

Dislocation dragging by electrons in metals

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The paper presents a review of the results of experimental and theoretical study of the influence of conduction electrons on the mobility of dislocations in normal metals and superconductors. The influence of the superconducting transition on the macroscopic mechanical properties of metals connected with dislocation motion, namely, plasticity and sound absorption, are also discussed.

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1. INTRODUCTION

Recently obtained experimental data show that a superconducting transition is accompanied by a unique softening of metals, namely, the plasticity of the metals is greatly increased by the transition^[1-4]. Since the crystal-lattice properties are not very sensitive to the superconducting transitions^[42-45], it is natural to ascribe the observed effect to the influence of the conduction electrons, the energy spectrum of which experiences very significant changes.

It is well known that the plasticity of metals is determined by the motion of the elementary "carriers" of the plastic deformation, the dislocations, so that an explanation of the softening effect must be sought in the influence of the electrons on the dislocation mobility. This point of view is advanced in almost all the experimental papers cited above, and is at present universally accepted.

The dislocation dragging mechanism is customarily divided into two groups, in accordance with the physical nature.

The first group includes mechanisms due to elastic interaction of dislocations with various structure defects, such as individual impurities, impurity clusters, other dislocations, etc. These defects are sources of potential barriers that hinder the gliding of the dislocations, and can play a twofold role. On the one hand, they are connected with long-range elastic fields that produce in the crystal a "ripple" of internal stresses with a certain amplitude σ_1 that depends on the defect density. As a result, dislocation motion becomes possible only when the external stress σ applied to the dislocation exceeds σ_1 . On the other hand, the defects that enter in the glide plane produce barriers localized at atomic distances. Such barriers, owing to their small spatial dimensions, are overcome by the dislocation even at a low level of external stress, with the aid of thermal or quantum fluctuations.

The discrete character of the crystal structure is the cause of the existence of unique potential barriers, called Peierls barriers. These barriers also have atomic dimensions in the direction of dislocation motion, and can be overcome with the aid of the fluctuations.

The second group of drag mechanisms is of dynamic origin, due to the interaction of the dislocations with the elementary excitations of the crystals, such as

phonons, conduction electrons, spin waves, etc. The moving dislocation collides with the quasiparticles, transfers their energy to them, and is therefore slowed down.

The drag force determined by the dynamic losses has a viscous character, vanishing when the dislocation velocity vanishes and increasing with increasing velocity. In spite of the fact that this force assumes the principal role only at very large velocities $\sim (10^2-10^1)s$ (s is the speed of sound), at which the dislocation kinetic energy exceeds the aforementioned potential barriers, it can play a rather appreciable role also in the fluctuation motion of dislocations through barriers^[46,47]. As a result, viscous drag exerts an influence on the dislocation mobility in the entire velocity interval.

The dissipative properties of metals at low temperatures, as is well known, are determined by the absorptivity of the conduction electrons. Under these conditions, the electron viscosity is the principal mechanism of dynamic dislocation loss, and any change in this viscosity (for example, a superconducting transition) should be accompanied by a change in the dislocation mobility and by the same token should influence those mechanical characteristics of metals which are connected with dislocation motion.

The first to point out the dislocation dragging by conduction electrons were apparently Tittman and Bommel^[48] and Mason^[49]. The first experimental data indicating this effect were obtained by Love and Shaw^[50] and by Tittman and Bommel^[48,51], who observed an amplitude dependence of the ultrasound absorption coefficient in lead following its transition to the superconducting stage. It was shown later^[52-57] that nonlinear absorption takes place also in the normal state, but at much larger sound amplitudes; the role of the superconducting transition reduces to a lowering of the critical amplitude of the effect. The nonlinear absorption observed in the cited papers is customarily connected with the breakaway of the dislocation from the impurity atoms that pin them^[58]. Since the critical amplitude of the sound at which the breakaway takes place depends in general on the magnitude of the drag forces that act on it, it was perfectly natural to assume^[48,49,51] that the observed lowering of the critical amplitude is due to the weakening of the electron drag force on the dislocation in the superconducting state in comparison with the normal state.

The shift of the critical amplitude of the nonlinear absorption is a rather "subtle" experimental effect.

It is very sensitive to peculiarities of the defect structure of the crystal and its detection calls for very precise measurements. Its observation hardly gave any ground for expecting the electron drag of dislocations to become noticeably manifest in "coarser" measurements such as ordinary mechanical tests of metals. However, the discovery of the weakening effects has shown that the influence of the superconducting transition on the mechanical behavior of metals is quite appreciable. In particular, it turned out that the superconducting transition is accompanied by a lowering of the elastic limit (the critical shear stress^[2]) by several dozen per cent, and by a decrease of the deformation stress in the case of plastic flow by several per cent^[3-21]. A rather large effect was observed in creep experiments, namely, the superconducting transition of the sample leads to an increase in the creep rate by dozens of times^[22-31]. The superconducting transition exerts also a rather strong influence on the stress relaxation^[32-39], on the glide-band mobility^[40,41] and on the twinning process^[24].

Special experiments have shown that within the limits of the measurement accuracy the superconducting transition exerts no influence on the elastic-deformation process; noticeable effects appear only during the stage of well-developed plastic deformation, when a large number of dislocations are displaced in the crystal. This circumstance is apparently the principal proof in favor of the assumption that the softening effects are based on the same physical mechanism as the shift of the critical amplitude of the nonlinear absorption of sound, namely the increase of dislocation mobility, which occurs in the superconducting transition as a result of the sharp decrease in the electron viscosity.

Thus, an analysis of the electron drag force on the dislocations and of its change in the superconducting transition is essential for the understanding of many experimentally observed features of the mechanical behavior of metals at low temperatures.

In this review we report from a unified point of view the results of theoretical papers devoted both to the study of the electron-drag force on individual dislocations in normal metals^[59-63] and in superconductors^[64-67], and to the analysis of the influence of this force on the dislocation absorption of sound^[49,51,61,66], and on the kinetics of plastic deformation^[47,69-72] of metals. We do not attempt to touch here upon all the questions pertaining to this problem. Principal attention is paid to the influence of electrons on the mobilities of individual dislocations; these questions constitute the contents of Chaps. 2 and 3. In the last two chapters of the review, using simplest models as examples, we show how the electrons influence certain macroscopic mechanical characteristics of metals.

2. ELECTRON DRAG OF DISLOCATIONS IN NORMAL METALS

The problem of electron drag of dislocations is equivalent in many cases to the problem of absorption of high-frequency sound $ql \gg 1$ (q is the wave vector of the sound wave and l is the electron mean free path). The moving dislocation produces in the crystal an alternating elastic-deformation field $u_{ik}(\mathbf{r}, t)$, which exerts on the conduction electrons a force determined by the deformation potential $\lambda_{ik}u_{ik}(\mathbf{r}, t)$ (the nonzero tensor components λ_{ik} are of the order of the width of

the electron band^[73]). Owing to this potential, the dislocation produces transitions in the electron system and loses its energy to the perturbation of this system. The drag force is equal in absolute magnitude to the energy absorbed by the electrons when the dislocation traverses a unit path.

In a concrete calculation of the drag force it is convenient, using a Fourier expansion, to represent the dislocation deformation field $u_{ik}(\mathbf{r}, t)$ in the form of a superposition of elastic waves. Within the framework of linear theory of elasticity, the action of each of these waves on the electrons can be regarded independently. Such an approximation does not take into account effects due to nonlinear deformation near the dislocation core; however, there are grounds for assuming that the relative magnitude of these effects is small.

The subsequent calculation is similar to the solution of the problem of absorption of ultrasound in metals^[73] and can be carried out in two ways. The first is kinetic^[59] and is based on the solution of the kinetic equations for the conduction electrons with the deformation potential of the dislocation as the external perturbing field, followed by calculation of the dissipative function. It turns out, and this is important, that the main contribution to the dissipative function is made by elastic waves with extremely large wave numbers $q \sim 1/a$ (a is the lattice constant). This means that the inequality $ql \gg 1$ is satisfied with a large margin for all the waves that must be taken into account. In this case, as shown in^[73], the interaction of the electron with the elastic medium can be regarded as a quantum-mechanical process of electron-phonon collision. This circumstance justifies the second, quantum method of calculating the dislocation drag force^[60]. Calculation of the drag force by this method reduces to a calculation of the number of electronic transitions with energy absorption.

The exposition that follows will be based on the quantum method, since it is much simpler and can be easily generalized to include the case of superconductors.

The qualitative features of electron drag can be established by considering a straight-line dislocation moving with constant velocity \mathbf{V} through an equilibrium gas of free electrons with isotropic quadratic dispersion law. In this case the strain tensor is $u_{ik} = u_{ik}(\mathbf{r} - \mathbf{V}t)$, where \mathbf{r} is a two-dimensional radius vector in a plane perpendicular to the dislocation line. Choosing the electron wave functions in the form plane waves normalized to the volume of the crystal, we can show that the Hamiltonian of the interaction of the electrons with the dislocation, expressed in the second-quantization representation, is

$$\mathcal{H}_{int} = \frac{1}{L_1 L_2} \sum_{\mathbf{q}, \mathbf{k}} \lambda_{in} u_{in}^{\mathbf{q}} e^{-i\omega_{\mathbf{q}} t} (a_{\mathbf{k}+\mathbf{q}}^{\dagger} a_{\mathbf{k}}^{\dagger} + a_{\mathbf{k}+\mathbf{q}}^{\dagger} a_{\mathbf{k}}), \quad (1)$$

$$\omega_{\mathbf{q}} = qV;$$

here $u_{in}^{\mathbf{q}}$ is the spatial Fourier component of the strain tensor and depends on the two-dimensional wave vector q lying in a plane perpendicular to the dislocation line; L_1 and L_2 are the transverse dimensions of the crystal; $a_{\mathbf{k}}^{\dagger}$ and $a_{\mathbf{k}}$ are the operators for the creation and annihilation of electrons in a state with the wave vector \mathbf{k} and the corresponding spin direction.

