Helical instability in electron-hole plasma in semiconductors

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A review is made of the experimental and theoretical investigations of the helical instability in semiconductors, beginning from the discovery of this effect by Ivanov and Ryvkin in 1958 and ending with papers published in 1973. A detailed analysis is made of the excitation of helical waves under oscillation and spatial amplification conditions, nonlinear effects in the excitation of the helical instability, possibility of using semiconductors for modeling processes occurring in gas plasmas, and influence of the band structure of semiconductors on the development of the helical instability. Special attention is paid to possible applications of the helical instability in semoconductors and to unsolved problems. Attentions is drawn to the need for further studies of the nonlinear effects which accompany the development of the helical instability and the effects due to the complex band structure of semiconductors.

CONTENTS

1. INTRODUCTION

The physics of plasma phenomena in solids is currently one of the leading subjects in solid-state physics. The subject has been reviewed on several occasions,[1] prominent investigators meet regularly at conferences, and one might have the impression that solid-state plasma is a new subject. However, the most interesting discoveries leading to this type of plasma were made in the late fifties and the early sixties. They were the foundation of this subject. This was clearly due to the rapid development of the physics of plasma of gaseous discharges which provided a conceptual base for the understanding of the properties of plasma-like media. Moreover, it was not accidental that the investigations in the physics of plasma in gas discharges and of solid-state plasma subsequently remained closely related.

Among the most important events in the history of the physics of solid-state plasma was the discovery of the helical instability,[2-4] which began intensive and systematic studies of instabilities in solid-state plasma. Over one hundred experimental and theoretical investigations of various aspects of the helical instability in semiconductors have been published so far and it seems to us that the moment is ripe to summarize the results of the most important investigations and outline the directions of future studies. This is the purpose of the present review. However, we must begin by considering briefly the main properties of an electron-hole plasma which will be used later in the review.

An electron-hole plasma is formed by electrons in the conduction band and holes in the valence band. Such a semiconductor plasma may be neutral or charged. An equilibrium neutral plasma exists only in very pure semiconductors at sufficiently low temperatures. In this case, the plasma density is governed entirely by the lattice temperature and band-structure parameters (intrinsic conduction), and semiconductors of this type are called intrinsic.

A nonequilibrium neutral plasma can be generated by carrier injection, impact ionization, or illumination. In this case, the plasma density is governed by the applied electric field or the intensity of the incident light.

An example of a charged plasma is an electron or hole plasma in a semiconductor that contains suitable impurities. In this case, one speaks of extrinsic n- and p-type conduction, respectively. Naturally, a crystal as a whole is electrically neutral because the ion matrix of impurity centers in a semiconductor compensates the plasma charge. When the impurity concentration is low, one can go over from extrinsic to intrinsic conduction by raising the temperature, i.e., by altering the ratio of the electron and hole densities.

A unipolar plasma composed of carriers of one sign may consist of several groups of carriers differing in respect of the mobility along a given direction. This feature is due to the many-valley structure of the conduction (or valence) band in semiconductors such as germanium, silicon, etc. For example, in silicon (Si), the constant-energy surfaces near the bottom of the conduction band are six ellipsoids of revolution oriented in pairs along three mutually perpendicular axes (⟨100⟩, ⟨010⟩, ⟨001⟩). The anisotropy is then quite strong: the ratio of the mobilities corresponding to the major and minor ellipsoid axes is $b_\perp/b_\parallel = 5$. Under normal conditions, when there are no external perturbations or when they are weak, all the valleys are equivalent (they are uniformly populated with electrons). At high temperatures (T = 77-300°K), the high rate of intervalley transitions ($\tau_i \approx 10^{-10}$-10^{-12} sec, where τ_i is the intervalley transition time) makes it possible to introduce a scalar mobility for all electrons and ignore the band structure. Rapid intervalley transitions destroy the differences between electrons belonging to different valleys. When the valley populations are nonuniform (for example, under uniaxial compression or in strong electric fields) and also at very low temperatures, when τ_i is high, the mobility anisotropy influences strongly the nature of plasma phenomena in semiconductors and the many-valley nature of the real band structure has to be allowed for in calculations.

The state of a sample's surface, usually represented by a phenomenological parameter which is the surface recombination velocity s, has a strong influence on the nature of plasma phenomena occurring in semiconductors. When the surface recombination velocity is high ("dirty" surface) so that $G_s = D_a/as \to 0$ (D_a is the ambipolar diffusion coefficient and a is the transverse size of a sample), the spatial distribution of a strongly nonequilibrium plasma drops steeply near the surface. The profile of the nonequilibrium carrier density becomes smoother, when the parameter G_s increases because the surface becomes "cleaner." Therefore, the state of the surface may have a strong influence on the development of drift instabilities, whose growth increments depend strongly on the initial plasma density gradient. One of the phenomena affected in this way is the helical instability.

We shall now consider this instability.

2. IVANOV AND RYVKIN'S DISCOVERY

In 1958, Ivanov and Ryvkin[2] reported oscillations of the current in long thin samples ($1 \times 1 \times 8$ mm) of n-type germanium (Ge) subjected to a fairly strong magnetic field (~10 kOe) and a parallel electric field. These oscillations were nearly sinusoidal (f = 10-15 kHz) only when the magnetic field and the current were parallel to within $\leqslant 10°$. The current-voltage characteristics of these samples were nonlinear, which indicated that an electron-hole plasma was injected into the sample. Etching in hydrogen peroxide, which reduced the surface recombination velocity, favored the appearance of these oscillations. A similar instability of the current was observed later[5] in samples of n-type indium antimonide (InSb) under impact ionization conditions. An important study of this unusual effect in samples of Ge, InSb, and Si was carried out by Larrabee and Steele,[6] who showed that the instability was not due to contact phenomena but was the result of the presence of an electron-hole plasma in the bulk of a sample and that this plasma could be generated by illumination, injection, or heating. The frequency of the oscillations observed in n-type InSb reached $\sim 10^7$ Hz. The critical magnetic field in which the instability was observed increased with decreasing electric field $H_{cr} \propto E^{-1}$. Etching in hydrogen peroxide (and other solutions) reduced the instability (oscillation) threshold governed by the value of EH. In some cases, the amplitude of the alternating component of the current was about 70% of the static current and the oscillations were nearly sinusoidal. This was why Larrabee and Steele suggested the name oscillistors for the samples in which the Ivanov-Ryvkin instability was observed. This name became generally accepted and the effect discovered by Ivanov and Ryvkin became known as the oscillistor effect. The experiments mentioned above were carried out in weak magnetic fields corresponding to $y = \omega_c\tau \ll 1$, where ω_c is the cyclotron frequency and τ is the carrier momentum relaxation time.

3. KADOMTSEV-NEDOSPASOV HELICAL INSTABILITY. PRINCIPAL RESULTS OF GLICKSMAN'S THEORY

The experimental material accumulated up to 1960 on the oscillistor effect was sufficient for a theoretical analysis. A theoretician well acquainted with the physics of semiconductors and gas-discharge plasmas found that the situation cleared up considerably when Kadomtsev and Nedospasov[3] showed convincingly in 1960 that the instability of a weakly ionized gas-discharge plasma, observed in a longitudinal magnetic field by Lehnert[7] in 1958, was due to what was later called the helical instability. It was sufficient to examine carefully the initial equations of the Kadomtsev-Nedospasov theory to see the great similarity between the transport equations for an electron-hole plasma and the corresponding equations for a weakly ionized gas-discharge plasma. In 1961, an American researcher, Glicksman,[4] explained the oscillistor effect using the Kadomtsev-Nedospasov helical instability theory. The mechanism of the appearance of this instability can be explained[8] with the aid of Fig. 1. Let us assume that quasineutral helical perturbations of density $n' = n_1(x) \exp(i\omega t - ik_z z - ik_y y)$ appear in an electron-hole plasma subjected to longitudinal electric and magnetic fields. The static electric field E_z shifts the electron perturbation of the density relative to the hole perturbation at a velocity $v_z^0 = -(b_e + b_h)E_z$, where $b_{e,h}$ are the electron and hole mobilities. When the electron and hole perturbations are coincident in space, this shift is equivalent to the rotation of the electron distribution n' relative to the hole distribution at an angular velocity of $k_z v_z^0$. This results in a charge separation and produces fields $E'_\perp$ which oppose this separation. In a longitudinal magnetic field and in fields $E'_\perp$, the perturbations n' may drift to a sample's surface. The question arises as to under what conditions does this drift result in the growth of the initial perturbation, i.e., when it produces an instability. We shall assume that, under steady-state conditions, the plasma density decreases near the surface. If a plasma layer ABCD (skew wave) inclined with respect to the Oz axis is displaced toward the surface (Fig. 1), the conductivity and current density in the relevant part of the crystal increase compared with the surrounding region. As a result, charges appear on the boundary surfaces of the ABCD layer and these charges oppose the increase in current. Such a charges produce a field $E''_\perp$. When the polarization of helical perturbations, governed by the signs of k_y and k_z and depending on the relative orientation of longitudinal electric and magnetic fields, is appropriate, the plasma drift in this field ($E''_\perp$) and in a longitudinal magnetic field is directed along the initial displacement in the field $E'_\perp$. If the intensities of the electric and magnetic fields are sufficiently high, the drift flux may exceed the diffusion flux and an instability may appear. A similar instability mechanism of helical waves also applies in a homogeneous plasma.[9] In the latter case, the instability is due to a helical surface wave excited by the steep fall of the plasma density near the surface of a sample. Such a wave is naturally excited if the surface recombination velocity is low. The amplitude of this wave grows exponentially toward the surface. For convenience in later considerations, we shall now introduce fairly arbitrary concepts of bulk and surface oscillistors (bulk and surface helical waves). In the case of a bulk oscillistor, the excitation of a helical instability is due to a steady-state plasma density gradient. A strong bulk oscillistor effect corresponds to a helical wave in a sample with a "dirty" surface and a high nonequilibrium plasma density. The limiting case of a surface oscillistor corresponds to a helical wave in an intrinsic semiconductor when the plasma distribution is uniform across the sample. Essentially, these concepts reflect the limiting approximations used

