# The Doppler method of measuring local velocities using lasers

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The article reviews research on a new optical method, based on the Doppler effect, of investigating flows of liquid and gases. The connection between the statistical characteristics of the photocurrent and the statistical characteristics of a stream contain light-scattering particles is discussed. Methods are considered for the separation of the Doppler frequency shift, which contains information on the local flow velocity, viz., photomixing, direct photodetection, and optical spectral analysis. The most frequently employed systems of Doppler optical velocity meters are described for laminary and turbulent flows of liquid and gases, at both subsonic and supersonic velocities.

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## 1. INTRODUCTION

Doppler predicted theoretically as early as 1842[1] the phenomenon of shift of the frequency of radiation when its source or receiver moves. This effect has found wide application in acoustics, radiophysics, and optics in the past century. The relative frequency shift of the radiation is proportional to the ratio of the speed of motion of the source (or receiver) to the speed of propagation of the vibrations. Hence the Doppler effect was initially verified in acoustics, where the speed of motion of the source can be comparable with the speed of propagation of sound.

The widespread application of the Doppler effect in the radiofrequency range arises from the fact that monochromatic radiation sources have been invented here, and special methods of measuring small frequency shifts have been developed. Thus, one can detect a speed of motion of the order of 3 cm/sec by detecting a frequency difference of 1 Hz in the 3-cm range.

Matters have been more complicated in using the Doppler effect in the optical range. Although the Russian physicists Belopol'skiĭ and Golitsyn[2,3] had verified the effect experimentally at the beginning of this century, as Fabry and Buisson[4] did later, yet the lack of powerful sources of monochromatic radiation and of methods of measuring small frequency shifts considerably hindered further studies in this field.[1)] Thus, the best of the optical devices, the Fabry-Perot etalon, has a resolving power of the order of $10^7$. However, owing to the high frequency of optical vibrations ($\sim 5 \times 10^{14}$ Hz) and the relatively broad spectrum of radiation of the usual sources, people could measure velocities no smaller than 100 m/sec. Hence, the Doppler effect in the optical range was mainly used in plasma diagnostics and astrophysics, where the velocities of motion of self-luminous objects are large, and also in the interpretation of the spectral composition of molecularly scattered light (the fine structure of the Mandel'shtam-Brillouin line).

We recall that the conclusion that the universe is expanding was made specifically by accounting for the Doppler effect, while experimental verification of the transverse Doppler effect contributed substantially to confirming the special theory of relativity.

The situation changed substantially when the method of detecting small frequency differences of two light waves was developed (the method of optical photomixing), and also coherent light sources in the optical range were invented (lasers). It became possible to go over to methods of active Doppler location, or measuring the velocities of motion of objects from the frequency shift in laser radiation that they scattered.

Righi[6] was involved with detecting beating of light with small frequency differences even at the end of the last century. However, the method of optical photomixing began to be widely used only after the studies of Gorelik[7] and Forrester.[8] In essence, it is analogous to the superheterodyne method of detecting oscillations in the radiofrequency range, and it permits one to detect frequency differences of two light waves down to 1 Hz. This means that one can detect the motion of an object having a velocity of 1 μm/sec, e.g., the rate of growth of bamboo or motion of glaciers; similar velocities are shown by particles in Brownian movement or liquids in natural convection when the temperature difference amounts to only about 0.01°C. Such potentialities could not help but be noted, and the optical Doppler method has taken a firm place in the experimental practice of many laboratories in the world. A new field of application of lasers began to develop: close-range Doppler optical location, measuring local velocities of flows of liquids and gases, and also of closely spaced moving objects.

One measures the local velocity of gas and liquid flows from the Doppler effect by probing points to be studied in the flow with a beam of coherent light. The flow must contain optical inhomogeneities, which can be either specially introduced fine particles, e.g., polystyrene of dimensions about 0.5 μm, or smoke particles for gases, or natural inclusions. Two-phase flows usually contain enough optical inhomogeneities that can scatter light. Owing to the Doppler effect, the light thus scattered has a different frequency, which depends on the velocity of motion. The frequency instability of lasers does not allow the frequency of the scattered light to serve as a quantitative characteristic of the motion of the object, especially at low velocities. Hence the frequency must be compared with the instantaneous frequency of the probe beam, so that one can then find their difference, from which the velocity can be found.

The Doppler method of measuring local velocities in liquid and gas flows has been widely applied since the studies of Yeh and Cummins,[9,10] who were the first to prove the possibility of photomixing of light scattered by bulk optical inhomogeneities. The light was scattered by monodisperse polystyrene particles of dimensions 0.5 μm that were specially introduced into a liquid flow. These studies were the basis of the stated method of measuring velocity by using laser Doppler velocimeters (LDV).

The fundamental lines of development, both in LDV systems and in applying them were actually established in studies conducted from 1964 to 1969. They are reviewed in[11-13]. During this period, the method was developed both along the line of refining and developing new optical systems, such as the single-beam system,[14,15] the double-beam system,[16] the differential system,[17-26] the system for measuring the velocity vector,[27] and systems using a Fabry-Perot interferometer;[28-30] and also along the line of extending the fields of application of it for studying laminar liquid flows,[31-36] gas flows,[33,37,38] non-Newtonian liquids,[39-41] turbulent flows,[42-45] and two-phase supersonic flows.[26,28,30,46]

In subsequent years researchers have directed efforts toward a rather complete theoretical basis for the method, especially in studying turbulent flows,[48-50] and in inventing an instrument capable of supplanting the thermoanemometer, which has been widely applied heretofore in research practice. This review will treat the further development of this method, using the articles that have been published up to August, 1972.

We should mention two possible treatments of the operation of the Doppler velocimeter. The first articles on this topic treated the appearance of an a.c. component of the photodetector current as resulting from nonlinear detection of the two waves, which have different frequencies, owing to the Doppler effect. However, Rudd[51] tried to prove that the operation of the LDV can be fully explained only on the basis of detection by the photodetector of running real or imaginary interference bands. Durst and Whitelaw[52] have even introduced a classification of optical velocimeters as belonging to Doppler or interference types. The equivalence of the two approaches from the standpoint of optics arouses no doubt, if the treatment is carried out with account taken of all phase and polarization effects arising upon scattering of the electromagnetic waves. An analysis of the operation of an LDV with account taken only of the intensity of the wave does not always correspond to reality.[53]

## 2. SPECTROSCOPIC ANALYSIS OF THE SCATTERED RADIATION

In the Doppler method of measuring velocity, the spectral characteristics of the scattered light are of greatest interest, since they contain the information on the motion of the object.

If coherent radiation having frequency $\omega_0$ and wave vector $\mathbf{k}_0$ falls on an object moving at velocity $\mathbf{u}$, and the scattered radiation is observed in a direction having wave vector $\mathbf{k}_S$, then in the non-relativistic case the frequency difference of the scattered radiation $\omega_D$ from the incident radiation will be[54]

$$\omega_D - \omega_0 = (\mathbf{k}_s - \mathbf{k}_0)\,\mathbf{u} \equiv \mathbf{K}\mathbf{u}. \qquad (1)$$

Thus, the frequency difference $\omega_D - \omega_0$ contains information on the projection of the velocity vector on the difference vector $\mathbf{K}$ (Fig. 1) that depends on the wavelength of the incident radiation and the angle between the vectors $\mathbf{k}_S$ and $\mathbf{k}_0$.

One can obtain the spectral characteristics of the scattered radiation fully only by treating the scattering of coherent light by the studied object. In close-range optical Doppler location, the objects of study are rough surfaces or gas or liquid flows containing optical inhomogeneities in the form of particles.

Light scattering by individual particles has been treated in the monographs[55,56]. The features of scattering of coherent light by bulk optical inhomogeneities have been given in[57] and scattering of light by a surface is treated in the review[58].

We should note that in treating light scattering as yet little attention has been paid to the phase and polarization characteristics, which are very important in this method.

