36]. Goldstone spinors[16,22] with zero mass appear, and the supercharges no longer annihilate the vacuum in the relation (2.16). These considerations suggest that supersymmetries may turn out to be important in the theory of gravitation[12,19,102].

c) Irreducible Representations of Supersymmetries

One of the Casimir operators of the superalgebra is the operator of the 4-momentum squared: $C_1 = P_\mu P^\mu$. The second Casimir operator, which generalizes the operator of the spin squared in the Poincaré group, is constructed on the basis of a generalization of the Pauli-Lyubanskiĭ vector[57,78]. Namely, we consider

$$K_\mu = \frac{1}{2}\varepsilon_{\mu\nu\lambda\rho} L^{\nu\lambda} P^\rho - \frac{1}{8}(\sigma_\mu)_{\alpha\dot\beta}[Q^\alpha, \bar{Q}^{\dot\beta}]. \quad (2.18)$$

3)The Campbell-Hausdorff identity in its simplest form expA·expB=exp(A+B)+½[A, B]), applicable when [A, [A, B]]=[B, [A, B]]=0, has been used.

4)Their concrete form in the Lagrangian theory is given below, in Sec. 4(c) on the supercurrent.

From (2.2) and (2.3) we find that the operator $K_{\mu\nu} = P_\mu K_\nu - P_\nu K_\mu$ commutes with all the spinor generators, and as the second Casimir operator we must take

$$C_2 = K_{\mu\nu}K^{\mu\nu}. \tag{2.19}$$

For states with $C_1 = P^2 = m^2 > 0$ the eigenvalues of C_2 are equal to $-2m^4Y(Y+1)$, where Y is an integer or a half-integer number, which we shall call the superspin[52,78]. In fact, in the rest-frame the normalized space components $(1/m^2)K_{0i} = Y_i$ form the algebra $SU(2)$: $[Y_i, Y_j] = i\epsilon_{ijk}Y_k$.

We consider an irreducible representation with mass m and superspin Y (a supermultiplet). In the rest frame ($P^i = 0$, $P^0 = m$) the algebra of (2.2) and (2.3) is transformed into a Clifford algebra for the spinor creation and annihilation operators. Introducing the operators with well-defined parity (cf. (2.5)):

$$q_\alpha = \frac{1}{\sqrt{4m}}(Q_\alpha + (\sigma_0)_{\alpha\dot\beta}\bar Q^{\dot\beta}), \quad \bar q_{\dot\alpha} = \frac{1}{\sqrt{4m}}(\bar Q_{\dot\alpha} + Q^\beta(\sigma_0)_{\beta\dot\alpha}), \qquad P: q_\alpha \to iq_\alpha, \quad \bar q_{\dot\alpha} \to -i\bar q_{\dot\alpha}. \tag{2.20}$$

we find

$$\{q_\alpha, q_\beta\} = \{\bar q_{\dot\alpha}, \bar q_{\dot\beta}\} = 0, \qquad \{\bar q_{\dot\alpha}, q_\beta\} = (\sigma_0)_{\beta\dot\alpha} (= \delta_{\beta\dot\alpha}). \tag{2.21}$$

Let the vectors of the states with spin J and parity η_P, $|J_3| \le J$, form the "Clifford vacuum"

$$\bar q_{\dot\beta}|J, J_3\rangle_0 = 0, \quad P: |J, J_3\rangle_0 \to \eta_P|J, J_3\rangle. \tag{2.22}$$

We shall construct the Fock space. The state vectors $q_\alpha|J, J_3\rangle$ will have spin-parity $(J - 1/2)^{i\eta P}$ and $(J + 1/2)^{i\eta P}$, while $q_\alpha q_\beta|J, J_3\rangle_0$ again has spin J, but parity $-\eta_P$, since q_α carries spin 1/2 and $q_\alpha q_\beta = 1/2\,\epsilon_{\alpha\beta}(qq)$ carries only spin 0. This exhausts the representation, since $q_\alpha q_\beta q_\gamma \equiv 0$ by virtue of the anticommutativity. Next we note that, as applied to $|J, J_3\rangle_0$, the space components of the generalized ((2.18)) and ordinary Pauli-Lyubanskiĭ vectors coincide. From this it follows that the superspin Y is equal to J for the representation under discussion. Thus, we have arrived at the conclusion that, for a nonzero mass, an irreducible supersymmetry representation with superspin Y is $4(2Y+1)$-dimensional and contains irreducible representations of the Poincaré group with spin-parity $J^P = (Y - 1/2)^{i\eta}$, Y^η, $Y^{-\eta}$ and $(Y + 1/2)^{i\eta}$ and a single mass m, e.g., for Y = 0, a scalar, pseudoscalar and spinor, and for Y = 1/2, $J^P = 0^{\pm 1}$, $1^{\pm 1}$, $1/2^{\pm i}$, $1/2^{\mp i}$ (the upper or lower sign is determined by the choice of $\eta = \mp i$).

For zero rest mass, in the reference frame in which $P^\mu = (p, 0, 0, p)$, the algebra (2.2), (2.3) takes the form

$$\{Q_\alpha, Q_\beta\} = \{\bar Q_{\dot\alpha}, \bar Q_{\dot\beta}\} = \{Q_1, \bar Q_{\dot 1}\} = \{Q_1, \bar Q_{\dot 2}\} = \{Q_2, \bar Q_{\dot 1}\} = 0, \tag{2.23}$$

and the only nonzero anticommutator is

$$\{Q, \bar Q_{\dot 2}\} = 4p. \tag{2.24}$$

Correspondingly, introducing the "Clifford vacuum" with helicity λ,

$$\bar Q_{\dot 1}|\lambda\rangle_0 = \bar Q_{\dot 2}|\lambda\rangle_0 = Q_1|\lambda\rangle_0 = 0, \tag{2.25}$$

we see that only two states arise, $|\lambda\rangle_0$ and $Q_2|\lambda\rangle_0$, with helicities differing by 1/2. Thus, for zero rest-mass[82], any irreducible representation contains states with helicities λ and λ + 1/2 (λ is any integer or half-integer). When there is invariance under spatial reflection, to these are added the mirror states with helicities −λ and −(λ + 1/2).

To construct relativistically invariant theories the concept of a field defined in space-time is introduced. The fields have simple transformation properties and describe particles from various irreducible representations of the Poincaré group. Analogously, to construct theories that are invariant with respect to the supersymmetries, it is worthwhile introducing the concept of a superfield[77] with simple transformation properties, and we shall now study this. Superfields describe particles from various irreducible representations of the supersymmetry group.

3. SUPERFIELDS

a) Superspace

The realization $P^\mu = -i\partial^\mu$, $L^{\mu\nu} = i(x^\mu\partial^\nu - x^\nu\partial^\mu)$ of the generators of the Poincaré group in the space of functions of the coordinates is well known. The anticommutator (2.3a) of the spinor generators gives the translations. Consequently, the supersymmetry transformations necessarily act on the coordinates of the Minkowski space. At the same time, it is impossible to construct the spinor generators in terms of the coordinates x_μ and differentiation operations. The realization of the spinor generators requires a spinor extension of the space. We shall introduce a superspace [77,82], whose elements are the coordinates x_μ and the elements of a Grassmann algebra[5)] – the anticommuting spinors θ^α and $\bar\theta^{\dot\alpha}$:

$$\{\theta^\alpha, \theta^\beta\} = \{\bar\theta^{\dot\alpha}, \theta^\beta\} = \{\bar\theta^{\dot\alpha}, \bar\theta^{\dot\beta}\} = 0. \tag{3.1}$$

The action of the Poincaré group in the space x, θ is written as

$$x_\mu \to \Lambda_\mu^\nu x_\nu + a_\mu, \quad \theta^\alpha \to A_\beta^\alpha(\Lambda)\theta^\beta, \quad \bar\theta^{\dot\alpha} \to \bar A_{\dot\beta}^{\dot\alpha}(\Lambda)\bar\theta^{\dot\beta}, \tag{3.2}$$

where $A(\Lambda)$ and $\bar A(\Lambda)$ denote the spinor representations (1/2, 0) and (0, 1/2) of the transformations of the Lorentz group. Under spatial reflections

$$x_0 \to x_0, \quad \mathbf{x} \to -\mathbf{x}, \quad \theta_\alpha \to \eta(\sigma_0)_{\alpha\dot\beta}\bar\theta^{\dot\beta}, \quad |\eta| = 1. \tag{3.3}$$

We can now determine the action of the supergroup in the superspace x, θ by postulating that the coordinates x and θ transform as its parameters, i.e., according to (2.13), $G(\zeta, 0)G(\theta, x) = G(\theta', x')$ where

$$x'_\mu = x_\mu + i(\theta\sigma_\mu\bar\zeta - \zeta\sigma_\mu\bar\theta), \qquad \theta' = \theta + \zeta, \quad \bar\theta' = \bar\theta + \bar\zeta. \tag{3.4}$$

The variables θ and the parameters ζ have dimensions $[L]^{1/2}$. It is interesting to note that the differential form

$$\omega_\mu = dx_\mu - (i\,(d\theta\sigma_\mu\bar\theta - \theta\sigma_\mu\, d\bar\theta) \tag{3.5}$$

is invariant under the transformations (6.4). The superspace generalization of the Minkowski-space interval $ds^2 = dx_\mu dx^\mu$ is the interval

$$ds_c^2 = \omega_\mu\omega^\mu, \tag{3.6}$$

which is invariant under the entire supersymmetry group. The "Cartan form" of ω_μ arises in the invariant expansion

$$G^{-1}(x, \theta)\, dG(x, \theta) = -i\,(\omega^\mu P_\mu + d\theta^\alpha Q_\alpha + \bar Q_{\dot\alpha}\, d\bar\theta^{\dot\alpha}). \tag{3.7}$$

5)We note that Berezin and Marinov [9b] have formulated a Hamiltonian approach to the classical dynamics of particles with spin by invoking the elements of a Grassmann algebra.

The realization (3.4) is not unique[90,99]6). Thus, with the change of variables

$$x_\mu^{(a)} = x_\mu + ia\theta\sigma_\mu\bar\theta \tag{3.8}$$

we find

$$x_\mu'^{(a)} = x_\mu^{(a)} + i\,(a+1)\theta\sigma_\mu\bar\zeta + i\,(a-1)\,\zeta\sigma_\mu\bar\theta + ia\zeta\sigma_\mu\bar\zeta. \tag{3.9}$$

When a = 1 (−1), only θ ($\bar\theta$) appears in (3.9). In the superspace we can define the γ_5 transformation

$$x'_\mu = x_\mu,\quad \theta' = \exp(i\lambda)\,\theta,\quad \bar\theta' = \exp(-i\lambda)\,\bar\theta, \tag{3.10}$$

the dilations

$$x'_\mu = lx_\mu,\quad \theta' = \sqrt{l}\,\theta,\quad \bar\theta' = \sqrt{l}\,\bar\theta, \tag{3.11}$$

the conformal transformation, etc.

In the transformations found, the coordinate x_μ must be interpreted as a certain even-parity element of the Grassmann algebra. The superspace can be defined as the Minkowski space on which is specified a function algebra with parameters x_μ, θ, $\bar\theta$. When considering the functions $\Phi(x, \theta, \bar\theta)$ we can keep x_μ as an ordinary c-number coordinate while representing the supersymmetry transformations as transformations between the coefficients of the expansion of $\Phi(x, \theta, \bar\theta)$ in a series in powers of θ, $\bar\theta$.

b) Concept of a Superfield

A superfield $\Phi(x, \theta, \bar\theta)$ is a field in the superspace x, θ, $\bar\theta$. The expansion of the superfield in a series in powers of θ, $\bar\theta$ terminates and a polynomial is obtained, since θ, $\bar\theta$ anticommute and the square of each of the components is equal to zero (e.g., $\theta_1\theta_1 \equiv 0$):

$$\begin{aligned}\Phi(x, \theta, \bar\theta) = A(x) + \theta\psi(x) + \bar\varphi(x)\,\bar\theta + \theta\theta F(x) + \bar\theta\bar\theta G(x)\\ + \theta\sigma_\mu\bar\theta B^\mu(x) + \theta\theta\bar\varkappa(x)\,\bar\theta + \bar\theta\bar\theta\theta\lambda(x) + \theta\theta\bar\theta\bar\theta D(x).\end{aligned} \tag{3.12}$$

Correspondingly, we can treat tensor and spin-tensor fields with outer vector- and spinor-indices, e.g., the tensor field $\Psi_{\mu\nu}(x, \theta, \bar\theta)$ or the spinor field $\Psi_\alpha(x, \theta, \bar\theta)$, by supplying each of the components of the expansions (3.12) with these indices (to give $A_{\mu\nu}$, $\psi_{\alpha,\mu\nu}$, etc.). Thus, a superfield describes a finite set of ordinary fields, namely, coefficients of its expansion in powers of θ, $\bar\theta$. These fields form specific representations of the Lorentz group, inasmuch as it is assumed that under the action of this group

$$\Psi'_A(x', \theta', \bar\theta') = D_{AB}(\Lambda)\,\Psi_B(x, \theta, \bar\theta),\quad x' = \Lambda x,\quad \theta' = A(\Lambda)\,\theta. \tag{3.13}$$

Fermion fields anticommute and boson fields commute with θ and $\bar\theta$. The introduction of anticommuting variables makes it possible to combine fields with different spins and statistics into one compact object—a superfield.

The scalar superfield (3.12) contains fermions (ψ, $\bar\varphi$, λ, κ) and bosons (A, F, G, B^μ, D). Under the transformations (3.4),

$$\Phi'(x_\mu, \theta, \bar\theta) = \Phi(x_\mu + i\theta\sigma_\mu\bar\zeta - i\zeta\sigma_\mu\bar\theta, \theta + \zeta, \bar\theta + \bar\zeta). \tag{3.14}$$

The superfield Φ is a representation of the full supersymmetry group. In accordance with (3.9), it is also convenient to introduce superfields in new realizations Φ_1 and Φ_2:

$$\Phi_1(x, \theta, \bar\theta) = (\exp - i\theta\hat\partial\bar\theta)\,\Phi(x, \theta, \bar\theta), \tag{3.15}$$

$$\Phi_2(x, \theta, \bar\theta) = (\exp\, i\theta\hat\partial\bar\theta)\,\Phi(x, \theta, \bar\theta), \tag{3.16}$$

for which

$$\Phi'_1(x, \theta, \bar\theta) = \Phi_1(x_\mu + 2i\theta\sigma_\mu\bar\zeta + i\zeta\sigma_\mu\bar\zeta, \theta + \zeta, \bar\theta + \bar\zeta), \tag{3.17}$$

$$\Phi'_2(x, \theta, \bar\theta) = \Phi_2(x_\mu - 2i\zeta\sigma_\mu\bar\theta - i\zeta\sigma_\mu\bar\zeta, \theta + \zeta, \bar\theta + \bar\zeta). \tag{3.18}$$

In the realization (3.14) we can consider real superfields

$$\Phi(x, \theta, \bar\theta)^* = \Phi(x, \theta, \bar\theta). \tag{3.19}$$

(The operation * indicates complex conjugation and interchange of anticommuting factors[6] and is called involution.)

On the other hand, it can be seen from (3.15), (3.16) that superfields of the type Φ_1 and Φ_2 go over into each other under complex conjugation. However, the realization (3.15) ((3.16)) is preferable in that θ ($\bar\theta$) does not appear in the transformation of the coordinate x_μ and so we can introduce "more-economical" superfields, depending only on θ ($\bar\theta$), e.g., the scalar chiral superfield

$$S(x, \theta) = A(x) + \theta\psi(x) + \theta\theta F(x) \tag{3.20}$$

($\theta_\alpha\theta_\beta\theta_\gamma \equiv 0$). For a nonzero mass, such a superfield describes an irreducible representation of the supersymmetry group with superspin Y ≡ 0 (the Casimir operator C_2 (2.19) gives zero) and describes the spinless complex fields A(x) and F(x) and the spinor field ψ(x).

Representing the infinitesimal transformations in the form

$$\delta\Phi = i\,(\zeta Q + \bar Q\bar\zeta)\,\Phi, \tag{3.21}$$

we find that for a superfield of the form Φ,

$$Q_\alpha = -i\frac{\partial}{\partial\theta^\alpha} - (\hat\partial\bar\theta)_\alpha,\quad \bar Q_{\dot\alpha} = i\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + (\theta\hat\partial)_{\dot\alpha}. \tag{3.22}$$

and correspondingly for superfields in the asymmetric realizations Φ_1 and Φ_2 (e.g., $Q_\alpha^{(1)} = -i\partial/\partial\theta^\alpha$, $\bar Q_{\dot\alpha}^{(1)} = i\partial/\partial\bar\theta^{\dot\alpha} + 2(\theta\hat\partial)_{\dot\alpha}$). The supercharges7) Q and $\bar Q$ satisfy all the relations of the superalgebra (2.1)–(2.3). Naturally, in $L_{\mu\nu}$ there appears an extra term $1/2(\theta_{\mu\nu}\partial/\partial\theta + \bar\theta\bar\sigma_{\mu\nu}\partial/\partial\bar\theta)$, rotating the spinor coordinates. The differentiation $\partial/\partial\theta^\alpha$ is defined[6] by the rules $(\partial/\partial\theta_\alpha)1 = 0$, $\{\partial/\partial\theta^\alpha, \theta^\beta\} = \delta_\alpha^\beta$. For example, $\partial\theta^\beta/\partial\theta^\alpha = \delta_\alpha^\beta$, $\partial(\theta^\beta\theta^\gamma)/\partial\theta^\alpha = \delta_\alpha^\beta\theta^\gamma - \delta_\alpha^\gamma\theta^\beta$, etc.