The frequency of the electronic transitions $\nu_{\mathbf{k}, \mathbf{k}+\mathbf{q}}$ with absorption of a quantum of energy $\hbar\omega_{\mathbf{q}}$, calculated

by perturbation theory with the matrix elements of the Hamiltonian (1), is given by

$$v_{k, k+q}^{(N)} = \frac{4\pi}{\hbar} \left| \frac{\lambda_{in} u_{in}^q}{L_1 L_2} \right|^2 [f(\epsilon_k) - f(\epsilon_{k+q})] \delta(\epsilon_{k+q} - \epsilon_k - \hbar\omega_q),$$

where $f(\epsilon) = (1 + e^{(\epsilon - \epsilon_F)/T})^{-1}$ is the equilibrium Fermi function¹⁾, $\epsilon_k = \hbar^2 k^2 / 2m$ is the energy of the electron in the state k , ϵ_F is the Fermi energy, m is the electron mass, and the index (N) indicates that this quantity pertains to the normal metal. Formula (2) takes into account the spin degeneracy of the electronic states.

The electron drag force per unit of dislocation length is determined by the relation

$$F = \frac{1}{L_3 V} \sum_{q, k} \hbar\omega_q v_{k, k+q}, \quad (3)$$

in which L_3 denotes the dislocation length. We note that formula (3) in this form is valid both for the case of the normal metal and for the case of the superconductor; the difference between these two cases lies in the concrete form of the transition frequency ν . Substituting (2) in (3), changing from summation to integration, and using the fact that in our case $\hbar\omega_q$, $T \ll \epsilon_F$ and $f(\epsilon) - f(\epsilon + \hbar\omega_q) \approx \hbar\omega_q \delta(\epsilon - \epsilon_F)$, we obtain for the dislocation electron-drag force in the normal metal the expression

$$F_N = \frac{2}{(2\pi)^4 V} \int d^2 q |\lambda_{in} u_{in}^q|^2 \omega_q^2 \int d^3 k \delta(\epsilon - \epsilon_F) \delta\left(\frac{\hbar}{m} \mathbf{k} \mathbf{q} + \frac{\hbar q^2}{2m} - \omega_q\right). \quad (4)$$

The presence of the small parameter $m\omega_q / \hbar k_F q \approx V/v_F \ll 1$ (k_F is the radius of the Fermi sphere in k -space and v_F is the Fermi velocity of the electron) makes it easy to integrate with respect to the variable k , after which we obtain

$$F_N = \frac{2m^2}{(2\pi)^3 \hbar^3 V} \int_{q < 2k_F} d^2 q \frac{\omega_q^3}{q} |\lambda_{in} u_{in}^q|^2. \quad (5)$$

The integration region in (5) is a circle with radius $2k_F$.

Expression (5) enables us to draw two important conclusions concerning the behavior of F_N even before we specify the form of the deformation potential. First, we see that the dislocation electron-friction force in the normal metal does not depend on the temperature. Second, since $\omega_q \sim V$, it follows that the force F_N depends linearly on the dislocation velocity V .

No further analysis is possible without specifying the explicit form of the strain tensor u_{in} . By way of example we consider a screw dislocation for which two components of the strain tensor differ from zero^[74]. In a coordinate system with Oz axis along the dislocation line and Ox axis along its velocity V , these are the components u_{xz} and u_{yz} . Using the well known formulas for these quantities^[74], we obtain the following expression for the Fourier component of the deformation potential:

$$\lambda_{in} u_{in}^q = ib \frac{\lambda_{xz} q_y - \lambda_{yz} q_x}{q^2}, \quad (6)$$

where b is the value of the dislocation Burgers vector.

Substituting (6) in (5), we can easily confirm the statement made above concerning the role of waves with large values of q . Indeed, the integrand in (5) does not depend on the modulus of the vector q , and consequently the integral is determined by the upper integration limit $2k_F \sim 1/a$.

Integrating in (5), we arrive at the following final

expression for the drag force of a screw dislocation in a normal metal:

$$F_N = B_N V, \quad B_N = \frac{m^2 b^2 \lambda^2 q_m^2}{(2\pi \hbar)^3}, \quad (7)$$

where $\lambda^2 = (1/4)(\lambda_{xz}^2 + 3\lambda_{yz}^2)$, $q_m = 2k_F$. A formula similar to (7) holds also for an edge dislocation, the only difference being that the quantity λ^2 was determined by a combination of other components of the tensor λ_{in} . Thus, the dislocation electron-drag force in the normal metal does not depend on the temperature and increases linearly with increasing dislocation velocity. It is easy to verify that if we assume $\lambda \sim \epsilon_F$, then we obtain for the drag coefficient B_N the estimate

$$B_N \sim \frac{bn\epsilon_F}{v_F},$$

where n is the concentration of the conduction electrons. For standard metals we have $B_N \sim 10^{-5}$ g/cm-sec.

A few remarks must be made concerning the accuracy of the result. First, we point out that in the expression for the deformation potential (6) we have used for the dislocation strain tensor the value obtained within the framework of linear elasticity theory. This means that expression (6) reflects correctly the behavior of the deformation potential only at values $q \lesssim 1/r_0$, where r_0 is the radius of the dislocation core. If furthermore $1/r_0 < 2k_F$, then the upper integration limit q_m in (5) should be chosen to be the quantity $1/r_0$, which is not rigorously defined. Second, the deformation-potential constant is a phenomenological parameter of the theory, for which only the order of magnitude is known. Finally, the structure of the energy spectrum of the conduction electrons in real metals (the shape of the Fermi surface, etc.) is much more complicated than in the very simple model (free-electron gas) used in the calculation. Although there are no grounds for expecting the foregoing circumstances to influence the main features of the force F_N , namely the linear dependence on the dislocation velocity V and independence of the temperature, nevertheless, by virtue of these circumstances, the numerical value of the drag constant B_N becomes somewhat indefinite. Therefore B_N should be regarded as a semiphenomenological parameter, the exact value of which must be determined from experiment.

The foregoing seems to explain the difference between the theoretical and experimental values of B_N and the large scatter of the theoretical estimates ($B_N \sim 10^{-6} - 10^{-4}$ g/cm-sec)^[51, 53, 55, 59, 62].

The external fields, by changing the state of the electron gas, naturally influence the electronic part of the dislocation drag force.

An external electric field applied to the metal causes a direct electric current \mathbf{j} to flow. Owing to the momentum transfer from the conduction electrons to the crystal lattice distorted by the presence of the dislocation, the dislocations should be dragged by the electrons in the direction of the current. The drag force can be calculated by replacing the Fermi function $f(\epsilon_k)$ in the expression for the transition frequency (2), by the quasi-equilibrium distribution function in the presence of a current $\tilde{f}(\mathbf{k})$, which, as is well known, is given by

$$\tilde{f}(\mathbf{k}) = f(\epsilon_k) + \delta(\epsilon_k - \epsilon_F) \hbar \mathbf{k} \mathbf{V}_0,$$

where $\mathbf{V}_0 = (1/ne)\mathbf{j}$ is the electron drift velocity (n is the electron concentration, \mathbf{j} is the current density, and e is the electron charge). In the simplest case when the

electric field is directed along the dislocation-motion direction, the total force exerted on the dislocation by the electrons is determined by the expression (V. Kravchenko^[61])

$$F_N = B_N (V - V_0). \quad (8)$$

The current increases the drag force if it is directed opposite to the dislocation motion ($V_0 < 0$) and decreases this force in the opposite case ($V_0 > 0$). An immobile dislocation is acted upon, in the direction of the current, by a dragging force $F = B_N V_0$, which in principle should cause motion of the dislocation²⁾. However, it is difficult to obtain in experiment large drift velocities V_0 for typical metals, and the question of observing this effect remains open (see, incidentally,^[76-80]).

A much greater effect on the electron drag of the dislocations is exerted by strong magnetic fields.

A magnetic field changes the structure of the energy spectrum of the conduction electrons, and this leads to different quantum oscillations (in particular, oscillations of the sound-absorption coefficient). There is no theory that takes into account such effects in the calculation of the dislocation-drag force. However, the magnetic field influences the propagation and absorption of sound in metals also in the classical approximation, since it increases the viscosity of the electron gas^[61]. Therefore the application of a magnetic field should lead to an increase in the dislocation electron-drag force.

We present here the results of a kinetic analysis (V. Kravchenko^[63]), without dwelling on the details. Naturally, the kinetic analysis is valid if the magnetic field is not quantizing, i.e., if $\hbar\Omega \ll T$ ($\Omega = eH/mc$ is the cyclotron frequency).

The influence of a weak magnetic field ($r_H \gg l$, where r_H is the Larmor radius) on the dragging of dislocations by the electrons is small, and there is no need to discuss it. In the case of a strong field ($r_H \ll l$), the electron-drag force is considerably altered, since it depends on the magnetic field and on the electron mean free path $l = v_F \tau$, (τ is the relaxation time).

In the simplest case, when the magnetic field is parallel to the dislocation line, the drag force is given by

$$F_N^H = \Omega \tau B_N V \Phi \left(\frac{V \tau}{b} \right), \quad \Phi(x) = \frac{\ln(x + \sqrt{1+x^2})}{x} + \frac{1 - \sqrt{1+x^2}}{x^2}, \quad (9)$$

$$\Omega \tau = \frac{l}{r_H} \gg 1.$$

It is easy to see that at low dislocation velocities $V \ll b/\tau \sim s/\omega_0 \tau$ (ω_0 is a frequency on the order of the Debye frequency), the drag force remains linearly dependent on the velocity:

$$F_N^H = B_N^H V, \quad B_N^H = \frac{1}{2} \Omega \tau B_N. \quad (10)$$

We note, however, that the drag coefficient B_N^H greatly exceeds B_N . At large velocities $V \gg b/\tau$, the dependence of the drag force on the velocity becomes weaker, logarithmic:

$$F_N^H = \Omega b B, \quad \ln \frac{V \tau}{b}. \quad (11)$$

So far we have considered electron drag of uniformly moving linear dislocations. We now discuss the influence of the nonstationary character of the motion and

the bending of the dislocation line on the electron drag force. It was shown above that the main contribution to the drag force is made by transition due to elastic waves with the maximum possible values of the wave number $q \sim 1/r_0$ (r_0 is the radius of the dislocation core). This means that effective exchange of energy in momentum between the electron and the dislocation occurs at distances on the order of r_0 from the dislocation line and after times on the order of r_0/v_F . As a rule, the characteristic curvature radii of dislocation line are $R \gg r_0$, and the characteristic periods of motion are $\tau_0 \gg r_0/v_F$. It is almost obvious that the corrections due to the nonstationary character of the motion and to the bending of the dislocation line should be small, of an order determined by the parameters $r_0/\tau_0 v_F \ll 1$ and $r_0/R \ll 1$.

