Development of electron avalances and streamers

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Usp. Fiz. Nauk 117, 493–521 (November 1975)

The article presents a review of the current state of the theory of avalanche-streamer processes, i.e., processes occurring during the development of discharges in dense gases. The principal attention is devoted to description of the physical picture of the phenomena. A systematic presentation is given of the theory of the main stages of formation of a discharge: the theory of electron avalanches, the theory of a self-maintaining discharge, the theory of the avalanche-streamer transition, and the theory of streamers. The question of breakdown of long spark gaps is discussed briefly.

PACS numbers: 52.80.Mg, 29.40.Hy

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INTRODUCTION

In recent years new interest has arisen in the theory of the gas discharge in spark gaps of length ~1 cm and pressures of the order of 1 atm. This is due to the development of laser technology, MHD generators, and also to use of spark and streamer chambers as elementary particle detectors.[1-6] However, Townsend's classical theory[7-12] of the breakdown of a gas, which discusses the discharge as the motion of electron avalanches arising as the result of potential ion-electron emission at the cathode, is not applicable in these regions of the variables N and d (N is the concentration of atoms or molecules of the gas, and d is the length of the discharge gap). In the literature it is usually assumed that the classical theory agrees satisfactorily with experiment at low pressures and not too large d, namely, for $Nd \lesssim 10^{19}$ cm^{-2}. If $Nd \gtrsim 10^{19}$ cm^{-2}, it follows from the experimental data that under these conditions such an important characteristic of the discharge as the breakdown voltage does not depend (or depends only weakly) on the cathode material, i.e., processes at the cathode no longer play the main role in establishment of a self-maintaining discharge, while in the theory of Townsend and others they are dominant. In addition, it has been found that the time of formation of a discharge at atmospheric pressure is roughly two orders of magnitude less than that predicted by the classical theory, which also indicates a change in the physical processes occurring during breakdown.

At the beginning of the nineteen-forties, Loeb, Meek, and Raether[11-20] advanced the hypothesis of changes in the discharge mechanism in the transition to large Nd. According to this hypothesis the Townsend avalanche breakdown with secondary processes at the cathode is replaced in this case by breakdown by a streamer, i.e., a narrow, highly conducting channel which propagates with a high velocity and is maintained by photoionization of the gas.

For a long time, however, the theory of Loeb, Meek, and Raether remained only a set of qualitative assumptions and crude quantitative estimates, and only in recent years has a rather accurate streamer theory been developed.[21-25] In addition, it has become clear that there is a certain intermediate region of values of N, d, and the external electric field strength E in which a self-maintaining discharge at large Nd originates without streamer formation, similar to avalanche breakdown but with photoionization of the gas as the secondary mechanism.[26]

Thus, at the present time gas discharge theory for parameters close to those of streamer chambers has advanced substantially and we believe that some results can be reported.

1. ELECTRON AVALANCHES AND THE THEORY OF A SELF-MAINTAINIG DISCHARGE

a). The Townsend Ionization Coefficient α

Townsend was the first to obtain theoretically and experimentally the dependence of the current between two plane parallel electrodes on the distance between them for a given field strength and gas pressure. He obtained this relation in the form

$$i = i_0 e^{\alpha d}, \quad \alpha = N\frac{\langle v\sigma_i\rangle}{u}, \tag{1.1}$$

where α is the number of pairs of charged particles produced by an electron per unit pathlength; σ_i is the cross section for ionization by electron impact; v is the velocity of the electrons; u is the drift velocity of the electrons; the averaging is carried out over the electron velocity distribution function. It follows from Boltzmann's kinetic equation[27-30] that the electron velocity distribution function depends on the ratio E/N. Therefore we obtain from Eq. (1) the well known Townsend relation

$$\frac{\alpha}{N} = f\left(\frac{E}{N}\right). \tag{1.2}$$

By means of this relation the results of experiments to measure α, carried out at low pressures, are then extrapolated to high pressures. However, we will show below that this is often impossible, since in some cases deviations from Townsend's law (1.2) are possible.[31,32]

Many experimental and theoretical studies have been made for the purpose of determining α in various gases (a detailed biography is contained in the book of Meek and Craggs,[12] and of the later studies we note refs. 33–43), but the result obtained should be considered with great caution for the following reasons.

First, the presence of even negligible fractions of easily ionized impurities can greatly change the value of α, and therefore some experimental data obtained when gas purification technology was not highly developed must be considered unreliable.

Appropriate experiments[44-48] and calculations[49-50] confirm the strong influence of impurities.

Second, some gases form molecular ions with a high probability as a result of associative ionization of the type

$$A^* + B \rightarrow (AB)^+ + e^-. \quad (1.3)$$

For example, for helium the cross section for the reaction

$$He^* + He = He_2^+ + e^- \quad (1.4)$$

has a value[51] $\sim 10^{-15}$ cm^2.

The cross sections in some other inert gases are also of the same order. Similar reactions occur in oxygen, nitrogen, carbon dioxide, and other gases,[52-59] but there are no data on the cross sections for these gases in the literature as yet.

When reaction (1.3) is taken into account, the Townsend coefficient should be determined from the relation[32]

$$\alpha = \alpha_i + \alpha^* = N\frac{\langle v\sigma_i\rangle}{u} + N\frac{\langle v\sigma^*\rangle}{u} W(\tau', T); \quad (1.5)$$

here α_i and α^* are the contributions to α from ionization by electron impact and associative ionization, respectively; σ^* is the combined cross section for excitation to levels entering into reaction (1.3); $W(\tau', T) = \tau'/(T + \tau')$ is the probability of occurrence of this reaction, which depends on the time in which it occurs $T = (Nv_M\sigma_T)^{-1}$ and on the characteristic time τ' for quenching of A^* without formation of an electron; v_M is the thermal velocity of the atoms or molecules; σ_T is the cross section for reaction (1.3).

Consequently, in the general case the Townsend relation (1.2) is not satisfied. If $T \gg \tau'$, we have a dependence of the form $\alpha \sim N^2 f(E/N)$. A dependence of this type has been observed in experiments by Daniel et al.[31] If $T \ll \tau'$, the Townsend relation is re-established, but the main contribution to α will be from associative ionization, since the electron velocity distribution function falls off rapidly in the tail, and the appearance of an electron as the result of associative ionization requires lower electron energies.

If we proceed on this basis to compare theoretical and experimental data on determination of α, it is necessary to know also at what pressures the experiment was carried out and what is the cross section for associative ionization in a given gas. From this point of view the data of Chanin and Rork[41] are of interest.

We note also that in calculation of α in a narrow range of E/N use is often made of the semiempirical relation

$$\alpha = ANe^{-BN/E}, \quad (1.6)$$

where the constants A and B are different in different intervals of E/N.

b). Electron Avalanches

The most important results in the study of electron avalanches were obtained by the Raether school.[61-64]

The first apparatus in which avalanches were observed was the Wilson cloud chamber. The study of electron avalanches with a cloud chamber is based on the fact that the ions arising in an avalanche serve as condensation centers for the supersaturated vapor with which the chamber is filled. Raether placed two plane parallel electrodes in a cloud chamber filled with a gas and water vapor. Synchronously with application of a voltage pulse to the electrodes, the chamber volume is rapidly expanded; as a result, the vapor becomes supersaturated and, in gases which have been cleaned of dust, condenses on the positive and negative ions of the avalanche. The negative ions are formed on attachment of electrons to neutral molecules.

Of the more recent investigations with cloud chambers, we should note the work of Allen and Phillips[65] who studied the development of electron avalanches in air, nitrogen, carbon dioxide, argon, water and oxygen, and also in the gases enumerated with additions of water vapor and various alcohols. By this method it is possible to measure quite reliably the mobilities of the electrons and ions and also to estimate their average energy and the diffusion coefficients.

Another method of studying electron avalanches is the electrical method, based on the fact that the electrons and ions of an avalanche produce a current pulse on crossing the discharge gap.[66-70] This pulse produces a voltage pulse in the resistance in the external circuit, which after amplification can be recorded with an oscilloscope.

If we measure the voltage V across this resistance as a function of time and make a semilogarithmic plot V(t), we obtain a straight line indicating an exponential rise in the number of current carriers. The slope of the straight line gives the buildup constant of the current.

A deficiency of the electrical method is the fact that it does not provide the possibility of establishing the spatial pattern of electron avalanche development.

In this area the best result is given by the optical method.[71-76] The essence of this method is that the electrons, in addition to ionization, produce an avalanche of excited molecules or atoms of the gas. The light emitted by the excited molecules is detected by a photomultiplier or image amplifier. In the latter case an image of the radiating avalanche is obtained on the screen of the image amplifier.

We shall now consider the mathematical aspect of the problem. The change in the concentrations of electrons and ions during development of an electron avalanche will be determined by the processes of ionization by electron impact, diffusion and mobility of the electrons and ions, and also by photoionization. Since the time of development of the discharge is $\sim 10^{-7}$ sec, recombination and electron attachment (if it can occur) can be neglected. We can also neglect diffusion and mobility of the ions. In this case the system of equations for the growth of the concentration of electrons N_e and of ions N_i in an avalanche which was initiated by a single electron near the cathode has the form[77]

$$\frac{\partial N_e(\mathbf{r}, t)}{\partial t} = \alpha u N_e(\mathbf{r}, t) + D\nabla^2 N_e(\mathbf{r}, t) - u\nabla N_e(\mathbf{r}, t)\cdot$$

$$+\lambda u\int K|\mathbf{r}-\mathbf{r}'|N_e(\mathbf{r}',t)\,d\mathbf{r}', \qquad (1.7)$$

$$\frac{\partial N_i}{\partial t}=\alpha u N_e(\mathbf{r},t); \qquad (1.8)$$

here D is the electron diffusion coefficient; λ is the probability of production of a photoelectron per unit electron pathlength; $K|r - r'|$ is the law of absorption of the photoionizing radiation. The initial conditions are written as

$$N_e(\mathbf{r},0)=\delta(\mathbf{r}), \qquad (1.9)$$

$$N_i(\mathbf{r},0)=0, \qquad (1.10)$$

where $\delta(\mathbf{r})$ is the Dirac delta function.

