

Kinetic equations for nonideal gas and nonideal plasma

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Recent experimental studies of kinetic processes in dense gases and in plasma have stimulated the development of the theory of nonideal gases or plasmas. By now, a kinetic theory has been developed for nonideal monatomic gases of medium density, as well as a kinetic theory of weakly nonideal fully ionized plasma. The present review is devoted to a systematic exposition of these questions. The kinetic theory of nonideal gases of low density is constructed on the basis of a generalized Boltzmann kinetic equation (in the pair-collision approximation) and the Cho-Uhlenbeck kinetic equation (in the triple-collision approximation). The kinetic equations for denser gases have an essentially different structure than the ordinary kinetic equations. This is caused by the impossibility of using the ordinary scheme, owing to the divergences of the collision integrals in the higher approximations in the density. The article presents a kinetic theory of a weakly nonideal plasma. Particular attention is paid to the possibility of constructing classical and quantum kinetic equations without assuming the particle interaction to be small at short distances, since the particle interaction at short distances plays an essential role in a nonideal plasma. A prominent place is occupied in the review by an exposition of the kinetic theory of fluctuations in nonideal gases and plasma. Certain problems in the kinetic theory of nonideal systems are considered.

CONTENTS

1. Introduction	512
2. Boltzmann Kinetic Equation for a Nonideal Gas in the Pair-Collision Approximation	517
3. The Boltzmann H-Theorem for a Nonideal Gas	519
4. Kinetic Equation for a Nonideal Gas with Allowance for Triple Collisions	520
5. Kinetic Equations for a Nonideal Plasma	520
6. Boltzmann's Equation for a Nonideal Plasma with Allowance for the Averaged Dynamic Polarization	521
7. Quantum Kinetic Equations for a Nondegenerate Nonideal Gas	521
8. Quantum Kinetic Equation for a Nonideal Plasma.	522
9. Kinetic Equations for Dense Gases	523
10. Kinetic Theory of Fluctuations in a Gas and a Plasma	524
11. Generalized Kinetic Equation	526
12. Conclusion	526
References	527

1. INTRODUCTION

We describe in this article the principles of modern kinetic theory of monatomic gases and of a fully ionized plasma.

The theory of non-equilibrium processes in gases is based on the Boltzmann kinetic equation, which is a closed equation for the distribution function $f_1(\mathbf{r}, \mathbf{p}, t)$ of one particle^[1-8]. The kinetic equation for a gas was derived by Boltzmann a hundred years ago on the basis of simple physical assumptions concerning the character of the interaction of particles in a low-density gas.

Boltzmann's equation is approximate. The most general statistical description of processes in a gas can be obtained on the basis of the Liouville equation, which is an equation for the distribution function of the coordinates and momenta of all the particles of the gas,

i.e., for the function $f_N(\mathbf{r}_1, \dots, \mathbf{r}_N, \mathbf{p}_1, \dots, \mathbf{p}_N, t)$. Solving the Liouville equation is equivalent to solving the system of equations of motion of all the particles of the gas, and therefore cannot be accomplished in the general case. There is, however, no practical need for this solution, since processes in a gas can usually be adequately described by knowing the distribution function of the variables of one or two particles. That is to say, the first (single-particle) distribution function $f_1(\mathbf{r}, \mathbf{p}, t)$ and the second (two-particle) $f_2(\mathbf{r}_1, \mathbf{p}_1, \mathbf{r}_2, \mathbf{p}_2, t)$.

The equation for the function f_1 can be obtained from the Liouville equation by integrating with respect to the variables of all the particles except one. The equation for the function f_1 , however, is no longer closed here, since it contains the function f_2 . An equation obtained in this fashion for the function f_2 contains the three-

particle distribution function f_3 , etc. The result is a chain of coupled equations for the distribution functions f_1, f_2, f_3, \dots , which is equivalent to the initial Liouville equation. It is called the system of Bogolyubov or BBGKY system of equations (Bogolyubov-Born-Green-Kirkwood-Yvon).

Owing to the extreme complexity of the chain of equations for the distribution function, it is natural to attempt to obtain approximate closed equations for the simplest distribution functions. An example of such an equation is indeed the Boltzmann kinetic equation for a gas.

The transition from the Liouville equation to the much simpler kinetic equation means, of course, a transition to a rougher description of the processes in the gases. This is possible because in a low-density gas the effective radius r_0 of the intermolecular forces is much smaller than the average distance $r_{av} \sim n^{-1/3}$ between the molecules (n is the concentration of the atoms), and r_{av} is in turn much smaller than the mean free path $l \sim 1/nr_0^2$, i.e., we have the inequalities

$$r_0 \ll r_{av} \ll l. \quad (1.1)$$

As a result we can introduce a small parameter, the density parameter

$$\epsilon = nr_0^3 \sim (r_0/r_{av})^3 \sim r_0/l. \quad (1.2)$$

The first approximation in ϵ corresponds to allowance for only double collisions, the second corresponds to triple collisions, etc.

The Boltzmann kinetic equation for a monatomic gas corresponds to the first approximation in ϵ , i.e., to the first-collision approximation. However, as will be shown below, the interaction of particles is incompletely taken into account in the Boltzmann equation. It is taken into account only in the dissipative terms that determine, for example, the establishment of the equilibrium state in a rarefied gas. The explicit contribution of the particle interaction is not taken into account in the nondissipative characteristics, for example in the expressions for the internal energy, the pressure, or the entropy. Therefore these expressions coincide at equilibrium with the corresponding expressions for an ideal gas. In this sense the Boltzmann equation is a kinetic equation for an ideal gas.

This raises the problem of how to modify the kinetic equation within the framework of a given approximation (the pair-collision approximation) so that it can take full account of effects due to the nonideal character of the gas, i.e., effects due to particle interaction.

