Global properties of matter in collapsed state ("black holes")

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The review deals with different external fields, the sources of which are contained in the matter of collapsing systems. An analysis is presented of papers, according to which scalar, meson, and neutrino fields vanish outside the black holes. In this connection, a number of questions are formulated, to which there is still no definite answer. It is assumed that exhaustive answers to these questions will be obtained when matchable external and internal solutions can be found for all these cases. At the present time the analysis of this problem is based mainly on considering the external solution and the hypothesis that a horizon exists and perturbation theory is applicable.

CONTENTS

1. INTRODUCTION

In light of recent theoretical investigations, an impression is gained that matter in the process of collapse and in the process of gravitational closure tends to minimize the variety of its global properties, and the variety of parameters characterizing the system as a whole.

Indeed, many global characteristics vanish during the collapse process, when the matter goes under the surface of the "event horizon," forming a "black hole." Thus, outside the "black hole" (collapsar) the magnetic dipole moment is lost, the higher gravitational multipoles vanish, and it is possible that the ability to excite certain external fields is lost, etc.

This situation was picturesquely characterized by Wheeler[1] by the phrase: "black holes have no hair." It is of considerable interest to understand what properties of the systems are characterized by this code term "hair." How does this "hair" disappear in the space around the "black hole," and under which situation does this hair, if we use this terminology, drop out, i.e., something is lost by the system during the process of the radiation prior to the instant of gravitational closure, and in which cases does this "hair" arrange itself into some, say, hairdo under the Schwarzschild sphere and becomes inaccessible to the external observer.

A macroscopic material system, for example a celestial body, can have various global characteristics.

We have in mind here such characteristics as the total mass of the system, its total electric charge, the total angular momentum, etc. A celestial body consisting, for example, of hydrogen gas, has tremendous baryon and lepton charges. The system can have strangeness.

If we have in mind electron-neutrino weak interactions, then the celestial body can be a source of neutrino-antineutrino field, that decreases like $1/r^5$. A macroscopic material system can in principle be a source of a scalar field. The system can have a magnetic dipole moment, higher gravitational moments, etc.

The discarding of such global characteristics, this unique "gravitational strip tease," can reach quite far.

There are also final states of material systems that are rid of all the global characteristics. We have in mind here systems with closed metrics, for which, in particular, the total mass[2)], the total angular momentum, and the total electric charge are equal to zero. "Black holes" and systems with closed metrics are two limiting states, which do not go over into each other (at least in classical physics), of systems with minimized global characteristics. A discussion of global characteristics that violate the metrics of closed systems and contradict the metrics[2] is very valuable, as we shall show, for the understanding of the gravitational strip tease in the formation of black holes. We recall that the investigations by Ginzburg[3] (1964) and by Ginzburg and Ozernoĭ[4] (1964) have shown that the magnetic dipole moment measured at a certain distance from a collapsar tends to zero in the course of time to the extent that the matter of the collapsar goes under the horizon of events in the course of the gravitational closure, i.e., as the surface of the star approaches the Schwarzschild surface[3)].

Doroshkevich, Zel'dovich and Novikov[5] (1965) have shown that collapsars do not have higher gravitational multipoles—they are radiated during the process of the collapse. (In this case the "hair" also falls out.)

Price[6] (1971) et al.[7] reached the conclusion that matter that carries sources of a long-range scalar field, on going under the Schwarzschild surface, do not excite a scalar field outside the black hole. This case is not connected with any radiation of the sources—the sources of the scalar field are buried in the black hole.

Hartle[8] arrives at the conclusion that the collapsar also fails to excite neutrino forces in the outer space. Thus, Hartle gives the following expression for the potential of neutrino forces from sources localized at a point a near a black hole[8a]:

$$V \sim \frac{G}{r^5}\left(1-\frac{M}{a}\right) \times \ldots \qquad (1)$$

If the place of the localization of the sources of the neutrino field (the electrons) approaches the Schwarzschild surface (a → M), then expression (1) vanishes: the black hole has no "neutrino hair." An interpretation of this case is more complicated and raises certain questions. There are statements in the literature that the black holes have no vector meson (baryon) field[7].

The question arises, what global properties of black holes are preserved?

There is a statement in the literature, according to which black holes can have only mass (M), electric charge (ϵ), and angular momentum (J). These quantities obey the conservation laws. The question is, however, what is the situation with the tremendous baryon charge of a collapsing star or with its lepton charge, which are also conserved?

Ruffini and Wheeler[9] attempt to explain the situation as follows:

"Electric charge is a distinguishable quantity because it carries a long-range force (conservation of flux, Gauss's law). Baryon number and strangeness carry no such long-range force. They have no Gauss's law . . . . [No one has] ever been able to give a convincing reason to expect a direct and spontaneous violation of the principle of conservation of baryon number. In gravitational collapse, however, that principle is not directly violated; it is transcended. It is transcended because in collapse one loses the possibility of measuring baryon number, and therefore this quantity can not be well defined for a collapsed object. Similarly, strangeness is no longer conserved." In exactly the same manner, Hartle et al. treat the impossibility of establishing experimentally, outside the black hole, the presence of a neutrino charge in the black hole. In a recent book by Zel'dovich and Novikov[10], the situation is formulated by the following phrase:

"The vanishing of the signals from the particles buried during the collapse does not mean that the particles are lost: after all, we do not suppose that a man is lost if he is hiding behind the corner of a building."

We wish to discuss subsequently in greater detail the extent to which the interpretation given by Ruffini and Wheeler is germane to the discussed situation in collapsing systems.

It is of interest to discuss the arguments that lead to the proof that black holes have no external scalar, massive vector and neutrino fields. But it is advisable first to consider one statement (a lemma, if you please) concerning systems described by a closed metric. The results of this analysis, as will be explained later on, are of great significance in the discussion of problems. The statement we have in mind is formulated in the following manner:

If a system containing sources (specific charges) of some field turns out to be incompatible with a closed metric, then the corresponding black hole has on its outside a field of the given sources.

It is assumed that the characteristics of the system (the critical density of matter etc.) are such that the metric becomes closed when the values of the discussed charges tend to zero.

As is well known, the Friedmann linear element[11]

$$ds^2 = a^2(\eta)\, d\eta^2 - a^2(\eta)\, d\chi^2 - a^2(\eta) \sin^2\chi\, (d\theta^2 + \sin^2\theta\, d\varphi^2) \qquad (2)$$

describes one of the models of a closed world.

The range of the variable χ is here

$$0 \leqslant \chi \leqslant \pi. \qquad (3)$$

The variable η is connected with the time t by the simple relation $c\,dt = a\eta$. The same form (2) describes the internal metric of a black hole, if the matter in the system is distributed in such a way that it fills the region of χ only to $\chi_0 \leq \pi/2$. Next, the external solution, which is Euclidean at infinity, should be "joined together" in some appropriate manner with this internal solution. The metric on the whole is nonstatic, but in a certain approximation the external metric can be a Schwarzschild metric.

Finally, if the matter fills the region $\pi/2 < \chi_0 < \pi$, then a system with a semiclosed metric is produced. If we draw around the point $\chi = 0$ spheres with certain values of χ, then the corresponding surface of any sphere is given by the expression

$$S = 4\pi a^2(\eta) \sin^2\chi. \qquad (4)$$

When χ increases to $\chi = \pi/2$, the surface area S of the sphere increases. But at values $\chi > \pi/2$ the dimensions of the sphere decrease, and at $\chi = \pi$ the sphere contracts to a point—the world becomes closed.

At $\chi_0 < \pi/2$ ("black hole")[12] the surface of the sphere at a given instant of time (η) increases monotonically with increasing radius: the quantity $a \sin\chi = r$ likewise assumes in the external metric the meaning of a monotonically increasing radius.

A semiclosed metric (the case $\pi/2 < \chi_0 < \pi$) is characterized by the existence of a minimal value of $r(\partial r/\partial\chi = 0,\ \partial^2 r/\partial\chi^2 > 0)$, in other words, by the presence in the metric of a specific throat ("molehill" in Wheeler's terminology), which connects the internal and external metrics.

The density of matter $\mu(t)$, integrated over the entire space of the closed world, yields the "bare" mass of the system, i.e., the total mass without allowance for the gravitational defect:

$$M_0 = 2\pi^2 \mu(t)\, a^3(t). \qquad (5)$$

This value of the bare mass determines the radius of the closed world (a_0) at the instant of its maximal expansion:

$$a_0 = \frac{\varkappa M_0}{3\pi c^2}. \qquad (6)$$

The last expression follows directly from Einstein's equation which a(t) satisfies[11]

$$\left(\frac{\dot a}{a}\right)^2 = \frac{2\pi\varkappa\mu}{3} - \frac{c^2}{a^2}, \qquad (7)$$

if we put in (7) $\dot a = da/dt = 0$ and in accordance with (5) $\mu_0 = M_0/2\pi^2 a_0^3$.

The total mass of that part of the closed world which is localized in the region from $\chi = 0$ to χ_0 (i.e., the bare mass minus its gravitational defect) is given by the expression[2,10]

$$M_{\text{tot}} = \frac{c^2}{\varkappa} a_0 \sin^3\chi_0. \qquad (8)$$

Thus, the total mass of the closed system ($\chi_0 = \pi$) is equal to zero.