An exception in these rules are kinks on dislocations^[82,83] in crystals with high Peierls barriers. In such crystals, individual segments of one and the same dislocation can be in neighboring valleys of the potential relief; the section of the dislocation line joining these segments is called a kink (Fig. 1). A kink can move along the dislocation, transferring it from one potential-relief valley to another. Obviously, in this case the dislocation mobility on the whole is determined by the mobility of the kinks, while the latter in metals at low temperatures should be determined in the main by the electron drag.

Since the radius of curvature of the dislocation at the location of the kink is comparable with the atomic distance, the force that decelerates the kink calls for a special analysis.

Calculation of the force of the electron fraction of the kink F_N^K reduces to calculation of the elastic dislocation field with a kink moving along it with velocity V_K ; in all other respects the calculation is almost identical with the calculation in the case of a straight-line dislocation. In a normal metal in the absence of external fields, this force is determined by the expression^[84]

$$F_N^K = B_N^K V_K, \quad B_N^K = \frac{a^2 B_N}{d}, \quad (12)$$

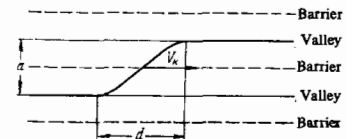
where d is the width of the kink and a is the lattice constant in the direction of the dislocation glide.

3. SINGULARITIES OF ELECTRON DRAG OF DISLOCATIONS IN SUPERCONDUCTORS

At a superconducting transition temperature $T = T_C$, a gap Δ appears in the energy spectrum of the conduction electrons, as a result of which the absorptivity of the electrons decreases strongly. It is therefore quite natural to expect also an abrupt decrease in the force of the dislocation drag by the electrons.

One might assume^[49,61] that the dislocation electron friction force in a superconductor decreases with temperature in accordance with the change of the concentration of the "normal" electrons, as does, for example, the ultrasound absorption coefficient^[85]. The actual

FIG. 1. Schematic representation of a kink on a dislocation. d —width of kink, v_K —velocity of kink, a —lattice constant.



situation is much more complicated for the following reason: As shown in the preceding chapter, the maximum energy transferred from the Fourier component of the elastic field of the dislocation to the electrons is of the order of $\hbar q_m V$. Very simple estimates show that at a velocity $V \sim 10^4$ cm/sec, which is quite easily attainable under the experimental conditions, this energy becomes comparable with the Cooper-pair binding energy 2Δ . Therefore a dislocation moving in a superconductor can cause not only transitions connected with the scattering of the "normal" excitations that exist at the given temperature, but also transitions connected with the creation of new excitations (breaking of the Cooper pairs). This circumstance leads to a complicated dependence of the electron-friction force in the superconductor on the dislocation velocity and on the metal temperature (^[64,65,66,67]).

A generalization of the calculation performed in the preceding chapter to include the case of a superconductor encounters no fundamental difficulties. Considering, as before, a uniformly moving straight-line dislocation and changing over in the Hamiltonian \mathcal{H}_{int} (1) from the electron operators $a_{\mathbf{k}}$ and $a_{\mathbf{k}}$ to the operators of the elementary excitations of the superconductor $\gamma_{\mathbf{k}}$ and $\gamma_{\mathbf{k}}$ with the aid of the Bogolyubov transformation (see, e.g., ^[68]), we obtain

$$\mathcal{H}_{int} = \frac{1}{L_1 L_2} \sum_{\mathbf{q}, \mathbf{k}} \lambda_{in} u_{in}^q e^{-i\omega_{\mathbf{q}} t} [(u_{\mathbf{k}+\mathbf{q}} u_{\mathbf{k}} + v_{\mathbf{k}+\mathbf{q}} v_{\mathbf{k}}) (\gamma_{\mathbf{k}+\mathbf{q}}^\dagger \gamma_{\mathbf{k}} + \gamma_{\mathbf{k}+\mathbf{q}} \gamma_{\mathbf{k}}^\dagger) + (u_{\mathbf{k}+\mathbf{q}} v_{\mathbf{k}} + v_{\mathbf{k}+\mathbf{q}} u_{\mathbf{k}}) (\gamma_{\mathbf{k}+\mathbf{q}}^\dagger \gamma_{\mathbf{k}}^\dagger + \gamma_{\mathbf{k}+\mathbf{q}} \gamma_{\mathbf{k}})] \quad (13)$$

Here $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ are transformation coefficients that satisfy the following system of equations:

$$\begin{aligned} u_{\mathbf{k}}^2 + v_{\mathbf{k}}^2 &= 1, \\ u_{\mathbf{k}} - v_{\mathbf{k}} &= \frac{\epsilon_{\mathbf{k}} - \epsilon_F}{\epsilon_{\mathbf{k}}^*}, \end{aligned}$$

where $\epsilon_{\mathbf{k}}^* = \sqrt{\epsilon_{\mathbf{k}} - \epsilon_F}^2 + \Delta^2$ is the excitation energy, $\epsilon_{\mathbf{k}} = \hbar^2 \mathbf{k}^2 / 2m$ denotes as before the electron energy, and $\Delta = \Delta(T)$ is the temperature-dependent gap. The physical meaning of the terms in the Hamiltonian (13) is obvious: the first and second terms describe the transition of the quasiparticle with corresponding spin direction from a state with a wave vector \mathbf{k} to a state $\mathbf{k} + \mathbf{q}$ via absorption of a quantum with energy $\hbar\omega_{\mathbf{q}}$; the third term describes the process of production of two quasiparticles in states $\mathbf{k} + \mathbf{q}$ and $-\mathbf{k}$ (breaking of a Cooper pair) with simultaneous absorption of a quantum of energy $\hbar\omega_{\mathbf{q}}$; the fourth term describes the annihilation of the pair of quasiparticles in the states \mathbf{k} and $-\mathbf{k} - \mathbf{q}$ and the production of a quantum of energy $\hbar\omega_{\mathbf{q}}$. The last two processes, naturally, are possible under the condition $\hbar\omega_{\mathbf{q}} > 2\Delta$. The transition frequencies for each of these processes separately have been written out in ^[67]. The total number of transitions with absorption of energy $\hbar\omega_{\mathbf{q}}$ is determined by the expression ^[68]

$$\begin{aligned} \nu_{\mathbf{k}, \mathbf{k}+\mathbf{q}}^{(\epsilon)} &= \frac{2\pi}{\hbar} \left| \frac{\lambda_{in} u_{in}^q}{L_1 L_2} \right|^2 \{ 2(u_{\mathbf{k}+\mathbf{q}} u_{\mathbf{k}} - v_{\mathbf{k}+\mathbf{q}} v_{\mathbf{k}})^2 \\ &\quad \times [\varphi(\epsilon_{\mathbf{k}}^*) - \varphi(\epsilon_{\mathbf{k}+\mathbf{q}}^*)] \delta(\epsilon_{\mathbf{k}+\mathbf{q}}^* - \epsilon_{\mathbf{k}}^* - \hbar\omega_{\mathbf{q}}) \\ &\quad + (u_{\mathbf{k}+\mathbf{q}} v_{\mathbf{k}} + v_{\mathbf{k}+\mathbf{q}} u_{\mathbf{k}})^2 [1 - \varphi(\epsilon_{\mathbf{k}}^*) - \varphi(\epsilon_{\mathbf{k}+\mathbf{q}}^*)] \\ &\quad \times [\delta(\epsilon_{\mathbf{k}+\mathbf{q}}^* + \epsilon_{\mathbf{k}}^* - \hbar\omega_{\mathbf{q}}) - \delta(\epsilon_{\mathbf{k}+\mathbf{q}}^* + \epsilon_{\mathbf{k}}^* + \hbar\omega_{\mathbf{q}})] \}, \end{aligned} \quad (14)$$

where $\varphi(\epsilon^*) = [1 + \exp(\epsilon^*/T)]^{-1}$ is the equilibrium distribution function of the normal excitations of the superconductor.

An expression that determines the dislocation electron-friction force F_S in the superconductor is obtained by substituting (14) in (3) and using the concrete form (6) of the deformation potential. Further

calculations become much easier if it is assumed that the dimensions of the Brillouin cell of the crystal are smaller than $2k_F$, and if the region of integration with respect to \mathbf{q} is confined to a rectangular Brillouin cell. Under these assumptions, the integral with respect to the projection q_y of the vector \mathbf{q} can be easily calculated, since the integration limits in this integral can be regarded as infinite without incurring a large error.

Omitting the rather cumbersome calculations, we present the final expression for the force F_S in a form convenient for analysis:

$$\begin{aligned} F_S = \frac{B_N}{\hbar q_m} \left\{ 2 \int_0^{\hbar q_m V} \frac{d\epsilon'}{\epsilon'} \int_{\Delta}^{\infty} d\epsilon \frac{e(\epsilon' + \epsilon) - \Delta^2}{\sqrt{\epsilon^2 - \Delta^2} \sqrt{(\epsilon' + \epsilon)^2 - \Delta^2}} [\varphi(\epsilon) - \varphi(\epsilon' + \epsilon)] \right. \\ \left. + \chi (\hbar q_m V - 2\Delta) \int_{2\Delta}^{\hbar q_m V} \frac{d\epsilon'}{\epsilon'} \int_{\Delta}^{\epsilon' - \Delta} d\epsilon \frac{e(\epsilon' - \epsilon) + \Delta^2}{\sqrt{\epsilon^2 - \Delta^2} \sqrt{(\epsilon' - \epsilon)^2 - \Delta^2}} \right. \\ \left. \times [1 - \varphi(\epsilon) - \varphi(\epsilon' - \epsilon)] \right\}; \end{aligned} \quad (15)$$

here B_N is the dislocation drag coefficient in the normal metal (7), q_m is the dimension of the Brillouin cell in the direction of motion of the dislocation, $\chi(x) = 1$ at $x > 0$, and $\chi(x) = 0$ at $x < 0$.

