Description of inelastic processes in dual models (B_5 phenomenology)

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A survey is presented of the application of the dual model to the description of inelastic scattering of hadrons (B_5 phenomenology). The properties of a five-point dual amplitudes are considered for the scattering of scalar particles. These amplitudes are generalized first to include mesons, and then to include the case of meson-baryon scattering. A number of examples of the application of the model to reactions of the type MB→MMB are considered (M and B stand for mesons and baryons, respectively). The predictions of the model are compared with experiment as well as with the predictions of earlier models. The advantages and shortcomings of B_5 phenomenology are discussed, particularly those connected with pion and vacuum exchange, and also with allowance for spin and unitarity. Further prospects for the development of the model, connected with its application to processes with larger multiplicity, and also connected with the use of a dual amplitude with Mandelstam analyticity, are considered. A table gives a complete summary of the results obtained by different workers within the framework of the B_5 phenomenology.

CONTENTS

1. INTRODUCTION

The experimental study and theoretical description of particle-production processes is one of the most timely fields in high-energy physics. Two methods are used in the investigation of these processes, inclusive and exclusive. In the former case one registers only a separated number of particles that participate in the reaction, and averaging is carried out over the remaining particles. This method is used in those cases when the total description of the process becomes impossible in view of the large multiplicity. In the latter method one registers all the particles that take part in the reaction.

The large number of variables on which the exclusive description of the production processes depends makes it difficult to study processes with large multiplicity.

Among the various models used to describe processes of the type 2 → 3, a special place is occupied by dual models (B_5 phenomenology). Within the framework of B_5 phenomenology it has become possible to describe for the first time, in a unified manner, different processes in a wide range of scattering energies and angles. The success of the model indicates that, in spite of the large number of approximations used in the calculations, it is at present the most consistent dynamic scheme for the exclusive description of inelastic scattering processes.

The results obtained by now are based on the use of the five-point dual amplitude B_5 as the invariant amplitude, which strictly speaking is valid only in the narrow-resonance approximation, in the idealized case of scattering of spinless particles. The fact that even approximate allowance for spin and unitarity makes it possible to obtain nontrivial predictions is evidence of the extensive capabilities of the model. Further theoretical investigations and experimental verification will undoubtedly lead to the construction of a more perfect dual model for the description of inelastic processes.

The purpose of this review is to acquaint the experimenters with the accomplishments, problems, and further prospects of B_5 phenomenology. Readers wishing to become better acquainted with the principles of duality and the consequences of dual models are referred to the reviews[1] and to the references cited therein.

2. DUAL MODELS

The concept of duality connects the region of high and low energies and denotes equivalence of the description of processes in terms of resonances in the direct s channel and in the crossing t channel (Fig. 1).

We recall that it is convenient to use resonance models in the region of low energies, where the scattering amplitude has a rich resonance structure. The

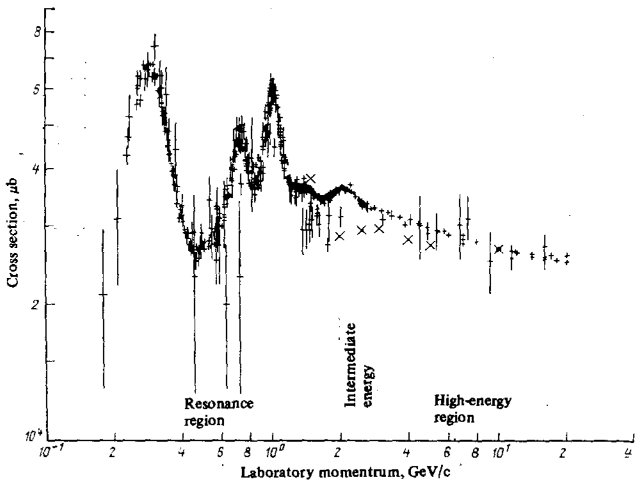


FIG. 1. Typical curve of total cross section of hadron-hadron interaction.

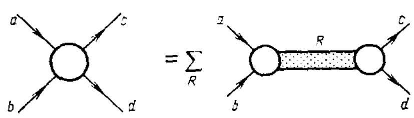


FIG. 2. Graphic illustration of resonance model.

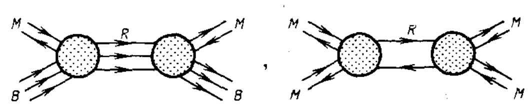


FIG. 3. Resonance model and quarks.

amplitude is approximated by the expression

$$A(s, t) \sim \sum_{R} \frac{g_R(t)}{s - s_R},$$

where s and t are the Mandelstam variables (Fig. 2).

If the s-channel resonances are nonexotic[1)], then in the language of the quark model the resonance dominance is expressed by the diagrams shown in Fig. 3.

In exchange models it is assumed that the scattering amplitude can be described with the aid of objects that are exchanged in the crossing t channel. For example, in the Regge-pole model which is used in the region of high energies, the scattering amplitude is given by

$$A(s, t) \sim \sum_{i} \beta_i(t)\, s^{\alpha_i(t)},$$

where $\alpha_i(t)$ is the Regge pole trajectory and $\beta_i(t)$ is the residue of the pole (Fig. 4).

If the particles exchanged in the t-channel have nonexotic quantum numbers, then we can construct in the quark model the diagrams shown in Fig. 5.

Many attempts were made to unify the two models. Thus, in the interference model it is assumed that the scattering amplitude is a sum of two terms

$$A = \Sigma \succ\!\!\prec + \Sigma\,)\!(= \Sigma A_{\text{res}} + \Sigma A_{\text{Regge}}$$

and in the resonance region A_{res} operates well while A_{Regge} "fades out," and vice versa. An essential shortcoming of the interference model is that at intermediate energies both terms operate equally well, and this leads to the so-called double allowances. This shortcoming is not possessed by in dual models, which are based on the deep connection between the s and t-channel resonances, between the low- and high-energy behavior of the scattering amplitude. The presence of such a connection is evidenced by the experimental data, (Fig. 6).

A convenient "laboratory" for the study of the connection between the scattering amplitude at low energies (resonance region) and high energies (Regge region) are the dispersion sum rules at finite energy[2]. The sum rules have played a fundamental role in the investigation of the dual properties of the theory and in the construction of dual amplitudes. It was demonstrated with their aid[3] that in some cases the resonance and exchange models describe equally well the imaginary part of the amplitude; consequently, we can write the symbolic equality

$$A = \Sigma \succ\!\!\prec = \Sigma\,)\!(= \Sigma A_{\text{res}} = \Sigma A_{\text{Regge}}.$$

Assuming that there are no exotic states in nature, we can represent the vertices BBM and MMM in the manner shown in Fig. 7. Then the dual properties of the amplitude are lucidly represented with the aid of the so-called dual diagrams[4] (Fig. 8).

The explicit form of an amplitude having a Regge

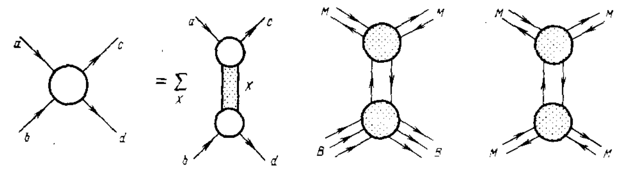


FIG. 4 FIG. 5

FIG. 4. Graphic illustration of exchange model.
FIG. 5. Exchange model and quarks.

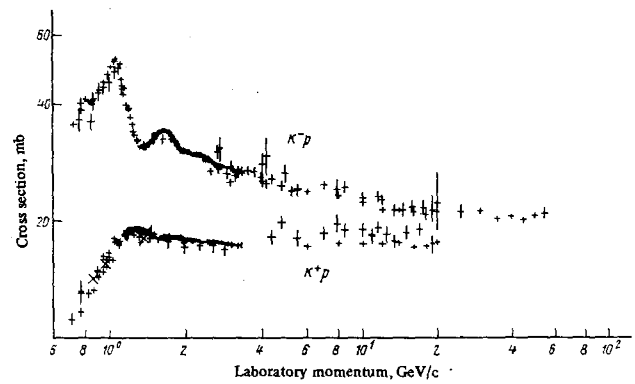


FIG. 6. Total cross sections for K^-p and K^+p scattering. In the K^-p system, at low energies, one observes a resonance structure, and an asymptotic decrease is observed at high energies. In the K^+p system there are no resonances at low energies (exotic channel); the cross section is constant at high energies.

