Hydrodynamic instability in solid-state plasma

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Usp. Fiz. Nauk **115**, 41–72 (January 1975)

We consider three groups of hydrodynamic instabilities of electron-hole plasmas in conducting solids, namely, helicoidal, two-stream, and overheating instabilities. The helicoidal instability is discussed in Chap. II and is due to the drift of electrons relative to the crystal lattice in constant electric and magnetic fields. It is manifested by the growth of transverse and longitudinal acoustic oscillations which interact with helicons. Particular attention is devoted to the explanation of the role of the magnetic field due to a constant current. In Chap. III, we give a summary of theoretical and experimental results on the interaction between a beam of electrons moving near the surface of a semiconductor and the associated electromagnetic waves. A detailed analysis is given of the size effect, i.e., the effect of the finite size of the specimen on the growth rates of two-stream instability. These growth rates exhibit a rapid rise in resonances. Chapter IV is concerned with the instability due to the heating of the electron gas by a constant electric field. Since the rate at which energy is transferred to the lattice is low, the static current-voltage characteristic includes a falling segment (negative differential resistance). This leads to an instability of temperature perturbations and the associated electromagnetic waves. Particular attention is devoted to the assessment of conditions under which overheating instability has an oscillatory character.

CONTENTS

I. INTRODUCTION

There has been increased interest during the last decade in studies of the electromagnetic properties of conducting solids. Various types of weakly attenuated electromagnetic waves in semiconductors and metals were discovered during this time, and various instabilities, nonlinear effects, interactions between waves, and interactions of waves with external fields were investigated. At the present time, these phenomena, which are combined under the general phrase "plasma phenomena in solids" are undergoing rapid development and are among the most lively topics in solid-state physics. This interest in plasma effects in solids is due to several factors. Firstly, unstable states of electron-hole plasma in semiconductors can be (and are) used for the generation, amplification, and transformation of electromagnetic waves in the broad range of wavelengths between radio and optical frequencies. Secondly, plasma effects in conductors are associated with the specific properties of solids and can therefore be used to investigate the energy spectrum, transport properties, and interactions of conduction electrons. Finally, solid-state plasma is convenient for the simulation of processes which occur in gas-discharge plasma. It is important to note that many of the electromagnetic properties of solids are similar to the properties of ordinary plasma. However, despite this similarity, there are also essential differences. The chief difference is that equilibrium solid-state plasma is absolutely stable, whereas the gas-discharge plasma usually has a short lifetime and decays rapidly. Other differences are due to the presence of the crystal lattice, quantum effects, anisotropic phenomena, and the possible variation of plasma parameters within very wide limits.

The study of instabilities is a very substantial part of plasma physics. It is well known that the numerous plasma instabilities can be divided into two groups, namely, kinetic and hydrodynamic instabilities. The former arise for long electron mean free paths, much greater than the wavelengths. They are due to resonant interactions between slow waves and individual particle groups, the velocities of which are close to the phase velocity of the wave. An example of kinetic instability is the inversion of Landau damping[1] during the motion of a beam in collisionless plasma.[2,3] Hydrodynamic instabilities, on the other hand, are connected with the ordered motion of macroscopic plasma volumes. As a rule, these instabilities develop at low frequencies and short mean free paths. The mathematical formalism used for investigating these instabilities is provided by the equations of hydrodynamics. A systematic account of the theory of hydrodynamic or, more precisely, hydromagnetic instability is given in the review paper of Kadomtsev.[4] A reasonably complete theory of plasma instabilities of both types is given in the recent book by Mikhaĭlovskii.[5]

The characteristic feature of electron-hole plasma in semiconductors is the relatively high collision frequency ν between current carriers and the scattering centers. The minimum value of ν is usually not less than 10^{11}—10^{12} sec^{-1}. It follows that the hydrodynamic approximation is valid up to infrared frequencies. In other words, the most common situation is that involving hydrodynamic instabilities.

In this review, we consider three groups of instabilities which, in our view, are important, namely, helicoidal, two-stream, and overheating instabilities.

Helicoidal instability is treated in Chap. II and is due to the drift of electrons relative to the crystal lattice in constant magnetic and electric fields. This instability is associated with the existence of a helical magnetic wave, i.e., the helicon.[6,7] The presence of elastic forces in the crystal lattice leads to the appearance of transverse and longitudinal acoustic oscillations which interact with the helicons. Under conditions of instability, this distinguishes solid-state plasmas from gas-discharge plasmas. We shall pay particular attention to the role of the magnetic field produced by a constant uniform current.

Chapter III gives a discussion of theoretical, and some experimental, results on the interaction between an electron beam moving near the surface of a semiconductor and intrinsic electromagnetic waves in the specimen. Particular attention is paid to the "size effect", i.e., the influence of the finite specimen size on the growth of the wave amplitude. The amplitude growth rates increase sharply near resonances where the wave frequency is equal to the oscillation frequency of electrons in the moving beam. The mechanism responsible for this instability is analogous to the two-stream instability in gas plasmas, and is connected with the transformation of the energy associated with the directed motion of the beam into the energy of oscillations.

Finally, Chapter IV is devoted to instabilities which appear as a result of the heating of electrons by a constant electric field. Since the rate at which energy is transferred from electrons to the lattice is low, the static current-voltage characteristic exhibits a descending section on which the differential resistance of the specimen is negative. This leads to the instability of temperature perturbations and the associated electromagnetic fields. Overheating instability is usually aperiodic. However, there is considerable interest in the elucidation of conditions under which this instability may have an oscillatory character.

The aim of this review is to draw attention to the above instabilities. They have not, so far, been extensively investigated experimentally, but they are interesting both from the standpoint of general physics and from the point of view of practical applications.

It is clear that our review will cover only a small part of the general problem of hydrodynamic instability in solid-state plasma. In particular, we shall not consider the helicoidal instability of two-component plasmas, the amplification and generation of sound in piezosemiconductors, the recombination-ionization instabilities, and the Gunn effect. Some of these are described in the reviews by Gurevich,[8] Pustovoĭt,[9] Volkov and Kogan,[10] and in the monograph by Bonch-Bruevich, Zvyagin, and Mironov.[11] Accordingly, the bibliography given at the end of this review is not intended to be complete: we cite only those literature sources which are directly relevant to the questions discussed.

II. HELICOIDAL INSTABILITY

1. Formulation of the problem. Equations and boundary conditions. To obtain a solution for the instability of magnetoactive plasma in a solid carrying a current, we must use Maxwell's equations and the equations of the theory of elasticity, as well as the constitutive equations relating the varying current to the electromagnetic and acoustic fields. The most complete and rigorous theory describing the interaction between conduction electrons and the lattice is given in[12-16]. In strong magnetic fields, in which the Larmor radius of electrons is much less than the wavelength of the electromagnetic or acoustic waves (the wavelength is also less than the mean free path), the predominant phenomenon is the induction mechanism which provides the coupling between electrons and the lattice.[14] We shall therefore confine our attention to this mechanism.

The complete set of equations has the following form:

$$\operatorname{rot}\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t},\quad \operatorname{rot}\mathbf{H}=\frac{4\pi}{c}\mathbf{j},\quad \operatorname{div}\mathbf{H}=0, \tag{1.1}$$

$$\mathbf{j}=Ne\left(\frac{\partial\mathbf{u}}{\partial t}-\mathbf{v}\right),$$

$$m\nu\left(\mathbf{v}-\frac{\partial\mathbf{u}}{\partial t}\right)=-e\left(\mathbf{E}+\frac{1}{c}[\mathbf{v}\times\mathbf{H}]\right),\quad \frac{\partial N}{\partial t}+\operatorname{div}(N\mathbf{v})=0, \tag{1.2}$$

$$\rho\left[\frac{\partial^2\mathbf{u}}{\partial t^2}-s_t^2\Delta\mathbf{u}+(s_t^2-s_l^2)\operatorname{grad}\operatorname{div}\mathbf{u}\right]$$
$$=Ne\left\{\mathbf{E}+\frac{1}{c}\left[\frac{\partial\mathbf{u}}{\partial t}\times\mathbf{H}\right]\right\}+m\nu N\left(\mathbf{v}-\frac{\partial\mathbf{u}}{\partial t}\right); \tag{1.3}$$

where **E** and **H** are the electric and magnetic fields, u is the lattice displacement vector, s_t and s_l are the velocities of transverse and longitudinal acoustic waves, ρ is the crystal density, and e, m, v, N, and ν are, respectively, the electron charge, effective mass, velocity, density, and collision frequency. The frequency is assumed to be low enough for inertial force to be neglected in the equations of motion for the electrons. We shall also assume that the quasineutrality condition is satisfied, i.e., the electron density is equal to the lattice charge density. The elastic properties will be taken to be isotropic, and dissipative terms in the equations of motion will take into account the conservation of momentum of the system during collisions between electrons and the lattice.

Equations (1.1)–(1.3) must be augmented with boundary conditions. These can be reduced to the following: the forces which are impressed on a separation boundary by two contacting but different media must be equal and opposite in direction:[17]

$$(\sigma_{ik}+T_{ik})\,n_k=(\sigma'_{ik}+T'_{ik})\,n_k, \tag{1.4}$$

where the primed and unprimed quantities refer to the different media, and the unit normal n has the same direction in both media. Repeated subscripts indicate summation between 1 and 3. Finally, σ_{ik} is the electric stress tensor[18] and T_{ik} is the Maxwell stress tensor. It is, of course, assumed that the electrodynamic boundary conditions are satisfied.

If the specimen is surrounded by a perfectly conducting surface (which is the case, for example, in a circular metal waveguide fully filled with a semiconducting material), the tangential electric field components vanish on the boundary. For a semiconductor rod surrounded by a nonconducting medium (vacuum, $\sigma'_{ik} = 0$), Eqs. (1.1)–(1.3) must be augmented by the Maxwell equations in vacuum:

$$\operatorname{rot}\mathbf{H}=0,\quad \operatorname{div}\mathbf{H}=0,\quad c\operatorname{rot}\mathbf{E}=-\frac{\partial\mathbf{H}}{\partial t}. \tag{1.5}$$

Displacement currents are assumed small in comparison with conduction currents in the conducting medium. When this is so, the normal components of the magnetic field and the tangential components of the electric field must be continuous across the boundary of the specimen. The tangential components of H may, in general, exhibit discontinuities due to the presence of surface currents.

Consider a cylindrical specimen of radius R, which is infinitely long along the axis of symmetry (z axis). The current $j_0 = j_{0z} = -N_0 e v_0$ and the external magnetic field H_{0z} are assumed to be uniform. In a state of equilibrium, the remaining quantities in (1.1)–(1.3) depend only on the variable r of the cylindric set of coordinates (r, φ, z). Equilibrium values are indicated by the subscript zero. From (1.1)–(1.3), we find that the azimuthal magnetic field $H_{0\varphi} = 2\pi j_0 r/c$ is related to the radial component of the lattice displacement vector by the expression

$$\rho s_l^2 \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(ru_{0r})\right] = \frac{j_0 H_{0\varphi}}{c} \equiv \frac{H_{0\varphi}}{4\pi r}\frac{\partial}{\partial r}(rH_{0\varphi}). \qquad (1.6)$$

Integrating this equation subject to the boundary condition (1.4) on the surface of the cylinder (we are assuming that the cylinder is located in the vacuum insofar as elastic properties are concerned, i.e., $\sigma'_{ik} = 0$), we can determine the deformation of the cylinder and the stress distribution in it.

It is clear from (1.6) that the relative deformation $u_{0r}/r \sim H_{0\varphi}^2/4\pi\rho s_l^2$ is very small in a solid. It is obvious that the elastic constants, for example, the velocity of sound, cannot undergo a substantial change as a result of such deformations. In the stationary state, $E_{0\varphi} = H_{0r} = 0$, $E_{0z} = -mv_0\nu/e$. Moreover, there is also the radial component of the electric field $E_{0r} = \nu_0 H_{0\varphi}/c$, which leads to very slight violation of the neutrality condition, namely,

$$\frac{\Delta N_0}{N_0} \sim \left(\frac{v_0}{c}\right)^2 \sim 10^{-6}.$$

Let us now consider small oscillations in which alternating increments are described by $A(r)\exp(ik_z z + il\varphi - i\omega t)$, where ω is the frequency and l, k_z are the azimuthal and axial wave numbers. Linearizing (1.1)–(1.3), we can readily obtain the equations for small oscillations in the following form:

$$-i(\omega - k_z v_0)\mathbf{H} - \frac{c^2\nu}{\omega_0^2}\Delta\mathbf{H} + \frac{c}{4\pi e N_0}\operatorname{rot}[\operatorname{rot}\mathbf{H}\times\mathbf{H}_0] = \omega k_z H_{0z}^*\mathbf{u} + i\omega\mathbf{H}_0\operatorname{div}\mathbf{u} - c\operatorname{rot}(\mathbf{E}_0\operatorname{div}\mathbf{u}), \qquad (1.7)$$

$$\omega^2\mathbf{u} + s_t^2\Delta\mathbf{u} + (s_l^2 - s_t^2)\operatorname{grad}\operatorname{div}\mathbf{u} = -\frac{1}{\rho c}[\mathbf{j}_0\times\mathbf{H}] - \frac{1}{4\pi\rho}[\operatorname{rot}\mathbf{H}\times\mathbf{H}_0], \qquad (1.8)$$

where $\omega_0^2 = 4\pi e^2 N_0/m$, $H_{0z}^* = H_{0z} + (e/k_z r)H_{0\varphi}$.