The total electric charge of the closed world is also equal to zero, owing to the conservation of the electric charge, and if an attempt is made to place an electric charge in a closed world, a contradiction arises between the Gauss theorem ($\int E_n\, dS = 4\pi e$) and the metric of the closed world[11]; this contradiction illustrates the lemma formulated above. The character of the deformation of the metric of the closed world by a small electric charge has been considered in[13,14]. For an arbitrarily small electric charge, an appreciable deviation from the metric of the closed world sets in only when χ is arbitrarily close to π. In other words, if we draw spheres with $\chi > 0$ around a small charge ϵ localized in the region $\chi = 0$, then these spheres will be characterized by expression (4) if the charge ϵ is small. When χ increases to $\chi = \pi/2$, the spheres increase. When $\chi > \pi/2$, if the charge is very small, the spheres will correspondingly decrease. But in the region $\chi > \pi/2$ the force lines of the electric vector (E_n) (the "hair" of the electromagnetic field) begin to condense. At $\chi > \pi/2$ the spheres play the role of converging lenses of sorts for the force lines of the electrostatic field.

A detailed analysis shows[14] that when the density of the force lines reaches a level such that the corresponding value for the electrostatic potential reaches a value

$$\varphi = \frac{c^2}{\sqrt{\varkappa}}, \tag{9}$$

then the described spheres begin again to increase with further increase of χ, and the metric goes over into the well known Nordstrom-Reissner metric, which in this case is characterized by the value of the Schwarzschild mass

$$M_{\text{tot}} = \frac{\varepsilon}{\sqrt{\varkappa}} \tag{10}$$

and the metric

$$ds^2 = \Phi c^2\, dt - \frac{dr^2}{\Phi} - r^2 (d\theta^2 + \sin^2\theta\, d\varphi^2), \tag{11}$$

where

$$\Phi = \left(1 - \frac{\varepsilon \sqrt{\varkappa}}{c^2 r}\right). \tag{12}$$

The minimal sphere radius admitted by the electric charge ϵ is proportional to the value of the charge

$$r_{\min} = \frac{\varepsilon \sqrt{\varkappa}}{c^2}. \tag{13}$$

The metric ceases to be closed at an arbitrary small electric charge. Figuratively speaking, the electric force lines ("hair") condense in the region of χ close to π, to such a degree that they "punch through" a "molehill" (throat) into the metric, into which the flux of the electric vector tends to flow, forming a Nordstrom-Reissner metric outside the given material system. On the whole, the metric, at arbitrarily small electric charge, has the character of the metric of a semiclosed ($r' = 0$, $r'' \gg 0$) world[4]. In beauty shop language, a "pony tail" hairdo is produced from without.

If the charge ϵ is large enough, however, then the minimal sphere may not exist. In this case a significant violation of the closed-world metric can occur already at $\chi \le \pi/2$. The given metric describes a black hole, but with an external electrostatic field.

We can "test" the proposed lemma without constructing a self-consistent solution that takes into account the effect of the sources of the considered field on the metric. It is possible, for example, for methodological purposes, to find and discuss solutions of Maxwell's equations for the electrostatic case $\epsilon \neq 0$ under conditions of a strictly closed metric, assume for simplicity in a Friedmann metric at the instant of maximal expansion of the world ($\dot{a} = 0$).

Under these conditions, the contradictions of the requirement $\epsilon \neq 0$ with closed metric become manifest in the appearance of a divergence for the potential as $\chi \to \pi$.

Indeed, as can be easily verified, in this case the solution for the electrostatic potential takes the form

$$\varphi = \frac{\text{const}}{a \sin\chi}. \tag{14}$$

For the energy density, naturally,

$$T_0^0 \sim \frac{1}{a^2 \sin^4\chi}. \tag{15}$$

As $\chi \to \pi$, expressions (14) and (15) diverge even in the case when the source (charge ϵ) is smeared out in the region $\chi \approx 0$ over a certain finite sphere. In other words, a singularity characteristic of a pointlike source, the presence of which in this place was not assumed by us, appears at $\chi = \pi$. The contradiction with the closed metric takes this form.

Let us apply the proposed test to a massive vector field. The question is, what situation arises in a closed world if, say, one extra neutron appears in the baryon-neutral matter of this system at the point $\chi = 0$. And let this neutron be a source, say, of a ρ-meson vector field.

The equations of a massive vector field (with mass m_ρ) in an arbitrary curved space take the form ($c = 1$)

$$F^{\mu\nu}_{;\nu} - m_\rho^2 \varphi^\mu = -4\pi j^\mu. \tag{16}$$

We consider a centrally-symmetrical solution of this equation in the absence of free waves.

We shall solve the problem under the same conditions as the preceding one, for the instant of maximal expansion of the Friedmann world ($a = a_0$, $\dot{a} = 0$).

Let $a_0 m_\rho > 1$, and let the metric be specified in the form (2) in such a way that $\sqrt{-g_0} = a_0^4 \sin^2\chi \sin\theta$. In this case the system (16) reduces to one equation

$$\frac{1}{\sin^2\chi} \frac{d}{d\chi}\left(\sin^2\chi \frac{d}{d\chi}\varphi_0\right) - m_\rho^2 a_0^2 \varphi_0 = -4\pi a_0^2 j^0. \tag{17}$$

At small χ_0, the charge g_0 and the charge density ρ are connected by the relation $g_0 = 4/3\pi\rho_0 a^3 \chi_0^3$; $j^0 = \rho_0/a_0$, if $\chi \le \chi_0$; $j' = 0$, if $\chi > \chi_0$.

Proceeding in the standard manner, we can easily obtain for φ_0, outside the location of the charge g_0, an expression of the type

$$\varphi_0 \sim \frac{\beta e^{-\lambda\chi}}{\sin\chi} \qquad \left(\lambda = \sqrt{a_0^2 m_\rho^2 - 1}\right), \tag{18}$$

which can be naturally regarded as the analog of the usual expression $\varphi \sim \exp(-m_\rho r)/r$ in Euclidean space.

On the basis of (18) we could conclude that the potential φ_0 diverges as $\chi \to \pi$ just as in the case of electrodynamics, and that the presence of sources of a massive vector field with total charge different from zero is incompatible with a closed metric.

But this reasoning is wrong—it is based on an error made by us: the point is that without thinking, we used in the solution, through force of habit, conditions that are usual for Euclidean space. These conditions that the solution be finite naturally pick out solutions that attenuate exponentially with increasing χ.

In the case of a closed world, there is no spatial infinity, so that a more general type of solution is possible[15]

$$\varphi_0 = \beta \frac{e^{-\lambda\chi}}{\sin\chi} + \gamma \frac{e^{+\lambda\chi}}{\sin\chi}. \tag{19}$$

The requirement that the solution be finite and continuous outside the source is satisfied by the condition imposed on the coefficients β and γ:

$$\beta e^{-\lambda\pi} + \gamma e^{+\lambda\pi} = 0.$$

Thus, outside a pointlike source we obtain a solution in the form

$$\varphi_0 = \frac{g_0 \operatorname{sh} \lambda (\pi - \chi)}{\operatorname{sh} \lambda\pi \sin \chi}. \quad (20)$$

Now φ_0 is finite as $\chi \to \pi$, and the field $(\partial\varphi_0/\partial\chi)$ vanishes as $\chi \to \pi$.

We arrive at the conclusion that the presence of sources of a massive vector field is compatible with a closed metric; that a closed world can contain a total baryon charge (a source of a massive vector field) different from zero. In[15], a closed metric was constructed with allowance for the influences exerted on the metric by a massive vector field. This analysis establishes the essential difference between a massless vector field and a massive one. Our result does not contradict the statement that there is no massive field outside black holes. Nor is it, however, a proof of this statement.

The point is that a lemma inverse to the lemma proposed above certainly does not hold; it suffices to recall that a mass, which is a source of a gravitational field, admits of a closed metric, and that a black hole excites a gravitational field in outer space.

As shown by Asanov[16], sources of a massless scalar field are also compatible with a closed Friedmann metric. In the metric $dS^2 = e^{\gamma(rt)}dt^2 - e^{\alpha(rt)}dr^2 - e^{\beta(rt)}d\Omega^2$ the equation for the scalar field

$$\nabla_\sigma \nabla^\sigma U = -4\pi j \quad (21)$$

takes the form

$$e^{-\alpha}\left[U'' + \left(-\frac{\alpha'}{2} + \beta' + \frac{\gamma'}{2}\right)U'\right] - e^{-\gamma}\left[\ddot{\varphi} + \left(\frac{\dot{\alpha}}{2} + \dot{\beta} - \frac{\dot{\gamma}}{2}\right)\right]\dot{U} = 4\pi\check{j}, \quad (22)$$

where $\check{j}$ is the invariant density of the sources of the scalar field.

In this metric, the Bianchi identity $\nabla_\sigma T_1^\sigma = 0$, using (22), yields the relation

$$\gamma'\rho + 2\check{j}U' = 0 \text{ (ρ is the mass density)}; \quad (23)$$

where the prime and the dot denote derivatives with respect to r and τ, respectively.