Enskog (see^[2]) generalized Boltzmann's collision integral for the hard-sphere model, with an aim at taking into account the finite dimensions of the gas atoms. This generalization makes it possible to take into account in the equation of state of the gas the deviation from the equation of state of an ideal gas due to the finite volume of the atoms.

Bogolyubov^[4] obtained a corresponding generalization of the expression for collision integral at an arbitrary character of the intermolecular repulsion forces.

The generalizations of Boltzmann's kinetic equation by Enskog and Bogolyubov are still insufficient for a complete allowance for the interaction of the particles within the framework of the pair-collision approximation. To take complete account of the non-ideal charac-

ter of the gas in the Boltzmann kinetic equations it is necessary to allow in the collision integral not only for the spatial inhomogeneity of the distribution functions at distances on the order of r_0 , but also the temporal delay of the distribution function by times on the order of r_0/v_T (v_T is the average thermal velocity of the gas particles).

For the classical case, within the framework of the pair-collision model, the interaction in the Boltzmann kinetic equation was completely taken into account in^[9].

The first attempt to take into account the contribution of the interaction to nondissipative characteristics in the quantum kinetic equation was made by Green^[10]. A more consistent derivation of the Boltzmann quantum kinetic equation for a nondegenerate nonideal gas in the pair-collision approximation was obtained by Kadanoff and Baym^[11], Grossmann and Baerwinkel^[12,13], and Klimontovich and Ebeling^[14].

Thus, within the framework of the pair-collision approximation (the first approximation in the density parameter ϵ), we can obtain for a gas a kinetic equation that takes into account the contribution of the interaction both to the dissipative and to the non-dissipative characteristics of the gas.

Is it possible to progress farther and to obtain a kinetic equation for a denser gas, when triple and more complicated collision processes must be taken into account, i.e., when higher approximations in powers of the parameter ϵ are necessary? To answer this question it is necessary to consider the principal ideas and premises used in the derivation of the kinetic equation for a gas.

We write down the first equation of the Bogolyubov chain of equations for the distribution functions f_1, f_2, \dots

$$\left(\frac{\partial}{\partial t} + v_1 \frac{\partial}{\partial r_1} + F \frac{\partial}{\partial p_1} \right) f_1 = n \int \frac{\partial \Phi_{12}}{\partial r_1} \frac{\partial g_2}{\partial p_1} dr_2 dp_2 \equiv J. \quad (1.3)$$

Here Φ_{12} is the potential energy of the interaction of two particles, $g_2 = f_2 - f_1 f_1$ is the correlation function, $n = N/V$ is the average particle concentration, and $F = -n \int (\partial / \partial r_1) \Phi_{12} f_1(2) dx_2$ is the average force. The normalization conditions for f_1 and f_2 are

$$\frac{1}{V} \int f_1 dx_1 = 1, \quad \frac{1}{V^2} \int f_2 dx_1 dx_2 = 1, \quad x = (r, p).$$

The right-hand side of this equation determines the contribution of the interaction of the gas particles. We denote it by J and call it the collision integral.

The collision integral is determined by the second distribution function f_2 . We write down the equation for this function (the second equation of the chain)

$$\begin{aligned} \left(\frac{\partial}{\partial t} + v_1 \frac{\partial}{\partial r_1} + v_2 \frac{\partial}{\partial r_2} - \frac{\partial \Phi_{12}}{\partial r_1} \frac{\partial}{\partial p_1} - \frac{\partial \Phi_{12}}{\partial r_2} \frac{\partial}{\partial p_2} \right) f_2 = \\ = n \int \left(\frac{\partial \Phi_{13}}{\partial r_1} \frac{\partial}{\partial p_1} + \frac{\partial \Phi_{23}}{\partial r_2} \frac{\partial}{\partial p_2} \right) f_3 dx_3 \equiv J_2. \end{aligned} \quad (1.4)$$

The right-hand side of this equation (which we denote by J_2) determines the contribution of the interactions of two particles (with indices 1 and 2) with one of the surrounding particles (index 3). It is determined by the distribution function f_3 . The equation for the function f_3 contains the functions f_4 etc. The distribution function f_3 in Eq. (1.4) can be represented in the form

$$f_3 = f_3/1 + f_1(1)g_2(2, 3) + f_1(2)g_2(1, 3) + f_1(3)g_2(1, 2) + g_3(1, 2, 3). \quad (1.5)$$

Here g_2 and g_3 are the correlation functions of the two and three particles, respectively.

In the pair-collision approximation $g_3 = 0$, since the function g_3 differs from zero when the three particles come closer together.

Let us examine the result of substitution of the remaining terms from (1.5) into the right-hand side of (1.4). The terms with $\Phi_{13}g_2$ (2.3), $\Phi_{23}g_2$ (1.3) can also be neglected, since they differ from zero when three particles approach simultaneously. Using the definition of the average force $F(r_1, t)$ and Eq. (1.3), we obtain an equation for the function f_2 in the pair-collision approximation

$$\left(\frac{\partial}{\partial t} + v_1 \frac{\partial}{\partial r_1} + v_2 \frac{\partial}{\partial r_2} - \frac{\partial \Phi_{12}}{\partial r_1} \frac{\partial}{\partial p_1} - \frac{\partial \Phi_{12}}{\partial r_2} \frac{\partial}{\partial p_2} + F \frac{\partial}{\partial p_1} + F \frac{\partial}{\partial p_2} \right) f_2 = \left(\frac{\partial}{\partial t} + v_1 \frac{\partial}{\partial r_1} + v_2 \frac{\partial}{\partial r_2} + F \frac{\partial}{\partial p_1} + F \frac{\partial}{\partial p_2} \right) f_1 f_1. \quad (1.6)$$

Thus, in the pair-collision approximation, the first two equations of the chain (1.3) and (1.6) become closed equations for the distribution functions f_1 and f_2 .