It is important to note that we can use the energy principle[4] to analyze the stability of (1.7)–(1.8) in the absence of dissipation. The idea of this method can be summarized as follows. If the initial set of equations is self-adjoint, it can be obtained from the least-action principle for the Lagrange function. Having determined the sign of the potential energy of the oscillations, we can use the general theorems of mechanics to judge the stability of the equilibrium of the system.

We shall, however, use the method of natural oscillations because it provides more complete information. When the external magnetic fields are sufficiently high (electron cyclotron frequency $\omega_H = eH_{0z}/mc$ much greater than the collision frequency and signal frequency), weakly damped electromagnetic waves (helicons) propagate through the conducting medium,[6,7] and there are also transverse and longitudinal sound waves. These waves are coupled,[14,16] and the strength of this coupling is characterized by a small parameter of the order of $H_{0z}^2/4\pi\rho s_{t,l}^2$. The degree of coupling may, however, rise sharply under resonance conditions when the frequencies and wave vectors of the initial (partial) waves are equal. This is accompanied by strong mutual wave transformation which results in coupled electromagnetic and acoustic oscillations. The presence of conduction electrons which drift with constant velocity v_0 then leads to a change in the electromagnetic wave spectrum. In particular, waves propagating with phase velocity smaller than the drift velocity are found to appear in the system. The resonant interaction between these electromagnetic oscillations and the sound waves leads to the growth of coupled oscillations with maximum growth rate. The partial electromagnetic waves are then the intermediaries which transform the energy of translational motion of electrons to the thermal energy of the entire system. We shall now investigate instabilities of this kind.

2. Helicoidal instability in an infinite medium. To begin with, we shall consider the case where the dimensions of the specimen are sufficiently large and much greater than the propagation wavelength. It is clear that surface effects can then be neglected and, if there is no interaction between conduction electrons and the lattice, (1.7) leads to the following dispersion relation which determines the spectrum and damping of the helicons:[1)]

$$\omega = k_z v_0 \pm \frac{ckk_z H_{0z}^*}{4\pi e N_0} - i\nu\frac{k^2c^2}{\omega_0^2}, \qquad (2.1)$$

where $k = (\kappa^2 + k_z^2)^{1/2}$ and κ is the transverse wave number which is determined by the boundary conditions.

It follows from (2.1) that two types of helicon can exist in drifting plasma, namely, a fast helicon, the phase velocity of which is greater than v_0, and a slow helicon whose velocity is less than the drift velocity. These helicons have different polarizations: curl H = ±kH. Both waves are, of course, damped. The instability can appear only in the presence of several groups of carriers with different mobility in the solid-state plasma,[20-22] or when the motion of ions in the gas-discharge plasma[9] is taken into account, or in the presence of a coupling between the electromagnetic wave and the lattice oscillations.[23-25]

Igitkhanov and Kadomtsev[26] note that the helicoidal instability discussed in[25], where the intrinsic magnetic field $H_{0\varphi}$ was neglected, was essentially the Kruskal-Shafranov instability[4] well known for the gas discharge plasma. We shall see later that the instability criterion is the same in both cases. Nevertheless, we would like to draw attention to the fact that the Kruskal-Shafranov instability in solid-state plasma has a number of distinguishing features.

Firstly, the elastic forces in the crystal lattice ensures that the instability can have an oscillatory character. Secondly, if the oscillation frequency is much greater than the growth rate, the instability may appear when $|H_{0\varphi}|$ is greater than H_{0z}. In other words, there is a range of parameters in solid-state plasma in which a sufficiently high magnetic field $H_{0\varphi}$ has no effect on instability development. The results reported in[23,25] are thus valid in this region even though they have been obtained without taking into account the field $H_{0\varphi}$.

Let us now consider the interaction between helicons with axial symmetry ($l = 0$) and transverse acoustic waves. Since $s_t < s_l$, one would expect that an instability involving the participation of transverse sound waves would appear at lower electron drift velocities. We note that when the linear dimensions of the specimen are large enough ($k_z^2 \gg \kappa^2 \sim \pi^2/R^2$), the condition for the existence of a slow helicon ($v_0 > ck_z H_{0z}/4\pi e N_0$) is equivalent to the requirement $H_{0\varphi} > H_{0z}$. However, the helicon dispersion relation for axially symmetric oscillations is independent of the azimuthal magnetic field.[23,25] Assuming that the waves propagate exclusively along the z axis ($\kappa \to 0$), and that the continuity condition div u = 0 is satisfied, we obtain the dispersion relation for coupled electromagnetic and transverse acoustic waves from (1.7)–(1.8) in the following form:[25]

$$\left(\frac{\omega^2}{s_t^2} - k_z^2\right)\left(\omega - k_z v_0 \mp \frac{ck_z^2 H_{0z}}{4\pi e N_0} + i\nu\frac{k_z^2c^2}{\omega_0^2}\right) = \frac{k_z^2 H_{0z}^2\omega}{4\pi\rho s_t^2}\left(1 \pm \frac{4\pi e N_0 v_0}{ck_z H_{0z}}\right). \qquad (2.2)$$

The upper sign refers to the circularly polarized wave with curl **H** = k_z**H**, and the lower sign corresponds to the wave with curl **H** = $-k_z$**H**. During the propagation of transverse acoustic waves with $\omega \approx k_z s_t$ the presence of the azimuthal magnetic field leads to the appearance of a longitudinal component of the lattice displacement vector

$$u_z = \frac{iH_{0\varphi}H_\varphi}{4\pi\rho k_z (s_l^2 - s_t^2)},$$

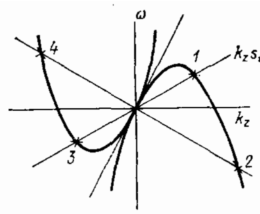

FIG. 1. Schematic plot of $\omega(k_z)$ for coupled helicon and acoustic waves in the presence of electron drift.

and, strictly speaking, the waves are no longer transverse. However, the contribution of u_z to (1.7) is negligible if

$$u_\varphi H_{0z} \gg u_z |H_{0\varphi}| \left|\frac{v_0}{s_t} - 1\right|. \tag{2.3}$$

It follows from (1.8) that

$$u_\varphi \approx \frac{iH_{0z}H_\varphi}{8\pi\rho s_t \delta\omega}\left(\frac{4\pi e v_0 N_0}{ck_z H_{0z}} - 1\right),$$

where $\delta\omega = \omega - k_z s_t$. Thus, the final condition that the oscillations be transverse can be written in the form

$$\frac{\omega}{|\delta\omega|} \gg 2\frac{H_{0\varphi}^2}{H_{0z}^2}\frac{s_t |v_0 - s_t|}{s_l^2 - s_t^2}\frac{ck_z H_{0z}}{|4\pi e N_0 v_0 - ck_z H_{0z}|}. \tag{2.3a}$$

If we use the order-of-magnitude result $|H_{0\varphi}| \sim H_{0z} k_z R$, we obtain the following range of parameters in which (2.2) is valid:

$$\frac{\omega}{|\delta\omega|}\frac{s_l^2 - s_t^2}{s_t|v_0 - s_t|}\left|\frac{4\pi e v_0 N_0}{ck_z H_{0z}} - 1\right| \gg k_z^2 R^2 \gg \pi^2. \tag{2.4}$$

During the resonant interaction between transverse acoustic waves and the helicon, the frequencies and wave vectors of both waves are equal. The corresponding resonance points are shown in Fig. 1. Consider, for example, the interaction between a slow electromagnetic wave and an acoustic wave. Suppose that $\omega > 0$, $v_z > 0$, and $v_0 > 0$. Clearly, the resonance condition takes the form

$$k_{z\,\mathrm{res}} = \frac{4\pi e N_0}{cH_{0z}}(v_0 - s_t), \quad \omega_{\mathrm{res}} = k_{z\,\mathrm{res}} s_t. \tag{2.5}$$

Hence, it follows that resonance occurs only when $v_0 > s_t$. The correction to the resonance frequency is determined by the right-hand side of (2.2) and is given by

$$\delta\omega_{\mathrm{res}} = \pm i\omega_{\mathrm{res}}\sqrt{\frac{H_{0z}^2}{8\pi\rho s_t (v_0 - s_t)}}. \tag{2.6}$$

It follows from this formula that one of the coupled waves grows and the other decays. The amplitude of circularly polarized oscillations will grow, forming a peculiar helical (helicoidal) structure. This type of instability is therefore referred to as helicoidal by analogy with the helical plasma instability discussed in[27]. There is no difficulty in calculating $\delta\omega_{\mathrm{res}}$ for the other resonance points as well. At points 2 and 4 (Fig. 1), there is no growth[2)] and at point 3 the quantity $\delta\omega_{\mathrm{res}}$ can be calculated from (2.6). In other words, among the coupled waves, the wave that will grow will be that for which the phase velocity is less than v_0. Its direction is the same as that of the electron drift velocity. In this case, the helicoidal instability mechanism is equivalent to the Cerenkov mechanism.

Let us now consider the character of the resulting instability. To do this, we must find the increment δK of the wave vector at fixed frequency. We have

$$\delta k^2 = -\frac{\omega_{\mathrm{res}}^2 H_{0z}^2}{8\pi\rho s_t^2 (v_0 - s_t)(\partial\omega/\partial k_z)_{\mathrm{res}}}, \tag{2.6a}$$

where $(\partial\omega/\partial k)_{\mathrm{res}}$ is the group velocity of helicons at resonance. It is readily shown[28] that when the helicon group velocity is negative ($v_0 > 2s_t$), the instability is absolute, and when it is positive, the instability is convective.

The formula given by (2.6) is obtained on the assumption of "strong" coupling, i.e., the degree of mutual transformation is so large that $\delta\omega_{\mathrm{res}}$ exceeds the helicon damping $\nu k_{z\,\mathrm{res}}^2 c^2/\omega_0^2$. In this case, the waves can no longer be divided into acoustic and electromagnetic, and coupled waves appear in the resonance region. If, on the other hand, the strong-coupling condition is not satisfied, the waves can be divided into helicons and sound. The helicons are then damped at a rate $\nu|v_0 - s_t|/\omega_H s_t$, and the acoustic waves grow at a rate

$$\delta\omega_{\mathrm{res}} = i\omega_{\mathrm{res}}\frac{H_{0z}^2\omega_H}{8\pi\rho(v_0 - s_t)^2\nu}. \tag{2.7}$$

Thus, for the semiconductor PbTe[3)] with $N_0 = 10^{18}\ \mathrm{cm}^{-3}$, $\rho = 3.5\ \mathrm{g\ cm}^{-3}$, $m \sim 10^{-9}$ g, $s_t \sim 10^5\ \mathrm{msec}^{-1}$, $\nu \approx 2\times 10^{10} - 10^{11}\ \mathrm{sec}^{-1}$, and with $H_{0z} = 1000$ Oe, $v_0 - s_t \sim s_t$ there is weak coupling between the waves. Under these conditions, $k_{z\,\mathrm{res}} \sim 20\ \mathrm{cm}^{-1}$, $\omega_{\mathrm{res}}/|\delta\omega_{\mathrm{res}}| \sim 10^5$, and the conditions given by (2.4) are satisfactorily fulfilled for $R \sim 1$ cm. In strong magnetic fields ($H_{0z} \sim 10^4$ Oe), and at helium temperatures, the strong coupling conditions are also satisfied ($k_{z\,\mathrm{res}} \sim 2\ \mathrm{cm}^{-1}$, $\omega_{\mathrm{res}}/|\delta\omega_{\mathrm{res}}| \sim 100$, $15 > k_z R > \pi$).

Well away from resonance (the frequencies of acoustic and electromagnetic waves not equal for given k_z), we can obtain a correction to the acoustic frequency $\omega = k_z s_t$ from (2.2) in the form

$$\frac{\delta\omega}{\omega} = \frac{H_{0z}^2}{4\pi\rho s_t^2}\left(1 - \frac{4\pi e v_0 N_0}{ck_z H_{0z}}\right)\left(1 - \frac{v_0}{s_t} - \frac{ck_z H_{0z}}{4\pi e s_t N_0} - \frac{i\nu c^2 k_z}{s_t \omega_0^2}\right) \times\left[\left(1 - \frac{v_0}{s_t} - \frac{ck_z H_{0z}}{4\pi e s_t N_0}\right)^2 + \frac{\nu^2 c^4 k_z^2}{\omega_0^4 s_t^2}\right]^{-1}. \tag{2.8}$$

The oscillations grow when

$$v_0 > \frac{ck_z H_{0z}}{4\pi e N_0}. \tag{2.9}$$

This condition corresponds to the requirement that the Lorentz force $(1/c)(\mathbf{j}_0 \times \mathbf{H})$ acting on the charged lattice exceeds the force $(1/c)(\mathbf{j} \times \mathbf{H}_0)$. It is important to emphasize that, in the absence of resonance, the inequality given by (2.9) is not connected with the relationships between the drift velocity v_0 and the phase velocity of the wave. Outside resonance, instability can therefore probably be detected not only in semiconductors but also in metals, where a high drift velocity cannot be produced. The instability condition given by (2.9) is equivalent to the Kruskal-Shafranov condition ($|H_{0z}|/H_{0z} > \pi R/L$, where L is the length of the system) which predicts the appearance of helical instability in a plasma column in a strong longitudinal field. We emphasize again, however, that the above condition is valid for a column with small transverse dimensions ($R \ll L$ and hence $|H_{0\varphi}| < H_{0z}$) whereas, in solid-state plasma, helicoidal instability may arise even when the intrinsic magnetic field is strong ($R \gg L$, $|H_{0\varphi}| > H_{0z}$).