If it is possible to introduce co-moving synchronous coordinates (i.e., where γ does not depend on r), we obtain as a consequence

$$jU' = 0. \quad (24)$$

Under these conditions, the scalar field should be either free (j = 0) or

$$U' = 0. \quad (25)$$

In the Friedmann metric, the condition (25) is a direct consequence of the homogeneity of the world. In Friedmann's world, only derivatives with respect to time remain in the equation for the scalar field (22).

Asanov[16] considered a closed-world model in which

$$\dot{U} = \frac{\dot{\alpha}}{2\sqrt{\varkappa}}, \quad (26)$$

where $e^\alpha = a_0(1 - \cos\eta)^2$, $0 < \eta < 2\pi$. Thus, he had in mind the synchronous form (ds^2) of the dustlike model of Friedmann's world[11].

Attention is called to the fact that the expression for the density of the sources of the scalar field is obtained in the form

$$j = \frac{2\sin^4(\eta/2) - 1}{32\pi\sqrt{\varkappa}\, a_0^3 \sin^8(\eta/2)}. \quad (27)$$

The density of the sources of the scalar field varies in a specific manner with time (cdt = a $d\eta$). The quantity j vanishes when $\sin^4(\eta/2) = 1/2$, and even reverses sign at certain instants of time. This behavior of the scalar-field sources can be attributed to the fact that, unlike the electric charge, the scalar-field charge is not subject to a conservation law.

Thus, a closed world can have a total nonzero scalar-field charge.

The situation is much more complicated with the neutrino field (B).

A preliminary analysis (by Berezin) shows that placement of a pointlike source of a neutrino field at a point $\chi = 0$ leads to the formation of a neutrino-field source (with the sign reversed) at $\chi = \pi$:

$$-B(\chi - \pi) = B(\chi).$$

If this result can withstand subsequent analysis, this will mean that a nonzero lepton charge is incompatible with the metric of a closed world. An external continuation of the metric is in this case inevitable and neutrino "hairs" outside the black holes must exist. In other words, this contradicts the previously cited papers by Hartle[8].

We turn now to consider the published proofs of the absence of scalar (U), massive-vector (φ), and neutrino (B) fields outside black holes.

All the published proofs of this type[6,7,8a] are based on a number of assumptions, the most significant of which are the following:

α) The considered system has an event horizon

$$g_{11} \to \infty \text{ and } r \to r_{gr}.$$

β) The potential of the considered field has a finite value on the surface of the event horizon.

γ) The influence of the considered field on the metric can be neglected (in the case of an arbitrarily weak field).

Let us examine in detail the situations of interest to us in the case of the different particular fields.

2. SCALAR LONG-RANGE FIELD

It should be recalled that a scalar field has a large number of very unique properties, which were noted, in particular, by Dicke. Some of these properties will also be discussed by us later on. We note first that in addition to the equation of the scalar field in the form (21)

$$\nabla_\sigma \nabla^\sigma u = -4\pi j,$$

there exists also another form[17,18]

$$\nabla_\sigma \nabla^\sigma u + \frac{R}{6}u = -4\pi j; \quad (28)$$

R is the scalar curvature.

The last equation is conformally invariant and has a number of advantages over the equation in the form (21).

A problem similar to the Nordstrom-Reissner problem was solved by Fisher[19] (for an equation in the form (21)) more than 20 years ago (1948). The metric obtained in that paper differs radically from the Nordstrom-Reissner metric and the Schwarzschild metric, in that in the case of a scalar field there is a striking absence of an event horizon. More accurately, the corresponding Schwarzschild sphere contracts to a point, and this occurs at an arbitrarily small charge of the scalar-field source.

This result in itself is so surprising and unlikely, that one seeks voluntarily some computational errors in the work or some natural limitations of the region of applicability of the static metric, just for the scalar field. We present first the particular results of[19]. The metric obtained by Fisher is

$$ds^2=\left(\frac{z-z_0}{z+z_1}\right)^p dt^2-\frac{r^2}{z^2}\left(\frac{z-z_0}{z+z_1}\right)^p dr^2-r^2(d\theta^2+\sin^2\theta\, d\varphi^2); \qquad (29)$$

here

$$z_{0,\,1}\equiv(\varkappa^2 m^2+\varkappa G^2)^{1/2}\mp\varkappa m, \text{ G is the scalar charge,} \qquad (30)$$

$$p=\frac{\varkappa m}{\sqrt{\varkappa^2 m^2+\varkappa G^2}} \qquad (31)$$

$$z(r)=re^{(\nu-\lambda)/2}\to r \text{ при } r\to\infty,\ g_{00}=e^{\nu},\ g_{11}=-e^{\lambda} \qquad (32)$$

and

$$(z-z_0)^{1-p}(z+z_1)^{1+p}=r^2. \qquad (33)$$

According to (29), g_{11} never becomes infinite and, just like g_{00}, tends to zero as $z\to z_0$. According to (33), g_{11} and g_{00} vanish at the point $r = 0$.

The potential obtained at this point at the point $r = 0$ has a logarithmic singularity

$$U=\frac{G}{2\sqrt{\varkappa^2 m^2+\varkappa G^2}}\ln\left(\frac{z+z_1}{z-z_0}\right). \qquad (34)$$

Fisher's calculations were independently repeated more than 20 years later by Janis, Newman, and Winicour[20]. They obtained a metric in the form

$$ds^2=\left[\frac{2R+r_0(\mu+1)}{2R-r_0(\mu-1)}\right]^{1/\mu}dR^2+r^2(d\theta^2+\sin^2\theta\, d\varphi^2)-\left[\frac{2R-r_0(\mu-1)}{2R+r_0(\mu+1)}\right]^{1/\mu}dt^2, \qquad (35)$$

where

$$r^2=\frac{1}{4}[2R+r_0(\mu+1)]^{1+(1/\mu)}[2R-r_0(\mu-1)]^{1-(1/\mu)}. \qquad (36)$$

This metric can be transformed in simple fashion into the metric (20), which is more convenient for analysis (since g_{11} in the latter is a coefficient of dr^2 and not of dR^2 as in (36)).

In both[19] and[20] there are certain errors in the analysis of the asymptotic behaviors of the metric[5)], but these errors do not concern the form of the linear element (29), which was calculated correctly. Thus, for a scalar field described by Eq. (21), there is no condition for the applicability of theorems that prove the absence of an external scalar field for black holes (if it is agreed that the metric (29) is correct in the case of a scalar field).

Indeed, in this case a system with scalar-field sources has no event horizon (the condition α is not satisfied) for the applicability of the theorems. More likely, there is no black hole in this case, and there is something in the nature of a "bare" singularity, into which the Schwarzschild sphere degenerates. But the potential becomes infinite on this degenerate Schwarzschild sphere (the condition β is not satisfied). The last remarks concern the criticism of proofs of the type given in[7,8a], which are based on consideration of a static metric.

An analysis of the problem, as a problem of a non-stationary and co-moving system of coordinates, treated for example in the substantial paper by Price[6], will be given somewhat later.

Of course, the metric (29) has many unexpected properties, which are difficult to reconcile with physical intuition. Foremost among them is the destruction of the Schwarzschild horizon in the case of an arbitrarily weak scalar field.

This raises the natural question: Can this property of the metric (29) be reconciled with the transition to the limit as $G\to 0$, which of necessity must lead to the Schwarzschild metric? It turns out that this requirement is satisfied.

Indeed, as $G\to 0$ we have (according to (30)) $z_0 = 0$ and $z = 2\kappa m$. According to (31) $p = 1$, and according to (33)

$$z+2\varkappa m=r. \qquad (37)$$

Consequently, as $G\to 0$ the metric (29) takes the form

$$ds^2=\left(1-\frac{2\varkappa m}{r}\right)dt^2-\left(1-\frac{2\varkappa m}{r}\right)^{-1}dr^2-r^2(d\theta^2+\sin^2\theta\, d\varphi^2). \qquad (38)$$

On the other hand, g_{00} and g_{11} tend monotonically to zero as $z\to 0$ at arbitrarily small G. A formal analysis shows that the metric does not have an analytic behavior as $r\to 0$ and $G\to 0$, namely $z(r\to 0)\to 0$, but at the limit itself ($r = 0$) it assumes jumpwise, according to (37), the value

$$z=-2\varkappa m. \qquad (39)$$

One can advance the hypothesis that the static metric (29) is applicable only up to certain values r_{cr}. It is perfectly possible that there exist physical not-yet analyzed causes which enable us to limit the internal solution in this metric (29) to arbitrarily small r.

It is possible that the basis for this should be sought in the following unique properties of the scalar field.

Consider, for example, in a certain Newtonian approximation, the total mass of a system distributed over a spherical region of radius r_0. Let the "bare" mass of the matter (mass without allowance for its gravitational defect) equal M_0, and let the total scalar charge distributed over this region be G.

Generalizing the well known Arnowitt-Deser-Misner relation[22] for the total mass of the system, we obtain

$$M_{\text{tot}}=M_0-\varkappa\frac{M_{\text{tot}}^2}{2c^2 r_0}-\frac{G^2}{2c^2 r_0}, \qquad (40)$$

or

$$M_{\text{tot}}=-\frac{2r_0c^2}{\varkappa}+\left(\frac{4r_0^2c^4}{\varkappa^2}+\frac{2r_0c^2M_0}{\varkappa}-\frac{G^2}{\varkappa}\right)^{1/2}. \qquad (41)$$

According to (41), the total mass of the system vanishes at

$$r_0=\frac{G^2}{2M_0c^2}. \qquad (42)$$

The considered system cannot be localized in the region $r < r_0$. In the case $r > r_0$, a negative value of the total mass is obtained: the gravitational attraction is replaced, as it were, by gravitational repulsion and the system retains its minimal dimensions.