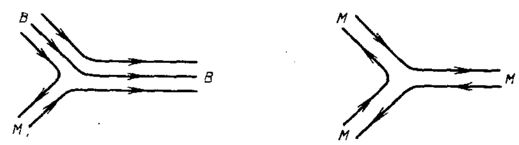


FIG. 7. In the construction of dual diagrams it is assumed that the quark lines enter and leave at different points and do not intersect.

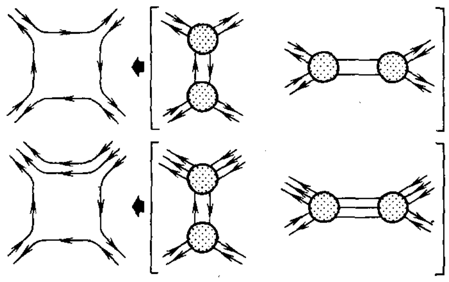


FIG. 8. Dual diagram for meson-baryon and meson-meson scattering.

asymptotic form and a pole structure at low energies (in other words, having simultaneously poles in the s and t channels) was obtained essentially[3] by using the sum rules at finite energy[2].

Historically, the development of dual models is connected with the narrow-resonance approximation, i.e., the use of linear real infinitely growing Regge trajectories (we shall return in Chap. 3 to a discussion of the trajectories in the model; a detailed analysis of the properties of Regge trajectories can be found in the review[5]).

The simplest of the many different narrow-resonance dual models is the model developed in[6] and based on the use of Euler B function as the invariant amplitude

$$V(s, t, u) = B[-\alpha(s), -\alpha(t)] + B[-\alpha(s), -\alpha(u)] + B[-\alpha(u), -\alpha(t)], \tag{2.1}$$

where

$$B[-\alpha(s), -\alpha(t)] = \frac{\Gamma[-\alpha(s)]\Gamma[-\alpha(t)]}{\Gamma[-\alpha(s)-\alpha(t)]} = \int_0^1 dx\, x^{-\alpha(s)-1}(1-x)^{-\alpha(t)-1};$$

$\alpha(s)$ is the Regge trajectory, and s, t, and u are the invariant Mandelstam variables.

We list the main properties of the amplitude (2.1).

a) Crossing symmetry (obvious from the construction).

b) Pole structure. When linear real Regge trajectories are used, the amplitude has an infinite number of poles located on the real axis and corresponding to infinitely narrow resonances. Under the principal trajectory there are located daughter levels. The residues at the poles with respect to one variable are polynomials with respect to the other. Deviation from linearity in the trajectories implies a nonpolynomial character of the residues, and consequently, the appearance of an infinite sequence of poles—"ancestors," corresponding to particles with arbitrarily large spins at a fixed mass (this contradicts unitarity).

c) Asymptotic behavior. The amplitude (2.1) has a Regge asymptotic form (for linear trajectories) in all directions with the exception of the physical one, i.e., the positive real axis. Here, however, the asymptotic behavior of the amplitude in the case of large-scattering angle is in gross contradiction with the limitations that follow from the unitarity condition.

The Regge asymptotic form can be preserved[7] by means of a certain special choice of Im α. In certain cases, the amplitude has also the correct behavior in large-angle scattering.

d) The model is unitary and has no Mandelstam analyticity.

Thus, the amplitude (2.1), while claiming to be quite general, has a number of significant shortcomings. From the point of view of the theory it can, however, be regarded as the starting term of a certain iteration procedure that is needed to improve its analytic and unitary properties. The similarity between the representation (2.1) and the Born term of the perturbation-theory series has stimulated the use of field methods for the construction of the iteration series. Many brilliant results were obtained in this direction (called dual field theory), but the principal difficulties in its path (connected in particular with the exponential divergence of the loop diagrams) cast doubts on the effectiveness of field methods in strong-interaction theory.

Another approach to the dual theory of strong interaction has been developing recently, based on methods of the analytic theory of the S matrix using the so-called dual amplitudes with Mandelstam analyticity (DAMA) as the starting term. A discussion of this approach, however, is beyond the scope of the present review (see[8] and the literature cited therein).

In spite of the rather arbitrary character of most of the properties of the representation (2.1), and in spite of the fact that this representation can be regarded strictly speaking only as an approximation to the true amplitude, its simplicity and appreciable generality make it attractive as a phenomenological amplitude for hadron scattering.

The extensive possibilities of the use of the Euler B function as a physical scattering amplitude were first demonstrated by Lovelace[6] with $\pi\pi$ scattering as an example. The scattering amplitude for this process is (for simplicity we confine ourselves to the (s, t) term of the amplitude)

$$A(s, t) = \lambda[\alpha(s)+\alpha(t)]\, B[1-\alpha(s), 1-\alpha(t)], \tag{2.2}$$

where $\alpha(s)(\alpha(t))$ is the ρ-meson trajectory. The arguments of the B function are shifted here by unity, so as to avoid the appearance of unphysical poles in the amplitude at negative values of s. The shift of the arguments violates the Regge-asymptotic B(s, t), but the "kinematic factor $[\alpha(s) + \alpha(t)]$ ensures the correct asymptotic behavior of the amplitude A(s, t).

One of the brightest applications of the model (2.2) is the description of the process $\bar{p}n \to \pi^+\pi^-\pi^-$. The fact that the $\bar{p}n$ system has the quantum numbers of the π^- meson makes it possible to consider formally the process within the framework of the four-point amplitude (2.2). Attention is called immediately to the similarity between the structure of the amplitude (2.2) (Fig. 9) and the hole structure of the Dalitz diagram (Fig. 10). A discussion of the quantitative predictions of the model (2.2) is, however, outside the scope of the present review (see, e.g.,[1]).

3. FIVE-POINT DUAL AMPLITUDE

The use of duality ideas has made it possible for the first time to write down for the amplitude of the processes of many-particle hadron scattering an analytic expression that is valid in the entire region of energies and scattering angles. The possibility of such a generalization has played an important role in the subsequent development of the dual approach. The five-point narrow-resonance dual amplitude for spinless particles is given by[9]

$$B_5(z)=\int_0^1\int_0^1 dx_{12}\,dx_{45}\,x_{12}^{z_{12}-1}x_{45}^{z_{45}-1}$$
$$\times(1-x_{45}x_{12})^{z_{51}-2}\left(\frac{1-x_{45}}{1-x_{45}x_{12}}\right)^{z_{34}-1}\left(\frac{1-x_{12}}{1-x_{45}x_{12}}\right)^{z_{23}-1} \qquad (3.1)$$
$$=\int_0^1\int_0^1 dx_{12}\,dx_{45}\,x_{12}^{z_{12}-1}x_{45}^{z_{45}-1}(1-x_{12})^{z_{23}-1}(1-x_{45})^{z_{34}-1}(1-x_{45}x_{12})^{z_{51}-z_{34}-z_{23}},$$

where

$$z_{i,\,i+1}=-\alpha(s_{i,\,i+1})=-\alpha(0)-\alpha' s_{i,\,i+1},\quad s_{i,\,i+1}=-(p_1+p_{i+1})^2.$$

The representation (3.1) has properties analogous to the properties (Fig. 11) of the four-particle dual amplitude (2.2). We consider the pole structure and the asymptotic behavior of the amplitude (3.1).

a) **Pole structure.** To study the pole structure of the amplitude it is convenient to use the series expansion proposed in [10]. We introduce the variables

$$w_{51}=z_{51}-z_{23}-z_{34}$$

and write

$$(1-x_{45}x_{12})^{w_{51}}=\sum_{n=0}^{\infty}(x_{45}x_{12})^n P(n,\,w_{51}), \qquad (3.2)$$

where $P(n, w_{51})$ is a polynomial of n-th degree in w_{51}.

Substituting (3.2) in (3.1) we obtain after integrating term by term

$$B_5(z)=\sum_{n=0}^{\infty}P_n(n,\,w_{51})\int_0^1 dx_{12}\,x^{z_{12}-1+n}(1-x_{12})^{z_{23}-1}\int_0^1 dx_{45}\,x_{45}^{z_{45}-1+n}(1-x_{45})^{z_{34}-1}$$
$$=\sum_{n=0}^{\infty}P(n,\,w_{51})\,B_4(z_{12}+n,\,z_{23})\,B_4(z_{45}+n,\,z_{34}). \qquad (3.3)$$

The series (3.3) is very useful in the study of the properties and consequences of the amplitude (3.1). First, it is possible to continue the amplitude analytically with its aid into the entire region of variation of the arguments (the integral representation (3.1) is valid only at

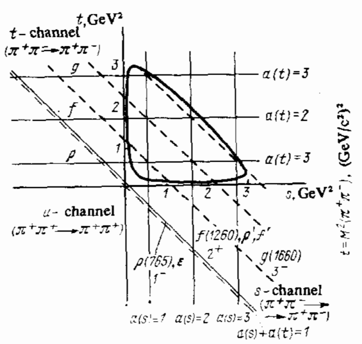


FIG. 9

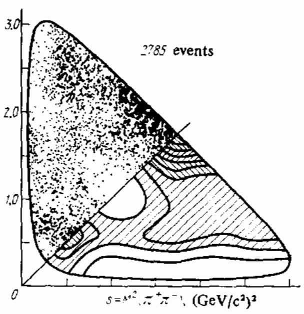


FIG. 10

FIG. 9. Structure of the s, t plane of the Veneziano-Lovelace amplitude. The solid lines marked $\alpha(s) = 1, 2, \ldots$ and $\alpha(t) = 1, 2, \ldots$ indicate the position of the poles of the amplitude with respect to s and t. The dashed lines $\alpha(s) + \alpha(t) = 1, 2, \ldots$ correspond to the zeroes of the amplitude. The curve in the region $s > 0$ and $t > 0$ is the boundary of the Dalitz diagram for the process $\bar{p}n \to \pi^+\pi^-\pi^-$.