Of course, one can speak of the growth of oscillations only when the growth rate exceeds the nonelectron (lattice) damping. Since the instability is expected for long-wave phonons, the condition $\omega\tau_{ph} \ll 1$ is satisfactorily fulfilled (τ_{ph} is the thermal phonon relaxation time). The lattice absorption is then described by the Akhiezer mechanism[30] and is proportional to the square of the wave vector. Comparison of the electron growth rate and the lattice absorption coefficient shows that, for $k_z < 4\pi e s_t N_0/cH_0$ ($v_0 \sim s_t$), absorption is negligible[31] for PbTe at helium temperatures.

When the drift velocity exceeds s_l, the presence of the azimuthal magnetic field $H_{0\varphi}$ leads to the interaction between the helicons and the longitudinal acoustic waves propagating in the z direction. This interaction is local and appears in the region well away from the axis of the cylinder, i.e., for $k_z r \ll 1$ and $H_{0z} \ll |H_{0\varphi}(r)|$. The formulas describing the dispersion properties and growth rates for coupled waves, in this case, are given in[32].

3. Resonance interaction between helicons and acoustic waves in a semiconductor rod with a perfectly conducting surface. The tangential components of the electric field are zero on a perfectly conducting surface. It is clear that this condition should be satisfied in the stationary case as well. Hence, we find that $E_{0z} = 0$ inside the specimen. The electron drift velocity v_0 is finite if, at the same time, we put $\nu = 0$. Hence, in the approximation involving a perfectly conducting surface, the static conductivity of the specimen in the direction of the magnetic field must be set equal to infinity, and the transverse conductivity to zero. The boundary conditions (1.4) for elastic stresses are then of the form

$$\sigma_{ir}|_{r=R} = 0. \qquad (3.1)$$

The components of the Maxwell stress tensor in the linear approximation are then removed from (3.1) because the electrodynamic boundary conditions are satisfied. If we express σ_{ir} in terms of the components of the tensor u_{ik}, the boundary conditions (3.1) can be written in the form (all quantities are taken for r = R)

$$\begin{gathered} s_l^2 r \frac{\partial u_r}{\partial r} + (s_l^2 - 2s_t^2)(ilu_\varphi + u_r + ik_z r u_z) = 0, \\ ik_z u_r + \frac{\partial u_z}{\partial r} = 0, \quad r\frac{\partial u_\varphi}{\partial r} - u_\varphi + ilu_r = 0. \end{gathered} \qquad (3.2)$$

It is clear from (3.2) that the boundary conditions "mix" the longitudinal and transverse acoustic waves even in the absence of electron-lattice coupling. However, for axially symmetric perturbations with $l = 0$, the equation for the component u_φ separates from the equations for u_z and u_r.

Let us therefore begin by considering the spectra of axially symmetric oscillations in a cylindric specimen in the absence of interaction between electromagnetic and acoustic waves. The helicon polarization plane rotates about the z axis, and the components of the alternating magnetic field are related by curl H = $\pm k_{01}$H, where $k_{01}^2 = \kappa_{01}^2 + k_z^2$. Hence,

$$H_r = -\frac{iAk_z}{\varkappa_{01}} J_1(\varkappa_{01} r), \quad H_\varphi = \mp i\frac{k_{01}}{k_z} H_r, \quad H_z = AJ_0(\varkappa_{01} r), \qquad (3.3)$$

where J_0 and J_1 are the Bessel functions. From the boundary condition $H_r(R) = 0$, we find that the radial wave number spectrum, (κ_{01}) is determined by the zeros of $J_1(\kappa_{01}R)$. The helicon frequency spectrum is described by (2.1) with $\nu = 0$.

Acoustic oscillations with polarization $u_\varphi(\sim J_1(\kappa_{02}r))$ are transverse, and their wave number is $\omega/s_t = k_{02} \equiv (\kappa_{02}^2 + k_z^2)^{1/2}$. The spectrum of radial wave numbers κ_{02} is found from the condition $J_2(\kappa_{02}R) = 0$. The dispersion properties of oscillations with u_z, $u_r \neq 0$ are described by more complicated expressions because the longitudinal and transverse waves do not separate out in the boundary conditions. For surface acoustic oscillations in a cylinder, one can readily obtain the well known dispersion relation

$$(\varkappa_t^2 + k_z^2)\frac{I_0(\varkappa_l R)}{I_1(\varkappa_l R)} - 4\varkappa_t\varkappa_l k_z^2 \frac{I_0(\varkappa_t R)}{I_1(\varkappa_t R)} + \frac{2\varkappa_l(k_z^2 - \varkappa_t^2)}{R} = 0, \qquad (3.4)$$

where $\kappa_{l,t}^2 = k_z^2 - \omega^2/s_{l,t}^2 > 0$, and I_0 and I_1 are the modified Bessel functions.

For large R, (3.4) transforms into the well-known dispersion relation for Rayleigh waves in a semibounded medium.[18] The velocity of the surface wave is $\omega/k_z = s_t\xi$, where $\xi < 1$ is a positive number depending on s_t/s_l (see[18]). For large but finite values of $k_z R$, the velocity of the Rayleigh wave is greater than $s_t\xi$ by an amount of the order of $1/k_z R$.

We must now consider the resonance interaction between helicons and transverse acoustic waves within the body of the specimen. The resonance condition in a bounded medium is that, in addition to the equality of the frequencies and of the z components of the wave numbers, an integral number of half-waves can be fitted into the radius R for each partial oscillation. In other words, $J_1(\kappa_{01}R)$ and $J_2(\kappa_{02}R)$ must both vanish. The corresponding resonance points without the interaction are shown in Fig. 2. The wave numbers k_{01} and k_{02} are not equal because of the difference between κ_{01} and κ_{02}. Assuming that in this interaction the coupled oscillations are transverse, we obtain the following characteristic equation which determines the frequency as a function of the total wave number:

$$\left(\frac{\omega^2}{s_t^2} - k^2\right)\left(\omega - k_z v_0 \mp \frac{ckk_z H_{0z}}{4\pi e N_0}\right) = \frac{k_z^2 H_{0z}^2 \omega}{4\pi\rho s_t^2}\left(1 \pm \frac{4\pi e v_0 N_0}{ckH_{0z}}\right). \qquad (3.5)$$

This is somewhat different from (2.2).

The upper sign refers to the coupled wave with polarization curl H = kH and the lower corresponds to curl H = −kH. To be specific, we assume that k, k_z, and ω are all positive. If we use the fact that in coupled waves $u_r = u_z = 0$, and introduce the boundary conditions for u_φ and H_r, we obtain a dispersion relation which describes the interaction between a slow helicon and sound:

$$J_1(\varkappa_1 R) J_2(\varkappa_2 R) = -\frac{H_{0z}^2}{4\pi\rho s_t^2} \frac{k_0^2 \varkappa_{01} k_{02} J_1(\varkappa_{02}R) J_2(\varkappa_{01}R)}{(k_{01} + k_{02})(k_{01} - k_{02})^2}, \qquad (3.6)$$

where

$$k_0 = \frac{4\pi e s_t N_0}{cH_{0z}}, \quad k_{01} = \frac{4\pi e N_0}{cH_{0z}}\left(v_0 - \frac{\omega}{k_z}\right), \quad k_{02} = \frac{\omega}{s_t}.$$

All these quantities are positive.

Equation (3.6) can be used to find the correction $\delta\omega$ to the frequency, since $\kappa_\alpha = \kappa_{0\alpha} + \delta\kappa_\alpha$ and

$$\delta\varkappa_\alpha \approx \frac{k_{0\alpha}}{\varkappa_{0\alpha}} \frac{\partial k_{0\alpha}}{\partial\omega} \delta\omega. \qquad (3.7)$$

In the case of the resonance interaction, we can neglect

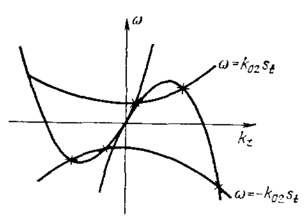

FIG. 2. Schematic plot of ω vs k_z for coupled helicon and transverse acoustic waves in a cylindrical specimen.

in (3.7) the corrections $\delta k_{0\alpha}$ due to the change in the wave numbers. These corrections are small relative to the parameter $(H_{0z}^2/4\pi\rho s_t^2)^{1/2}$. Resonance occurs when the following condition is satisfied:

$$k_{02}s_t = k_{z\,\mathrm{res}}\left(v_0 - \frac{ck_{01}H_{0z}}{4\pi eN_0}\right). \tag{3.8}$$

Expanding the left hand side of (3.6) in powers of $\delta\omega$, and recalling the resonance condition (3.8), we finally obtain

$$(\delta\omega)^2 = \frac{H_{0z}^2}{4\pi\rho s_t^2}\,\frac{\varkappa_{01}^2 k_0^2 s_t\,\partial\omega/\partial k_{01}}{R^2 k_{01}(k_{01}+k_{02})(k_{01}-k_{02})^2}. \tag{3.9}$$

The sign on the right hand side of (3.9) is the same as the sign of the derivative $\partial\omega/\partial k_{01}$, i.e., it is determined by helicon dispersion. Since a slow helicon has anomalous dispersion ($\partial\omega/\partial k_{01} < 0$), its resonant interaction with the transverse volume sound leads to the excitation of coupled oscillations. The growth of the waves does not occur during the interaction between a fast helicon and sound because $\delta\omega^2 \sim \partial\omega/\partial\omega_{01} > 0$.

The criterion for the applicability of (3.9) can readily be obtained by estimating the relative role of the terms in the "equations of motion" (1.7) which contain the magnetic field $H_{0\varphi}$ due to the current. Simple estimates[32] show that the necessary and sufficient condition for the oscillations to be transverse is

$$\frac{\omega}{|\delta\omega|} \gg \left(\frac{k_{01}cH_{0\varphi}}{4\pi eN_0}\right)^2 \frac{1}{s_l^2 - s_t^2}. \tag{3.10}$$

The degree of "depolarization" of the sound waves as a result of the interaction between the waves, i.e., the relative magnitude of the component u_r, is small and is of the order of $|\delta\omega|/\omega$.

Let us now consider the interaction between helicons and acoustic waves when the lattice displacement vector has nonzero components u_r and u_z, and $u_\varphi = 0$. From (1.7) and (1.8) it follows that the coupling coefficient between the induced axially-symmetric acoustic oscillations and helicons is independent of $H_{0\varphi}$, and for longitudinal sound it is a function of $H_{0\varphi}$ and H_{0z}. Since we are assuming that $H_{0\varphi}(R) \gg H_{0z}$, it will be sufficient in estimating the growth rate to restrict our attention to the interaction between the helicons and the longitudinal acoustic waves, and to neglect the constant magnetic field H_{0z} in the coupling coefficient (the coupling between the transverse and longitudinal acoustic waves is then achieved through conditions on the boundary).

The resulting set of equations can be solved by the method of successive approximations, using the small parameter

$$\frac{H_{0\varphi}^2}{4\pi\rho s_l^2} \ll 1. \tag{3.11}$$

The result is a dispersion relation which describes coupled Rayleigh and helical electromagnetic waves in a semiconductor cylinder[32]. However, we shall not attempt to write out this complicated equation, and will reproduce only the final expression for the correction to the frequency near resonance. The resonant interaction between helicons and Rayleigh sound waves occurs when

$$s_l\xi = v_0 \pm \frac{ck_{01}H_{0z}}{4\pi eN_0}, \quad k_{01} = \sqrt{\varkappa_{01}^2 + k_z^2}, \tag{3.12}$$

where $\kappa_{01}R$ is the root of $J_1(\kappa)$. The frequency corrections are given by

$$(\delta\omega)^2 = \frac{H_{0\varphi}^2}{4\pi\rho s_l^2}\,\frac{\partial\omega}{\partial k_{01}}\,\frac{\omega k_{01}\varkappa_{01}^2}{R(\varkappa_l^2+\varkappa_{01}^2)^2}\left[\frac{1}{\varkappa_l} + \left(\frac{s_l}{s_t}\right)^2 \frac{\varkappa_l k_z^2 - (\varkappa_t^2 + k_z^2)\varkappa_t}{\varkappa_l^2 k_z^2}\right]^{-1}. \tag{3.13}$$

Since in Rayleigh waves $\kappa_l > \kappa_t$ and $k_z > \kappa_t$, the quantity in the square brackets is positive and the sign of the right hand side is the same as the sign of the helicon group velocity $\partial\omega/\partial k_{01}$. Consequently, the instability of the coupled oscillations is possible only for an interaction between Rayleigh sound waves and slow helicon with anomalous dispersions. The situation is essentially the same as in the case of resonance between a helicon and transverse volume sound. However, the growth of the oscillations in this case occurs at smaller drift velocities than in the case of the interaction between a helicon and volume sound waves.

In a thin rod, the transverse dimensions of which are small in comparison with the wavelength and the depth of penetration of the waves into the specimen, the dispersion relation (3.4) leads to the well known dispersion law for longitudinal oscillations

$$\omega = k_z s_t \sqrt{2(1+\sigma)}, \tag{3.14}$$

where σ is the shear modulus. The phase velocity of this wave is less than the velocity of the longitudinal volume sound. The interaction between the oscillations (3.14) and helicons is possible only through the magnetic field due to the constant current, provided the radial wave number κ_{01} of the helicon is large in comparison with k_z. The growth rate of the coupled waves [slow helicon plus the wave (3.14)] is determined by

$$(\delta\omega)^2 = \frac{H_{0\varphi}^2}{2\pi\rho s_l^2}\,\frac{\partial\omega}{\partial k_{01}}\,\frac{(k_z s_t)^2(1-\sigma)}{\omega\varkappa_{01}R^2}. \tag{3.15}$$

The resonance condition can be used to determine the magnetic field H_{0z} for given specimen size, drift velocity, and electron density. If the transverse dimensions of the specimen are smaller than the wavelength in the radial direction ($\kappa_{01}R > 1$), the amplitude of the electromagnetic wave is strongly reduced and the interaction between the sound waves and the helicons is reduced to zero.