If there is no photoionization ($\lambda = 0$) and the distortion of the field by the space charge of the avalanche is small, then the solution of Eq. (1.7) is a Gaussian distribution function expressed in coordinate system moving with velocity u along the z axis[12]

$$N_e(\mathbf{r},t)=\frac{1}{(4\pi Dt)^{3/2}}\exp\left(-\frac{x^2+y^2+(z-ut)^2}{uDt}+\alpha ut\right). \qquad (1.11)$$

It follows from Eq. (1.11) that in this case the radius of the avalanche is determined by electron diffusion and is given by

$$r_D=\sqrt{\int_0^\infty r^2N_e(\mathbf{r},t)\,d\mathbf{r}\Big/\int_0^\infty N_e(\mathbf{r},t)\,d\mathbf{r}}=\sqrt{6Dt}. \qquad (1.12)$$

If $\lambda \neq 0$, then for solution of Eq. (1.7) it is necessary to determine the explicit form of the absorption law for the photoionizing radiation, for which it is necessary to know the nature of this radiation.

Existence of photoionizing radiation emitted by an electron avalanche was demonstrated experimentally for the first time by Raether.[78] Raether made a small opening in the cloud chamber to one side of the electrodes. At some distance from the opening he placed an additional spark gap of brass or aluminum spheres, so that the narrow beam of photons produced in a discharge between the spheres could pass midway between the electrodes in the cloud chamber and parallel to them. From the number of avalanches per unit length with increasing distance from the opening, and with the assumption that the radiation absorption law is exponential, it was possible to deduce the absorption coefficient of the ionizing radiation in the gas. It turned out that the absorption coefficient in various gases, converted to atmospheric pressure, was of the order $\sim 1\ \text{cm}^{-1}$. Thus, in air $\kappa \approx 1.8\ \text{cm}^{-1}$, in oxygen $\kappa \approx 1\ \text{cm}^{-1}$, in hydrogen $\kappa \approx 0.8\ \text{cm}^{-1}$, and so forth.

The experiments of Raether and other investigators[78-80] convincingly proved the existence of ionizing radiation formed by an electron avalanche, but the mechanism of this radiation remained unknown for a long time. We shall discuss in principle what processes can lead to formation of ionizing photons.

For homogeneous gases the processes which can lead to production of photons with energy exceeding the ionization energy are the excitation of ions and recombination. However, these processes are quadratic in the electron concentration, and since the electron concentration is low, they can be neglected. Estimates show that the intensity of these processes during the short discharge time of $\sim 10^{-7}$ sec is practically zero. There is another possible process, which is linear in the electron density. This is the collision of an electron with a neutral molecule with simultaneous ionization and excitation of the residual molecule. However, this process requires such large electron energies that it is highly improbable, since the electron velocity distribution function falls off rapidly at the tail of the distribution.

Ionization of excited molecules by photons with energies less than the ionization energy also is an effect quadratic in the electron density.

In a mixture of gases, in particular, in air, as a result of the difference in the ionization potentials of nitrogen and oxygen, the possibility arises of ionization of a molecule of oxygen by a photon emitted by a molecule of nitrogen excited to a level with energy greater than the oxygen ionization energy. For a long time during the absence of another explanation this mechanism was accepted by most investigators,[11,12] but it is quite clear that it cannot explain the existence of the penetrating photons in Raether's experiments.

The fact is that photons with energy greater than 12.2 eV formed in the radiation of nitrogen and capable of ionizing oxygen are resonant with respect to nitrogen. Their absorption coefficients at atmospheric pressure for the central part of the spectral line will be of the order of $\sim 10^6\ \text{cm}^{-1}$. In addition, since these photons are produced on radiation of the upper excited levels of nitrogen, their lifetime will be small, $\sim 10^{-7}$ sec, as a result of the transition of the molecule to a lower energy state with energy less than 12.2 eV. During this time the photons, diffusing through the gas, are practically confined to the volume of the avalanche.

There remains a small fraction of these photons which, as a result of the finite width of the spectral line, act quite differently from the main fraction—they escape from the resonance—and therefore they have a small absorption coefficient for the same line. However, the energy of these photons will be not much greater than the ionization energy of oxygen and therefore they will be strongly absorbed by oxygen[81-83] with an absorption coefficient $\sim 100\ \text{cm}^{-1}$. Thus, we reach the conclusion that among the processes occurring in a discharge in homogeneous gases and also in mixtures of gases, there are none which could lead to production of photons with energy exceeding the ionization energy of the gas and at the same time capable of traveling large distances during the short time of the discharge development.

In an earlier article[21] we suggested that secondary electrons in a discharge without involvement of the cathode arise from action of photons with energy less than the ionization energy by means of some chemical reaction. Let us consider this hypothesis in more detail.

In the development of an electron avalanche, an avalanche of excited atoms or molecules is also formed, a portion of these atoms or molecules being excited to such a level that on collision of an excited molecule with a neutral molecule an ion-molecule chemical reaction of associative ionization (1.3) can occur. In section a) we showed that at atmospheric pressure the major portion of such molecules, which have not been able to radiate a photon, enter into this reaction.

However, some portion of the excited molecules nevertheless do radiate a photon. Since these photons are resonance, their absorption coefficient is very high, $\sim 10^6\ \text{cm}^{-1}$. A small fraction of the emitted photons, as a result of the finite width of the spectral line, deviate in their behavior from the main fraction and can travel a great distance before the first absorption.

In order to carry out appropriate evaluations, it is necessary first of all to establish the law of absorption of resonance photons with allowance for the finite width of the spectral line.

Let a photon of frequency ω be emitted at the origin. Then, if the spectral line were infinitely narrow, the probability for the photon to traverse a distance r without absorption would be

$$W(r) = e^{-\varkappa r}. \tag{1.13}$$

When the finite width of the line is taken into account, this probability is given by the relation

$$W(r) = \int_0^\infty e^{-\varkappa(\omega) r} P(\omega)\, d\omega; \tag{1.14}$$

here $P(\omega)$ is the shape of the spectral line and $\kappa(\omega)$ is the frequency-dependent absorption coefficient of the photons. For further calculations it is necessary to specify a definite spectral line shape.

Estimates show that in a gas under normal conditions the broadening of the spectral line occurs mainly as the result of molecular collisions, so that we can choose the Lorentz collision shape for the spectral line.[81] We have

$$P(\omega) = \frac{(\Gamma/2\pi)}{(\omega-\omega_0)^2+(\Gamma^2/4)}; \tag{1.15}$$

here $\Gamma = Nv_M\sigma_{eff}$ is the collision width of the spectral line; σ_{eff} is the effective cross section for collision of the molecules.

Here

$$\varkappa(\omega) = \frac{\varkappa_0\Gamma^2/4}{(\omega-\omega_0)^2+(\Gamma^2/4)}; \tag{1.16}$$

here κ_0 is the absorption coefficient of a central photon of frequency ω_0.

Substituting (1.15) and (1.16) into (1.14), carrying out the integration, and taking into account that $\kappa_0 r \gg 1$, we obtain[84]

$$W(r) = \frac{1}{\sqrt{\pi\varkappa_0 r}}. \tag{1.17}$$

The probability that the photon is absorbed in a unit solid angle in a distance from r to r + dr from the place of radiation is accordingly

$$K(r) = -\frac{1}{4\pi r^2}\frac{dW(r)}{dr} = \frac{1}{(4\pi)^{3/2} r^3 (\varkappa_0 r)^{1/2}}. \tag{1.18}$$

Now we have removed the inconsistency between the absorption coefficient values obtained, on the one hand, in Raether's experiments and, on the other hand, from the theory and experiments of other authors.

In fact, if the photoionizing radiation has the nature discussed above, it is easy to explain the existence of photons with a long mean free path. As follows from (1.14), the absorption coefficient of photons whose frequency is sufficiently different from the resonance frequency can take on rather low values. In addition, in Raether's calculations he assumed beforehand an exponential law for absorption of the photoionizing radiation, whereas this is not the case.

If we plot the experimental points for the number of avalanches produced by photoionizing radiation in Raether's experiments as a function of the distance from the source of radiation, we obtain the pattern shown in Fig. 1. We can see that within the statistical error, which is equal to $\sqrt{n}$, good agreement is observed between theory and experiment. It is necessary also to take into account that the absorption law (1.17) is valid in a homogeneous gas or in any case in a mixture where there is no component with ionization potential less than $\hbar\omega_0$. There is no guarantee that no such component was present in Raether's experiments, which could somewhat distort the results. Therefore it is desirable to repeat Raether's experiment in a pure gas.

