On some results of statistical physics (from the materials of the 4th Conference on Statistical Physics, L'vov, Sept. 4-10, 1972)

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At present the following problems occupy the center of attention of many researchers in the field of statistical mechanics: 1) The statistical theory of equilibrium systems of interacting particles. 2) Non-equilibrium statistical thermodynamics. 3) General problems of solid-state theory. Phase transitions.

This is precisely the set of problems that was the topic of the 4th yearly Conference on Statistical Physics that was held in the L'vov Institute of Theoretical Physics of the Academy of Sciences of the Ukrainian SSR.

It is germane to decipher somewhat the notice advertising the conference. The fundamental core of the conference, as was reflected in most of the papers, was the application and development of the ideas, concepts, and methods of N. N. Bogolyubov in the field of statistical mechanics. Remarkably, as we see it, all of N. N.

Bogolyubov's concepts and methods in statistical physics (and some are already of mature "age," being of the same age as a good half of the researchers working in this field of science) thus far have been used, discussed, and substantiated, or in a word, have "operated" with ever growing efficiency. And indeed, "there is nothing more practical than a good theory!"

Turning to the content of the papers, we note first of all that most of the reports, as we see it, were interesting, they elicited broad discussion—mostly business-like reports with a limited number of authors, and in this sense, the reports were useful not only to the speakers. If, as the Organizing Committee has agreed, I shall take up below certain reports, this is explained mainly by the limited size of this note, which permits me to characterize in more detail only some of the papers of rather general nature. (The materials of this conference, as those of the preceding ones, will be fully published by the publisher "Naukova Dumka" in Kiev. This will make it possible for those who wish to acquaint themselves with all the presented papers to do so.)

The paper by D. Ya. Petrina was devoted to proving the existence of a solution of N. N. Bogolyubov's well-known chain of equations (in the classical and quantum cases), and it aroused a vigorous discussion. This is natural. The chain of equations for the distribution function is used widely and ubiquitously. Yet the problem of existence and uniqueness of the limiting distribution functions and their dependence on the density, which is closely connected with the admissible class of potential functions, has relatively recently been posed in a strict mathematical format and solved, first of all, by N. N. Bogolyubov himself (and also by his students D. Ya. Petrina and B. I. Khatset). D. Ya. Petrina was able to prove rigorously the analyticity of the free energy (as a function of the density), as well as the existence and analyticity of the correlation functions as a definite class of potentials. These studies are currently being continued along the line of studying a situation involving potentials that rather realistically approximate physical reality.

The approach developed by N. N. Bogolyubov Jr. and his associates seems interesting and important. These studies have been concerned with a procedure of replacing various model Hamiltonians with approximating Hamiltonians (both for the free energy and for the quasi-averages). In particular, this permits one to construct exactly solvable models. Here a new approach has been developed for calculating the means over a Hamiltonian that is invariant with respect to some transformation group. The degeneracy with respect to the latter is not removed (i.e., the means are calculated without recourse to the concept of "quasi-averages"). This can be used for studying systems with "spontaneous breakdown of symmetry". The developed formalism is illustrated by a model Hamiltonian of the BCS type. A paper by B. I. Sadovnikov and V. K. Fedyanin was devoted to the problem of specific ordering, as treated on the basis of N. N. Bogolyubov's inequalities for the Green's commutator functions. We emphasize that the mathematical apparatus of N. N. Bogolyubov's work on quasi-averages can be used effectively for studying problems of degeneracy of statistical-equilibrium states arising from the invariance of the Hamiltonian of the system with respect to some transformation group. This is directly related to the problem of second-order phase transitions, which is the central unsolved problem of modern statistical mechanics. New features in the above-cited paper were the use of N. N. Bogolyubov's inequalities in an Ising model and a rigorous proof of absence of crystalline ordering in one- and two-dimensional systems characterized by Ruelle behavior of the free energy (the interaction at infinity declines as r^{-n}, with $n > d$). The fact that any of the spectral functions in the Ising model are sums of δ-functions whose coefficients are certain definite combinations of correlators (S. V. Tyablikov and V. K. Fedyanin first established these results in 1966) has the result that N. N. Bogolyubov's inequalities give rise to a direct and rigorous algorithm for establishing highly non-trivial inequalities for the correlation functions. These inequalities are valid throughout the range of temperatures and magnetic field intensities. In turn, this is essential in using the Ising model in the theory of second-order phase transitions. The rigorous proof of lack of crystalline ordering is based on defining the latter as the existence of "condensation points," and in no way does it use any of the approximations of lattice theory.

A number of studies were devoted to applying the displacement method and collective variables in the theory of the electron gas in metals and in the theory of Bose systems. Thus, the paper by I. R. Yukhnovskiĭ and R. N. Petrashko analyzed the well-known difficulty involving the negativity of the second correlation function at metallic densities (as $r \to 0$), and they constructed exact group quantum expansions (quantum analogs of the virial expansions) for degenerate Fermi systems. The latter permitted them to obtain throughout the range of r a positive definite binary correlation function. A paper by I. R. Yukhnovskiĭ and I. A. Vakarchuk determined by a single method the wave functions of the ground and weakly-excited states of Bose systems, and they developed an effective perturbation theory based on successive inclusion of interactions arising from ever larger numbers of transfer vectors.