The spectral characteristics of coherent radiation scattered by moving optical inhomogeneities have also been studied by a number of authors.[48-50,59-61] However, problems of the interaction of laser radiation with a medium containing optical inhomogeneities having random characteristics requires further study. First of all, this involves the breakdown of spatial coherence when radiation propagates through such a medium.

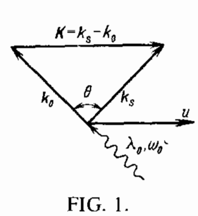


FIG. 1.

Let us study the spectrum of radiation scattered by individual discrete particles that move along with the flow being studied, and whose trajectories of motion are $\mathbf{r}(t)$. If the electromagnetic wave $E_0(\mathbf{r}, t)$ falls on a particle, then the scattered field at the distance $\mathbf{R}$ will be:

$$E_s(\mathbf{R}, t) = \frac{E_0(\mathbf{r}, t)\,\sigma \exp(ik|\mathbf{R}-\mathbf{r}|)}{|\mathbf{R}-\mathbf{r}|}, \quad (2)$$

Here $\sigma$ is the complex scattering function of the particle, $k = 2\pi/\lambda$, and $\lambda$ is the wavelength. The case of greatest interest is the one in which the dimensions of the scattering volume are considerably smaller than the distance to the observation point. Then we can replace the denominator by R, and keep only the linear terms in the exponential.

Let the incident radiation be monochromatic:

$$E_0(\mathbf{r}, t) = \sqrt{P}\,\mathrm{Re}\,W(\mathbf{r}) \exp[-i(\omega_0 t - \mathbf{k}_0\mathbf{r})], \quad (3)$$

where P is the power of the incident beam, and $W(\mathbf{r})$ is the normalized weighting function of the amplitude distribution of the light wave. The scattered field is a function of the distance R and the direction of observation $\mathbf{k}_s$, i.e., a function of the wave vector $\mathbf{K}$. In the single-scattering case, we can represent the scattered field from many particles as a sum of the scattered fields from each particle:

$$E(\mathbf{K}, t) = \mathrm{const}\cdot\sum \sigma_n \exp[i\mathbf{K}\mathbf{r}_n(t)] \exp(-i\omega_0 t)\, W(\mathbf{r}). \quad (4)$$

Let the particle be transported at the constant velocity $\mathbf{u}$. Then $\mathbf{r}_n(t) = \mathbf{r}'_n(t) + \mathbf{u}t$, where $\mathbf{r}'_n(t)$ depends on the time, since the scattering particle can undergo random migration owing to Brownian movement or turbulent pulsations in the flow. Then we can write (4) in the form

$$E(\mathbf{K}, t) = \mathrm{const}\cdot\sum \sigma_n \exp\{i\mathbf{K}[\mathbf{r}'_n(t) + \mathbf{u}t]\} \exp(-i\omega_0 t)\, W[\mathbf{r}'_n(t) + \mathbf{u}t]. \quad (5)$$

The summation in this expression is performed over all the particles that are scattering light at the given instant. An adequate characteristic of the scattered random field is its statistical mean, the correlation function $R(\mathbf{K}, \tau)$, which for a steady-state process is

$$R(\mathbf{K}, \tau) = \mathrm{Re}\,\langle\langle E(\mathbf{K}, 0)\, E(\mathbf{K}, \tau)\rangle\rangle, \quad (6)$$

Here the angle brackets denote averaging over the ensemble, including averaging over the initial positions of the scattering centers and averaging over their displacements with respect to the liquid.

If the particles are distributed randomly, and their displacement with respect to the liquid does not depend on their initial positions, then the correlation function of the scattered field is[49]

$$R(\mathbf{K}, \tau) = \mathrm{const}\cdot\mathrm{Re}\,[\exp(i\omega_0\tau) \int_{-\infty}^{\infty} G(\Delta\mathbf{r}, \tau) \exp(-i\mathbf{K}\,\Delta\mathbf{r}) Q(\Delta\mathbf{r} + \mathbf{u}\tau)\, d(\Delta\mathbf{r})], \quad (7)$$

Here $G(\Delta\mathbf{r}, \tau)$ is the space-time correlation function of Van Hove, which shows the density of probability that the scattering particle will move the distance $\Delta\mathbf{r}$ during the time interval $\tau$, $\Delta\mathbf{r}(\tau) = \mathbf{r}(\tau) - \mathbf{r}(0)$, and $Q(\Delta\mathbf{r} + \mathbf{u}\tau)$ is the autocorrelation function of the amplitude distribution of the field, which is defined as follows:

$$Q(\Delta\mathbf{r} + \mathbf{u}\tau) = \mathrm{const}\cdot \int_0^{\infty} W[\mathbf{r}(0)]\, W[\mathbf{r}(0) + \Delta\mathbf{r} + \mathbf{u}\tau] \exp(-i\mathbf{K}\mathbf{u}\tau)\, d\mathbf{r}(0). \quad (8)$$

According to the Wiener-Khinchin theorem for random steady-state processes, the power spectrum of the scattered light can be found by Fourier transformation of its correlation function:

$$S(\mathbf{K}, \omega) = \int_0^{\infty} R(\mathbf{K}, \tau) \exp(-i\omega\tau)\, d\tau. \quad (9)$$

It is interesting to note that one cannot get the power spectrum by Fourier transformation of the square of the amplitude of the field, since the quantity thus obtained is concentrated at the frequency $2\omega_0$. The spectral amplitude also cannot serve to characterize the scattered field, since it is a random quantity.

Accounting for the diffusional motion of the particles in an infinite volume leads to the following expression for the Van Hove function:[49]

$$G(\Delta\mathbf{r}, \tau) = (4\pi D\tau)^{-3/2} \exp\left[-\frac{(\Delta\mathbf{r})^2}{4D\tau}\right]. \quad (10)$$

where D is the diffusion coefficient of the particle.

If the incident radiation is homogeneous, then

$$S(\mathbf{K}, \omega) = \frac{\mathrm{const}\cdot K^2 D}{(K^2 D)^2 + (\omega_0 - \omega - \mathbf{K}\mathbf{u})^2}. \quad (11)$$

This equation shows that the spectrum of the scattered radiation is shifted in frequency with respect to the incident wave by the amount $\mathbf{K}\cdot\mathbf{u}$, and it has a Lorentzian form. The half-width of the spectrum will be $2K^2D$, and it will depend only on the diffusion coefficient of the scattering particle and the wave vector $\mathbf{K}$. In the case of a finite volume, the spectrum of the scattered light also depends on its dimensions.[49]

One must account for diffusional motion when measuring the transport velocity only when the latter is small. In order that the Doppler frequency can be distinguished, the following condition must be obeyed:

$$\mathbf{K}\mathbf{u} > K^2 D. \quad (12)$$

When we take account of the fact that

$$K = \frac{4\pi n}{\lambda_0} \sin\frac{\theta}{2}, \quad (13)$$

where n is the refractive index of the medium, the condition (12) takes on the following form:

$$u > 4\pi n D \lambda_0^{-1} \sin\frac{\theta}{2}. \quad (14)$$

For the typical values $\theta = 45°$, $\lambda = 6328$ Å, and $D = 10^{-8}$ cm$^2$/sec, the minimum detectable velocity will be $u > 0.001$ cm/sec.