FIG. 10. Experimental distributions of events on the Dalitz plot for the process $\bar{p}n \to \pi^+\pi^-\pi^-$.

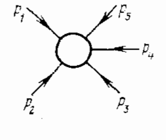


FIG. 11. Reaction with participation of five particles. The momenta of the particles are regarded as incoming, i.e., $\sum_i^5 p_i = 0$.

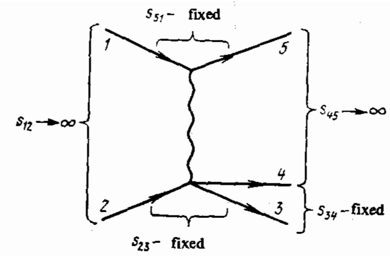


FIG. 12. Simple Regge limit of a five-point diagram.

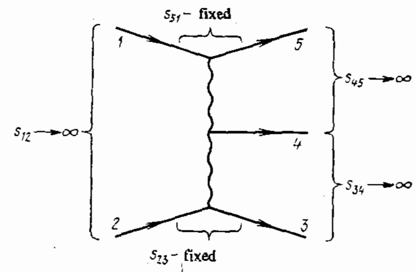


FIG. 13. Double Regge limit of a five-point diagram.

positive values of the arguments); second, it is convenient in the calculations (B_5 phenomenology); finally as will be shown below, it is convenient also in the study of the pole structure of the amplitude $B_5(z)$.

Using the series expansion of the amplitude $B_4(z)$, we obtain

$$B_4(z_{12},\,z_{23})=\sum_{n=0}^{\infty}P(n,\,z_{23})\frac{1}{z_{12}+n},$$

$$B_5(z)=\sum_{k=0}^{\infty}P(k,\,w_{51})\sum_{n=0}^{\infty}(n,\,z_{23})\frac{1}{z_{12}+n+k}B_4(z_{45}+k,\,z_{34}) \qquad (3.4)$$
$$=\sum_{n=0}^{\infty}\frac{1}{z_{12}+n}\sum_{k=0}^{m}P(k,\,w_{51})\,P(n-k,\,z_{23})\,B_4(z_{45}+k,\,z_{34}).$$

From this we obtain the residues at the poles

$$\operatorname*{Res}_{z_{12}=0}B_5(z)=B_4(z_{45},\,z_{34}),$$
$$\operatorname*{Res}_{z_{12}=-1}B_5(z)=(1-z_{23})\,B_4(z_{45},\,z_{34})\,(1-w_{51})\,B_4(z_{45}+1,\,z_{34})\ \text{etc.}$$

It follows from (3.4) that Res $B_5(z)$ at $z_{12} = -n$ is a polynomial of degree n in terms of the angle variable in the s_{12} channel. Thus, as in the case of $B_4(z)$, the amplitude $B_5(z)$ contains at the pole point a set of particles ("parent" and "daughter" levels) and is free of "ancestors" (see Chap. 1).

b) **Asymptotic behavior of $B_5(z)$.** We recall that in the simple Regge limit one fixes one of the two-particle invariant masses in the final state (Fig. 12). In the double Regge limit, all three two-particle invariant masses of the final state tend to infinity (Fig. 13). In the Regge-pole model, the amplitude of the scattering behaves in the simple Regge limit like $T \sim (s_{12})^{\alpha(s_{51})}$, $s_{12} \to \infty$, and in the double limit like $T \sim (s_{34})^{\alpha(s_{23})}(s_{45})^{\alpha(s_{51})}$, $s_{12} \to \infty$ (without allowance for the signature factor).

We shall show that the representation (3.1) has the correct asymptotic behavior. We consider the simple Regge limit $z_{12} \to \infty$, $z_{45} \to \infty$; $H \equiv z_{45}/z_{12}$, z_{23}, z_{34}, and z_{51} are fixed. After substituting in (3.1) the expressions

$$x_{12}=\exp\left(-\frac{y_{12}}{z_{12}}\right),\quad x_{45}=\exp\left(-\frac{y_{45}}{z_{45}}\right)$$

we obtain

$$B_5(z) = (z_{12}z_{45})^{-1} \int_0^\infty \int_0^\infty dy_{12}\, dy_{45} e^{-y_{12}} e^{-y_{45}} \times$$
$$\times [1 - \exp(-y_{12}/z_{12})]^{z_{23}-1} \left\{ \frac{1-\exp(-y_{45}/z_{45})}{1-\exp[(-y_{12}/z_{12})-(y_{45}/z_{45})]} \right\}^{z_{34}-1}$$
$$\times \left[1 - \exp\left(-\frac{y_{12}}{z_{12}} - \frac{y_{45}}{z_{45}}\right)\right]^{z_{51}-z_{23}-1}.$$

Expanding in a series all the exponentials containing the variable y_{12} in the arguments, and retaining only the first terms of the expansion, we obtain

$$B_5(z) \approx (z_{12}z_{45})^{-1} (z_{12})^{-z_{23}+1} (z_{12})^{-z_{51}+z_{23}+1}$$
$$\times \int_0^\infty \int_0^\infty dy_{12}\, dy_{45} e^{-y_{12}} e^{-y_{45}} y_{12}^{z_{23}-1} \left[\frac{y_{45}/H}{y_{12}+(y_{45}/H)}\right]^{z_{34}-1} \left(y_{12} + \frac{y_{45}}{H}\right)^{z_{51}-z_{23}-1}. \tag{3.5}$$

Since the factors preceding the integral reduce to $H^{-1}z_{12}^{-z_{51}}$, and the integral itself does not depend on variables that tend to infinity, formula (3.5) corresponds to the Regge asymptotic form of the amplitude.

The presence of a double Regge limit of the amplitude is proved in similar fashion.

The properties discussed above hold only if linear real infinitely-growing Regge trajectories are used. This approximation, as in the case of a four-point diagram, is naturally in gross contradiction to unitarity.

We note, finally, that the amplitude $B_5(z_{12}, z_{23}, z_{34}, z_{45}, z_{51})$ has the property of cyclic and anticyclic symmetry with respect to permutations of the arguments.

4. B_5 PHENOMENOLOGY

Many problems are encountered on the way to a direct application of the model discussed in the preceding section, namely:

1) The vacuum trajectory cannot be included in the model together with the ordinary Regge trajectories[2)].

2) Linear real trajectories are used in the model, corresponding to infinitesimally narrow resonances.

3) The trajectories in the model contain a particle with zero spin (actually, the first to appear on the leading boson trajectories are vector particles).

4) The spin and isospin of the external particles are not taken into account in the model.

There exists as yet no consistent method of including the vacuum trajectory in the dual narrow-resonance models (some of the possibilities will be discussed below), so that the best way out in this case is to describe reactions in which the vacuum exchange is negligibly small (we note that inclusion of a vacuum trajectory in dual models with Mandelstam analyticity does not encounter any fundamental difficulties[12]).