Let us consider the additional assumptions under which it is possible to obtain from the system (1.3) and (1.6) a closed equation (the Boltzmann kinetic equation) for f_1 .

We denote by $\tau_0 = r_0/V_T$ the time of interaction of a particle pair, and by $\tau_{col} = l/V_T$ the free path time (the relaxation time of the function f_1). It follows from the inequality (1.1) that $\tau_0 \ll \tau_{col}$, and from (1.2) that $\tau_0/\tau_{col} \sim \epsilon$.

The solution of Eq. (1.6) with the functions f_1 given can be represented as a sum of two parts, the solutions of the homogeneous and inhomogeneous equations. The first part of the solution is determined by the distribution function f_2 at the initial instant t_0 . Since $f_2 = f_1 f_1 + g_2$, this solution depends on the initial particle correlation.

The first additional limitation encountered when deriving the Boltzmann kinetic equation from the system (1.4), (1.6) is the use of condition of complete weakening of the initial correlation, introduced by Bogolyubov^[4].

Realistically, in the derivation of the kinetic equation one can neglect only those correlations for which the characteristic times are $\tau_{cor} \ll \tau_{col}$. By the same token, the condition of total weakening of the initial correlations corresponds to the assumption that the long-lived correlations (with $\tau_{cor} \geq \tau_{col}$) play no essential role in the kinetic theory.

When this condition is used, the solution of the homogeneous equation (1.6) is determined only by the initial values of the functions f_1 at the instant $t_0 = t - \tau$ ($\tau = t - t_0$).

Thus, under condition of complete weakening of the correlations, the function $f_2(x_1, x_1 t)$ is determined by the values of the functions f_1 at earlier instants of time $t - \tau$, i.e., a temporal delay takes place.

The second limitation consists of complete neglect of the temporal delay, i.e., $f_1(t - \tau) \rightarrow f_1(t)$. In the same approximation, we can neglect the right-hand side of (1.6) and the spatial variation of the functions f_1 over distances on the order of r_0 .

As a result, the solution of (1.6) takes the form

$$f_2(x_1, x_2, t) = f_1(r_1, P_1(-\infty), t) f_1(r_1, P_2(-\infty), t) \equiv S_{-\infty}^2(1, 2) f_1 f_1. \quad (1.7)$$

$P_{1,2}(-\infty) = \lim_{\tau \rightarrow \infty} P_{1,2}(-\tau)$ are the initial momenta of the two particles "1" and "2," which collide at the instant of time t , and $S_{-\tau}^2(1, 2)$ is the operator of the shift of the variables x_1 and x_2 by a time interval $-\tau$. The transition to the limit as $\tau \rightarrow \infty$ is possible by virtue of the inequality $\tau_0 \ll \tau_{col}$.

To find an expression for the collision integral J it is necessary to substitute expression (1.7) in the right-hand side of (1.3). Thus,

$$J(x_1, t) = n \int \frac{\partial \Phi_{12}}{\partial r_1} f_1(r_1, P_1(-\infty), t) f_1(r_1, P_2(-\infty), t) dx_2. \quad (1.8)$$

Bogolyubov has shown that this expression differs only in the notation from the expression for the collision integral in Boltzmann's equation (see^[4-8]).

A similar conclusion of the Boltzmann kinetic equation was given by Born and Green^[15]. The analysis of different derivations of the Boltzmann kinetic equation is the subject of^[16] (see also^[6]).

Let us recall the properties of the collision integral in the Boltzmann equation. We multiply (1.8) by the function $\varphi(p_1)$ and integrate with respect to p_1 . Then

$$\int \varphi(p_1) J dp_1 = 0 \quad \text{at } \varphi = 1, p_1, p_1^2/2m. \quad (1.9)$$

The first equation (at $\varphi = 1$) ensures satisfaction of the particle-number conservation law, the second the momentum conservation law, and the third the conservation of the kinetic energy of the gas particles.

If we choose φ in the form $\varphi = -\kappa \ln f_1$ (κ is Boltzmann's constant), then we get $-\kappa \int \ln f_1 \cdot J dp_1 \geq 0$!

On this basis, Boltzmann proved the so-called H-theorem, from which follows the law that the total entropy of the gas increases as the state of the gas approaches equilibrium. According to Boltzmann, the entropy density of the entire system is given by the expression

$$S_B(t) = -\kappa n \int \ln f_1 \cdot f_1 \frac{dx_1}{V} \quad (1.10)$$

and according to the H theorem we have $dS/dt \geq 0$, i.e., the total entropy of the gas can either increase or remain constant. The latter takes place after the equilibrium state is reached.

Let us examine the degree to which the assumptions made in the derivation of Boltzmann's equation are fundamental, and whether more general kinetic equations for a gas can be obtained.

Within the framework of the pair-collision model, one can construct a kinetic equation only under one condition of the weakening of the initial correlation, i.e., with allowance for the change in the distribution function within the time τ_0 and over the distance r_0 . Since the relations for the parameters of a gas are $\tau_0/\tau_{col} \sim r_0/l \sim \epsilon$, it suffices to take into account in the pair-collision approximation only the first-order terms in τ_0/τ_{col} and r_0/l (see Chap. 2).

The equation obtained in this manner takes into account the contribution of the interaction to both the dissipative and nondissipative characteristics of the gas, so that it can be called the Boltzmann kinetic equation for a nonideal gas (in the pair-collision approximation).