If the incident radiation has a Gaussian amplitude distribution with a beam radius w, then for a laminar flow, the form of the spectrum of scattered radiation will also be Gaussian:

$$S(\mathbf{K}, \omega) = \mathrm{const}\cdot\frac{w}{u} \exp\left[-\frac{w^2}{2u^2}(\omega_0 - \omega - \mathbf{K}\mathbf{u})^2\right], \quad (15)$$

with a half-width at the 1/e level of

$$\Delta\omega = \frac{2\sqrt{2}\,u}{w}. \quad (16)$$

One cannot calculate the spectrum of the scattered radiation for turbulent flows in the general case because one does not know the Van Hove function. Although these problems have been treated partially in[60,61], there are still many unsolved problems here. Indeed, in certain special cases, e.g., for a homogeneous and isotropic turbulent flow, the spectrum of the scattered radiation is determined by the degree of turbulence of the flow, and is used in finding the latter.[42,62]

## 3. METHODS OF DETERMINING THE DOPPLER FREQUENCY SHIFT

One can extract the information on the motion of an object conveyed in the light that it scatters by subjecting the latter to spectral analysis. One can easily detect large Doppler shifts by analyzing the optical frequencies with narrow-band filters. Small Doppler shifts can be detected only by analyzing in the radiofrequency range. To do this, the frequency of the scattered light must be shifted into this range by non-linear mixing. We shall treat below the possible methods of realizing this method.

a) Photomixing. In this case, two waves are directed onto the surface of the photodetector. Its response is quadratic with respect to the field. Hence, if the two waves have different frequencies, then the output current of the detector will contain a component at the difference frequency. The reference wave to which the Doppler frequency shift is referred is a portion of the laser radiation probing the studied object. This involves the frequency instability of the laser radiation.

In optical photomixing, the scattered radiation must mix on the surface of the photocathode with a portion of the unscattered laser radiation. If the radiation is linearly polarized and depolarization upon scattering is not taken into account, then we can express the instantaneous field at the photocathode by the scalar quantity E(t). The photocurrent of the detector at any instant of time t, neglecting noise, is proportional to the square of the incident field:

$$i(t)=\text{const}\cdot\overline{|E(t)|^2}. \qquad (17)$$

The superior bar denotes averaging over a time that is large in comparison with the period of the light oscillations. The overall field E(t) is the sum of the scattered $E_s(t)$ and reference $E_0(t)$ beams. Hence,

$$i(t)=\text{const}\cdot\overline{|E_s(t)+E_0(t)|^2}. \qquad (18)$$

Usually the observed quantity is the power spectrum $J(\omega)$ of the photocurrent, which is equal to the Fourier transform of its autocorrelation function:

$$J(\omega)=\operatorname{Re}\int_0^\infty \langle i(0)\, i(\tau)\rangle \exp(-i\omega\tau)\, d\tau, \qquad (19)$$

where the angle brackets denote averaging over the ensemble.

If we assume Gaussian statistics, upon taking account of (18) and (9), we can write Eq. (19) in the form: [49]

$$J(\omega)=\text{const}\cdot\Big[\int_{-\infty}^{\infty} S(\mathbf{K},\omega')\, S_0(\omega-\omega')\, d\omega' + \int_{-\infty}^{\infty} S_0(\omega')\, S_0(\omega-\omega')\, d\omega' + \int_{-\infty}^{\infty} S(\mathbf{K},\omega')\, S(\mathbf{K},\omega-\omega')\, d\omega'\Big]. \qquad (20)$$

The first integral in this expression describes the beating between the signal and reference waves. The power spectrum of the photocurrent has a maximum at a frequency equal to the Doppler shift $\mathbf{K}\cdot\mathbf{u}$. The remaining two terms describe the self-beating of the reference and scattered waves. Their spectra of photocurrent power lie near the zero frequency. Usually the power of the reference wave is considerably greater than that of the scattered wave. Hence we can neglect the last term in (20).

Figure 2 shows the power spectrum $S_0(\omega)$ of the incident radiation from the laser, which consists of two

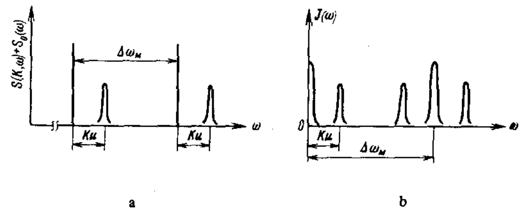


FIG. 2. a) Radiation spectra of the laser and of the scattered light; b) Spectrum of the photocurrent. Ku – Doppler frequency shift, $\Delta\omega_M$ – intermode beating frequency.

modes having a frequency difference $\Delta\omega_m$, the power spectrum $S(\omega)$ of the scattered radiation, and the power spectrum $J(\omega)$ of the photocurrent.

Let us examine the spatial conditions in photomixing. The efficiency of conversion of light energy into electrical energy at the Doppler frequency depends substantially on the amplitude and phase distributions of the waves incident on the photodetector. These problems have been treated by various authors both for a definite distribution of the amplitudes and phases of the waves being mixed, [63–67] and for waves having random amplitude and phase distributions. [68–71] As we should expect, conversion will be efficient only when the wave fronts are in phase over the entire surface of the photodetector.

We can calculate the total photodetector current by integrating the square of the total field over the area of the photodetector. Thus, if the quantum yield is constant over the entire surface, then the signal current at the Doppler frequency will be

$$i_s(t)=\text{const}\cdot\int_A \overline{E_s(x,y,t)\, E_0^*(x,y,t)}\, dx\, dy, \qquad (21)$$

where A is the area of the photodetector.

If two plane waves of uniform amplitude distribution fall on a photodetector of diameter D, one perpendicular to the surface and the other at the angle $\gamma$ to the first, then the amplitude of the signal current $i_s$ will be

$$i_s=\text{const}\cdot 2J_1\Big(\frac{\pi D\sin\gamma}{\lambda}\Big)\Big/\Big(\frac{\pi D\sin\gamma}{\lambda}\Big), \qquad (22)$$

where $J_1(x)$ is a first-order Bessel function.

The signal current is appreciable only when the angle of deviation of the wave fronts satisfies the equation

$$\sin\gamma<\frac{1.22\lambda}{D}. \qquad (23)$$

For a photocathode of 100 $\mu$m dimensions and wavelength $\lambda = 0.63\ \mu$m, this condition gives $\gamma < 2\times 10^{-3}$ radians.

We can explain the physical meaning of this condition as follows (Fig. 3). The interference of the waves having different frequencies on the surface of the photodetector gives rise to running interference fringes having the period $\Lambda = \lambda/\sin\gamma$. The a.c. component of the photocurrent will be considerably larger than the d.c. component if the dimensions of the photoreceiver are smaller than $\Lambda$, which is equivalent to the condition (23). We note that we can get the admissible angle of deviation by considering the photodetector to be a radiating source having the same dimensions. Then its diffractional angle of divergence gives the admissible angle of deviation.

Siegman[69] has proved an antenna theorem that states that such a detector having a given area $A_R$ efficiently receives waves over the solid angle $\Omega_R$, or that if the solid angle $\Omega_R$ is fixed, not all of the surface of the photodetector operates efficiently, but only a certain part $A_R$. Here the relation between $A_R$ and $\Omega_R$ (Fig. 4) is:

$$A_R\Omega_R \approx \lambda^2. \tag{24}$$

All of the optical elements lying on the path of the incident beams can alter only the relationship between $A_R$ and $\Omega_R$, but they cannot change the value of their product.

We can also obtain the properties of a photodetector operating in a photomixing system by treating such a detector as a spatial filter:[72,73] there will be a signal current only when the spatial spectrum of the signal wave overlaps the spatial spectrum of the reference wave. This implies that the signal current does not vary if:

a) a phase filter is placed before the photodetector;

b) the surface of the photodetector is shifted for some distance along the axis;

c) an equivalent reference beam is used instead of any passive spatial filters placed in the path of the signal beam.

We can easily derive from the following arguments the spatial conditions in photomixing of light scattered by a set of particles. Let us represent the light scattered by the particles as the spatially incoherent radiation from a set of point sources. According to the van Cittert-Zernike theorem,[74] a plane source of radius $\rho$ illuminates at the distance R an area of radius $r_0$ almost coherently, and here

$$r_0 \approx \frac{\lambda R}{\rho}. \tag{25}$$

It is important in photomixing of scattered light what part of the illuminated volume contributes to the signal current. We find from (25) that this part has the area:

$$A_c \approx \frac{\lambda^2 R^2}{A}, \tag{26}$$

where A is the area of the photodetector.