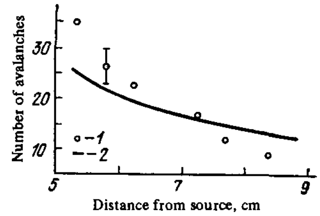


FIG. 1. Number of avalanches as a function of the distance from the source of photons. 1—experiment [78]; 2—theory [21].

In regard to the photoionization mechanism in a mixture of gases, in each specific case it is necessary to make a special study of the composition of the mixture and of the excitation and ionization potentials of each component.

In oxygen, apparently, the main role is played by reaction (1.3) with participation of oxygen, for example,[60]

$$\begin{aligned} &O_2^* + N_2 \rightarrow NO + NO^+ + e^-,\\ &O_2^* + O_2 \rightarrow O_3 + O^+ + e^- \end{aligned} \tag{1.19}$$

Similar reactions with excited nitrogen for production of long-range photons in air do not play an important role, since the resonance photons which deviate from the main frequency will, as discussed above, be absorbed by oxygen in the avalanche itself.

Now that the explicit form of the absorption law is known, we can obtain a solution[77] of Eq. (1.7). To a sufficiently good approximation we can write this solution in the form

$$N(r,t) = \frac{e^{(\alpha+\lambda)ut}}{2\pi^2}\int_0^\infty \frac{k\sin kr_1}{r_1}\exp\left(-k^2D+\lambda u\sqrt{\frac{k}{\varkappa_0}}\right) t\, dk. \tag{1.20}$$

Here the radius vector r_1 has components $\{x, y, z - ut\}$. Further solution can be only numerical, if the constants entering into Eq. (1.20) are known.

However, some estimates of interest can be made without resorting to numerical calculations.

The first term in the exponential under the integral sign in (1.20) provides broadening of the electron avalanche as the result of electron diffusion, i.e., the diffusion radius of the avalanche is

$$r_D \sim \frac{1}{k} \sim \sqrt{Dt}. \tag{1.21}$$

The second term in the exponential is responsible for broadening of the avalanche as the result of photoionizing radiation. Here

$$r_P \sim \frac{1}{k} \sim \frac{\lambda^2u^2t^2}{\varkappa_0}. \tag{1.22}$$

We shall estimate values of r_D and r_P for breakdown of a centimeter gap in air. Here $D \sim 10^2$ cm^2/sec, $u \sim 10^7$ cm/sec, $\kappa_0 \sim 10^6$ cm^{-1}, $\lambda \leq \alpha \sim 20$ cm^{-1}, and $t \sim 10^{-7}$ sec.

Substituting these parameter values into (1.21) and

(1.22), we find that $r_D \sim 10^{-2}$ cm; $r_P \sim 10^{-4}$ cm, i.e., $r_D \gg r_P$. Thus, photoionization broadening of an avalanche in this case can be neglected. It is true r_D increases with time as $\sqrt{t}$, while r_P increases as t^2, and therefore with passage of time r_P can exceed r_D. However, for large t the use of the formulas obtained becomes risky, since it will hardly be possible in this case to neglect the effect of space charge. In this case it is necessary to solve Eq. (1.7) with allowance for the dependence of its coefficients on the field, which is an extraordinarily complicated mathematical problem which at the present time has not yet been solved. However, a very important conclusion which can be drawn from expression (1.20) is the fact that, as numerical evaluations show, after a time $\sim 10^{-7}$ sec from the start of the initial electron, another electron arises at the location of its start, which as we know means that the condition of a self-maintaining discharge is satisfied.

Consequently, at atmospheric pressure a self-maintaining discharge is possible without participation of cathode processes and without formation of a streamer. The role of secondary processes in such a discharge is played by associative ionization.

In addition, as we will see subsequently, it is just the departure of λ from zero which can lead to the formation and propagation of an inverse (cathode) streamer.

While in a Townsend discharge with secondary processes at the cathode the time of formation of the discharge is determined by the drift time of the positive ions and also by the diffusion of metastables or resonance radiation to the cathode, which at pressures of the order of one atmospheric and $d \sim 1$ cm amounts to 10^{-4}–10^{-5} sec, on the other hand in a discharge with associative ionization as the secondary mechanism the time will be determined by the motion of the electrons from the cathode to the anode, $\sim 10^{-7}$ sec, since the time of formation of secondary electrons in this case can be neglected, and this value is observed experimentally.

c) Theory of a Self-Maintaining Discharge

Beginning with some distance d between electrodes at a given value of E/N, the rise of current according to Eq. (1.1) is no longer valid. With increasing gap length the current rises more rapidly. To explain this effect Townsend suggested that additional electrons arise on bombardment of the cathode by positive ions of the avalanche.

It can be shown[11,12] that in this case the rise of the electron current will be described by the formula

$$i = \frac{i_0 e^{\alpha d}}{1 - \gamma (e^{\alpha d} - 1)}. \qquad (1.23)$$

If the condition

$$\gamma (e^{\alpha d} - 1) = 1 \qquad (1.24)$$

is satisfied, the current will rise to infinity if i_0 is finite.

Townsend interpreted the physical meaning of this fact to be that if condition (1.24) is satisfied the number of electrons leaving to the anode is completely regenerated by liberation of electrons at the cathode by impacts of positive ions and by ionization of the gas. In this way the discharge will become self-maintaining and it is no longer necessary to have an initial current i_0 different from zero. From Eqs. (1.2) and (1.24) one obtains Paschen's breakdown law, according to which the breakdown voltage depends on the product of the concentration of molecules by the length of the discharge gap, and not on each of these quantities independently. However, since the relation $\alpha/N = f(E/N)$ is not always satisfied, Paschen's law also has limited applicability. The existence of impurities in the gases and also the cathode material may have an important effect on the value of the breakdown voltage, since in this case the coefficients α and γ change. We note that it is possible for the coefficient γ not to be a function of E/N, and nevertheless it is difficult to observe deviations from Paschen's law, since from Eqs. (1.6) and (1.24) it follows that in the expression for the breakdown voltage V = Ed the quantity γ occurs twice in the argument of the logarithm. In breakdown of air γ varies from 10^{-4} at relatively low pressures to 10^{-8} at atmospheric pressure. Here the breakdown voltage changes only by $\sim 10\%$. This was the reason that for a long time the theory being discussed was considered applicable to breakdown both at low and high pressures, since it gave approximately correct values of the breakdown voltages. However, it has been found more recently that at pressures of the order of 1 atm the breakdown voltage does not depend on the cathode material, and at the present time the criterion of a self-maintaining discharge in the form of (1.24) is considered applicable only to breakdown of a gas at reduced pressure.

We note that expression (1.23) is quite crude and simplified and cannot be applied to the transition stage of electron avalanche development, which represents breakdown, since when the condition (1.24) is satisfied the current goes to infinity.

Further investigations showed that electrons can be knocked out of the cathode not only by positive ions, but also by photons and by atoms or molecules excited to resonance or metastable states and diffusing to the cathode.[85,86] Phelps[87] attempted to solve the problem of current rise for helium with inclusion of additional cathode processes, but his analysis was hindered by absence in the literature of data on the values of the constants needed.

We shall now obtain the condition for a self-maintaining discharge[26] in which photoionization is the seconary mechanism and processes at the cathode are ineffective, which usually occurs[12] for $Nd \gtrsim 10^{19}$ cm^{-2}.

Let there be two infinite plane parallel electrodes a distance d apart and let there be an initial electron current i_0 from the cathode which we will later let approach zero. The z axis we will choose along the direction of the applied field E.

Then the change of the electron current in a layer dz will be equal to the change resulting from ionization of the gas by electron impact and from photoionization of the gas.

Thus, for the established discharge we have the equation (with the previous designations)

$$\frac{di(z)}{dz} = \alpha i(z) + \lambda \int_0^d K|z - z'|\, i(z')\, dz'; \qquad (1.25)$$

here $K|z - z'|$ is obtained from (1.18) by averaging over angles.

The boundary condition for this equation is written as follows:

$$i(0) = i_0. \quad (1.26)$$

Consequently, the problem of finding the established current in this case reduces to solution of the integro-differential equation (1.25) with the boundary condition (1.26).

The parameter λ in this case plays the role of the characteristic value of the integro-differential equation.

If the discharge becomes self-maintaining, then the integro-differential equation (1.25) for some characteristic value of λ can have a finite solution different from zero even for $i_0 = 0$, i.e., with zero current from the cathode.

The relation between the constants λ, α, and d and also the constants characterizing $K|z - z'|$ for the condition of existence of a nontrivial solution (1.25) with boundary condition $i_0 = 0$ obviously will be the condition of a self-maintaining discharge.[1)]

Corresponding calculations, which were carried out in an earlier article,[26] give the following expression:

$$\frac{\alpha^* T}{T+\tau}\frac{e^{\alpha d}}{3\sqrt{\pi \varkappa_0 d}\,\alpha^2 d} = 1. \quad (1.27)$$

The factor $T/(T+\tau)$ in Eq. (1.27) is the probability that an excited molecule will radiate a photon from the head of the avalanche without having taken part in reaction (1.3). Here τ is the lifetime of the excited state with respect to the transition to the ground state. The system of equations (1.27) and (1.5) replaces the system (1.24) and (1.1) in Townsend's theory. This system permits calculation of the gap breakdown voltage $V = Ed$ as a function of N and d. From analysis of this system of equations we can conclude that in the discharge considered Paschen's law should not be followed, but the magnitude of the deviation from this law depends on the type of gas and is not necessarily large. We note that deviations from Paschen's law are also observed experimentally.[88-90] The theory presented here is valid if the cross section for associative ionization is sufficiently large. We have already pointed out that as yet there are no data in the literature on these cross sections for most gases. If it turns out that the cross section for associative ionization for some gas is small, and the secondary ionization processes associated with it are not effective, but in this gas a discharge with a short formation time is nevertheless observed, it will evidently be possible to explain this as follows: Resonance photons which deviate from the main frequency as the result of the finite width of the spectral line traverse a distance from the avalanche head to the cathode without absorption and eject secondary electrons from the cathode. It is easy to understand that the time of formation of a discharge with this secondary mechanism will also be determined by the time of motion of the electrons from the cathode to the anode.