When the nonideality effects are taken into account, a change takes place, in particular, in the expression for the entropy (the Boltzmann H-function)^[21]. A term that takes into account the correlation of the pairs of interacting particles in the gas is added to the Boltzmann expression (1.10) (see Chap. 3).

Expression (1.7) for the second distribution function can be regarded as the zeroth term of the expansion of the exact distribution function with respect to density (with respect to the parameter ϵ).

Bogolyubov^[4] has developed a method that makes it possible, using the condition of weakening of the initial correlation of any number of interacting particles, to obtain in succession the terms of the expansion of the distribution function in terms of the density.

Expression (1.7) determines the zeroth approximation in the density. The next term of the expansion (the first approximation in the density) contains contributions of two types. It determines the influence of triple collisions on the dissipative characteristics of the gas. The second contribution is of a different nature. It includes terms that determine the influence of pair collisions on the non-dissipative characteristics of the gas, i.e., for example, the corrections to the thermodynamic functions due to the interaction of the particle pairs. This is equivalent (see Chap. 2) to allowance for terms of first order in τ_0/τ_{col} and r_0/l in the thermodynamic functions.

Using for the function f_2 the expression that takes into account terms linear in the density, we can, naturally, obtain also a more accurate expression for the collision integral in the kinetic equation. This was done by Cho and Uhlenbeck (see^[5]).

It is possible to construct a kinetic equation for a nonideal gas in the triple-collision approximation. This was done in^[22] (see Chap. 4).

If we take into account the terms of the next-higher approximation in the density in the expression for f_2 , then we can obtain an even more general kinetic equation for the gas. This equation takes additional account of the contribution of the quadrupole collisions to the dissipative characteristics of the gas and of the triple collisions to the nondissipative characteristics.

It appears thus that there exists a method of constructing kinetic equations for a gas in any approximation in the density. In each approximation in the density, only one assumption is made concerning the complete weakening of the initial correlation over times much shorter than τ_{col} . However, fundamental difficulties are encountered when it comes to realization of this program.

Information carried out independently by Weinstock^[17], Goldman and Frieman^[18], and Dorfman and Cohen^[19] (see the reviews by Cohen^[20]), have shown that the contribution made to the collisions integrals by the first approximation in the density contain divergent integrals (with respect to time).

It is interesting that the character of the divergences turns out to be different in the two-dimensional and three-dimensional models of the gas.

In the two-dimensional model, even the first-order correction in the density (allowance for the triple collisions) leads in the corresponding collision integral to

a logarithmic divergence ($\ln t/\tau_0$). Terms of order ϵ^n ($n \geq 2$) lead to a divergence of the type $(t/\tau_0)^{n-1}$.

In the three-dimensional case in the first approximation in ϵ (triple collisions), there is no divergence. The divergence (proportional to $\ln t/\tau_0$) results when quadrupole collisions are taken into account. Terms of order ϵ^n in f_2 ($n \geq 3$) lead to divergences of the type $(t/\tau_0)^{n-2}$.

This shows that the construction of the kinetic equations for dense gases by direct utilization of the method of successive approximation in powers of the density is impossible. There are several papers (see the review^[20]) in which it is shown, by means of particular examples, that the logarithmic divergence can be eliminated by taking into account the contribution of the most divergent diagrams that are produced when the expansion in terms of the density is used. This leads to a cutoff of the region of integration with respect to t at $t \sim \tau_{col}$. As a result, the collision integral turns out to be proportional to the quantity $\ln(\tau_{col}/\tau_0) \sim \ln(1/nr_0^3)$, which has a non-analytic dependence on the density. As a result, a virial expansion of the kinetic coefficients (in powers of the density) is impossible.

The problem of constructing kinetic equations for dense gases can be solved in another manner without summing divergent diagrams in the expansion in the density. To this end, however, it is necessary to dispense with the condition of complete weakening of the initial correlations over times much shorter than τ_{col} (Chap. 9).

Attention to the fact that the condition of total weakening of the initial correlations is approximate was called relatively long ago (see, e.g.,^[23]). All that is weakened is the influence of the small-scale fluctuations, for which, in the pair-collision approximation, the correlation length l_{cor} and the correlation time τ_{cor} are such that

$$\tau_{cor} \ll l, \quad \tau_{cor} \ll \tau_{col}. \quad (1.11)$$

In the general case there is only partial weakening of the correlations. The large-scale fluctuations do not attenuate sufficiently rapidly and should consequently be taken into account when a kinetic theory is constructed. Their role becomes manifest, in particular, in the fact that the distribution function f_1 , for which the kinetic equation is written, is not strictly determined (see Chap. 9).

The arbitrary boundary separating the regions of fast and slow fluctuations can be defined in the following manner^[24].

We consider the pair-correlation approximation. We denote by l_{ph}^3 and τ_{ph} the volume and the time interval which can be regarded as physically infinitesimally small for the Boltzmann equation. Here $\tau_{ph} = l_{ph}/V_T$.

The quantity l_{ph} is determined from the condition of the continuity of the process of pair collisions, namely, the interval between successive collisions of any of the particles in a physically infinitesimally small volume is equal to τ_{ph} , i.e.,

$$\frac{\tau_{col}}{nl_{ph}^3} = \tau_{ph} = \frac{l_{ph}}{V_T}. \quad (1.12)$$

Hence

$$l_{ph} = \sqrt{\epsilon} l \ll l, \quad \tau_{ph} = \sqrt{\epsilon} \tau_{col} \ll \tau_{col}, \quad \frac{l_{ph}^3}{V_T} \sim \frac{1}{\sqrt{\epsilon}} \gg 1. \quad (1.13)$$

We shall define as small-scale those fluctuations for which $\tau_{\text{COR}} \leq \tau_{\text{ph}}$ and $\tau_{\text{COR}} \lesssim l_{\text{ph}}$. It follows from (1.13) that they attenuate within the time on the order of $\sqrt{\epsilon} \tau_{\text{COL}} \ll \tau_{\text{COL}}$.