Usually the area $A_c$ is called the coherence area, while the solid angle that it subtends from the photodetector is called the coherence cone.

We find upon comparing Eqs. (24) and (26) that they are practically equivalent. Hence, the antenna theorem is implied by the van Cittert-Zernike theorem for the photomixing process.

Thus, all the particles occurring in a cylinder of area $A_c$ contribute to the signal current, while all the rest of the particles contribute only to the d.c. component of the photocurrent. The area $A_c$ is small under ordinary conditions. Hence the laser beam must be focused on the studied point in the flow.

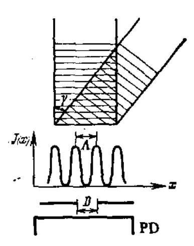


FIG. 3.

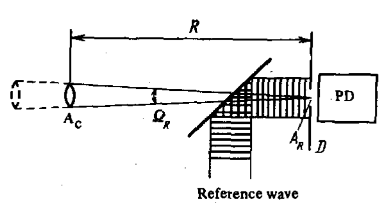


FIG. 4.

FIG. 3. Spatial conditions in photomixing of plane waves. D – diaphragm, PD – surface of photodetector.

FIG. 4. Spatial conditions in photomixing of a plane reference wave with a scattered wave.

Use of the photomixing method in order to distinguish the Doppler frequency shift will succeed when the signal-noise ratio at the photodetector output is large enough. Various authors[66,69,25,76] have studied this problem, and they have established that the noise power consists of thermal and also shot noise power caused by the action of the signal and reference waves, the background exposure, and the dark current of the photodetector.

In Doppler location, people mainly use as the photodetectors photomultipliers having a high internal amplification. At a signal power of the order of $10^{-7}$–$10^{-9}$ watt, as is most typical in an LDV, the major noise source as a rule is the shot noise of the reference wave. Hence, we can write the signal-noise ratio in the form[66]

$$\frac{S}{N} = \frac{\varepsilon P_s}{h\nu\Delta f\,[1+P_s/P_0]}, \tag{27}$$

Here $\epsilon$ is the quantum efficiency of the photoreceiver, h is Planck's quantum constant, $P_S$ is the power of the signal wave, $P_0$ is the power of the reference wave, and $\Delta f$ is the band width of the analyzing apparatus.

We see from this formula that S/N is directly proportional to the quantum efficiency of the photodetector, and when $P_0 \gg P_S$, the signal power u is inversely proportional to the band width of the photodetector. This formula has been derived under the assumption that the width of the signal spectrum is less than the band width of the analyzing apparatus, which is not always true.

When $P_0 \gg P_S$, as Eq. (27) implies, S/N does not depend on the power of the reference wave. This will be true whenever the reference wave does not contain intrinsic noise. In the converse case, the reference wave power must always be limited. In actual LDV systems, owing to laser noise, the power of the reference wave must not exceed the power of the signal wave by more than severalfold.

b) Direct photodetection. At first glance, it seems impossible to use direct photodetection in Doppler location, since here all of the frequency and phase information is lost: the photodetector is not sensitive to frequency or phase modulation of the optical carrier. Hence one must convert the frequency modulation into amplitude modulation. One can do this in a system with two light beams that are incident at different angles. Then the intensity of the resulting scattered field will be modulated with a frequency that is the difference of the Doppler frequencies, independently of the direction of observation.[17-26,77-81] Hence, LDV's that use this principle are called differential LDV's. The fundamental merit of direct photodetection is the simplicity of the detection apparatus, since problems of registry of the wave fronts do not arise here. Hence, the scattered light can be collected over a large solid angle.

Let us consider the intersection of two coherent beams having the wave vectors $\mathbf{k}_0'$ and $\mathbf{k}_0''$ (Fig. 5). In the region of intersection, an interference field is formed in which the fringes of equal intensity are directed along the bisectrix of the angle $\alpha$ and perpendicular to the plane of the incident beams. If the frequencies of the incident beams differ, then a running interference field is formed. In this case, the light intensity depends on the time and the coordinates according to the law:

$$I(\mathbf{r},\ t)=I_1(\mathbf{r})+I_2(\mathbf{r})+\sqrt{I_1(\mathbf{r})\,I_2(\mathbf{r})}\cos[(\omega_1-\omega_2)\,t-\mathbf{Kr}],$$

where $I_1(\mathbf{r})$ and $I_2(\mathbf{r})$ are the intensities of the incident waves, and $\omega_1$ and $\omega_2$ are their frequencies.

The distance $\Lambda$ between the equal-intensity fringes is

$$\Lambda=\frac{\lambda}{2\sin(\alpha/2)} \tag{28}$$

Motion of a particle in the standing interference field ($\omega_1 = \omega_2$) has the result that the intensity of the scattered light proves to be modulated at the frequency f, which is equal to

$$f=\frac{u_x}{\Lambda} \tag{29}$$

where $u_x$ is the projection of the velocity vector on the perpendicular to the bisectrix of the angle $\alpha$.

This frequency does not depend on the direction of observation. If there are several particles in the region of intersection of the beams, then additional terms appear in the expression for the scattered field that are due to interference of the fields scattered by the individual particles. This problem has been studied in detail in[77].

One gets a good signal-noise ratio in the direct-photodetection method because the scattered light is collected over a wide solid angle, and hence its power is rather large. Thus, the measurements of the signal-noise ratio in[78] with a scattered power of $10^{-8}$ watt gave a value S/N = 31 dB, while S/N = 17 dB for photomixing under the same conditions.

The limiting signal-noise ratio in the direct-photodetection method is 3 dB smaller than in photomixing.[66] When there is a high concentration of particles, the S/N ratio is lower in direct photodetection than in photomixing.[77]

We note in conclusion that the modulus of the difference of the two frequencies is recorded, both in photomixing and in direct photodetection.

c) Optical spectral analysis. The essence of this method has been presented well in the monograph[83]. The promise of using the method in an LDV has been shown in[21]. Optical spectral analysis can prove to be especially effective in measuring high velocities.[26,28-30,47,82,84,85]

In order to make a direct estimate of the energy spectrum $S(\omega)$, the scattered radiation must be passed through a narrow-band filter having a known frequency characteristic $H(\omega)$. Then the response of the photodetector placed behind the filter will be determined by the following relationship:[86]

$$i_s=\rho\int_0^\infty S(\mathbf{K},\ \omega)\,|H(\omega)|^2\,d\omega, \tag{30}$$

where $\rho$ is the sensitivity of the photodetector.

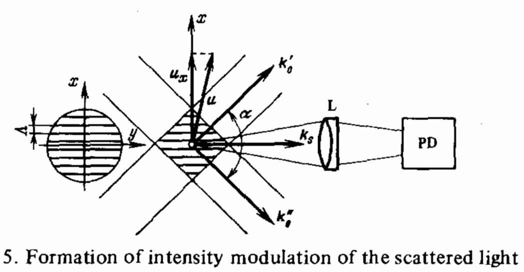


FIG. 5. Formation of intensity modulation of the scattered light when a moving object is probed by two beams. L – lens, PD – photodetector.

As we see from Eq. (30), with (11) taken into account, the response of the detector will be appreciable only when the transmission frequency of the filter coincides with the frequency $\omega_0 - \mathbf{K}\cdot\mathbf{u}$, i.e., the Doppler frequency. For optical Doppler location, the most suitable narrow-band filter is the Fabry-Perot interferometer, plane or confocal,[87-89] whose frequency characteristic has the form[86]

$$|H(\omega)|^2=\sum_{n=-\infty}^{\infty}\frac{A^2}{\beta^2+(\omega-\omega_n)^2}, \tag{31}$$

where $\beta$ and A are coefficients that depend on the parameters of the interferometer.