For axially nonsymmetric oscillations, the calculation of the growth rate becomes exceedingly laborious. We shall therefore restrict ourselves to the following note. From the resonance condition (2.1) for slow helicons and sound waves it is clear that, for nonsymmetric oscillations, the effective magnetic field H_{0z}^* can be less the external field H_{0z}. This leads to a reduction in the drift velocity near resonance. We note that, in the case of a perfectly conducting surface, there are no surface helicons with both $l = 0$ and $l \neq 0$. The conclusion reported in[32,33] that there are axially nonsymmetric surface helicons is therefore incorrect.

In conclusion, let us briefly consider the instability of coupled waves in a specimen placed in a vacuum. In this case we must take into account both the helicon and the surface oscillations in the medium[34,35] because helicons alone are insufficient to satisfy the boundary conditions. Allowance for the surface oscillations, despite their strong damping, leads to collisionless helicon losses, and the presence of drift leads to the collisionless growth of oscillations. The reason for these losses can be found in the loss of energy through the excitation of surface oscillations.

III. TWO-STREAM INSTABILITY

4. Instability in an infinite medium. We have considered the interaction between waves and conduction electrons drifting with constant velocity under the action of an electric field. However, since the drift

velocity in semiconductors is quite low ($v_0 \approx 10^6-10^7$ cm/sec), drift instability is restricted to relatively low frequencies and slow waves. New possibilities appear in the case of the interaction of oscillations and electron beams moving in vacuum above the surface of a conducting solid. The two-stream instability in gas-discharge plasmas was predicted by Pearce[36], Akhiezer and Faĭnberg[37], and by Bohm and Gross,[2] and has now been extensively investigated[5]. On the other hand, two-stream instability in solids has been studied to a much lesser extent. One of the first papers in this area was the review by Lopukhin and Vedenov[38]. These authors discussed the principle of a resistive amplifier based on the phase shift between the electron current in a beam and the alternating field of a wave propagating in solid-state plasma. It is clear that this interaction is much more effective when natural oscillations are excited in the solids, e.g., electromagnetic waves[39], excitons[40], spin waves[41], surface plasmons[42a,43,44], and so on. The growth rate increases rapidly as we approach resonance where the wave frequency is equal to one of the natural oscillation frequencies in the moving beam

$$\omega = kv_0, \quad \omega = kv_0 - \Omega_H; \tag{4.1}$$

where v_0 is the velocity of the beam and $\Omega_H = |e|H/m_0c$ is the electron cyclotron frequency.

In this section we shall review the theory and discuss some of the experiments on instability due to the resonance interaction between electron beams and surface[42a,43-46] and volume waves[46-48] in solid-state plasma. The beam will be assumed to be completely compensated, and the thermal motion of the particles in the beam will be neglected. In the unperturbed state, the densities and velocities of the light and heavy components of the beam are equal. Variable fields perturb only the electron component, and the ion density and velocity remains unaltered. To determine the nonequilibrium part of the density n and the perturbation in the velocity v we use the linearized equations of hydrodynamics in the form

$$\begin{aligned} &\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}_0\nabla)\,\mathbf{v} = \frac{e}{m_0}\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}_0 \times \mathbf{H}] + \frac{1}{c}[\mathbf{v} \times \mathbf{H}_0]\right) \quad (e > 0),\\ &\frac{\partial n}{\partial t} + n_0 \operatorname{div} \mathbf{v} + (\mathbf{v}_0\nabla)\, n = 0; \end{aligned} \tag{4.2}$$

where m_0 is the mass of a free electron, while n_0 and v_0 are the equilibrium values of the density and velocity of the beam.

The physical origin and the properties of two-stream instability can be illustrated by the example of the interaction between transverse excitons and particles in an infinite medium in the absence of a constant magnetic field[40]. In this case, the Maxwell equations assume the form

$$\begin{aligned} &\operatorname{rot} \mathbf{H} = -\frac{4\pi e}{c}(n\mathbf{v}_0 + n_0\mathbf{v}) + \frac{1}{c}\frac{\partial}{\partial t}(\mathbf{E} + 4\pi\mathbf{P}),\\ &\operatorname{rot} \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \end{aligned} \tag{4.3}$$

where $\mathbf{P}$ is the electric polarization vector which is related to the electric field by[49,50]

$$\frac{\partial^2 \mathbf{P}}{\partial t^2} + \omega_e^2 \mathbf{P} - \alpha\Delta\mathbf{P} = \gamma\mathbf{E}; \tag{4.4}$$

where ω_e is the frequency of exciton absorption, the parameter α describes the spatial dispersion ($\alpha \approx \hbar\omega_e/m$), and the constant $\gamma \approx Ne^2/m$ is proportional to the strength of the oscillator N with frequency ω_e. The dispersion relation for a beam with excitons is

$$[\omega^2\varepsilon(\omega, \mathbf{k}) - k^2c^2](\omega - \mathbf{k}\mathbf{v}_0)^2 = \Omega_b^2\left[\omega^2 - \frac{c^2(\mathbf{k}\mathbf{v}_0)^2}{v_0^2\varepsilon(\omega, \mathbf{k})}\right], \tag{4.5}$$

where $\Omega_b = \sqrt{4\pi e^2 n_0/m_0}$ is the Langmuir frequency of the beam, and

$$\varepsilon(\omega, \mathbf{k}) = 1 - \frac{4\pi\gamma}{\omega^2 - \omega_e^2 - \alpha k^2} \tag{4.6}$$

is the permittivity of the medium with allowance for spatial dispersion. If we assume that the beam density is small and $\Omega_b \to 0$, Eq. (4.5) splits into two:

$$\omega = \mathbf{k}\mathbf{v}_0, \quad \omega^2 = k^2c^2\varepsilon(\omega, \mathbf{k}). \tag{4.7}$$

The first of these describes the oscillations of a beam with vanishing density, and the second, the natural electromagnetic waves in the medium. When $k^2c^2 > \omega_e^2$, the second equation in (4.7) yields the spectra of photons and excitons:

$$\omega_1^2 \approx k^2c^2 + 4\pi\gamma, \quad \omega_2^2 = (\omega_e^2 + \alpha k^2)\left(1 - \frac{4\pi\gamma}{k^2c^2}\right).$$

At resonance, the frequencies and wave numbers of the beam oscillations and the excitons are equal. When the beam density is small but finite, the growth rate of the coupled wave is, in accordance with (4.5),

$$\eta = \operatorname{Im}\omega = \sqrt{3}\,\omega\sqrt[3]{\frac{\pi\gamma\Omega_b^2\sin^2\theta}{4k^4c^4}}, \tag{4.8}$$

where θ is the angle between $\mathbf{k}$ and $\mathbf{v}_0$. From the resonance condition we see that the beam velocity v_0 must be greater than the phase velocity of the wave ω/k. This means that the instability mechanism is connected with Cerenkov radiation. If we take the typical parameter values $k \approx 10^5$ cm^{-1}, $\omega_e \sim 4\pi\gamma \approx 10^{14-15}$ sec^{-1}, $v_0 = 0.1$ c, and $n_0 = 3 \times 10^{10}$ cm^{-3}, we find that the relative growth rate turns out to be of the order of 10^{-5}. Since for a number of materials[49-50] the relative damping may be lower by an order of magnitude, excitons can, at least in principle, be amplified by the beam. However, this effect has not as yet been observed experimentally. We note that in optically active media such as quartz and cinnabar[12] the growth rate may turn out to be much greater.

The character of the instability depends on the sign of the group velocity, i.e., the sign of α. In the case of anomalous dispersion ($\alpha < 0$) the instability defined by (4.8) is absolute, and for normal dispersion ($\alpha > 0$) it is convective.

In addition to transverse waves, the beam can exite and amplify longitudinal excitons, the spectrum of which is determined by the zeros of the function $\epsilon(\omega, \mathbf{k})$. The dispersion equation and growth rate for longitudinal excitons can readily be found from (4.5) by substituting $c = \infty$.

The excitation of excitons should of course be carried out in thin specimens. This means that we must analyze the interaction between a beam and waves in the bounded medium.

5. Boundary conditions. In an inhomogeneous system with a sharp boundary the question of boundary conditions is an important one. Let us begin by considering a quasineutral beam of electrons moving in vacuum above the plane surface $y = 0$ of a semiconductor. The tangential components of the electric field must then be continuous across the separation boundary. The particular feature of such problems is the appearance of surface currents which are due to the transport of perturbations in the electron density of the beam along the boundary. The normal components of the induction vector D_y are then no longer continuous, and the discontinuity is proportional to the beam velocity v_0.[51,52] Mathematically, this is connected with the fact that the tangential components of the induction vector contain derivatives with respect to the normal coordinate because the varying current in the beam

$$j_z \sim -|e| v_0 n = -\frac{|e| v_0}{i(\omega - k_z v_0)} \frac{\partial (n_0 v_y)}{\partial y} \qquad (5.1)$$

is proportional to $\partial(n_0 v_y)/\partial y$. It follows that when the equation div D = 0 is integrated over an infinitesimally thin transition layer, the integrals containing the tangential component D_z provide a finite contribution. If we express v_y in terms of the electric fields through the equations of motion (4.2), the boundary condition for the discontinuity in D_y can be written in the form

$$[D_y^{(2)} - D_y^{(1)}]_{y=0} = -\frac{\Omega_b^2 k_z v_0}{\omega^2 \Delta}\left[\omega E_y^{(2)} + i\Omega_H E_x + \frac{ik_x v_0 \Omega_H}{\omega - k_z v_0} E_z - k_z v_0 \left(E_y^{(2)} + \frac{i}{k_z}\frac{\partial E_z^{(2)}}{\partial y}\right)\right]_{y=0};$$
$$\Delta = \Omega_H^2 - (\omega - k_z v_0)^2. \qquad (5.2)$$

Here and henceforth the indices 1 and 2 refer to regions occupied by the semiconductor and the beam respectively[4)].

For semi-infinite systems, the boundary condition (5.2) must be augmented by the condition that all the variable fields vanish as $y \to \infty$. In a bounded specimen, additional boundary conditions are necessary for other surfaces. By solving (4.2) simultaneously with Maxwell's equations we obtain from the boundary conditions the dispersion relations describing the interaction of the beam with surface and volume oscillations.

The Maxwell equations in the beam have the form given by (4.3) when **P** = 0, and the electromagnetic properties of the medium are characterized by the permittivity

$$\varepsilon_{ik} = \varepsilon_0 \delta_{ik} + \varepsilon_{ik}^*. \qquad (5.3)$$

In this expression ϵ_0 is the lattice part of the permittivity of the specimen, and ϵ_{ik}^* is its electron part. The nonzero elements of the tensor δ_{ik}^* are given by the well known expressions

$$\varepsilon_{xx}^* = \varepsilon_{yy}^* = -\frac{\omega_0^2(\omega + i\nu)}{\omega[(\omega + i\nu)^2 - \omega_H^2]}, \quad \varepsilon_{xy}^* = \frac{i\omega_0^2 \omega_H}{\omega[(\omega + i\nu)^2 - \omega_H^2]},$$
$$\varepsilon_{zz}^* = -\frac{\omega_0^2}{\omega(\omega + i\nu)}, \qquad (5.4)$$

where ω_0 is the plasma frequency of the current carriers in the semiconductor and the vector **H** is parallel to the z axis.

6. Interaction between potential oscillations and a beam. In the case of potential peturbations the variable magnetic field is much smaller than the electric field because the conduction and displacement currents almost completely compensate one another. Assuming that at large distances from the separation boundary the equilibrium electron density is constant, we obtain the characteristic equation in the form

$$k_i k_j \varepsilon_{ij} = 0, \qquad (6.1)$$

If we take into account the symmetry properties of the tensor ϵ_{ij} then we can only find k_y from (6.1):

$$k_y^2 = -k_x^2 - k_z^2 \frac{\varepsilon_{zz}^*}{\varepsilon_{xx}}, \qquad (6.2)$$

where the signs of k_y are chosen so that the fields inside and outside the specimen are damped out, i.e., $\mathrm{Im}\, k_y^{(2)} > 0$.