The last considerations can offer evidence in favor of the fact that it is apparently not legitimate to continue the external metric (29), a metric in vacuum, to arbitrarily small distances (like the static metric).

If we analyze the structure of the metric (29) in light of the last considerations, then we can state the following. As $r \to \infty$ we get $z \to r$ and also

$$-g_{11} = +e^{\lambda} = \frac{r^2}{z^2}\left(\frac{z-z_0}{z+z_1}\right)^p \to 1, \qquad r \to \infty. \tag{43}$$

On the other hand, using (33), we can rewrite e^{λ} in the form

$$e^{\lambda} = \frac{(z-z_0)(z+z_1)}{z^2}. \tag{44}$$

Alternately, expanding the values of z_0 and z_1,

$$e^{\lambda} = 1 + \frac{2\varkappa m}{z} - \frac{\varkappa G^2}{z^2}. \tag{45}$$

At large but finite r, and consequently z, we have $e^{\lambda} > 1$, just as in the case of the Schwarzschild and Nordstrom-Reissner metrics.

Starting with certain r or z, however, e^{λ} begins to decrease again and tends to zero monotonically as $r \to 0$. But in the course of this change (and this is most unexpected) e^{λ} never becomes infinite; in other words, in this case we do not encounter an event horizon[6)]. Attention is called, however, to the fact that when r is varied e^{λ} assumes a value unity twice: as $z \to \infty$ (i.e., as $r \to \infty$), and as

$$z \to \frac{G^2}{2m} = z_c. \tag{46}$$

In accordance with the meaning of (42), relation (46) can be interpreted in the following manner: if matter charged by a scalar charge is localized in a region $z_c = G^2/2m$, then the total mass of the system in the region from $z = 0$ to $z = z_c$ is equal to zero. In other words, if the Schwarzschild mass measured at $r \to \infty$ turns out to be equal to m, then it is all localized only in the region $z > z_c$.

From everything stated above concerning the metric (29) and the specific properties of the scalar field, it follows that the problem of the scalar field in the case of a collapsing system still (let us be cautious) awaits its solution. This situation arises when an internal solution of a collapsing system is obtained (with allowance for the influence of the scalar field on the metric) and the external solution can be made continuous with it. This, most importantly, should be obtained outside the framework of perturbation theory. This problem is essentially nonstatic. It is made complicated by the fact that, unlike electrodynamics, centrally-symmetrical motion of matter can be accompanied by monopole radiation of a scalar field, which changes the mass (and the gravitational radius of the system).

We have considered above a scalar field that satisfies Eq. (21).

Another form of a scalar equation, besides (28), leads to a metric that contains an event horizon in a particular case[24]. This particular case is characterized by a definite connection between the total mass (m) and the total charge (G) of the system

$$G^2 = 3\varkappa m^2. \tag{47}$$

In this case there arises an external metric which is perfectly analogous to the Nordstrom-Reissner metric in its particular case, when the total mass (m) is equal to the total electric charge (ϵ):

$$g_{00} = e^{\nu} = e^{-\lambda} = \left(1 - \frac{a}{r}\right)^2, \tag{48}$$

$$a = \varkappa m = \sqrt{\frac{\varkappa G^2}{3}}. \tag{49}$$

However, unlike the electrostatic case, in this metric the scalar potential becomes infinite on the surface of the horizon event:

$$U = -\frac{G}{r-a}. \tag{50}$$

In this case we have in explicit form a counter-example, where the corresponding black hole has an external scalar field.

Since the potential U vanishes at infinity on the event horizon, the theorem according to which the field vanishes outside the black hole is not applicable to this case (the condition which we designated before by β is violated).

On the other hand, the fact that no external field can exist is corroborated in this case by the general analysis of Chase[25], according to which the potential on the event horizon must have infinitely large values in this case.

Generally speaking, Chase's result[25] is obtained by the following elementary procedure.

If in the case of electrostatics, when the potential transforms like the fourth component of a vector, the first integral of the equation yields a derivative of the electrostatic potential with respect to the coordinate ("a field") in the form

$$\varphi' = \frac{e}{r^2} e^{(\lambda+\nu)/r} = -\frac{e}{r^2}\sqrt{g_{00}g_{11}}, \tag{51}$$

then we have when the field U is scalar[19]

$$U = -\frac{g}{r^2} e^{(\lambda-\nu)/2} = \frac{g}{r^2}\sqrt{\frac{g_{11}}{g_{00}}}. \tag{52}$$

In the Nordstrom-Reissner metric we have

$$g_{00}g_{11} = e^{\lambda+\nu} = 1.$$

In a similar metric or in a more general one, but also with a horizon event, when $e^{\lambda} \to \infty$ as $r \to r_g$ and g_{00} is bounded, the field U′ should inevitably acquire infinitely large values on the event horizon. This is the peculiarity of U and the peculiarity of its scalar nature.

Generally speaking, certain similar cases, which are characterized by infinite values of certain physical quantities on the Schwarzschild surface, can be "disqualified" as unphysical. They cannot, for example, be limiting cases of a physical collapse. In the course of collapse, there should be no singularity that cannot be eliminated by a coordinate transformation in the metric on the event horizon: the known invariants should not have singularities on the horizon.

From this point of view, the metric (49), is perfectly correct, since it coincides formally with the Nordstrom-Reissner metric. No singularities of the invariants (for example, $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$) occur on the event horizon.

Moreover, even though the potential U diverges as $r \to a$, the energy density (T_0^0) vanishes at $r = a$. This occurs because of a unique[24] dependence of the tensor T_i^{μ} on U and its derivatives.

$$L = -\sqrt{1 - \frac{v^2}{c^2}}\,(m + U).$$

In addition, the peculiarity of the scalar field lies in the fact that in the expression for the Lagrangian the scalar potential is added to the mass[26,27].

This means that, from the point of view of an observer of a system that moves with velocity[7)] v, the scalar potential U is given by

$$U' = \sqrt{1-\frac{v^2}{c^2}}\,U. \tag{53}$$

If it is recognized that, in accordance with (49),

$$\sqrt{1-\frac{v^2}{c^2}} = \sqrt{g_{00}} = \left(1-\frac{a}{r}\right), \tag{54}$$

and the potential (50) is given by

$$U = -\frac{G}{r}\frac{1}{1-(a/r)},$$

then the potential of the scalar charge localized on the surface of the horizon remains finite everywhere for an observer crossing the horizon in a freely falling system of coordinates.

Leaving aside the particular case (48), the general solution (for which $\nu' + \lambda' \neq 0$) contains, in analogy with (29), a singularity only at $r \to 0$.

In light of the foregoing, it is advantageous to consider the very process of collapse of matter charged by sources of a scalar field. This means that it is necessary to consider a nonstatic problem: find a nonstatic internal solution (in a region occupied by matter), and the solution joined with it in vacuum, for example in a falling system of coordinates.

A program of this type is carried out in Price's paper, but within the framework of perturbation theory.

Price[6] believes that it is possible, assuming the scalar field to be weak, to disregard the influence of the scalar field on the metric. He considered the model of dustlike matter. The internal metric of the star is described by a linear Friedmann element (2). The external (vacuum) solution is given by a spherically symmetrical linear element in a co-moving (synchronous) coordinate system[11].

$$ds^2 = dT^2 - \frac{(\partial r/\partial R)^2}{1+2E(R)}dR^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2).$$

The system begins to collapse from a distance

$$r_{st} = 4M.$$

At this initial instant, the scalar field (Φ) is assumed static ($d\Phi/dT = 0$; $d^2\Phi/d\tau^2 = 0$). The internal initial form of the potential is chosen, for convenience in the solution of the problem, with a definite dependence on χ. Indeed, the particular solution that depends on χ is assumed in the form

$$\Phi_s = \frac{\sqrt{2}}{8}\cos\chi\,(-11+\sin^2 2\chi). \tag{55}$$

The author shows that at the instant the potential of the scalar field and its derivatives remain finite and do not vanish when the surface of the star crosses the horizon of events.

Thus, it is proved that the scalar field perturbs the metric little.

Price[6] ignores completely the discussion of the static metric (29), which seemingly does not admit of application of perturbation theory to Price's problem.

Thus, all that can be stated so far is that there is a contradiction between the two approaches to the problem. In addition, however, there is also, as we have seen above, a direct counter-example to Price's result, an example in the form of the metric (49), where there is a horizon but the scalar potential on the horizon becomes infinite. In the particular case of (49) we have $G^2 = 3\kappa m^2$. This means that the scalar and gravitational forces are of the same order. Moreover, the zeroth approximation (there exists an event horizon, there is no scalar field) has no meaning ($G = 0$ implies $m = 0$). The particular Price model also is subject to doubt.