The intrinsic frequencies $\omega_n$ are determined by the distance L between the mirrors of the interferometer, and they are equal to

$$\omega_n=2\pi\frac{nc}{2L} \tag{32}$$

for a plane interferometer, and

$$\omega_n=2\pi\frac{nc}{4L} \tag{33}$$

for a confocal interferometer. Here c is the speed of light, and L is the distance between the mirrors. The width of the frequency characteristic, which determines the minimum measurable Doppler frequency shift,[2)] does not depend on the order number of the intrinsic frequencies, and is equal to

$$\Delta\omega\approx\frac{2(1-r)}{L}c, \tag{34}$$

where r is the reflective coefficient of the mirrors.

One can determine the spectrum of the scattered radiation and the value of the Doppler frequency by varying the frequency characteristic of the interferometer. One directs a reference wave onto the interferometer along with the scattered wave in order to increase the accuracy of measurement (Fig. 6).

We should note the advantages of using a confocal interferometer, which has a narrower frequency characteristic (the resolving power is as great as $10^8$). This permits one to measure velocities as low as 1 m/sec.

We should also list among the advantages of the spectroscopic method the simplicity of constructing the instruments and of measuring large velocities. Since one directly measures the Doppler frequency here, no problem arises of determining the direction of motion.

Unfortunately, the lack of corresponding commercially-produced spectroscopic instruments hinders the spread of this method in Doppler location, although its possibilities have not yet been exhausted.

## 4. PRACTICAL LASER DOPPLER VELOCIMETER SYSTEMS

a) Optical systems of LDV's. All optical LDV systems can be classified into the following groups accord-

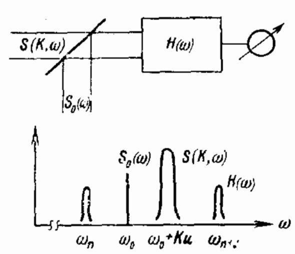


FIG. 6. Direct spectral analysis of the scattered radiation using a narrow-band filter.

ing to the method of distinguishing the Doppler frequency shift:

a) systems with a reference beam using the photomixing method;

b) differential systems based on direct photodetection;

c) systems having optical spectroscopic instruments.

Figs. 7 and 8 show the systems (in which the Doppler frequency shift is distinguished by the photomixing method) proposed in[16,33]. The He–Ne laser beam 1 (Fig. 7) is limited by the diaphragm 2 and is focused by the lens 3 onto the point under study in the flow. The scattered light is collected by the lens 4, and is directed by the mirror 6 onto the photodetector 8. From the same point, the transmitted laser beam is directed by the lens 9, the semitransparent plate 7, and the mirror 10, which serves to equalize the optical paths of the scattered wave and the direct beam. A fundamental defect of this system is its complexity of adjustment, since the direct and scattered waves must be aligned with a high degree of accuracy.

A widely-used system is one having two probe beams.[16] Here the laser beam 1 (Fig. 8) is divided by the semitransparent plate 2 into two beams. The mirrors 3 and 4 and the focusing lenses 5 and 6 direct these beams to the studied point in the flow. The set of diaphragms 7 and 8 is placed in the path of one of the beams. The wave fronts of the scattered light and the reference wave are relatively easily aligned on the surface of the photodetector 9, since they are propagated in the same direction. The neutral filter 10 is inserted in order to get a definite ratio between the powers of the reference and scattered beams.

Some authors use systems in which the reference beam passes alongisde the object being studied.[15,90-94] Thus the flow does not interfere with the spatial coherence of the reference beam.

LDV systems exist in which the reference wave is laser radiation scattered at a different angle.[52,78]

A differential LDV system was proposed in[17-26] in which the wave fronts of the scattered waves are automatically aligned, and hence the photodetector operates in a direct photodetection system.

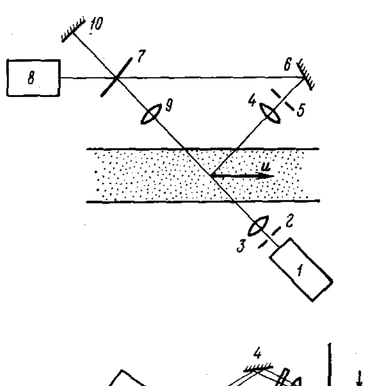

FIG. 7. A single-beam LDV system.[33] 1 – laser, 2, 5 – diaphragms, 3, 4, 9 – lenses, 6, 10 – mirrors, 7 – semitransparent mirror, 8 – photodetector.

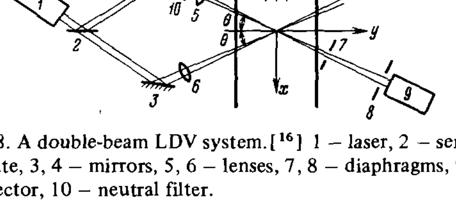

FIG. 8. A double-beam LDV system.[16] 1 – laser, 2 – semitransparent plate, 3, 4 – mirrors, 5, 6 – lenses, 7, 8 – diaphragms, 9 – photodetector, 10 – neutral filter.

Figure 9 shows one of these systems.[95] The light beam 1 from the continuous-wave laser is divided by the reflecting prism 2 into two identical beams, which are directed to the studied point 7 in the flow by means of the mirrors 3 and 4 and the lenses 5 and 6. In contrast to the systems of Fig. 8, the scattered light is collected over a large solid angle by the objective 8, and is directed to the photoreceiver 10, before which the diaphragm 9 is placed. This system measures the velocity at the point of intersection of the two beams, where an interference field is formed that has an alternation of maxima and minima of intensity.

In the differential system proposed in[22,51], the transmitted beams are also directed onto the photodiode, in addition to the scattered beams. However, one gets a higher signal-noise ratio without the transmitted beams[96] or if one uses a differential amplifier[97] to diminish the effect of laser noise.

Figure 10 shows an LDV system in which a confocal Fabry-Perot interferometer is used to distinguish the Doppler frequency shift.[84] The radiation from the argon laser 1, which operates in a single-frequency mode, is divided into two beams by the plate 2, and is focused by the lenses 5 and 6 onto the studied point 7 of the flow. The scattered light and the direct light attenuated by the filter 3 are directed onto the confocal interferometer 10. The lens 8 and the diaphragm 9 serve to adjust the incident radiation to the interferometer 10. The radiation from the interferometer is applied to the photodetector 12 through the limiting diaphragm 11. The signal from the photodetector is applied to the amplifier 13 and observed on the oscillograph 14, whose sweep voltage is applied to the scanning interferometer. A scanning confocal interferometer was used in[84] that had a resolution of 7.5 MHz in the region of dispersion of the device of 2 GHz. It turned out that one could measure velocities up from 25 cm/sec with the developed

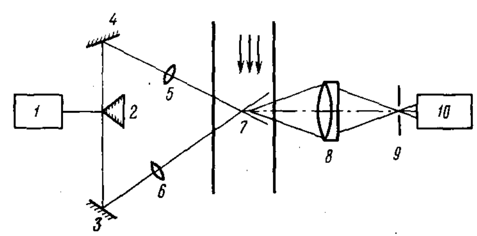

FIG. 9. A differential LDV system.[95] 1 – laser, 2 – reflecting prism, 3, 4 – mirrors, 5, 6 – lenses, 7 – flow being studied, 8 – objective, 9 – diaphragm, 10 – photodetector.

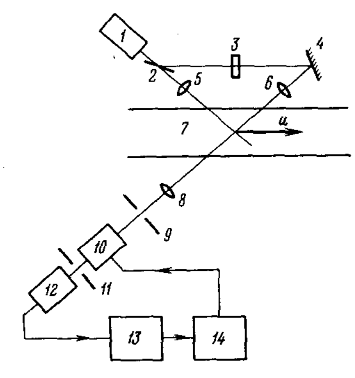

FIG. 10. An LDV system having a Fabry-Perot interferometer.[84] 1 – laser, 2 – semitransparent plate, 3 – neutral filter, 4 – mirror, 5, 6 – lenses, 7 – flow being studied, 9, 11 – diaphragms, 10 – confocal interferometer, 12 – photodetector, 13 – amplifier, 14 – oscillograph.