The condition for a self-maintaining discharge in this case changes its form somewhat. It can now be written in the form[91]

$$\frac{2}{3}\frac{\lambda^*}{\alpha}\gamma_P\frac{e^{\alpha d}}{\sqrt{\pi \varkappa_0 d}} = 1, \quad (1.28)$$

where λ^* is the number of atoms excited to a resonance state, produced by an electron per unit pathlength; γ_P is the probability of the photoeffect.

If the contributions to the secondary processes from the associative ionization mechanism and the photoeffect are comparable, we can in principle write in addition a generalized condition for a self-maintaining discharge.[91]

The question of which of the conditions which we have written down for a self-maintaining discharge with a short formation time is preferable must be solved for each specific gas-cathode combination for the condition that the cross section for associative ionization is also known in the given gas.

If we analyze expressions (1.27) and (1.28), we can observe that they have a form similar to Townsend's criterion (1.24). The difference lies in the pre-exponential factors, in which are reflected the actual physical processes responsible for the appearance of secondary electrons in the discharge.

At atmospheric pressure and a discharge gap length of ~1 cm the pre-exponential factor in (1.27) has a value $\sim 10^{-5}$. This means, in accordance with what we have said above, that the breakdown voltage calculated from the system of equations (1.27) and (1.24) will be in good agreement with experiment. However, we have already pointed out that the breakdown voltage is not very sensitive to the value of the pre-exponential factor. Therefore agreement with experiment here is not the main criterion of suitability of the theory. In our opinion the main advantage of the relations obtained is that they permit rather simple explanation of the existence of a self-maintaining discharge with a short rise time, and calculation of its parameters.

The discharge considered here is called a dark discharge. A dark discharge is obtained when the field of the space charge of the electron avalanche can be neglected in comparison with the field applied to the discharge gap. We shall obtain a criterion for realization of a dark discharge. Let the gap length be d and the field applied to it E_0. In development of an electron avalanche a space charge arises and its magnitude can be found from the Poisson equation

$$\frac{dE}{dz} = 4\pi e N_e. \quad (1.29)$$

Since the space charge field of the discharge can be neglected in comparison with the applied field, we obtain the inequality

$$\frac{E_0}{d} \gg 4\pi e N_e. \quad (1.30)$$

The average energy of the electrons under the conditions of a dark discharge is less than the ionization energy of the atoms or molecules. On the other hand, it is clear that the work done by the electric field in a length d exceeds the ionization energy, i.e.,

$$eE_0 d \gg \varepsilon_i \gg kT_e. \quad (1.31)$$

From Eqs. (1.30) and (1.31) it follows that

$$d \ll \sqrt{\frac{kT_e}{4\pi e^2 N_e}} = r_D. \quad (1.32)$$

Consequently, a dark discharge is realized when the length of the discharge gap is much less than the Debye radius of the gas discharge plasma.

2. THE AVALANCHE-STREAMER TRANSITION

a) Effect of Space Charge. Initiation of Anode and Cathode Streamers

Up to this time we have neglected the effect of space charge on the development of breakdown; however, as shown by experiment, in the transition to longer gaps at

[1)]The conditions $i(z) > 0$ and $di/dz \geqslant 0$ must also be satisfied; λ can be expressed in terms of α^* (see Eq. (1.5)).

pressures of the order of an atmosphere the space charge field begins to exert a substantial influence on the discharge and in some cases leads to appearance of a completely new form of discharge, so-called streamer breakdown of the gas. After analyzing a large amount of experimental data and also the difficulties of the Townsend theory for $Nd \gtrsim 10^{19}$ cm^{-2}, Loeb and Meek[11,12] proposed the following requirements for a new theory:

1) the breakdown mechanism should depend substantially on the electron motion; ions can be assumed stationary during the short breakdown time;

2) the discharge must begin with one electron and propagate along a narrow channel;

3) the discharge must depend on secondary processes in the gas volume and cannot be associated with processes at the cathode;

4) a correctly chosen mechanism of discharge development must be preferred at high pressures and can include processes due to space charge.

In accordance with these requirements, Loeb and Meek developed a streamer theory.[11,12] At the present time it is only of historical interest and we will not present it here, but it should be noted that, in spite of the fact that the theory of Loeb and Meek cannot be considered satisfactory in its quantitative aspects and also in some qualitative relations,[2)] its general representation of the transition of the avalanche stage to the streamer stage in a discharge is valid and can be considered experimentally proved. Subsequently a number of authors[92-100] have attempted to improve the theory of Loeb and Meek, but no particular success has been achieved in this direction. Generally speaking, an exact solution of the problem of transition of an avalanche to a streamer must contain a solution of the system of equations (1.7) and (1.8) in combination with the Poisson equation

$$\operatorname{div} \mathbf{E}' = 4\pi e (N_e - N_i) \tag{2.1}$$

and with relations expressing the dependence of the coefficients of the equations on the resultant field $\mathbf{E} = \mathbf{E}_0 + \mathbf{E}'$. The combination of these equations determines in principle the pattern of the entire process from appearance of the initial electron near the cathode to the avalanche-streamer transition and the subsequent propagation of the cathode and anode streamers. The moment of the immediate avalanche-streamer transition is determined by the appearance of some region of space inside which the electric field strength turns out to be much less than outside. It is clear from general considerations that this region will approximately have the form of a sphere. Unfortunately, solution of this problem is not yet possible even by computer. As a result, papers exist in the literature in which attempts are made to solve the one-dimensional problem[101-103] of the avalanche-streamer transition. It must, however, be noted that although solution of the problem in a one-dimensional formulation permits some useful information to be obtained, the main parameters of the avalanche-streamer transition can be obtained only on solution of the three-dimensional problem. The furthest advance in this direction has been made by Firsov,[104] who created an approximate three-dimensional model of an avalanche-streamer transition.

Before turning to a brief exposition of the main results obtained on the basis of this model, it should be noted that the experiments[105,69] have shown a significant increase in the effective diameter of an avalanche in comparison with the diffusion diameter when the number of charge carriers exceeds 10^5–10^6. The observed expansion can be explained in principle by ionization of the gas by radiation, which produces a plasma more efficiently near the head of the avalanche, and also by electrostatic repulsion of the electron cloud of the head. However, we have shown above in Chap. 2 that in development of an avalanche $r_P \ll r_D$. In regard to the electrostatic repulsion, it can be estimated in the following way.[77]

The rate of diffusion expansion of an avalanche is

$$v_D = \frac{dr_D}{dl} \sim \frac{r_D b E_0}{z}. \tag{2.2}$$

The rate of expansion of an avalanche under the influence of space charge can be estimated from the formula

$$v_{E''} = bE''; \tag{2.3}$$

here b is the electron mobility and E'' is the space charge field of the electron avalanche, which can be assumed approximately spherically symmetric.

From Eqs. (2.2) and (2.3) it follows that for

$$\frac{E''}{E_0} \sim \frac{r_D}{z} \sim \sqrt{\frac{l}{z}} \tag{2.4}$$

these velocities are equal. Here l is the electron mean free path. In air at atmospheric pressure $l \approx 3 \times 10^{-5}$ cm. Consequently, for a gap length ~1 cm even for a space charge field of the order 1% of the external field E_0 the expansion of the avalanche as the result of electrostatic repulsion of the electrons becomes greater than the diffusion expansion, and therefore beyond that point diffusion can be neglected. Experiments[64] confirm the estimates made above.

Let us consider now a sphere of radius r described around the center of the electron avalanche. Let r increase in such a way that dr/dt coincides with the radial velocity of the electrons in the coordinate system in which the center of the avalanche is at rest. The average radial velocity of the electrons is due to the action of the electric field of the charge q contained inside the sphere, and to the diffusion flow. The latter can be neglected, according to the discussion above, if $E''/E_0 \gtrsim \sqrt{l/z}$. Then we have

$$\frac{dr}{dt} = bE'' = b\frac{q}{r^2} = \frac{be \exp(\alpha z)}{r^2}. \tag{2.5}$$

Integrating Eq. (2.5) with allowance for the fact that $z = bE_0 t$, we obtain

$$r = \left(\frac{3e}{\alpha E_0}\right)^{1/3} \exp\left(\frac{\alpha z}{3}\right), \tag{2.6}$$

i.e., in this case the avalanche radius increases exponentially with time.