Spatial reflection transforms the representation (1/2, 0) into the representation (0, 1/2) of the Lorentz group:

$$\begin{aligned}x'_\mu = (x_0, -\mathbf{x}),\quad \theta' = i\sigma_0\bar\theta,\\ P\colon \Phi(x, \theta, \bar\theta) \to \Phi^P(x', \theta', \bar\theta') = \eta_P\,[\Psi(x, \theta, \bar\theta)]^*,\end{aligned} \tag{3.23}$$

where $|\eta_P|^2 = 1$, and the superfield Ψ, generally speaking, can differ from Φ.

The charge-conjugation operation can be defined by

$$C\colon \Phi(x, \theta, \bar\theta) \to \Phi^C(x, \theta, \bar\theta) = \eta_C\Psi(x, \theta, \bar\theta). \tag{3.24}$$

Under Wigner time reversal

$$\begin{aligned}x'_\mu = (-x_0, \mathbf{x}),\quad \theta'_\alpha = \epsilon_{\alpha\beta}\theta_\beta,\\ T\colon \Phi(x, \theta, \bar\theta) \to \Phi^T(x', \theta', \bar\theta') = \eta_T\Phi(x, \theta, \bar\theta).\end{aligned} \tag{3.25}$$

If the superfield $\Phi(x, \theta, \bar\theta)$ is intended to describe truly neutral particles, then $\Psi(x, \theta, \bar\theta) = \Phi(x, \theta, \bar\theta)$.

6)The different realizations of the supergroup correspond to parametrizations of its group element that differ from (2.12). From a general point of view, they are all equivalent.

7)Q_α and $\bar Q_{\dot\alpha}$ are Hermitian conjugates with respect to the specially-defined scalar product on the Grassmann algebra [75].

Furthermore, under the γ_5 transformation (3.10),

$$\Phi'(x', \theta', \bar\theta') = \exp(i\alpha m)\,\Phi(x, \theta, \bar\theta), \qquad (3.26)$$

where m is an arbitrary number.

Under the dilations (3.11),

$$\Phi'(x', \theta', \bar\theta') = l^n\Phi(x, \theta, \bar\theta). \qquad (3.27)$$

where n defines the weight of the superfield. It is also possible to define a conformal and a superconformal transformation[12,28,33,63].

c) Covariant Spinor Derivatives

It is obvious that $\partial^\mu\Phi$ is also a superfield, with the same superspin values as Φ has. It is also possible to define spinor "covariant" derivatives D_α and $\bar{D}_{\dot\alpha}$ which incorporate differentiation with respect to the spinor coordinates and anticommute with the supercharges Q_α and $\bar{Q}_{\dot\alpha}$[82,99]:

$$\{D_\alpha, Q_\beta\} = \{D_\alpha, \bar{Q}_{\dot\beta}\} = \{\bar{D}_{\dot\alpha}, Q_\beta\} = \{\bar{D}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0. \qquad (3.28)$$

Then, because of the anticommutation properties of the parameters ζ, $\bar\zeta$,

$$[D_\alpha, \zeta Q + \bar{Q}\bar\zeta] = [\bar{D}_{\dot\alpha}, \zeta Q + \bar{Q}\bar\zeta] = 0, \qquad (3.29)$$

and if Φ is a superfield, then both $D_\alpha\Phi$ and $\bar{D}_{\dot\alpha}\Phi$ will also be superfields and the superspin values are unchanged, since D_α and $\bar{D}_{\dot\alpha}$ commute with the Casimir operator C_2 (2.19). In explicit form,

$$\left.\begin{aligned} D_\alpha &= \frac{\partial}{\partial\theta^\alpha} + i(\hat\partial\bar\theta)_\alpha, & \bar{D}_{\dot\alpha} &= -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - i(\theta\hat\partial)_{\dot\alpha},\\ D^{(1)}_\alpha &= \frac{\partial}{\partial\theta^\alpha} + 2i(\hat\partial\bar\theta)_\alpha, & \bar{D}^{(1)}_{\dot\alpha} &= -\frac{\partial}{\partial\bar\theta^{\dot\alpha}},\\ D^{(2)}_\alpha &= \frac{\partial}{\partial\theta^\alpha}, & \bar{D}^{(2)}_{\dot\alpha} &= -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - 2i(\theta\hat\partial)_{\dot\alpha}. \end{aligned}\right\} \qquad (3.30)$$

The covariant derivatives D form the same algebra as the supercharges:

$$\begin{gathered} \{D_\alpha, D_\beta\} = \{\bar{D}_{\dot\alpha}, \bar{D}_{\dot\beta}\} = [P^\mu, D_\alpha] = [P^\mu, \bar{D}_{\dot\alpha}] = 0,\\ \{D_\alpha, \bar{D}_{\dot\beta}\} = 2(\sigma_\mu)_{\alpha\dot\beta}P^\mu. \end{gathered} \qquad (3.31)$$

In view of this algebraic structure, we can form only 16 independent operators from products of D_α and $\bar{D}_{\dot\alpha}$. Useful identities and the multiplication rules are given in the Appendix.

When D_α acts on a product,

$$D_\alpha(\Psi_1\Psi_2) = (D_\alpha\Psi_1)\Psi_2 \pm \Psi_1 D_\alpha\Psi_2. \qquad (3.32)$$

The upper sign is taken if Ψ_1 is a tensor superfield and the lower sign is taken if it is a spin-tensor superfield (tensor superfields commute and spin-tensor fields anticommute with the spinor coordinates). An analogous situation holds for $\bar{D}_{\dot\alpha}$.

d) Superfields and Representations of the Supersymmetry Group

A general superfield describes a reducible representation of the supersymmetry group. Suppose that the rest-mass is nonzero and the superfield satisfies additional conditions that distinguish the given spin j by outer indices, e.g., $\partial^\mu\Psi_\mu = 0$ for a superfield with a vector index. Then such a superfield includes irreducible representations of the supersymmetry group that correspond to superspins Y = j + 1/2, j, j and j − 1/2[89]. The average superspins are distinguished by the conditions

$$\bar{D}_{\dot\alpha}\Psi = 0 \text{ and } D_\alpha\Psi = 0 \qquad (3.33)$$

respectively. These conditions have a simple meaning. In the realization (3.15), a superfield satisfying (3.33) does not depend on $\bar\theta$: $\Psi_1(x, \theta, \bar\theta) = \Psi_1(x, \theta)$. For example, the scalar superfield $S_1(x, \theta)$ (3.20) will be irreducible. Analogously, the superfields $\Psi_2(x, \bar\theta)$ will be irreducible.

For the highest superspin Y = j + 1/2, it is necessary that the conditions

$$(DD)\Psi = (\bar{D}\bar{D})\Psi = 0, \qquad (3.34)$$

be fulfilled and also, when outer vector indices are present, the conditions

$$D^\alpha(\sigma^\mu)_{\alpha\dot\beta}\Psi_{\mu\ldots} = \sigma^\mu_{\alpha\dot\beta}\bar{D}^{\dot\beta}\Psi_{\mu\ldots} = 0, \qquad (3.35)$$

while for a spinor superfield $\Psi_{\alpha\ldots}$ ($\Psi_{\dot\alpha\ldots}$) with spinor index α ($\dot\alpha$) the conditions

$$D^\alpha\Psi_{\alpha\ldots} = 0 \qquad (\bar{D}^{\dot\alpha}\Psi_{\dot\alpha\ldots} = 0). \qquad (3.36)$$

must also be fulfilled. For a scalar superfield, which has no outer indices, a projection operator that separates out superspin 1/2 and ensures that (3.34) is fulfilled has been constructed by Salam and Strathdee[82]:

$$\left.\begin{gathered} \Pi_V = \frac{1}{16\partial^2}(D^\alpha(\bar{D}\bar{D})D_\alpha + \bar{D}_{\dot\alpha}(DD)\bar{D}^{\dot\alpha}),\\ DD\Pi_V = \Pi_V DD = \bar{D}\bar{D}\Pi_V = \Pi_V\bar{D}\bar{D} = 0. \end{gathered}\right\} \qquad (3.37)$$

The superfield $V = \Pi_V\Phi$ describes, for nonzero mass, an irreducible supermultiplet of fields with Y = 1/2. The superfield $V(x, \theta, \bar\theta)$ is often called a vector field (with respect to the largest spin).

The superfield $(1 - \Pi_V)\Phi = \Pi_C\Phi$ describes two irreducible chiral superfields with Y = 0, which can be separated out by the projection operators.

$$\left.\begin{gathered} \Pi_+ = -\frac{1}{16\partial^2}DD\cdot\bar{D}\bar{D}, \quad \Pi_- = -\frac{1}{16\partial^2}\bar{D}\bar{D}\cdot DD,\\ \Pi_V + \Pi_+ + \Pi_- = 1, \quad \Pi_m\Pi_n = \delta_{mn}\Pi_m. \end{gathered}\right\} \qquad (3.38)$$

The projection properties of the operators Π_m can be verified by means of the formulas for the covariant derivatives D_α (see Appendix 2). Projection operators for the general case, ensuring fulfilment of all the conditions (3.34)–(3.36), have been constructed by Sokachev[89].

To conclude the general discussion of superfields we underline the following rules, which are useful in the construction of invariant Lagrangians:

1) Spinor transformations on the superfields are realized as "translations" with respect to the usual (x_μ) and spinor (θ, $\bar\theta$) coordinates (cf. (3.14), (3.17), (3.18)).

2) Consequently, a product of superfields in a given realization is again a superfield, in the same realization:

$$\Psi(x, \theta, \bar\theta)\Psi'(x, \theta, \bar\theta) = \Psi''(x, \theta, \bar\theta), \qquad \Psi_1(x, \theta, \bar\theta)\Psi'_1(x, \theta, \bar\theta) = \Psi''_1(x, \theta, \bar\theta). \qquad (3.39)$$

3) The superfield Ψ^* is again a superfield, of the type Ψ. The superfield $(\Psi_1)^*$ is a superfield of the type Ψ_2. In order to obtain, again, a superfield of the type Ψ_1 from $(\Psi_1)^*$ we must perform the translation

$$\Psi'_1(x, \theta, \bar\theta) = \exp(-2i\theta\hat\partial\bar\theta)(\Psi_1(x, \theta, \bar\theta)^*). \qquad (3.40)$$

4) The ordinary (∂^μ) and covariant (D_α, $\bar{D}_{\dot\alpha}$) derivatives of a superfield are also superfields.

e) Transformations of Boson and Fermion Fields

In order to obtain the explicit form of the transformations of the fermion and boson fields in a given superfield, we must take the expansion of the superfield in powers of θ, $\bar\theta$, use (3.14) or (3.15), (3.16), and again expand in powers of θ and $\bar\theta$. In doing this we use the simple rule for interchange of two-component spinors (see Appendix 1): $\theta^\alpha\theta^\beta = -1/2\,\epsilon_{\alpha\beta}(\theta\theta)$. Thus, for the chiral superfield

$$\Phi_1(x,\theta) = A_1(x) + \theta^\alpha\psi_\alpha(x) + (\theta\theta)F_1(x) \qquad (3.41)$$

under infinitesimal transformations (cf. (3.17)),

$$\begin{aligned}\delta A_1(x) &= \zeta^\alpha\psi_\alpha(x),\ \delta F_1(x) = -i\partial^\mu\psi(x)\sigma_\mu\bar\zeta,\\ \delta\psi_\alpha(x) &= 2i(\sigma_\mu\bar\zeta)_\alpha\partial^\mu A_1(x) + 2\zeta_\alpha F_1(x).\end{aligned} \qquad (3.42)$$

If the superfield $\Phi_1(x,\theta)$ has dimensions L^{-1} (in units $\hbar = c = 1$), then the scalar field $A_1(x)$ and spinor field $\psi_1(x)$ have canonical dimensions L^{-1} and $L^{-3/2}$ respectively, while the field $F_1(x) \sim L^{-2}$. Putting the phase factor η_P in (3.23) equal to 1, we find that under spatial reflection ($x' = (x_0, -\mathbf{x})$),

$$P:\ A_1'(x') = A_1^*(x),\ F_1'(x') = F_1^*(x),\ \psi_\alpha'(x') = i(\sigma_0)_{\alpha\dot\beta}\bar\psi^{\dot\beta}(x), \qquad (3.43)$$

and under charge-conjugation (according to (3.24)),

$$C:\ A_{1C}(x) = A_1(x),\ F_{1C}(x) = F_1(x),\ (\psi_\alpha(x))_C = \psi_\alpha(x). \qquad (3.44)$$

Thus, if parity is conserved in the theory,

$$A_1(x) = \frac{1}{2}(A(x) - iB(x)),\quad F_1(x) = \frac{1}{2}(F(x) + iG(x)), \qquad (3.45)$$

where A(x) and F(x) are real scalar fields, and B(x) and G(x) are real pseudo-scalar fields.

As was discussed above, in Sec. 2(c), an irreducible representation with Y = 0 contains one scalar, one pseudo-scalar, and one field with spin 1/2. The equations of motion enable us to express the extra fields F and G in terms of the fields A and B. In terms of these fields and the Majorana spinor $\psi(x)$, the transformations (3.42) are written as

$$\left.\begin{aligned}&\delta A(x) = \bar\zeta\psi(x),\quad \delta B(x) = i\bar\zeta\gamma_5\psi(x),\\ &\delta F(x) = i\bar\zeta\hat\partial\psi(x),\ G\delta(x) = -\bar\zeta\gamma_5\hat\partial\psi(x),\\ &\delta\psi(x) = i(\partial^\mu A(x) - i\gamma_5\partial^\mu B(x))\gamma_\mu\zeta + (F(x) + i\gamma_5 G(x))\zeta\end{aligned}\right\} \qquad (3.46)$$

(here and below, $\hat\partial$ denotes $\gamma^\mu\partial_\mu$). In the original papers of Wess and Zumino[12] these transformations were guessed as a four-dimensional generalization of the supergauge transformations in dual models. The use of the superalgebra and superfields[77] makes it possible to obtain them consistently and algorathmically. Even in the simplest case of an irreducible chiral superfield $\Phi_1(x,\theta)$, the form of the transformations embraces much more, and is much more economical, in terms of the superfield than in terms of the fields. We also give the explicit form of the transformations for a general real scalar superfield:

$$\begin{aligned}V(x,\theta,\bar\theta) = {}& C + i\theta\varkappa - i\bar\theta\bar\varkappa - \frac{i}{2}\theta\theta(M+iN) - \frac{i}{2}\bar\theta\bar\theta(M-iN)\\ &- \theta\sigma_\mu\bar\theta v^\mu + i(\theta\theta)\bar\theta\left(\bar\lambda + \frac{i}{2}\hat\partial\varkappa\right) - i(\bar\theta\bar\theta)\theta\left(\lambda + \frac{i}{2}\hat\partial\bar\varkappa\right)\\ &+ (\theta\theta)(\bar\theta\bar\theta)\left(\frac{1}{2}D - \frac{1}{4}\partial^2 C\right).\end{aligned} \qquad (3.47)$$

Under infinitesimal transformations, applying (3.13) we find, directly in terms of the Majorana spinors,

$$\begin{aligned}&\delta C = i\bar\zeta\gamma_5\varkappa,\\ &\delta M = \bar\zeta\lambda + i\bar\zeta\hat\partial\varkappa,\ \delta N = i\bar\zeta\gamma_5\lambda + \bar\zeta\gamma_5\hat\partial\varkappa,\\ &\delta\varkappa = \gamma_5\gamma_\mu\zeta\partial^\mu C + i\gamma_\mu\zeta v^\mu + (M + i\gamma_5 N)\zeta,\\ &\delta v_\mu = i\bar\zeta\gamma_\mu\lambda + \bar\zeta\partial_\mu\varkappa,\\ &\delta\lambda = i\gamma_5\zeta D + \frac{1}{2}\gamma_\mu\gamma_\nu\zeta(\partial^\mu v^\nu - \partial^\nu v^\mu),\\ &\delta D = \bar\zeta\gamma_5\hat\partial\lambda.\end{aligned} \qquad (3.48)$$

These transformation laws are rather complicated. As we have seen above, a general scalar superfield carries two zero-superspins Y = 0 and one superspin Y = 1/2. The decomposition is achieved by applying the projection operators Π_+, Π_- and Π_Y (cf. (3.37) and (3.38)):

$$\Phi = \Phi_+ + \Phi_- + \Phi_Y. \qquad (3.49)$$

To this decomposition of the superfields correspond

$$\begin{aligned}C &= C_+ + C_- &&- C_\perp,\\ \varkappa &= \varkappa_+ &&+ \varkappa_\perp,\\ M + iN &= (M + iN)_+,\\ v_\mu &= i\partial_\mu C_+ - i\partial_\mu C &&+ v_{\mu\perp}\quad (\partial^\mu v_{\mu\perp} = 0),\\ \lambda &= &&-\frac{1}{2}\hat\partial\varkappa_\perp,\\ D &= &&+\frac{1}{2}\partial^2 C_\perp.\end{aligned} \qquad (3.50)$$

Changing from v_μ to $v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu$ in the last three lines of (3.50), we see that

$$\left.\begin{aligned}&\delta v_{\mu\nu} = i\bar\zeta(\gamma_\mu\partial_\nu - \gamma_\nu\partial_\mu)\lambda,\\ &\delta\lambda = i\gamma_5\zeta D + \frac{1}{2}\gamma_\mu\gamma_\nu\zeta v^{\mu\nu},\\ &dD = \bar\zeta\gamma_5\hat\partial\lambda,\end{aligned}\right\} \qquad (3.51)$$

i.e., the fields $v_{\mu\nu}$, λ and D transform via themselves and constitute an irreducible multiplet with Y = 1/2.