LDV. Such LDV systems are conveniently applied in studying supersonic flows. Their defects include complexity of reduction of the information, especially with use of a computer.

Measurement of the velocity vector has been treated in[27,98-102]. In all of the systems treated above, the modulus of the projection of the velocity vector on a chosen direction is determined. In order to measure the direction, one must shift the frequency of one of the beams. In order to do this, one uses single-band modulation of the beam with electrooptic crystals,[103] and light diffraction by ultrasonic waves[9,10,91,104] and by a moving diffraction grating.[105-107]

He-Ne, argon, and $CO_2$ lasers operating in a continuous-wave mode are currently used as the sources of coherent radiation in LDV's. When one uses photodetectors to distinguish the Doppler frequency shift, lasers with good spatial coherence are required. When several longitudinal modes exist, one must equalize the optical paths of the direct and scattered waves to an accuracy of several cm.[108] When spectroscopic instruments are used, then lasers with good time coherence are required.

Actually, all LDV systems require apparatus for separation or combination of light beams, for which one can use semitransparent plates,[109] beam-splitting cubes, polarizing apparatus,[17] diffraction gratings,[119] reflecting prisms, Fresnel biprisms, and light diffraction by two slits.

An important characteristic of an LDV is its spatial resolution, i.e., the volume from which the velocity information arises, especially when one is studying boundary layers of a liquid, flow next to a wall, and flows having a high velocity gradient.

Calculation of the velocity measurement volume amounts to finding the dependence of the size of the Doppler signal on the position of the particle. For the system shown in Fig. 8, it has the form[48]

$$i_s(t) = \text{const}\cdot\exp\left(-\frac{x^2\cos^2\theta + y^2\sin^2\theta + z^2}{w^2}\right)\cos(\Omega_D t + \Phi);$$

Here w is the radius of the beam at which the intensity declines by a factor of e.

If the ratio of the minimum detectable current to the maximum is denoted by $\exp(-\eta^2)$, then the measurement volume is bounded by the surface

$$\frac{x^2\cos^2\theta + y^2\sin^2\theta + z^2}{w^2} = \eta^2 \qquad (35)$$

This is an ellipsoid having the axes

$$\left.\begin{aligned} \Delta x &= \frac{2w\eta}{\cos\theta}, \\ \Delta y &= \frac{2w\eta}{\sin\theta}, \\ \Delta z &= 2w\eta. \end{aligned}\right\} \qquad (36)$$

For a laser beam of radius 50 $\mu$m, $\theta = 10°$, $\eta = 1$; the measurement volume has the dimensions $100 \times 600 \times 100$ $\mu$m, and the actually attainable size is $3 \times 3 \times 10$ $\mu$m. Here the largest dimension must coincide with the direction of the velocity being measured.

Thus, the volume of measurement is determined by the optical parameters w and $\theta$, and also by the r.f. measuring part of the LDV system (the parameter $\eta$). The spatial resolution has been tested experimentally in[21,33].

We should state that many articles have cited the dimensions of the measurement region of a developed system without saying anything about how these dimensions were determined. As we see from the presented arguments, it makes no sense to speak of the spatial resolution without doing this.

In a differential system, the measurement volume is determined by the region of intersection of the two beams, the parameters of the objective that collects the scattered light, and the dimensions of the diaphragm.

The optical systems shown in Figs. 7–9 permit one to distinguish Doppler frequency shifts that give information on the velocity structure of a flow. Various r.f. measuring systems are used to treat the Doppler signal taken from the photodetector. Rather than going on to discuss them, let us analyze the parameters of the Doppler signal. It has been shown in[48-50] that it is a narrow-band random Gaussian signal having an amplitude that fluctuates even for laminar flows. The amplitude modulation arises from the discreteness of the scattering centers and the finite scattering volume, and also from the size distribution of the particles.

The Doppler signal is non-monochromatic, and it has a finite spectral width (see Eqs. (11) and (15)). We can classify the causes of broadening of the Doppler signal into two classes:

a) the effect of physical processes that occur in the flow;

b) the effect of certain factors of the optical part of the system.

The first group of causes of broadening stems from the following facts:

1) the velocity of the scattering particles can vary during the time of data reduction of the signal,

2) a velocity gradient of flow exists within the volume of measurement;[21,49,112]

3) the particles undergo Brownian movement.[13,49]

These factors fully pertain to the mechanics of the liquid, and in principle, they give information on the flow. For example, the spectrum of the scattered light in a flow that has a velocity gradient along the z axis has the form[49]

$$S(\mathbf{K}, \omega) = \text{const}\cdot\frac{l_x}{l_z}\int_{-\infty}^{\infty} u^{-1}\exp\left[-\frac{2l_x^2}{u^2}(\omega_0 - \omega - \mathbf{K}\mathbf{u})^2 - \frac{(z - z_0)^2}{2l_z^2}\right]dz, \qquad (37)$$

where $l_x$ and $l_z$ are the characteristic dimensions of the scattering volume.

In order to obtain the final form of the spectrum, one must integrate Eq. (37) with a given function u(z).

We can consider the apparatus broadening caused by the optical part of the LDV system in two ways: as the finite time packet of the scattered wave, whose duration is determined by the linear dimension of the intersection region in the direction of flow, i.e., w/u, or as the angular indeterminacy in the direction of the focused light waves, $\Delta\alpha/\alpha$.[112,113] It was shown in[48,49] that both of these approaches to treating broadening are equivalent.

We see from Eqs. (7) and (11) that the power spectrum of the scattered field for a laminar flow without accounting for diffusion is fully determined by the autocorrelation function of the amplitude distribution of the field. In particular, if the incident light has a Gaussian intensity distribution, then as was noted above, the spectrum of the scattered light is also Gaussian in form.

Hence, according to (21), the spectrum of the photocurrent will be the convolution of two Gaussian curves, and its width will be $\sqrt{2}$ times greater than that of the spectrum of the scattered radiation.

We should especially note that the width of the photocurrent spectrum does not depend on the concentration of particles in the flow, as has been stated erroneously in[119].

The width of the recorded photocurrent spectrum will arise from the finite width of the radiation spectrum of the laser, the apparatus broadening of the analyzing apparatus, and the width of the spectrum of the scattered field.

b) R. f. measurement systems of LDV's. Frequency-tracking systems, systems having a frequency discriminator, and also spectral or correlation analysis are used for data reduction of the Doppler signal taken from the photodetector. Reduction of the Doppler signal in an LDV is a typical problem of statistical r.f. technology.[120] Hence all of the already-developed methods can be applied.

People most often use spectrum analyzers. One measures the mean frequency and records the spectrum of the Doppler signal by scanning a narrow-band filter over the spectrum under study. By using it, one can study both laminar and turbulent steady-state flows. Fig. 11 shows typical spectra for laminar (Fig. 11a) and turbulent (Fig. 11b) flow. In spite of its simplicity and good supply of apparatus, this method has substantial defects:[121]

1) a relatively large error of measurement, which amounts to about 1% for the mean velocity and 5% for the width of the spectrum for turbulent flows,

2) the method is not applicable for studying non-steady-state flows.

It would seem that this method will be used in the future for monitoring signals, but not for measurement.

Correlation analysis of Doppler signals is less widespread than spectroscopic analysis, apparently, because of complexity of measurement and lack of the appropriate apparatus. Its potentialities of use have been studied in[111,122-125]. Correlation analysis has the same defects as spectroscopic analysis, and for a random Gaussian process, it does not give new information as compared with the latter.