It is readily shown that, for potential oscillations, the components ϵ_{xx} and ϵ_{zz} in the beam are given by

$$\varepsilon_{xx}^{(2)} = \varepsilon_{yy}^{(2)} = 1 + \frac{\Omega_b^2}{\Delta}, \quad \varepsilon_{zz}^{(2)} = 1 - \frac{\Omega_b^2}{(\omega - k_z v_0)^2}. \qquad (6.3)$$

From the boundary conditions (5.2) and the continuity of the tangential components of **E** we obtain the dispersion relation for the potential oscillations[45] in the form

$$k_y^{(2)}\varepsilon_{yy}^{(2)} - k_y^{(1)}\varepsilon_{yy}^{(1)} + k_x\left[\varepsilon_{yx}^{(1)} - \frac{i\Omega_H \Omega_b^2}{\Delta(\omega - k_z v_0)}\right] = 0. \qquad (6.4)$$

In the absence of the constant magnetic field, the dispersion relation (6.4) describes the interaction between the beam and surface plasmons

$$\left[\varepsilon_0 + 1 - \frac{\omega_0^2}{\omega(\omega + i\nu)}\right](\omega - k_z v_0)^2 = \Omega_b^2. \qquad (6.5)$$

We note that when $\epsilon_0 = 1$, $\nu = 0$ this equation describes the instability of glancing beams[5]. At resonance, when

$$\omega_p \equiv \omega_0(\varepsilon_0 + 1)^{-1/2} = k_z v_0, \qquad (6.6)$$

the growth rate of the coupled wave, in the approximation in which it is assumed that the beam density is low, is given by

$$\mathrm{Im}\,\omega = \frac{\sqrt{3}}{\sqrt[3]{2^4}} \frac{\sqrt[3]{\omega_0 \Omega_b^2}}{\sqrt{\varepsilon_0 + 1}} \propto \sqrt[3]{n_0}. \qquad (6.7)$$

This formula is valid in the case of strong coupling between the beam and the plasmons when $\mathrm{Im}\,\omega \gg \nu$. It can be shown that the above instability is convective in character because the group velocity of the surface plasmons is positive[17]. In the case of weak coupling ($\mathrm{Im}\,\omega \ll \nu$) the growth of the oscillations is also possible. In this case

$$\delta\omega \equiv \omega - k_z v_0 = \pm \Omega_b \frac{k_z v_0 (k_z v_0 + i\nu)}{k_z^2 v_0^2 (\varepsilon_0 + 1) - \omega_0^2 + i\nu k_z v_0}. \qquad (6.8)$$

The instability given by (6.8) is in a sense analogous to the "resistive" instability reviewed in[38]. However, because of the frequency dispersion of the permittivity $\epsilon(\omega) = \epsilon_0 - \omega_0^2/\omega^2$ the growth rate is a maximum under the resonance conditions (6.6). When $\omega \gg \nu$, the growth rate at the maximum is given by

$$\mathrm{Im}\,\delta\omega = \Omega_b \sqrt{\omega_0/2\nu}. \qquad (6.9)$$

It is interesting to note that this type of instability will also arise on the boundary between two plasma-like media if in one of them the electrons drift under the action of a constant electric field. According to[42b] the dispersion relation for potential oscillations is somewhat different to that given in (6.5) and takes the form

$$\left(\varepsilon_0 - \frac{\omega_0^2}{\omega(\omega + i\nu)}\right)(\omega - k_z v_0) = -4\pi i\sigma, \qquad (6.10)$$

where $\epsilon_0 = \epsilon_{01} + \epsilon_{02}$, v_0 is the drift velocity, and σ is the static conductivity of the specimen. Hence, when the conductivity σ is small, the growth rate is given by

$$\mathrm{Im}\,\delta\omega = \frac{4\pi\sigma (k_z v_0)^2 [\omega_0^2 - \varepsilon_0 (k_z^2 v_0^2 + \nu^2)]}{(\varepsilon_0 k_z^2 v_0^2 - \omega_0^2)^2 + \varepsilon_0^2 \nu^2 k_z^2 v_0^2}. \qquad (6.11)$$

The growth of drift oscillations occurs when the real part of the effective permittivity $\epsilon_0 - [\omega_0^2/k_z v_0 (k_z v_0 + i\nu)]$ is negative (the expression in the square brackets in (6.11) is positive). Although the growth rate given by (6.11) is proportional to the first power of the density of the drifting plasma, its absolute magnitude may turn out to be greater than in the case of (6.9) because of the greater concentration of current carriers in the semiconductor plasma.

We must now consider the possible amplification of surface waves in semiconductors by an electron beam. In the absence of collisions, the phase velocity of a surface wave $\omega/k_z = c\sqrt{1-|\epsilon_1|^{-1}}$ can be as low as desired. In particular, when $\omega = \omega_0(\epsilon_0+1)^{-1/2}$ this velocity is zero. When collisions are taken into account, the wave exhibits relatively small attenuation for $|\epsilon_1| - 1 \gg \epsilon_1^{(i)}$ where $\epsilon_1^{(i)} = \omega_0^2\nu/\omega^3$. For typical values of the parameters, i.e., $\nu \approx 10^{11}$ sec^{-1}, $\omega_0 \approx 3\times 10^{12}$ sec^{-1}, and $\omega \approx \omega_0\epsilon^{-1/2} \approx 10^{12}$ sec^{-1}, the quantity $\epsilon_1^{(i)}$ turns out to be of the order of unity. This means that in such specimens the minimum velocities of waves in the millimeter band may reach up to $c/\sqrt{2}$, i.e., a little less than c. In other words, the strong resonance coupling (6.6) between oscillations in the millimeter band is achieved only in the case of beam relativistic velocities. For velocities $v_0 \approx 0.1c$, only the weak (nonresonant) coupling (6.8) can be realized in this region. In the submillimeter band, the phase velocity of the wave decreases and resonance amplification can be achieved with a nonrelativistic beam ($v_0 \lesssim 0.1c$).

We note that the velocity of surface waves of exciton origin in the optical region may turn out much less, i.e., of the order of 10^9 cm/sec. The dispersion relation (6.5) remains valid in this case. If the quantity $\epsilon_0 - [\omega_0^2/\omega(\omega+i\nu)]$ is replaced by $1 - 4\pi\gamma[\omega(\omega+i\nu) - \omega_e^2]$. Under these conditions $\nu/\omega_e \approx 10^{-6}$ and the amplication conditions become favorable from the experimental standpoint.

Electron beams are usually focused with a magnetic field $H_0 \sim 10^4$ Oe. Since in such fields the cyclotron and plasma frequencies of electrons are of the same order, one would expect some new instability features.

Let us begin by considering the interaction of a beam with "oblique" potential oscillations[43] in which

$$k_x^2 \gg k_z^2 |\varepsilon_{zz}/\varepsilon_{yy}|.$$

Substituting $k_y^{(1,2)} = \pm i k_x (k_x > 0)$ in (6.4), we obtain

$$\left[\omega(\omega-\omega_H+i\nu) - \frac{\omega_0^2}{\varepsilon_0+1}\right](\omega-k_z v_0)(\omega-k_z v_0+\Omega_H) = \Omega_b^2 \frac{\omega(\omega-\omega_H+i\nu)}{\varepsilon_0+1}. \tag{6.12}$$

In the limit as $\Omega_b \to 0$, the relation given by (6.12) splits into three independent equations describing the surface waves in a semiconductor and the charge-density waves in the beam (4.1) in the presence of a magnetic field. By setting the expression in the square brackets in (6.12) to zero we obtain the limiting frequencies of surface waves:

$$\omega^{\pm} = \frac{\omega_H - i\nu}{2} \pm \sqrt{\left(\frac{\omega_H - i\nu}{2}\right)^2 + \frac{\omega_0^2}{\varepsilon_0+1}} \tag{6.13}$$

where the root is positive for $\nu \to 0$. In a strong magnetic field

$$\omega^+ = \omega_H + \frac{\omega_0^2}{\omega_H(\varepsilon_0+1)}, \quad \omega^- = -\frac{\omega_0^2}{\omega_H(\varepsilon_0+1)}\left(1+i\frac{\nu}{\omega_H}\right). \tag{6.14}$$

We note that oscillations with frequency ω^- exist in a broad range of frequencies $\omega \ll \omega_H$ independently of the ratio of ω to ν (the relative growth rate is ν/ω_H).

We must now consider the resonance interaction between surface waves and beam oscillations $\omega = k_z v_0 - \Omega_H$ under the conditions of anomalous Doppler effect. At resonance, when $\omega^+ = k_z v_0 - \Omega_H$ the imaginary corrections to the frequency are given by

$$\delta\omega \equiv \omega - \omega^+ = \frac{i}{2}\left[-\nu \pm \sqrt{\nu^2 + \frac{4\Omega_b^2\omega_r^+(\omega_r^+-\omega_H)}{(\varepsilon_0+1)\,\Omega_H\,(\omega_r^+-\omega_r^-)}}\right]. \tag{6.15}$$

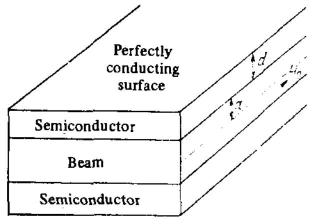

FIG. 3

In this expression the subscript r represents the real part of the frequency (6.14). It is clear that the instability arises when the second term under the square root is positive. In the case of strong coupling ($\nu \to 0$), the growth rate is proportional to $\Omega_b \sim \sqrt{n_0}$. In the case of weak coupling, the square root in (6.15) can be expanded in powers of the second term, and the growth rate turns out to be proportional to $\Omega_b^2 \sim n_0$. Similar results are obtained for the interaction between the waves (6.14) and the oscillations in the beam $\omega = k_z v_0$.

We have investigated potential oscillations with large values of k_x. In the opposite limiting case $k_x \ll k_z$ there are no surface waves in the region of strong magnetic fields. In fact it follows from (6.4) that the quantities k_i are real, i.e., the oscillations are volume oscillations. Let us therefore consider a sandwich-type structure (Fig. 3) consisting of a beam and two semiconducting layers[6)]. This geometry corresponds to the experimental conditions described in[45]. The dispersion properties of this system are described by

$$\varepsilon_{yy}^{(2)} k_{y2} \operatorname{ctg}\left(k_{y2}a - \alpha\frac{\pi}{2}\right) + \varepsilon_{yy}^{(1)} k_{y1} \operatorname{ctg}\left(k_{y1}d - \beta\frac{\pi}{2}\right) = 0. \tag{6.16}$$

The values $\alpha = 0$ (and $\alpha = 1$) correspond to the symmetric and antisymmetric distribution of the electric field in the beam, respectively ($|y| \leq a$); $\beta = 0$ corresponds to perfectly conducting outer boundaries $|y| = a + d$, and $\beta = 1$ corresponds to the boundary with vacuum. Suppose that $\alpha = 1$, $\beta = 0$. In a strong magnetic field ($\omega_H \gg \omega$) $\epsilon_{yy}^{(1)} = \epsilon_0$, $\epsilon_{yy}^{(2)} = 1$. If $\mathrm{Im}(k_{y1}d) \gg 1$, the equation given by (6.16) can be written in the form

$$\varepsilon_0 k_{y1} \operatorname{ctg}(k_{y2}a) = i k_{y2}, \tag{6.17}$$

where

$$k_{y1} = k_z\sqrt{\frac{\omega_0^2}{\varepsilon_0\omega(\omega+i\nu)} - 1}, \quad k_{y2} = k_z\sqrt{\frac{\Omega_b^2}{(\omega-k_z v_0)^2} - 1},$$
$$\mathrm{Im}\,k_{y1} > 0, \quad \mathrm{Re}\,k_{y2} < 0.$$

To simplify the final formulas we suppose that $\epsilon_0 \gg 1$. In that case,

$$\Delta\omega \approx \frac{i k_{y2}^3(\omega-k_z v_0)}{\varepsilon_0 k_{y1} a (k_z^2+k_{y2}^2)}, \quad k_{y2}a \approx \pi\left(n+\frac{1}{2}\right) \tag{6.18}$$

Hence it is clear that the maximum growth rate is achieved for $\omega \approx \omega_0\epsilon_0^{-1/2}$.

In the limiting case of a "thin" beam ($k_{y2}a \ll 1$) and a thin plate ($k_{y1}d \ll 1$) placed in a vacuum ($\beta = 1$), the dispersion relation (6.16) for symmetric oscillations ($\alpha = 1$) can readily be transformed with the aid of (6.2) so that it assumes the form

$$\varepsilon_{zz}^{(2)} a + \varepsilon_{zz}^{(1)} d = 0. \tag{6.19}$$

Since the components ϵ_{zz} are independent of H_0, this equation describes the one-dimensional interaction between longitudinal plasma waves and the beam. It differs from (6.5) by the fact that the quantities ϵ_0 and ω_0^2 contain the additional factor d/a. Therefore, when $d < a$, the growth rate increases by a factor of $(a/d)^{1/3}$, in accordance with (6.7).

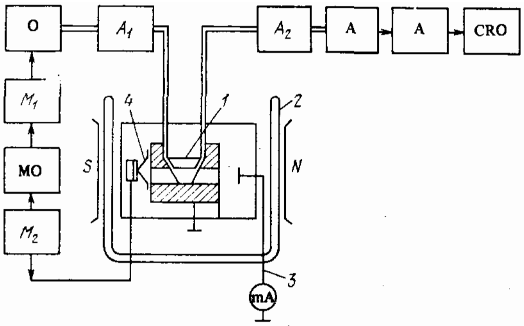

FIG. 4. Measuring system [4]: O—oscillator, $A_{1,2}$-attenuators, $M_{1,2}$-modulators, MO—master oscillator, A–amplifier, 1–specimen, 2–dewar, 3–collector, 4–electron gun.

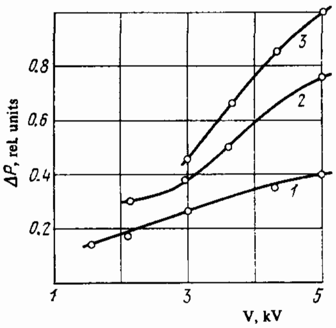

FIG. 5

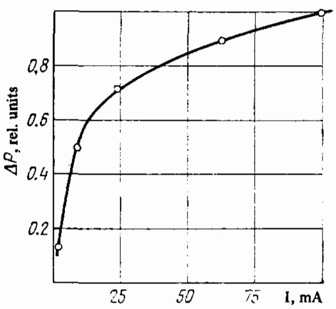

FIG. 6

FIG. 5. ΔP as a function of V. Beam current (mA): 90 (1), 120 (2), 200 (3).