The transformation properties of the boson and fermion fields from other supermultiplets can be found analogously. An important general rule holds:

5) In supersymmetry transformations the change in the last term in the expansion of any superfield (e.g., δF in (3.46) and δD in (3.48)) is a total derivative.

In fact, the last term undergoes a change only on account of the translation of the penultimate terms with respect to the coordinate x_μ.

Corollary. The four-dimensional integral of the F- and D-components of the superfields $\Phi_1(x,\theta)$ and $V(x,\theta,\bar\theta)$ is invariant not only under the Poincaré group but also under the supersymmetry group.

4. LAGRANGIAN THEORY OF SUPERFIELDS

a) Manifestly Invariant Integrals of Superfields

The rule (5) is applied in a number of papers (e.g., [75,81,83,84]) to construct invariant Lagrangians. Bilinear, trilinear, etc., combinations are formed from the given superfields, such that they form scalar superfields. The four-dimensional integral of the corresponding F- and D-components is taken as the invariant Lagrangian. This separation of the F- and D-components implies loss of the explicit invariance under the supersymmetry group; the invariance is satisfied only implicitly. However, this procedure can be given a manifestly supersymmetry-invariant form by two equivalent methods. The first of these was proposed by Salam and Strathdee[82] and consists in the following. For the F-component of the chiral superfield (3.20),

$$\int d^4x F(x) = \frac{1}{2}\int d^4x \frac{\partial}{\partial\theta_\alpha}\frac{\partial}{\partial\theta^\alpha}\Phi_1(x,\theta) = -\frac{1}{2}\int d^4x\, DD\Phi_1(x,\theta), \quad (4.1)$$

since the derivative $\partial/\partial\theta_\alpha$ differs from the covariant derivative D^α by the total derivative $(\partial\bar{\theta})^\alpha$. Analogously,

$$\int d^4x D(x) = \frac{1}{4}\int d^4x\,(D^\alpha D_\alpha)(\bar{D}_{\dot\alpha}\bar{D}^{\dot\alpha})\,V(x,\theta,\bar{\theta}). \quad (4.2)$$

In formulas (4.1) and (4.2) the terms depending on θ and $\bar{\theta}$ are space-time derivatives and fall out on integration. The explicit supersymmetry is conserved, but the coordinates x and θ appear on an unequal footing and high powers of the covariant derivatives appear in the Lagrangians.

Another method, proposed independently by Fujikawa and Lang[107] and the authors[61], seems to us to be preferable. This method is equivalent to that described, but preserves the similarity between x and θ and does not introduce extra derivatives. It is based on the concept, introduced by Berezin[6], of an integral on a Grassmann algebra. The formal Grassmann integral is defined by the following rules:

$$\int d\theta_\alpha = 0, \quad \int d\theta_\beta\theta^\alpha = \delta^\alpha_\beta, \quad (4.3)$$

where $d\theta_\alpha$ are anticommuting "differentials"[8] (of dimensions $L^{-1/2}$):

$$\{d\theta_\alpha, d\theta_\beta\} = \{d\theta_\alpha, \theta_\beta\} = 0. \quad (4.4)$$

These rules suffice, since $\{\theta_\alpha, \theta_\beta\} = 0$ and therefore none of the components $\theta\alpha$ can appear as the square. A multiple integral is taken as a repeated integral. We can define the generalized delta-functions

$$\delta^G(\theta-\theta') = (\theta_2-\theta_2')(\theta_1-\theta_1') \quad (4.5)$$

and

$$\delta^G(\bar{\theta}-\bar{\theta}') = (\bar{\theta}_2-\bar{\theta}_2')(\bar{\theta}_1-\bar{\theta}_1'). \quad (4.6)$$

While being polynomials, they possess, at the same time, the properties of ordinary δ-functions. Thus, for any function $f(\theta)$,

$$\delta^G(\theta-\theta')f(\theta) = \delta^G(\theta-\theta')f(\theta'), \quad (4.7)$$
$$\int d\theta_1 d\theta_2\,\delta(\theta) = 1 = \int d\bar{\theta}_1 d\bar{\theta}_2\,\delta(\bar{\theta}).$$

However, unlike ordinary δ-functions,

$$\delta^G(0) \equiv 0, \quad (\delta^G(\theta-\theta'))^2 \equiv 0. \quad (4.8)$$

It is precisely these properties that enable us to trace, rapidly and effectively, the cancellation of divergences in supersymmetry models, as we shall see below. The delta-functions (4.5) and (4.6) can be represented in the form

$$\delta^G(\theta) = \frac{1}{2}\theta^\alpha\theta_\alpha, \quad \delta^G(\bar{\theta}) = -(\delta(\theta))^* \quad (4.9)$$

and are scalars under Lorentz transformations, just like $d^4\theta = d\theta_1 d\theta_2 d\bar{\theta}_1 d\bar{\theta}_2 = 1/4\, d\theta^\alpha d\theta_\alpha d\bar{\theta}\dot{\alpha} d\bar{\theta}_{\dot\alpha} = d^2\theta d^2\bar{\theta}$. We emphasize that Grassmann integration is equivalent to Grassmann differentiation:

$$\int d\theta_\alpha f(\theta) = \frac{\partial}{\partial\theta^\alpha}f(\theta). \quad (4.10)$$

A Grassmann integral is invariant under translations:

$$\int d^2\theta f(\theta+\alpha) = \int d^2\theta f(\theta).$$

It is now easy to write integrals that are explicitly invariant with respect to the supersymmetries:

$$\left.\begin{aligned} \int d^4x\, d^4\theta\,\delta(\bar{\theta})\,S_1(x,\theta) &= 2\int d^4x F(x),\\ \int d^4x\, d^4\theta\,\delta(\theta)\,S_1^+(x,\bar{\theta}) &= -2\int d^4x F^+(x),\\ \int d^4x\, d^4\theta V(x,\theta,\bar{\theta}) &= -4\int d^4x D(x).\end{aligned}\right\} \quad (4.11)$$

The invariance of these integrals is obvious, since, on the superfields, the supersymmetry transformations are realized as translations in the space-time and spinor coordinates (cf. rule (1)). The two types of coordinate appear in these definitions on an equal footing and extra high powers of the covariant derivatives do not appear. In the following we shall use this approach and base the theory on the rule:

6) To construct invariant Lagrangians we must take the integrals (4.9)–(4.11) of appropriately selected bilinear, trilinear, etc., combinations of superfields over the superspace x, θ, $\bar{\theta}$.

b) Lagrangians for Simple Superfields

In this subsection we shall discuss the Lagrangian and equations of motion in the model of Wess and Zumino, which describes the self-interaction of a scalar chiral superfield with a dimensionless coupling constant, and the free theory of a vector superfield.

We shall apply the rules (2)–(4) and (6). For a scalar superfield $\Phi_1(\mathbf{x}, \theta)$, the mass term has the form

$$S_m = \frac{m}{2}\int d^4m\, d^4\theta\,[\delta^G(\bar{\theta})\,\Phi_1^2(x,\theta) - \delta^G(\theta)\,\Phi_1^{+2}(x,\bar{\theta})], \quad (4.12)$$

or, in terms of the fields (3.41), (3.45), using the Majorana spinor we have

$$S_m = m\int d^4x\left(AF+BG-\frac{1}{2}\bar{\psi}\psi\right), \quad (4.13)$$

and the kinetic term is written as

$$\begin{aligned} S_k = &-\frac{1}{4}\int d^4x d^4\theta\delta^G(\bar{\theta})\,\Phi_1(x,\theta)\,\bar{D}_{\dot\alpha}\bar{D}^{\dot\alpha}\exp(-2i\theta\hat{\partial}\bar{\theta})\,\Phi_1^+(x,\bar{\theta})\\ &= -\frac{1}{4}\int d^4x d^4\theta\Phi_1(x,\theta)\exp(-2i\theta\hat{\partial}\bar{\theta})\,\Phi_1^+(x,\bar{\theta}) =^{9)} \quad (4.14)\\ &= \int d^4x\cdot\frac{1}{2}[(\partial_\mu A)^2+(\partial_\mu B)^2 - i\bar{\psi}\hat{\partial}\psi + F^2 + G^2].\end{aligned}$$

The form of S_m and S_k is uniquely determined by the condition that they not contain derivatives of boson and fermion fields of higher order than is required. The only self-interaction with a dimensionless coupling constant is

$$\begin{aligned} S_1 = \frac{2g}{3}&\int d^4x d^4\theta\,[\delta^G(\bar{\theta})\,\Phi_1^3(x,\theta) - \delta^G(\theta)\,\Phi_1^{+3}(x,\bar{\theta})] \quad (4.15)\\ &= g\int d^4x\,[(A^2-B^2)F + 2ABG - \bar{\psi}(A-i\gamma_5 B)\psi].\end{aligned}$$

Then the fields satisfy the equations of motion

$$\left.\begin{aligned} \Box A &= mF + 2gAF + 2gBG - g\bar{\psi}\psi, & F &= -mA - g(A^2-B^2),\\ \Box B &= mG - 2gBF + 2gAG + ig\bar{\psi}\gamma_5\psi, & G &= -mB - 2gAB,\\ &-(i\hat{\partial}+m)\psi = 2g(A-i\gamma_5 B)\psi. \end{aligned}\right\} \quad (4.16)$$

---

[8]The differential $d\theta_\alpha$ is written with a lower index in order that (4.3) have an explicitly Lorentz-invariant form (the authors are grateful to V. Akulov, who drew their attention to the desirability of this form). Under linear changes of variables, $d\theta_\alpha$ and θ^β are transformed by mutually-inverse matrices [6]. Formulas for a general change of variables in Grassmann-algebra integrals have been found by Berezin [8], and in integrals over ordinary and over anticommuting variables—by Pakhomov [69]. These formulas are useful in discussing superconformal symmetry and for a possible spinor generalization of the group of general coordinate transformations.

[9]The equality is verified by integrating by parts taking into account the identity $\bar{D}_{\dot\alpha}D^\alpha\delta^G(\bar{\theta})=2$.

As was noted above, the auxiliary fields F and G occur in these equations without derivatives, and can be eliminated. Using the fact that

$$\frac{\delta\Phi_1(x,\theta)}{\delta\Phi_1(x',\theta')}=\delta(x-x')\,\delta^{G}(\theta-\theta'), \qquad (4.17)$$

by varying the first lines of (4.12)–(4.14) we can find the equations of motion directly in terms of the superfields:

$$\frac{1}{4}\bar{D}\bar{D}\exp(-2i\theta\hat{\partial}\bar{\theta})\Phi_1^{+}(x,\bar{\theta})=m\Phi_1(x,\theta)+2g\Phi_1^{2}(x,\theta) \qquad (4.18)$$

and its Hermitian conjugate, or

$$\frac{1}{4}\bar{D}\bar{D}\Phi^{+}=m\Phi+2g\Phi^{2},\quad \frac{1}{4}DD\Phi=m\Phi^{+}+2g\Phi^{+2} \qquad (4.19)$$

for the symmetric realization

$$\Phi(x,\theta,\bar{\theta})=(\exp i\theta\hat{\partial}\bar{\theta})\,\Phi_1$$
$$=A+\theta\psi+\theta\theta F+i\theta\sigma_\mu\bar{\theta}\partial^\mu A+\frac{i}{2}(\theta\theta)\bar{\theta}\,\tilde{\partial}\psi-\frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\partial^2 A \qquad (4.20)$$

of a chiral superfield. The equations of motion (4.18) or (4.19) for the superfields contain, in compact and manifestly invariant form, the equations of motion (4.16) for the fields, By virtue of the equations of motion, the spin-vector current[10]

$$J^{\mu}_{\alpha}(x)=\tilde{\partial}(A-i\gamma_5 B)\gamma^\mu\psi+i(F+i\gamma_5 G)\gamma^\mu\psi,\quad \partial_\mu J^\mu_\alpha=0. \qquad (4.21)$$

is conserved. The total action S is

$$S=S_{SV}+S_I=S_k+S_m+S_I. \qquad (4.22)$$

If we eliminate the auxiliary fields F and G, the total action will have the standard form

$$S=\int d^4x\left\{\frac{1}{2}[(\partial_\mu A)^2+(\partial_\mu B)^2-i\bar{\psi}\,\hat{\partial}\psi-m\bar{\psi}\psi-m^2A^2-m^2B^2]\right.$$
$$\left.-mgA(A^2-B^2)-\frac{g^2}{2}(A^2+B^2)^2-g\bar{\psi}(A-i\gamma_5 B)\psi\right\}. \qquad (4.23)$$

The fields A, B and ψ have a single mass m, and the renormalizability of the theory is obvious (Yukawa couplings and trilinear and quadrilinear couplings, without derivatives, appear). However, in this form the supersymmetry is masked and is expressed in the conservation of the spin-vector current (4.21) in which F and G are expressed in terms of the fields A and B.

An investigation of this simplest supersymmetry model will be described in detail below.

We now discuss the theory of a free superfield with superspin 1/2. The real vector superfield $V(x,\theta,\bar{\theta})$ (3.47) contains a superspin 1/2 and two superspins 0 (analogy: the vector field contains spin 1 and spin 0). We write the action in the form

$$S_{SV}=-\frac{1}{4}\int d^4x d^4\theta\,(V(x,\theta,\bar{\theta})\,\Box\Pi_V V(x,\theta,\bar{\theta})+m^2V^2(x,\theta,\bar{\theta})), \qquad (4.24)$$

where Π_V is the projection operator separating out superspin 1/2. (Analogy: for the vector field A_μ the Lagrangian density can be written in the form $1/2 \times A^\mu\Box\Pi_{\mu\nu}A^\nu-(m^2/2)(A_\mu)^2$, where $\Pi_{\mu\nu}=\eta_{\mu\nu}-(\partial_\mu\partial_\nu/\Box)$ is the projection operator separating out spin 1.) The equations of motion for the superfields are obtained by varying (4.24) with respect to V:

$$(\Box\Pi_V+m^2)V(x,\theta,\bar{\theta})=0. \qquad (4.25)$$

It follows from them that $DDV=\bar{D}\bar{D}V=0$ (analogously to $\partial_\mu A^\mu=0$).

We give an expression for the action in terms of the fields:

$$S_{SV}=\int d^4x\left\{-\frac{1}{4}(F_{\mu\nu})^2-\frac{i}{2}\bar{\lambda}\,\hat{\partial}\lambda+\frac{1}{2}D^2\right.$$
$$\left.+m^2\left[\frac{1}{2}V_\mu^2+CD+(\partial_\mu C)^2+(M^2+N^2)+\bar{\varkappa}\lambda+\frac{i}{2}\bar{\varkappa}\,\hat{\partial}\varkappa\right]\right\}. \qquad (4.26)$$

After the auxiliary fields M, N and D have been eliminated, there arises the usual Lagrangian describing the free fields V_μ, $mC(1/\sqrt{2})(\lambda+m\kappa)$ and $(1/\sqrt{2})(\lambda-m\kappa)$ with the same mass m.

In the case of zero mass the spinor field κ and scalar field C drop out, and the theory describes only the "photon" V_μ and the Majorana spinor λ (as we should expect, knowing the content of the irreducible representation for zero mass (Sec. 2(c))). It can be seen from the projection properties (3.37) – (3.38) of Π_V that for m = 0 the theory becomes gauge-invariant with respect to the replacement

$$V(x,\theta,\bar{\theta})\to V(x,\theta,\bar{\theta})+\frac{1}{i}(S(x,\theta,\bar{\theta})-S^{+}(x,\theta,\bar{\theta})) \qquad (4.27)$$

(analogy: the gauge invariance of electrodynamics), where S is an arbitrary chiral scalar superfield. The specific form of the additive extra term is connected with the fact that the parity of the superfield $V(x,\theta,\bar{\theta})$ is assumed to be negative: $Vp(x',\theta',\bar{\theta}')=-V(x,\theta,\bar{\theta})$. Under (4.27) the vector field V_μ goes over into $V_\mu+\partial_\mu A$. With the help of the gauge transformations (4.27), all the auxiliary fields, except for the field D which is not affected by these transformations, can be eliminated from $V(x,\theta,\bar{\theta})$.