If one uses a frequency discriminator,[126-128] then the frequency of the Doppler signal is converted into an analog potential. Two difficulties arise here:

1) the frequency discriminator must be linear over a broad range, which is a complicated problem;

2) the signal-noise ratio must be large.

It is evident from what has been stated that in the future frequency discriminators may be used only as part of instruments for tracking the frequency of the Doppler signal.

It would seem that frequency-tracking systems will be widely used for measuring and data-reduction of Doppler signals. Such a system was first used in[33] for measuring laminar flow of a liquid with a maximum velocity of 5 m/sec. Turbulent flows,[27,98,124-133] non-steady-state flows,[134] and correlation of velocities[135] have subsequently been studied using these systems.

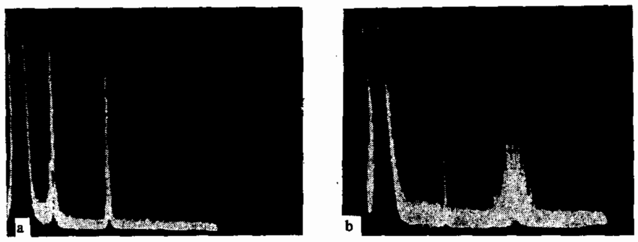


FIG. 11. Doppler signal spectra. a) Laminar mode, Re = 300, with the mark at the right at 200 kHz; b) Turbulent mode, Re = 3000, with the mark in the middle at 200 kHz. Exposure time 20 sec.

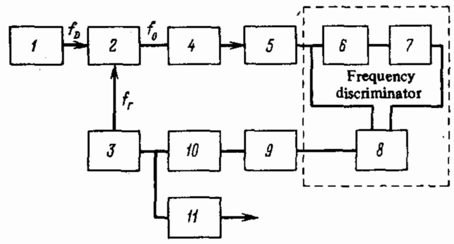


FIG. 12. Fundamental diagram of a tracking system for determining the parameters of a turbulent flow.[131] 1 – photodetector, 2 – mixer, 3 – controlled oscillator, 4, 6 – narrow-band filters, 5, 7 – limiters, 8 – phase-comparison instrument, 9 – integrator, 10 – amplifier, 11 – adjusting cascade.

Figure 12 shows a block diagram of a tracking system developed by the authors of[131]. The Doppler signal taken from the photodetector 1 is applied to the input of the tracking system. This signal has amplitude and frequency modulation. The voltage at the output of the tracking system is proportional to the velocity. The Doppler signal is mixed in the modulator 2 with a signal from the controlled oscillator 3. The signal at the intermediate frequency further passes through the narrow-band filter 4 and the limiter 5, which eliminates amplitude fluctuations inherent in the Doppler signal, and it enters the frequency discriminator, which consists of a narrow-band filter 6, the limiter 7, and the phase-comparison instrument 8. The output voltage is proportional to the deviation of the intermediate frequency about the fixed value $f_0$. After an appropriate smoothing with a large time constant $\tau$ in block 9 and amplification in block 10, the resultant discrepancy voltage V is applied through a feedback circuit to the controlling input of the oscillator. The result of action of the feedback circuit is that, when the amplification is sufficient, the oscillator frequency tracks the frequency of the Doppler signal, and maintains an almost constant difference from it that is equal to the intermediate frequency $f_0$. Thus, the voltage V is an electrical analog of the Doppler frequency, and hence also of the velocity of flow. This voltage is output through the supplementary divider 11 for further treatment.

A feature of the discussed system is the memory block, since the Doppler shift strongly fluctuates, and there are intervals of time when the signal level is insufficient. For example, when the concentration of particles is so small that on the average the illuminated volume contains less than one particle, the signal will totally vanish between successive passages of particles. However, the signal can fluctuate to a level below the minimum detectable threshold even at a high particle concentration. When this happens, the memory block has at its output a discrepancy voltage equal to the previous value.[3)]

Unfortunately, along with the definite advances in developing tracking systems, there are still many difficulties here involving breakdown of tracking, especially in studying turbulent flows and at small signal-noise ratios. We also note that tracking systems operate at a higher signal-noise ratio than non-tracking systems do.[120]

## 5. APPLICATION OF THE METHOD IN AERO- AND HYDRODYNAMIC EXPERIMENTS

a) Study of Brownian movement of particles in a liquid. The first study on Doppler location involved light scattering by particles in Brownian movement.[7] Here, a spectral analysis of the scattered light, as Eq. (11) shows, permits one to determine such valuable characteristics as the diffusion coefficient of the particles, from which one can calculate their dimensions. A fundamental feature of studying Brownian movement is that here one analyzes spectrally only the scattered wave (without a reference wave). This substantially simplifies the experimental methodology.

This method is also applied for measuring the thermal conductivity of ordinary liquids, the thermal conductivity near the critical point in liquids; in chemistry: for determining the rates of chemical reactions; and in biophysics: for determining the diffusion coefficients of biological macromolecules. This line of application of the method has been developing intensively. The reviews[13,136] are devoted to it.

b) Study of the velocity distribution in subsonic flows. The first studies on application of the method in hydrodynamic experiments involved studying laminar flows in liquids and gases. It later turned out that measurement of the mean velocity of turbulent flows using this method entails no additional difficulties. Photomixing or differential LDV's are used to measure subsonic velocities. There is no fundamental difference in studying liquid or gas flows. The distinction consists only in the fact that it is more difficult to introduce optical inhomogeneities into a gas flow. Moreover, the velocity of the introduced particles does not always agree with the velocity of the flow, as is especially noted in studying turbulent flows.

The velocity distribution of liquid flows in channels has been measured in[13,31-36,93-95,137-139], in elements of jet automation in[45,134,140,141], and between rotating cylinders in[42]. Here the range of velocities is from several $\mu$m/sec[137] to 20 m/sec. Non-steady-state flows have been measured by this method in[134,143,144], and non-Newtonian liquids in[39-41,145-147]. We note that one cannot use a thermoanemometer in the latter case, so that the Doppler method still remains the only one for studying the velocity structure of flows with non-Newtonian viscosity, and apparently, it allows one more fully to explain the Toms effect (reducing hydraulic resistance upon adding infinitesimal concentrations of polymers to a liquid), and also to elucidate other anomalous phenomena in the behavior of non-Newtonian liquids.

We should especially distinguish the studies applying the method to measure velocities in a boundary layer.[21,31,41,112,137,140,141] Here the advantages of the LDV over the traditional methods of measurement are most fully manifested; the high spatial resolution permits one to conduct such studies without introducing disturbances into the flow being studied. The existence of a velocity gradient within the measurement volume can also be used to get the velocity distribution in the boundary layer in a liquid flow.

A series of authors[33,37,38,98,148,153] have also extended the method to study gas flows in the velocity range from several cm/sec to 300 m/sec. Thus, a turbulent air jet was studied in[98], and all three components of the velocity vector were measured, while in[152] they studied the possibility of using an LDV with a 20-watt single-frequency $CO_2$ laser to study vortices in the atmosphere that were produced by airplanes at a distance up to 1 km.

We should note that as yet most of the above-cited studies on applying the method have been illustrative in nature, since they have mainly demonstrated the potentialities of measuring velocities with the apparatus developed by the authors. However, some studies have obtained results that could not be obtained by other methods. This is especially true of studying non-Newtonian liquids and boundary layers. For example, Fig. 13 shows the velocity distribution in a laminar film of liquid 0.95 mm thick flowing over a vertical surface as obtained in[31] by using a differential LDV system.