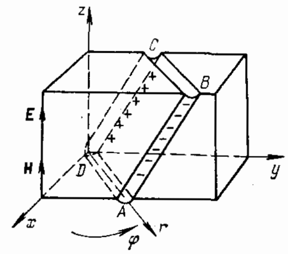

FIG. 1. Mechanism of excitation of the helical instability. [8]

in the helical instability theory. Naturally, when the problem is solved exactly and the dispersion equation is derived allowing correctly for the boundary conditions and plasma inhomogeneity, concepts of this kind do not arise. However, in view of the great mathematical difficulties encountered in the rigorous solution of the problem, it is usual to consider these limiting variants which are frequently close to experimental situations. In particular, the Kadomtsev-Nedospasov theory corresponds to a bulk helical wave (in the plasma of a positive column in a gas discharge, the density drops quite strongly near the surface of the discharge tube). Glicksman[4] derived the dispersion equation describing the excitation of an oscillistor. He applied the Kadomtsev-Nedospasov calculation method to a strongly nonequilibrium quasineutral (n = p) plasma in a semiconductor. In this case, one uses linearized (with respect to small perturbations) equations of motion and continuity for electron and holes

$$\left.\begin{aligned} &\mathbf{v}_e = -b_e\mathbf{E} - \frac{D_e}{n}\nabla\mathbf{n} - \frac{b_e}{c}[\mathbf{v}_e\mathbf{H}],\\ &\mathbf{v}_h = b_h\mathbf{E} - \frac{D_h}{n}\nabla\mathbf{n} + \frac{b_h}{c}[\mathbf{v}_h\mathbf{H}],\\ &\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}_e) = \gamma n, \quad \frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}_h) = \gamma n, \end{aligned}\right\} \qquad (1)^*$$

where γ is the nonequilibrium-carrier generation coefficient. The potential($\mathbf{E}' = -\nabla\varphi'$), quasineutral ($n^1 = p^1$) perturbations of the type $A' = A_1(r)\exp(-i\omega t + ikz + im\varphi)$, are considered and for $|m| = 1$ these perturbations correspond to helical perturbations in cylindrical geometry. The inertial terms in the equations of motion are ignored since $\omega\tau \ll 1$. It is assumed that $n \ll n_0$ and $n \ll p_0$, where n_0 and p_0 are the equilibrium electron and hole densities, and the equilibrium plasma background is ignored in the calculations. It is also assumed that the diffusion length $L_D = \sqrt{D_a\tau_p}$, where τ_p is the carrier lifetime, exceeds the transverse size of the sample ($L_D > a$) so that the bulk recombination can be ignored. Under steady-state conditions, the density distribution is described by a zeroth-order Bessel function: $n = N_0 J_0(\beta_0 r)$, where

$$\beta_0^2 = \frac{\gamma}{bD_h + D_e}[1 + y_e^2 + b(1 + y_h^2)], \quad b = \frac{b_e}{b_h}.$$

The constant β_0 is found from the condition

$$J_0(\beta_0 a) = \delta, \qquad (2)$$

where the parameter $\delta = n(a)/n(0)$ governs the degree of "cleanness" of a sample surface. The parameter δ can be expressed directly in terms of the surface recombination velocity,[10] if use is made of the boundary condition corresponding to the equality of the ambipolar diffusion flux and the surface recombination flux on the surface of a sample:

$$G_s x = \frac{J_0(x)}{J_1(x)}, \qquad (3)$$

where $x = \beta_0 a$ and $G_s = D_a/as$.

In the case of a "dirty" surface ($G_s \to 0$), we have $J_0(x) \to 0$ and $\delta \to 0$, whereas, in the case of a "clean" surface ($G_s \to \infty$), we have $x \to 0$ and $\delta \to 1$.

In a positive column in a gas discharge, we have $\delta \to 0$ and $(\beta_0 a)$ is governed by the first root of the function J_0.

The initial equations for small perturbations can be reduced to a system of two linear second-order differential equations with coefficients variable because of the spatial inhomogeneity of the initial distribution, and these equations describe the amplitudes of the perturbations of the density $n_1(r)$ and potential $\varphi_1(r)$. It is not possible to solve these equations because the density and ambipolar potential profiles are quite complex. Therefore, in deriving the dispersion equations, Glicksman used the approximate Kadomtsev-Nedospasov method, which was one of the modifications of the Galerkin method.[11] In this case, the amplitudes of the perturbations $n_1(r)$ and $\varphi_1(r)$ are specified by a known function which should satisfy certain requirements that follow from the symmetry of the problem, the boundary conditions, the nature of the steady-state solutions, and the type of instability. In the Glicksman and Kadomtsev-Nedospasov theories, the steady-state density profile is governed by the function J_0. In the case of a positive column in a gas discharge, this function always vanishes on the surface of the discharge tube, whereas, in the case of a semiconductor, the density on the surface may vary within wide limits which are governed by the surface recombination velocity. Allowance for possible density distributions and for $n_1(r)$ and $\varphi_1(r)$ is the only additional complication in the Glicksman theory compared with the Kadomtsev-Nedospasov theory. However, the theoretical analysis of a helical instability in a semiconductor is simplified considerably in the Glicksman theory because the electric and magnetic field in a semiconductor are independent parameters. In a positive column in a gas discharge, the longitudinal electric field in the plasma is a function of the magnetic field and the final stability criteria must be obtained using the energy balance equation, which complicates the problem considerably.

The profiles $n_1(r)$ and $\varphi_1(r)$ used in [3,4] are chosen in the form $(\hat{n}_1, \hat{\varphi}_1) J_1(\beta r)$, where $\hat{n}_1$ and $\hat{\varphi}_1$ are constants. The parameter β is selected in such a way that the steady-state density profile and the functions n_1 and φ_1 vanish at the same point. This selection of the radial profile of the perturbed quantities has some justification. The Kadomtsev-Nedospasov instability is of the drift type so that the amplitudes are $n_1 \propto \varphi_1 \propto \nabla n \propto J_1$. On the other hand, the perturbations should be small in the region of vanishingly small densities. A rigorous mathematical justification of the Kadomtsev-Nedospasov method was given by Johnson and Jerde.[12]

Multiplying the initial equations by the function $J_1(\beta r)$ and integrating over the whole section of a sample, we obtain a system of two algebraic equations (for the constants $\hat{n}_1$ and $\hat{\varphi}_1$) and then we find the dispersion equation by equating the determinant of the system to zero. This equation is easily solved for ω (the initial equations are of the first order in respect of time!) and the conditions $\mathrm{Im}\,\omega = 0$, $d(\mathrm{Im}\,\omega)/dk = 0$ allow us to find the minimum excitation threshold of he-

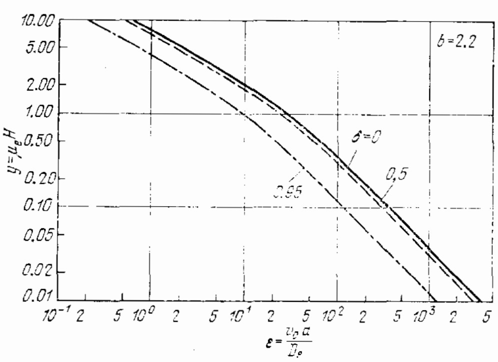

FIG. 2. Electric-field dependences of the lowest magnetic field in which the helical instability is excited, plotted for different values $\delta = n(a)/n(0)$; [4] $v_0 = b_e E$; $y_e = (b_e/c)H$; $b = b_e/b_h$.