We use the following reasoning for collisions of three and four particles: We denote the corresponding relaxation times by $\tau_{\text{COL}}^{(3)}$ and $\tau_{\text{COL}}^{(4)}$. They are connected with τ_{COL} in the following manner: $\tau_{\text{COL}}^{(3)} \sim \tau_{\text{COL}}/\epsilon$ and $\tau_{\text{COL}}^{(4)} \sim \tau_{\text{COL}}/\epsilon^2$. We substitute the quantities $\tau_{\text{COL}}^{(3)}$ and $\tau_{\text{COL}}^{(4)}$ in place of τ_{COL} in (1.12). As a result we obtain $\tau_{\text{ph}}^{(3)} \sim \epsilon^{1/4} \tau_{\text{COL}} \ll \tau_{\text{COL}}$ and $l_{\text{ph}}^{(3)} \sim (\epsilon)^{1/4} l \ll l$, but $\tau_{\text{ph}}^{(4)} \sim \tau_{\text{COL}}$ and $l_{\text{ph}}^{(4)} \sim l$, so that when quadrupole collisions are taken into account the small-scale correlations are those for which $\tau_{\text{COR}} \lesssim \tau_{\text{COL}}$ and $r_{\text{COR}} \lesssim l$. It follows therefore that a correct construction of the integral for quadrupole collisions is impossible without taking into account the correlations with $\tau_{\text{COR}} \sim \tau_{\text{COL}}$ and $r_{\text{COR}} \sim l$. Consequently, the collision integral depends on τ_{COL} (or on l). This gives rise to the additional dependence on the density.

We note that for the two-dimensional case, we have $\tau_{\text{ph}} \sim \tau_{\text{COL}}$ and $l_{\text{ph}} \sim l$ even for triple collisions (in (1.12) we have $\tau_{\text{COL}}/n_{\text{ph}}^2 = \tau_{\text{ph}} = l_{\text{ph}}/V_{\text{T}}$). In this case, therefore, the triple-collision integral cannot be constructed without taking into account the correlation with $\tau_{\text{COR}} \sim \tau_{\text{COL}}$ and $l_{\text{COR}} \sim l$. This explains why expansion in terms of the density begins earlier in the two-dimensional case than in the three-dimensional case.

Thus, the position of the boundary that separates the regions of the fast and slow fluctuations depends on the considered approximation.

It follows from the foregoing that when the kinetic equation is derived for dense gases, only a partial weakening of the correlations that play an important role in the construction of the kinetic equation takes place within times $\tau \ll \tau_{\text{COL}}$.

It is shown in^[24] that when the condition for partial weakening of pair correlations is used, the Boltzmann kinetic equation does not follow from the Liouville equation. The simplification of the initial Liouville equation consists only in the fact that instead of the equation for the exact distribution function f_{N} one obtains an approximate equation for a distribution function \tilde{f}_{N} that is smoothed out over the intervals $\tau_{\text{ph}} \sim \tau_{\text{COL}}$ and l_{ph}^3 (see Chap. 9).

From the equation for the function f_{N} there follows again a chain of equations, but now this chain is for the smooth distribution functions $\tilde{f}_i \equiv f_1, f_2, f_3, \dots$. This chain of equations differs from the BBGKY chain in that it already takes into account the dissipation due to the pair collisions, and all the functions are slowly varying.

It will be shown in Chap. 9 how to use this chain of equations to construct a kinetic equation for dense gases.

In the derivation of the kinetic equations for a gas in various approximations, it is assumed that the large-scale fluctuations (with $\tau_{\text{COR}} \gtrsim \tau_{\text{ph}}$ and $l_{\text{COR}} \gtrsim l_{\text{ph}}$) do not play any role. However, the large-scale fluctuations, naturally, exist. They cause the distribution functions for which the kinetic equations are written to be in fact not to be strictly determined.

An investigation of the large-scale fluctuations has recently become of considerable interest in connection with the investigation of noise in various devices.

In the investigation of large-scale fluctuations in gases, two approaches are used. In the first approach one starts with kinetic equations that are regarded as Langevin equations with random sources. This approach for gases has been developed in the papers of Kadomtsev^[34], Gor'kov, Dzyaloshinskiĭ, and Pitaevskii^[35], and Kogan and Shul'man^[36].

The second approach is based on an approximate solution of the chain of equations for the distribution functions or the corresponding equations for the Green's function^[57, 59-61, 24] (see Chap. 10).

At the present time, a different method of obtaining generalized kinetic equations is widely discussed. It is based, on the one hand, on the work of Prigogine, Balescu, Resibois, Henin, George, and Wallenborn^[25-29], and on the other hand on the work of Zubarev and Peletminskiĭ and of Tsukanov, Kalashnikov, and Novikov^[30-33]. In these papers, the authors strive to obtain immediately the most general kinetic equations that take into account arbitrary particle interactions.

Chapter 11 will deal with a generalized kinetic equation of this type, with allowance for both dissipative and nondissipative term. The method employed will be that used in Chaps. 2 and 4 to derive Boltzmann's equations for a nonideal gas. A generalization of the kinetic equation is much simpler than a generalization of the Liouville equation, since it is an equation for the first distribution function. It is nevertheless still so complicated that its practical utilization calls for further significant simplifications. Nonetheless, an approach of this type can be quite fruitful.

Problems analogous to those considered here are investigated also in plasma theory.