The second problem is solved by the so-called phenomenological unitarization of the amplitude, i.e., by introducing into the trajectory an imaginary part Im (s) corresponding to the width of the resonances:

$$\operatorname{Im} \alpha(s)|_{s\to s_{\text{res}}} = \frac{d \operatorname{Re} \alpha(s)}{ds}\bigg|_{s\to s_{\text{res}}} M_{\text{res}} \Gamma_{\text{res}},$$

where M_{res} and Γ_{res} are the mass and total width of the resonance. To avoid the appearance of "ancestors" (i.e., poles corresponding to particles with spin exceeding the value of the trajectory at the point of the pole), it is assumed that the imaginary part is equal to zero below the threshold s_n of the two-particle state with which this resonance is connected. Thus,

$$\alpha(s) = \alpha(0) + \alpha' s + i\theta(s - s_n) \operatorname{Im} \alpha(s).$$

The solution of the third problem is connected with the so-called kinematic factor E(1, 2, 3, 4, 5) (the analog of $[\alpha(s) + \alpha(t)]$ in the case of elastic scattering (see Chap. 1)), which we shall discuss below. Just as in the Veneziano–Lovelace model, the arguments of the B_5 function must be shifted by unity, i.e., we must replace $-\alpha(s)$ by $1 - \alpha(s)$. This shift calls for the introduction of a kinematic factor that must be:

a) cyclically symmetrical;

b) linear in the angle variables (at a fixed energy variable (in order that the first energy pole appear in the P wave (vector meson));

c) linear in the energy variable when the latter tends to infinity at a fixed momentum transfer (to "correct" the asymptotic behavior of the amplitude, which is violated by the shift of the argument).

The only factor having the above-listed properties is

$$E(1, 2, 3, 4, 5) = \varepsilon_{\mu\nu\rho\sigma} (p_1)^\mu (p_2)^\nu (p_3)^\rho (p_4)^\sigma,$$

where $\epsilon_{\mu\nu\rho\tau}$ is a completely antisymmetrical tensor. We recall that the choice of this factor is connected with the assumption of vector exchange in the intermediate channels.

We have already touched upon the last problem in part, since the entire information on the spin is contained in the kinematic factor.

The spin of the external particles is usually neglected (the spin of the external bosons can be taken into account by introducing several invariant amplitudes, which would greatly complicate the calculations); allowance for the spin of fermions is a complicated problem, which has not been solved within the framework of the narrow-resonance dual models (which encounters no difficulty, however, within the framework of the DAMA). A half-integer fermion spin in the intermediate channels is taken into account by shifting the corresponding arguments of the B functions by 1/2 or 3/2. For simplicity, only one of all the possible isospin states is chosen in the model.

There exists a certain leeway in the choice of particles that are exchanged in the intermediate channels. This choice can nevertheless be made reasonable by making use of experimental data.

In analogy with the case of elastic scattering, the amplitude is a sum of three terms (see Chap. 1)

$$V(s, t, u) = B(s, t) + B(s, u) + B(t, u),$$

for a reaction in which five particles take part we have $(N - 1)!/2 = 12$ diagrams (Fig. 14) which are not connected by cyclic or anticyclic permutation. The presence of channels in which exotic particles are exchanged greatly simplifies the calculations, since the corresponding diagrams make no contribution to the amplitude.

The total cross section of the process is calculated from the formula

$$\sigma_{\text{tot}} = c \frac{1}{\sqrt{(p_a p_b)^2 - m_a m_b^2}} \int |A|^2 \delta(p_a + p_b - p_1 - p_2$$
$$- p_3) \delta(p_1^2 - m_1^2) \delta(p_2^2 - m_2^2) \delta(p_3^2 - m_3^2)\, d^4p_1\, d^4p_2\, d^4p_3,$$

where p and m are respectively the momenta and masses of the external particles and A is the amplitude

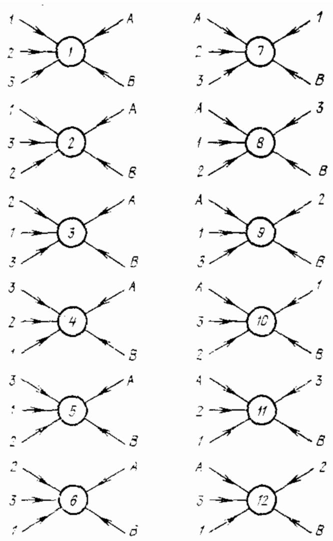


FIG. 14. Twelve "non-equivalent" permutations for a five-point diagram.

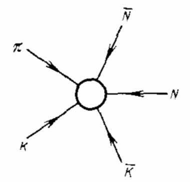


FIG. 15. Reaction with participation of $\pi K\bar K N\bar N$.

of the process. The only arbitrary constant c in the model is determined by fitting to the experimental data.

To calculate the functions B_5, we use the series expansion[10]

$$B_5(x) = B_4(x_1+n,\ x_2)\, B_4(x_4,\ x_5) \times \sum_{k=0}^{\infty} \frac{(x_2)_k\,(x_4)_k}{(x_1+x_2+n)_k\,(x_4+x_5)_k\,k!}\,[A_n\,(z_3)_k + B_n\,(z_3-1)_k],$$

where $z_3 = x_1 - x_3 + x_5$, and the coefficients A and B are determined with the aid of the recurrence formulas

$$A_{n+1} = A_n\left[\frac{z_1(z_3-1)+(z_4+n)(x_1+n)}{(x_1+n)(x_3+n)}\right] + B_n\frac{z_3-1}{x_1+n},\quad A_0 = 1,$$
$$B_{n+1} = A_n\frac{z_1(z_5+n+1)}{(x_1+n)(x_3+n)} + B_n\frac{z_5+n+1}{x_1+n},\quad B_0 = 0,$$

where $z_1 = x_4 - x_1 + x_3$; the remaining relations are connected by cyclic permutation of the indices.

The calculations are carried out by the Monte Carlo statistical method with a computer. They make it possible to obtain the statistical distributions of physical quantities measured in experiment (the differential cross section, the angular distribution, the effective-mass spectrum, etc.). By summing the distributions it is possible to obtain the cross sections of processes accurate to a normalization constant, and the mass spectra make it possible to calculate the resonant-production cross sections.

5. EXAMPLES

a) **Description of the system $\pi K\bar K N\bar N$.** A class of reaction that can be conveniently investigated is shown in the diagram of Fig. 15.

The class of reactions is convenient for two reasons: first, the presence of exotic channels simplifies the calculation; second, in view of the absence of identical external lines there is a large number of different reactions connected by crossing symmetry.

Figure 15 shows 14 charge configurations. Using isotopic invariance, we can separate five groups of reactions

(I) $\pi^0K^+K^-p\bar p$,
(II) $\pi^-K^+\bar K^0p\bar p$,
(III) $\pi^-K^+K^-p\bar n$,
(IV) $\pi^0K^0K^-p\bar n$,
(V) $\pi^0K^0\bar K^0p\bar p$.

Of greatest interest among them are groups (I) – (III), in each of which there are contained several reactions that have been experimentally investigated:

(I) $K^+p \to K^+\pi^0p$, (5.1)
$^-p \to K^-\pi^0p$, (5.2)
$\bar pp \to K^+K^-\pi^0$; (5.3)
(II) $K^+p \to K^0\pi^+p$, (5.4)
$K^-p \to \bar K^0\pi^-p$, (5.5)
$\pi^+p \to K^+\bar K^0p$, (5.6)
$\pi^-n \to K^0K^-n$, (5.6′)
$\pi^-p \to K^0K^-p$, (5.7)
$\pi^+n \to \bar K^0K^+n$, (5.7′)
$K^+n \to K^0\pi^+n$, (5.8)
$K^-n \to \bar K^0\pi^-n$, (5.9)
$\bar pp \to K^0K^-\pi^+$; (5.10)
(III) $K^+p \to K^+\pi^+n$, (5.11)
$K^-p \to K^-\pi^+n$, (5.12)
$\pi^+n \to K^+K^-p$, (5.13)
$\pi^-p \to K^0\bar K^0n$, (5.13′)
$\pi^-p \to K^+K^-p$, (5.14)
$\pi^+n \to K^0\bar K^0p$, (5.14′)
$K^+n \to K^+\pi^-p$, (5.15)
$K^-n \to K^-\pi^-p$, (5.16)
$\bar pn \to K^+K^-\pi^-$. (5.17)

Using reactions (5.4), (5.5), and (5.7) as an example, for which good experimental data are available, let us illustrate the symmetry properties of the model[3)] which were mentioned at the beginning of the chapter.

It is assumed that the reactions

$$K^+p \to K^0\pi^+p,$$
$$K^-p \to \bar K^0\pi^-p,$$
$$\pi^-p \to K^0K^-p$$

are described by one amplitude that is a combination of 12 terms corresponding to the 12 nonequivalent permutations of the external lines. It is easy to note that only four diagrams do not contain exotic channels (Fig. 16). The choice of the trajectories in the intermediate states is based on the following considerations:

1) In the $\bar pp$ channel, the choice of ω is based on experimental data on K^+ meson production[13].

2) $Y_1^*(1386)$ is the only resonance of the Y_1^* family which is strongly coupled to the system $\bar K^0p$.