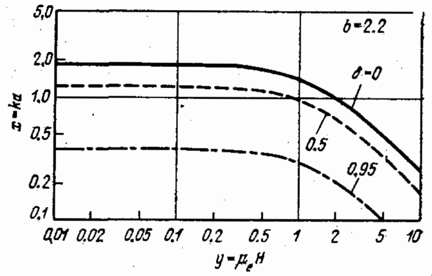

FIG. 3. Magnetic-field dependences of the wave vector corresponding to the minimum excitation threshold of the helical instability, plotted for different values of δ. [4]

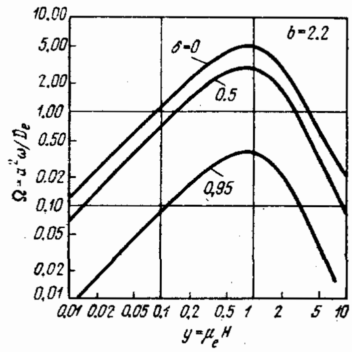

FIG. 4. Magnetic-field dependences of the oscillation frequency (Re ω) at the excitation threshold of the helical instability.

lical waves (Im $\omega \gg 0$) and their wave vector **k**. The value of Re ω governs the oscillation frequency. Figure 2 shows the electric-field dependences of the lowest (threshold) value of the magnetic field in which the oscillistor effect is still excited, plotted for different values of the parameter δ. It is clear from Fig. 2 that the excitation threshold H_{th} rises with decreasing electric field. This dependence can easily be explained by the following simple considerations. If the magnetic field is weak ($y_{e,h} \ll 1$), the main role in the suppression of the instability is played by the transverse diffusion flux which is then independent of the magnetic field. The instability-causing drift flux is proportional to kEH and at the excitation threshold we have $H \propto E^{-1}$. In strong magnetic fields ($y_i \gg 1$), the dispersal of helical perturbations occurs not only due to the transverse diffusion (proportional to H^{-2}) but also to the longitudinal diffusion which depends strongly on the perturbation wavelength (wave vector **k**). Since the drift flux is proportional to kE/H (in the $y_i \gg 1$ case), the softest excitation conditions correspond to the longest perturbation wavelengths and at the excitation threshold we again have $H \propto E^{-1}$. Naturally, in the case of a "clean" surface ($\delta \to 1$), the role of the transverse diffusion becomes weaker and the excitation threshold decreases (Fig. 2).

The magnetic-field dependences of the wave vector for which the excitation threshold of the helical instability is lowest are plotted in Fig. 3 for different values of δ.

The existence of an optimal perturbation wavelength along the applied field is due to the fact that, at high values of λ, the instability-causing drift flux is small, whereas, at low values of the λ (high values of k), the longitudinal diffusion flux predominates. In the case of a "clean" surface, the role of the longitudinal diffusion becomes stronger and, therefore, the wavelength increases with increasing δ. It should be pointed out that all these results are only valid for very long samples. It is shown in [3,4] that, in the case of short discharge tubes or short semiconductors, when the wavelength λ becomes comparable with the length of the tube or sample, the criteria for the excitation of the helical instability may become more stringent. The spatial structure of the wave changes correspondingly. Similar dependences on the length of the sample appear in the case of semiconductors subjected to strong magnetic fields ($y_i \gg 1$). We shall return to this problem later.

It must be stressed that the excitation criterion of an oscillistor (criterion of the absolute instability of helical waves) corresponds to the condition Im $\omega > 0$ only if there is no ambipolar drift of quasineutral perturbations along the electric field. This drift appears for unequal background electron and hole densities ($n_0 \neq p_0$).[13] Naturally, in the presence of this drift, the absolute instability criterion becomes more stringent and we can no longer find this criterion using the condition Im $\omega > 0$ but we must investigate the evolution in time of a small perturbation limited in space.[14] The instability is absolute if a perturbation appearing at any point in space grows in the limit $t \to \infty$ to an unbounded value at the same point (naturally, this is true within the linear approximation) so long as the nonlinear effects do not limit the rise of the perturbation amplitude. In this case, the probes located along the axis of a sample can detect excitation of spontaneous oscillations at each point. In the presence of the drift described above, the absolute instability criterion can be found[14] using the change in a wave packet $U(t) = \int e^{i\omega(k)t}\,dk$ with time. If $U(t) \to \infty$ for $t \to \infty$, the true absolute instability occurs. As stressed earlier, in the case of samples with unequal background electron and hole densities, there is a drift of perturbations along the electric field at an ambipolar velocity $v_a = b_a E_z$, where

$$b_a = \frac{b_e b_h (n_0 - p_0)}{b_e n_0 + b_h p_0}$$

is the ambipolar mobility.

When the velocity of the drift is sufficiently high, a perturbation at each point in space remains finite in amplitude and the instability is convective. Spontaneous oscillations do not then appear and perturbations with a helical structure can only be amplified from point to point along the axis of the sample. The convective instability of helical waves appears in range of parameters corresponding to the criterion Im $\omega > 0$ when the true absolute instability criterion is not satisfied.

Figure 4 shows the magnetic-field dependence of the oscillation frequency (Re ω) near the oscillistor excitation threshold. In the case of a completely neutral plasma, the oscillation frequency is governed by the frequency of rotation of helical perturbations in a longitudinal magnetic field and in transverse electric fields of ambipolar origin (for equal electron and hole mobilities, the oscillation frequency is zero). Therefore, in weak fields ($y_i \ll 1$), we have Re $\omega \propto H$, whereas, in strong fields we have Re $\omega \propto H^{-1}$. In the case of a charged plasma, when $n_0 \neq f_0$, there is an additional rotation mechanism due to the ambipolar drift of quasineutral perturbations in an electric field, which is equivalent to the azimuthal rotation. The correction to the oscillation frequency due to such drift is $\omega' = kv_a$. If $n_0 > p_0$, the sign of this correction is opposite to the sign of the frequency in a quasineutral plasma. If the background is far from compensation, this correction may govern the sign of the frequency (direction of rotation of the perturbations).

4. GROWTH OF INVESTIGATIONS OF HELICAL INSTABILITY. SURFACE WAVES

The Glicksman theory results were in qualitative and even quantitative agreement with the experimental results available at the time. The results we have in mind are the threshold dependence $E \propto H^{-1}$ for very long samples and the dependence of the oscillistor excitation criterion on the degree of "cleanness" of the surface of a sample. The occasionally observed disagreement (particularly, in respect of the dependence of the oscillation frequency on the magnetic field) between the theory and experiment was more likely to be due to the uncontrolled nature of some of the experimental parameters such as the injection rate, uncompensated background ($n_0 \neq p_0$), and surface recombination velocity. Some of the difficulties encountered in comparison of the theory and experiment were resolved by Holter,[15] who considered a bulk helical wave for unequal background electron and hole densities and a variable injection rate η ($\eta = \Delta n/p_0 + bn_0$),[15] where Δn is the excess density on the axis of a sample.

Nevertheless, the success of the Glicksman theory which explained the principal dependences found by Ivanov and Ryvkin[2] was quite clear. Glicksman's paper was follwed immediately by new experiments. The interesting question was whether helical waves were also excited in oscillistors. This question was also underlying the gas-discharge investigations after the appearance of the paper by Kadomtsev and Nedospasov. In 1962, the American investigators, Paulikas and Pyle,[16] recorded the time dependences of the radiation emitted by a positive column in a helium plasma under instability conditions and showed convincingly that the structure of perturbations which occurred in this column was indeed helical. A suitable selection of the discharge parameters made it possible to stop the motion of the helix (Re ω = 0) and examine it visually.[16b] In 1962, two groups of Japanese researchers[17,18] showed independently and by different methods that the structure of the density and potential perturbations in oscillistor experiments was helical. The experiments were carried out on n-type Ge under injection conditions. Misawa and Yamada[17] made phase measurements using the reflected microwave signal method, whereas Okamoto, Koike, and Tosima[18] measured the phase shifts between two pairs of probes located in the same azimuthal positions and shifted in phase by $\pi/2$. The frequency of a signal picked up from either probe pair was equal to the oscillation frequency of the current in the external circuit. Oscillation of the transverse voltage in the sample also appeared for an exactly parallel orientation of the electric and magnetic fields, whereas oscillations of the current in the external circuit appeared only when there was a slight deviation ($\sim 0.5°$) from the parallel orientation, similar to that found in the experiments of Larrabee and Steele.[6] The direction of rotation of a helical wave was found to be opposite to that predicted theoretically,[4] but this "contradiction" could be easily removed—as pointed out by Okamoto et al.[18] —by allowing for the correction to the frequency due to the ambipolar drift.

Thus, in four years since the discovery of the oscillistor effect by Ivanov and Ryvkin[2] it finally became clear that the effect was due to the excitation of the helical instability. The oscillistor oscillations in the external circuit could not be explained by the linear theory[3,4] utilizing the azimuthal symmetric form of perturbations because the alternating component of the total current should be zero. The appearance of oscillations in the external circuit was evidently due to nonlinear effects[9] or various deviations from the azimuthal symmetry[18,19] appearing, for example, for a small angle between the directions of the electric and magnetic fields. There is as yet no generally accepted view on this subject.

The convincing establishment of the cause of the oscillistor oscillations engendered additional interest in this effect. Gurevich and Ioffe[20] developed a theory of the helical instability for an inhomogeneous distribution of the density established by a transverse magnetodensity effect[21] appearing because of a deviation of the electric from the magnetic field, inhomogeneous distribution of impurities, and similar causes.