Formula (15) shows that the dependence of the electron-friction force $F_S(V, T)$ on the dislocation velocity V and on the superconducting temperature T is quite complicated, and its explicit form can be established only in limiting cases ^[66]. Let us consider some of them.

1) At absolute zero temperature ($T = 0$, $\Delta = \Delta_0$) we have

$$F_S = \chi (\hbar q_m V - 2\Delta_0) \frac{B_N}{\hbar q_m} \int_{2\Delta_0}^{\hbar q_m V} \frac{d\epsilon'}{\epsilon'} \int_{\Delta_0}^{\epsilon' - \Delta_0} d\epsilon \frac{e(\epsilon' - \epsilon) + \Delta_0}{\sqrt{\epsilon^2 - \Delta_0^2} \sqrt{(\epsilon' - \epsilon)^2 - \Delta_0^2}}. \quad (16)$$

Obviously, in this case the electron-friction force is equal to zero at dislocation velocities $V < V_c = 2\Delta_0/\hbar q_m$. At $V > V_c$, the friction force differs from zero, and if $V - V_c \ll V_c$, then

$$F_S = \frac{\pi}{2} B_N (V - V_c). \quad (17)$$

The appearance of the critical velocity V_c is a consequence of the existence of a gap in the energy spectrum of the superconductor, and has the same nature as the threshold absorption of the electromagnetic ^[65] and acoustic ^[67] energy, due to the breaking of the Cooper pairs.

The character of the singularity of $F_S(V)$ near the critical velocity must be discussed in greater detail. The point is that the relation (17) is not universal, and was obtained as a result of a special choice of the form $u_{\mathbf{k}}$ and as a result of the simplifying assumptions made in the integration with respect to \mathbf{q} . A detailed analysis of this question is contained in ^[67]. In the general case it is apparently necessary to replace (17) by ^[68]

$$F_S = \tilde{B}_N V_c \left(\frac{V}{V_c} - 1 \right)^\rho, \quad V - V_c \ll V_c, \quad (17')$$

where $\tilde{B}_N \sim \tilde{B}_N$, and ρ is a positive parameter, the value of which is determined by the form of the dislocation, by the symmetry of the crystal, etc.

At large velocities $V \gg V_c$, the friction force F_S in the superconductor tends asymptotically to the value of the friction force F_N in the normal metal (Fig. 2a).

2) At nonzero temperatures, special interest attaches to the case of small dislocation velocities

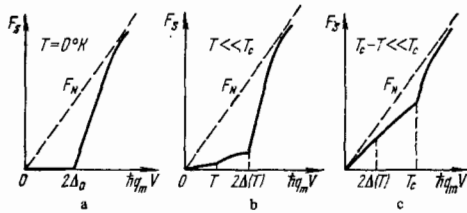


FIG. 2. Schematic plots of the dislocation electron-drag force in a superconductor against the dislocation velocity at different temperatures (solid curves); the dashed lines show the electron-drag force in the normal metal [66].

$V \ll T/\hbar q_m$, $\Delta(T)/\hbar q_m$. In this case the ratio of the friction force in the conducting and the normal states behaves, in accordance with the assumption made by Mason [49], like the ratio of the ultrasound-absorption coefficients in the BCS theory [85]

$$F_S = B_S V, \quad \frac{B_S}{B_N} = \frac{2}{1 + e^{\Delta/T}}. \quad (18)$$

When using (18), it must be remembered that the velocity interval in which this formula is valid depends strongly on the temperature. For intermediate temperatures, this interval is quite large (0–10³ cm/sec), but it becomes much narrower as $T \rightarrow 0^{\circ}K$ and $T \rightarrow T_c$.

3) If the superconductor temperature is low in comparison with the critical temperature, $T \ll T_c$, then the small linear section (18) on the plot of $F_S(V)$ of Fig. 2b is replaced with increasing velocity ($T \ll \hbar q_m V \ll T_c$) by a parabolic section:

$$F_S = 2B_N \sqrt{\frac{\pi T V}{\hbar q_m}} e^{-\frac{\Delta}{T}}. \quad (19)$$

Further increase of the dislocation velocity $\hbar q_m V > 2\Delta \approx 2\Delta_0$ turns on a threshold drag mechanism, namely the breaking of the Cooper pairs. At velocities $\hbar q_m V \gg 2\Delta_0$, the friction force in the superconductor differs only negligibly from the friction force in the normal metal:

$$F_S \approx B_N V \left(1 - \frac{2\Delta_0}{\hbar q_m V} \ln \frac{\hbar q_m V}{2\Delta_0} \right). \quad (20)$$

The value of the force F_S at the point $V = V_c$ is determined in this case by the expression

$$F_S(V_c) \approx \pi B_N V_c \sqrt{\frac{\pi T}{2\Delta_0}} e^{-\frac{\Delta}{T}} \quad (21)$$

4) At temperatures close to T_c ($T_c - T \ll T_c$) and at low velocities $\hbar q_m V \ll 2\Delta \approx 6.4 T_c [1 + (T/T_c)]^{1/2}$, formula (18) is valid. At higher velocities $2\Delta(T) \ll \hbar q_m V \ll T_c$ we have

$$F_S = B_N V \left[1 - \frac{\Delta(T)}{T_c} \right]. \quad (22)$$

With further increase of the velocity ($\hbar q_m V \gg T_c$) we have

$$F_S = B_N V \left[1 - \frac{2\Delta(T)}{\hbar q_m V} \ln \frac{\hbar q_m V}{T_c} \right]. \quad (23)$$

It is obvious that as $T \rightarrow T_c$ the force $F_S \rightarrow F_N$ in the entire velocity interval (Fig. 2c).

With the aid of formulas (17)–(23) it is easy to find the friction-force discontinuity $\delta F_{NS} = F_N - F_S$ [66], which determines the change of the dislocation mobility in the superconducting transition. The temperature dependence of δF_{NS} coincides in the limiting cases with the temperature dependence of the gap $\Delta(T)$, but there is no single linear dependence of δF_{NS} on $\Delta(T)$.

The formulas cited above, together with Fig. 2, present a qualitative picture of the velocity and temperature dependences of the dislocation electron-friction in the superconductor. Some quantitative characteristics of this dependence, obtained by a computer analysis of formulas of the type (15), are given in [64, 67].

The analysis results given above for the electron friction in a superconductor are valid, generally speaking, for linear dislocations. However, as expected, on the basis of considerations advanced at the end of the preceding chapter, allowance for the bending of the dislocation [67] hardly affects these results if the curvature radius is much larger than atomic dimensions. Electron drag of kinks on dislocations in a superconductor, just as in the case of a normal metal (see Chap. 2) calls for a special analysis.

In concluding this chapter we note that in a superconductor, unlike a normal metal, it is more important to take into account effects connected with the dislocation core. First, the crystal deformations are nonlinear near the dislocation axis, so that the elastic-wave spectrum should contain also multiple frequencies, in addition to the fundamental frequency $\omega_q = q \cdot V$, and this leads to a lowering of the threshold velocity V_c . In addition, if account is taken of the discrete structure of the crystal, then the partial spectrum of the stationary elastic field of the dislocation contains frequencies that are multiples of the quantity V/a (V is the average dislocation velocity, which is constant in time, and a is the lattice parameter in the glide direction); this circumstance should also affect the ‘‘prethreshold’’ drag [3].

4. MECHANISMS OF NONLINEAR ABSORPTION OF ULTRASOUND IN METALS

Absorption of sound in typical metals at low temperatures is determined mainly by the conduction electrons. Transfer of energy from the acoustic wave to the electrons is effected in two ways—by direct interaction of the wave with the electrons and by electron drag of the dislocations that are actuated by the wave. The dislocation part of the absorption depends, naturally, on the dislocation density and can therefore be easily separated. In addition, this component of the absorption has a number of distinguishing features, the most essential of which is the appearance of an amplitude dependence at sound amplitudes at which the direct absorption is still linear.

The pure well-annealed single crystals usually employed in ultrasonic measurements contain a three-dimensional dislocation grid. [58, 66] whose nodes are ‘‘rigid’’ for the pinning of the dislocation lines. When impurities are introduced, the dislocations are additionally pinned by the impurity atoms that settle on them. Consequently, there are two characteristic lengths of the dislocation segments, L determined by the grid and l determined by the impurities (Fig. 3a). At sufficiently small sound-wave amplitudes the dislocation segments bounded by the impurities vibrate like elastic strings in a viscous medium (Fig. 3b). The absorption coefficients determined by these vibrations do not depend on the amplitude and have a resonant dependence on the frequency, reaching a maximum near the natural frequency of the segments. With increasing sound intensity, starting with a certain critical amplitude σ_c of the stresses in the sound wave, the dislocations break away from the impurities (Fig. 3c), as a result of which the

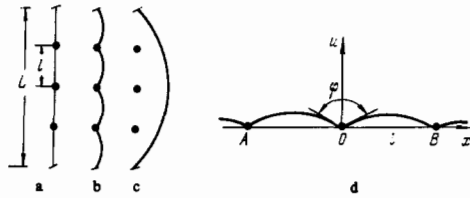


FIG. 3. Successive stages of the motion of a pinned dislocation in the field of a sound wave when the amplitude of the sound oscillations is increased [58]. L —distance between the nodes of the dislocation grid, l —distance between the impurities that settle on the dislocation, u —displacement of dislocation from the equilibrium position x —coordinate of the dislocation element.

dislocation absorption increases strongly and becomes nonlinear. The critical amplitude depends not only on the binding energy of the dislocation with the impurity atoms, but is also sensitive to a certain degree to the drag force that attenuates the oscillations of the dislocation segment. Therefore the change in the dislocation electron friction in the superconducting transition or when a strong magnetic field is turned on should lead to a shift of σ_c .