3) Y^{*0} in the K^+p channel can be either the Λ resonance or the $Y_1^*(1385)$ resonance (the experimental data favor Λ).

4) The quantum numbers admit of a great leeway in the choice of the N^{*0} trajectories in the π^-p channel; there are some grounds for giving preference to the N_α trajectory. Thus, the scattering amplitude takes the form

$$A=c\varepsilon_{\mu\nu\rho\sigma}(p_\pi)^\mu(p_{\bar{K}})^\nu(p_k)^\rho(p_{\bar{p}}^-)^\sigma(P+Q+R+S),$$

where

$$P=B_5\left(1-\alpha_{K^*},\ 1-\alpha_{A_2},\ \tfrac{1}{2}-\alpha_\Lambda,\ 1-\alpha_\omega,\ \tfrac{1}{2}-\alpha_N\right),$$

$$Q=B_5\left(1-\alpha_{K^*},\ \tfrac{1}{2}-\alpha_\Lambda,\ 1-\alpha_\omega,\ \tfrac{3}{2}-\alpha_{Y_1^*},\ 1-\alpha_{K^*}\right),$$

$$R=B_5\left(1-\alpha_{K^*},\ 1-\alpha_{A_2},\ \tfrac{3}{2}-\alpha_{Y_1^*},\ 1-\alpha_\omega,\ \tfrac{3}{2}-\alpha_\Delta\right),$$

$$S=B_5\left(\tfrac{3}{2}-\alpha_\Delta,\ \tfrac{1}{2}-\alpha_\Lambda,\ 1-\alpha_{A_2},\ \tfrac{3}{2}-\alpha_{Y_1^*},\ \tfrac{1}{2}-\alpha_N\right),$$

where, for example,

$$\alpha_\Lambda(s)=-0.71+0.9s+i\theta(s-s_0)\cdot 0.09(s-s_0),\ s_0=(m_\Sigma+m_\pi)^2.$$

Different charge configurations of this system were investigated in detail in a wide range of energies by various groups of workers (see the table).

b) **Description of the system $K^+K^-\pi^-p\bar{n}$.** It is known[53] that pion exchange predominates in the reaction $K^-p \to K^-\pi^+n$. We consider below the crossing-symmetrical descriptions of the reactions

$$K^+p\to K^+\pi^+n, \qquad (5.11)$$

$$K^-p\to K^-\pi^+n, \qquad (5.12)$$

$$K^+n\to K^+\pi^-p, \qquad (5.15)$$

$$K^-n\to K^-\pi^-p, \qquad (5.16)$$

$$\pi^-p\to K_1^0K_1^0n \qquad (5.13)$$

and discuss the role of the pion exchange. We consider reactions in that phase-space region where the contribution from the exchange of the vacuum pole and the baryon exchange can be neglected. Following[26], we can assume that the amplitude with pion exchange describes the reactions (5.11)–(5.16) at small momentum transfers $|t_{pn}|$. To this amplitude we add an amplitude that contains vector-meson exchange, the contribution from which is appreciable at large momentum transfer. The scattering amplitude with pion exchange (Fig. 17) is expressed in the form

$$A_\pi=\beta_\pi\bar{u}(p_{\bar{n}})\gamma_5u(p_p)(\alpha_\rho+\alpha_{K^*}-1)B_4(1-\alpha_\rho,\ 1-\alpha_{K^*})\frac{s^{\alpha_\pi}}{\alpha_\pi},$$

($u(p_N)$ is the spinor function of the nucleon), i.e., we consider the amplitude in the pion pole. We note that the kinematic factor in this case differs from the kine-

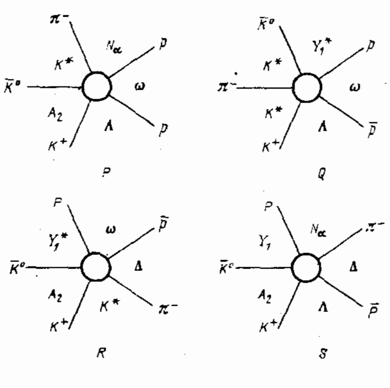


FIG. 16. Diagrams that contribute to the scattering amplitude.

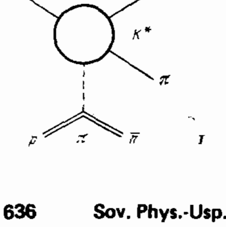


FIG. 17. Five-point amplitude in pion pole.

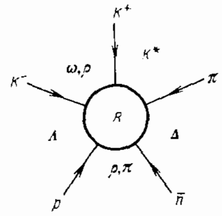


FIG. 18. Inclusion of pion exchange in the dual scheme.

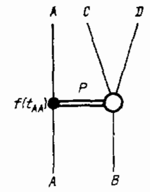


FIG. 19. Vacuum exchange.

matic factor in vector-meson exchange.

The amplitude with exchange of vector mesons is written in the form

$$A_\rho=\beta_\rho\varepsilon_{\mu\nu\lambda\sigma}(p_K)^\mu(p_{\bar{K}})^\nu(p_\pi)^\lambda(p_{\bar{n}})^\sigma B_5\left(1-\alpha_\omega,1-\alpha_{K^*},\tfrac{3}{2}-\alpha_\Delta,1-\alpha_\rho,\tfrac{1}{2}-\alpha_\Lambda\right).$$

This amplitude can be added to A_π.

We can attempt to include the pion in the common dual scheme of the resonances, by expressing the amplitude in the form

$$A'_\pi=\beta_\pi\bar{u}(p_{\bar{n}})\gamma_5u(p_p)(\alpha_\rho+\alpha_{K^*}-1)B_5$$
$$\times\left(1-\alpha_\rho,\ 1-\alpha_{K^*},\ \tfrac{3}{2}-\alpha_\Delta,\ 1-\alpha_\pi,\ \tfrac{1}{2}-\alpha_\Lambda\right),$$

which corresponds to the diagram shown in Fig. 18, (at small momentum transfers $|t_{pn}|$ this amplitude reduces to A_π).

The results of the calculations show, however, that the assumption of duality of the π meson leads to the prediction of baryon resonances with large widths, which do not exist in nature.

c) **Vacuum exchange.** In diffraction dissociation processes, for example in processes like

$$NN\to N(\pi N),$$
$$KN\to K(\pi N),$$
$$KN\to(K2\pi)N,$$
$$\pi N\to\pi(\pi N),$$
$$\pi N\to(3\pi)N,$$
$$\gamma N\to(2\pi)N,$$

vacuum-pole exchange predominates. To describe such processes, an amplitude was proposed in the following[42] (Fig. 19)

$$T\sim f(t_{AA})\,\bar{s}V_{PB\to CD},$$

where $f(t_{AA})$ is the form factor of the hadron-pomeron vertex, determined by factorization from the elastic scattering, and $\bar{s}=(p_A+p_B)$; finally, $V_{PB\to CD}$ denotes the Veneziano-Lovelace amplitude for a certain fictitious process $P+B\to C+D$.

Of particular interest to experimentors is the study of the process $\gamma p\to p\pi^+\pi^-$. In the $\pi^+\pi^-$ mass spectrum, the peak of the ρ meson predominates, so that this reaction has been customarily investigated within the framework of the vector-dominance model. The presence of a background, however, and also the asymmetry of the ρ-meson peak, indicates that in this reaction one cannot neglect the three-particle final states.

The model of[45] makes it possible to take full ac-

Reactions of type 2 → 3, investigated within the framework of the B_5 phenomenology