The point is that in the synchronous coordinate system we have, on the basis of the Bianchi identity $U' = 0$, and consequently the potential should not depend on χ (see relations (23), (24), and (25)). In general, Eq. (22) for the potential in the system used by Price should not contain derivatives with respect to the spatial coordinate. We do not know the extent to which this particular dependence on χ in the given model is a permissible approximation and whether the initial conditions $\Phi_t = 0$ and $\Phi_{tt} = 0$ are permissible, bearing in mind Eq. (22), which should contain in the synchronous system, strictly speaking, only derivatives with respect to time.

Price's analysis of the role of the effective potential in the suppression of the higher multipoles of different fields is very convincing. It seems, however, that the simplest case of monopole radiation of a scalar field still is not free of objections.

The scalar field has one more surprising property: the mass of the particle of the source of the scalar field should be a function of the scalar potential. The time variation of the scalar potential changes the mass of the system, and vice versa. This property of the scalar field was also indicated by Dicke[26], namely

$$\frac{d}{d\tau}(mv_i) + \frac{dm}{du}u \quad (i=0), \tag{56}$$

where v_i are the components of the velocity vector

$$m = m(u), \tag{57}$$

From this point of view it is not clear whether it is sufficiently convincing to treat the scalar field in collapse as only a "free" field of long waves reflected from an effective barrier.

But the main and possibly greatest interest attaches to the short-range scalar field, the field of scalar mesons with rest mass different from zero. And Price's arguments concerning the role of the effective potential in the reflection of monopole radiation cannot have any bearing on this case of "short-range" fields.

In the case of scalar mesons, with relatively large masses, it appears that the monopole radiation in collapse can be neglected.

Thus, an investigation of a scalar field under conditions of a collapsing system is much more complicated than the electrodynamic problem. In our opinion, it has not yet been exhaustively solved.

The study of the behavior of massive scalar fields in the course of collapse is also very important. Such fields have been discussed of late in the theory of elementary particles and play a fundamental role in attempts to construct unified theory of weak and electromagnetic interactions[28].

3. MASSIVE VECTOR FIELD

Unfortunately, no solution of the Nordstrom-Reissner type has been obtained as yet for a massive vector field.

The example of the scalar field shows how important it is to have the concrete form of the metric in order to analyze the state of the external field of collapsing systems. Having no concrete form of the metric, it is difficult to guarantee the absence of some unexpected surprises in this case.

Let us examine the example of a massive vector field in the form of a ρ-meson field. The sources of the ρ-meson field are nucleons, say for simplicity neutrons. The field $F_{\mu\nu} = \partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu$ and the potential φ_μ satisfy the equation

$$F^{\mu\nu}_{,\nu} - m_\rho^2 \varphi^\mu = -4\pi j_1^\mu,$$

where m_ρ is the ρ-meson mass and j^μ is the baryon-current vector.

In this case, unlike the case of a scalar field, the baryon charge ρ satisfies a conservation law. However, unlike electrodynamics, there is no Gauss theorem in vector mesodynamics.

It is assumed that it is precisely this difference from electrodynamics which leads to the absence of an external mass of vector field for black holes.

It should be pointed out that in the case of a massive vector field there is a certain unique analog of the Gauss theorem, more accurately, its unique generalization. A more detailed examination of the relations given here allows us to suggest that also in the case of a massive vector field the question of the external metric of a black hole is still, cautiously speaking, awaiting its solution.

Let us examine the mesodynamic analog of the electrodynamic Gauss theorem, for simplicity in a Euclidean metric.

Let the baryon charge ρ be localized in a certain region such that

$$\rho \neq 0, \quad r < r_0;$$
$$\rho = 0, \quad r > r_0.$$

For the flux of the mesodynamic vector E_n through a closed surface surrounding the charge we obtain, on the basis of Eq. (51), the following expression:

$$\int E_n\, dS = m_\rho^2 \int \varphi^0 dv - \int \rho\, dv. \tag{58}$$

Drawing a circle of radius $r \geq r_0$ around the charge, we obtain the flux of the vector E_n through the sphere in the form

$$\int E_n\, dS = 4\pi m_\rho^2 \int_0^r \varphi(r) r^2 dr - 4\pi g; \tag{59}$$

if $\varphi^0(r) = ge^{m_\rho r}/r$, then

$$\int E_n\, dS = 4\pi g \left[m_\rho^2 \int_0^r e^{-m_\rho r} dr - 1 \right]. \tag{60}$$

In electrodynamics, the flux of the vector E_n has the same value on a sphere of arbitrary radius surrounding the charge. But in mesodynamics the total flux of the vector E_n decreases with increasing radius of the sphere. At a sphere radius tending to infinity, the right-hand side of (60) vanishes: the flux E_n is completely extinguished.

If it is appropriate to use the term "force lines" in mesodynamics, then in the case of a massive vector field the force lines do not terminate, as in the case of electrodynamics, at charges of opposite sign. Instead they are extinguished, figuratively speaking, by a specific "field charge" which is effected by the field potential which is always continuously distributed over all of space. We have in mind the integral in the right-hand side of (60).

A nonzero flux of the vector E_n passes through a sphere of finite radius r—this is the "hair" of the baryon field. The statement that the flux of the vector E_n vanishes outside the black hole, means that the flux of the vector E_n vanishes inside the black hole on approaching the surface of the event horizon. This means that in some manner the "charge of the field," integrated over the internal space of the black hole, increases to such a degree that it is capable to compensate for that vanishing of the potential which has occurred outside the black hole. Without a corresponding increase of the potential in the internal space, it is impossible to annihilate the flux through the Schwarzschild sphere.

If we turn to the situation with the discussed flux in a closed world, then it is easy to verify that a potential in the form (20) ensures the vanishing of the flux on the boundary of the world at $\chi = \pi$.

This possibility arises because inside a closed world a term with an increasing exponential is added to the expression of the potential. Because of this, the density of the "field charge" increases enough to compensate for the baryon charge. In this arbitrary sense, the total "baryon charge" (the right-hand side of (58)) is equal to zero in a closed world. Owing to the change of the potential, the "hair" of the ρ-meson field is located inside the closed world.

This raises the question of what makes it possible for the integral of the massive vector field potential to increase inside the black hole, for after all the boundary conditions at Euclidean infinity do not change for a collapsing system. In other words, it is impossible to cause the external field of the black hole simply to disappear, it must, figuratively speaking, to be "driven inside" the black hole, precisely in such a way as to increase the integral $m_\rho^2 \int \varphi^0 dv$ inside the black hole to a value that compensates for the total baryon charge (G).

At any rate it is still not clear how the generalized Gauss theorem is satisfied in the collapsed process, and it cannot be stated that black holes have no external baryon fields.

4. NEUTRINO FIELD

The situation is much more complicated with neutrino forces. Indeed, if the $((e\nu)(e\nu))$ interaction exists, then a system consisting, say, of hydrogen should excite in the surrounding space a neutrino-antineutrino field with a potential

$$B \sim \frac{1}{r^5}.$$

We have in mind the vector variant of the interaction, for which there is a conservation law for the sources of this field—the lepton charge conservation law. The forces here are repulsion forces.

The quantity not subject to annihilation is

$$L = [n(e^-) - n(e^+)] + [n(\nu_e) - n(\tilde{\nu}_e)].$$

If the corresponding spinors are designated by

$$\psi_e \text{ and } \psi_\nu,$$

then the lepton-charge conservation law is given by the equation

$$\partial_\alpha (\bar{\psi}\Gamma_\alpha\psi) = 0,$$

where

$$\psi = \frac{\psi_e}{\psi_\nu}, \quad \Gamma_\alpha \begin{vmatrix} \psi_\alpha & 0 \\ 0 & \gamma_\alpha \end{vmatrix}$$

and γ_α are the corresponding Dirac matrices.

For the considered case, the situation is made complicated by the fact that there is a frequently cited paper by Hartle[8a], where it is stated that the potential of the neutrino forces vanishes outside a black hole.

Within the framework of the approximations assumed in that paper, this is seemingly correct. If all of the matter of the black hole becomes neutronized during the course of gravitational closure, then the questions of interest to us no longer arise, for in this case the lepton charge is emitted.

In principle, however, situations are possible in which the density of matter is very small at a large mass.

This raises the question whether a metric (for example the metric of our universe) can be closed if the total lepton charge is not equal to zero[2].

Unfortunately, the field referred to here is not a solution of any equation of the Maxwell type, and the lepton conservation law is not connected here explicitly with any analog of the Gauss theorem. We can attempt, however, to construct formally a certain generalized Maxwell field for the case of electrostatic forces, with a dependence $1/r^5$ and higher. Such a generalization was obtained by Berezin and the author[29]. In such a consistent formalism, a Gauss theorem is automatically formulated for a certain tensor. The formalism includes static potentials of the type

$$V \sim \frac{1}{r^{2l+1}}. \tag{61}$$

Just as in the case of an electrostatic field, the generalized vector field does not vanish outside the black hole. The outer metric of such a collapsar is a generalization of the Nordstrom-Reissner metric. This example is instructive because the potential with such a high dependence on r does not exclude the corresponding Gauss theorem[8)].

One could assume that the absence of neutrino "hair" outside the black holes is connected in some manner with the rapid decrease of the potential with distance. An example with a suitably generalized Maxwell field, however, demonstrates that this situation here is more complicated.