In plasma, the role of the effective radius of action is played by the Debye radius r_{D} . Unlike in gases, in a plasma we have in place of (1.1)

$$r_{\text{av}} \ll r_{\text{D}} \ll l, \quad (1.14)$$

and consequently a sphere with radius r_{D} contains many particles, so that the small parameter is the quantity $\mu = 1/nr_{\text{D}}^3 \sim 1/N_{\text{D}}$, where N_{D} is the number of particles in a sphere with radius r_{D} . The quantity μ is called the plasma parameters.

The possibility of using kinetic equations for a plasma is due not to the smallness of the density, as in a gas, but to the fact that each particle of the plasma interacts directly with a large number of particles, and the interaction with an individual particle is weak. Consequently the fluctuations of the phase density of each component and of the field intensities in the plasma are small. The zeroth approximation in the plasma parameter corresponds to complete neglect of the fluctuations. In this case one obtains a closed system of self-consistent equations (the Vlasov equations) for the first distribution functions f_a (a is the index of the plasma component) and for the average fields.

The equation of zeroth order in the plasma parameter is insufficient for the description of the processes that take care when the equilibrium state is established. This follows, in particular, from the fact that the total

entropy of the plasma does not vary with time in this approximation.

In the first approximation in μ , there appears in the equation for f_a an additional term, which is determined by the pair correlation function of the charged particles of the plasma or by the associated correlation of the phase density and the field (Chap. 5). This term, in analogy with Boltzmann's equation, is also called a collision integral although, as follows from the definition of the parameters ϵ and μ , the approximations that lead to the kinetic equations are opposite in the theory of gases and in plasma theory.

In plasma theory, two expressions are used for the collision integrals. These are the Landau collision integral and the Balescu-Lenard collision integral^[6, 8, 37, 38]. In the Landau collision integral, the collective character of the particle interaction in the plasma is obtained by approximately taking into account the static polarizability of the plasma. In the Balescu-Lenard collision one takes into account the plasma dynamic polarizability produced by the moving particles.

The Landau and Balescu-Lenard kinetic equations, like the Boltzmann equation, take into account the interaction of the particles in first order in μ only in the dissipative characteristics, and in this sense are kinetic equations for an ideal plasma.

Classical kinetic equations in first order in μ for a fully ionized nonideal plasma were obtained in^[39], and the corresponding quantum equations were obtained in^[14]. They are considered in Chaps. 5 and 8.

The collision integrals J_a in the kinetic equations for an ideal plasma possess the properties

$$\sum_a n_a \int \varphi_a J_a dp_a = 0 \text{ and } \varphi_a = 1, \quad p_a, \quad p_a^2/2m_a, \quad (1.15)$$

which are analogous to (1.9) (n_a is the concentration of the component a).

These properties ensure satisfaction of the conservation laws for the particle number, the total momentum, and the total kinetic energy of all the plasma components.

For a nonideal gas and a nonideal plasma, only the first pairs of equations in (1.9) and (1.15) are satisfied. At $\varphi_a = p_a^2/2m_a$, the expression $\sum_a n_a \int \varphi_a J_a dp_a$ does not vanish, since the total kinetic energy is not an integral of the motion (Chaps. 2, 4, and 5).

The kinetic equations for a plasma, obtained in the first approximation in the plasma parameter, do not take into account with sufficient accuracy the particle interaction at short distances. This results in a logarithmic divergence in the Landau and Balescu-Lenard collision integrals at short distances.

The Boltzmann collision integral, to the contrary, takes correct account of the particle interaction at short distances in the case of particles with Coulomb interaction, but contains a logarithmic divergence at large distances. This is a consequence of the fact that in the pair-collision interaction one does not take into account the collective character of the particle interaction in the plasma.

De Witt^[41], Hubbard^[42], Aono^[43], and Alyamovskii^[44] have proposed to use as a collision integral for a plasma

a combination of the Boltzmann, Landau, and Balescu-Lenard collision integrals, and by the same token take into account simultaneously the contributions of the pair and collective interaction of the particles.

Allowance for the effects of non-ideality in the kinetic equations with collision integrals proposed in^[41-44] shows that the corresponding thermodynamic functions do not satisfy the necessary limiting conditions (see Chap. 6).

This problem is solved in^[39] in another way: a Boltzmann kinetic equation is obtained for a non-ideal plasma with account taken of the dynamic polarization averaged over the velocities (Chap. 6). The collision integral in this equation converges at small and large distances. The spatial correlation function, which determines the contribution of the interaction to the thermodynamic functions, coincides at short distances with the correlation function in the pair-collision approximation, and at large distances with the Debye correlation function. This yields also the correct limiting values for the thermodynamic functions.

The corresponding results for a quantum nondegenerate plasma were obtained by Klimontovich and Ebeling^[14] and are briefly described in Chap. 8. (See note added in proof at the end of the article.)

In Chap. 9 we consider certain problems in the theory of nonideal gases and plasma. One of the problems is connected with the construction of the kinetic theory of high-density gases, when we cannot limit ourselves to the approximation of the pair and triple collisions.

2. BOLTZMANN KINETIC EQUATION FOR A NONIDEAL GAS IN THE PAIR-COLLISION APPROXIMATION

In the pair-collision approximation, we can describe nonequilibrium processes in a gas by using the closed system of equations (1.3), (1.6) for the distribution functions f_1 and f_2 .

Under the condition of complete weakening of the correlations, the function f_2 can be expressed in terms of f_1 ^[9]:

$$f_2(x_1, x_2, t) = f_1(X_1(-\tau), t-\tau) f_1(X_2(-\tau), t-\tau) + \int_0^\tau \left[\left(\frac{\partial}{\partial t} + \mathbf{v}_1 \frac{\partial}{\partial \mathbf{r}_1} + \mathbf{v}_2 \frac{\partial}{\partial \mathbf{r}_2} \right) f_1(x_1, t) f_1(x_2, t) \right]_{x_1(-\tau), x_2(-\tau), t-\tau} dt' \quad (2.1)$$

Here $X_1(-\tau)$ and $X_2(-\tau)$ are the coordinates and momenta of two interacting particles at the instant of time $t - \tau$.