The radial field produced by the avalanche electrons can be written in the form

$$E''_r = E''_R \frac{r}{R}. \tag{2.7}$$

It has a maximum value "at the edge" of the sphere for r = R. Here, as follows from Eq. (2.6), the ratio of this field to the external field will be

2)Loeb and Meek incorrectly identify the breakdown criterion and the criterion for streamer formation for $Nd \geqslant 10^{19}$ cm^{-2}. As was shown above, breakdown under these conditions can occur even without a streamer. A detailed critical analysis of the theory of Loeb and Meek is contained in Ref. 91.

$$\frac{E''}{E_0}=\frac{e\exp(\alpha x)}{R^2E_0}=\frac{\alpha R}{3}. \qquad (2.8)$$

However, the avalanche radius cannot grow forever according to Eq. (2.6), especially because in derivation of these formulas we have neglected the field produced by the space charge of the positive ions left to themselves by the moving avalanche. This can be done as long as $R \ll 1/2\alpha$. If $R \approx 1/2\alpha$, then already about half of all the ions are within the boundary of the avalanche, and the charge of the ions decreases the charge and consequently also the field of the avalanche by at least a factor of two. With further increase of R the charge of the avalanche should stop growing and soon the avalanche radius itself should practically stop growing.

For $R = 1/2\alpha$ it follows from Eq. (2.8) that the space charge field of the electrons of the avalanche reaches $E_0/6$. In this case it is already necessary to take into account the dependence of the ionization coefficient α on the field strength.[91]

For this purpose it is necessary to take into account that the electrons of the avalanche, in addition to the action of the uniform field E_0, experience the action of the spherically symmetric field E'' produced by the space charge of the electrons and the field E' produced by the space charge of the positive ions left to themselves by the avalanche on its motion to the anode.

If for convenience we imagine the space charge as two spheres, one of which is charged negatively (on the anode side) and the other positively (on the cathode side), it is easy to deduce that the resulting field is stronger than the external field near the surface of the negative sphere turned toward the anode, and that of the positive sphere turned toward the cathode, and weaker than the external field near the surface of the spheres where they are turned toward each other.

Thus, inside the avalanche the field is weakened. However, in spite of the fact that the ionization coefficient α decreases greatly with decrease of the field, it is still not apparent that on the average the electrons will ionize less, since α as a function of E has a greater value than the second derivative $d^2\alpha/dE^2$ in the region of E values corresponding to breakdown. The change in the number of electrons in the avalanche per unit time can be determined from the formula

$$\frac{dn_e}{dt}=\int N_e\alpha u\,d\tau=\int N_e\alpha u_0\frac{E}{E_0}\,d\tau; \qquad (2.9)$$

here $d\tau$ is the element of volume and the integration is carried out over the entire avalanche; $\mathbf{E} = \mathbf{E}_0 + \mathbf{E}' + \mathbf{E}''$ is the resultant field. To evaluate the change in ionization in the avalanche we can assume that

$$N_e=\begin{cases}\frac{\exp(\alpha x)}{4\pi R^3/3}=\frac{\alpha E_0}{4\pi} & \text{for } r\leqslant R,\\ 0 & \text{for } r>R.\end{cases} \qquad (2.10)$$

Here we have utilized Eq. (2.8). It follows from the relations (2.10) that, to a first approximation, the concentration of electrons remains constant during development of an avalanche.

The electric field which is created by the positive ions in the avalanche can be considered approximately uniform and directed opposite to the external field E_0, since all of the ions are for the most part behind the avalanche. If the number of electrons in the avalanche is n_e and its diameter is 2R, then on movement of the avalanche toward the anode the ionization of the gas by electrons will leave behind the avalanche a track of positive ions whose diameter is of the order of 2R and in which the number of ions per unit length in the part of the track directly adjacent to the avalanche is of the order αn_e. With increasing distance from the avalanche in the direction to the cathode the density of ions falls off exponentially and we can assume that the field produced by the ions in the avalanche is approximately

$$E'=\frac{\alpha e n_e}{R}. \qquad (2.11)$$

If we now choose the origin at the center of the head of the avalanche and designate by θ the angle between the vectors **r** and $\mathbf{E}_0$, the expression for the resultant field E acting on the electrons of the avalanche can be written in the form

$$E=\sqrt{(E_0-E')^2+(E'')^2-2(E_0-E')E''\cos\theta.} \qquad (2.12)$$

Substituting Eqs. (2.12) and (1.6) into (2.9) and integrating over the volume of the avalanche, we can define an average ionization coefficient $\bar{\alpha}$, where

$$\bar{\alpha}=\frac{1}{u_0 n_e}\frac{dn_e}{dt}. \qquad (2.13)$$

Up to this point the discussion has been of a general nature. To obtain quantitative estimates it is necessary to choose a definite gas and a definite pressure. For air at atmospheric pressure the appropriate calculations were done in Ref. 91, where the following approximate expression was obtained for the change in ionization in the avalanche:

$$\frac{\bar{\alpha}-\alpha_0}{\alpha_0}=\left[-2.8+2.6\frac{B}{E_0}-0.1\left(\frac{B}{E_0}\right)^2\right]\left(\frac{E''}{E_0}\right)^2; \qquad (2.14)$$

here B is the coefficient in Eq. (1.6), which for air at atmospheric pressure is about 200 kV/cm over a wide range of field strength, including the breakdown strength.

The expression in square brackets in Eq. (2.14) reaches its maximum values at $B/E_0 = 13$ and 19.7. In air at atmospheric pressure this corresponds to $E_0 \approx 15$ kV/cm.

Thus, at a field strength $E_0 \sim 30$ kV/cm corresponding to the ordinary spark breakdown in air at atmospheric pressure, when $E''/E_0 = 1/6$ and $R = 1/2\alpha_0$, the "true" value of α according to Eq. (2.11) is decreased by about a factor of two. However, for $E''/E_0 = 1.6$ the deviation from Eq. (2.6) is still small, since the rate of increase of R is determined by the already created charge of the avalanche, and the principal part of it and consequently an appreciable change in α are encountered only in the last segment of the avalanche path of the order $\sim 1/\alpha_0$.

However, with further development of the avalanche, when $E''/E_0 \gtrsim 1/6$, the intensity of ionization in the front part of the avalanche in this case is already more than three times greater than that in the rear part. A significant decrease in the ionization coefficient inside the avalanche sets in, and the avalanche radius increases much more slowly. It should practically stop growing as a result of the fact that α is decreasing and the ions occupy almost the same volume as the electrons, since $R \approx 1/2\alpha_0$. However, it now generally becomes impossible to distinguish the electron avalanche from the ion track.

Actually, the electrons which are moving from behind are in a significantly weakened electric field as a result of the rapidly rising space charge of the electrons located in front and of the positive ions located behind. These electrons practically stop ionizing and come to rest. Electrons which are moving forward are in an electric field strengthened by the space charge of the rear electrons; they move rapidly than in the field E_0 and ionize the gas very strongly, leaving behind them continuously new masses of newly created electrons and ions and forming a conducting quasineutral plasma. The electrons in this plasma should smooth out the potential gradient in it in so far as it changes on motion of the forward ions.

Thus, the avalanche goes over into a positive (anode) streamer.

In order to explain the rapid propagation of the cathode streamer, it is necessary to assume the existence of rather intense photoionizing radiation which produces secondary electrons near the streamer. These electrons, moving in a strong electric field, near the positive end from short but intense avalanches which flow into the streamer and leave behind them a large positive charge which extends the end of the streamer to the cathode. Naturally, between this positive charge and the cathode end of the streamer, electron motion and ionization will occur, until the field between this end and the positive charge becomes rather small. Each such avalanche produces radiation necessary to form the following avalanches by photoionization of the gas. For sufficiently intense photoionization of the gas, the rate of formation of positive charge by the electron avalanches produced by photoionization will obviously be determined by the rate of motion of these avalanches in the field of the streamer. Consequently, the velocity of propagation of the cathode end should be approximately the same as the velocity of its anode end. However, with insufficient photoionization this velocity can be somewhat smaller, and for sufficiently strong ionization by photons having a long mean free path in the gas the velocity can be somewhat larger, since in this case there can be formed colliding streamers, which then fuse. Thus, everything that has been said regarding propagation of the anode streamer is valid to a substantial degree also with respect to the propagation of the cathode streamer. There is, however, a certain difference. Departure of electrons from the anode end occurs in large quantity continuously and is determined by the presence of a quite definite distribution of field strength. This field is maximal on the streamer axis, which coincides in direction with the lines of force of the applied field. Therefore the propagation of the anode streamer should occur more or less strictly along the field lines, i.e., in a uniform electric field it should occur in a straight line.

Propagation of the cathode end occurs by the growth of new electron avalanches in the strong field of the cathode end, produced by a small number of initial electrons. The place of formation and the density of these electrons are to some degree statistical. The motion of avalanches formed by secondary electrons does not occur along the lines of force of the external field, but is determined more by the stronger field of the streamer. Therefore, although electron avalanches produced in the path where the lines of force of the external field and the field of the streamer coincide are developed more efficiently, as a result of the statistical nature of the production of secondary electrons a curving of the path of the positive end is possible, and sometimes even looping or sharp bends in the streamer. However, as a rule, only curving of the streamer path occurs.

Thus, what we call a streamer is a thread-shaped conducting channel along which the field strength is small in comparison with the field strength outside this channel at a sufficiently large distance from it. Naturally, this formation is due to space charge or, more accurately, approximately line-shaped free charges of different signs. Sometimes this channel can terminate in one of the electrodes. In this case it can serve as the extension of a needle-shaped electrode; then there will be no charge of the other sign at the point of junction with the electrode, and the entire streamer will be charged with one sign. The field produced by the streamer will be determined by the distribution of charge in it and by the charge of the reverse sign of its electrical image in the electrode. If the streamer is produced in the middle of the gap far from the electrodes, the image fields can be neglected.