We shall analyze this phenomenon in the simplest case, by considering an aggregate of dislocations of the same type belonging to one glide system, and assuming that all the dislocation segments have the same length l .⁴⁾ To describe the absorption on an individual segment, we use the Kohler-Granato-Lucke "string" model of the dislocation [58], according to which small vibrations of a pinned dislocation in the field of a sound wave are described by the equation

$$M \frac{\partial^2 u(x, t)}{\partial t^2} - C \frac{\partial^2 u(x, t)}{\partial x^2} + F(V) = b\sigma \sin \omega t, \quad (24)$$

$$u(0, t) = u(l, t) = 0, \quad \sigma < \sigma_c.$$

Here $u(x, t)$ is the displacement of the dislocation from the Ox axis passing through the pinning point (Fig. 3d); $F(V)$ is the drag force and depends on the velocity $V = \partial u / \partial t$; σ is the amplitude of the component of the stress tensor in the glide plane of the dislocation; M and C are respectively the effective-mass linear density and the coefficient of linear tension in the dislocation.

It was shown in the two preceding chapters that in a number of cases the dislocation electron drag force in the normal metal and in the superconductor depend linearly on the velocity: $F(V) = BV$ (the drag coefficient B can depend on the temperature or on the magnetic field). In such cases the solution of (24) takes the form

$$u(x, t) = \frac{4b\sigma}{\pi M} \sum_{k=0}^{\infty} \frac{1}{(2k+1) \sqrt{(\omega_k^2 - \omega^2)^2 + \gamma^2 \omega^2}} \sin \frac{(2k+1)\pi}{l} x \cos(\omega t + \delta_k), \quad (25)$$

where

$$\gamma = \frac{B}{M}, \quad \omega_k = (2k+1)\omega_0, \quad \tan \delta_k = \frac{\omega\gamma}{\omega_k^2 - \omega^2}, \quad \omega_0 = \frac{\pi}{l} \sqrt{\frac{C}{M}}.$$

The energy W absorbed by an individual dislocation segment during one period is

$$W = \int_0^l dx \int_{-\pi/\omega}^{\pi/\omega} dt VF(V), \quad (26)$$

and the dislocation component Γ of the sound damping decrement is connected with W by the relation

$$\Gamma = \frac{GNW}{I\sigma^2}, \quad (27)$$

where G is the shear modulus and N is the dislocation density. Substitution of (25) in (26) and (27) yields

$$\Gamma = \frac{8NGb^2}{\pi M} \sum_{k=0}^{\infty} \frac{\gamma\omega}{(2k+1)^2 [(\omega_k^2 - \omega^2)^2 + \gamma^2 \omega^2]}. \quad (28)$$

Expression (28) describes the dislocation component of the sound absorption in metals at sufficiently low amplitudes $\sigma < \sigma_c$ (in the linear region). We recall that the region of applicability of this formula is connected, in particular, with the conditions under which the force $F(V)$ depends linearly on the velocity V , namely the condition $|\partial u(x, t) / \partial t| \ll T / \hbar q_m$, $\Delta(T) / \hbar q_m$ in superconductors and the condition $|\partial u(x, t) / \partial t| \ll b / \tau$ in metals placed in a magnetic field, if $\Omega\tau \gg 1$ (see (9) and (10)). Incidentally, estimates show that in the cases of practical interest these limitations are not too stringent.

Let us find now the amplitude σ_c at which the dislocation breaks away from the impurities and the absorption becomes nonlinear. In this model, the criterion of breakaway from the pinning point O (Fig. 3d) is equality of the angle φ to the critical value $\varphi_c = \pi - (E/bC)$, where E is the binding energy of the dislocation with the impurity atom. Since the dislocation vibrations are assumed to be small, it follows that $\varphi = \pi - 2(\partial u / \partial x)_x = 0$; using this circumstance, we obtain

$$\sigma_c = \frac{EMl}{8b^2C} \left[\sum_{k=0}^{\infty} \frac{1}{\sqrt{(\omega_k^2 - \omega^2)^2 + \gamma^2 \omega^2}} \right]^{-1}. \quad (29)$$

This formula determines the dependence of the critical amplitude σ_c on the dislocation drag coefficient $\gamma = B/M$.

In a superconductor we have $\gamma = \gamma_S = (2B_N/M) \times [1 + e\Delta/T]^{-1}$, so that σ_c should decrease sharply when the temperature is lowered. This effect should be particularly large if the inequalities $\omega \ll \omega_0$ and $\gamma\omega \gg \omega_0^2$ are satisfied (under experimental conditions we usually have $\omega \sim 10^7 - 10^8 \text{ sec}^{-1}$ and $\omega_0 \sim 10^9 - 10^{10} \text{ sec}^{-1}$, and the value of γ for typical metals in the normal state is of the order of $\gamma_N \sim 10^{10} - 10^{11} \text{ sec}^{-1}$).

We note that the qualitative character of the behavior of the quantities Γ and σ_c can be obtained in many cases of practical importance by retaining only the first terms of the series in (28) and (29).

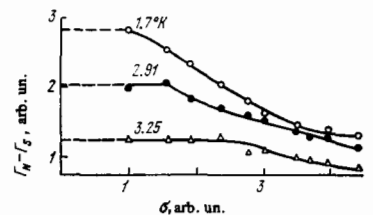
Figure 4 shows the character of the nonlinear-absorption critical-amplitude variation observed experimentally in superconducting indium [52]. The observation of nonlinear absorption in superconductors [48, 50, 51, 53-57], and the fact that it is not observed in the same crystals in the normal state, pertain apparently to a case when the sound amplitude σ satisfied the inequality

$$\sigma_c(\gamma_N) > \sigma > \sigma_c(\gamma_S).$$

If the metal is placed in a strong magnetic field, $\Omega\tau \gg 1$, then the coefficient γ , together with the critical amplitude σ_c , depends on the field intensity H , namely $\gamma = \gamma_N^H = \gamma_N \tau eH / mc$. Insofar as we know, there are no experimental data on this effect at present.

In superconductors at low temperatures $T \ll T_C$

FIG. 4. Dependence of the difference $\Gamma_N - \Gamma_S$ between the sound-damping decrements in normal and superconducting indium on the amplitude of the acoustic oscillations at different temperatures [52] (the damping decrements and the amplitudes are shown in arbitrary units). The absorption in the normal state was independent of the amplitude in the studied interval of σ .



there is in addition to the nonlinear absorption connected with the breakaway of the dislocations from the impurities also another nonlinear-absorption mechanism^[68] due to the essentially nonlinear dependence of the friction force $F_S(V)$ on the dislocation velocity V .

To simplify the analysis, we confine ourselves, following^[68], to consideration of extremely low temperatures ($T = 0^\circ\text{K}$). In this case there is no direct absorption of sound by electrons at all frequencies $\omega < 2\Delta_0/\hbar \sim 10^{11} - 10^{12} \text{ sec}^{-1}$.^[67] As to the absorption due to the dislocation electron friction, it should also be absent at sufficiently low sound amplitudes at which the amplitude of the velocity of the vibrating segment V is lower than the threshold velocity $V_C = 2\Delta_0/\hbar q_m$, inasmuch as $F_S(V) = 0$ at these velocities (see (16)). At sufficiently large amplitudes, starting with a certain critical amplitude $\tilde{\sigma}_C$, the velocity of the segment exceeds V_C during a certain part of the period, and the dislocation experiences during that time electron drag. As a result, at amplitudes $\sigma > \tilde{\sigma}_C$, the dislocation absorption should acquire an increment that is connected with the electron drag and increases with increasing amplitude.

At $T = 0^\circ\text{K}$ and $\sigma < \tilde{\sigma}_C$, the only cause of the damping of the vibrations of the dislocation segments are the losses to phonon emission (radiative losses^[69]), which are small enough to be neglected in other cases. We shall show below that for the phenomenon in equation greatest interest attaches to the frequencies $\omega \sim \omega_0$. In this case, when solving Eq. (24), we can confine ourselves with sufficient accuracy to the first term of the series:

$$\frac{\partial u(x, t)}{\partial t} = V(\sigma, \omega) \sin \frac{\pi}{T} x \cos(\omega t - \delta_R), \quad (30)$$

$$V(\sigma, \omega) = \frac{4b\sigma}{\pi M} \frac{\omega}{\sqrt{(\omega_0^2 - \omega^2)^2 + \gamma_R^2 \omega^2}}, \quad \text{tg } \delta_R = \frac{\gamma_R \omega}{\omega_0^2 - \omega^2},$$

where γ_R is the radiative-damping coefficient, which has, in accordance with^[69], a value $\gamma_R \sim 10^{-1} \omega$.

Equating the dislocation-velocity amplitude $V(\sigma, \omega)$ to the threshold velocity V_C and solving the obtained equation relative to σ , we obtain

$$\tilde{\sigma}_c = \frac{4MV_C}{4b\omega} \sqrt{(\omega_0^2 - \omega^2)^2 + \gamma_R^2 \omega^2}. \quad (31)$$

It is easily seen that the minimum value of the threshold amplitude is reached at a frequency ω close to the resonant frequency of the segment ω_0 :

$$\min[\tilde{\sigma}_c(\omega)] \sim \tilde{\sigma}_c(\omega_0) \sim 10^{-1} \frac{MV_C \omega_0}{b}. \quad (32)$$

Assuming the amplitude of sound σ to be close to the threshold amplitude ($0 < (\sigma - \tilde{\sigma}_c)/\sigma_C \ll 1$) and using formulas (17'), (26), (27), and (30), we can obtain for the electronic part of the damping decrement Γ_S in the superconductor, at $T = 0^\circ\text{K}$, the expression

$$\Gamma_s = \frac{64\tilde{B}_N N G b^2}{\pi^2 M^2 (\rho + 1)} \frac{\omega}{(\omega_0^2 - \omega^2)^2 + \gamma_R^2 \omega^2} \left[\frac{\sigma - \tilde{\sigma}_c(\omega)}{\tilde{\sigma}_c(\omega)} \right]^{\rho+1}. \quad (33)$$

We note that the character of the singularity in the amplitude dependence of Γ_S near $\tilde{\sigma}_C$ is directly connected with the character of the singularity of $F_S(V)$ near V_C .