One of the most interesting theoretical and experimental investigations of the helical instability in semiconductors was carried out by Hurwitz and McWhorter,[9] who investigated the excitation of a helical surface wave when the plasma was homogeneous under steady-state conditions (the case of an equilibrium plasma or a very clean surface of a sample). In contrast to the bulk case, the problem of the surface oscillistor effect can be solved quite rigorously. The initial equations for the potential perturbations $A' = A_1(r) \exp(-i\omega t + im\varphi + ikz)$ in weak magnetic fields ($y_i \ll 1$) are of the form

$$D_a \Delta n_1 + i(\omega - kb_a E) n_1 = 0, \quad \Delta\varphi_1 + A\Delta n_1 = 0, \tag{4}$$

and the solutions of these equations are

$$n_1(r) = c_1 I_m(\beta r), \quad \varphi_1(r) = Ac_1 I_m(\beta r) + c_2 I_m(kr); \tag{5}$$

here,

$$D_a = \frac{n_0 b_e D_h + p_0 b_h D_e}{n_0 b_e + p_0 b_h}$$

is the ambipolar diffusion coefficient; b_a is the ambipolar mobility; A and β are constants which depend on the plasma and wave parameters, and on the electric field intensity; I_m is a Bessel function with an imaginary argument.

In the absence of a steady-state density gradient, a longitudinal magnetic field occurs only in the boundary conditions which express the equality of the perturbed radial electron and hole fluxes on the surface of a sample to the surface recombination flux. If the surface recombination velocity is low ($G_s \to \infty$), these conditions are

$$\begin{aligned} n_0 b_e \left(\frac{d\varphi_1}{dr} - \frac{im}{r} y_e \varphi_1 \right) - D_e \left(\frac{dn_1}{dr} - \frac{im}{r} y_e n_1 \right) \Big|_{r=a} &= 0, \\ p_0 b_h \left(\frac{d\varphi_1}{dr} + \frac{im}{r} y_h \varphi_1 \right) + D_h \left(\frac{dn_1}{dr} + \frac{im}{r} y_h n_1 \right) \Big|_{r=a} &= 0. \end{aligned} \tag{6}$$

Substituting the solutions (5) into the boundary conditions (6) and eliminating the integration constants (c_1, c_2), we obtain the dispersion relationship.

We shall only give the final results. The criterion of the excitation of the helical instability ($|m| = 1$, Im $\omega > 0$) is of the form

$$HE \geqslant -\frac{6k_c D_a}{mb_M^2} c \qquad \left(k_c^2 = \frac{4}{3a^2} \right), \tag{7}$$

where

$$b_M^2 = \frac{n_0 p_0 b_e b_h (b_e + b_h)^2}{(n_0 b_e + p_0 b_h)^2}.$$

The oscillation frequency (Re ω) near the excitation threshold of an almost intrinsic sample ($|n_0 - p_0|/(n_0 + p_0) \ll y_{e,h}^2$) is given by the expression

$$f_c = \frac{\mathrm{Re}\,\omega}{2\pi} = \frac{20mD_a}{9\pi a^2}(b_e - b_h)\frac{H}{c}. \tag{8}$$

If we arbitrarily select the magnetic field direction as the positive axis ($H > 0$), we find that for $b_e > b_h$ the frequency is positive definite for $m > 0$ (or for $m < 0$ if $H < 0$).

Therefore, as is clear from the criterion (7), for $E > 0$ (or $E < 0$, if $H < 0$), if the fields E and H are parallel, right-handed helical waves are excited (k_c and m have different signs), but, if $E < 0$ (fields E and H antiparallel), left-handed waves are excited (k_c and m have the same sign). The corresponding constant-phase lines represent either right-handed rotation (changes in φ and z positive) or left-handed rotation (changes in φ and z of different sign).

It is clear from the condition (7) that, if $n_0/p_0 \to 0$ or ∞, the excitation threshold is $EH \to \infty$, i.e., the oscillistor effect cannot be excited in a strongly charged plasma. This is a natural result because a charged plasma is at rest in a coordinate system moving with the drift velocity of the unipolar component. In the case of extrinsic samples, ($|n_0 - p_0|/(n_0 + p_0) \gg y^2_{e,h}$)

$$f_c = \frac{k_c}{2\pi} b_a E, \tag{9}$$

i.e., the phase velocity of helical waves is equal to the velocity of ambipolar drift. The polarization of helical waves is then governed only by the relative orientations of the electric and magnetic fields and it is independent of the sign of the ambipolar mobility.

As pointed out earlier, in the presence of a drift of perturbations in a longitudinal electric field ($b_a \neq 0$), the absolute instability criterion governing the oscillistor excitation threshold does not agree with the condition $\mathrm{Im}\,\omega > 0$ [criterion (7)] and becomes more stringent, irrespective of the ambipolar drift direction. If we use the Landau and Lifshitz approach[14] mentioned earlier, we find that the true absolute instability criterion of helical waves is given by the simple relationship

$$\left(\frac{2}{3}\frac{b_M^2 EH}{c}\right)^2 \geqslant (b_a E)^2 + \left(\frac{8D_a}{a\sqrt{3}}\right)^2, \tag{10}$$

i.e., the square of the velocity of the instability-causing drift flux should be greater than the sum of the squares of the ambipolar velocity and ambipolar diffusion velocity. This form of the absolute instability criterion is quite natural in the presence of the ambipolar drift because this criterion should be independent of the sign of the ambipolar mobility.

If $b_a = 0$, the criterion (10) becomes identical with the criterion (7). We can easily show that the condition (10) is satisfied beginning only from a certain value of the magnetic field

$$H_{\min} > \frac{3|b_a|c}{2b_M^2}. \tag{11}$$

The oscillistor oscillations are not excited in weaker magnetic fields.

In the range

$$\left(\frac{8D_a}{a\sqrt{3}}\right)^2 < \left(\frac{2}{3}b_M^2\frac{EH}{c}\right)^2 < (b_a E)^2 + \left(\frac{8D_a}{a\sqrt{3}}\right)^2 \tag{12}$$

the convective instability of helical waves is observed, i.e., in this range of values of EH and of plasma parameters, it should be possible to obtain spatial amplification of waves of suitable polarization. If the frequency ω in the initial dispersion relationship is regarded as a real quantity, the wave vector is given by the following expression

$$-k = -\frac{\omega}{b_a E} + i\frac{8D_a}{3a^2 b_a E}\left[2\frac{f}{f_c}\frac{H}{H_c} - \left(\frac{f}{f_c}\right)^2 - 1\right]. \tag{13}$$

This expression is obtained subject to the following restrictions:

$$b_a E \gg \frac{D_a}{a}, \quad \frac{|n_0 - p_0|}{n_0 - p_0} \gg y^2_{e,h}, \quad (b_a E)^2 \gg \omega D_a. \tag{14}$$

The real part of Eq. (13) gives the phase velocity of a wave and the imaginary part of this equation gives the spatial gain (attenuation coefficient). The spatial amplification appears when

$$2\frac{H}{H_c} > \frac{f}{f_c} + \frac{f_c}{f}. \tag{15}$$

The gain maximum corresponds to the frequency $f_m = (H/H_c)f_c$, which increases linearly with the magnetic field. It is clear from Eq. (13) that the gain has a bell-shaped frequency dependence.

Hurwitz and McWhorter[9] investigated experimentally the surface oscillistor effect in cylindrical samples of n-type Ge and in samples of square cross section. The ratio of the electron and hole densities (n_0, p_0) could be varied by altering the temperature. The main results were obtained under spatial amplification conditions. The excitation of the initial signal with an angular dependence $\sin\varphi$ was achieved with four probes located at the same distances along the azimuth. The gain was measured using point probes soldered along a sample. Figure 5 shows the calculated and experimental dependences of the gain on the frequency (Fig. 5a) and magnetic field (Fig. 5b). The highest measured value of the gain was $\approx 4\ \mathrm{cm}^{-1}$, which corresponded to 35 dB/cm. Figure 6 shows the dependence of the critical frequency (9) on the electric field. An increase in the slope of the lines in Fig. 6 with decrease in temperature was due to an increase in the ambipolar drift mobility. Measurements of this kind could be used to determine the ambipolar drift mobility in semiconductors.

It is clear from Figs. 5 and 6 that the calculations[9] agreed excellently with the experimental results. The paper of Hurwitz and McWhorter had a strong influence on the subsequent investigations of the helical instability in semiconductors.

An interesting feature of the surface oscillistor effect in weak magnetic fields ($y_i \ll 1$) was observed by Balakirev[22] in n-type Ge. He found that, when the ambipolar mobility was sufficiently low, the threshold excitation frequency in the oscillistor effect tended to zero for some particular value of the magnetic field. This was explained by assuming that the ambipolar drift and rotation in a magnetic field corresponded to frequencies of different signs in n-type samples, so that a helix at rest was observed. This feature was found in nearly intrinsic n-type Ge (Fig. 7).