It is known that the Lagrangian of a massless vector field is singular, and therefore, on quantization, the quantity $(1/2\alpha)(\partial_\mu v^\mu)^2$, which fixes the gauge in a relativistically invariant way, is added to it. In our case the addition of such a quantity would violate the explicit supersymmetry (just as the addition of $(1/2\alpha)(\partial v)^2$ would violate the explicit relativistic invariance). To fix the gauge in a manifestly supersymmetric approach we must introduce the term $(1/2\alpha)DDV\bar{D}\bar{D}V$ into the Lagrangian density. Extra components of the fields then "appear" (just as longitudinal and scalar photons "appear" in electrodynamics), which have no observable effects.

Thus, the concept of a superfield enables us to work in a formalism that is manifestly invariant under the supersymmetries at each stage.

c) Supercurrent

In discussing the algebra of supersymmetries in Sec. 2(b) we have already noted the remarkable fact that the conserved spin-vector current and energy-momentum tensor are connected by a supersymmetry transformation. Ferrara and Zumino[102] showed how to define a supercurrent—a real superfield with one space-time index and one non-space-time index—containing both these important quantities. For the model (4.12)–(4.14) of a chiral superfield the supercurrent has the form

$$V_{\alpha\dot{\alpha}}=i\Phi\overleftrightarrow{\partial}_{\alpha\dot{\alpha}}\Phi^{*}+\frac{1}{2}D_\alpha\Phi\bar{D}_{\dot{\alpha}}\Phi^{*}. \qquad (4.28)$$

From the equations of motion (4.19) the relation[11]

$$D^\alpha V_{\alpha\dot{\alpha}}=\frac{m}{2}\bar{D}_{\dot{\alpha}}(\Phi^{*})^2 \qquad (4.29)$$

and its Hermitian conjugate follow. The right-hand side of (4.29) depends on the model and on the choice of defi-

[10]In calculating the spin-vector current by Noether's theorem it must be kept in mind that the Lagrangian density is invariant to within a 4-divergence, the explicit form of which must be taken into account.

[11]Relations from Appendix 2 have been used.

nition[12]. In all known models the supercurrent satisfies the equation

$$D^{\alpha}V_{\alpha\dot{\alpha}} = \bar{D}_{\dot{\alpha}}S^{*}, \quad D_{\alpha}S^{*} = 0, \tag{4.30}$$

where S is a certain left-handed chiral superfield. For a free vector superfield,

$$V_{\alpha\dot{\alpha}} = (\bar{D}\bar{D}D_{\alpha}V)(DD\bar{D}_{\dot{\alpha}}V) + \frac{16m^2}{3}(V[D_{\alpha}, \bar{D}_{\dot{\alpha}}]V - D_{\alpha}V\bar{D}_{\dot{\alpha}}V) \tag{4.31}$$

and from the equations of motion (4.25) we find

$$D^{\alpha}V_{\alpha\dot{\alpha}} = \bar{D}_{\dot{\alpha}}\left(\frac{4}{3}m^2 DDV^2\right). \tag{4.32}$$

Applying the operation ∂ to (4.30), we obtain a model-independent relation for the supercurrent $V_{\mu} = \tilde{\sigma}_{\mu}{}^{\dot{\alpha}\alpha}V_{\alpha\dot{\alpha}}$:

$$\bar{D}_{\dot{\alpha}}DDV_{\mu} = 4i\partial_{\mu}(\sigma^{\nu})_{\alpha\dot{\alpha}}D^{\alpha}V_{\nu}. \tag{4.33}$$

The expansion in the components of the supercurrent V_{μ} differs from the expansion (3.47) only by the addition of the vector index μ. Correspondingly, to obtain the form of the transformations of the fields appearing in V_{μ}, we must supply all the fields in (3.48) with the vector index μ.

The invariant condition (4.30) and its conjugate limit the number of independent superfield components $V_{\alpha\dot{\alpha}}$. Using the Majorana spinors, we obtain the relations

$$\begin{aligned} &M_{\mu} = \partial_{\mu}\tilde{A}, \quad N_{\mu} = \partial_{\mu}\tilde{B}, \quad v_{\mu\nu} - v_{\nu\mu} = -\frac{1}{2}\varepsilon_{\mu\nu\lambda\rho}(\partial^{\lambda}c^{\rho} - \partial^{\rho}c^{\lambda}), \\ &\lambda_{\mu} = -\tilde{\partial}\varkappa_{\mu} + \partial_{\mu}(\gamma\cdot\varkappa), \quad D_{\mu} = -\Box C_{\mu} + \partial_{\mu}(\partial C) \end{aligned} \tag{4.34}$$

and the conservation laws

$$\partial_{\mu}(t^{\mu\nu} - \eta^{\mu\nu}t^{\rho}_{\rho}) = 0, \quad t_{\mu\nu} = \frac{1}{2}(v_{\mu\nu} + v_{\nu\mu}), \quad \partial^{\mu}(\varkappa_{\mu} - \gamma^{\lambda}\gamma_{\mu}\varkappa_{\lambda}) = 0. \tag{4.35}$$

The relations (4.34) imply that the fields λ_{μ}, D_{μ} and the antisymmetric part of $v_{\mu\nu}$ are expressed in terms of other fields, while the vector fields M_{μ} and N_{μ} can be replaced by the spinless fields $\tilde{A}$ and $\tilde{B}$.

The independent quantities $t_{\mu\nu}$, κ_{μ}, c_{μ}, $\tilde{A}$ and $\tilde{B}$ are combined into one supermultiplet. From the law of transformation of $V_{\mu}(x, \theta, \bar{\theta})$, when the conditions (4.34) and (4.35) are taken into account, it follows that

$$\begin{aligned} &\delta\tilde{A} = i\bar{\xi}\gamma^{\mu}\varkappa_{\mu}, \quad \delta\tilde{B} = \bar{\xi}\gamma_5\gamma^{\mu}\varkappa_{\mu}, \quad \delta c_{\mu} = i\bar{\xi}\gamma_5\varkappa_{\mu}, \\ &\delta\varkappa_{\mu} = \gamma^{\lambda}t_{\lambda\mu}\xi - \varepsilon_{\mu\nu\lambda\rho}\partial^{\nu}c^{\rho}\gamma^{\lambda}\xi - \partial_{\lambda}c_{\mu}\gamma^{5}\gamma^{\lambda}\xi + \partial_{\mu}(\tilde{A} - i\gamma_5 B)\xi, \\ &\delta t_{\mu\nu} = \frac{i}{2}\bar{\xi}\partial_{\mu}\varkappa_{\nu} + \frac{i}{2}\bar{\xi}\gamma_{\mu}\partial_{\nu}\gamma^{\lambda}\varkappa_{\lambda} - \frac{i}{2}\bar{\xi}\gamma_{\mu}\tilde{\partial}\varkappa_{\nu} + (\mu \leftrightarrow \nu). \end{aligned} \tag{4.36}$$

Inspection shows that the components of the supercurrent are connected in a simple way with the conserved spin-vector current and the energy-momentum tensor. Thus, in the model of Wess and Zumino,

$$\begin{aligned} &\theta_{\mu\nu} = -\frac{2}{3}(t_{\mu\nu} - \eta_{\mu\nu}t^{\rho}_{\rho}), \\ &J^{\mathrm{imp.r}}_{\mu} = -\frac{4}{3}(\varkappa_{\mu} - \gamma_{\mu}\gamma^{\lambda}\varkappa_{\lambda}), \end{aligned} \tag{4.37}$$

where $\theta_{\mu\nu}$ is the improved energy-momentum tensor [39,111] $\theta_{\mu\nu} = T_{\mu\nu} - (1/6)(\partial_{\mu}\partial_{\nu} - \eta_{\mu\nu}\Box)(A^2 + B^2)$ ($T_{\mu\nu}$ is the canonical energy-momentum tensor of the model), and J^{impr}_{μ} is the improved conserved spin-vector current

$$J^{\mathrm{impr}}_{\mu} = J_{\mu} + \frac{1}{3}[\gamma^{\mu}, \tilde{\partial}]\{(A - i\gamma_5 B)\psi\}. \tag{4.38}$$

The quantity C_{μ} is related to the axial current:

$$C_{\mu} = \frac{1}{2}J^{5}_{\mu} = \frac{1}{2}\left(A\overleftrightarrow{\partial}_{\mu}B - \frac{i}{4}\bar{\psi}\gamma_5\gamma_{\mu}\psi\right). \tag{4.39}$$

The use of the "improved" quantities simplifies the transition to zero mass, at which superconformal invariance and the corresponding new conservation laws arise, and the supercurrent $V_{\alpha\dot{\alpha}}$ satisifes the condition $D^{\alpha}V_{\alpha\dot{\alpha}} = 0$, which is more restrictive than (4.29).

We note that the above derivation[102] of the expression for the supercurrent is based on felicitous guesses and is too operational; it is worth trying to find a better one. The existence of the supercurrent seems to be important from various points of view—in particular, in the light of a possible supersymmetry generalization of Einstein's theory of gravitation[13].

5. INVARIANT PERTURBATION THEORY

An invariant perturbation theory can be formulated directly in terms of the superfields. As was noted above, the superfields contain spare components (just as the massive vector field A_{μ} contains the spare component A_0). Some of the equations of motion turn out to be coupling conditions which express the spare components in terms of the other fields, after which the supersymmetry remains in implicit form, manifesting itself in the conservation of the spin-vector current. (Analogously, in the vector-field theory the Hamiltonian and propagators lose their manifestly Lorentz-invariant form after A_0 is eliminated, but the angular-momentum tensor is conserved as before.)

To construct the invariant perturbation theory we shall base the derivation on a generating functional for the Green functions that has the form of a continuous integral with integration over all the fields. We could integrate out the "spare" fields, but the manifest invariance under the supersymmetry would then be lost (integration over A_0 in the theory of the vector field also leaves the Lorentz invariance in implicit form only).

The covariant quantization and the Feynman rules for the superfields are defined in an analogous way to that usually used in quantum field theory.

a) Propagators

For a free chiral superfield in the presence of sources

$$J_{-}(x, \theta) = J_F(x) + \theta\eta + \theta\theta J_A, \quad J_{+}(x, \bar{\theta}) = J^{*}(x, \bar{\theta}), \tag{5.1}$$

the equations of motion (cf. (4.18))

$$\frac{1}{4}\bar{D}\bar{D}\exp(-2i\theta\hat{\partial}\bar{\theta})\Phi_1^{*}(x, \bar{\theta}) - m\Phi(x, \theta) = J_{-}(x, \theta) \tag{5.2}$$

and their Hermitian conjugate have the solution

$$\begin{aligned} \Phi_1(x, \theta) = -\int d^4x d^4\theta' \, [&G_{--}(x, \theta, x', \theta')J_{-}(x', \theta')\delta^{G}(\bar{\theta} - \bar{\theta}') \\ &+ G_{-+}(x, \theta, x', \bar{\theta}')J_{+}(x', \bar{\theta}')\delta^{\Gamma}(\theta - \theta')]. \end{aligned} \tag{5.3}$$

The Green functions (propagators) of Eqs. (5.2) are written as

$$G_{-+}(x, \theta, x', \bar{\theta}') = \frac{1}{2}\exp(-2i\theta\hat{\partial}\bar{\theta}')\Delta_C(x - x'), \tag{5.4a}$$

$$G_{--}(x, \theta, x', \theta') = 2m\delta(\theta - \theta')\Delta_C(x - x'), \tag{5.4b}$$

where

---

[12]Thus, in the model under discussion it is possible to introduce the modified supercurrent $V^{\mathrm{mod}}_{\alpha\dot{\alpha}} = V_{\alpha\dot{\alpha}} - (im/4g)\,\delta_{\alpha\dot{\alpha}}(\Phi - \Phi^{*})$, for which $D^{\alpha}V^{\mathrm{mod}}_{\alpha\dot{\alpha}} = -(m^2/4g)\Phi^{*}$, by analogy with the condition for partial conservation of the axial current.

[13]Could the supercurrent take the place of the energy-momentum tensor? In principle, the supercurrent could be the source for a superfield of the type $\mathcal{H}_{\alpha\dot{\alpha}}$, which, at zero mass, contains only two particles—one with helicity ±2 and another with helicity ±3/2.

$$\Delta_C(x-x') = \int \frac{d^4k}{(2\pi)^4}\frac{\exp[ik(x-x')]}{m^2-k^2-i\varepsilon} \tag{5.5}$$

is the causal Green function for a scalar field with mass m.

This can be verified by substituting (5.4) into (5.2) and taking into account the identities

$$\exp(-2i\theta\hat{\partial}\bar{\varkappa}) = 1 - 2i\theta\hat{\partial}\bar{\varkappa} + 4\delta^{G}(\theta)\,\delta^{G}(\bar{\varkappa})\,\Box \tag{5.6}$$

and

$$\varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^{\alpha}}\frac{\partial}{\partial\theta^{\beta}}\exp(-2i\theta\,\hat{\partial}\bar{\varkappa}) = -8\delta(\bar{\varkappa})\,\Box. \tag{5.7}$$

The Green functions G_{+-} and G_{--} contain the propagators of the boson and fermion fields us coefficients of the expansion in θ, $\bar{\theta}$. Thus, for the Fermi field, from (5.4a) and (5.4b) we find

$$\langle T\psi_{\alpha}(x)\,\psi_{\beta}(x')\rangle_0 = -im\varepsilon_{\alpha\beta}\Delta_C(x-x'),\quad \langle T\psi_{\alpha}(x)\,\bar{\psi}_{\dot{\beta}}(x')\rangle_0 = +(\hat{\partial})_{\alpha\dot{\beta}}\Delta_C(x-x'), \tag{5.8}$$

and for the Majorana spinor $\psi = (\psi_{\alpha}/\bar{\psi}^{\dot{\alpha}})$ we obtain the usual propagator

$$\langle T(\psi(x)\,\bar{\psi}(x'))\rangle = -i(-i\hat{\partial}+m)\,\Delta_C(x-x'). \tag{5.9}$$

In switching on the interaction we need a manifestly invariant regularization, which is achieved by adding[37] to (4.2.2) (sic) the term

$$\frac{\xi}{4}\Box^2\Phi(x,\theta)\exp(-2i\theta\,\hat{\partial}\bar{\theta})\,\Phi^*(x,\bar{\theta}). \tag{5.10}$$

The regularized Green functions are calculated analogously to the unregularized ones (5.4), depend in exactly the same way on the spinor coordinate, and differ from (5.4) only by the replacement of the causal function Δ_C by D^1_R in (5.4a) and by D^2_R in (5.4b):

$$\begin{aligned} D^1_R(x) &= \int \frac{d^4k}{(2\pi)^4}\frac{\exp(ikx)\,[1-\xi(k^2)^2]}{m^2-k^2[1-\xi(k^2)^2]^2-i\varepsilon},\\ D^2_R &= \int \frac{d^4k}{(2\pi)^4}\frac{\exp(ikx)}{m^2-k^2[1-\xi(k^2)^2]^2-i\varepsilon}. \end{aligned} \tag{5.11}$$

In momentum space, for large k, $(G_{+-})_R \sim k^{-4}$ and $(G_{--})_R \sim k^{-10}$. The regularization temporarily removes the divergences and makes the calculations meaningful, but in the final results we must let the regularization parameter ξ tend to zero. This invariant method is analogous to that proposed by Slavnov[86] for Yang-Mills theories.

The propagator of the vector superfield $V(x, \theta, \bar{\theta})$ (3.7) can be defined as the Green function of the free equations of motion in the presence of a source—the neutral vector superfield $J(x, \theta, \bar{\theta})$:

$$\frac{1}{2}(\Box\Pi_V + m^2)\,V(x,\theta,\bar{\theta}) = J(x,\theta,\bar{\theta}). \tag{5.12}$$

Using the projection property $\Pi_V^2 = \Pi_V$, it is easy to solve this equation ($\Pi_C = 1 - \Pi_V$),

$$V(x,\theta,\bar{\theta}) = \int d^4x\,d^4\theta\,G(x,\theta,\bar{\theta},x,\theta',\bar{\theta}')\,J(x',\theta',\bar{\theta}), \tag{5.13}$$

where

$$\begin{aligned} G(x,\theta,\bar{\theta},x',\theta',\bar{\theta}') &= 2\left(1+\frac{\Box}{m^2}\Pi_c\right)(m^2+\Box)^{-1}\delta(x-x')\,\delta^{G}(\theta-\theta')\,\delta^{G}(\bar{\theta}-\bar{\theta}')\\ &= 2\left[1-\frac{1}{16m^2}(\bar{D}\bar{D}DD+DD\bar{D}\bar{D})\right]\Delta_C(x-x',m^2)\,\delta^{G}(\theta-\theta')\,\delta^{G}(\theta-\theta'). \end{aligned}$$

We emphasize the complete analogy with the vector field A_{μ}, for which the equations of motion are written in the form

$$(\Box(\Pi_V)_{\mu\nu} + m^2\eta_{\mu\nu})\,A^{\nu} = J_{\mu}(x), \tag{5.14}$$

where $(\Pi_V)_{\mu\nu} = \eta_{\mu\nu} + (\partial_{\mu}\partial_{\nu}/\Box)$ is the projection operator separating out the spin 1 in A_{μ}, and the causal Green function is defined as

$$(G_C)_{\mu\nu} = \left(\eta_{\mu\nu}+\frac{1}{m^2}\partial_{\mu}\partial_{\nu}\right)\Delta_C(x,m^2), \tag{5.15}$$

where $\partial_{\mu}\partial_{\nu} = \Box(\Pi_C)_{\mu\nu}$, and $(\Pi_C)_{\mu\nu} = \partial_{\mu}\partial_{\nu}/\Box$ is the projection operator separating out the spin 0 in the field A_{μ}.