Although the generalized Maxwell field referred to above is a vector field, like the neutrino-antineutrino field, the static potentials in both cases have one and the same power-law dependence; the analogy between these fields is so far limited to this fact only. The theorems proved for the generalized Maxwell field cannot be simply transferred to Hartle's case.

An attempt to arrive in Hartle's case, by analogy with the electrostatic field, at a contradiction (or an agreement) with a closed metric by a direct calculation of the values of the neutrino-antineutrino field as $\chi \to \pi$, seems to lead actually to singularities just as in the case of electrostatics (Berezin).

For a pointlike source of neutrino forces, localized at the point $\chi = 0$ of a closed world, a mirror image of the source is produced at $\chi = \pi$:

$$B(\chi) = -B(\pi - \chi).$$

In other words, it would seem that the corresponding neutrino "hair" should not disappear. A more thorough verification of this result is necessary. To be sure, if the lepton-charge conservation law is not violated in strong gravitational fields, then it seems that it is difficult to conceive of a "mechanism" with the aid of which it would be possible to locate the neutrino "hair" in the space of a closed world or under the Schwarzschild sphere. A neutrino vector field has in a certain sense properties close to those of a Maxwellian field, and requires unlike scalar fields but like the Maxwellian field, the inevitable existence of particles and antiparticles as its sources. Unlike the mesodynamic field, the neutrino field (as in electrostatics) cannot be made to vanish on a surface surrounding its source by using some boundary condition.

Yet this vanishing of the neutrino field inside the black hole as the horizon is approached from the interior of the black hole should be realized in some manner, if the neutrino field is cut off outside the black hole. In other words, in the case of a neutrino field, just as in the preceding cases of the scalar and vector meson fields, the final answer concerning the behavior of these fields outside black holes will also be given after joinable internal and external solutions of the considered systems are obtained.

As a result of such an analysis of these problems, which are not static in nature (collapse), the entire situation with the presence of horizons, "bare" singularities (if they exist), and the behavior of fields outside and (of necessity) inside black holes becomes clear.

Leaving open for the time being the question of the existence of scalar and baryon fields for black holes, we can state more definitely that there can exist a world with a closed metric and with sources of a neutral scalar field. There can exist a closed world with an unequal number of nucleons and antinucleons.

We must emphasize the circumstance that the closed world can exist in principle with very small dimensions (small $a_0 = a_{max}$) and can contain matter with very small masses M_0. But the necessary homogeneous density of matter at the instant of maximal expansion of the system should satisfy the relation

$$\mu_0 \sim \frac{c^6}{\varkappa^3 M_0^2}. \tag{62}$$

Thus, for a mass on the order of the solar mass ($M_\odot \sim 10^{33}$ g), the maximum dimension of the closed world is

$$a_0 \sim 1 \text{ km},$$

and the density is

$$\mu_0 \sim 10^{18} \text{ g/cm}^3.$$

The limit of applicability of classical (non-quantum) theory for the formation of systems with a closed metric lies in the mass region

$$M_0 \sim 10^{-5} \text{ g}$$

and in the closed-world dimension region

$$\frac{\hbar}{M_0 c} \sim a_0 \sim 10^{-33} \text{ cm}.$$

Were we able in our world to construct artificially such a system out of matter surrounding us, or could such systems arise spontaneously under some conditions, then unique situations would arise. These systems with closed metric would be characterized on the outside by a complete absence of any type of "hair." All the properties of matter would be "buried" in such systems without any external traces ("hair") and irrevocably. Since a closed system, as we have verified above, can in principle consist of neutrons only (i.e., without an equal number of antineutrons), it follows that the formation of such systems would denote the vanishing of some number of neutrons from our experiment. This would be a direct violation of the baryon-number conservation. In this situation, it would not be appropriate to use the term "transcended" introduced by Wheeler. This situation is fully equivalent to the term "violated." Following the text of the book by Zel'dovich and Novikov, it is impossible to apply to this example the picturesque comparison with a man "hiding around the corner." No such corner would remain in this case.[9)] However, as is well known, the formation of systems with closed metric from the matter surrounding us is impossible.

A black hole, the semi-closed system, and a closed world—all these objects can be described by one and the same linear element (2). A semi-closed system in a closed world comprises lower energy states of systems that consist, in principle, of one and the same number of say, neutrons. However, transitions of black holes into a state of a semi-closed system are nevertheless forbidden.

The point is that the surface of the event horizon (the Schwarzschild sphere, for example) is, as it were, a surface of a unilaterally transmitting membrane, which can only absorb matter, and after absorbing it, can only increase in size. A system that is formed under the gravitational radius cannot decrease its total mass and cannot radiate energy.

The transition of a black hole into the state of a system with a semi-closed metric is impossible[10)]. For the same reason, the transition of a semi-closed system into a closed system is also impossible. The impossibility of these transitions is apparently a unique manifestation of the law of baryon-charge conservation.

From the methodological point of view, it is very instructive, in the development of many of the concepts referred to above, to discuss the example of a collapse of very small masses, the possibility of which was illustrated by Zel'dovich[30]. In this example it is shown how it is possible to arrange a specified and furthermore arbitrarily small number of baryons (N) in such a way that the total mass, measured by an external observer, is arbitrarily small. Indeed, the total mass M for matter of density μ at rest is given by the expression

$$M = 4\pi \int_0^R \mu(r) r^2 dr, \qquad (63)$$

and the total number of particles N is given by the integral

$$N = 4\pi \int_0^R n(r) e^{\lambda/2} r^2 dr, \qquad (64)$$

where n(r) is the particle-number density and $e^{\lambda/2} = \sqrt{g_{11}}$.

If we choose for the distribution of μ the expression

$$\mu = \frac{a}{r^2}, \quad r < R, \text{ and } \mu = 0, \quad r > R,$$

then we have in the case of an ultrarelativistic gas

$$\mu = \frac{3}{4} \hbar (3\pi^2)^{1/3} \cdot \frac{1}{2} n^{4/3},$$

and the expression for the total mass is

$$M = \text{const}\, N^{2/3} a^{1/2} \sqrt[3]{1 - \frac{8\pi\varkappa}{c^2} a} \quad \text{at } r < R, \ \lambda = \text{const}. \qquad (65)$$

According to (65), the mass M tends to zero for any specified N if $a \to c^2/8\pi\kappa$.

Zel'dovich notes that it would be possible in principle to construct a machine capable, by realizing tremendous compressions, of bringing the system to the required configuration with an extremely large gravitational mass defect, such that an energy close to the total self-energy of the system is released. In nuclear reactions, approximately only 1% of the system mass is released. In this case we are interested not in Zel'dovich's fantastic machine itself, but only in an instructive occasion for discussing the limiting case of a system with zero total mass.

Assume that we have made up a system of this type of N neutrons, or that Zel'dovich has succeeded in constructing such a machine which, by realizing the case $a = c^2/8\pi\kappa$, brings the system to a state with zero total mass. If such a case were to be realized, then such a system would completely vanish from our experimental setup. The realization of such a machine would lead to a decrease in the number (i.e., to annihilation) of baryons in the universe. In this sense the result is perfectly equivalent to the formation of the closed systems which were discussed above.

It is appropriate to continue here, using this concrete example, the discussion of the generalized Gauss theorem for a massive vector field. The neutrons are sources, say, of ρ-meson forces which are completely neglected in Zel'dovich's formulas.

In the context of attempts to "organize" a collapse, say, of one gram of neutrons, then when the neutrons are localized in a region of dimensions much smaller than $\hbar/m_\rho c$, the numerical value of the second term in formula (60) ("of the field" or "charge") becomes negligibly small. When the localization region is still many orders of magnitude larger than the gravitational radius of the system ($r_{gr} \sim 10^{-28}$ cm), when the gravitational forces can still be neglected, the generalized Gauss theorem takes in practice the form

$$\int E_n \, dS = -4\pi g. \qquad (66)$$

In other words, a pure electrodynamic analog of the Gauss theorem is obtained, with all its characteristic properties, particularly with respect to the baryon "hair."

If the collapse of such a system were to be realized, then by virtue of the applicability of relation (66), the corresponding black hole would have baryon "hair" and consequently a nonzero mass. We have used here once more the opportunity to return to the illustration of the importance of the analog of the Gauss theorem in mesodynamics. As to the discussed machine, although the corresponding calculations within the framework of the idealization assumed by the author are perfectly correct, it must be noted that such a machine is in principle impossible when applied to real neutron matter. Indeed, if the system is compressed to dimensions so much smaller than $\hbar/m_\rho c$, the potential of the repulsion forces of this system of N neutrons reaches values

$$V \sim N \frac{g^2}{\hbar} m_n c \quad \text{for} \quad \frac{V}{c^2} \sim N \frac{g^2}{\hbar c} m_n,$$

where m_n is the neutron mass.

Since $g^2/hc \geq 1$, the energy or mass localized in this field at distances larger than $\alpha\hbar/m_n c$, $\alpha < 1$ turns out to be larger than the total rest mass of the neutrons making up the given system[11]. Further localization of the system of N neutrons (i.e., their packing into a smaller region) only increases the energy of the meson field of the given system on the outside. The total mass of the system of N neutrons, localized in the field when the machine operates, is always larger than Nm_n and never vanishes. From this point of view, the machine described by Zel'dovich, one that annihilates baryons, is impossible.