If we substitute expression (2.1) in the right-hand side of (1.3), then we obtain a most general expression (in the pair-collision approximation) for the collision integral. It takes into account the spatial variation of the distribution functions $f_1(x_1, t)$ and $f_1(x_2, t)$ within the limits of the operating radius of the intermolecular forces, and also the time delay.

Let us consider particular cases.

1) The distribution function is spatially homogeneous and the time delay is not taken into account. Under these assumptions, expression (2.1) coincides with (1.7), and the collision integral is determined by (1.8), which differs only in the notation from the collision integral in Boltzmann's equation.

2) We neglect the time delay, but take into account the effective spatial inhomogeneity in the first term of (2.1). This corresponds to the assumption that the function f_2 is determined by the values of the functions $f_1(x_1, t)$ and $f_1(x_2, t)$ at the same instant of time. As a result we arrive at the generalized Boltzmann equation obtained by Bogolyubov (Eq. (9.17) of [4]). For the hard-sphere model, it coincides with the generalized Boltzmann equation proposed by Enskog (see Chap. 16 of the book [2]).

The difference between these equations and the Boltzmann equations becomes manifest, in particular, on going over to the hydrodynamic approximation. When the generalized equations are used, it is possible to obtain corrections to the equation of state and to the kinetic coefficients in the form of series in the density [5].

We recall that these equations were obtained in the pair-collision approximation. The obtained corrections to the kinetic coefficients are therefore not exact, since when account is taken of more complicated interactions there will be obtained additional contributions of the same order (see Chap. 4 concerning this subject). For the same reason, the first-order correction in the density will be the only correct additional term in the equation of state of an ideal gas.

It follows therefore that when the spatial inhomogeneity is taken into account it suffices to retain only the first-order terms in $r_0/l \sim \epsilon$ in (2.1) in order to obtain the kinetic equation.

The generalized kinetic equations considered in this section do not take into account the time delay, so that on going over to the gasdynamics equations the functions under the $\partial/\partial t$ sign contain no corrections due to the interaction. Thus, the interaction is not taken into account in the expression for the internal energy. To take all the contributions of order ϵ into account, it is necessary to obtain from (2.1) an expression for f_2 with allowance for the terms of first order in r_0/l and τ_0/τ_{rel} .

3) Spatially-homogeneous distribution. Allowance for terms of first order in τ_0/τ_{rel} . In this case we can rewrite (2.1) in the form [6]

$$f_2^{(tr)}(x_1, x_2, t) = f_1(P_1(-\infty), t) f_1(P_2(-\infty), t) - \frac{\partial}{\partial t} \int_0^\infty \tau \frac{d}{d\tau} f_1(P_1(-\tau), t) f_1(P_2(-\tau), t) d\tau. \quad (2.2)$$

When this expression is substituted in the right-hand side of (1.3), the collision integral is represented as a sum of two parts:

$$J = J_{(1)} + J_{(2)}. \quad (2.3)$$

The first part coincides with expression (1.8) and consequently possesses the properties (1.9). For $J_{(2)}$, we have the expression

$$J_{(2)} = -\frac{\partial}{\partial t} \int_0^\infty \int \frac{\partial \Phi_{12}}{\partial x_1} \frac{\partial}{\partial p_1} \tau \frac{d}{d\tau} f_1(P_1(-\tau), t) f_1(P_2(-\tau), t) d\tau dx_2. \quad (2.4)$$

This part of the collision integral has the properties (1.9) only at $\varphi = 1$ and p_1 . At $\varphi = p_1^2/2m$ it follows from (2.4) that

$$n \int \frac{p_1^2}{2m} J dp_1 = -\frac{n^2}{2} \frac{\partial}{\partial t} \int \Phi_{12} f_1(P_1(-\infty), t) f_1(P_2(-\infty), t) \frac{dx_1 dx_2}{V}.$$

As a result, it is not the kinetic energy of the gas which is conserved, but the total energy in the pair-

collision approximation [6]. We thus obtain for the internal-energy density

$$U = n \int \frac{p_1^2}{2m} f_1 dp_1 + \frac{n^2}{2} \int \Phi_{12} f_2^{(tr)}(x_1, x_2, t) \frac{dx_1 dx_2}{V}. \quad (2.5)$$

This expression determines the first two terms of the expansion of the nonequilibrium internal energy of the gas in terms of the density.

We note that the distribution functions themselves can also be expanded in series in the density. In the pair-collision approximation, we can obtain the first two terms of the expansion of the function f_1 in the density, so that the function f_1 can be represented in the form $f_1 + f_1^0 + f_1^1$. Here f_1^0 is the distribution function of an ideal gas. The function f_1^1 is determined by the correlation function $g_2 = f_2 - f_1 f_1$. The connection between f_1^1 and g_2 is determined by the expression

$$f_1(x_1, t) = f_1^0 + n \int \left[g_2 dp_2 - \int g_2 dp_1 dp_2 f_1^0(x_1, t) \right] \frac{dx_1 dx_2}{V}. \quad (2.6)$$

It follows from (2.5) and (2.6) that under nonequilibrium states, when the correlation of the distributions with respect to the coordinates and momenta differs from zero, we have $f_1 \neq f_1^0$, and consequently the interaction of the gas particles determines in (2.5) not only the potential energy, but also part of the kinetic energy.