Since the streamer channel, in view of the equations of electrostatics, is unavoidably associated with strong fields radial with respect to the channel, this formation cannot exist stably with time. The streamer must break up, especially at the ends and, as we have already noted, especially at the cathode end. Since the strongest field exists at the ends of the streamer at a distance of its radius of curvature R, then on lengthening by a radius the streamer on the average deviates by an amount θR from the field line, where $\theta \sim 1$. The total deviation of the streamer $\overline{\Delta x^2}$ from motion along a field line is the statistical sum of these random deviations. Consequently, in a length L it amounts to $(\theta R \sqrt{L/\theta R})^2$, i.e.,

$$\overline{\Delta x^2} = \theta L R. \qquad (2.15)$$

We will discuss streamer stability more in detail in the next chapter.

The electrons, being in the weak field inside the streamer, gradually lose energy and recombine until practically only the excess charge remains. At the same time the molecules excited during formation of the streamer radiate. Therefore the streamer is always highly luminous.

On the other hand, in photographs we may mistake for a streamer the track of an intense electron avalanche, which is not a streamer in the sense indicated.

In contrast to the criterion for the occurrence of a dark or Townsend discharge (1.32), transition of an avalanche to a streamer occurs when the Debye radius of the plasma becomes much less than the size of the avalanche.

Specifically, if we use Eqs. (2.6) and (2.10) and also assume that in the transition of an avalanche to a streamer the radius of the avalanche is $R \approx 1/2\alpha_0$, we obtain for the ratio of the Debye radius to the avalanche diameter the expression

$$\frac{r_D}{2R} = \sqrt{\frac{\alpha k T_e}{e E_0}}. \qquad (2.16)$$

If we substitute into this the appropriate parameter values for air[12]: $\alpha \approx 20\ \mathrm{cm}^{-1}$, $d \sim 1$ cm, $E_0 = 3 \times 10^4$ V/cm = 100 CGS units, $kT_e \approx 3.6$ eV $\approx 6 \times 10^{-12}$ erg, we obtain

$$\frac{r_D}{2R} \approx 4 \cdot 10^{-2} \ll 1. \qquad (2.17)$$

In regard to the quantity αz_{cr}, where z_{cr} is the distance traveled by the avalanche before its transition to a streamer, it follows from Eqs. (2.6) and (2.10) that the resulting space charge field becomes comparable with the applied breakdown field for a centimeter gap when $\alpha \approx 20\ \text{cm}^{-1}$. Consequently, in this case

$$\alpha z_{cr} \approx 20, \tag{2.18}$$

which is in agreement with the experimental value.[64]

The value of αz_{cr} changes only slightly with increase of the gap length.

In concluding our discussion of the theory of the avalanche-streamer transition, we should note that the physical picture of the processes occurring in transition of an avalanche to a streamer has been studied quite well. At the same time, the mathematical aspect of the theory has been inadequately developed. As we have seen above, all of the main parameters of the avalanche-streamer transition can still be determined only in order of magnitude. Because of the importance of knowing these parameters for the development of streamer theory, attempts are being made at the present time to construct a rigorous theory of the avalanche-streamer transition. The difficulties in this task are mainly of a theoretical nature and there is hope that this problem will soon be solved by computer.

b) Breakdown of Long Spark Gaps

In concluding this chapter we shall dwell briefly on the theoretically little studied question of the breakdown of long spark gaps.

As has been shown, for $\alpha z \sim 20$ the electric field produced by the space charge of an avalanche of electrons and positive ions is such that in the head of the avalanche of size $\sim 1/\alpha$ the resultant field is equal to zero, while in front of and behind the head of the avalanche the field increases sharply; however, the total number of electrons increases significantly more slowly than exponentially. This decrease in the number of electrons automatically leads to a decrease in the intensity of photoionization. Therefore if prior to formation of a streamer the condition for a self-maintaining discharge is not satisfied for the entire gap, then on formation of a streamer, when $\alpha z \gtrsim 20$, it will never be satisfied. Firsov[104] has suggested that a discharge can occur in this case if the strengthening of the field and the increase of α associated with it compensates the decrease in the intensity of photoionization, i.e., the equivalent of a self-maintaining discharge arises in the region of enhanced field.

In this connection let us estimate the electric field which is produced by the positive ions left by the first avalanche of electrons after the electrons have been drawn to the anode. Here the positive ions are located mainly in a region near the anode of extent $\sim 1/\alpha_0$ along the field and $\sim 1/\alpha_0$ in the transverse direction. With increasing distance from the anode the density of ions falls approximately exponentially, as $\exp \alpha (d - z)$.

If we do not take into account the change in α, the maximum of the field produced by the ions and directed identically with the applied field should be at a distance of the order $\sim 1/\alpha_0$ from the anode.[91] The total charge of the part of the ion track of extent $\sim 1/\alpha_0$ adjacent to the anode will be $\sim e^- \exp(\alpha_0 d)(1 - 1/e)$, while the field produced by this charge will be $e^- \exp(\alpha_0 d)(1 - 1/e) 4\alpha_0^2$ (here e^- is the electronic charge and $e = 2.718\ldots$).

Hence it is necessary to subtract the field produced by the charge $e^- \exp(\alpha_0 d)/e$ of the remaining part of the ion track, which will be $\sim (4e^- \alpha^2/e)\exp(\alpha_0 d)$, and, finally, it is necessary to subtract the field of the electrical image of the charges in the anode, which is approximately equal to $e^-(\alpha_0^2/2)\exp(\alpha_0 d)$. As a result one obtains (these relations are derived in Ref. 91)

$$E'' = 4\alpha_0^2 e^- \exp(\alpha_0 d)\left(1 - \frac{2}{e} - \frac{1}{8}\right) \approx \frac{\alpha_0^2 e^- \exp(\alpha_0 d)}{2}. \tag{2.19}$$

Since $\alpha_0 d$ is large, $(\alpha_0 d \sim 20)$, the size of the region of enhanced field is $\sim 1/\alpha_0 \ll d$. In order for a self-maintaining discharge to arise in this region, the ionization coefficient α must be much greater than α_0. This is possible only in the case when the additional field turns out to be of the order of the applied field. However, it follows from Eq. (2.19) that for $R = 1/2\alpha_0$ we have

$$E' = \frac{e^- \exp(\alpha_0 d)}{8R^2};$$

in this case the quantity $e^- \exp(\alpha_0 d)/R^2$, as follows from (2.8), is $E_0/6$. Consequently,

$$E' = \frac{1}{48} E_0. \tag{2.20}$$

In order for the additional field to become of the order of the applied field, it is necessary, if we do not take into account the decrease of α, that the avalanche traverse a further length $\sim 4/\alpha_0$. However, as was shown in Section a), in this distance the avalanche ionizes several times more weakly (in air 2–3 times) as a result of the fact that it is moving in a field weakened by the charge of the ions, subsequently going over to a streamer. Thus, equating $E' = E_0$, we must make a correction to Eq. (2.19). It is necessary to replace $\exp(\alpha_0 d)$ by $\exp \int_0^d \bar{\alpha} dz$, where $\bar{\alpha}$ is defined by Eq. (2.13).

Since $\bar{\alpha}$ changes substantially only in the last pathlength Δz, in which the charge of the avalanche should increase by about 48 times, in this pathlength we have for the average value

$$\bar{\alpha}\,\Delta z \approx \ln 48 \approx 4. \tag{2.21}$$

We can now write

$$\int_0^d \bar{\alpha}\, dz = \int_0^{d-\Delta z} \alpha_0\, dz + \int_{d-\Delta z}^{d} \bar{\alpha}\, dz = \alpha_0 d - C, \tag{2.22}$$

where $C = \Delta z(\alpha_0 - \alpha)$.

It should also be noted that as a consequence of the growth of the avalanche into a streamer a fraction of the electrons is not absorbed by the anode and therefore the additional field produced by space charge will be somewhat less. This decrease, and also the numerical factor 1/2 in Eq. (2.19) have an extremely insignificant effect on the value of the breakdown voltage. Therefore, if we take into account the approximate nature of this derivation, it does not make sense to complicate the equations. It is sufficient to include the change produced by these factors in the uncertainty of the quantity C. Thus, the breakdown condition, which consists in equality of the space charge field to the external field, takes the form

$$e\alpha_0^2 \exp(\alpha_0 d - C) = E_0. \tag{2.23}$$

For purposes of calculation it is more convenient to give the expression for the value of d corresponding to breakdown at a given field E_0. We have

$$d = \frac{1}{\alpha_0}\left(C + \ln\frac{E_0}{e\alpha_0^2}\right). \qquad (2.24)$$

The breakdown voltage values calculated from Eq. (2.24) for air at atmospheric pressure (in this case the parameter C calculated by means of Eq. (2.14)[91] is (5 ± 1)) are in good agreement with the experimental data given by Meek and Craggs[12] for gap lengths $d \gtrsim 3$ cm. The maximum discrepancy does not exceed 4%. For $d \lesssim 3$ cm the theoretical values of the breakdown strengths are less than the experimental values. This is due to the fact that with such gap lengths in air at atmospheric pressure breakdown can occur without streamer formation and the breakdown voltage in this case must be calculated from the criterion (1.27). Specifically, for example, for $d = 1$ cm we have $E_0 = 31.6$ kV/cm and $\alpha = 17\ \text{cm}^{-1}$. In this case, as follows from Eq. (2.19), the additional field is

$$E' = 540\ \text{V/cm} = 1.6\%\ E_0.$$

It is clear that with this additional field a streamer is not formed and breakdown occurs in accordance with the mechanism which has been discussed in detail in Sec. b of Chap. 1. After analyzing Eq. (2.24), we can note that Paschen's law also is not satisfied here. Use of the criterion obtained for super-long sparks such as lightning is entirely inappropriate, since lightning is propagated in a nonuniform field whose geometry is unknown. In addition, the primary streamer which breaks down the gap between the cloud and the Earth, or as it is usually called, the leader, is propagated in individual steps with a time interval between steps of ~50 μsec. At the present time there is no satisfactory explanation of this stepwise leader propagation, which is apparently produced by nonlinear effects associated with the propagation of the ionization wave. Nevertheless, we shall estimate the length of the breakdown gap in air in a uniform field of strength 10 kV/cm. In such fields the empirical formula (1.6) is not accurate and we shall use the data of Saunders (see Ref. 11), which are more appropriate in this case, and which for atmospheric pressure give a value $\alpha_0 = 2 \times 10^{-3}\ \text{cm}^{-1}$. Substituting the parameter values chosen into Eq. (2.24), we find that $d \approx 2 \times 10^4$ m. This length corresponds to lightning. Of course, this estimate is quite crude, but it permits estimation of the order of magnitude of the field in a lightning discharge.