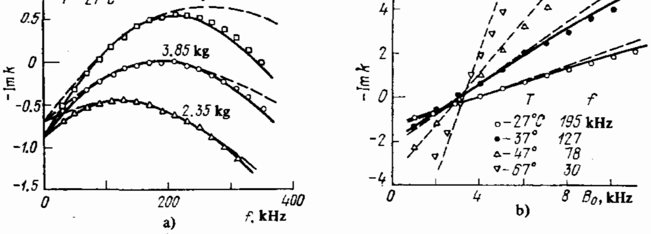

FIG. 5. a) Dependences of the spatial gain (attenuation coefficient) on the frequency of a helical wave obtained in different magnetic fields [continuous curves represent the exact calculations and dashed curves are the approximate results obtained using Eq. (13)]; [9] b) dependences of the gain on the magnetic field at different temperatures. [9]

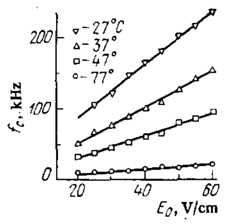

FIG. 6

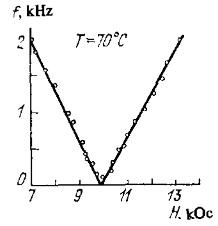

FIG. 7

FIG. 6. Dependences of the critical frequency f_c, corresponding to the onset of amplification of a helical wave, on the electric field E_0 at different temperature. [9]

FIG. 7. Magnetic-field dependences of the oscillistor frequency at the excitation threshold of n-type Ge with a nearly intrinsic conductivity. The symbols are the experimental results and the continuous curve is calculated. [22]

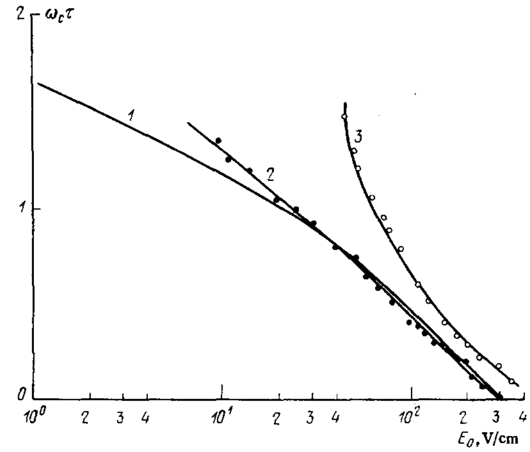

FIG. 8. Dependence of the electric field $E_0 = b_e Ea/D_e$ on the magnetic field $[\omega_c\tau = (b_e/c)H]$ at the excitation threshold of the helical instability: [33] 1) $L/a \to \infty$; [4] 2) $L/a = 30$; 3) $L/a = 6$.

Vladimirov and Shanskiĭ [10] carried out a theoretical and experimental study the influence of the surface recombination velocity on the oscillistor excitation criterion of a strongly nonequilibrium plasma in Ge. They found that the bulk oscillistor effect changed to the surface form and the excitation threshold fell strongly when the surface recombination velocity decreased. The calculated dependence of the excitation criterion on the surface recombination velocity was in good agreement with the experimental results. These results were used to suggest a new method for the determination of the surface recombination velocity in semiconductors. Pataki[23] studied the influence of the field effect on the excitation criterion and the oscillistor frequency of Ge. These oscillistor properties depended strongly on the direction and intensity of the electric field applied between the surface of the sample and an external electrode. Thus, a study of the influence of the field effect on the oscillistor oscillations gave information on the kinetics of surface recombination centers. Kuniya[24] studied the possibility of using the helical instability in semiconductors under convective instability conditions for delaying hf signals. The measurements were carried out on n-type Ge under injection conditions at room temperature. The input signal band was 30-200 kHz. The delay time was independent of the magnetic field and frequency and was given by $t = L/v_a$, where L is the sample length and v_a is the ambipolar drift velocity. Lautz and Schulz[25] investigated the helical instability in n-type Ge under convective and absolute instability conditions. Special attention was paid to the transition between these two types of instability. Schulz[26] developed a theory of a surface helical wave in an injected plasma for the case when a density inhomogeneity appeared along the electric field. Vikulin et al.[27,28] studied the influence of the injection rate on the excitation threshold of the oscillistor effect and obtained the dependence of the oscillistor frequency (at the excitation threshold) on the magnetic field. The results of these experiments were in agreement with the theoretical calculations.[4,15]

Several investigations were made of the dependences of the oscillistor characteristics on the angle between the electric and magnetic fields.[19,29] The excitation of the surface oscillistor effect in planar samples was considered theoretically by Volkov[30] and Gilinskiĭ.[31] The interesting problem of the resonance interaction between an acoustic wave and an oscillistor was discussed by Gilinskiĭ and Sultanov.[32] The helical instability in semiconductors was also studied by Gurevich and Ioffe.[20] A series of papers published in 1969 and 1970[33-38] reported theoretical and experimental studies of the helical instability in semiconductors subjected to strong magnetic fields ($y_i \gg 1$). Dubovoĭ and Shanskiĭ [33] carried out experiments on a plasma in Ge in magnetic fields up to 150 kOe. In contrast to weak magnetic fields ($y_i \ll 1$), when the threshold electric field decreased with increasing intensity of the magnetic field, the opposite tendency for the threshold electric field to increase was noted in strong magnetic fields (Fig. 8). This tendency appeared at higher magnetic fields in long samples. An absolute minimum of $E_{th}(T)$ in strong magnetic fields was found experimentally by Tsipivka at al.[34] and by Meĭlikhov.[35] This behavior of the threshold dependences was due to the finite longitudinal dimensions of the samples, which caused a spatial change in the wave structure in strong fields.[4] The wavelength increased[4] and became comparable with the length of the sample when the magnetic field intensity was increased. In this case, the spatial structure of the wave was governed entirely by the geometry of the sample and the longitudinal diffusion flux, which played the main role in the suppression of the helical instability in strong magnetic fields, ceased to depend on the magnetic field. Since the drift flux causing the instability was proportional to E/H, it was found that, in strong fields, $E_{th} \propto H$, i.e., the threshold electric field increased with increasing magnetic field intensity. Direct measurements of the dependence of the wavelength on the magnetic field in samples of finite dimensions were carried out by Dubovoĭ and Shanskiĭ[33] and confirmed these conclusions. A rigorous mathematical description of these effects, subject to the boundary conditions on the ends, was given by Vladimirov.[38] The problem was also discussed in [39,40]. One should also mention the work of Balakirev and Bogdanov[36] and of Uspenskiĭ[37] on the surface oscillistor effect in strong magnetic fields.

One of the most interesting phenomena associated with the helical instability in semiconductors was discovered by Glicksman and Steele[41] in 1959 in a study of the pinch effect in an impact-ionized electron-hole plasma in InSb. The pinch effect was the compression of the plasma by the intrinsic magnetic field of the current flowing through a sample.[42] The plasma was compressed toward the axis of the sample when the radial drift velocity of carriers in the azimuthal magnetic field of the current and in a longitudinal electric field exceeded the ambipolar diffusion velocity. The pinch effect in the electron-hole plasma in InSb was always accompanied[41,43] by an anomalous resistance due to enhancement of the electron-hole scattering and to the quadratic bulk recombination under strong compression.[44] The effect found by Glicksman and Steele was the disappearance of the pinch effect (disappearance of the anomalous resistance accompanying the pinch effect) in a longitudinal magnetic field comparable in intensity with the intrinsic magnetic field of the plasma filament current. The destruction of the pinch in a longitudinal magnetic field was later observed by other workers[43] and it was suggested that the effect was due to the development of a helical instability in the pinch channel. In 1967, Glicksman and Ando[45] determined the spatial structure of the perturbations appearing under these conditions and demonstrated experimentally that the destruction of the pinch effect in a longitudinal magnetic field was due to the development of a helical instability. This instability produced an anomalous diffusion flux toward the surface of the sample and the radial compression effect disappeared. It was interesting to note that such apparently unlike effects as those discovered practically simultaneously by Ivanov and Ryvkin[2] and Glicksman and Steele[41] were found to be closely related because it was the helical instability in the experiments of Glicksman and Steele that destroyed the pinch effect in a longitudinal magnetic field but this instability was not manifested explicitly. The theory of the helical instability under strong pinch effect conditions was developed by Vladimirov and Shchedrin[48] allowing for the intrinsic magnetic field of the current and for the complex spatial distribution of the plasma density in the pinch effect. It was found that, to excite helical instability under these conditions, it was necessary to ensure that the longitudinal magnetic field was twice as high as the maximum intrinsic magnetic field of the current. The longitudinal length of helical waves decreased with increasing current and this was due to the enhancement of the role of the transverse diffusion under strong compression conditions.