For zero mass this propagator loses its meaning, since the gauge invariance (4.27) arises. The gauge can be fixed in a manifestly invariant form by adding the expression $(1/2\alpha)DDV\bar{D}\bar{D}V$ to the Lagrangian. Then the propagator is written as

$$\begin{aligned} &i\langle TV(x,\theta,\bar{\theta})\,V(x',\theta',\bar{\theta}')\rangle\\ &\quad = 2\left(1-\frac{1-\alpha}{16\Box\alpha}\{DD,\bar{D}\bar{D}\}\right)\Delta_C(x-x')\,\delta^{G}(\theta-\theta')\,\delta^{G}(\bar{\theta}-\bar{\theta}'). \end{aligned} \tag{5.16}$$

Here, $\alpha = 1$ corresponds to the Fermi gauge in electrodynamics.

b) Feynman Rules in the Model of Wess and Zumino

To calculate the matrix elements of the different processes it is convenient, using Grassmann integration, to represent the S-matrix in the form

$$= T\exp\left\{i\int d^4x\,d^4\theta\,\frac{2g}{3}\left[\delta^{G}(\theta)\,\Phi^{*3}(x,\bar{\theta}) - \delta^{G}(\bar{\theta})\,\Phi^3(x,\theta)\right]\right\}. \tag{5.17}$$

The Feynman diagrams are drawn in the usual way. There are two types of vertex: $(\Phi^+)^3$ and Φ^3. We shall call these left and right vertices and denote them in the diagrams by the symbols ⊗ and ⊖ respectively. At each vertex the 4-momentum is conserved and a factor $(2\pi)^4\delta(\sum_i p_i)$ is associated with the vertex. With each internal line we associate a propagator: for a line joining two right vertices,

$$\underset{\theta_2}{\ominus} - \overset{p}{\rightarrow} - - \underset{\theta_1}{\ominus} \rightarrow \frac{im\delta^{G}(\theta_1-\theta_2)}{p^2-m^2+i\varepsilon}, \tag{5.18}$$

for a line joining two left vertices,

$$\underset{\bar{\theta}_2}{\oplus} - - \overset{p}{\rightarrow} - - \underset{\bar{\theta}_1}{\ominus} \rightarrow -\frac{im\delta^{G}(\bar{\theta}_1-\bar{\theta}_2)}{p^2-m^2-i\varepsilon}, \tag{5.19}$$

and for a line joining a right and a left vertex,

$$\underset{\bar{\theta}_2}{\oplus} \underset{p}{\rightarrow} - \ominus \rightarrow \frac{1}{2}\frac{\exp(2\theta_1\hat{p}\bar{\theta}_2)}{p^2-m^2-i\varepsilon}. \tag{5.20}$$

With each external line we associate a wavefunction $\Phi_{ext}(p, \theta)$ or $\Phi^+_{ext}(p, \bar{\theta})$:

$$\Phi_{ext}(p,\theta) = A_{ext}(p) + \theta\psi_{ext}(p) + (\theta\theta)\,F_{ext}(p). \tag{5.21}$$

Integration is performed over all the momenta of the internal lines and Grassmann integration is performed over all the spinor coordinates θ, $\bar{\theta}$ of the internal vertices (i.e., the vertices with no external lines). This procedure gives the S-matrix element for the superfields.

In order to obtain the S-matrix element of a process for boson and fermion fields[83] we must also take the Grassmann integral over the spinor coordinates θ, $\bar{\theta}$ of all the external vertices. In doing this we must replace F(p) by $-mA^*(p)$ in Φ_{ext} and Φ^+_{ext}, in accordance with the equations of motion for the free fields. The supersymmetry connects the spin amplitudes of processes with a fixed number of particles (e.g., the two-particle amplitudes: $0^{\pm}\,1/2 \to 0^{\pm}\,1/2$, $0^{\pm}0^{\pm} \to 0^{\pm}0^{\pm}$ and $1/2\,1/2 \to 1/2\,1/2$).

The method of superfields makes it possible to perform calculations and exhibit the general properties of supersymmetry theories in the clearest and most economical way, since each diagram with superfields incorporates in an invariant manner a whole set of diagrams with boson and fermion fields. We shall give a

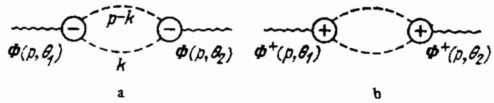

FIG. 1

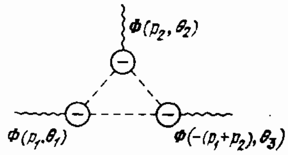

FIG. 2

FIG. 3

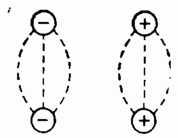

FIG. 4

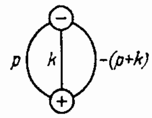

FIG. 5

few examples and, in particular, show how easily, and with practically no calculations, we can see the cancellation of divergences in the diagrams in the lowest orders of perturbation theory.

We consider the diagram for the proper-mass counterterm (Fig. 1a). This counterterm is equal to zero, since the product of two G_{--} contains $(\delta^G(\theta_1 - \theta_2))^2 \equiv 0$ (cf. (4.8)). By the same arguments, the diagram of Fig. 1b also gives no contribution. Also equal to zero is the contribution from the diagrams for the counterterm in the renormalization of the coupling constants (Fig. 2), because

$$\delta^G(\theta_1 - \theta_2)\, \delta^G(\theta_2 - \theta_3)\, \delta^G(\theta_3 - \theta_1) = [\delta^G(\theta_1 - \theta_2)]^2\, \delta^G(\theta_3 - \theta_1) = 0$$

and the same is true (with interchange of θ and $\bar{\theta}$) for the vertex $(\Phi^+)^3$.

In the calculations the integrals over the momenta of internal lines can be assumed to be regularized in accordance with (5.11). In the lowest orders of perturbation theory the vanishing of the counterterms in the renormalization of the mass and coupling constant happens because of the remarkable properties of the Grassmann delta-function and does not require any calculations. To perceive this same fact in the formalism with ordinary Fermi and Bose fields, using the Lagrangian (4.15), it is necessary to study a large number of different diagrams and establish that the contributions from them mutually cancel.

We return to the examples. Also equal to zero are the "tadpoles" (Fig. 3), since $\delta^G(0) = 0$, and the contributions from vacuum loops consisting only of right (or only of left) vertices (Fig. 4). It is easy to show that any diagram which contains a closed circuit consisting only of right (or only of left) vertices gives zero contribution.

The contribution from a diagram with vacuum loops containing right and left vertices (Fig. 5) also vanishes. In this case, this happens because of the fact that the integrand does not depend on θ and $\bar{\theta}$, and in the Grassmann integration $\int d\theta = 0$. Below we shall convince ourselves that vacuum loops give zero in arbitrary order of perturbation theory; here we have cited the

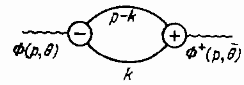

FIG. 6

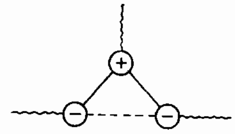

FIG. 7

simplest examples. Finally, the diagram of Fig. 6 gives the contribution

$$Z\Phi(p,\ \theta)\exp(-2\theta\hat{p}\bar{\theta})\,\Phi^+(p,\ \bar{\theta}), \tag{5.22}$$

where Z diverges logarithmically. Thus, in orders g^2 and g^3 of perturbation theory only one divergence appears—the logarithmic divergence in the manifestly invariant counterterm in the wavefunction renormalization.

We note also that the vertex diagram depicted in Fig. 7 gives a finite contribution. In higher orders all diagrams give a finite contribution except for diagrams with two external lines of which one is joined to a (−) vertex and the other to a (+) vertex. Such diagrams can diverge only logarithmically and their aggregate determines the only necessary counterterm of a supersymmetry-invariant form in the theory under discussion. We proceed now to the proof of this surprising fact, and start from identities that follow from the supersymmetry; in accordance with tradition, we shall call them Ward identities.

c) Ward Identities

We consider a Feynman diagram of general form. Suppose that it contains n external right vertices θ_j and m external left vertices $\bar{\theta}_k$, at which there are incoming external lines with momenta p_j and q_k, respectively. Then the corresponding amplitude is written as

$$\begin{aligned} &A((p\theta),\ (q,\ \bar{\theta})) \\ &= \int dl_1 \ldots dl_a d^2\theta_{i_1} \ldots d^2\theta_{i_p} d^2\bar{\theta}_{i_1} \ldots d^2\theta_{i_s} K((p\theta),\ (q,\ \bar{\theta}),\ (\theta_i,\ \bar{\theta}_i,\ l)), \end{aligned} \tag{5.23}$$

where the integral is taken over the momenta l_a of the internal loops and over the spinor variables of the internal left (θ_i) and right $(\bar{\theta}_i)$ vertices, and we have used the abbreviations

$$\begin{aligned} (p\theta) &= (p_1\theta_1,\ \ldots,\ p_n\theta_n), \quad (q\bar{\theta}) = (q_1\bar{\theta}_1\ \ldots\ q_m\bar{\theta}_m), \\ (\theta_i,\ \bar{\theta}_i,\ l) &= (\theta_{i_1},\ \ldots,\ \bar{\theta}_{i_1},\ \ldots,\ l_1,\ \ldots,\ l_a). \end{aligned} \tag{5.24}$$

We shall show that the kernel $K((p\theta)(q\theta)(\theta_i\bar{\theta}_i l))$ of the amplitude for any given Feynman diagram satisfy the following important identities[107]:

$$\begin{aligned} &K((p\theta),\ (q\bar{\theta})(\theta_i\bar{\theta}_i,\ l)) \\ &\equiv \exp\Big(-2\sum_{j=1}^{m}\zeta\hat{q}_j\bar{\theta}_j\Big) K((p,\ \theta+\zeta),\ (q\bar{\theta})(\theta_i+\zeta,\ \bar{\theta}_i,\ l)) \\ &\equiv \exp\Big(+2\sum_{k=1}^{n}\theta_k\hat{p}_k\bar{\zeta}\Big) K((p,\ \theta),\ (q,\ \bar{\theta}+\bar{\zeta})(\theta_i,\ \bar{\theta}_i+\bar{\zeta},\ l)) \\ &\equiv \exp\Big[2\sum_{k=1}^{n}\theta_k\hat{p}_k\bar{\zeta} - 2\sum_{j=1}^{n}\zeta\hat{q}_i\bar{\theta}_j + \zeta\Big(\sum_{k=1}^{n}p_k - \sum_{j=1}^{m}q_i\Big)\bar{\zeta}\Big] \\ &\qquad \times K((p,\ \theta+\zeta),\ (q,\ \bar{\theta}+\bar{\zeta})(\theta_i+\zeta,\ \bar{\theta}_i+\bar{\zeta},\ l)). \end{aligned} \tag{5.25}$$

First we shall prove the identities (5.25). The kernel K is a product of propagators. Under a general spinor translation $\theta \to \theta + \zeta$ the propagators G_{++} and G_{--} do not change, while the propagators G_{+-} acquire factors that depend on the variables $\bar{\theta}$. Therefore, it is sufficient to consider only the left vertices; each group of external left vertices joined by propagators $G_{++} \sim \delta^G \times (\bar{\theta}_l - \bar{\theta}_m)$ can be "contracted" into one effective left

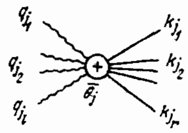

FIG. 8

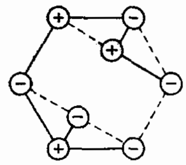

FIG. 9

vertex (Fig. 8). When $\theta \to \theta + \zeta$ each of the propagators $G_{+-} \sim \exp(-2\theta_{j_2}\hat{k}_{j_2}\bar{\theta}_j)$ acquires a factor $\exp(-2\zeta\hat{k}_{j_r}\bar{\theta}_j)$, and, on account of all the external left vertices, the factor $\exp(-2\sum\zeta\hat{q}_j\bar{\theta}_j)$ multiplying the kernel K in the identity (5.25) appears (here we have taken into account the four-momentum conservation law $(\sum_r k_{j_r} = -\sum_l q_{j_l})$). Internal left vertices joined by external propagators G_{++} have already been considered. In the others we again contract each group of left internal vertices joined together by propagators G_{++} into one effective vertex from which only (+−)-lines emerge. The sum of the 4-momenta in the G_{+-} of these lines is equal to zero, and, therefore, each such effective internal vertex makes no contribution to the change of the kernel $(\exp(2\zeta\sum_l \hat{k}_{j_l}\theta_j) = 1)$, since $\sum_l k_{j_l} = 0$).

This completes the proof of the first identity of (5.25). The middle identity is proved analogously. Finally, the third identity is obtained by combining the first two. It is important to note that the identities found are valid for each individual diagram. They are stronger than the Ward identities for the Green functions

$$G_{nm}((x\theta), (y\bar{\theta})) = G_{nm}((x'\theta'), (y'\bar{\theta}')), \tag{5.26}$$

which follow from the invariance of the vacuum under the supersymmetry transformations (3.17) and (3.18), where

$$G_{nm}((x\theta), (y\bar{\theta})) = \langle T\Phi(x_1\theta_1) \ldots \Phi(x_n\theta_n)\Phi^+(y_1\bar{\theta}_1) \ldots \Phi^+(y_m\bar{\theta}_m)\rangle_0. \tag{5.27}$$

For the Fourier transform $\tilde{G}_{nm}$ in momentum space these identities are written as

$$\tilde{G}_{nm}((p, \theta+\zeta), (q, \bar{\theta}+\bar{\zeta})) = \exp\left[-2\sum_{k=1}^{n}\theta_k\hat{p}_k\bar{\zeta} + 2\sum\zeta\hat{q}_j\bar{\theta}_j + \zeta\left(\sum_{j=1}^{m} q_j - \sum_{k=1}^{n} p_k\right)\bar{\zeta}\right]\tilde{G}_{nm}((p\theta)(q\bar{\theta})). \tag{5.28}$$

They are contained in, and can be obtained from, the third identity of (5.25).

Corollaries:

1) Any vacuum loop vanishes (e.g., Fig. 9). In fact, in the absence of external vertices and lines, it follows from (5.25) that the kernel K depends only on the differences $\theta_{i_k} - \theta_{i_l}$ and $\bar{\theta}_{i_m} - \bar{\theta}_{i_n}$. The amplitude of vacuum-vacuum transitions vanishes, since the number of Grassmann integrations exceeds the number of independent Grassmann variables and $\int d\theta = 0$ (4.3). We illustrated this fact above in very simple examples (cf. Figs. 4 and 5). We note that Zumino[36] managed to exhibit this striking property of the supersymmetry model by summing the contributions of many diagrams with Bose and Fermi fields.

2) For any diagram with no external left vertices the kernel K depends on differences of the variables θ (the upper identity of (5.25)), and the amplitude depends on differences of the external variables θ. The analogous

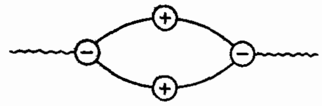

FIG. 10

statement (with θ replaced by $\bar{\theta}$) is valid for diagrams with no external right vertices.

3) For a diagram with no external left vertices the amplitude vanishes when all $p_k = 0$. In fact, in this case (the middle identity of (5.25)), the kernel depends on differences of the internal variables and the number of Grassmann integrations over $\bar{\theta}_i$ exceeds the number of independent variables $\bar{\theta}_i$; $A(0, \theta) = 0$. Analogously, $A(0, \bar{\theta}) = 0$.

In particular, in the theory under discussion not only do the vacuum loops vanish automatically, but so too do "tadpoles" of arbitrary complexity. Indeed, diagrams with one external ⊗ or ⊖ vertex and zero external momentum correspond to "tadpoles."

4) If all the external momenta tend to zero ($p_i = 0$, $q_k = 0$), the kernel will depend only on differences of the variables θ and differences of the variables $\bar{\theta}$, i.e., the number of independent right and left vertices is reduced by one.

5) For any diagram with two right vertices, for all ζ,

$$K((p, \theta_1), (-p, \theta_2), (\theta_i\bar{\theta}_i l)) = \exp(2(\theta_1 - \theta_2)\hat{p}\bar{\zeta})\,K((p, \theta_1)(-p, \theta_2)(\vartheta_i, \theta_i + \zeta, l)). \tag{5.29}$$

Putting $\bar{\zeta} + \bar{\theta}_i = 0$ for one of the internal $\bar{\theta}_i$ and integrating over the internal variables $(\int d^2\bar{\theta}_i \exp[2(\theta_1 - \theta_2)\hat{p}\bar{\theta}_i] = -4\,\delta^G(\theta_1 - \theta_2)p^2)$, we find

$$A(p, \theta_1; -p, \theta_2) = \delta^G(\theta_1 - \theta_2)\,p^2 f(p^2). \tag{5.30}$$

and analogously, with the replacement $\theta_k \to \bar{\theta}_k$, for any $(\Phi^+)^2$ diagram. As an example, the reader can calculate the amplitude for the diagram of Fig. 10.