FIG. 6. ΔP as a function of beam current (V = 4 kV).

The interaction between a beam and plasma oscillations were investigated experimentally in germanium[54] and in indium antimonide[45]. In the latter case, the single crystal specimen of n-InSb had a mobility of $7\times10^5\ \mathrm{cm^2V^{-1}sec^{-1}}$ and concentration $N_0 = 5\times10^{13}\ \mathrm{cm^{-3}}$ at liquid-nitrogen temperatures. The effective mass in these specimens was $m_e \approx 0.015\, m_0$ and the collision frequency was $\nu = 1.7\times10^{11}\ \mathrm{sec^{-1}}$. The static permittivity was $\epsilon_0 = 16$.

Plasma resonance was detected at the frequency $\omega_0/\sqrt{\epsilon_0} = 8.1\times10^{11}\ \mathrm{sec^{-1}}$ which corresponds to a vacuum wavelength $\lambda = 2.3$ mm. The resonance was observed by measuring the reflection coefficient as a function of frequency in the millimeter range. The specimen was placed in a rectangular waveguide (7.2 × 3.4 mm) and its thickness was chosen to be sufficiently large for the wave reflected from the other boundary to become negligible.

The interaction between the beam and the specimen was investigated with the apparatus illustrated in Fig. 4. The transverse dimensions of the beam were $0.4\times3.5\ \mathrm{mm^2}$, and the beam was focussed by a magnetic field. Acceleration was achieved with a power unit producing a variable pulse length between 0.2 and 100 μsec. The InSb specimen was in the form of a rectangular plate, 6 mm long, 2.5 mm thick, and 3.5 mm wide. It had wedge-shaped slots so that matching could be achieved with the input and output of the waveguide. The oscillators were backward-wave tubes, operating in the 12, 5, 2 and 1.5 mm bands under pulsed conditions (the oscillator pulse length was somewhat greater than the accelerating pulse length).

Figures 5 and 6 show the power increment ΔP as a function of the voltage and current in the beam at the resonance wavelength $\lambda = 2.3$ mm. This increment is the difference between the output power of the system when a beam is present and the power P_0 in the absence of the beam. For small values of the ratio $\Delta P/P_0$ this ratio is proportional to the growth rate. We have observed a monotonic increase in ΔP with increasing voltage and current. The dependence of the increment on the beam current is essentially given by (6.18). In fact, substituting the expression for $\omega - k_z v_0$ through $k_{y2}a \approx \pi(n+1/2)$, we obtain

$$-\mathrm{Im}\,k_z = \mathrm{Re}\,\frac{\Omega_b\omega\cdot(k_{y2}v_0)^2}{a\omega_0\varepsilon_0^{1/2}(\omega^2+k_{y2}^2v_0^2)^{3/2}}\left(1-\frac{\omega^2\varepsilon_0}{\omega_0^2}-i\frac{\nu}{\omega}\right)^{-1/2}. \quad (6.20)$$

Since the beam density n_0 enters only through the Langmuir frequency Ω_b, the dependence of the growth rate on the current J_b takes the form $|\mathrm{Im}\,k_z| \propto \sqrt{J_b}$.

The formula given by (6.20) can also explain the increase in the growth rate with increasing voltage. If we suppose that $k_{y2}v_0 < \omega$, then for J_b = const the quantity $|\mathrm{Im}\,k_z|$ will be proportional to $v^{3/2} \propto U^{3/4}$, which is in qualitative agreement with experiment. The experimental results reported in[45] are thus explained by the interaction between a finite beam and quasipotential oscillations in a strong magnetic field.

The output power was found to increase with increasing constant magnetic field, but from $H_0 \approx 2$ kOe onward, the power ceased to depend on H_0. This is not predicted by the above formulas which show that when the specimen is not too thin the growth rate decreases with increasing H_0. Under the conditions of the above experiment, the increase in H_0 was accompanied by an increase in the current density near the specimen surface, and an increase in the length of the region in which the beam-wave interaction occurred. The observed increase in the output power with H_0 is connected with this effect.

Because of the relatively high collision frequency ($\nu/\omega \sim 0.1$), we did not unfortunately succeed in separating the interaction with the volume and surface plasmons (their frequencies differ by only 3%). For the same reason, only the case of weak beam-wave coupling was realized in the experiment. Hence amplification due to the presence of the beam only partially compensated the general energy losses experienced by the electromagnetic wave in the semiconductor, and the generation of oscillations could not be achieved.

It is important to note that large conversion losses occur at the output of the system during the transformation of potential into electromagnetic waves. The excitation of nonpotential oscillations by the beam is therefore of considerable interest from this point of view.

7. Interaction between nonpotential waves and a charged particle beam in a strong magnetic field. We must first show that nonpotential surface waves are present near the boundary between the semiconductor and vacuum. We shall assume that a strong magnetic field parallel to the surface of the specimen is present, and that the semiconductor plasma is magnetized. In other words, we shall suppose that

$$|\varepsilon_{xx}| \ll |\varepsilon_{xy}| \ll |\varepsilon_{zz}|, \quad (7.1)$$

where ϵ_{ik} is given by (5.3) and (5.4). In this approximation, the Maxwell equations

$$\left[k^2\delta_{ik} - k_ik_k - \frac{\omega^2}{c^2}\varepsilon_{ik}(\omega, \mathrm{H})\right]E_k = 0 \quad (7.2)$$

lead to the two types of wave. One of them is a helicon and the other a potential wave. The characteristic equation for the helicon (without taking damping into account) can be written in the form

$$k_{yh}^2 = -k_x^2 - k_z^2 - \frac{\omega^4 \varepsilon_{xy}^2}{k_z^2 c^4}. \tag{7.3}$$

From the wave equation for potential oscillations $k_i k_k \epsilon_{ik} = 0$ we find, assuming a strong magnetic field ($\epsilon_0 \gg |\epsilon_{xx}^*|$),

$$k_{ys}^2 = k_z^2 \frac{\omega_0^2}{\varepsilon_0 \omega (\omega + i\nu)}, \quad k_x^2 \ll k_{ys}^2, \quad \operatorname{Im} k_{ys} > 0. \tag{7.4}$$

It is clear that the helicon wave (7.3) will become the surface wave in the case of "oblique" propagation if

$$k_x^2 \gg k_z^2 \quad \frac{\omega^4 |\varepsilon_{xy}^2|}{c^4 |k_z^2|}, \text{ i.e. } k_{yh} = -i|k_x|. \tag{7.5}$$

It is important to note that, near the separation boundary, the helicon is always accompanied by the potential oscillations (7.4). This follows from the fact that, when the wave (7.4) is ignored, we cannot satisfy the boundary conditions on the $y = 0$ plane.

The dispersion law and the damping of nonpotential surface waves can readily be found by using the continuity of the magnetic field across the separation boundary. The result is[48]

$$\omega - 2\omega_H \left(\frac{k_z c}{\omega_0}\right)^2 \operatorname{sgn} k_x = 2i \frac{k_z^2 c^2 |k_x|}{\omega_0 \varepsilon_0 k_{ys}}. \tag{7.6}$$

The right-hand side of this equation describes the damping of the surface wave. The relative damping rate is given by

$$\left|\frac{\delta\omega}{\omega_r}\right| = \left|\frac{\varepsilon_{xy} k_x}{\varepsilon_0 \operatorname{Re} k_{ys}}\right|. \tag{7.7}$$

The condition that the damping is small is more readily satisfied with increasing static lattice permittivity ϵ_0 in the magnetic field and decreasing effective mass of the carriers. A suitable material from this point of view is InSb or PbTe.

In InSb with $N_0 = 5 \times 10^{14}$ cm^{-3}, $\epsilon = 16$, $m = 0.01 m_0$, $\omega = \nu = 3 \times 10^{11}$ sec^{-1}, $H_0 = 10$ kOe, and $k_x = 2k_z$ we have $k_z = 40(1 + 0.3i)$ cm^{-1}. In PbTe in which $N_0 = 10^{17}$ cm^{-3}, $\epsilon = 400$, $m = 0.02 m_0$, $H_0 = 50$ kOe, $k_x = 2k_z$, and $\omega = 5 \times 10^{11}$ sec^{-1}, we have $k_z = 330(1 + 0.15i)$ cm^{-1}.

The surface helicon wave (7.6) was predicted in[42b] and was recently detected experimentally by Baĭbakov and Datsko[55,56] in n-InSb.

The wave was observed in a plate of $5 \times 13 \times 15$ mm at room temperature at frequencies between 20 and 1500 MHz and in magnetic fields up to 25 kOe. The electron density was 1.2×10^{16} cm^{-3} and the mobility was 5×10^4 cm^2 V^{-1} sec^{-1}. The dispersion relation was found by a resonance method based on the excitation of a standing surface wave at the edge of the plate and the detection of the fundamental harmonic of its dimensional resonance.

Figure 7 shows the dependence of the wave frequency on k_z for different values of the magnetic field. It is clear that the theoretical curve based on the formula $\omega = 2\omega_H (k_z c/\omega_0)^2$ is close to the experimental result for the density corresponding to the specimen which was investigated. The quadratic dependence of the frequency on k_z means that the detected wave was in fact a surface helicon. The departure of the experimental curve from the theoretical prediction is due to the fact that the frequency corrections on the right-hand side of (7.6) have been neglected.

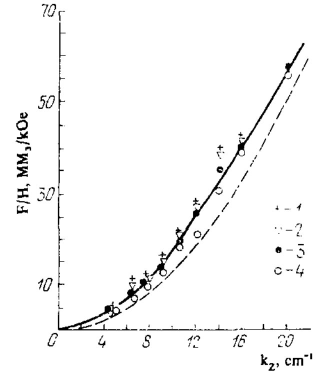

FIG. 7. Dispersion characteristic of a helicon in n-InSb. [56] H(kOe): 23 (1), 15 (2), 10 (3), and 5 (4). The theoretical curve (dashed) refers to Eq. (7.6) with $N_0 = 1.2 \times 10^{16}$ cm^{-3}.

The existence of the helicon is known to be due to the Hall effect in a strong magnetic field. The surface helicon will therefore disappear in compensated plasma. At the same time, Alfven-type surface waves[48] which are analogous to the volume waves[57] may appear. The growth rates for nonpotential surface waves interacting with a beam are found in[43,46,48]. The excitation of helicon-type volume oscillations excited by a beam can evidently also be produced in sandwich-type structures. This problem was considered theoretically in,[47,58] and the corresponding experiments are described in[59,60].

IV. OVERHEATING INSTABILITY

We have so far considered some of the instabilities of solid-state plasma due to the ordered (translational) motion of electrons. Here we shall consider another type of instability which originates from the thermal motion of electrons in external electric and magnetic fields.

We shall thus be concerned with the so-called overheating instability. The mechanism responsible for it may be summarized as follows:

In the stationary state, the mean momentum and energy lost by electrons as a result of collisions with the lattice are compensated by the effect of the constant electric field. At low temperatures, the transfer of energy from electrons to the lattice is impeded because the effective phonon mass T/s^2 is much greater than the electron mass m. Therefore, even at relatively low field strengths, the electron gas is heated up,[61] and the mean electron kinetic energy exceeds the temperature of the crystal. For certain scattering mechanisms, the frequency of electron collisions with momentum transfer decreases as the mean energy increases. This means that the Joule heat absorbed by electrons increases with increasing field. At the same time, there is a reduction in the fraction of energy transferred to the lattice and this, in turn, produces too great a heating of the electron gas, and so on. The result of all this is the appearance of a descending segment on the current-voltage characteristic (negative differential conductivity)[10,11,62-68] 7). This overheating of the electron gas leads to instability. The overheating instability of gas-discharge plasmas was described by Kadomtsev,[4] whilst the corresponding phenomena in semiconductors were considered in[10,11,65,68,72-82]. This instability is usually aperiodic. We shall show below that, under certain conditions, the overheating instability in semiconductors assumes an oscillatory character.

8. Basic equations. To analyze the overheating instability in external electric and magnetic fields, we

must first establish the expressions for the current density j and the heat flow **Q**. This problem can be solved on the basis of the transport equation. We shall not derive these formulas in all their detail, but will confine our attention to the most important approximation and then reproduce the final result (for details see, for example,[72]).

The scattering of electrons by electrons, phonons, and impurities must be taken into account in the collision integral. It will be assumed that the mass of the impurities is infinite and, therefore, the electrons are scattered elastically by them. Moreover, we shall assume that electron-electron collisions leading to the Maxwellization of the electron distribution function play the most important role. In this way, we can derive the effective electron temperature $\Theta(r, t)$, and write the energy conservation law in the form

$$\frac{3}{2}\frac{\partial}{\partial t}(N\Theta)+\operatorname{div}\mathbf{Q}=\mathbf{jE}-\mathcal{P}(N,\Theta), \tag{8.1}$$

where $\mathcal{P}$ is the power transferred by the electron system to the lattice. The criterion for the validity of the definition of the effective electron temperature takes the usual form:

$$N \gg N_{\text{crit}}=\frac{(m\Theta)^{3/2}s^2\varepsilon_0^2}{2\pi L_c T e^4\tau_{\text{ph}}}, \tag{8.2}$$

where T is the temperature of the lattice (T and Θ are measured in energy units), L_C is the Coulomb logarithm, and τ_{ph} is the electron-phonon momentum relaxation time. The inequality given by (8.2) is the condition that the characteristic electron-electron Coulomb relaxation time $\tau_e \approx \sqrt{m}\,\theta^{3/2}\epsilon_0^2/2\pi L_C N e^4$ is small in comparison with the energy relaxation time τ_{ph}/δ ($\delta \approx ms^2/T$ is the small inelasticity during the scattering of electrons by acoustic phonons). Numerical estimates show that the critical concentration N_{crit} for typical values of the semiconductor parameters is of the order of 10^{12}—10^{14} cm^{-3}[8]).