Reaction	p_{lab}	Calculated values and degree of agreement with experiment	Remark	Reference
		1. KN interactions		
$K^-p \to \Lambda\pi^+\pi^-$	3–10	(01, 01P, 11, 13P, 21, 21P, 22, 23, 33)+	Pioneer work on B_5 phenomenology	14
(a)	3–10	01+, (11, 23)−, (21, 22)±	Attempt to take the spin into account	15
(b)	10	(11, 13, 23, 33)+	Free parameters introduced (coefficients in the diagrams)	16
(c)	3–10	(01P, 11)+	Unified description of a group of reactions connected by crossing symmetry	17
$K^+p \to pK^0\pi$	2,5–13	01+ (+ for large p_{lab})	Ditto	13
		(01P, 11, 13, 13P, 23, 31P, 32P)+		18
(d)	1–20	01+ 11+	Exchange of vacuum trajectory taken into account	19
	12	(11, 13, 31P, 32P)±	Amplitude contains free parameters	20,21
$K^-p \to p\bar{K}^0\pi^-$	2,5–13	(01, 01P, 11, 13P, 31P, 32P)+	See (c)	13
	3–10	(01, 11, 23)+, (13, 13P)±	See (b)	22
(e)	3.3	(11, 13, 21, 31P, 32P)+	Pion exchange taken into account	23
$K^+p \to pK^+\pi^0$	1–20	01±, 11+		19
$K^-p \to pK^-\pi^+$		01±, (11, 13)+	See (d)	
$K^-p \to nK^-\pi^+$	10	(01, 01P)−(13, 13P, 23, 25, 33)+	See (b), (d), (e)	24
	5–16	11+ (13P, 21P)±	See (d), (e)	25
	3–10	(01P, 13P, 31P, 32P)+	See (c), (e)	26
	1.33–11.2	(0,1, 01P)+ (11, 12, 13, 13P, 21, 21P, 31P, 32P)+	See (e)	27
$K^-n \to pK^-\pi^-$	3–10	(11, 13, 32P)+	See (c), (e)	26
	3–12,6	(11, 13, 31P, 32P)+	Ditto	28
$K^+n \to pK^+\pi^-$	3–10	(01P)?	Ditto	26,29
$K^+p \to nK^+\pi^+$	3	(11, 13P, 32P)+	Ditto	28
	3	(11, 32P)+	Ditto	28
$K^-n \to \Lambda\pi^0\pi^-$ (f)	3	11±	Unified description of group of reactions connected by isospin symmetry	17
$K^+p \to N^{*++}K^+\pi^-$	5–16	(11, 13P, 21P, 31P)+	See (d), (e)	25
$K^+p \to \Lambda p\bar{p}$ $K^-p \to \Lambda p\bar{p}$	5–15	(01, 11, 12, 21)+	See (d)	30
$K^-p \to nK^{*-}\pi^+$	6–10	(12, 13)+	See (a), (d)	31
$K^+p \to pK^+\omega$	4.6; 9.0	(11, 13)+	Study of double Regge limit without allowance for vacuum exchange	32
$K^-p \to \Lambda K^+K^-$	2.24–5.5	01± (11, 21, 31P, 32P)+		33
$K^-n \to pK^{*-}\pi^-$	4.5	11+, 21±		34
$K^+n \to nK^0\pi^+$	2–4.5	(11, 51)+	Unified description of crossing and charge-symmetrical reactions	35
$K^-n \to n\bar{K}^0\pi^-$ $K^-p \to \Delta^{++}K^{*-}\pi^-$	4.25; 10	(11, 21)+		36
		2. πN interactions		
$\pi^+p \to \Lambda K^+\pi^+$	1.5–10	(01P, 11, 13P, 32P)+	Sec (c)	37
	8	(11, 13, 23)+	See (b)	16
	8	(01P, 23)+	See (c)	17
$\pi^-p \to pK^0K^-$	2.5–1.3	01− 11+ (31P, 32P)∓ for 3.1 Gev and for 4 Gev	Ditto	13
	2–8	01+ (+ for large p_{lab}) (11, 13, 21)+	Neglect of baryon exchange	38,39
$\pi^-p \to nK^0\bar{K}^0$	2–8	01± (+ for large p_{lab}) 11− (13, 21)+	Ditto	38,39
	12	11− (12, 13, 31P)±	Region of multiregge limit separated	40
	3–12	(01P, 31P, 32P)+	Region of resonances separated	26
$\pi^+p \to pK^+\bar{K}^0$	8	11+	See (c)	13
$\pi^-p \to \Lambda K^+\pi^-$ $\pi^-p \rightleftarrows \Lambda K^0\pi^0$	2–8	(01, 11, 13, 21)+		
	3–6	(01, 11)+ (21P, 31P, 32P)±	See (c), (e)	17
$\pi^-p \to \Sigma^0K^+\pi^-$	2–8	(01, 11)+		38
$\pi^-p \to n\pi^0\pi^0$	2–10	(01, 13, 21)+ 01P±		41
$\pi^-p \to n\pi^+\pi^-$ $\pi^+p \to N^{*++}\pi^+\pi^-$	5–16	(01P, 11, 13, 21P, 41)+	See (d), (e)	25
		3. NN and $\bar{\text{N}}$N interactions		
$pp \to pn\pi^+$	28.5	(11, 13)+	Exchange of vacuum trajectory taken into account	42
$\bar{p}n \to \bar{\Delta}^{--}p\pi^0$	7	(11, 13, 23, 31P, 32P)+	Estimate of role of kinematic factor, comparison with double Regge limit	43
$\bar{p}n \to \pi^+\pi^-\pi^-$	1.0–1.6	(11, 51)+	See (a)	44
$\bar{p}p \to K_1^0K^\pm\pi^\mp$	1.1–5.7	(0,1, 11, 21)±	Modified kinematic factor	
$pp \to p\Lambda K^+$	6.6	(12, 13, 21)+	See (c)	30

Reaction	p_{lab}	Calculated values and degree of agreement with experiment	Remark	Reference
		4. Photoproduction		
$\gamma p \rightarrow p\pi^+\pi^-$	2,5—5.8	(11, 13, 31, 32)+ 42?	See (d)	45
$\gamma p \rightarrow pK^+K^-$	2.5—17.8	(11, 13)+	Ditto	46
		5. Processes of type 2 → 2		
$K^-p \rightarrow n\bar{K}^0$	2,5—13	(01, 13)+	Amplitude for 2 → 3 processes at pole	47
$K^+n \rightarrow pK^0$				
$\pi^-p \rightarrow \Lambda K^0$				
$K^-n \rightarrow \Lambda\pi^-$				
$\pi^-p \rightarrow n\rho^0$	1.6—16	13+	$B_5 \rightarrow B_4 \times B_4$ (See (e))	48
$pp \rightarrow n\Delta^{++}$				
$\pi^+p \rightarrow \Delta^{++}\rho^0$				
$K^-p \rightarrow K^{*-}p$				
		6. Processes to type 2 → 4		
$\bar{p}p \rightarrow 4\pi$		(11, 21, 34)+	$B_6 \rightarrow B_5$ at pole ($\bar{p}p$)	49 50
$ep \rightarrow ep\pi^+\pi^-$		11+	See (d)	51
$K^-p \rightarrow nK^0\pi^+\pi^-$	4.6; —5	(11, 31, 32)+	$B_6 \rightarrow B_5$. See (e)	52

Notation

01 $\sigma(p_{lab})$	–dependence of total cross section on p_{lab}
11 $d\varepsilon/dM_{eff}$	–effective-mass spectra
12 $d\sigma/ds$	–spectra of squares of effective masses
13 $d\sigma/dt$	–spectra of squares of momentum transfer
21 $d\sigma/d\cos\theta^*$	–distributions with respect to production angle in c.m.s.
22 $d\sigma/dp^*$	–distribution with respect to the modulus of the momentum in the c.m.s.
23 $d\sigma/dp_L^*$	–distributions with respect to the longitudinal momentum in the c.m.s.
24 $d\sigma/dp_\perp$	–distributions with respect to the transverse momentum in the c.m.s.
25 $\langle p_\perp \rangle (\omega)$	–ω is the Van-Hove angle
31 $d\sigma/d\Phi_{TY}$	–distribution with respect to the Treiman-Yang angle
32 $d\sigma/d\theta_J$	–distribution with respect to the Jackson angle
33 $d\sigma/d\omega$	–distribution with respect to the radial angle ω on the Van-Hove diagram
34 $d\sigma/d\chi$	–distribution with respect to the Goldhaber angle
41 $(F-B)/(F+B)$	–asymmetry parameter
42 b	–diffraction-cone slope parameter
51	–Dalitz plots

The letter P following a symbol denotes that this quantity is calculated for the quasi-two-particle process of resonance production. The sign that follows shows the degree of agreement of the predicted model with experiment; "+"–good, "±"–satisfactory, "–"–poor.

count of the three-particle final state, including multiperipherism, resonant production, and also the presence of a background, in a wide interval of energies.

6. DISCUSSION OF RESULTS OF B_5 PHENOMENOLOGY

In this chapter we present and discuss some of the most characteristic results obtained within the framework of B_5 phenomenology.

Figure 20 shows plots of the total cross section as a function of the momenta of the incident particles. The normalization constant was fitted to the experimental point of the total cross section of the reaction $K^+p \rightarrow K^0\pi^+p$ at 5 GeV/c. The theoretical predictions agree with the experimental data both with respect to the energy dependence and in absolute magnitude. Analyzing the predictions of the model for the total cross sections, we can draw the following conclusions:

a) At momentum values less than 3 GeV/c, the calculated curves lie much lower than the experimental points. This discrepancy can be due to an increase in the contribution of the pion exchange at low energies, a contribution not accounted for in the model.

b) For the reaction $\pi^-p \rightarrow K^0K^-p$, the predictions of the theory disagree with the experimental data by almost a factor of 2 (the theory predicts the correct energy dependence of the cross section). It should be noted that even such a rough agreement between theory and experiment should be regarded as a success of the theory; if we take into account the large (20-fold) difference between the cross sections of this reaction and the corresponding values for the reaction to which the amplitude was normalized.