Investigations were made relatively recently of the influence of the energy band structure of semiconductors on the helical instability.[47,48] It was shown theoretically[47] that the helical instability could appear in unipolar semiconductors if electrons (or holes) were not all characterized by the same parameters but consisted of several groups with different mobilities in the direction of the electric field. As pointed out earlier, this situation appears in semiconductors with the many-valley band structure (Si, Ge, etc.) if the intervalley transition time is sufficiently long (if the diffusion length of the intervalley scattering $l_i = \sqrt{D_a \tau_i}$ is greater than the transverse dimensions of the sample and the characteristic oscillation frequency is higher than the frequency of the intervalley transitions $1/\tau_i$) so that electrons of different valleys participate independently in the transport processes. In unipolar semiconductors, the helical instability is always convective and, consequently, this instability can be observed only under amplification conditions. The numerical calculations reported in [47] were carried out for silicon. Bondar et al.[48] investigated experimentally and theoretically the influence of the intervalley redistribution of electrons caused by uniaxial compression (elongation) of a crystal on the criterion of the excitation of the helical instability in electron-hole plasma in silicon. It is worth recalling that the constant-energy surfaces of silicon near the bottom of the conduction band are described by six ellipsoids arranged in pairs along three mutually perpendicular axes (⟨100⟩, ⟨010⟩, ⟨001⟩). The compression or elongation of a crystal along one of these directions causes the greatest redistribution of the electrons between the valleys (it enhances or reduces the populations of the valleys located along the direction of deformation).[49] This redistribution is due to a relative change in the forbidden band width along the compression (elongation) axis. No redistribution of electrons takes place when the stress is applied along the ⟨111⟩ directions. In the case of a nonuniform valley population, the mobility anisotropy affects strongly the oscillistor characteristics even when the intervalley transition time is short. The calculations reported in [48] are carried out within the framework of the two-valley model; it is assumed that the electron gas consists of two ensembles. The electron mobilities in the first ensemble are $b_{\parallel}$ and $b_{\perp}$ along and at right-angles to the direction of compression (this ensemble is equivalent to two valleys oriented along the ⟨100⟩ axis). In the second ensemble, corresponding to four equivalent valleys oriented along the ⟨010⟩ and ⟨001⟩ axes, the electron mobility is assumed to be isotropic. The surface oscillistor effect is considered theoretically (the surface of the sample is assumed to be clean). It is shown that the intervalley redistribution of electrons increases the ambipolar mobility. Therefore, the drift of perturbations along the electric field becomes greater and the criterion of the absolute instability, governing the appearance of the oscillistor oscillations (during compression or elongation), becomes correspondingly more stringent. The experiments reported in [48] were carried out at liquid nitrogen temperature in order, on the one hand, to raise the carrier mobility and, on the other, to ensure a sufficient redistribution of electrons under slight compression. A low surface recombination velocity was ensured by etching the samples in a mixture of hydrofluoric and nitric acids. Figure 9 shows the dependences of the oscillistor excitation criterion of p-type silicon on the pressure applied at different rates of injection of nonequilibrium carriers (curves 1 and 2 are experimental and curves 3-5 are theoretical). Even a slight redistribution of ≈ 10% (P = 100 kgf/cm²) caused a strong rise in the threshold value of EH. When the injection rate (n/p) was increased, the pressure dependence of the excitation threshold became less steep due to a reduction in the ambipolar drift mobility. When silicon crystals were compressed along the ⟨111⟩ directions, the threshold fields were practically unaffected. The corresponding experiments carried out on Ge[50] under compression and elongation confirmed the main predictions of the theory (the oscillistor threshold increased). It is worth recalling that the constant-energy surfaces of Ge are four ellipsoids of revolution oriented along axes

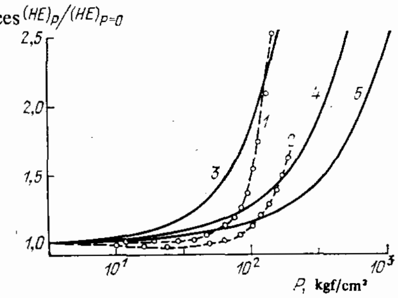

FIG. 9. Pressure dependences $(HE)_p/(HE)_{p=0}$ of the excitation criterion of the oscillistor effect in p-type silicon. The dashed curves are the experimental results (n_0/p_0 = 0.7 and 0.8 correspond to curves 1 and 2); [48] the continuous curves are the results of calculations (n_0/p_0 = 11/15, 12/15, 13/15 correspond to curves 306, respectively).

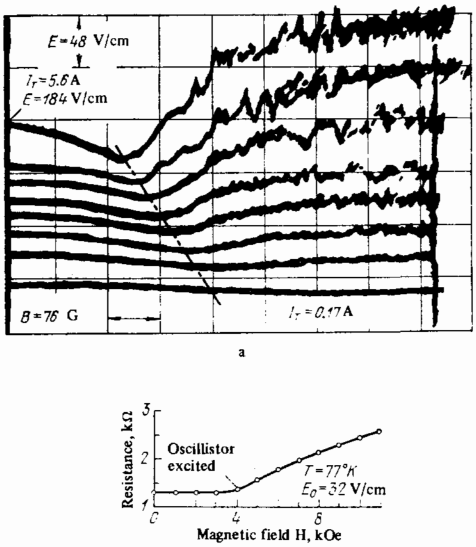

FIG. 10. Magnetic-field dependences of: a) the electric field in p-type InSb under injection conditions (for a fixed current); [53] b) the resistance of Ge with nearly intrinsic conductivity during the excitation of the helical instability. [9]

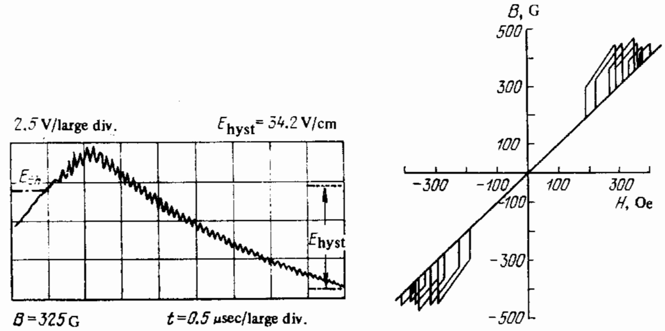

FIG. 11. Hysteresis of the threshold electric field in the excitation of the helical instability in InSb (a) and hysteresis of the threshold magnetic field (b). [53]

of the ⟨111⟩ type. In this case, calculations can also be made within the two-valley framework but they are much more complex than in the case of silicon.

These investigations of the oscillistor effect are still in their early stages and further effort is justified. New possible applications (for example, new types of strain gauge) as well as new methods for investigating the band structure of semiconductors merit attention.

5. NONLINEAR EFFECTS

We shall now recount the most interesting nonlinear effects which accompany helical instability. Ancker-Johnson,[51] Hurwitz and McWhorter,[9] and several other investigators[22,52] discovered a strong rise of the resistance during the excitation of oscillistor oscillations. Figure 10 shows typical dependences for a plasma injected into InSb[51] (Fig. 10a) and for an intrinsic plasma in Ge[9] (Fig. 10b). When the helical instability is pronounced, the great majority of carriers experiences rotation in a helix of finite amplitude.[9] Therefore, the path traveled by carriers between the contacts increases and the resistance becomes higher. When the supercriticality parameter ($H - H_{th}$) becomes larger, an increasing number of carriers joins the helical perturbation. In a nonequilibrium plasma (injection or impact ionization), a strong rise in the resistance may be also explained by the anomalous diffusion of the plasma toward the surface of the sample and by the annihilation of carriers as a result of surface recombination. Therefore, a higher electric field is needed to maintain a given current. It should be noted that the dependences $E_{th}(H)$ shown in Fig. 10a were observed in 1958 by Lehnert[7] in an investigation of the instability of a positive column in a gas discharge subjected to a magnetic field.

In 1963, Ancker-Johnson[53] discovered a strong hysteresis of the oscillistor threshold of p-type InSb under injection conditions. In a fixed magnetic field (325 Oe), a sample was subjected to triangular voltage pulses (Fig. 11a). The oscillations were excited in a field $E \approx 74$ V/cm and the instability disappeared in a field $E \approx 40$ V/cm. Thus, the hysteresis was about 46%. Somewhat weaker hysteresis of the threshold magnetic field (Fig. 11b) was observed when the electric field was fixed. This hysteresis of the threshold indicated that the excitation conditions were hard.[54] The amplitude of helical waves jumped suddenly when the threshold conditions were exceeded.

We shall now briefly consider a phenomenological theory of the soft and hard excitation of waves of finite amplitude.[54,55] The amplitude of small perturbations which develop in an unstable solid-state plasma grows exponentially with time so that the square of the amplitude η satisfies the differential equation

$$\frac{d\eta}{dt} = 2\gamma\eta, \tag{16}$$

where γ is the increment in the linear theory. As the perturbation grows, the rate of its growth varies and Eq. (16) becomes invalid. If η is small and the resultant pulsations are regular, i.e., if they have a definite frequency and wavelength, the growth rate can be found by expanding the above equation in terms of the perturbation amplitude. In this way, we obtain

$$\frac{d\eta}{dt} = 2\gamma_H\eta, \quad \gamma_H = \gamma + a\eta + b\eta^2 + \ldots, \tag{17}$$

which differs from Eq. (16) by the replacement of the linear-theory increment γ with the nonlinear increment γ_H, which depends on the amplitude η. Equation (17) describes several phenomena which occur in an unstable system when the supercriticality is small, i.e., when the increment γ is small. The soft excitation regime corresponds to $a < 0$ in Eq. (17); in this case, the square of the amplitude of the steady-state motion which appears in an unstable solid-state plasma is $\eta = -\gamma/a$ and it increases smoothly from zero on transition from the stability ($\gamma < 0$) to instability ($\gamma > 0$) when the supercriticality parameter ($x - x_{cr}$) is increased. Any quantity y averaged over the pulsations then varies continuously when ($x - x_{cr}$) is increased.