Since the energy transfer between electrons and phonons is slow, the corresponding collision integral can be simplified by expanding it in terms of the small inelasticity[61]. The isotropic part of the distribution function $F_0(\epsilon)$ is then the solution of the Fokker-Planck equation in which the collisional term has the form

$$\hat{\nu}\{F_0\}=\frac{1}{v}\frac{\partial}{\partial\varepsilon}\left[A(\varepsilon)\,v\left(\frac{\partial F_0}{\partial\varepsilon}+\frac{F_0}{T}\right)\right], \tag{8.3}$$

where $A(\epsilon)$ is the diffusion coefficient in energy space.

For the anisotropic part of the distribution function $\mathbf{F}_1\cdot\mathbf{p}/p$ the collision integral deduces to the relaxation time $\tau(\epsilon)$, namely,

$$\nu\{F_1\}=\frac{F_1}{\tau(\varepsilon)}. \tag{8.4}$$

The function $\tau(\epsilon)$ defines the variation of the electron momentum with energy ϵ.

In real semiconductors there may be several scattering mechanisms and, practically always, the functions $A(\epsilon)$ and $\tau(\epsilon)$ take the form

$$A(\varepsilon)=A_0(T)\left(\frac{\varepsilon}{T}\right)^r,\qquad \tau(\varepsilon)=\tau(T)\left(\frac{\varepsilon}{T}\right)^q. \tag{8.5}$$

The values of the constants r and q which characterize the type of scattering are listed in the table (taken from[67]). Thus if collisions with acoustic phonons result in a change in momentum, and energy is transferred to piezophonons, then $q = -1/2$ and $r = 1/2$.

We must now formulate the basic equations and the boundary conditions. The complete system consists of the energy balance equation (8.1), the Maxwell equations

$$\operatorname{rot}\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t},\qquad \operatorname{rot}\mathbf{H}=\frac{4\pi}{c}\mathbf{j}+\frac{\varepsilon_0}{c}\frac{\partial\mathbf{E}}{\partial t} \tag{8.6}$$

and the continuity equations

$$-e\frac{\partial N}{\partial t}+\operatorname{div}\mathbf{j}=0,\qquad e>0. \tag{8.7}$$

On the separation boundary we have, besides to the usual electrodynamic boundary conditions, the additional conditions for the momentum and energy flux associated with the particles. It is quite obvious that these additional conditions (which are distinct from the electrodynamic conditions) are not universal in character and must be formulated for each particular physical problem.

Suppose the semiconductor is placed in constant electric and magnetic fields. In the stationary state, the electron temperature Θ_0 is determined by the requirement that the right-hand side of (8.1) should vanish with $\mathcal{P}$ and j replaced by the expressions

$$\mathcal{P}_0=\frac{2N_0(\Theta_0-T)}{\sqrt{\pi}\,T\Theta_0^{5/2}}\int_0^\infty d\varepsilon A(\varepsilon)\,\varepsilon^{1/2}\exp\left(-\frac{\varepsilon}{\Theta_0}\right), \tag{8.8}$$

$$\mathbf{j}_0=\hat{\sigma}\mathbf{E}_0\equiv e^2\int_0^\infty d\varepsilon\,\varepsilon^{3/2}F_0(\varepsilon)\,\hat{\beta}(\varepsilon)\cdot\mathbf{E}_0; \tag{8.9}$$

where $F_0(\epsilon) \propto \exp(-\epsilon/\Theta_0)$ is the Maxwell distribution function with temperature Θ_0 and the tensor $\hat{\beta}$ is given by

$$\hat{\beta}(\varepsilon)=\frac{8\pi\sqrt{2m}\,\tau(\varepsilon)}{3\Theta_0(1+\omega_H^2\tau^2(\varepsilon))}\begin{pmatrix}1, & -\omega_H\tau(\varepsilon), & 0\\ \omega_H\tau(\varepsilon), & 1, & 0\\ 0, & 0, & 1+\omega_H^2\tau^2(\varepsilon)\end{pmatrix}, \tag{8.10}$$

(the z axis is parallel to $\mathbf{H}_0$).

In the dynamic situation there are both constant and varying magnetic fields. We shall suppose that the varying quantities are proportional to $\propto \exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t)$. According to,[72] the expressions for the oscillatory parts of the current and heat densities can be written in the form

$$\mathbf{j}(\omega,\mathbf{k})=\hat{\sigma}\left\{\mathbf{E}+\frac{1}{\omega}[\mathbf{v}_0,[\mathbf{k}\times\mathbf{E}]]\right\}+\frac{N'}{N_0}\hat{\sigma}\left(\mathbf{E}_0+\frac{i\mathbf{k}\Theta_0}{e}\right)+\frac{\Theta'}{\Theta_0}\left[\left(\hat{\lambda}-\frac{5}{2}\hat{\sigma}\right)\mathbf{E}_0-i\frac{\Theta_0}{e}\left(\frac{3}{2}\hat{\sigma}-\hat{\lambda}\right)\mathbf{k}\right], \tag{8.11}$$

$$Q'(\omega,\mathbf{k})=-\frac{\Theta_0}{e}\hat{\lambda}\left\{\mathbf{E}+\frac{1}{\omega}[\mathbf{v}_0\times[\mathbf{k}\times\mathbf{E}]]\right\}-\frac{N'\Theta_0}{N_0 e}\hat{\lambda}\left(\mathbf{E}_0+i\mathbf{k}\frac{\Theta_0}{e}\right)+\frac{\Theta'}{\Theta_0}\left[\left(-\hat{\varkappa}+\frac{5\Theta_0}{2e^2}\hat{\lambda}\right)e\mathbf{E}_0+i\Theta_0\left(\frac{3\Theta_0}{2e^2}\hat{\lambda}+\hat{\varkappa}\right)\mathbf{k}\right]. \tag{8.12}$$

Primed quantities represent variable additions; the

Scattering mechanism	r	q
Acoustic oscillations	3/2	−1/2
Optical oscillations, $T < T_D$*)	1	0
Optical oscillations, $T > T_D$	−1/2	1/2
Peizoacoustic oscillations	1/2	1/2
Polar semiconductors, scattering by optical oscillations, $T > T_D$	−1/2	3/2
Neutral impurities	—	0
Charged impurities	—	3/2
Dipole impurities	—	1/2

*T_D—Debye temperature.

thermoelectric and thermal conductivity tensors are given by

$$\hat{\lambda}=\frac{e^2}{\Theta_0}\int_0^\infty d\varepsilon\varepsilon^{5/2}F_0(\varepsilon)\hat{\beta}(\varepsilon),\quad \hat{\varkappa}=\frac{1}{\Theta_0}\int_0^\infty d\varepsilon\varepsilon^{7/2}F_0(\varepsilon)\hat{\beta}(\varepsilon). \quad (8.13)$$

The rate of stationary drift is determined by the constant current $\mathbf{j}_0 = -N_0 e \mathbf{v}_0$.

The combinations of the tensors $\hat{\sigma}$, $\hat{\lambda}$, $\hat{\varkappa}$ in (8.11) and (8.12) describe the contributions of varying fields to the currents, and also the contributions due to drift motion, diffusion, thermal diffusion, and thermal conduction.

The formulas given by (8.13) do not take into account spatial and temporal dispersion of the dissipative coefficients, and are valid provided

$$\omega\tau,\ kl,\ kv_0\tau \ll 1. \quad (8.14)$$

However, even in the hydrodynamic approximation defined by (8.14), the analysis of overheating instability in a constant magnetic field with allowance for all the dissipative processes is very laborious. We shall therefore confine our attention to the most interesting special cases.

9. Instability of isotropic plasma. In the absence of the external magnetic field H_0, the tensors $\hat{\sigma}$, $\hat{\lambda}$, $\hat{\varkappa}$ become scalars.

$$\sigma=\frac{N_0e^2}{m\nu(\Theta_0)}\frac{\Gamma(q+2.5)}{\Gamma(2.5)},\quad \lambda=(q+2.5)\sigma,\quad \varkappa=\frac{\Theta_0}{e^2}(q+3.5)\lambda, \quad (9.1)$$

where $\nu(\Theta_0) = (1/\tau_0)(T/\Theta_0)^q$ is the electron collision frequency at temperature Θ_0 and $\Gamma(x)$ is the Euler function. The energy balance equation (8.1) in the stationary state (when $\Theta_0 \gg T$) assumes the form

$$\frac{e^2E_0^2\tau_0}{m}=\frac{3\Gamma(r+1.5)}{2\Gamma(q+2.5)}\frac{A(T)}{T}\left(\frac{\Theta_0}{T}\right)^{r-q}. \quad (9.2)$$

For the slow processes discussed below, the potential and induced oscillations can be separated. In the case of potential oscillations we may suppose that $H = 0$ and $c = \infty$, while the relation between the longitudinal electric field and the nonequilibrium concentration is given by the Poisson equation. The final result is the following dispersion relation:

$$\omega^2+i\omega\left(\frac{4\pi\sigma}{\varepsilon_0}+B\right)-\frac{4\pi\sigma}{\varepsilon_0}\left[\frac{\gamma(r-q)k_\perp^2+(r+q)k_\parallel^2}{k^2}+i\mathbf{k}\mathbf{v}_0+\frac{8\pi\lambda}{3\varepsilon_0}(kr_D)^2\right]-iC=0. \quad (9.3)$$

In these expressions

$$B=(r-q)\gamma+\frac{2\alpha}{3}\left(i\mathbf{k}\mathbf{v}_0+\frac{4\pi\sigma}{\varepsilon_0}k^2r_D^2\right),$$

$$C=\mathbf{k}\mathbf{v}_0\left[\gamma(r-q)+\frac{16\pi\lambda}{3\varepsilon_0}(kr_D)^2\right]+\frac{2}{3}i\left\{v_0^2\left[k_\parallel^2\left(q+\frac{7}{2}\right)+k^2(q-r)\right]-\lambda\sigma\left(\frac{4\pi k^2r_D^2}{\varepsilon_0}\right)^2\right\}, \quad (9.4)$$

$$r_D^2=\frac{\varepsilon_0\Theta_0}{4\pi N_0e^2},\quad \gamma=\frac{2\sigma E_0^2}{3N_0\Theta_0},\quad \alpha=5+3q+q^2;$$

where $k_\parallel$ and $k_\perp$ are the wave vector components along and across the drift velocity $\mathbf{v}_0 = \sigma E_0/N_0 e$ and r_D is the Debye length of electrons. The parameter γ is the energy relaxation frequency in the electron system.

The fact that (9.2) is a quadratic equation is due to oscillations in the electron temperature and density. It is clear that these oscillations are coupled.

If the displacement current is small, i.e., the Maxwell relaxation time $\epsilon_0/4\pi\sigma$ is the smallest among all the characteristic times ($4\pi\sigma \gg |\epsilon_0 B|$), the solutions of (9.3) take the form

$$\omega_1=-i\frac{4\pi\sigma}{\varepsilon_0},\quad \omega_2=\mathbf{k}\mathbf{v}_0-i\gamma\frac{(r-q)k_\perp^2+(r+q)k_\parallel^2}{k^2}-i\frac{8\pi\lambda}{3}(kr_D)^2. \quad (9.5)$$

It is clear from the above table that $r + q$ is always positive while $r - q$ may be negative (for example, in the case of energy scattering by optical phonons and momentum scattering by charged impurities). Overheating instability in the case of small $\epsilon_0/4\pi\sigma$ is therefore possible only for current perturbations with $k_\perp^2 > [(r + q)/|r - q|]k_\parallel^2$ which are "elongated" along the current. The perturbation length must be sufficiently large to ensure that thermal conduction and diffusion, which are described by the last term in ω_2, do not impede instability. Consequently, the instability criterion is

$$\left(q+\frac{5}{2}\right)(kr_D)^2<\frac{\varepsilon_0E_0^2}{4\pi N_0\Theta_0}\ll\frac{3}{2}. \quad (9.6)$$

Since $\gamma > |\mathbf{k}\mathbf{v}_0|$, the instability is aperiodic.

As in the case of gas-discharge plasmas,[4] the overheating instability may give rise to a stratification of the specimen into domains which are elongated in the direction of the current and have different conductivities[68].

In the case of transverse longwave perturbations ($k_\parallel \ll k_\perp \ll r_d^{-1}$, eE_0/θ_0), density and temperature oscillations can be separated, and the overheating instability arises independently of the ratios of γ and $4\pi\sigma/\epsilon_0$. The electron density then decreases at the rate $4\pi\sigma/\epsilon_0$, and temperature fluctuations increase at the rate

$$\omega_2 = i\gamma(q-r),\quad q>r. \quad (9.7)$$

It can be shown that short-wave perturbations are damped out because of diffusion and thermal conduction.

Let us now analyze the evolution of longitudinal perturbations in the opposite limiting case of large Maxwell times ($4\pi\sigma/\epsilon_0 \ll |r - q|\gamma$).