6) For any diagram with one right and one left vertex the identity (5.25) gives, for any $\zeta, \bar{\zeta}$,

$$A(p, \theta; -p, \bar{\theta}) = \exp[2(\theta\hat{p}\bar{\zeta} + \zeta\hat{p}\bar{\theta} + \zeta\hat{p}\bar{\zeta})]\,A(p, \theta+\zeta; -p, \bar{\theta}+\bar{\zeta}).$$

Putting $\theta + \zeta = \bar{\theta} + \bar{\zeta} = 0$, we establish the general structure

$$A(p, \theta; -\bar{p}, \theta) = \exp(-2\theta\hat{p}\bar{\theta})\,u(p^2), \tag{5.31}$$

where $u(p^2) = A(p, 0; -p, 0)$.

d) Cancellation of Divergences in Higher Orders

We shall ascertain which of the diagrams diverge, and give an estimate of their index of divergence[107]. In its form, the theory under discussion is a φ^3 theory, but the exponential in the propagator G_{+-} (5.20) gives an extra momentum-squared. The index of a diagram can be written in the form

$$\omega(\tau) = \omega_3(\tau) + \omega_{+-}(\tau), \tag{5.32}$$

where $\omega_3(\tau)$ is the index of divergence of the diagram τ in the φ^3 theory and $\omega_{+-}(\tau)$ is the correction due to the propagators G_{+-}. If the diagram has N vertices (N_+ right and N_- left vertices) and n_e external lines, then, as is well-known[10],

$$\omega_3(\tau) = \frac{3N - n_e}{2}\cdot\frac{1}{2}\cdot 4 - 4(N-1) = 4 - N - n_l. \tag{5.33}$$

We shall give now an estimate for $\omega_{+-}(\tau)$. We contract each group of left vertices joined by propagators $G_{++} \sim \delta(\bar\theta_j - \bar\theta_k)$ into one effective left vertex $\bar\theta_j$, and each group of right vertices joined by propagators $G_{--} \sim \delta(\theta_i - \theta_k)$ into one effective vertex θ_i. The number of effective left and right vertices will be, respectively, $\tilde N_+ = N_+ - n_{++}$ and $\tilde N_- = N_- - n_{--}$, where n_{++} is the number of propagators G_{++} and n_{--} is the number of propagators G_{--}. Several propagators G_{+-} attached to the same effective vertex give the same additional second power of the momentum as one propagator: e.g.,

$$\exp[2\bar\theta_i(\hat p_j\bar\theta_j + \hat p_k\bar\theta_k)] = 1 + 2\theta_i(\hat p_j\bar\theta_j + p_k\bar\theta_k) - 4\delta^{G}(\theta_i)[\delta^{G}(\bar\theta_j)p_j^2 - \delta^{G_{--}}(\bar\theta_k)p_k^2 - \theta_j\theta_k p_j p_k].$$

Therefore,

$$\omega_{+-}(\tau) \leqslant 2\min(\tilde N_+, \tilde N_-) = \tilde N_+ - \tilde N_- - |\tilde N_+ - \tilde N_-| = N - n_{++} - n_{--} - |N_+ - N_- - n_{++} + n_{--}|, \tag{5.34}$$

and, taking (5.33) into account, we find

$$\omega(\tau) \leqslant 4 - n_e - n_\delta - |N_+ - N_- - n_{++} - n_{--}|, \tag{5.35}$$

where $n_\delta = n_{++} + n_{--}$.

For a diagram with five or more external lines, $\omega(\tau) < 0$, and they certainly make a finite contribution. For diagrams with four external lines, $\omega(\tau) \le 0$. For Φ^3 and $(\Phi^+)^3$ diagrams, again, $\omega(\tau) \le 0$, because either $n_\delta = 0$ but $|\tilde N_+ - \tilde N_-| = 1$, or $n_\delta \ge 1$. In the case of $\Phi\Phi\Phi^+$ (and $\Phi\Phi^+\Phi^+$) diagrams, combinatoric arguments show that $n_\delta \ge 1$, and, consequently, for these $\omega(\tau) \le 0$ again. The greatest degree of divergence is given by the highest powers of momenta of internal loops. In order to consider only these, we put the external momenta equal to zero. But then (consequence (4) of the Ward identities), the number of independent right and left vertices is decreased by unity, since the kernel of the diagram depends on differences of the spinor variables and this lowers the degree of the highest divergence by at least 2. Thus, all diagrams with 4 and 3 external lines contain no divergences.

It remains for us to consider diagrams with two external lines. For diagrams of the $\Phi\Phi$ type, $\omega(\Phi\Phi) \le 0$, since for these either $n_\delta \ge 2$, or $n_{++} = 1$, $|\tilde N_+ - \tilde N_-| \ge 1$ (the case $n_\delta = 0$ is excluded: the nonsense $3N_- - 2 = 3N_+$ would appear). However, according to consequence (5) of the Ward identities, the corresponding amplitude contains the square of the external momentum as a factor, and therefore such diagrams (and, analogously, all $\Phi^+\Phi^+$ diagrams) are free from divergences.

In the case of $\Phi\Phi^+$ diagrams, both n_δ and $|\tilde N_+ - \tilde N_-|$ can be equal to zero, and for these $\omega(\tau) \le 2$. But consequence (6) of the Ward identities says that the corresponding counterterm has the form

$$\Phi(p)\exp(-2\theta\hat p\bar\theta)\Phi^+(p)\,u(p^2). \tag{5.36}$$

The exponential behaves at large momenta like p^2. Therefore, $u(p^2)$ for any diagram diverges logarithmically, at most.

In all the calculations it is assumed that the propagators have been previously regularized in accordance with (5.11).

It has been proved, then, that in the model of Wess and Zumino, only diagrams of the $\Phi\Phi^+$ type lead to divergences, the divergences being logarithmic irrespective of the complexity of the diagram, and the structure of their contributions is the same and has the invariant form (5.36). Going over to x-space, we write the contribution from all $\Phi\Phi^+$ diagrams in a form identical to the kinetic part of the Lagrangian:

$$\frac{Z-1}{4}\int d^4x\, d\theta\,\Phi(x,\theta)\exp(-2i\theta\,\hat\partial\bar\theta)\,\Phi^+(x,\bar\theta), \tag{5.37}$$

where $Z - 1$ is a logarithmically divergent constant.

The constant Z is included in the renormalization of the wavefunction of a scalar superfield. We convince ourselves that the renormalized quantities in the model of Wess and Zumino are defined in accordance with

$$\Phi_r(x,\theta) = Z^{-1/2}\Phi(x,\theta),\quad m_r = Zm,\quad g_r = Z^{3/2}g. \tag{5.38}$$

Only one counterterm and, correspondingly, only one renormalization constant appear. This striking fact was first noticed and proved in [13,37]. It is this fact that has stimulated interest in supersymmetries. The model of Wess and Zumino is the simplest supersymmetry theory; everything in it is clear and so the largest number of papers have been devoted to it. The invariant perturbation theory for this model has been developed at different levels in [30,40,41,54,61,83,95,107]. Comparatively simple play with only the spinor variables θ, $\bar\theta$ makes it possible to exhibit the unusual properties of supersymmetry theories—the clear-cut cancellation of divergences, the vanishing of vacuum loops, etc.—while avoiding summing the mutually cancelling contributions of a large number of diagrams with Bose and Fermi fields.

The renormalization-group equations are simple for the model under discussion, since there is only one renormalization constant (e.g., it follows from (5.38) that $\partial g_r/\partial m = (3/2)\partial \ln Z/\partial m$). They have been investigated in articles by Ferrara, Iliopoulos and Zumino[98] and Shafi[85] (in the latter, in terms of superfields). The model does not possess asymptotic freedom—the effective coupling constant grows without limit as the energy increases.

We note that an analysis of the theory of a scalar field with self-interaction $(\lambda/4!)(\Phi^4(x,\theta)\delta(\bar\theta) + \text{h.c.})$ has shown[55] that it is not renormalizable even though it contains fewer divergences than one might expect. The theory of a chiral spinor superfield interacting with a chiral scalar superfield (with a dimensional coupling constant) was studied in the article[2]. It is also not renormalizable, but many of the divergences cancel. Renormalizable theories of a scalar superfield describing a vector multiplet[14)], generalizing the gauge theories of Yang and Mills, will be discussed briefly in the following section.

6. SUPERSYMMETRIC GENERALIZATION OF YANG-MILLS THEORIES

In this section we discuss the inclusion of internal symmetries and attempts to generalize the gauge theories in a supersymmetry model[14,80,100]. It is fairly simple to introduce a global internal symmetry, e.g., the unitary symmetry SU(n). The scalar chiral superfield $S(x,\theta)$ describes neutral particles A, B and ψ with a definite parity. Therefore, to construct complex representations of the internal-symmetry group it is sufficient to introduce a set of complex superfields (i = 1, 2, . . . , k)

$$S_i(x,\theta) = S_i^{(1)}(x,\theta) + iS_i^{(2)}(x,\theta) \tag{6.1}$$

14)Likhtman [58] has recentely studied a supersymmetric renormalizable field-theory model in which a massive vector field interacts with a nonconserved current.

and postulate the transformation law

$$S_i'(x, \theta) = \exp(-i\alpha_m t_m)_{ij} S_j(x, \theta), \qquad (6.2)$$

where α_m are the parameters, and t_m the matrices of a given representation of the internal symmetry. We shall attempt to progress further and generalize the local transformations, the parameters of which depend on the coordinates of the field. From the requirement that a transformed superfield be again a superfield, we conclude that the parameters themselves should be chiral superfields: $\alpha_m \to \alpha_m(x, \theta) = \alpha_m(x) + \theta^\alpha \alpha_{m,\alpha}(x) + \theta\theta\beta_m(x)$, i.e., that the symmetry should also be "local" in the superspace. As compared with ordinary local symmetries, a dependence on the spinor coordinates θ is added. Then,

$$S'(x, \theta) = \exp(-i\Lambda(x, \theta)) S(x, \theta) \quad (\Lambda(x, \theta) = \alpha_m(x, \theta) t_m). \qquad (6.3)$$

Under spatial reflection (cf. (3.23)),

$$S(x, \theta) \to S_P(x', \theta') = T(x, \bar\theta) = S^{(1)*}(x, \bar\theta) + iS^{(2)*}(x, \bar\theta), \qquad (6.4)$$

$$\Lambda_P(x, \theta) \to \Lambda_P(x', \theta') = \Lambda^*(x, \bar\theta). \qquad (6.5)$$

Under local transformations,

$$T(x, \bar\theta) \to T'(x, \bar\theta) = \exp(-i\Lambda^*(x, \bar\theta)) T(x, \bar\theta). \qquad (6.6)$$

The kinetic and mass terms of the Lagrangian, invariant under global transformations, have the form[15]

$$S^*(x - i\theta\sigma_\mu\bar\theta, \bar\theta) S(x + i\theta\sigma_\mu\bar\theta' \theta) + T^*(x + i\theta\sigma_\mu\bar\theta' \theta) T(x - i\theta\sigma_\mu\bar\theta, \bar\theta) + m[S^* T\delta^G(\theta) - T^* S\delta^G(\bar\theta)]. \qquad (6.7)$$

The mass term is also invariant under the local transformations (3.5), while the kinetic term is not invariant. We shall follow the philosophy of gauge fields[1]. We introduce the vector superfields $\tilde V_i(x_\mu, \theta, \bar\theta)$, transforming according to the adjoint representation of the internal-symmetry group ($\tilde V = t_i \tilde V_i$, $\alpha = t_i\alpha_i$):

$$\tilde V'(x, \theta, \bar\theta) = \exp(-i\alpha) \tilde V(x, \theta, \bar\theta) \exp(i\alpha). \qquad (6.8)$$

Under infinitesimal <u>local</u> transformations we assume that

$$\delta\tilde V = -i\Lambda^+\tilde V + i\tilde V\Lambda - \frac{i}{g}(\Lambda - \Lambda^+), \qquad (6.9)$$

as a generalization of ordinary local transformations:

$$\delta v_\mu = -i\alpha v_\mu + iv_\mu\alpha + \frac{1}{g}\partial_\mu\alpha.$$

Then under finite local transformations,

$$1 + g\tilde V' = \exp(-i\Lambda^+)(1 + g\tilde V)\exp(i\Lambda). \qquad (6.10)$$

We shall now reestablish the local invariance by introducing a "compensating" superfield $\tilde V$ and replacing the kinetic term by

$$S^*(1 + g\tilde V) S + T^*(1 + gV)^{-1} T. \qquad (6.11)$$

Under spatial reflection $(1 + gV) \to (1 + gV)^{-1}$, i.e., V transforms essentially nonlinearly. It is convenient to make the equivalence transformation

$$1 + g\tilde V' = \exp(gV). \qquad (6.12)$$

Under spatial reflection the vector superfield $V(x, \theta, \theta')$ simply changes sign, like an ordinary vector field, and, in turn, the gauge transformations (6.9) for V become nonlinear. The kinetic term in (6.7) acquires the form

$$S^* \exp(gV) S + T^* \exp(-gV) T. \qquad (6.13)$$

[15]If the superfield S is described by a real representation of SU(n), then the matrix Λ is antisymmetric, we can proceed without $S^{(2*)}$, and $T \equiv S^*$.

In order to find the Lagrangian for the self-interaction of the superfield V we form the spinor superfield

$$\Psi_\alpha = -\frac{i}{g}\exp(-gV) D_\alpha \exp(gV), \qquad (6.14)$$

which transforms according to the law (cf. (6.10) and (6.12))

$$\Psi_\alpha' = \exp(-i\Lambda)\Psi_\alpha \exp(i\Lambda) - \frac{i}{g}\exp(-i\Lambda) D_\alpha \exp(i\Lambda), \qquad (6.15)$$

since $D_\alpha\Lambda^+ = 0$ by definition. Applying the covariant derivative $\bar D_\alpha$ twice, we obtain a chiral spinor superfield

$$W_\alpha = \bar D\bar D\Psi_\alpha, \quad \bar D_\gamma W_\alpha = 0 \qquad (6.16)$$

with a uniform sign of the transformation:

$$W_\alpha' = \exp(-i\Lambda) W_\alpha \exp(i\Lambda). \qquad (6.17)$$

The self-interaction of the vector superfield $V(x, \theta, \bar\theta)$ can now be represented in a manifestly invariant form

$$\delta^G(\bar\theta)\,\mathrm{Tr}(W^\alpha W_\alpha) + \text{h.c.} \qquad (6.18)$$

Gauge invariance has been achieved, but the model constructed is extremely nonlinear and the kinetic terms (6.13) for the scalar fields and the self-interaction (6.18) are also nonlinear. In this form it is difficult to assess the renormalizability. In order to represent the model in a manifestly renormalizable form, Wess and Zumino[14] fix the gauge such that, of the entire aggregate of fields appearing in the vector superfield (cf. (3.47) and (6.7)), only the fields V_μ, λ and V remain:

$$\tilde V(x, \theta, \bar\theta) = -\theta\sigma_\mu\bar\theta V^\mu + \theta\theta\bar\theta\bar\lambda + \bar\theta\bar\theta\theta\lambda + \theta\theta\theta\theta D. \qquad (6.19)$$

In this gauge the self-interaction of the vector superfield is described by the Lagrangian density

$$L = \mathrm{Tr}\left\{-\frac{1}{4}(F_{\mu\nu})^2 - \frac{i}{2}\bar\lambda\gamma^\mu\nabla_\mu\lambda + \frac{1}{2}D^2\right\}, \qquad (6.20)$$

where

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + ig[V_\mu, V_\nu], \qquad (6.21)$$

$$\nabla_\mu\lambda = \partial_\mu\lambda + ig[V_\mu, \lambda]. \qquad (6.22)$$

Except for the term containing the auxiliary field D, (6.20) coincides exactly with the Lagrangian for a Yang-Mills field interacting with a Majorana spinor λ belonging to the adjoint representation of the internal-symmetry group. If the scalar superfield S is also described by the adjoint representation, then its interaction with a vector superfield is described in terms of the component fields:

$$L = \mathrm{Tr}\left\{\frac{1}{2}[(\nabla_\mu A)^2 + (\nabla_\mu B)^2 - i\bar\psi\gamma^\mu\nabla_\mu\psi + F^2 + G^2] + g\bar\lambda[A - i\gamma_5 B, \psi] + igD[A, B] + m\left(FA + GB - \frac{1}{2}\bar\psi\psi\right)\right\}, \qquad (6.23)$$

where $\nabla_\mu A = \partial_\mu A + ig[V_\mu, A]$, etc. Again, the familiar Yang-Mills couplings have arisen. In addition, couplings of the Yukawa type have appeared, and also, after the auxiliary field D is eliminated from (6.20) and (6.23), quadrilinear couplings of the fields A and B. In all these interactions there appears one universal coupling constant g. The theory obtained is manifestly renormalizable, but the special choice of gauge (6.19) deprives it of its manifestly invariant form under the supersymmetries.