Under hard excitation conditions ($a > 0$), the turbulence appears in the following way. In the limit, $x \to x_{cr}$, we have $\gamma \to 0$ and the system goes over to the instability range. If $\gamma = +0$, the amplitude of the perturbations suddenly rises to a finite value η_1, which is determined by the vanishing of the nonlinear increment γ_H (Fig. 12):

$$a\eta_1 + b\eta_1^2 + \ldots = 0, \quad \eta_1 = -\frac{a}{b}. \tag{18}$$

If the parameter x now decreases below the critical value

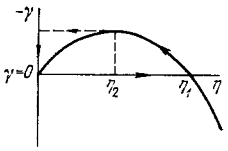

FIG. 12. Dependence of the oscillation amplitude on the instability increment. [54]

($x < x_{cr}$), the motion in the plasma does not stop; the amplitude $\sqrt{\eta}$ suddenly falls to zero only when $x = x'_{cr} < x_{cr}$ is reached. The solution for the amplitude $\eta = \eta_0$ is stable if $\partial\gamma_H/\partial\eta|_{\eta=\eta_0} < 0$, and unstable if $\partial\gamma_H/\partial\eta|_{\eta=\eta_0} > 0$, so that perturbations are suppressed when $\partial\gamma_H/\partial\eta = 0$, i.e., when $\eta = \eta_2 = -a/2b$ (Fig. 12).

Thus, under hard excitation conditions, a hysteresis is observed, namely, the appearance and disappearance of oscillations in a plasma occur for different values of the external parameters. Any quantity y averaged over the pulsations exhibits discontinuities and a hysteresis loop appears in its dependence on the parameter x when the excitation conditions are hard.

A hysteresis of the threshold conditions was also observed in an investigation of the helical instability in the presence of a strong pinch effect in an impact-ionized plasma in InSb.[45] The hysteresis was due to a strong paramagnetism of the plasma in the presence of a pronounced helical instability, where almost all the carriers were gathered into a helix. The azimuthal part of the helical current increased the internal magnetic field by an amount exactly sufficient to ensure that a finite-amplitude perturbation did not break up because of the anomalous diffusion. Holter and Johnson[56] developed a nonlinear theory of the helical instability in an impact-ionized plasma and estimated the paramagnetism (the change in a longitudinal magnetic field) and anomalous resistance of a sample. Unfortunately, the calculated quantities were found to be an order of magnitude smaller than those obtained experimentally. Therefore, the question of the existence of such a strong plasma paramagnetism in the oscillistor effect remains unresolved. A strong hysteresis of the threshold conditions should naturally be accompanied by the appearance of higher azimuthal harmonics with excitation thresholds higher than that of the $|m| = 1$ mode. Therefore, in the nonlinear theory, one has to allow for the interaction between these harmonics and the fundamental mode. A correct nonlinear theory of the helical instability capable of giving a quantitative explanation of the hard excitation conditions must also allow for the intrinsic magnetic field of the current of finite-amplitude helical perturbations. These factors are not considered in [56]. Undoubtedly, considerable difficulties may be expected when these factors are taken into account but only then can a correct description be obtained of the hard excitation of the oscillistor effect in a nonequilibrium plasma in InSb. It should be pointed out that the paramagnetic properties of a positive-column plasma in a gas discharge under helical instability conditions are much weaker (by several orders of magnitude) than the analogous properties of an electron-hole plasma in InSb.[57]

A hysteresis of the threshold conditions of the oscillistor effect in InSb may find application in computer memory elements.[53] For example, if a sample subjected to parallel electric and magnetic fields of intensities somewhat lower than the critical values for the excitation of the oscillistor effect is excited by a sawtooth voltage pulse, oscillistor oscillations should be observed and these should not disappear even after the end of the pulse because of the hysteresis effect. In this way, the system can retain a memory of the initial signal which generated the oscillistor effect.

Hysteresis of the threshold conditions was not observed for the helical instability in Ge.[22,25] In this case, the nonlinear conditions are soft[55] and the perturbation amplitude rises smoothly when the supercriticality parameters ($H - H_{th}$, $E - E_{th}$) are increased.

The theory of the soft excitation of the helical instability, applicable to the experimental results obtained for Ge, was developed by Uspenskiĭ[58] for a helical surface wave.

We shall summarize this section by stressing once again the need for further theoretical and experimental studies of the nonlinear effects associated with the helical instability in semiconductors. It would be extremely desirable to develop a microscopic theory of the hard excitation of the oscillistor effect in an InSb plasma. The high reproducibility and simplicity of the oscillistor experiment should help theoreticians in the selection of correct models and solutions and in the development of a consistent nonlinear theory of the oscillistor effect. Nature has provided an excellent plasma effect in the form of the oscillistor and the development of a general theory of turbulent plasmas may be helped by reliable experimental and theoretical information on the nonlinear properties of the oscillistor.

6. METHODS FOR SUPPRESSING HELICAL INSTABILITY

One of the most interesting directions of research into the oscillistor effect was founded by the pioneer investigations of Ancker-Johnson.[59,60] She investigated the suppression of helical instability in semiconductors by electric and magnetic stabilization systems of the type used in research on controlled nuclear fusion. In 1964, Ancker-Johnson[59] suppressed the helical instability in an injected InSb plasma by a "minimum-B" magnetic trap produced by currents in "Ioffe bars."[61] The apparatus and magnetic field configuration were as shown in Fig. 13. A current produced a magnetic field which increased in all directions away from the axis of the system (min B); the instability was effectively suppressed when the current in the "bars" was ≈ 11 A and the maximum intensity of the additional magnetic field was only a few tens of oersteds. When a high-frequency current (f ≈ 84 MHz) was used, stabilization occurred when the amplitude of this current was ≈ 0.8 A. It was

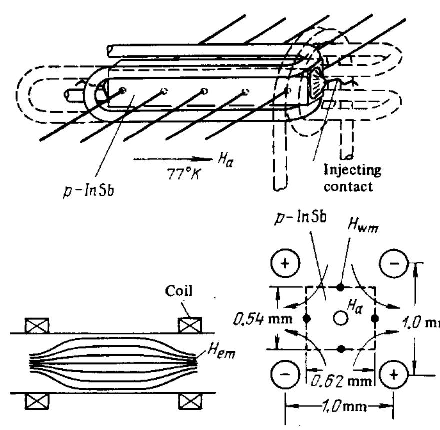

FIG. 13. Semiconductor model of the Ioffe magnetic trap. [60]

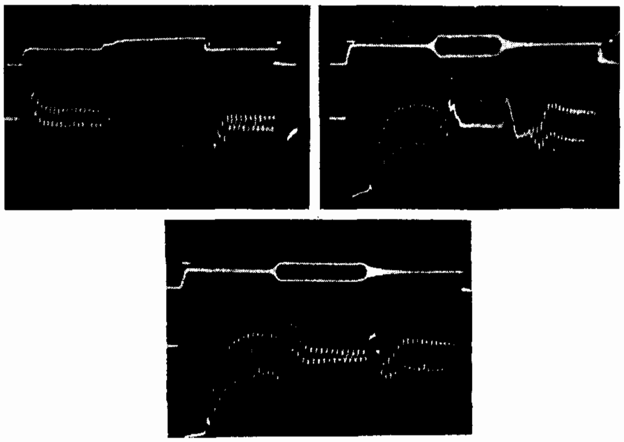

FIG. 14. High-frequency stabilization of the helical instability. The upper trace represents the external electric field and the lower trace the instability current. The experiments were carried out on Ge. [64]

interesting to note that the carrier lifetime and plasma density in a trap rose strongly when the helical instability was suppressed.[60] This confirmed very convincingly the stabilizing influence of the Ioffe magnetic trap. The experiments of Ancker-Johnson were carried out under conditions satisfying the inequality $\omega_{ce}\tau_e \gtrsim 3$ ($H \approx 400$ Oe), whereas in Ioffe's experiments the inequality $\omega_{ce}\tau_e \gg 1$ was obeyed. Unfortunately, a theory of the stabilization of the oscillistor effect in this type magnetic trap has not yet been developed.

A simpler oscillistor stabilization system, utilizing a high-frequency longitudinal electric field, was described in 1965 by Dubovoĭ and Shanskiĭ,[62] who carried out experiments on plasma injected into Ge. The stabilization occurred in samples of reasonable length for a relatively weak modulation of the electric field ($\eta_c = \tilde{E}/E \approx 1$-$2$), which was proportional to L/a, where L is the length of the sample. Typical oscillograms demonstrating this high-frequency stabilization effect are shown in Fig. 14. The effect observed by Dubovoĭ and Shanskiĭ[62] could not be explained by the theory of dynamic stabilization[63] because the inertia of the particles involved had no influence on the helical instability mechanism (the equations describing the helical instability are of the first order in time[3,4]). This is why the initial equations of the oscillistor effect in the presence of a high-frequency electric field cannot be reduced directly to the Mathieu-Hill equations describing dynamic stabilization. The starting point of a theory of the high-frequency stabilization effect in semiconductors is the experimentally established dependence of the efficiency of the stabilization effect on the length of a sample. It is shown in [64,65] that the main role in the high-frequency stabilization effect is played by the helical waves reflected from the ends of a sample, which are present in addition to the main wave and are manifested by a multimode excitation regime. The temporal correlation between these phase-shifted modes in the presence of a high-frequency electric field stabilizes or destabilizes a given mode at a given moment. During the first half-period, the high-frequency field amplifies the first mode and weakens the second (if two modes are present), whereas, in the second half-period the reverse is true. A similar situation occurs in hard-focusing accelerators. Under certain conditions, the overall effect of a high-frequency field is to suppress the instability. In long samples, the temporal correlation of the spatial modes of a helical wave is weak and the high-frequency stabilization is absent. A correct description of the high-frequency stabilization effect can be given by solving the initial differential equation in terms of partial derivatives (variables z, t) describing density perturbations under certain boundary conditions on the ends of a sample. The equation for $n_1(z, t)$ deduced from the Glicksman theory[4] for the $|m| = 1$ mode and reduced to the dimensionless form is (for $y_i \ll 1$)

$$\hat{L}n_1 = \frac{\partial^4 n_1}{\partial \tilde{z}^4} - n_1, \qquad (19)$$

where

$$\hat{L} = \frac{\partial}{\partial\theta}\left(\frac{\partial^2}{\partial\tilde{z}^2} - 1\right) + \mu\left(\frac{\partial^2}{\partial\tilde{z}^2} - 1\right) - i\alpha(\theta)\frac{\partial}{\partial\tilde{z}},$$

θ is the time and $\tilde{z}$ is the spatial coordinate in the direction of the electric and magnetic fields, $\alpha(\theta) = \alpha_c(1 + \eta\sin\beta\theta)$, $\alpha_c \propto EH$, β is the frequency of the hf field, and μ is a parameter which depends on the surface recombination velocity.