At sound amplitudes $\sigma \gg \tilde{\sigma}_C$, we have $|\partial u / \partial t| \gg V_C$, over practically the entire period of the oscillations, and consequently we can put $F(V) \approx B_N V$ in (24). Therefore at large sound amplitudes the damping decrement should approach asymptotically the amplitude-independent value Γ_N determined from

formula (28) with $\gamma = \gamma_N$.

Two mechanisms of nonlinear absorption of ultrasound are thus possible in a superconductor at low temperatures (as $T \rightarrow 0^\circ\text{K}$). The threshold amplitudes of these mechanisms are determined by different parameters and therefore, generally speaking, they do not coincide, so that these mechanisms can be separated. It should be noted that for not very pure crystals one should expect nonlinear absorption connected with the threshold velocity V_C to manifest itself at amplitudes larger than those needed for the dislocations to break away from the impurity; only after breakaway from the dislocations do the dislocation-segment lengths become large enough to be able to satisfy the inequality $V(\sigma, \omega) > V_C$.

We know of no attempts of separating the second nonlinear-absorption mechanism by analyzing the experimental data.

5. CONDUCTION ELECTRONS AND PLASTICITY OF METALS

We shall analyze the influence of electron drag of dislocations on the plasticity of metals by using as an example the simplest dislocation model of plastic deformation.

Assume that the crystal contains one effective glide system with a mobile-dislocation density N which is uniform along the crystal. In this case the fundamental equation that determines the kinetics of the plastic-deformation process is

$$\dot{\epsilon}_p = bN\bar{V}(\sigma, \sigma_1); \quad (34)$$

here $\dot{\epsilon}_p$ is the instantaneous value of the plastic-deformation rate, and $\bar{V}(\sigma, \sigma_1)$ is the average velocity of an individual dislocation and depends on the instantaneous value of the external deforming stress σ and on the average level of internal stress σ_1 ; the velocity V should be averaged along the direction of motion of the dislocation, over distances in which its motion has a non-stationary character (for example, atomic distances in a crystal without defects or the characteristic distances between local barriers produced by structure defects).

We must stipulate at once that Eq. (34) with N constant is highly idealized and does not take into account many singularities of the real deformation process. These are, first, the multiplicity of the glide systems, each of which has values of N and V that are in general different from those of the others. Second, the mobile-dislocation density N , even if homogeneous along the crystal, can in the general case depend on the time and on the external and internal stresses; the assumption $N = \text{const}$ is equivalent to the assumption that the dislocation sources operate in a regime in which each mobile dislocation that goes out of play is immediately replaced by a new one. Finally, owing to the long-range character of the elastic dislocation field, it is very important to take correct account of their interaction with one another. This circumstance is taken into account in (34) only by introducing a certain characteristic internal-stress level σ_1 .

Nevertheless, experience shows that Eq. (34) makes it possible to describe, at least qualitatively, most of the important features of the plastic deformation of crystals. Since the superconducting transition does not lead to excessively large changes in the deformation process (the integral value of the effects is $\lesssim 10\%$), one can hope Eq. (34) to remain applicable, in the sense indicated above, after the superconducting transition.

To use (34) to describe the deformation it is necessary, of course, to have the explicit form of $\bar{V}(\sigma, \sigma_i)$, and also to know the law governing the hardening, i.e., the connection $\sigma_i = \sigma_i(\epsilon_p)$ between the internal stresses σ_i and the value of the plastic deformation ϵ_p .

The $\bar{V}(\sigma, \sigma_i)$ dependence is determined by drag mechanisms that control the dislocation mobility; as already noted in the Introduction, in real crystals the physical nature of these mechanisms is different at high and low velocities.

The hardening law depends on the type of mechanism controlling the multiplication of the defects when the crystal is plastically deformed. There is at present no consistent theory of strain hardening; only certain particular cases have been considered (see, e.g., [88,90]). Experiment shows that at low temperatures and not too high strains, a good approximation for most metals is a linear hardening law described by the following phenomenological relation [88]:

$$\sigma_i = \sigma_i^0 + k(\epsilon_p - \epsilon_0), \quad (35)$$

where σ_i^0 and ϵ_0 are respectively the internal stress and strain at the yield point, and k is the hardening coefficient⁵⁾.

Let us note an experimental fact of importance in the exposition that follows: at low temperatures, the hardening coefficient of most metals is not sensitive to the temperature and does not depend on the rate of plastic deformation in a wide range of rates. This coefficient retains its value also after a superconducting transition.

The assumption that the parameters N and k in (34) and (35) are not altered by the superconducting transition greatly simplifies the problem of determining the influence of the transition on the plastic deformation process, reducing it to the problem of the change in the mobility of an individual dislocation.

It must be borne in mind, however, that one cannot exclude cases when the deformation process is governed to a considerable degree by the rate at which the dislocation sources operate. In this case, the cause of the softening should be sought in the influence of the superconducting transition on the work of the sources. Unfortunately, this process has not yet been sufficiently studied; there is only one paper [72] in which the influence of the transition on the initial stage of the work of a Frank-Read source is analyzed. According to [72], this influence is also a consequence of the change of the mobility of an individual dislocation (which breaks away in this case from the source).

Depending on the deformation conditions, the role of the variable quantity to be determined from (34) can be assumed either by the plastic deformation ϵ_p or by the deforming stress σ . For an experimental study of the plasticity, the following types of mechanical tests are most frequently used:

Creep: deformation of the sample under the influence of a constant stress ($\sigma = \text{const}$). The plastic deformation ϵ_p is then a monotonically increasing function of the time (Fig. 5a).

Active deformation: strain at a constant rate governed by the testing machine ($\dot{\epsilon} = \dot{\epsilon}_p + \dot{\epsilon}_e = \text{const} \neq 0$, where $\dot{\epsilon}_e$ is the rate of the elastic deformation); the variable quantity is the stress σ producing a specified deformation rate (Fig. 5b).

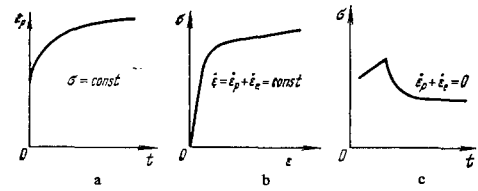


FIG. 5. Time dependence of the quantities characterizing plastic deformation of crystals in different types of mechanical tests ($\epsilon = \dot{\epsilon}t$).

Stress relaxation: the testing machine is stopped and the deformation of the sample is the result of elastic stresses existing in the sample and in the machine; in this case $\dot{\epsilon}_p + \dot{\epsilon}_e = 0$, and the stress σ decreases monotonically with time (Fig. 5c).

In the simplest case when the internal stresses are negligibly small and the external stress is so large that the dislocation motion is above the barrier, the function $\bar{V}(\sigma)$ is determined by the balance between the external force $b\sigma$ acting on the dislocation and the dynamic drag force $F(V)$:

$$b\sigma = F(\bar{V}). \quad (36)$$

As already noted in the Introduction, the principal dynamic drag mechanism in normal metals at low temperatures is electron viscosity. In this case relation (36) takes the form (see formula (7) of Chap. 2)

$$b\sigma = B_N \bar{V}. \quad (37)$$

In a superconductor we have

$$b\sigma = F_S(\bar{V}, T), \quad (38)$$

where F_S is determined by the formulas of Chap. 3.

Under the experimental conditions, the superconducting transition is produced most frequently by rapidly turning on or off the magnetic field that destroys the superconductivity. The electron drag force on the dislocation in this transition changes jumpwise⁶⁾ by an amount $\delta F_{NS} = B_N \bar{V} - F_S(\bar{V}, T)$, and this should naturally lead to an abrupt change in the deformation process. It is easily seen that the transition of the metal to the superconducting state under creep conditions should be accompanied by a sharp increase in the strain rate $\dot{\epsilon}_p$, and under conditions of active deformation the deforming stress σ should decrease sharply in such a transition; in either case, the plasticity of the metal is sharply increased.

The extent to which the plasticity is increased in the considered "dynamic" case is determined only by the jump in the drag force δF_{NS} , and can be obtained in principle with the aid of formulas (34), (37), (38), and the formulas of Chap. 3 for $F_S(\bar{V}, T)$. However, it is difficult to examine these quantities analytically because of the complicated form of the function $F_S(\bar{V}, T)$, and we shall not dwell on the analysis here. It should furthermore be noted that a purely dynamic situation is apparently rarely realized in plastic deformation. Large dislocation rates are realized, for example, in the heads of glide bands propagating under the influence of large pulsed loads [40,41], and possibly, in the case of a high active strain rates induced by large deforming stresses.

It is much more difficult to take into account the effect of the electrons on the plasticity of metals in the case when the dislocation mobility proceeds via fluctuation surmounting of local barriers. We confine our-

selves here to a discussion of crystals with negligibly low Peierls barriers⁷⁾ and assume that the main obstacles to dislocation motion are barriers produced by defects such as impurities or "forest" dislocations^[88].

The elementary act of plastic deformation in motion of this type is shown schematically in Fig. 6. If the dislocation moving under the influence of internal and external stresses has assumed the metastable configuration AOB, then after a certain time, owing to quantum or thermal fluctuations of required magnitude, the dislocation breaks away from the local barrier at the point Θ , after which the stress brings it to the position AO₁B. The average dislocation velocity \bar{V} in formula (34) is then given by

$$\bar{V}(\sigma, \sigma_1) \approx l w(\sigma, \sigma_1), \quad (39)$$

where l is the characteristic distance between the barriers and w is the average frequency at which the individual barrier is surmounted. Formula (39) is of course meaningful only so long as the average time w^{-1} required to overcome the barrier is much larger than the time of the free motion of the dislocation between neighboring barriers. To this end it suffices as a rule to satisfy the inequality $\sigma - \sigma_1 \ll \sigma_C^{(0)}$, where $\sigma_C^{(0)} = \pi E / 8b^2 l$ is the value, at $\omega = 0$, of the breakaway stress σ_C introduced in Chap. 4 (at $\sigma - \sigma_1 \geq \sigma_C^{(0)}$, the dislocation breaks away from the pinning point by purely mechanical means).