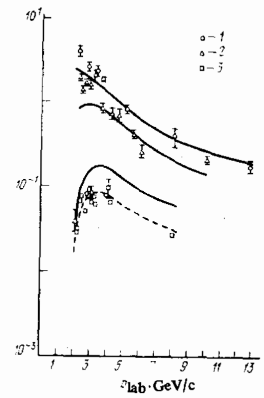


FIG. 20. Plots of total cross section for the reactions $K^+p \rightarrow K^0\pi^+p$ (1), $K^-p \rightarrow \bar{K}^0\pi^-p$ (2) and $\pi^-p \rightarrow K^0K^-p$ (3) (solid lines).

c) The predictions of the scattering cross sections for the processes (5.5) and (5.7) with the aid of the reaction (5.4) make it possible to analyze the crossing-symmetry properties of the amplitude.

Plots of the distribution with respect to the effective masses are shown in Fig. 21. The advantage of the dual models over the Regge-pole model, due to the allowance for the resonance effects, is obvious here. Because of this property, the dual models, as seen from the figure, reproduce the main features of the experimental data with distributions with respect to the effective masses.

Figures 22 and 23 show plots of the cross sections for the production of the dominant resonances in the reactions $K^+p \rightarrow K^0\pi^+p$ and $K^-p \rightarrow \bar{K}^0\pi^-p$.

Figure 24 shows the angular distributions for the decay of the resonances. In this model, these angular

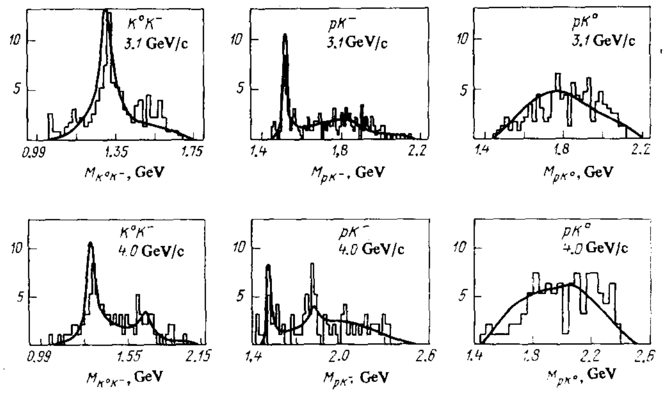
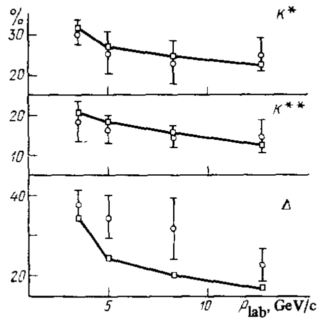


FIG. 21. Distribution with respect to the effective masses in the reaction $\pi^- p \to K^0 K^- p$.



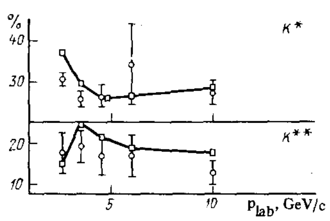
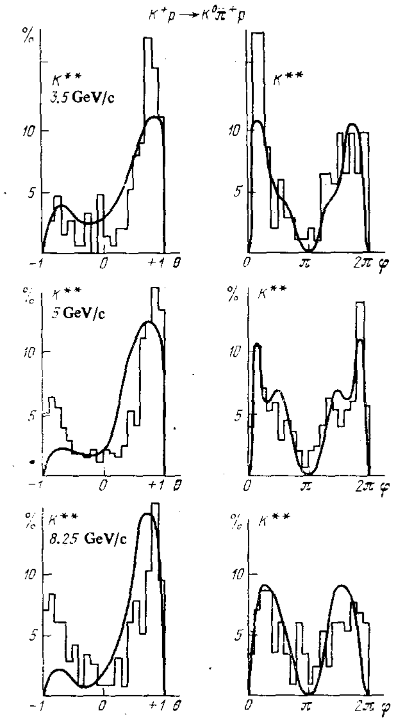


FIG. 22 FIG. 23

FIG. 22. Cross sections for resonance production in the reaction $K^+ p \to K^0 \pi^+ p$.

FIG. 23. Cross sections for resonance production in the reaction $K^- p \to \bar{K}^0 \pi^- p$.

FIG. 24. Angular distributions for the resonance decays.



distributions for the principal resonances on each trajectory depend on the kinematic factor and can thus serve as a criterion for verifying the correctness of the assumptions made concerning the particles exchanged in the intermediate channels (vector, pseudoscalar, etc. exchange).

We note that the predictions of the model of Chan et al.[13] deviate from the experimental data on the total cross sections of the reactions $K^0 p \to K^+ \pi^- p$ and $\bar{K}^0 p \to K^- \pi^+ p$. This discrepancy is due apparently to an incorrect choice of the amplitudes and (or) the kinematic factor[54].

The examples presented here constitute a negligible fraction of all the results obtained within the framework of the B_5 phenomenology. A complete summary of the results, with references to the original papers, is given in the table.

7. CONCLUSION

The predictions obtained within the framework of the B_5 phenomenology have a qualitative rather than quantitative character, although in many cases there is agreement with the experimental data. It must be borne in mind here that within the framework of this model it was possible, for the first time, to describe in a unified manner different features of inelastic scattering processes (double peripherism, resonance production, background, etc.), which previously were regarded as unconnected. Using an amplitude containing a single arbitrary constant, it is possible to describe many processes (which are interconnected by crossing symmetry) in a wide interval of energies and scattering angles.

Further development of this trend will depend apparently on the solution of the following problems. The nonunitarity of the model is its main shortcoming (in particular, according to this model the contribution of the daughter resonances is equal to the contribution of the resonances on the principal trajectory, and this distorts strongly the scattering picture). The construction of an invariant amplitude having good unitary properties and suitable for calculations would constitute the solution of the main problem of B_5 phenomenology.

The remaining problems are more technical in character, and their solution depends to a considerable degree on the progress made in the unitarization of the dual model. This includes the problem of including the fermion spin and diffraction (vacuum exchange). An important role in the solution of the foregoing problems can be played by dual amplitudes with Mandelstam analyticity (DAMA)[8]. This class of amplitudes does not contradict the postulates of the analytic of the S-matrix theory, and has at the same time attractive unitary and analytic properties which are not possessed by the narrow-resonance dual models[4)]. An essential factor on these models is the use of nonlinear trajectories, which make it possible to include in natural fashion fermion trajectories[55] and vacuum exchange[12].

The use of a maximum number of B_5 functions in the amplitude (i.e., neglect of the minimal number of diagrams) should lead in principle to a more correct description of the processes, but is accompanied by cumbersome calculations, which affect adversely the accuracy of the results.

We note finally that the possibilities of B_6 phenomenology are now under investigation. At present time, however, it is impossible to obtain results with reasonable accuracy using a reasonable amount of computer time.

On the whole we can state that the B_5 phenomenology, owing to its generality and further development possibilities, is a highly promising approach in the exclusive description of inelastic hadron-scattering processes.

[1)]Resonances are defined as nonexotic if they can be constructed from a quark-antiquark pair (mesons) or three quarks (baryons).

[2)]This trajectory differs from the usual ones, in particular, in the fact that no resonances are observed on it [11], so that it cannot be a dual trajectory with an infinite number of resonances.

[3)]The crossing-symmetry properties, naturally, obtain only in the approximation of infinitesimally narrow resonances.

[4)]Whereas the narrow-resonance dual amplitude is a meromorphic function of its arguments, which are the Regge trajectories, the DAMA depends on these trajectories in a functional manner.

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[1]L. L. Jenkovszky, V. V. Kukhtin, and V. P. Shelest, Dual Resonance Models, I. Preprint ITF-70-43, Kiev, 1970; L. L. Jenkovszky and V. V. Timokhin, Dual Resonance Models, II, Preprint ITF-71-118r, Kiev, 1971; A. V. Kaidalov, Usp. Fiz. Nauk 105, 97 (1971) [Sov. Phys.-Usp. 14, 600 (1972)].

[2]A. A. Logunov, L. D. Soloviev, A. N. Tavkhelidze, Phys. Lett. **B24**, 181 (1967).