The Zel'dovich machine is impossible precisely by virtue of the "hair," the source of which is the conserved charge[12]. When discussing the work of the machine no account was taken also of the fact that the tremendous meson fields produced upon compression lead to the production of neutron-antineutron pairs. The antineutrons of the produced pairs will be attracted by this system, decreasing its baryon charge, and the produced neutrons will be removed from the system by the repulsion forces. When the system is fully neutralized upon further compression with respect to the baryon charge, the system can become in principle gravitationally closed with a zero mass.

In this case the system will actually not have any baryon "hair" (a meson field), but this circumstance is in agreement with the baryon-number conservation law. Unfortunately, in this case the machine does not give the expected energy release from matter, and owing to the work of the machine the same number of neutrons will remain in the surrounding space. Roughly speaking, all the neutrons will turn out to be "driven out" of the system.

It is perhaps appropriate to note at this point that owing to the vector meson repulsion forces, it appears that the collapse of a star cannot develop without limit.

The role of the short-range nuclear forces in the development, gravitational collapse has been discussed many times. Usually, however, the situation considered was one in which the dimension of the collapsing system (R) is much larger than the range of action of the nuclear forces ($\hbar/m_n c$).

In this case ($R \gg \hbar/m_n c$) one can introduce the concept of pressure, since the considered nuclear energy enters additively in the summation of the smaller volumes of which this system consists. A general thermodynamic analysis shows that during this stage of development of the collapse ($R \gg hm_n c$) the nuclear forces do not stop the gravitational compression of the system.

At $R < \hbar/m_n c$, the phenomenon lies outside the framework of the thermodynamic analysis, and the subsequent course of the collapse must be considered dynamically, just as in the case of the presence of electrostatic forces. As noted by Novikov[31], electrostatic forces, being long-range, are capable of stopping the gravitational collapse. When the system is localized in a region $R < h/m_n c$, the considered field become practically long-range and a complete analogy with electrostatic forces arises in the treatment of the possible stopping of the collapse[32]. The density of matter becomes in this case, of course, tremendous. Indeed, the critical mass with which a collapse of a star is possible, is $M \sim M_\odot \sim 10^{33}$ g.

In a sphere of radius $h/m_n c$, the density of such a mass is

$$\mu \sim \frac{10^{33}}{(\hbar/m_n c)^3} \text{ g/cm}^3 \sim 10^{74} \text{ g/cm}^3.$$

It is curious that this density of matter, at which it is assumed that the collapse of a star stops, is smaller by almost 20 orders of magnitude than the so-called critical (quantum) density

$$\mu_{cr} \sim 10^{93} \text{ g/cm}^3,$$

with which the possible stopping of collapse of a star is associated in certain hypotheses[13].

It appears that a real problem of unlimited development of the collapse of a star to its pointlike limit simply does not exist.

To be sure, the problem of expansion of the star after the stopping of the collapse arises in a co-moving coordinate system. This expansion, considered classically, cannot be an expansion in the same space[31].

We have discussed above the impossibility of formation of closed and semiclosed systems within the framework of classical physics. It is not excluded, however, that in quantum theory there are situations in which the analogs of such systems can be realized. We have in mind the possibilities, for example, of rare quantum fluctuations of systems with small numbers of nucleons. We can refer here, of course, to our ignorance of the laws in this region of physical phenomena. Apparently, however, fluctuation formation of closed systems would be violation of the law of baryon conservation. Fluctuation (spontaneous) appearance of microscopic semiclosed systems or of microscopic black holes does not contradict any conservation laws in principle, but at small dimensions of these systems only extremely small (single-charges (electric, baryon) are energywise profitable, i.e., systems such as Friedmann's[2]. Such small systems with large charges are unstable as a result of pair production and vacuum polarization in strong fields near practically pointlike field sources.

However, in modern elementary-particle theory there exists a situation where the discussion of the possible formation of microscopic black holes and perhaps also of microscopic semiclosed systems, can be significant. We have in mind the so-called intermediate states in modern quantum perturbation theory as applied to elementary particles.

The state of semiclosed systems or the states of black holes should seemingly be "listed" in the complete set of sets that can arise spontaneously in these cases. Moreover, these states are energy wise the lowest ones and this, as follows from the sequel, is essential.

Indeed, if a particle in the intermediate states emits a quantum of mass $m = \hbar\nu/c^2$, then this mass, according to the Heisenberg principle[14] is localized in a region

$$l \sim \frac{\hbar}{mc} \quad \text{and} \quad m \sim \frac{\hbar}{lc}. \tag{67}$$

A complete set of intermediate states includes states with arbitrarily high energies, and consequently masses. A surprising violation of logic has been historically legitimized in modern theory of elementary particles, namely, states have been introduced with arbitrarily large masses, and at the same time their gravitational effects have been completely neglected.

If the necessary mass of order

$$m \sim \sqrt{\frac{\hbar c}{\varkappa}},$$

appears in an intermediate state, then the gravitational radius of this mass is

$$r_{gr} = \frac{2\varkappa m}{c^2} = \frac{2\sqrt{\hbar c}\sqrt{\varkappa}}{c^2}. \qquad (68)$$

On the other hand, the size of the region where the mass is localized (according to (67))

$$l \sim \frac{\hbar}{mc} \sim \frac{\sqrt{\hbar c}\sqrt{\varkappa}}{c^2},$$

coincide for this mass with the gravitational radius of the object in this state. With further increase of the intermediate-state energy, the gravitational radius should accordingly increase.

On the other hand, however, the region of localization of the energy of the intermediate state, according to the Heisenberg relation, should correspondingly decrease, and at $m > \sqrt{\hbar c}/\sqrt{\kappa}$ it would become smaller than the gravitational radius. If such a situation were to arise in the region where classical physics is applicable, we would say that we are dealing with a system whose mass is under the Schwarzschild gravitational sphere. In other words, we would be dealing with a system in a collapsed state. This could be either a black-hole state, or more readily the state of a system with a semiclosed metric, if the "bare" mass of the intermediate state decreases strongly as a result of the gravitational defect. At the present time we do not know the extent to which the concepts concerning the metric remain in force in this state. We do know, however, that the region of localization of the mass decreases with increasing energy of the intermediate state, in accordance with the Heisenberg relation; consequently, owing to the large mass concentration the gravitational mass defect should increase, and should accordingly decrease the total mass of the intermediate state. It appears that only as a result of taking the gravitational mass defect into account is it impossible for the gravitational radius of the system to exceed the dimensions allowed by the Heisenberg relation, i.e., the discussed contradiction can be resolved in this manner.

If it were permissible to use here for the total mass of the intermediate-state estimates of the type given above by relation (40) then, according to this estimate, the total mass of the intermediate state would be obtained from the relation

$$M_{tot} = m_p + \frac{\hbar}{r_0 c} - \varkappa\frac{M_{tot}}{2r_0 c},$$

where m_p is the particle mass and $\hbar/r_0 c$ is the mass of the emitted energy quantum in the intermediate state. Hence

$$M_{tot} = -\frac{r_0 c^2}{\varkappa} + \sqrt{\frac{r_0^2 c^4}{\varkappa^2} + \frac{\hbar c}{\varkappa} + \frac{2r_0 c^2 m_p}{\varkappa}}.$$

As $r_0 \to 0$ we have

$$M_{tot} \to \sqrt{\frac{\hbar c}{\varkappa}},$$

where the last expression for the mass is the maximum of the possible values of the total masses (energies) of the intermediate states.

Of course, an adequate quantum description of collapsing systems can introduce significant corrections, but apparently only with respect to their space-time description. It can hardly change significantly the energy picture of these states, or more accurately, it hardly influences significantly such an effect as the gravitational mass defect localized in this region. Yet it is only this effect which was discussed above. If the advanced considerations actually turn out to be significant in elementary particle theory, this will be one of the rare cases where the discussion of properties of collapsing cosmic bodies initiates a discussion of fundamental problems in the theory of elementary particles.

In conclusion I consider it my duty to express gratitude to my co-workers R. A. Asanov, V. A. Berezin, and V. P. Frolov for numerous discussions as a result many factors became clear or, to the contrary, lost a previously seeming clarity.

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1) Reported at the Third All-Union Gravitational Conference, October 14, 1972 (Erevan).

2) It is assumed that the so-called Λ term in Einstein's equations is equal to zero.

3) When the star surface approaches the Schwarzschild surface (the "event horizon") during the course of the gravitational collapse a discarding of free electrons are also discarded, and their number reaches a minimum value n: $\kappa Mme/r_{gr} \sim ne^2/r_{gr}$, i.e., for a critical mass $M \sim M_\odot$: we have $n_{max} \sim \kappa M_\odot m_e/e^2 \sim 10^{18}$. In other words, there remains one electron for 10^{16} g ~ 10^{10} tons of matter. The density of matter in this case is $\mu \sim 10^{18}$ g/cm^3.