We note that a deficiency of the criterion (2.24) is the fact that it does not explicitly involve the photon absorption coefficient. The difficulty in obtaining a more general criterion is due to the difficulties already mentioned in solving Eq. (1.7) when its coefficients are not constant. Therefore, further progress in the theory of the breakdown of long gaps involves solution of this equation, and it will apparently be necessary to include in it additional terms describing recombination and electron attachment, since in long gaps the discharge time increases and it will no longer be possible to neglect these processes.

3. STREAMERS

a) Results of Experimental Studies in Streamer Chambers

It is well known that one of the principal deficiencies of spark chambers as detectors of elementary particles is the existence of an identified direction in the chamber[2,3]—the direction of the applied electric field E_0. This means that a spatial anisotropy is observed in the chamber properties such as ability to detect particles traveling at various angles to the electric field vector, difference in the nature of particle tracks, and so forth. Complete achievement of isotropy of the chamber properties in the presence of an identified direction is impossible, but some possibilities exist for improving the isotropy of these properties. This is achieved in the contemporary particle detector known as the streamer chamber, which was first suggested by Dolgoshein and Luchkov[4] and Mikhaĭlov et al.[5] and at the present time has come into extensive use.

At the moment when the electrons formed by an ionizing particle begin to move in the electric field, forming an avalanche, the isotropy of the chamber properties is already destroyed, since the avalanche has a nonspherical shape—its dimension along the electric field is greater than the transverse dimension. However, this anisotropy is still not very great, and it can be reduced practically to zero if the avalanche is short and also if it is taken into account that the brightest part, recorded on a photographic film, is the head of the avalanche. Therefore the detection of particle tracks at the streamer stage provides the greatest possibilities for isotropy of the chamber properties. Unfortunately, in this case the conditions for photography of the weakly luminous streamers (in contrast to bright sparks) deteriorate somewhat.

In order to stop the development of the streamer at a length of several millimeters, it is necessary to shape the duration of the high voltage pulse with an accuracy of ~1 nsec. The most convenient means for supplying voltage to a chamber of small size is the Arkad'ev-Mark generator, together with a special shaping element—a cutoff spark gap.[4,106]

The main deficiency of the streamer chamber—the weak brightness of the track, which makes photography difficult—is removed by the following means: increase of the streamer length,[4,107] increase of the electric field strength,[108,109] by means of an image amplifier,[110] and by introduction of various hydrocarbon additives.[111-112]

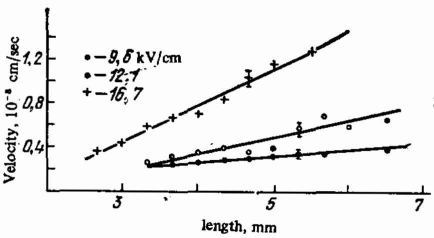
The streamer chamber, in addition to its main purpose—detection of elementary particles, is a unique instrument for study of the properties of streamers. The results of measurements of the main parameters of streamers in streamer chambers have to a large extent made possible a clarification of the picture of the physical processes occurring in the development of a streamer. In Figs. 2–6 we have shown the results of several experiments on measurement of the lengths, diameters, and velocities of streamers, and also their brightness.

The main conclusions which can be drawn from the graphs shown are as follows:

1) The streamer velocity is a linear function of its length and the external field strength (Fig. 2);[113,116]

2) on achievement of some critical length or critical external field strength an acceleration of the streamer is observed (Fig. 3);[114,116]

3) the streamer diameter increases approximately in proportion to the square root of its length (Fig. 4);



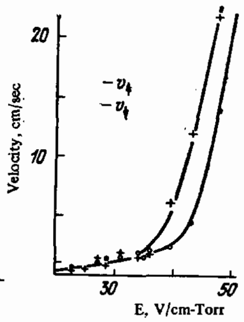
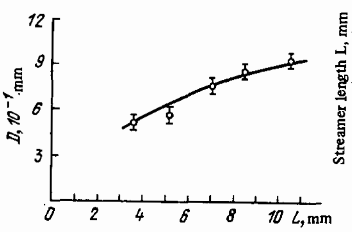


FIG. 2

FIG. 3

FIG. 2. Dependence of streamer velocity on its length for different electric field strengths [113].

FIG. 3. Dependence of streamer velocity on E/P [114].



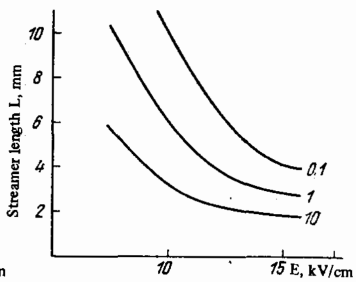


FIG. 4

FIG. 5

FIG. 4. Streamer diameter as a function of its length [115].

FIG. 5. Streamer brightness as a function of electric field and streamer length [115]. The curves 0.1 and 10 correspond to brightnesses a factor of ten smaller and larger than for curve 1.

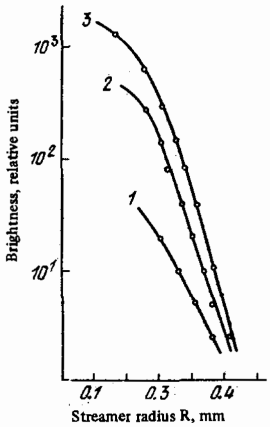


FIG. 6. Distribution of brightness over the radius of the streamer for different streamer lengths [115]. L=2mm for curve 1, 3.5mm for curve 2, and 5.5mm for curve 3.

4) the brightness of a streamer increases rapidly with its length, and the field strength and falls off with radial distance from its axis (Figs. 5–6).[115]

It is possible to explain almost all of these regularities in terms of the model developed in Ref. 23, which we will now discuss.

b) Mathematical Formulation of the Problem of Streamer Motion

We shall present the qualitative discussions of Sec. b of Chap. 2 in mathematical form.

Assume that at an initial time $t = 0$ in a discharge gap to which a field E_0 is applied, at the origin we have some region of highly conducting quasineutral plasma, i.e., the head of an avalanche at the moment of the avalanche-streamer transition. Then the field in the surrounding space is distorted and can be determined by solution of Laplace's equation for the potential,

$$\Delta\Phi(\mathbf{r}, t) = 0. \tag{3.1}$$

Since the plasma existing in the head of the avalanche has a high conductivity, its surface can be considered an equipotential. Thus, the boundary condition for Eq. (3.1) has the form

$$\Phi(\tilde{\mathbf{r}}, t) = 0, \tag{3.2}$$

where $\tilde{\mathbf{r}}$ is the radius vector of the points of the boundary of the region.

On advance of the plasma to the anode and cathode, its boundary remains as before an equipotential. Consequently, we have an additional boundary condition:

$$\left.\frac{d\Phi}{dt}\right|_{r=\tilde{r}} = 0. \tag{3.3}$$

The condition (3.3) can be rewritten in a somewhat different form. We have

$$\frac{d\Phi}{dt} = \frac{\partial\Phi}{\partial t} + \mathbf{v}\nabla\Phi = 0.$$

Since

$$\mathbf{v} = b\mathbf{E} = \pm b\nabla\Phi, \tag{3.4}$$

where the + sign is chosen for the anode end of the plasma and the − sign for the cathode end, and the mobility is practically independent of the field E under breakdown conditions over a wide range of variation of the field, we can write that at the plasma boundary the following condition is satisfied:

$$\frac{\partial\Phi}{\partial t} \pm b(\nabla\Phi)^2 = 0. \tag{3.5}$$

At a large distance from the plasma the potential should go over to the potential of a uniform field E_0, i.e.,

$$\Phi|_{r\to\infty} \to -E_0 z. \tag{3.6}$$

The initial condition depends on the shape of the plasma region at the moment of the avalanche-streamer transition.

If we assume that this region is a sphere of radius R_0, then by solving the problem of the potential distribution in a uniform field in which a quasimetallic sphere is placed, we can obtain[117]

$$\Phi(r, 0) = -E_0 r\cos\theta\cdot\left(1 - \frac{R_0^3}{r^3}\right), \tag{3.7}$$

where θ is the angle between the radius vector of a given point and the vector E_0. In actual fact, the shape of the avalanche head obviously differs somewhat from spherical, but its true shape and, consequently, a more accurate initial condition can be obtained only on the basis of an exact solution of the problem of the avalanche-streamer transition.