The variables in Eq. (19) are not separable and it is not possible to find the general solution of Eq. (19). If we select the solution in the form of a plane wave: $n_1 \sim [n_1(t)]\exp(i\tilde{k}\tilde{z})$, i.e., if we consider just one mode, we find that Eq. (19) for $n_1(t)$ does not describe the high-frequency stabilization effect. In fact, the plane-wave solution also fails to satisfy zero boundary conditions assumed in this problem:

$$n_1|_{\tilde{z}=0,\,\tilde{z}_L} = 0, \quad \tilde{z}_L \sim \frac{L}{a}. \qquad (20)$$

These zero boundary conditions are, strictly speaking, only justified for ohmic (weakly injecting) contacts. Subsequent studies of the spatial structure of perturbations in the high-frequency stabilization effect, carried out by Dubovoĭ and Shanskiĭ,[66] demonstrated that the zero boundary conditions were satisfied well in their experiments.

The coordinate basis of the reduced equation

$$Ln_1 = 0 \qquad (21)$$

can be found quite easily because the variables (z, θ) in this equation are separable. The basis is of the form

$$\Phi_n(\tilde{z}) = \exp(i\rho_n\tilde{z})\sin\varkappa_n\tilde{z}, \qquad (22)$$

where

$$\rho_n = \sqrt{1 + \varkappa_n^2}, \quad \varkappa_n = \frac{\pi n}{\tilde{z}_L}, \qquad n = 1, 2, 3, \ldots$$

The exponential factor (22) describes the fine structure of a helical wave whose period is comparable with the transverse dimensions of the sample and the harmonic factor corresponds to the long-wavelength envelope of the fine structure. The period of this envelope corresponds to the harmonics of the length of the sample. It should be pointed out that Eq. (21) also fails to describe the high-frequency stabilization effect because there is no temporal correlation of modes.

If the initial equation (19) is modified by substituting the basis functions (22), it is found that the right-hand side of Eq. (19) becomes proportional to z_L^{-1} and for long samples this part is small ($\sim a/L$). Therefore, we can apply the perturbation theory to the solution of Eq. (19). The solution of Eq. (19) then becomes an expansion in terms of the basis (22) with coefficients depending on time. Using the Galerkin methods,[11] we can reduce the initial equation in terms of partial derivatives to a system of ordinary differential equations for these coefficients. In the zeroth approximation ($a/L \to 0$), there is no high-frequency stabilization because each of the modes, labeled by a number n, is described by an equation of the first order in time due to the absence of correlation with other modes. Therefore, the high-frequency stabilization effect should not appear in very long samples. The temporal mode correlation is obtained in the first order of the perturbation theory when terms $\sim a/L$ are included in Eq. (19). In the two-mode approximation, the initial equations describing the time evolution of the modes reduce to the generalized Hill equations for each of the modes. A study of the range of stable solutions of these equations gives information on the high-frequency stabilization of the helical instability. We shall give only the main results of the theory of this stabilization effect.[64,65] The effect can appear only for a certain range of values of EH near the oscillistor excitation threshold. This is due to the fact that well beyond the threshold the mode growth increments are so large that the correlation effect cannot give rise to the stabilization effect. The width of the stabilization zone decreases with increasing length of a sample (L/a). The stabilizing modulation of the electric field is $\eta_c \sim L/a$. For samples with a clean surface ($G_s \gg 1$), the stabilization zone is considerably wider than in the $G_s \ll 1$ case and the values of η_c are smaller. The results of the theoretical calculations were confirmed later in several experiments by Dubovoĭ and Shanskiĭ.[65,66,67] The stabilization zone was determined experimentally in [65]. The measurements were carried out on samples of Ge of different lengths (L/a = 6, 9, 12, 15) ranging from 6 to 15 mm. The surface recombination velocity was reduced by etching in hydrogen peroxide. The instability was recorded with point probes located on a lateral surface of a sample. Pulse operating conditions (the duration of electric field pulses was $\approx 100\ \mu$sec and the duration of the high-frequency field pulses was $+50\ \mu$sec) avoided heating of the lattice and carriers when the currents were considerable. The oscillistor oscillation frequency was $\approx 10^5$ Hz and the frequency of the stabilizing field was $\approx 10^6$ Hz. The carrier heating was minimized, the threshold of the excitation zone was reached, and the stabilization zone was entered by increasing the magnetic field to 13 kOe (the electric field was kept constant). The value η_c in the stabilization zone remained almost constant when the magnetic field was increased and then rose strongly (by a factor of 3-5) as shown in Fig. 15. The value of EH in which this strong rise of η_c was observed was regarded as the upper limit of the stabilization zone. The dashed lines in Fig. 15 are the theoretical limits of the stabilization zone. We found that when the ratio L/a was increased, the width of the stabilization zone decreased. A strong rise of the stabilizing modulation

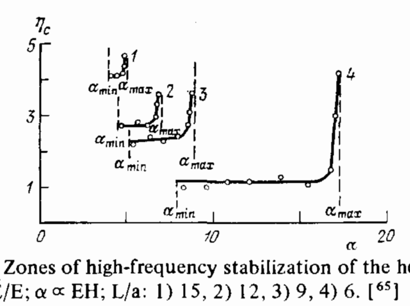

FIG. 15. Zones of high-frequency stabilization of the helical instability; $\eta_c = \tilde{E}/E$; $\alpha \propto EH$; L/a: 1) 15, 2) 12, 3) 9, 4) 6. [65]

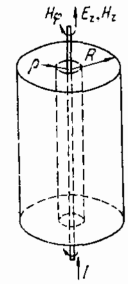

FIG. 16. Geometry of a system used in an investigation of the stabilization of the helical instability in tubular samples of Ge. [68]

coefficient η_c near the upper limit of the stabilization zone was a natural consequence of a transition from the region where the system was stable to the region where it became unstable. This considerable increase in the value of the modulation coefficient or the amplitude of the high-frequency field outside the limit of the stabilization zone caused carrier heating and, consequently, it reduced the mobility and increased the diffusion coefficient, which was equivalent to the displacement of the upper limit of the stabilization zone in the direction of stronger magnetic fields.

A very important investigation of the anomalous diffusion of a nonequilibrium electron-hole plasma under helical instability conditions was carried out by Dubovoĭ and Shanskiĭ.[67] Laser diagnostics was used to show that the diffusion coefficient of carriers was several times higher than the classical value. Under the high-frequency stabilization conditions, were was no such anomalous diffusion and this provided further support in favor of the existence of the high-frequency stabilization effect.

A theoretical and experimental study of the helical instability was carried out[68] in the Triax thermonuclear geometry.[69] The instability was excited in tubular Ge samples (Fig. 16). An azimuthal magnetic field of an axial conductor passing through a cavity in a sample suppressed the instability when the current was sufficiently high and this happened irrespective of the direction of the currents in the sample and in the axial conductor. The stabilization effect was due to a strong magnetodensity effect which appeared in the sample in the presence of the field E and H_φ. Depending on the directions of the axial current and the current in the sample, the plasma was compressed either at the external or internal surface of the tubular sample, which enhanced the transverse diffusion and suppressed the instability. Similar experiments were carried out earlier in a gaseous plasma.[70]

In spite of the apparent success of the model experiments, there is still an element of doubt associated with the incomplete analogy between semiconductor plasmas (where usually the mean free path of carriers is considerably shorter than the sample) and hot low-pressure discharge plasmas. However, as pointed out in [71], under anomalous resistance conditions, when the effective collision frequency increases, the helical instability may appear even in a "collisionless" plasma of the type used in thermonuclear research. In this case, the model experiments on semiconductor plasmas may be applied directly to the thermonuclear fusion program. It should be pointed out that modeling with the aid of semiconductor plasma units is of considerable intrinsic interest. The most important aspect is the study of the nature of the diffusion of particles in an electron-hole plasma in the case of spontaneous growth of fluctuations and under stable conditions, including the determination of the main stabilization mechanisms. In this sense, the model investigations can enrich considerably the range of tools available to investigators working on thermonuclear fusion.

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