For long waves ($k \ll eE_0/\theta_0$) we have

$$\omega_1=i\gamma(q-r), \quad (9.8)$$

$$\omega_2=kv_0-i\frac{4\pi\sigma(r+q)}{\varepsilon_0(r-q)}\left[1+(kr_D)^2\frac{(r+q+q^2)}{r+q}\right]. \quad (9.9)$$

These formulas (with $k^2r_D^2 \ll 1$) were obtained in[72,73,75]; they were also obtained by Gulyaev[81] and Chavchanidze[82] with allowance for the terms including $k^2r_D^2$. It is clear that, when $r < q$, both oscillation branches are growing ones. Despite the fact that $|\omega_1| > |\omega_2|$ (and the growth of fluctuations is determined largely by ω_1), let us consider the terms including $k^2r_D^2$, since there are conflicting conclusions in the literature. It is clear from (9.9) that, for low-frequency ($\omega \ll |\omega_1|$) longitudinal perturbations, the true electrical conductivity is replaced by the differential conductivity $\sigma_{diff} = \sigma(r + q)/(r - q)$ which may change sign for certain types of energy and momentum. Scattering the term including $k^2r_D^2$ in (9.9) appears not only as the result of diffusion and thermal conduction, but is also due to the spatial transport of electron energy and density perturbations under the action of the constant electric fields. This was first noted by Gulyaev[81]. Analysis of (9.3) shows that, in the longwave region ($k_\parallel \ll eE_0/\theta_0$), the transport effects dominate and are responsible for the change in the sign of $k^2r_D^2$ in (9.9) (when $r < q$). Hence the conclusion reported in,[82] namely, that diffusion damping changes sign simultaneously with σ_{diff} is strictly speaking incorrect. This misunderstanding is due to the fact that the authors of[82] considered the longwave region ($k_\| \ll eE_0/\theta_0$) in which diffusion and thermal conduction are not very important. The statement given in[80] that diffusion leads to a growth in space-charge fluctuations is a consequence of a particular assumption about mobility as a function of current, which is in fact invalid. (The shortcomings of this model are discussed in[10]). Diffusion and thermal conduction appear in full measure only in the region of shortwave perturbations and always suppress the growth of fluctuations. Thus, when $k_\| \gg eE_0/\theta_0$, both oscillation branches are damped out at the rate

$$\omega_{1,2} = -i\frac{4\pi\sigma}{3\varepsilon_0}k^2 r_d^2\left(\alpha \mp \sqrt{\alpha^2 - 3(5+2q)}\right), \qquad (9.10)$$

where $\alpha^2 > 3(5 + 2q)$.

Therefore, when the condition

$$1 + k^2 r_D^2 \ll |r-q|\frac{\varepsilon_0 E_0^2}{4\pi N_0 \Theta_0} \qquad (9.11)$$

is satisfied for solid-state plasma there is both the aperiodic instability (9.8) and oscillatory instability (9.9). However, the development of the oscillatory instability is greatly impeded because $|\omega_1| \gg |\omega_2|$. This will probably seriously impede attempts to use overheating instability for the generation of microwaves[9)]. It is expected that, after instability develops, a specimen with large transverse dimensions will probably become stratified into regions with different electron concentration and temperature, and these domains will be highly elongated across the current.

An interesting result is obtained when the Maxwell time $\epsilon_0/4\pi\sigma$ is of the order of the energy time $1/\gamma$ [75]. When $r < q$ and

$$\left(\frac{4\pi\sigma}{\varepsilon_0} + \operatorname{Re} B\right)^2 \ll \frac{4\pi\sigma}{\varepsilon_0}\gamma(r+q), \quad kv_0 \ll \gamma, \qquad (9.12)$$

the growth due to the overheating of electrons and the damping of fluctuations due to Maxwell relaxation, diffusion, and so on, is almost completely compensated. The result of this is the appearance of coupled temperature-density oscillations which are either slightly damped or grow. Their spectrum and damping are of the form

$$\omega_{1,2} = \pm\Omega_\Theta + \frac{\alpha}{3}kv_0 - i\frac{2\pi\sigma}{\varepsilon_0}\left[1 + \frac{(r-q)\varepsilon_0 E_0^2}{6\pi N_0\Theta_0} + \frac{2}{3}\alpha k^2 r_D^2\right], \qquad (9.13)$$

where

$$\Omega_\Theta^2 = \frac{4\pi N_0 e^2}{\varepsilon_0 m_{\text{eff}}}, \quad m_{\text{eff}} = \frac{3\Theta_0}{2v_0^2(r+q)}. \qquad (9.14)$$

The intrinsic frequency of these oscillations is analogous to the Langmuir frequency of charged particles with mass $m = m_{\text{eff}}$.

In the derivation of the above dispersion relations we did not take into account the magnetic field due to the constant current j_0. It is clear that this is admissable provided $(\tilde{\omega}_H\tau)^2 \ll 1$ where $\tilde{\omega}_H \approx \omega_0^2 v_0 a/c^2$ is the cyclotron frequency corresponding to the magnetic field, and a is the maximum linear dimension of the specimen. For typical values of the parameters ($N_0 \approx 10^{14}$ cm^{-3}, $v_0 \approx 10^6$ cm sec^{-1}, $m \approx 10^{-28}$ g, $\nu \approx 10^{12}$ sec^{-1}) the quantity $\tilde{\omega}_H\tau$ is less than unity even for $a \approx 5$ m.

10. Overheating instability in a magnetic field. Hot magnetoactive plasma is of interest for the generation and amplification of oscillations because helicons can be present in such plasma. It is expected that the effective conductivity which governs the damping of helicons will change sign as a result of heating, and instability will set in. Let us therefore begin by considering the properties of induced perturbations in a highly overheated plasma with one type of carrier. Suppose that the z axis is parallel to the constant magnetic field $\mathbf{H}_0$ and the y axis is parallel to $\mathbf{E}_0$. In a strong field ($\omega_H\tau \gg 1$) the nonzero components of the conductivity tensor are

$$\sigma_{xx} = \sigma_{yy} = \frac{N_0 e^2}{m\omega_H^2\tau_0}\left(\frac{T}{\Theta_0}\right)^q \frac{\Gamma(2.5-q)}{\Gamma(2.5)},$$
$$\sigma_{xy} = -\sigma_{yx} = -\frac{N_0 e^2}{m\omega_H}, \quad \sigma_{zz} = \frac{N_0 e^2\tau_0}{m}\left(\frac{\Theta_0}{T}\right)^q \frac{\Gamma(2.5+q)}{\Gamma(2.5)}. \qquad (10.1)$$

The thermoelectric and thermal conductivity tensors λ_{ik} and κ_{ik} can be expressed in terms of σ_{ik} as follows:

$$\lambda_{xx} = \lambda_{yy} = (2.5-q)\sigma_{yy}, \quad \lambda_{zz} = (2.5+q)\sigma_{zz}, \quad \lambda_{xy} = -\lambda_{yx} = 2.5\sigma_{xy},$$
$$\varkappa_{xx} = \varkappa_{yy} = (3.5-q)\frac{\Theta_0}{e^2}\lambda_{xx}, \quad \varkappa_{zz} = (3.5+q)\frac{\Theta_0}{e^2}\sigma_{zz}, \quad \varkappa_{xy} = \frac{7\Theta_0}{2e^2}\lambda_{xy}. \qquad (10.2)$$

In the stationary state, the energy conservation law (8.1) assumes the form

$$\sigma_{yy}E_0^2 = \mathcal{P}(N_0, \Theta_0) \equiv \frac{2A(T)}{\pi^{1/2}T}N_0\left(\frac{\Theta_0}{T}\right)^r\Gamma(r+1.5). \qquad (10.3)$$

We shall confine our attention to sufficiently high electron densities and will neglect displacement currents. We shall also assume that spatial dispersion is weak, and that the perturbations propagate along $\mathbf{H}_0$. Moreover, we shall suppose that the drift velocity v_0 is much less than the phase velocity ω/k.[10)] The electric current $\mathbf{j}$ and the heat flux $\mathbf{Q}$ are then expressed in terms of field and temperature perturbations as follows:

$$\mathbf{j} = \hat{\sigma}\mathbf{E} + \frac{\Theta'}{\Theta_0}\left(\hat{\lambda} - \frac{5}{2}\hat{\sigma}\right)\mathbf{E}_0, \quad \mathbf{Q} = -\frac{\Theta_0}{e}\hat{\lambda}\mathbf{E} - \frac{\Theta'}{\Theta_0}\left(\hat{\varkappa} - \frac{5\Theta_0}{2e^2}\hat{\lambda}\right)e\mathbf{E}_0. \qquad (10.4)$$

From the Maxwell equations (8.6) with $\epsilon_0 = 0$, and the linearized energy balance equation (8.1), we can readily obtain the dispersion relation for the coupled electromagnetic and temperature waves:

$$(k^2c^2 - 4\pi i\omega\sigma_{yy})\left[k^2c^2 - 4\pi i\omega\sigma_{yy}\frac{(r-q)\gamma_H - i\omega}{(r+q)\gamma_H - i\omega}\right] = (4\pi\omega\sigma_{xy})^2, \qquad (10.5)$$

where $\gamma_H = 2\sigma_{yy}E_0^2/3N_0\Theta_0$ is the energy relaxation frequency in a strong magnetic field. We shall solve this equation, assuming that γ_H is much greater than the helicon frequency $\omega_h = k^2c^2/4\pi|\sigma_{xy}|$. Since $\sigma_{yy} \ll |\sigma_{xy}|$, we can readily obtain the following spectrum branches[75].

$$\omega_1 = -i(r+q)\gamma_H,$$
$$\omega_{2,3} = \left(\pm 1 - i\frac{r}{r+q}\frac{\sigma_{yy}}{|\sigma_{xy}|}\right)\omega_h. \qquad (10.6)$$

The first formula describes the aperiodic damping of temperature perturbations. These formulas do not contain the varying electric and magnetic fields, and the varying current appears as a result of the transport of temperature perturbations in constant fields.

Helicons may turn out to be unstable, and the necessary condition for this is $r < 0$ ($r + q$ is always positive). In particular, the growth of helicons is possible in polar semiconductors for which $r = -1/2$. The instability has an oscillatory character and its appearance is due to the fact that, at low frequencies $\omega \approx \omega_h \ll \gamma_H$ the temperature can "follow" the variation of the electromagnetic field, and its dependence on the field determines whether or not the amplitude of the helicon will grow[11)]. It is quite obvious that at high frequencies $\omega \gg \gamma_H$ the temperature does not vary and the helicon is damped at a rate proportional not to the differential but the static conductivity σ_{yy}.

For perturbations $\mathbf{E} = (E_x, E_y, 0)$, $\mathbf{H} = (0, 0, H_z)$ with wave vector $\mathbf{k} = (0, k, 0)$ the dispersion relation assumes the form

$$\omega^2 + i\omega\left[(r+q)\gamma_H + \frac{\sigma_{yy}}{4\pi}\left(\frac{kc}{\sigma_{xy}}\right)^2\right] - (r-q)\gamma_H\frac{\sigma_{yy}}{4\pi}\left(\frac{kc}{\sigma_{xy}}\right)^2 = 0. \quad (10.7)$$

Temperature fluctuations may grow when $(kc/\sigma_{xy})^2 \gg 4\pi\gamma_H(r+q)/\sigma_{yy}$:

$$\omega_1 \approx -i(r-q)\gamma_H, \quad \omega_2 = -i\frac{k^2c^2\sigma_{yy}}{4\pi\sigma_{xy}^2}. \quad (10.8)$$

In contrast to the case defined by (10.6), temperature perturbations are accompanied by the y-component of the varying electric field, and $j_y = 0$. Hence the "transport" of temperature fluctuations takes place in a different way than in the case described by (10.6). If the first term in the square brackets in (10.7) is much greater than the second, then the electromagnetic field oscillations turn out to be unstable:

$$\omega_1 \approx -i(r+q)\gamma_H, \quad \omega_2 = -i\frac{k^2c^2}{4\pi\sigma_{xy}^2}\sigma_{yy}\frac{r-q}{r+q}. \quad (10.9)$$

It is readily shown that perturbations with wave vector k lying along the x axis are always stable.

Thus, the necessary condition for the development of periodic instability of the form (10.6), and suppression of the aperiodic (10.8), is that the linear dimensions of the specimen in the direction of the magnetic field H_0 should be a minimum.

It can be shown that the instability of potential perturbations in a magnetic field is also oscillatory. Instabilities of this kind are discussed in[76,79].

1)It is important to note that this spectrum is retained by the helicons even when the amplitude is large. [19]

2)At these points the oscillations are damped with a relative rate proportional to ν/ω_H.

3)All the calculations will henceforth refer to PbTe because the interaction between helicoidal and acoustic waves has been found experimentally in this material. [29]

4)In all cases where we are concerned with surface waves it will be assumed that the frequency of surface recombination is small in comparison with ν, and the wave penetration depth exceeds the characteristic inhomonogeneity length connected with this recombination.

5)Interesting results on the interaction between an axially nonsymmetric potential wave and a beam are reported in [53].

6)The heating of electrons in gas-discharge and solid-state plasmas was described in [69,70] and in Ginzburg's monograph [71].

7)We note that if the condition given by (8.2) is violated, the isotropic part of the electron distribution function ceases to be Maxwellian. This situation is discussed in detail in the review by Kikvidze and Rukhadze [65].

8)This does not, however, exclude the possibility of devices such as those described in [83] where the current-carrying specimen is placed in a resonator.

9)This removes the question of the role the magnetic field due to the constant current and the frequency shift due to electron drift.

10)We note that, in this case, potential perturbations in the density are rapidly damped out and, consequently, do not affect the oscillatory character of helicon instability.

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Translated by S. Chomet