[3]R. Dolen, D. Horn, C. Schmid, Phys. Rev. **166**, 625 (1967); M. Ademolo, H. R. Rubinstein, G. Veneziano, M. Virasoro, ibid. 176, 1904 (1968).

[4]T. Matsuoka, K. Ninomiya, S. Sawada, Progr. Theor. Phys. **42**, 56 (1972); H. Harari, Phys. Rev. Lett. **22**, 562 (1969); J. Rosner, ibid., p. 689.

[5]D. V. Shirkov, Usp. Fiz. Nauk 102, 87 (1970) [Sov. Phys.-Usp. 13, 599 (1971)].

[6]G. Veneziano, Nuovo Cimento A57, 190 (1968); C. Lovelace, Phys. Lett. B28, 265 (1968).

[7]R. Z. Roskies, Phys. Rev. Lett. 21, 1851 (1969); K. V. Vasavada, Lett. Nuovo Cimento 6, 453 (1969).

[8]A. Bugrij, G. Cohen-Tannoudji, L. Jenkovszky, A. Kobylinsky, Preprint ITP-72-184E, Kiev, 1972; Fortschr. Phys. 21, (9) (1973).

[9]K. Bardakci, H. Ruegg, Phys. Lett. **B28**, 242 (1968); M. Virasoro, Phys. Rev. Lett. 22, 37 (1969).

[10]J. F. L. Hopkinson, Preprint DNPL/P21. Daresbury, 1969.

[11]V. V. Ilyin, L. L. Jenkovszky, N. A. Kobylinsky, V. P. Shelest, Preprint RIFP-139, Kyoto, 1972.

[12]A. I. Bugrij, L. L. Jenkovszky, N. A. Kobylinsky, V. P. Shelest, Lett. Nuovo Cimento **6**, 577 (1973).

[13]Chan Hong-Mo, R. O. Raitio, G. H. Thomas, N. A. Törnqvist, Nucl. Phys. **B19**, 173 (1970).

[14]B. Petersson, N. A. Törnqvist, ibid. **B13**, 629 (1969).

[15]S. A. Adjei, P. A. Collins, B. J. Hartley, R. W. Moore, K. J. M. Moriarty, Phys. Rev. D3, 150 (1971).

[16]H. J. Schreiber, Preprint PHE 70-4, Berlin-Zeuthen, 1970.

[17]P. Hoyer, B. Petersson, N. A. Törnqvist, Nucl. Phys. B22, 497 (1970).

[18]S. Shafee, Cavendish Lab. Preprint HEP 71-4, Cambridge, 1971.

[19]K. Kajantie, S. Papageorgiou, Nucl. Phys. **B22**, 31 (1970).

[20]V. Waluch, S. M. Flatte, J. H. Friedman, D. Sivers, Phys. Rev. D5, 4 (1972).

[21]V. Waluch, Preprint UCRL-LBL-736, 1972.

[22]J. Bartsch, M. Deutschman, R. Honecker, R. Schulte, R. Steinberg, H. Böttcher, U. Gensch, S. Nowak, H. Schiller, A. Angelopoulos, V. P. Cocchoni, P. F. Dalpiaz, J. D. Hansen, W. Kittel, D. R. O. Morrison, K. Paler, H. J. Schreiber, H. Töfte, P. J. Dorman, S. J. Goldsack, M. J. Losty, M. E. Mermikides, A. Frölich, M. Markytan, G. Otter, P. Schmid, H. Wahl (Aachen–Berlin–CERN–London–Vienna Collaboration), Nucl. Phys. B20, 63 (1970).

[23]P. A. Schreiner, D. H. Stork, A. G. Saxon, L. Lyons, ibid. **B24**, 157 (1970).

[24]J. Bartsch, M. Deutschman, R. Honecker, R. Schulte, R. Steinberg, H. Böttcher, U. Gensch, J. Kalltwasser, D. Pose, A. Angelopoulos, V. T. Cocchoni, P. F. Dalpiaz, J. D. Hansen, W. Kittel, D. R. O. Morrison, K. Paler, H. J. Schreiber, H. Töfte, P. J. Dorman, S. J. Goldsack, M. J. Losty, M. E. Mermikides, A. Frölich, M. Markytan, G. Otter, P. Schmid, H. Wahl (Aachen–Berlin–CERN–London–Vienna Collaboration), ibid. **B23**, 1.

[25]S. Pokorski, M. Szeptycka, A. Zieminski, Nucl. Phys. **B27**, 568 (1971).

[26]P. Hoyer, B. Peterson, A. L. Lea, J. E. Paton, G. H. Thomas, ibid. B32, 285 (1971).

[27]M. D. Lyberg, L. Gislén, Univ. of Lund Preprint, Lund, 1971.

[28]M. D. Lyberg, Univ. of Lund Preprint, Lund, 1971.

[29]K. Buchner et al., Preprint Max-Planck-Institute Phys. and Astrophys., München, 1971.

[30]W. M. Dunwoodie, J. K, Tuominiemi, Preprint CERN (D. Ph.), PHYS 70-59, Geneva, 1970.

[31]S. A. Adjei, P. A. Collins, B. J. Hartley, R. W. Moore, K. J. M. Moriarty, Phys. Rev. D5, 139 (1972); D. Barkai, K. J. M. Moriarty, Preprint ICTP/71/17, London, 1971.

[32]J. A. Jerome, W. A. Simmons, Nucl. Phys. **B24**, 623 (1970).

[33]P. A. Schreiner et al., ibid. **B28**, 85 (1971).

[34]K. Palers, A. C. Amman, D. D. Carmony, A. F. Garfinkel, L. J. Gutay, D. H. Miller, W. L. Yen, ibid. **B21**, 407 (1970).

[35]R. O. Raitio, ibid., p. 427.

[36]R. Baier, H. Kühnelt, F. Widder, ibid. **B27**, 372 (1971).

[37]N. A. Törnqvist, ibid. **B18**, 530 (1970).

[38]L. L. Jenkovsky, V. V. Kukhtin, V. V. Timokhin, Preprint ITP-70-77, Kiev, 1970.

[39]Yu. A. Budagov, V. B. Vinogradov, L. L. Jenkovszky, S. V. Klimenko, G. Martinska, V. V. Timokhin, and L. Sandor, JINR Preprint R2-5943, 1971.

[40]P. A. Collins et al., Nucl Phys. B22, 150 (1971).

[41]B. A. Budagov, V. B. Vinogradov, L. L. Jenkovszky, S. V. Klimenko, V. V. Kukhtin, N. K. Kutsidi, G. Martinska, and V. V. Timokhin, Zh. Eksp. Teor. Fiz. 62, 836 (1972) [Sov. Phys.-JETP 35, 443 (1972)].

[42]S. Pokorski, H. Satz, Nucl. Phys. B19, 113 (1970).

[43]P. Antich, A. Callahan, R. Carson, C.-Y. Chien, B. Cox, D. Denegri, L. Ettlinger, D. Feiock, G. Goodman, J. Haynes, R. Mercer, A. Pevsner et al., John Hopkins Univ. Preprint, Baltimore, 1970.

[44]A. Bettini, M. Cresti, M. Mazzucato, L. Peruzzo, S. Sartori, G. Zumerle, M. Alston-Garnjost, R. Huesman, R. Ross, F. T. Solmitz, L. Bertanza, R. Carrara, R. Casali, P. Lariccia, R. Pazzi, G. Borreani et al., Nuovo Cimento A1, 333 (1971).

[45]H. Satz, K. Schilling, ibid. **A67**, 511 (1970).

[46]H. Satz, K. Schilling, lett. Nuovo Cimento, **3**, 723 (1970).

[47]B. Petersson, G. H. Thomas, Nucl. Phys. **B20**, 451 (1970).

[48]A. J. Buras, Contribution to XVth Intern. Conference on High-Energy Physics, Kiev, 1970.
[49]J. E. L. Hopkinson, R. G. Roberts, Lett. Nuovo Cimento 2, 466 (1969).
[50]M. Chaichian, J. F. L. Hopkinson, ibid. 4, 616 (1970).
[51]B. Bartl, C. Iso, Preprint DESY 70/56, Hamburg, 1970.
[52]L. Gislén, Preprint LPTHE 71/6, Orsay, 1971.
[53]G. Belletini, Rapporteur's Talk at 14th Intern. Conference on High-Energy Physics, Vienna, 1968, p. 354.
[54]E. L. Berger, Proc. of the Conference on "Phenomenology in Particle Physics" (1971), Caltech, March 1971.
[55]L. L. Jenkovsky, N. A. Kobylinsky, A. B. Prognimak, Preprint ITP-73, Kiev, 1973.

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