An analysis, within the framework of fluctuation theory, of the process whereby the dislocation surmounts the local barriers via fluctuations, shows that in the simplest cases the following expression holds for the quantity $w(\sigma, \sigma_1)$:

$$w = \nu \exp \left[- \frac{U - \nu(\sigma - \sigma_1)}{T^*} \right]. \quad (40)$$

Here U is the activation energy and is directly connected with the energy of interaction between the dislocation and the defect, ν is the activation volume, ν is a frequency factor (the frequency of attempts, and T^* is the effective temperature and is a function of the usual temperature T ($T^* = T^*(T)$)).

The quantities T^* and ν should be discussed in greater detail.

At sufficiently high temperatures, the breakaway of dislocations from barriers is due to thermal fluctuations, and T^* goes over into the ordinary temperature T ^[92,93]. At extremely low temperature (as $T \rightarrow 0^\circ\text{K}$), the principal role is assumed by quantum fluctuations, and T^* tends to a constant value determined by the probability of breakaway as a result of quantum fluctuations^{[92-94]B}.

The pre-exponential factor in (40), namely the attempt frequency ν , cannot be obtained within the framework of thermodynamic fluctuation theory (unlike the argument of the exponential), and its calculation calls for an approach based on methods of nonequilibrium

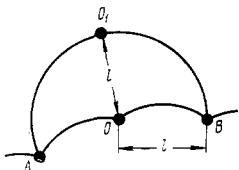


FIG. 6. Schematic representation of the elementary act of plastic deformation by fluctuation motion of the dislocation through an aggregate of local barriers.

statistical mechanics. Usually, on the basis of purely phenomenological considerations, it is assumed that this factor is close in magnitude to the characteristic frequency of the thermal fluctuations of the dislocation segment, and the latter is identified with its natural frequency $\omega_0 = (\pi/l) \sqrt{C/M} \sim s/l$ ^[88]. A simple analysis shows, however, that the factor ν depends essentially on the mobility of the segment, i.e., in final analysis, on the viscous drag force acting on it (see^[46]). This dependence can be introduced phenomenologically, assuming as before that ν coincides with the frequency of the thermal oscillations of the dislocations, but taking into account the renormalization of the latter by the drag force. An analysis based on the fluctuation-dissipation theorem (see, e.g.,^[47]) shows that the role of the characteristic frequency of the thermal oscillations of a dislocation string experiencing viscous drag is played by the quantity $\sqrt{\omega_0^2 - (\gamma^2/4)}$ at $\gamma \ll 2\omega_0$ and $(\gamma - \sqrt{\gamma^2 - 4\omega_0^2})/2$ at $\gamma \gtrsim 2\omega_0$. Considering for simplicity the limiting cases of weak and strong friction, we have

$$\nu \approx \begin{cases} \omega_0, & \gamma \ll 2\omega_0, \\ \frac{\omega_0^2}{\gamma}, & \gamma \gtrsim 2\omega_0. \end{cases} \quad (41)$$

In a normal metal we have $\gamma = \gamma_N = B_N/M$. In a superconductor, formula (41) remains meaningful so long as the electron-friction force depends linearly on the velocity, i.e., the condition for the applicability of formula (18) is satisfied. In this case $\gamma = \gamma_S = B_S/M$. Using for the estimates the rms fluctuation as the characteristic rate of thermal motion of the segment, we obtain the following criterion for the applicability of formula (41) to the case of superconductors^[47]:

$$\sqrt{\frac{2T}{lM}} \ll \frac{\Delta(T)}{lq_m}, \quad \frac{T}{lq_m}.$$

It is easily seen that this condition is satisfied in practically the entire temperature interval in which superconductivity exists. Exceptions are small vicinities of absolute zero ($T = 0^\circ\text{K}$) and of the superconducting-transition temperature ($T = T_C$).

Combining formulas (34), (35), (39), and (40), we obtain the following equation for the description of the kinetics of plastic deformation of a metal at low temperatures and small external stresses ($\sigma - \sigma_1 \ll \sigma_C^{(0)}$):

$$\dot{\epsilon}_p = Av(T) \exp \left\{ - \frac{U - \nu[\sigma - \sigma_1^{(0)} - k(\epsilon_p - \epsilon_0)]}{T^*(T)} \right\}, \quad (42)$$

where $A = bNl$, and the function $\nu(T)$ is determined, according to (41), by the temperature dependence of the dislocation electron drag coefficient B .

With the aid of (42) it is easy to analyze the changes that occur in the plastic deformation and accompany the superconducting transition of the metal^[47].

a) Creep ($\sigma = \text{const}$). In the normal state, we have an unsteady logarithmic creep:

$$\epsilon_p(t) = \epsilon_0 + \frac{T^*}{kv} \ln(\alpha \nu_N t + 1), \quad \dot{\epsilon}_p(t) = \frac{\alpha \nu_N T^*}{kv(\alpha \nu_N t + 1)}, \quad (43)$$

$$\alpha = \frac{kvA}{T^*} \exp \left[- \frac{U - \nu(\sigma - \sigma_1^{(0)})}{T^*} \right].$$

The superconducting transition is accompanied by a jump in the strain rate $\delta \dot{\epsilon}_{NS}$ and by additional deformation of the sample $\delta \epsilon_{NS}$ (Fig. 7a):

$$\delta \dot{\epsilon}_{NS} = \dot{\epsilon}_N(t_0) \left(\frac{\nu_S}{\nu_N} - 1 \right), \quad \delta \epsilon_{NS} = \frac{T^*}{kv} \ln \frac{\nu_S}{\nu_N}. \quad (44)$$

It is seen from Fig. 7b that the theoretical conclusion that the jump of the deformation rate $\delta \dot{\epsilon}_{NS}$ is propor-

tional to the rate $\dot{\epsilon}_N(t_0)$ at the instant of the transition agrees qualitatively with the experimental data^[24].

Experiment shows^[22, 24, 91] that the strain rate increases not simultaneously with the superconducting transition, but after the lapse of a certain time (delay time). It is natural to assume^[91] that the characteristic time of transition from one deformation regime to another is close in magnitude to the average time w^{-1} needed by the dislocation to surmount the local barriers. An analysis based on formulas (40) and (42) confirms this assumption^[69].

When the conclusions of the theory are compared with experiment, it must be borne in mind that Eq. (42) and the ensuing formulas (43) and (44) describe only the stage of unsteady creep; the case of steady-state creep calls for a separate analysis.

b) Active deformation ($\dot{\epsilon} = \dot{\epsilon}_p + \dot{\epsilon}_e \approx \dot{\epsilon}_p = \text{const}$). In a normal state, plastic flow takes place with linear hardening:

$$\sigma(t) = \sigma_0^{(0)} + \frac{U}{v} - \frac{T^*}{v} \ln \frac{A v_N}{\epsilon_p} + k(\dot{\epsilon}_p t - \epsilon_0). \quad (45)$$

The superconducting transition decreases the deforming stress by an amount $\delta\sigma_{NS}$ (Fig. 8):

$$\delta\sigma_{NS} = \frac{T^*}{v} \ln \frac{v_S}{v_N}. \quad (46)$$

The experimental values of the jump $\delta\sigma_{NS}$ of the deforming stress are $\lesssim 10^6 - 10^7$ dyn/cm². Thus, e.g., for lead we have $\delta\sigma_{NS} \lesssim 6 \times 10^6$ dyn/cm²^[15]. The independently measured parameter T^*/v of lead is equal to 10^6 dyn/cm²^[91]. Recognizing that the factor $\ln(v_S/v_N)$ in (46) can reach a value of several units (see formula (50) below), we can state that the theoretically predicted order of magnitude of $\delta\sigma_{NS}$ agrees with the experimental data.

c) Stress relaxation ($\dot{\epsilon}_p + \dot{\epsilon}_e = 0$). The elastic deformation ϵ_e is connected with the stress by the relation $\epsilon_e = \mathcal{X}^{-1}\sigma$, where \mathcal{X} is the effective elastic modulus of the testing machine. The equation describing the relaxation process is therefore

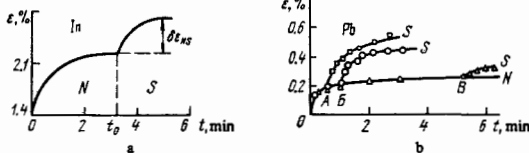


FIG. 7. Change of creep curve in superconducting transition. a) Jump of creep rate and additional elongation of an indium crystal following the superconducting transition at the instant of time t_0 ^[29]; b) dependence of the jump in the creep rate on the instantaneous rate at the instant of the superconducting transition: at the points A, B, and C, a transition from the normal to the superconducting state was effected respectively in three identical lead samples^[24].

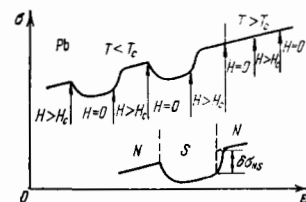


FIG. 8. Upper curve—section of deformation curve of lead with a magnetic field $H > H_c$ repeatedly turned on and off (H_c is the critical superconductivity-destroying field)^[15]. The lower curve shows a schematic representation of the jump in the deforming stress when the metal goes from the normal to the superconducting state and back^[4].

$$\dot{\epsilon}_p + \mathcal{X}^{-1}\dot{\sigma} = 0. \quad (47)$$

Assume that during the time preceding the instant t_0 at which the testing machine is stopped the sample had been deformed at a constant rate $\dot{\epsilon}_p = \text{const}$ and was in the normal state. Expressions describing the process of stress relaxation after the stopping of the machine can be easily obtained with the aid of (42) and (47):

$$\sigma(t) = \sigma_0 - \frac{T^*}{v} \ln [\beta(t - t_0) + 1], \quad \dot{\sigma}(t) = -\frac{\mathcal{X}\dot{\epsilon}_p}{\beta(t - t_0) + 1}, \quad (48)$$

$$\beta = \frac{\mathcal{X}v\dot{\epsilon}_p}{T^*}, \quad t > t_0.$$

A superconducting transition of the metal at the instant $t = t_1$ is accompanied by an increase $\delta\dot{\sigma}_{NS}$ in the relaxation rate and by an increase $\delta\sigma_R$ in the depth of relaxation (Fig. 9):

$$\delta\dot{\sigma}_{NS} = \dot{\sigma}_N(t_1) \left(\frac{v_S}{v_N} - 1 \right),$$

$$\delta\sigma_R = \begin{cases} \frac{T^*}{v} \ln \left[\beta \frac{v_S}{v_N} (t - t_1) \right], & \beta\tau \ll 1, \\ \frac{T^*}{v} \ln \left(\frac{v_S}{v_N} \frac{t - t_1}{\tau} \right), & \beta\tau \gg 1, \end{cases} \quad t - t_1 \gg \tau. \quad (49)$$

The proportionality of the jump of the relaxation rate $\delta\dot{\sigma}_{NS}$ to the quantity $\dot{\sigma}_N(t_1)$ was observed in^[39] (Fig. 10), and the logarithmic dependence of $\delta\sigma_R$ on the time τ at large values of τ was observed in^[34, 39] (Fig. 11).