A. V. Svidzinskiĭ and his associates have conducted a very promising study. It dealt with current states in superconducting contacts (superconductor—normal metal—superconductor). They showed that N. N. Bogolyubov's equations can be solved exactly in a model having a piecewise-constant value of the ordering parameter, if one has performed a preliminary "smoothing" over lengths of atomic order. The methods developed here are also applicable for studying other types of superconducting contacts. A paper by W. Ebeling (East Germany) dealt with constructing equations of state that clearly distinguish between the contributions of free and bound states. While the former contribute in the form of a certain density function, the latter appear as expansions in terms of the activity. Of course, this is closely connected with the differing natures of the forces caused by the existence of these two types of states. Papers by K. D. Tovstyuk and his associates were devoted to different models of a metal; an essential point here is the use of group-theoretical methods in the many-electron treatment of the solid state. In particular, M. F. Olekseeva and K. D. Tovstyuk treated the problem of constructing a model Hamiltonian based on pair interaction with group-theoretical selection rules taken into account. A paper by A. S. Davydov and A. A. Serikov was devoted to different possible ways of accounting for the phonon background in the sum rules. A paper by A. S. Davydov on an exciton mechanism of muscle contraction attracted a very broad audience (physicists, biologists, and chemists). The model proposed by A. S. Davydov is simple, free from many unnatural assumptions that are characteristic of the existing utilitarian models, and allows the treatment of both boundary and temperature effects. Of course, as the author of the model himself emphasized, as well as in much discussion from the floor, the deciding argument here must be experiment. In any case, the model seems very promising, and the discussion was very vigorous.

Several of the papers discussed concrete problems of magnetism. On the basis of various spin Hamiltonians with widespread use of the well-known Bogolyubov-Tyablikov splitting and its corresponding analogs, they discussed both the possible ways to get out of this framework and various interpolations in the low- and high-temperature regions.

The discussion papers and comments from the floor on problems of non-equilibrium statistical mechanics naturally reflected the presence at the conference and active participation in it of Yu. L. Klimontovich and S. V. Peletminskiĭ. As we know, Yu. L. Klimontovich is developing ways of possible generalization of Boltzmann's equation that take account of slow, non-equilibrium fluctuations (with a correlation time comparable with the time of free flight). For this purpose, he constructs an equation for the smoothed distribution functions in which the contributions from large- and small-scale correlations are separated. For the latter, he uses the well-known Bogolyubov's principle of correlation weakening. S. V. Peletminskiĭ has constructed a coarsened density matrix, which, in full accord with N. N. Bogolyubov's fundamental ideas, proves to depend on the time via some quite definite set of parameters. Here S. V. Peletminskiĭ starts with a density matrix that satisfies in a certain way the formulated principle of correlation weakening (thus, e.g., the means of the product of quasi-local operators calculated from the spatially-homogeneous density matrix break down into products of means upon separation of the times, etc.) The set of parameters itself is ultimately determined by the Hamiltonian of the system. A new point here was a formalism developed in S. V. Peletminskiĭ's paper for constructing operators for fluxes of physical quantities in terms of the density operators of physical quantities. Further, by using N. N. Bogolyubov's concept of quasi-averages, he was able to find the means of the flux operators in terms of the thermodynamic potential of a superfluid liquid (he illustrated the entire formalism with the example of this model).

Both approaches are promising, and they permit one to discuss a broad class of concrete problems, as has been demonstrated in a paper by V. V. Krasil'nikov, S. V. Peletminskiĭ, and A. A. Yatsenko, who studied the interaction of electrons with paramagnetic impurities, and in a paper by V. V. Belyĭ, V. I. Emel'yanov, and Yu. L. Klimontovich, who treated non-equilibrium fluctuations in semiconductors. A number of papers here dealt with quite concrete problems: lattice thermodynamics based on the local-equilibrium ensemble; the role of dispersion in propagation of ultrasound in a liquid—here the corresponding moduli were approximated within the framework of the similarity hypothesis; application of the method of projection operators to the theory of Brownian movement, etc.

Three seminars of generally tutorial nature were conducted at the conference: 1) methods of nonequilibrium statistics (directed by Yu. L. Klimontovich), 2) elementary excitations in Fermi and Bose systems (directed by A. S. Davydov and I. R. Yukhnovskiĭ), 3) methods of modern quantum statistics in the Ising model (directed by V. K. Fedyanin). To judge from the rather numerous collection of participants and their activity (questions, comments, etc.), the seminars proved useful. (The materials of the seminars will be published by the Institute of Theoretical Physics of the Academy of Sciences of the Ukrainian SSR, Kiev.)

The organization of the conference is to be esteemed most highly. Basically, they strictly observed the time rule for papers (from 20 to 40 minutes) and short communications (5—10 minutes). As a rule, the general discussion time was not limited, and this made it possible to discuss comments from all sides. The problems of arrangement of participants, auditoriums, leisure time, etc., were excellently solved. All the participants will surely recall the excursion into the Carpathians to the "Dobusz Cliffs" and to the trout hatchery.

Contacts and discussions facilitated elucidation and explanation of a number of fundamental problems of modern statistical physics, and it seems to me that the experience gathered at the four conferences will facilitate the holding of subsequent conferences at the same high level.

In lieu of concluding remarks. The International Conference on Mathematical Problems of Field Theory and Quantum Statistics was held from Dec. 12 to 19, 1972 in the V. A. Steklov Mathematical Institute of the Academy of Sciences of the USSR. It was called on the initiative of Academician N. N. Bogolyubov, and both Soviet (about 300 participants) and foreign researchers (about 90 persons) were broadly represented. Naturally, this event requires a separate note with a field-theoretical "slant": the overwhelming number of papers were on mathematical problems of elementary-particle theory. Yet it is pertinent to note that the material of the L'vov conferences on equilibrium aspects of statistical physics (the first five conferences that we mentioned above) was used to some extent also by the participants at the December conference (in the quantum-statistics section). These papers were met with interest, both by the Soviet and the foreign scientists (in particular, the latter were represented by such well-known researchers as Cohen, Guggenholz, Ruelle, Hepp, and Lanford). This redounds once more to the credit of the yearly conferences conducted by the Academy of Sciences of the Ukrainian SSR.

Translated by M. V. King