Slavnov[87] has proved the renormalizability when the gauge is fixed in a supersymmetry-invariant form (cf. also [54,105]). In supersymmetric electrodynamics the anomalous magnetic moment of the electron vanishes[101].

It is interesting[80,100] that in the single-loop approximation for a vector superfield interacting with k superfields S belonging to adjoint representations, the Callan-Symanzik function $\beta(g)$ is equal to $-(g^3/16\pi^2) \times (3-k)G_2$, where G_2 is the magnitude of the square of the Casimir operator for the adjoint representation. For $k < 3$—in particular, for the self-interaction of a vector superfield—the theory is asymptotically free. For $k = 3$ the renormalization of the coupling constant is finite[16)].

We note[80,100] that if we put the mass in (6.23) equal to zero ($m = 0$), eliminate the field D from the equations of motion ($D = -ig[A, B]$) and combine the Majorana spinors λ and ψ into the Dirac spinor $\varphi = (1/\sqrt{2})(\lambda + i\psi)$, then (6.20) and (6.23) are written as

$$\mathrm{Tr}\left\{-\frac{1}{4}(F_{\mu\nu})^2+\frac{1}{2}[(\nabla_\mu A)^2+(\nabla_\mu B)^2-i\bar{\varphi}\gamma^\mu\overleftrightarrow{\nabla}_\mu\varphi]-g\bar{\varphi}[A-i\gamma_5 B,\ \varphi]+\frac{g^2}{2}[A,B]^2\right\}. \tag{6.24}$$

In formula (6.24) an additional invariance $\varphi \to e^{i\alpha}\varphi$ has appeared, which we can attempt to associate with the conservation of baryon number. Analogously, in the case of chiral internal symmetries, V-A gauge theories [29] arise in which attempts are made to introduce a conserved fermion number, and the scalar superfields must be assinged to the adjoint representation. The masses of all the particles should then vanish, and there is no invariant way of making the particles massive; this creates difficulties.

On the whole, despite the fact that, by the method described above, Wess, Delburgo, Zumino, Ferrara, Salam and Strathdee have succeeded in finding a supersymmetric generalization of the Yang-Mills theory, there remain certain unsatisfactory aspects. The theories have turned out to be highly nonlinear in the manifestly supersymmetric form, and the procedure is not without artificiality. The most important point is the following. For Yang-Mills fields the conserved vector currents serve as the sources. For supersymmetric models conserved spin-vector supercurrents are characteristic. In the approach described, they remain out of play and do not serve as sources of the corresponding gauge spin-vector fields. It is not ruled out that this is connected with the artificial choice of a vector superfield as the gauge field. We shall give the arguments.

The law of transformation (6.15) of the spinor superfield (6.14) is the exact analog of the Yang-Mills transformation

$$V'_\mu = \exp(-i\alpha)V_\mu \exp(i\alpha) - \frac{i}{g}\exp(-i\alpha)\,\partial_\mu \exp(i\alpha), \tag{6.25}$$

differing from it by the replacement of the ordinary derivative $\partial/\partial x^\mu$ by the spinor derivative D_α. The analog of the stress tensor $F_{\mu\nu}$ (6.21) ($F'_{\mu\nu} = \exp(-i\alpha) \times F_{\mu\nu}\exp(i\alpha)$) for the superfield is (with the natural replacement of commutators by anticommutators)

$$\Psi_{\alpha\beta} = D_\alpha\Psi_\beta + D_\beta\Psi_\alpha + ig\{\Psi_\alpha,\ \Psi_\beta\}, \tag{6.26}$$

$$\Psi'_{\alpha\beta} = \exp(-i\Lambda)\,\Psi_{\alpha\beta}\exp(i\Lambda). \tag{6.27}$$

The analogy is complete. Of greatest interest is the fact that the stress $\Psi_{\alpha\beta}$ vanishes identically if we express Ψ_α in terms of the vector superfield V by (6.14). In the Yang-Mills case the situation is exactly the same[68]: $F_{\mu\nu}$ vanishes identically if we substitute the "vector" field in the form (cf. (6.14))

$$V_\mu = -\frac{i}{g}\exp(-ig\varphi)\,\partial_\mu \exp(ig\varphi), \tag{6.28}$$

where $\varphi(x)$ is a gauge scalar field with the transformation law $\exp(ig\varphi)' = \exp(ig\varphi)\exp(i\lambda)$, which ensures the correct transformation properties (6.25) of the field V_μ (6.28). It seems to us, therefore, that the nonlinear generalization of Yang-Mills theories with a vector superfield, discussed above, is in a certain sense orthogonal to the philosophy of Yang and Mills, and that it is very important to study other possibilities—especially the gauge spinor superfield Ψ_α. In this case it is possible to hope for the appearance of well-grounded theories of the spin-vector field $\psi_{\mu\nu}$ appearing in the superfield Ψ_α (renormalizable theories of the vector field, other than the Yang-Mills theory, do not exist).

We have not succeeded in constructing, using the stress tensors (6.26) and the gauge group (6.15), a Lagrangian theory for the superfield Ψ_α such that the equations for the spinor fields would be first-order and those for the boson fields second-order. Evidently, the problem is that the gauge group (6.15) with a chiral scalar superfield $\Lambda(x, \theta, \bar{\theta})$ in place of the parameters is too limited. However, we can extend it and consider formally the same transformation law

$$\Psi'_\alpha = \exp(-i\Lambda)\,\Psi_\alpha \exp(i\Lambda) - \frac{i}{g}\exp(-i\Lambda)\,D_\alpha \exp(i\Lambda), \tag{6.29}$$

but regard Ψ_α as a Majorana spinor superfield and $\Lambda(x, \theta, \bar{\theta})$ as the general scalar superfield (3.47). The group properties are not lost when we do this.

Just as the gauge invariance in the Yang-Mills theory corresponds to replacement of the parameters of the transformation by general scalar fields (functions of x), so (6.29) corresponds to their replacement by general scalar superfields (scalar functions of x, θ and $\bar{\theta}$). Naturally, the interaction with chiral superfields cannot be included here, but the interaction with real scalar superfields can be included and is obtained by the usual technique of extending the covariant Majorana spinor derivative $D_\alpha \to D_\alpha + ig\Psi_\alpha$. It is also possible to write down a self-interaction Lagrangian invariant under (6.29). All these questions, the renormalizability of the theory and the effect of the Higgs type in which the field Ψ_α acquires mass, etc., are now being investigated by É. Sokachev and the authors of this review article.

7. UNIFICATION OF INTERNAL SYMMETRIES AND SUPERSYMMETRIES

The direct product of the internal symmetries and supersymmetries is simply realized and was discussed at the beginning of the preceding section. Their nontrivial unification is also, in principle, realizable [20,33,81]. Namely, the generators Q_α of the supersymmetries can be supplied with internal-symmetry indices i and the algebra (2.23) can be generalized in a consistent manner as follows:

$$\{Q_{\alpha i},\ Q_{\beta j}\} = \{\bar{Q}^i_{\dot\alpha},\ \bar{Q}^j_{\dot\beta}\} = 0, \tag{7.1a}$$

$$\{Q_{\alpha i},\ \bar{Q}^j_{\dot\beta}\} = 2\delta^j_i(\sigma_\mu)_{\alpha\dot\beta}P^\mu, \tag{7.1b}$$

$$[Q_{\alpha i},\ P_\mu] = [\bar{Q}^i_{\dot\alpha},\ P_\mu] = 0. \tag{7.1c}$$

Thus, the subscripts i, j can refer to the quark n-dimensional representation of the SU(n) group, and the

16)Kalashnikov and Fradkin [47] have shown that with a special summation of the higher orders these results remain valid. They also calculated the asymptotic form of the Green functions (scaling arises for k=3). In a model with spontaneous breaking of the internal symmetry [82] the asymptotic freedom disappears [47].

superscripts to the antiquark representation. The internal-symmetry generators are scalars and commute with the generators of the Poincaré group. In an article entitled "All possible generators of the supersymmetry of the S-matrix"[17)], Haag, Lopuszanski and Sohnius[109] state that the algebra (7.1), under rather general requirements, turns out to be practically[18)] the only possible algebra in a theory with massive particles[19)].

We introduce anticommuting spinor coordinates $\theta^{\alpha i}$, $\bar{\theta}_i{}^{\dot{\alpha}}$, in complete analogy with the simple supersymmetry group discussed above; the spinor generators can be realized in the form

$$Q_{i\alpha} = -i\frac{\partial}{\partial\theta^{i\alpha}} - (\hat{\partial}\bar{\theta}_i)_\alpha, \quad \bar{Q}^i_{\dot{\alpha}} = i\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}_i} - (\theta^i\hat{\partial})_{\dot{\alpha}}, \tag{7.2}$$

and satisfy the algebra (7.1). In the superspace x, $\theta^{\alpha i}$, $\bar{\theta}_i{}^{\dot{\alpha}}$ the spinor transformations are analogous to (3.4):

$$x'_\mu = x_\mu + i(\theta^i\sigma_\mu\bar{\zeta}_i - \zeta^i\sigma_\mu\bar{\theta}_i), \quad \bar{\theta}^{\dot{\alpha}\prime}_i = \bar{\theta}^{\dot{\alpha}}_i + \bar{\zeta}^{\dot{\alpha}}_i, \quad \theta^{\alpha i\prime} = \theta^{\alpha i} - \zeta^{\alpha i}. \tag{7.3}$$

The superfields are also defined analogously[33], but they contain more terms in the expansion in the spinor coordinates, since the number of these coordinates is increased. We shall not dwell on these questions, but turn to the discussion of the irreducible representations in the rest-frame. We note first of all that the spinor generators commute with the generators of translations (7.1c), and, therefore, in this amalgamation of the internal symmetries with the supersymmetries, O'Raifeartaigh's theorem[51,71] remains valid: the masses of all particles within a given representation should coincide. In the rest-frame the algebra (7.1) becomes a Clifford algebra of 2n spinor creation and annihilation operators. We introduce the notation $(1/\sqrt{2M})Q_{i\alpha} = Q_A$, $(1/\sqrt{2M})\bar{Q}^j_\beta = \bar{Q}_B$, where the indices A and B take 2n values. Then for $P_\mu = (M, 0)$ the algebra (7.1) is written as

$$\{Q_A, \bar{Q}^B\} = \delta^B_A, \quad \{Q_A, Q_B\} = \{\bar{Q}^A, \bar{Q}^B\} = 0. \tag{7.4}$$

In terms of Q_A and $\bar{Q}^B$ we can form operators F^B_A commuting with the Hamiltonian and forming the algebra of the U(2n) group:

$$F^B_A = \bar{Q}^B Q_A, \quad [F^B_A, F^D_C] = \delta^D_A F^B_C - \delta^B_C F^D_A. \tag{7.5}$$

Thus, in the rest-frame the group U(2n) arises. As in Sec. 2c, we define the "Clifford vacuum" $|\Omega\rangle_0$ by the condition

$$Q_A|\Omega\rangle_0 = 0. \tag{7.6}$$

The space of an irreducible representation will be formed by the states obtained by the action of the operators $\bar{Q}^B$, to first and higher (nonvanishing) powers, on $|\Omega\rangle_0$. For each $|\Omega\rangle_0$ there will be 2^{2n} states, since only antisymmetrized products of $k \le 2n$ operators $\bar{Q}^B$ are nonzero. For example, in the case of internal SU(3) symmetry[81][20)], by choosing the one-particle state with zero values of the spin, isospin and hypercharge as $|\Omega\rangle_0$, we find the fundamental representation, containing the $2^6 = 64$ states

$$|\Omega\rangle_0, \ \bar{Q}^B|\Omega\rangle_0, \ \bar{Q}^{B_1}\bar{Q}^{B_2}|\Omega\rangle_0, \ \ldots, \bar{Q}^{B_1}\ldots\bar{Q}^{B_6}|\Omega\rangle_0 \ldots \tag{7.7}$$

These states are the antisymmetric D(6) tensors, and we have the expansion 64 = 1 + 6 + 15 + 20 + 15 + 6 + 1.

The ground state $|\Omega\rangle_0$ can have nonzero quantum numbers with respect to the internal symmetry SU(3) and the group O(3) (spin) of rotations in the rest-frame, and we come to the conclusion that, in the rest-frame, the vast group U(6) × SU(3) × O(3) arises. The appearance of the SU(6) classification in the rest-frame is very specific, and the connection with the Gürsey-Radikati-Sakita SU(6) group (or the Wigner SU(4) group in the case of internal SU(2) symmetry) is not entirely clear: 1) the fundamental representation 64 contains quarks (representation 6) and the higher antisymmetric tensors 15, 20, ... (diquarks, triquarks); 2) on each state with given SU(3) and O(3) quantum numbers the representation 64 is constructed anew. The group U(6) × SU(3) × O(3) appears. The last factor can be interpreted as corresponding to the l-excitation. But the generators of SU(3) as subgroups of SU(6) do not coincide with the generators of the SU(3) internal-symmetry group. The symmetry that arises seems to be too wide.

In conclusion we shall describe the intuitive construction by Volkov and Akulov (cf. [26]) of spinor extensions of the Poincaré group. As is well-known, the group combination law for the parameters $((\hat{a}_1, \Lambda_1)(\hat{a}_2, \Lambda_2) = (\hat{a}_1 + \Lambda_1\hat{a}_2\Lambda_1^+, \Lambda_1\Lambda_2)$, where $\hat{a} = \sigma_\mu a^\mu)$ can be represented as the matrix multiplication of (4 × 4)-matrices:

$$\begin{pmatrix} \Lambda & -\frac{i}{2}\hat{a}\Lambda^{+-1} \\ 0 & \Lambda^{+-1} \end{pmatrix} = \begin{pmatrix} 1 & -\frac{i}{2}\hat{a} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \Lambda & 0 \\ 0 & \Lambda^{+-1} \end{pmatrix}. \tag{7.8}$$

The parameters a form a homogeneous space, and, assuming that the coordinate x_μ (in matrix form, $\hat{x} = \sigma_\mu x^\mu$) transforms like these parameters, we obtain the usual law $\hat{x}' = \Lambda\hat{x}\Lambda^+ + \hat{a}$. We move the blocks of this matrix apart and place the (n × n) matrix of the parameters of the internal symmetry U at the center. This corresponds to the direct product of the Poincaré group and the internal-symmetry group. We fill in the upper free (2 × n) block and the free (n × 2) block to the right with the spinor parameters ζ^i_α and $\bar{\zeta}_{\dot{\alpha} i}$, respectively, and modify the upper right corner.

The matrix thus constructed

$$\begin{pmatrix} \Lambda & \zeta U & \frac{1}{2}(\zeta\bar{\zeta} - i\hat{a})(\Lambda^+)^{-1} \\ 0 & U & \bar{\zeta}(\Lambda^+)^{-1} \\ 0 & 0 & (\Lambda^+)^{-1} \end{pmatrix} = \begin{pmatrix} 1 & \zeta & \frac{1}{2}(-i\hat{a} + \zeta\bar{\zeta}) \\ 0 & 1 & \bar{\zeta} \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \Lambda & 0 & 0 \\ 0 & U & 0 \\ 0 & 0 & \Lambda^{+1} \end{pmatrix} \tag{7.9}$$

corresponds to supersymmetries mixed nontrivially with the internal symmetries. A product of such matrices reproduces the group-combination law. The parameters ζ, $\bar{\zeta}$ and a form a homogeneous "superspace." Identifying the coordinates of this superspace with the coordinates of the superspace x_μ, θ^i_α and $\bar{\theta}_{\dot{\alpha} i}$, we arrive at the transformations (7.3).

8. ATTEMPTS TO BREAK THE SUPERSYMMETRIES

The masses of the fermions and bosons in one superfield coincide, and therefore, to construct future

[17)]This paper generalizes the results of Coleman and Mandula [43] to the case of supersymmetries.

[18)]"Practically"—in the sense that the conceivable modifications of it reduce to substituting into the right-hand side of (7.1) certain operators that commute with all the other generators and with each other.

[19)]In the recently published article [51b] by Konopel'chenko the structure of spinor extensions of the Poincaré algebra, with inclusion of higher spinor generators, is studied, and the above statement by the authors of [109] is put in doubt.

[20)]We note that the simplest Lagrangian model with a nontrivial unification of supersymmetry and SU(2) internal symmetry has been found to be unrenormalizable [42].

realistic models, it is necessary to be able to break the supersymmetry in a suitable way while preserving the renormalizability properties. Adequate methods have not yet been found. A particularly intensive search is being made for a successful spontaneous breaking of the supersymmetry. We shall discuss the principal attempts.