4) If our universe were to have a density $\mu \sim 10^{-29}$ g/cm^3, which in the case of electric neutrality of matter could lead to a closed metric, then the presence of but one single extra electron in the universe would make the universe open with a throat of radius $r_{min} \sim e\sqrt{\kappa}/c^2 \sim 10^{-33}$ cm, and the total mass as seen by an external observer would be $M_{tot} \sim e\sqrt{\kappa} \sim 10^{-6}$ g. A system that turns into a system with a closed Friedmann metric when the electric charge tends to zero, was called an electrostatic "fridmon" [2], and an external metric of the type (11) with Φ in the form (12) was called a fridmon metric. It should be noted, however, that an external metric of this type has also another internal continuation, which describes the Papapetrou model with the same ratio $M = \epsilon/\sqrt{\kappa}$. But in this case the external solution describes a static system, in which the gravitational and electrostatic forces balance each other. The dimensions of the material system must then be larger than its gravitational radius [15]. The fridmon metric is the limiting case of a metric of a semiclosed world ($M_{tot} > \epsilon/\sqrt{\kappa}$) as $M_{tot} \to \epsilon/\sqrt{\kappa}$.

5) As noted by Asanov [21], the gravitational constant was omitted from the symbol $r_0 = 2m$ used in the formulas of [20]. This raised difficulties in the interpretation of the asymptotic form of the metric as $\kappa \to 0$. Indeed, Fisher has $1/p = [1 + (G^2/\kappa m^2)]^{1/2} \to \infty$, $p \to 0$. In [20] they have $1/p = [1 + (\kappa G^2/r_0^2)]^{1/2} = [1 + (\kappa G^2/m^2)]^{1/2} \underset{x\to 0}{\to} 1$. In the former case the metric (29) becomes Euclidean as $\kappa \to 0$.

6) In a recent paper, Asanov [23] constructed a model with sources of a scalar and electrostatic field for a Schwarzschild mass m, requiring Euclidean behavior at the pint r = 0. A numerical solution shows that there is no event horizon, in any case down to $r \sim 0.9\ \kappa m$, where $e^\lambda > 1$ and where there is no advantage in joining the internal and external solutions.

7) In the equation $\nabla_\sigma \nabla\sigma U = -4\kappa j$, both U and the charge density are invariant, but the total scalar charge $G = \int j\, dv$ is not invariant and transforms like a volume: $G = [1 - (v^2/c^2)]^{1/2} G_0$.

8) The classical equation of motion for a given charge is written, as was done by Hartle [8a]

$$m_0 \frac{du^\mu}{dt} = \frac{G}{c^2\sqrt{2}} u^\nu (\partial^\mu B_\nu - \partial_\nu B^\mu), \quad F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu,$$

where B^ν is the vector potential and u^μ is the velocity vect . Like Hartle, we choose an interaction Lagrangian

$$\mathcal{L}_w = -\frac{1}{c^2} j_\mu B^\mu, \ j_\mu \text{ is the current}$$

But the interaction Lagrangian of the free field is now written in the form

$$\mathscr{L}_f = -\alpha\,(-F_{\mu\nu}F^{\mu\nu})^k, \quad \alpha \text{ is a constant.}$$

and the generalized Maxwell equation takes the form

$$\frac{\partial}{\partial x^i}\left[(-F_{\mu\nu}F^{\mu\nu})^{k-1}F^{i\nu} = \frac{1}{4\alpha k c^2}\,j^\nu.\right.$$

The Gauss theorem is satisfied for the tensor

$$D^{i\nu} = (-F_{\mu\nu}F^{\mu\nu})^{k-1}F^{i\nu}.$$

At $k = 1$, the formalism leads to Maxwell's theory.

9) If any analogies in human terms are possible here at all, we are dealing more readily with Lieutenant Kije, who, as is well known, "has no figure."

10) A different situation arises in the case of anticollapse, when radiation from the system is possible.

11) The gravitational mass defect during this stage of compression (distances $\hbar/m_n c \sim 10^{-14}$ cm) is still negligibly small.

12) It must be emphasized that we are dealing with systems that are far from microscopic in their initial state, albeit far from the critical masses of celestial bodies. A gravitational radius $\hbar m_n c$ is possessed by a mass $M_0 \sim \hbar/m_n c)(c^2/2\kappa) \sim 10^{14}$ g $\sim 10^8$ tons.

13) These considerations consist in the fact that at such densities the non-quantum-mechanical approach is no longer valid, and there remains in principle the hope that other laws will prevent further development of the collapse at these densities.

The density $\mu \sim 10^{93}$ g/cm^3 is reached for masses 10^{20} times larger than $M_\odot$, i.e., for $M \sim 10^{53}$ g. It may be accidental that this mass coincides approximately with the mass of our universe.

At such compressions it is necessary to take into account the space-time picture of pair production, the intermixing of charge particles, and the motion of the pair components towards the periphery. It is not excluded that this circumstance can essentially alter the entire situation.

14) When a quantum of energy $E = \hbar\nu$ is emitted, the particle using Fermi's words [33], "borrows" an energy $E = mc^2$. According to the uncertain relation, the "borrowing" time cannot be longer than $\hbar/mc^2$. During that time, the emitted energy quantum cannot move farther than $\sim \hbar/mc$ away from the particle.

[1] A. Wheeler, Cortona Symposium on Weak Interactions, Rome, 1971.

[2] M. A. Markov, Ann. Phys. (N.Y.) **59**, 109 (1970); M. A. Markov, Cosmology and Elementary Particles (Lecture Notes), Trieste, IC/71/33, pt. I–II.

[3] V. L. Ginzburg, Dokl. Akad. Nauk SSSR **153**, **43** (1964).

[4] V. L. Ginzburg and L. M. Ozernoi, Zh. Eksp. Teor. Fiz. **47**, 1030 (1964) [Sov. Phys.-JETP **20**, 689 (1965)].

[5] A. G. Doroshkevich, Ya. B. Zel'dovich, and I. D. Novikov, ibid. **49**, 170 (1965) [**22**, **122** (1966)].

[6] B. Price, Phys. Rev. **D5**, 2419 (1972); K. Torne, Preprint OAP-236 (1970).

[7] J. Bekenstein, Phys. Rev. Lett. **28**, 452 (1972); C. Teitelboim, Lett. Nuovo Cimento **3**, 326, 397 (1972).

[8] J. B. Hartle, a) Preprint, 1971; b) Phys. Rev. **D3**, 2938 (1971).

[9] R. Ruffini and A. Wheeler, Phys. Today **24** (1), 36 (1971).

[10] Ya. B. Zel'dovich, and I. D. Novikov, Teoriya tyagoteniya i évolyutsiya zvezd (Gravitation Theory and Stellar Evolution), Nauka, 1971.

[11] L. D. Landau and E. M. Lifshitz, Teoriya Polya (Field Theory (Nauka, 1967) [Pergamon, 1971].

[12] F. Hoyle, W. Frowler, P. Burbridge and E. M. Burbridge, Quasistellar Sources and Gravitational Collapse, Chicago—London, Univ. of Chicago Press, 1965.

[13] V. P. Frolov, Thesis, Moscow State Univ. 1969.

[14] M. A. Markov and V. P. Frolov, Teor. Mat. Fiz. **3**, 3 (1970).

[15] M. A. Markov and V. P. Frolov, ibid. **16**, 70 (1973).

[16] R. A. Asanov, ibid. **2**, 280 (1970).

[17] R. Penrose, translation in: Gravitatsiya i topologiya (Gravitation and Topology), Mir, 1966.

[18] N. A. Chernikov and E. A. Tagirov, Ann. Inst. H. Poincaré **A9**, 109 (1968).

[19] I. Z. Fisher, Zh. Eksp. Teor. Fiz. **18**, 636 (1948).

[20] A. I. Janis, E. T. Newman and J. Winicour, Phys. Rev. Lett. **20**, 878 (1968); Phys. Rev. **186**, 1729 (1969), A. I. Janis, Q. Q. Robinson and J. Winicour, ibid. **176**, 1507 (1968).

[21] R. A. Asanov, Zh. Eksp. Teor. Fiz. **53**, 673 (1967) [Sov. Phys.-JETP **26**, **424** (1968)].

[22] R. Arnowitt, S. Deser and C. Misner, Phys. Rev. **120**, 313 (1960).

[23] R. A. Asanov, JINR Preprint R2-6564, Dubna, 1972.

[24] N. M. Bocharova, K. A. Bronnikov, and V. M. Mel'nikov, Vestn. MGU, ser. III (Fiz., Astron.), **11**, 706 (1971).

[25] J. E. Chase, Comm. Math. Phys. **19**, 276 (1970).

[26] R. H. Dicke, in: Gravitation and Relativity, ed. by Chiu and Hoffman, Benjamin, 1964.

[27] A. D. Sakharov, Zh. Eksp. Teor. Fiz. **27**, 686 (1947).

[28] S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967).

[29] V. A. Berezin and M. A. Markov, Teor. Mat. Fiz. **12**, 153 (1972).

[30] Ya. B. Zel'dovich, Zh. Eksp. Teor. Fiz. **42**, 641 (1962) [Sov. Phys.-JETP **15**, 446 (1962)].

[31] I. D. Novikov, Astron. zh. **43**, 911 (1968) [Sov. Astron.-AJ **10**, 231(1969)].

[32] V. A. Berezin and M. A. Markov, Teor. Mat. Fiz. **3**, 161 (1970).

[33] E. Fermi, Elementary Particles (Russ. transl.), IL, 1953, p. 53.

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