The problem (3.1)–(3.7) completely determines the development of the anode and cathode streamers from the time of the avalanche-streamer transition.

The exact solution of this problem presents great mathematical difficulties, but the main parameters of the streamer can be obtained without an exact solution. The point is that the main role in development of the streamer is played by its anode and cathode ends, where the region of enhenced field is concentrated. This permits a substantial simplification of the problem.

Specifically, by expanding the potential Φ in series near the anode or cathode end of the plasma region boundary and retaining terms to third order, we can obtain two model-independent expressions for the radius of curvature of the streamer near its ends[23]:

$$R = \frac{2\Phi_z}{\Phi_{zz}}, \tag{3.8}$$

$$\frac{dR}{da} = 2\left(\frac{\Phi_z \Phi_{zzz}}{\Phi_{zz}^2} - 2\right); \tag{3.9}$$

here a is half the length of the streamer and the derivatives Φ_z, Φ_{zz}, and Φ_{zzz} are taken at the end points of the streamer. For a further solution we must know the values of these derivatives, i.e., we must have a definite model of the streamer shape. The criterion of suitability of the model chosen will be agreement with experiment of the parameters calculated on the basis of this model.

If we analyze most streamer photographs, we can note that the shape of the streamer surface recalls to a first approximation an ellipsoid of revolution drawn out along the external field direction. In Ref. 22 we made an appropriate calculation and showed that near the ends of the streamer its surface differs very little from an ellipsoid of revolution. Therefore we shall take as a model of a streamer an ellipsoid of revolution with a major semiaxes a (along the external field E_0) and a focal distance f.

The problem of the potential distribution around a quasimetallic ellipsoid placed in a uniform field E_0 has been solved exactly,[23] and therefore we can obtain values of Φ_z, Φ_{zz}, and Φ_{zzz} at the point $z = a$, $\rho = 0$.

Substituting these values into Eq. (3.9), we obtain

$$\frac{dR}{da} = 0. \tag{3.10}$$

It follows from this that

$$R = \text{const} = R_0. \tag{3.11}$$

Consequently, for the model chosen the radius of curvature of the streamer surface near its ends remains constant during the development of the streamer and equal to the initial radius R_0 of the sphere at the moment of the avalanche-streamer transition.

This result permits us to obtain immediately analytic expressions for the velocity, length, and width of the streamer in the course of its development.

Specifically, for $a \gg R_0$ the streamer velocity is determined by the relation

$$v = |b\Phi_z| = bE_0 \frac{a/R_0}{\ln[(2/e)\sqrt{a/R_0}]}; \tag{3.12}$$

here e = 2.718...

The effective streamer diameter L in this model coincides with the minor axis of the ellipsoid of revolution, and therefore we can write

$$L = 2\sqrt{a^2 - f^2} = 2\sqrt{aR_0}. \tag{3.13}$$

Thus, on the basis of the model of a streamer in the form of an ellipsoid of revolution drawn out along the external field direction, we have obtained the following relations:

1) the velocities of the cathode and anode streamers are identical and increase linearly with the external field E_0 and approximately linearly with the length;

2) the brightness of the streamer increases with its length, since the energy expended per unit volume is $\sim E^2$, and E increases approximately linearly with the streamer length;

3) the brightness of the streamer falls off along the radius, since the field at the surface of the streamer near its end is equal to the field at the tip multiplied by the cosine of the angle between the z axis and the normal to the surface[22];

4) the streamer width increases as the square root of its length.

Consequently, in spite of the simplicity of the model, the results obtained are in good qualitative agreement with experiment. The experimental observation[114,116] of a break in the linear dependence v(a) at certain critical values of a and E_0 finds no explanation in this model, since such a break is evidently due to the appearance of the streamer surface instability we mentioned above. Quantitative comparison with experiment at the present time is difficult, since the most important parameter of the streamer, which enters into all of the formulas, is the initial radius R_0 which is so far known only in order of magnitude.

In addition, this model requires some quantitative improvement, for the following reason.

From the initial condition (3.7) it follows that in the plane $z = 0$ the field is equal to zero, and since $v = bE$, all points of this plane remain stationary during the streamer development, i.e., a "neck" is formed. Succeeding cross sections of the streamer surface during its development will therefore have a form somewhat different from an ellipsoid of revolution (Fig. 7). The effect of the neck on development of the streamer, as has been shown in Ref. 24, does not change the qualitative relations obtained.

c) Investigation of Stability of the Streamer Surface

Let us assume, in the first approximation, that the streamer surface is a plane conductor moving in the direction of the z axis with a velocity $v = bE_0$. In this case the potential distribution will be given by the formula

$$\Phi = -E_0(z - vt). \tag{3.14}$$

If a wave with frequency ω and wave number k is propagated along the streamer surface, the vertical displacement of the points of the streamer surface will be

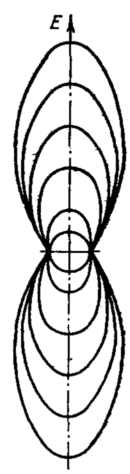


FIG. 7. Cross section of streamer surface [22].

$$\xi(x,t)=\xi_0 e^{i(kx-\omega t)}. \quad (3.15)$$

Using standard methods of studying stability,[91] it can be shown that for the problems (3.1)—(3.7)

$$\omega = ibkE_0 = ikv. \quad (3.16)$$

Since the imaginary part of ω turns out to be positive, it follows from Eq. (3.15) that the oscillations arising will grow without limit, i.e., the streamer surface turns out to be unstable, the characteristic time for development of an instability with wavelength $\lambda \sim 1/k$ being

$$t_I \sim \frac{1}{\omega} \sim \frac{\lambda}{v}. \quad (3.17)$$

If we substitute here for purposes of estimation of a λ of the order of the radius of curvature of the streamer surface, which has a value $\sim 10^{-1}$—10^{-2} cm, and a value $v \sim 10^7$ cm/sec, it turns out that $t_I \sim 10^{-8}$—10^{-9} sec, while the development of the streamer occupies $\sim 10^{-7}$ sec, i.e., it is one or two orders of magnitude greater.

Nevertheless, experiments show that streamers in their initial stage develop as stable formations and only later begin to curve and loop. Therefore we must assume that for some reason the instability is suppressed in the initial stage of streamer development.

In Ref. 22 we suggested that the stability of the streamer with time is apparently due to the finite conductivity of the plasma inside it and correspondingly to the finite thickness of the surface charge. In fact, the growth of a thinner streamer from the head of the main streamer is accompanied by a rapid increase in the current density, while the conductivity to a first approximation remains constant.

Since the field E' inside the streamer is approximately given by

$$E' = \frac{j}{\sigma}, \quad (3.18)$$

where σ is the conductivity of the plasma and j is the current density, this leads to an increase of the field inside the streamer, which in turn leads to a drop in the field strength at the head of the new thin streamer and will prevent its further development. When the thickness of the surface charge becomes much less than the radius of curvature of the streamer, it will become unstable against loops and bends. Subsequently, in Ref. 25, these qualitative discussions were put into mathematical form.

In Refs. 25 and 118—121 the problem of streamer development as an ionization wave, similar to Eqs. (1.7)—(1.8), has been solved, but in a one-dimensional formulation. The one-dimensional formulation is quite adequate for study of streamer stability. This investigation was carried out[25] by the method used by Barenblatt and Zeldovich[122] in the problem of stability of a flame front. Allowance for the finite thickness of the front, which has a magnitude $\sim\sqrt{D/\alpha b E_0}$, leads to the following relation, instead of Eq. (3.16):

$$\omega = -iDk^2. \quad (3.19)$$

In this case the front turns out to be stable with respect to infinitely small perturbations. The physical meaning of (3.19) is that the instability is suppressed by diffusion of electrons from the ionized region.

A similar picture exists in the problem of flame front stability. As Landau[123] has shown, a flame, considered as the surface of an explosion, is unstable with an increment kv.

At the same time, allowance for the finite width of the front by Barenblatt, Zel'dovich, and Istratov[124] showed that as a result of the thermal conductivity (neglecting diffusion of the fuel) the front is stable with respect to infinitely small perturbations. The two approaches discussed do not give solutions which go over to each other smoothly in the limit when the wavelength is greater than the width of the front. As A. A. Vedenov has pointed out, this evidently means that the approximation of an infinitely thin front corresponds to investigation of perturbations whose amplitude is large in comparison with the thickness of the front.

As a result it may turn out that in the initial stage when the width of the front is large the streamer is stable with respect to infinitely small perturbations of the front. In the later stage when the front becomes thin, it is unstable with respect to perturbations larger than the width of the front.

CONCLUSION

As follows from the material discussed above, the theory of a discharge in dense gases has been significantly advanced at the present time. It permits calculation of breakdown strengths and also of the main parameters of electron avalanches and streamers. At the same time many important unsolved problems remain. We shall enumerate the principle of these: the theory of the avalanche-streamer transition; the theory of breakdown in inhomogeneous fields, in particular, the theory of the corona discharge in a constant field; the theory of the step-like leader, and also of the subsequent phases of a lightning discharge, except for the theory of the spark channel, which has been developed quite well by Drabkina[125] and Braginskiĭ.[126]

In conclusion the author expresses his sincere indebtedness to B. M. Smirnov and O. B. Firsov for helpful discussions which have permitted the author to set forth more clearly the content of this review.

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Translated by Clark S. Robinson