It is seen from (44), (46), and (49) that the softening effect is determined to a considerable degree by the ratio v_S/v_N . This ratio depends significantly on the temperature, owing to the temperature dependence of the coefficient B_S . According to (18) and (41) we have (assuming that $l \ll l_0 = (2\pi/B_N)\sqrt{MC}$)

$$\frac{v_S}{v_N} = \begin{cases} 1, & T > T_c, \\ \frac{1}{2}(1 + e^{\Delta/T}), & T_0 < T < T_c, \\ \frac{l}{l_0}, & T < T_0. \end{cases} \quad (50)$$

The temperature T_0 is determined here from the equation

$$\frac{\Delta(T)}{T} = \ln \left(\frac{2l}{l_0} - 1 \right).$$

We note, however, that the temperature dependence

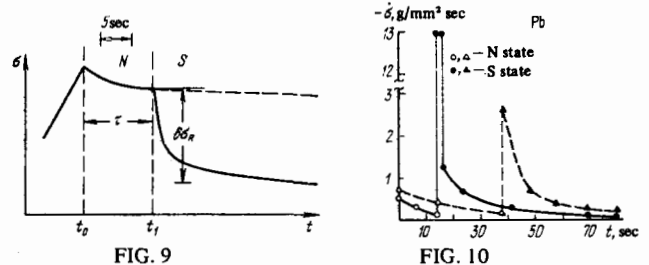
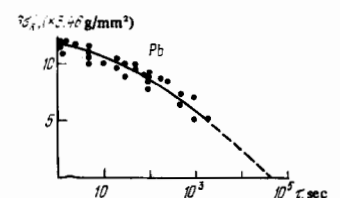


FIG. 9. Variation of stress relaxation in a lead crystal during the course of a superconducting transition^[34]: τ —duration of relaxation in the normal state, $\delta\sigma_R$ additional depth of relaxation.

FIG. 10. Jump of the stress relaxation rate, in the transition of a lead crystal from the normal to the superconducting state, vs the instantaneous relaxation rate at the instant of the transition^[39].

FIG. 11. Additional depth $\delta\sigma_R$ of the stress relaxation in the superconducting transition vs the duration τ of the relaxation in the normal state for lead crystals^[34].



of the softening is sensitive also to the form of the function $T^*(T)$, which is presently known only for zinc^[92].

This circumstance and the fact that the experiments were performed in a rather small temperature interval (there are practically no measurements at $T < 0.3T_C$) make, in our opinion, a detailed discussion of the temperature dependence of the softening effects premature. We note only that under reasonable assumptions concerning the form of the function $T^*(T)$, formulas (44), (46), and (49) yield relations that agree qualitatively with those observed in experiment.

If the characteristic distances between the barriers that hinder the dislocation glide are sufficiently small ($l \ll l_0 = (2\pi/B_N) \sqrt{MC}$), then we have ν_S/ν_N at all temperatures; obviously the softening effects should vanish in this case. For typical metals, the length is $l_0 \sim 10^{-5}$ cm; incidentally, this estimate is quite approximate, in view of the uncertainty in the values of B_N , M , and C . A decrease of the effect with increasing impurity concentration was observed in a study of the creep of lead with antimony impurity^[31]. When the impurity was in a nonequilibrium solid-solution state obtained by quenching the crystal from a high temperature, increasing its concentration from 1% to 3% decreased the value of $\delta\epsilon_{NS}$ by several times. This decrease was determined mainly by the increase of the hardening coefficient k , which was measured independently. After prolonged aging, the effect practically vanished in the same crystals ($\delta\epsilon_{NS} = 0$), but an increase of k could no longer ensure such a decrease. It appears that aging has led to the formation of impurity clusters and by the same token to a sharp decrease of the effective length of the dislocation segments.

We call attention to one important circumstance that must be remembered when the results of the theory are compared with the experimental data.

Many experimental data demonstrating the dependence of the softening effects on the number and form of the impurities, on the magnitude of the deforming stress, or on the preliminary plastic deformation, have been obtained by now^[19,21,24,26,31]. The theoretical formulas presented above do not contain these relations in explicit form. They are, however, implicitly contained in the relations between the aforementioned factors and phenomenological parameters of the theory such as the hardening coefficient k and the activation volume v . These parameters can be easily determined by experiment, and the theoretical conclusions can be verified by studying their dependence on the number and type of impurities and on the total plastic deformation. At the same time, a more detailed theoretical study of these relations would be quite useful.

In addition to the mechanism described above, one more mechanism was proposed for the influence of the superconducting transition on the surmounting of local barriers by dislocations; this mechanism employs the inertial properties of the dislocations^[70,71], and unlike the preceding mechanism it becomes manifest only in the case of weakly damped dislocation segments ($l < l_0$) and in a stress interval that is bounded both from above and below ($\sigma^{(0)}/2 < \sigma < \sigma_C^{(0)}$, if it is assumed, following^[70,71], that there are no internal stresses). Owing to the inertial forces, the dislocation can surmount local barriers by purely mechanical means at external stresses $\sigma < \sigma_C^{(0)}$, and the role of the inertial effects increases because of the decrease in the dis-

location drag coefficient in the superconducting transition. However, no analysis based on a consistent account of the inertial effects in (34) has been made so far of the influence of this mechanism on the plasticity of metals, and we are therefore unable to assess this mechanism. It must only be emphasized that the inertial mechanism becomes manifest in the intermediate stress region ($\sigma \approx \sigma_C^{(0)}$), when both mechanisms of surmounting the obstacles by the dislocation play approximately equal roles. In addition, the time to surmount an individual obstacle becomes comparable in this case to the time of travel between neighboring obstacles. Therefore the separation of the inertial mechanism in pure form seems to be difficult. It should also be noted that the authors of^[20,70,71] use in the analysis of the inertial mechanism an expression linear in the velocity for the drag force in the superconductor, $F_S(V, T) = B_S(T)V$, in spite of the fact that in this case the realized dislocation velocities are quite high, and the function $F_S(V, T)$ is essentially nonlinear (see Chap. 3).

The foregoing theoretical premises concerning the influence of electron dragging of dislocations on the plasticity of metals are based on a rather simplified model of plastic deformation, in which no account is taken of a number of factors inherent in a real crystal, such as the multiplicity of the glide systems, the inhomogeneity of the deformation along the sample, the uneven distribution of the distances between barriers, etc. A number of phenomenological and semiphenomenological relations such as (35) and (40) have been used, so that the results contain phenomenological parameters that depend significantly on the structure and internal state of the crystal. This means that the existing theory can claim only a qualitative description of the softening effects. Nevertheless, a comparison of the results obtained in this obviously qualitative theory with experiment shows that this theory "works"! To be sure, some of the conclusions of the theory (they were noted above) still await their experimental confirmation, primarily when it comes to the dependence of the softening effects on the temperature and on the defect structure of the crystal.

Solid-state theory is customarily divided presently into several distinctly delineated branches: electron theory of metals and semiconductors, dynamic theory of crystal lattices, plasticity and strength theory, etc. The development of each of these branches is accompanied, naturally, by "junctions," "intersections," and "overlaps." It seems to us that the present review offers evidence of the appearance of a new junction, between the electron theory of metals and plasticity theory.

We take the opportunity to thank V. V. Pustovalov for great help in the selection of the literature on the questions considered in the review.

¹Strictly speaking, the electrons become redistributed around the dislocation in the metal, so that the Fermi function contains in place of ϵ_F the coordinate-dependent chemical potential. This refinement, however, leads only to a certain renormalization of the deformation-potential tensor in the final results and is therefore inessential.

²Dislocation dragging by an electron stream in a particular case of electron wind, which is the displacement of crystal defects under the influence of translational motion of the conduction electrons^[75].

³The authors are indebted to A. M. Kosevich for these two concluding remarks.

⁴Actually there is a certain scatter of the segment length, and allowance

for this scatter greatly complicates the picture of the phenomena from the quantitative point of view.

- ⁵Generally speaking, the deformation of the crystal from the start of the plastic flow to the failure goes through several (most frequently two) stages of linear hardening, with different hardening coefficients. In these cases the quantities $\sigma^{(0)}$ and ϵ_0 should pertain to the start of the considered stage.
- ⁶The time required to change the drag force is determined by the time of the superconducting transition in the sample. As a rule, this time is much shorter than the characteristic times of the deformation process. This is confirmed by the results of studies of the effect of delay of the deformation in the course of a superconducting transition under creep condition [^{66,91}] and the kinetics of the transient process under active-deformation conditions [^{17,18}].
- ⁷Softening effects were observed in the metals Pb, In, Sb, Tl, Hg and Nb and in a number of superconducting alloys. There are grounds for assuming that almost all these metals have low Peierls barriers; the only possible exception is niobium.
- ⁸It should be noted that the problem of the influence of quantum, fluctuations on the breakaway of a dislocation from the defect that pins it is in essence a particular case of a more general problem of the influence of quantum fluctuations on the kinetics of phase transitions, which has been under a lively discussion of late [^{95,96}]. The role of quantum fluctuations in the course of generation of double kinks on dislocations in crystals with large Peierls barriers is analyzed in [^{97,98}].
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