It is interesting that spontaneous breaking, in its most striking form in nonlinear realizations of the supersymmetries, was investigated by Volkov and Akulov[16,20] approximately a year and a half before the appearance of the papers of Wess and Zumino[12] which initiated the wide interest in supersymmetries. According to the general approach to nonlinear realizations of spontaneously broken symmetries[17,45], we introduce a Goldstone field with the quantum numbers of those charges whose conservation we wish to violate. In our case it is necessary to introduce the spinor Goldstone field $\theta(x)$, which one should identify with the spinor coordinates regarded as functions of x_μ. In other words, we must consider a certain "surface" $\theta = \theta(x)$, $\bar{\theta} = \bar{\theta}(x)$ in the superspace. The supersymmetry transformations (3.4) are then written as

$$\theta'(x') = \theta(x) + \zeta,\quad \bar{\theta}'(x') = \bar{\theta}(x) + \bar{\zeta},\quad x'_\mu = x_\mu + i(\theta(x)\sigma_\mu\bar{\zeta} - \zeta\sigma_\mu\bar{\theta}(x)) \tag{8.1}$$

or, infinitesimally,

$$\delta\theta(x) = \zeta + i(\zeta\sigma_\mu\bar{\theta}(x) - \theta(x)\sigma_\mu\bar{\zeta})\,\partial^\mu\theta(x) \tag{8.2}$$

and analogously for $\bar{\theta}(x)$. It is convenient to assign dimensions $[L]^{-3/2}$ to the spinor fields and parameters. After the corresponding redefinition, we rewrite (8.2) in the form

$$\delta\theta(x) = \zeta - \frac{ia}{2}(\zeta\sigma_\mu\bar{\theta}(x) - \theta(x)\sigma_\mu\bar{\zeta})\,\partial^\mu\theta(x), \tag{8.3}$$

where the constant a has the dimensions of the fourth power of a length. The nonlinear transformations obtained possess the necessary group structure; their commutator gives translations. The presence of the spinor constant in the right-hand sides of (8.1)–(8.3) emphasizes the Goldstone nature of the "neutrino" field $\theta(x)$. For $a = 0$, the free equation for the neutrino, $\sigma_\mu \partial^\mu \theta(x) = 0$, is invariant under (8.3). The phenomenological Lagrangian for $a \neq 0$ is constructed with the aid of the differential forms ω_μ (3.5) which are invariant under (8.3):

$$\omega_\mu = dx_\mu + \frac{ia}{2}(\theta\sigma_\mu d\bar{\theta} - d\theta\sigma_\mu\bar{\theta}) = dx_\nu\left[\delta^\nu_\mu + \frac{ia}{2}(\theta\sigma_\mu\partial^\nu\bar{\theta} - \partial_\nu\theta\sigma_\mu\bar{\theta})\right] = dx_\nu z^\nu_\mu.$$

The invariant action is given in the form

$$S = \frac{1}{a}\int d^4x\,\mathrm{Det}\,\|z^\mu_\nu\| \tag{8.4}$$

and corresponds to an invariant four-dimensional volume in the superspace. As in all Goldstone theories, Adler's principle is satisfied (cf. [20]). Interactions with other fields can also be included. Unfortunately, the theory is highly nonlinear, and derivatives of the Goldstone "neutrino" appear to high powers[21)]. It is very important to find spontaneous breaking in field theories that are of the usual form and renormalizable (the analog of the σ-model in chiral symmetry, which, when the σ-particle mass tends to infinity, corresponds to nonlinear realizations and becomes unrenormalizable).

Iliopoulos and Zumino[37] have studied soft induced breaking in the model of Wess and Zumino[22)], adding the linear breaking term $-cA(x)$ to the Lagrangian of (4.22). The renormalization properties are not changed by this, and, as before, only one counterterm is needed. However, the masses of the field A, B and ψ cease to be equal and in the "tree approximation" there arises the mass formula (higher corrections to which are finite and calculable)

$$m_A^2 + m_B^2 = 2m_\psi^2. \tag{8.5}$$

This mass formula is valid for any value of the parameter c, and, in particular, for c = 0. In the limit of exact symmetry, $m_A = m_B = m_\psi$. On spontaneous breaking, $m_\psi = 0$, ψ becomes a Goldstone fermion, and a spinor-constant correction appears in the transformation of ψ. However, the Goldstone regime is not stable[79]: $m_A^2 = -m_B^2$, i.e., the square of the mass of one of the particles, A or B, is negative. Taking the higher approximations into account[44] (the effective potential in the single-loop approximation[108]) does not reestablish the stability of the Goldstone regime. A model with spontaneous breaking of the supersymmetry should differ from the model of Wess and Zumino[23)]. Thus, Fayet and Iliopoulos[96a] have shown that a stable Goldstone regime and spontaneous breaking arise in supersymmetric electrodynamics when the linear term $\xi/gD(x)$, violating parity but invariant under transformations of the supersymmetry and gauge types, is added to the Lagrangian. In this case $\langle D\rangle = -\xi/g$ and $\delta\lambda = -(\xi/g)\gamma_5\zeta + \ldots$. The masses of the scalar fields become unequal and, under certain conditions, spontaneous breaking of the gauge invariance also arises, in which the vector field acquires mass by way of the Higgs mechanism and the Goldstone fermion becomes a linear combination of the Majorana spinors participating in the model. A V-A variant does not arise. The model is not generalized to the gauge theories associated with non-Abelian semi-simple algebras. To reproduce the calculations would take up too much space, and we refer the reader to the primary source[96].

We note also that the characteristics of spontaneous breaking of internal symmetries in supersymmetry models have been investigated by Salam and Strathdee[82] and O'Raifeartaigh[73a][24)]. The problem of a realistic spontaneous breaking of the supersymmetries remains unsolved at present.

9. CONCLUSION

How, then, have supersymmetries enriched elementary-particle physics? The simplest supersym-

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21)In the gauge version of this model [19] the gauge fields have spin 3/2, the Higgs effect is possible and a coupling with gravitation appears.

22)The algebra of supercharges in the presence of breaking is discussed in the article [32] by de Wit.

23)The heuristic argument, given in the paper [37], that supersymmetry can never be spontaneously broken turned out to be unjustified and was refuted by the authors themselves in [36,96].

24)After this review had gone to press, the paper [96b] by Fayet appeared, in which was proposed a semi-realistic model for the unified theory of the electromagnetic and weak interactions of the electron, electron neutrino and heavy leptons, with spontaneous breaking of the supersymmetries and of the gauge symmetry SU(2)×U(1). The electron neutrino serves as the Goldstone fermion and is combined with the photon in one supermultiplet. Unfortunately, there is no place for the muon neutrino with zero rest-mass in the framework of this model. We note also that in a new article by O'Raifeartaigh [73b] a generalized Wess-Zumino model, describing the interaction of N chiral scalar superfields, is considered. Spontaneous breaking of the supersymmetries can arise only for N≥3.

metry models, which appeared in 1971 and were studied intensively in 1974, possess many unexpected and attractive properties, arising from the connections between the Green functions of the fermions and bosons:

a) A sharp reduction in the number of divergences is observed.

b) All diagrams with vacuum loops, and the vacuum average of the energy-momentum tensor, vanish.

c) The conserved vector currents play a large role in elementary-particle physics. In the supersymmetry models an entirely new object appears – the conserved spin-vector currents.

d) The conserved spin-vector current of the supersymmetries appears with the energy-momentum tensor in the same superfield.

e) A nontrivial unification of the internal symmetries with the space-time symmetries is theoretically possible.

The problem lies in the construction of realistic physical theories. Hopes of a unified renormalizable asymptotically-free theory of the weak and electromagnetic (and, perhaps, the strong and gravitational) interactions, in which the neutrino would be the Goldstone particle, are emerging. Of the more technical problems, we note the search for well-grounded theories of the spin-3/2 particles associated with the conserved spin-vector currents[25)], the development of the differential geometry of superspace[5], etc. The incorporation of these supersymmetries into physics requires the assimilation of the new concept that has arisen, and a search for a suitable spontaneous supersymmetry-breaking and an adequate mechanism for conserving the number of baryons and fermions.

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APPENDIX

1. Useful Relations of the Spinor Formalism

The matrices $\tilde{\sigma}_\mu$ (see "Notation" at the end of the Introduction) are expressed in terms of σ_μ:

$$(\tilde{\sigma}_\mu)^{\dot{\alpha}\beta}=\varepsilon^{\dot{\alpha}\dot{\beta}}\varepsilon^{\beta\alpha}(\sigma_\mu)_{\alpha\dot{\beta}},\quad (\sigma_\mu)_{\alpha\dot{\beta}}=\varepsilon_{\alpha\beta}\varepsilon_{\dot{\beta}\dot{\alpha}}(\tilde{\sigma}_\mu)^{\dot{\alpha}\beta}.$$

The completeness relation is written as

$$(\sigma_\mu)_{\alpha\dot{\alpha}}(\tilde{\sigma}^\mu)^{\dot{\beta}\beta}=2\delta_\alpha^\beta\delta_{\dot{\alpha}}^{\dot{\beta}},\quad (\sigma_\mu)_{\alpha\dot{\alpha}}(\sigma^\mu)_{\beta\dot{\beta}}=2\varepsilon_{\alpha\beta}\varepsilon_{\dot{\alpha}\dot{\beta}},$$

$$\sigma_\mu\tilde{\sigma}_\nu+\sigma_\nu\tilde{\sigma}_\mu=2\eta_{\mu\nu},\quad \mathrm{Sp}\,(\sigma_\mu\tilde{\sigma}_\nu)=2\eta_{\mu\nu},$$

$$\sigma_\mu\tilde{\sigma}_\nu=\eta_{\mu\nu}-\frac{i}{2}\varepsilon_{\mu\nu\lambda\rho}\sigma^\lambda\tilde{\sigma}^\rho,\quad (\sigma_{\mu\nu})_\alpha^\beta(\sigma^{\mu\nu})_\rho^\tau=4(2\delta_\alpha^\tau\delta_\rho^\beta-\delta_\alpha^\beta\delta_\rho^\tau).$$

The product of two different spinors can be reduced:

$$\zeta_\alpha\bar{\xi}_{\dot{\beta}}=\frac{1}{2}(\sigma_\mu)_{\alpha\dot{\beta}}\zeta\sigma^\mu\bar{\xi},\quad \left(\left(\frac{1}{2},0\right)\otimes\left(0,\frac{1}{2}\right)=\left(\frac{1}{2},\frac{1}{2}\right)\right),$$

$$\zeta_\alpha\xi_\beta=\frac{1}{2}\zeta^\gamma\xi_\gamma\cdot\varepsilon_{\alpha\beta}-\frac{1}{2}\zeta\sigma^{\mu\nu}\xi(\sigma_{\mu\nu})_{\alpha\beta}\left(\left(\frac{1}{2},0\right)\otimes\left(\frac{1}{2},0\right)=(0,0)+(1,0)\right).$$

For the interchange of identical anticommuting spinors, the following simple rules hold:

$$\xi_\alpha\xi_\beta=\frac{1}{2}\varepsilon_{\alpha\beta}\xi^\gamma\xi_\gamma,\quad \xi^\alpha\xi^\beta=-\frac{1}{2}\varepsilon^{\alpha\beta}\xi^\gamma\xi_\gamma,\quad \bar{\xi}_{\dot{\alpha}}\bar{\xi}_{\dot{\beta}}=-\frac{1}{2}\varepsilon_{\dot{\alpha}\dot{\beta}}\bar{\xi}_{\dot{\gamma}}\bar{\xi}^{\dot{\gamma}}.$$

In the review article we often have to go over from bilinear combinations of spinors to bilinear combinations of Majorana bispinors $\psi=(\psi_\alpha/\bar{\psi}^{\dot{\alpha}})$. We give the corresponding connections:

$$\bar{\psi}\varkappa=\bar{\varkappa}\psi=\psi^\alpha\varkappa_\alpha+\bar{\psi}_{\dot{\alpha}}\bar{\varkappa}^{\dot{\alpha}},$$

$$\bar{\psi}\gamma_5\varkappa=\bar{\varkappa}\gamma_4\psi=\psi^\alpha\varkappa_\alpha-\bar{\psi}_{\dot{\alpha}}\bar{\varkappa}^{\dot{\alpha}},$$

$$\bar{\psi}\gamma_\mu\varkappa=-\bar{\varkappa}\gamma_\mu\psi=\psi\sigma_\mu\bar{\varkappa}-\varkappa\sigma_\mu\bar{\psi},$$

$$\bar{\psi}\gamma_5\gamma_\mu\varkappa=\bar{\varkappa}\gamma_5\gamma_\mu\psi=\psi\sigma_\mu\bar{\varkappa}+\varkappa\sigma_\mu\bar{\psi},$$

$$\bar{\psi}\gamma_\mu\gamma_\nu\varkappa=\bar{\varkappa}\gamma_\nu\gamma_\mu\psi=\psi\sigma_\mu\tilde{\sigma}_\nu\varkappa+\bar{\psi}\tilde{\sigma}_\mu\sigma_\nu\bar{\varkappa}.$$

2. Products of Covariant Derivatives

The covariant derivatives D_α, $\bar{D}_{\dot{\alpha}}$ obey the commutation relations (3.31). Because of this, any product of D_α and $\bar{D}_{\dot{\alpha}}$ reduces to a linear combination of the 16 independent elements

$$1,\ D_\alpha,\ \bar{D}_{\dot{\alpha}},\ DD,\ \bar{D}\bar{D},\ D\sigma_\mu\bar{D},\ (DD)\bar{D}_{\dot{\alpha}},\ (\bar{D}\bar{D})D_\alpha,\ (DD)(\bar{D}\bar{D})$$

with coefficients that include ordinary derivatives ∂_μ. Thus,

$$D_\alpha D_\beta=\frac{1}{2}\varepsilon_{\alpha\beta}(DD),\quad \bar{D}_{\dot{\alpha}}\bar{D}_{\dot{\beta}}=-\frac{1}{2}\varepsilon_{\dot{\alpha}\dot{\beta}}\bar{D}\bar{D},$$

$$\{D_\alpha,\bar{D}_{\dot{\beta}}\}=(\sigma_\mu)_{\alpha\dot{\beta}}D\sigma^\mu\bar{D}-2i\hat{\partial}_{\alpha\dot{\beta}},$$

$$D_\alpha D_\beta D_\gamma\equiv 0,\quad \bar{D}_{\dot{\alpha}}\bar{D}_{\dot{\beta}}\bar{D}_{\dot{\gamma}}\equiv 0,$$

$$[\bar{D}_{\dot{\alpha}},DD]=4i\hat{\partial}_{\beta\dot{\alpha}}D^\beta,\quad [D_\alpha,\bar{D}\bar{D}]=-4i\hat{\partial}_{\alpha\dot{\beta}}\bar{D}^{\dot{\beta}},$$

$$D_\alpha(D\sigma_\mu\bar{D})=-\frac{1}{2}(\sigma_\mu)_{\alpha\dot{\beta}}(DD)\bar{D}^{\dot{\beta}},$$

$$\bar{D}_{\dot{\alpha}}(D\sigma_\mu\bar{D})=-\frac{1}{2}(\sigma_\mu)_{\alpha\dot{\alpha}}(\bar{D}\bar{D})D^\alpha-4i\partial_\mu\bar{D}_{\dot{\alpha}},$$

$$[DD,\bar{D}\bar{D}]=16\Box-8i\partial^\mu D\sigma_\mu\bar{D},$$

$$\bar{D}_{\dot{\alpha}}(DD)\bar{D}^{\dot{\alpha}}-D^\alpha(\bar{D}\bar{D})D_\alpha=(DD)(\bar{D}\bar{D})+(\bar{D}\bar{D})(DD)+16\Box.$$

Products of five and more D are reduced with the help of these formulas and the relations

$$(DD)(\bar{D}\bar{D})D_\alpha=4i\hat{\partial}_{\alpha\dot{\beta}}\bar{D}^{\dot{\beta}}.$$

For example,

$$(\mathrm{DD})(\bar{\mathrm{D}}\bar{\mathrm{D}})(\mathrm{DD})=-16\Box(\mathrm{DD}),\ (\bar{\mathrm{D}}\bar{\mathrm{D}})(\mathrm{DD})(\bar{\mathrm{D}}\bar{\mathrm{D}})=-16\Box\bar{\mathrm{D}}\bar{\mathrm{D}},$$

$$[(\mathrm{DD})(\bar{\mathrm{D}}\bar{\mathrm{D}})]^2=-16\Box(\mathrm{DD})(\bar{\mathrm{D}}\bar{\mathrm{D}})$$

etc.

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For general orientation, we list the principal papers: investigations of the supersymmetry algebra and its representations[26,27,57,20,12,77,78,97,99,75,72,23,59,65]; the model of Wess and Zumino[13,37]; invariant perturbation theory in the approach of Salam and Strathdee[82,40,30,31,54,83,41], and in the approach with Grassmann integration[61,107]; gauge theories[14,100,80,87,54,105]; unification of supersymmetries and internal symmetries[20,78,33,109,42,15]; attempts at spontaneous breaking[16,18-20,79,37,82,73,22,108]; the problems of the baryon and fermion numbers[100,80,29,84,24]; the supercurrent and the connection with the energy-momentum tensor[36,102]; the renormalization group and asymptotic freedom[98,100,80,93,85,47,21]; mathematical papers[6,7-9,69,49,50,92,53,75,76,46,60]; review articles[35,15,106,82,52,46,74].

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