Studying the small-scale flows in elements of a jet automation has made it possible to specify the laws of motion of the liquid more precisely and to devise a method for calculating them.

c) Study of the velocity distribution in supersonic flows. Application of the LDV to study supersonic gas flows has a number of peculiarities. First of all, there are the large Doppler shifts, of the order of 1 GHz for ordinary optical systems. Since the frequency characteristic of the photodetectors declines in a band about 100 MHz, this hinders use of the photomixing and direct-detection methods. Indeed, one can considerably reduce the Doppler frequency shift by using special optical systems, e.g., by choice of the angle between the beams. One can use a differential system in which the beam is divided with a Fresnel biprism. Here the angle between the beams is as much as 30′, which gives 10 MHz at 1000 m/sec. However, this impairs the other parameters of the LDV, e.g., the spatial resolution.

There are currently relatively few data on use of this method to study supersonic flows, although here we can expect very interesting results, especially in studying flow in the boundary layer, and in detachment zones where theoretical calculation is as yet difficult. Two-phase flows have been relatively little studied by this method.

Distinguishing of the Doppler signal using photodetectors has been carried out in studies on measuring velocity in shock waves[25,154] and gas flows.[155,157]

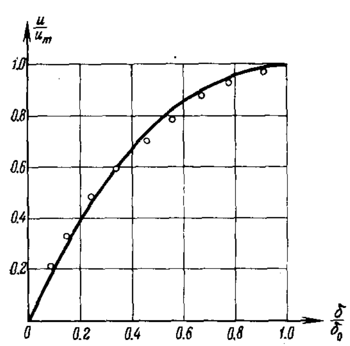


FIG. 13. Velocity distribution in a laminar film of liquid.[31] $u_m$ = 5.0 cm/sec, film thickness $\delta_0$ = 0.97 mm.

As we have noted above, apparently the most promising method of distinguishing the Doppler signal for supersonic flows is optical spectroscopic analysis using high-resolution instruments.[26,28,30,48,84,157,158] Thus, in[84] they treated the problem of using an argon laser with a confocal interferometer to study supersonic gas flows, measuring the mean velocity and degree of turbulence. In[28] they studied supersonic two-phase flows at velocities up to 3000 m/sec.

Application of interferometers is as yet also complicated by the relative difficulty of reducing the information gained by using them.

d) Measurement of parameters of turbulent flows. Experimental study of the velocity of turbulent flows is a complicated problem, for whose solution only the thermoanemometer has been used until recently. We can say with assurance that the Doppler method of measuring local velocities will become widespread here, and instruments based on it will be invented that greatly surpass the thermoanemometer.

One must determine the following quantities in studying turbulent flows: the mean velocity, the degree of turbulence, the frequency of velocity pulsations, the correlation of velocities at different points, and the correlation of velocities at a single point but in different directions. One can get these parameters with an LDV without introducing disturbances into the studied flow.

One can measure the mean velocity of turbulent flows with any of the r.f. measurement systems treated above, just as for laminar flows.

One can determine the degree of turbulence by spectral analysis of the Doppler signal. This method was first used in[42] and later in[43,62,118,145,159].

We can represent the velocity at any point of a turbulent flow as the sum of two components: the mean velocity of flow $\bar{u}$ averaged over a long period of time and the pulsatile component u',[162] i.e.,

$$u(t) = \bar{u} + u'(t). \tag{38}$$

Hence we can represent the frequency of the scattered light as

$$f_D(t) = f_0 + f_{\text{dev}}(t), \tag{39}$$

where $f_0$ is the carrier frequency corresponding to the velocity u, and $f_{\text{dev}}(t)$ is the frequency deviation corresponding to the velocity u'.

Hence the degree of turbulence is defined as

$$\frac{\sqrt{\overline{u'^2}}}{\bar{u}} = k\frac{\Delta f}{f_0},$$

where $\Delta f$ is the broadening of the spectrum caused by the turbulent pulsations, and k is a coefficient that depends on the level to which the width of the spectrum is referred.

However, we can determine the degree of turbulence in this way only under the condition that the ratio of the r.m.s. value of the frequency deviation to the maximum pulsation frequency is considerably greater than unity, as was first pointed out in[62].

Figure 14 gives the results of measurements in an axially symmetric submerged turbulent liquid jet as obtained with a differential LDV with spectral analysis of the signal.[145]

When one uses an LDV with a tracking system, the degree of turbulence is defined as the ratio of the root-mean-square voltage taken from the filter to its d.c. component.[131]

One can get the frequency spectrum of the velocity pulsations only by using an LDV having a tracking system to make a spectral analysis of the voltage taken from the filter. In[98,160], they compared measurements of the degree of turbulence and the frequency spectrum of velocity pulsations using an LDV and a thermoanemometer.

One can measure the spatial correlation of velocities in two ways. In the first of these, the photocurrent near the zero frequency is analyzed spectrally.[50] In the second, one uses two LDV's that measure the instantaneous velocities at two points and then one calculates their correlation.[128,135]

Reynolds stresses (the mean value of the product of pulsatile velocities at a given point in two mutually-perpendicular directions) have been measured both with a single-channel[132,161] and with a double-channel LDV.[133] Figure 15 shows the relationship between the Reynolds stresses and the cross-section of the flow.[133]

## 6. CONCLUSION

Thus, the Doppler method of measuring velocities can be successfully applied in various fields of science and technology. In this review we have not been able to touch on such important problems as measuring the velocity of motion of various surfaces: rolled iron sheets, rotating cylinders, etc.

The amplitude and polarization characteristics of the scattered light can be used to give supplementary information on the object being studied. For example, one can determine the dimensions of particles from the size of the Doppler signal.[163]

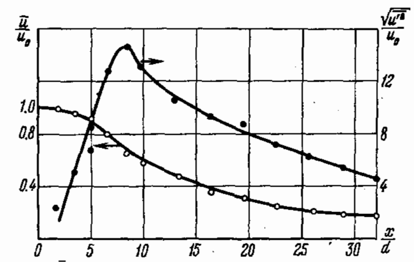


FIG. 14. Velocity distribution and degree of turbulence along the axis of a submerged liquid jet.[145] $u_0$ – initial efflux velocity of the jet, d – nozzle diameter (d = 3.0 mm), x – coordinate along the axis of the jet.

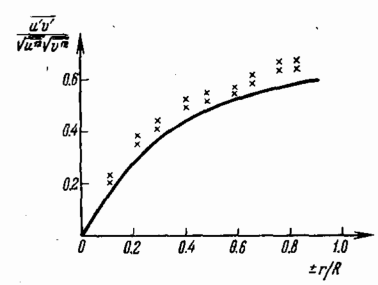


FIG. 15. Distribution of Reynolds stresses in a tube at Re = 18,000.[133] u', v' – components of the pulsatile velocity vector, R – radius of the tube, r – distance from the axis of the flow, X – experimental data, — – curve obtained by Laufer[163] at Re = 50,000.

The propagation of the method is facilitated by its positive aspects, such as the lack of disturbances, freedom from contact, lack of need to calibrate, and also the fact that the quantity to be measured is coded in the frequency of an electrical signal. The latter permits one to apply well-known methods of data reduction of the signal, including a computer. This favorably distinguishes this method from other methods of measuring velocities, including holographic methods.[164,165]

Solution of a number of insufficiently elucidated problems, such as the optimum statistical treatment of the Doppler signal with account taken of the parameters of the medium (the particle concentration, their dimensions, and the dimensions of the flow), the operation of the tracking system at high turbulences of flow and at low signal-noise ratios, and reduction of data taken with a Fabry-Perot interferometer with the use of a computer will facilitate even more widespread application of this promising method of measuring velocities.

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1)The history of the discovery of the Doppler effect is well presented in[5].

2)The width of the frequency characteristic unambiguously determines the minimum measurable frequency shift according to Rayleigh's criterion only when the interference pattern is recorded photographically. As has been shown in[167, 168], application of radiophysical methods of recording the interference pattern permits one to increase the resolving power of the spectroscopic devices considerably.

3)By using this principle of treating the Doppler signal, the "Disa Elektronik" company is producing regularly a laser anemometer (Type 55L) with limits of measurement from 1 mm/sec to 300 m/sec and error ± 1%.


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