In the state of local equilibrium we have $f_1 = f_1^0$ or a Maxwellian distribution. Expression (2.5) then becomes

$$U = n \frac{3}{2} \kappa T + \frac{n^2}{2} \int \Phi_{12} e^{-\Phi_{12}/\kappa T} \frac{dx_1 dx_2}{V}. \quad (2.7)$$

For a quantum gas we have $f_1 \neq f_1^0$ even in the equilibrium state (see Chap. 7).

4) Allowance for the spatial inhomogeneity and for the time delay in first order in r_0/l and τ_0/τ_{COR} . In this approximation, the collision integral is represented as a sum of three parts [6]

$$J = J_{(1)} + J_{(2)} + J_{(3)}. \quad (2.8)$$

The first part differs from (1.8) in that $f_1(\mathbf{P}(-\infty), t)$ is replaced by $f_1(\mathbf{r}, \mathbf{P}(-\infty), t)$. The second differs from (2.4) in that the operator $\partial/\partial t$ is replaced by $\partial/\partial t + (\mathbf{v}_1 + \mathbf{v}_2)/2 \cdot \partial/\partial \mathbf{r}_1$, and $J_{(3)}$ is determined by expression (3.4) of [6].

The complete collision integral has the property (1.9) only at $\varphi = 1$. This property ensures satisfaction of the particle-number conservation law.

At $\varphi = p_1$ we have in the case of the collision integral (2.8)

$$n \int p_{1i} J dp_1 = -\frac{\partial}{\partial r_j} \Delta P_{ij}, \quad (2.9)$$

where

$$\Delta P_{ij} = -\frac{n^2}{2} \int \frac{r_{1j}}{r} \frac{\partial \Phi_{12}}{\partial r} f_1(\mathbf{r}, \mathbf{P}_1(-\infty), t) f_1(\mathbf{r}, \mathbf{P}_2(-\infty), t) d\mathbf{r}_1 dp_1 dp_2$$

is the increment, due to the particle interaction, to the stress tensor P_{ij} .

In the state of local equilibrium we have

$$\Delta P_{ij} = \delta_{ij} p, \quad \Delta p = -\frac{n^2}{6} \int r \frac{\partial \Phi_{12}}{\partial r} e^{-\Phi_{12}/\kappa T} d\mathbf{r}. \quad (2.10)$$

For the model of spheres with weak attraction, when

$$\Phi(r) = \begin{cases} \infty & \text{at } r \leq r_0, \\ \Phi(r) < 0, & |\Phi(r)| \ll \kappa T \text{ at } r > r_0, \end{cases}$$

expressions (2.7) and (2.10) take the form

$$U = \frac{3}{2} n \kappa T - n^2 a, \quad p = n \kappa T \left[1 + n \left(b - \frac{a}{\kappa T} \right) \right].$$

Here

$$b = \frac{2\pi}{3} r_0^3, \quad a = 2\pi \int_{r_0}^{\infty} |\Phi(r)| r^2 dr$$

are the Van der Waals constants.

Expression (2.9) determines the contribution of the interaction to the momentum-density balance equation. In the presence of spatial inhomogeneity, an additional contribution appears, naturally, also in the internal-energy balance equation. The integral $n \int p_{i1}^2 / 2m J_{(2)} dp_1$, is now determined by expression (2.4) in which $\partial/\partial t$ is replaced by $\partial/\partial t + (\mathbf{v}_1 + \mathbf{v}_2)/2 \cdot \partial/\partial \mathbf{r}_1$. In the local-equilibrium state we have

$$n \int \frac{p_{i1}^2}{2m} J_{(2)} dp_1 = - \left(\frac{\partial \Delta U}{\partial t} + \frac{\partial}{\partial \mathbf{r}} (U \Delta U) \right), \quad (2.11)$$

where

$$\Delta U = \frac{n^2}{2} \int \Phi_{12} e^{-\Phi_{12}/\kappa T} dr$$

is the contribution of the interaction to the internal-energy density.

Finally, for the last part of the collision integral we have in the local-equilibrium approximation

$$n \int \frac{p_{i1}^2}{2m} J_{(3)} dp_1 = - \frac{\partial}{\partial \mathbf{r}} (U \Delta p). \quad (2.12)$$

The quantity Δp is defined in (2.10).

We see that the total contribution of the interaction between the particles to the energy flux in the state of local equilibrium is determined by the expression $U(\Delta U + p)$. The expression in the parentheses is the density of the thermal function (enthalpy).

In the pair-collision approximation, the viscous-stress tensor $\pi_{ij} = P_{ij} - \delta_{ij} p$ and the vector of the thermal flux S are determined in the usual manner using only the first part of the collision integral $J_{(1)}$ in (2.8).

We see that in the pair-collision approximation, there follows from the Boltzmann equation with the collision integral (2.8) a system of gasdynamic equations for a nonideal gas. The density of the internal energy and the pressure are determined by the expressions

$$U = n \frac{3}{2} \kappa T + \Delta U, \quad p = n \kappa T + \Delta p.$$

In the gasdynamic approximation, calculation of the quantities ΔU and Δp can be carried out in the local-equilibrium approximation.

We have retained here in the gasdynamics equations only the terms linear in two small parameters: the density parameter $\epsilon = nr_0^3$, and the gasdynamic parameter $\epsilon_g = l/L \sim \epsilon^{-1} r_0/L$ (L is a characteristic parameter of length in the gasdynamic description). The question of the higher approximations in the parameters ϵ and ϵ_g will be considered in Chap. 4.

3. THE BOLTZMANN H-THEOREM FOR A NONIDEAL GAS

According to the Boltzmann H-theorem, the entropy of a gas, defined by expression (1.10), increases when the state of the gas approaches equilibrium.

For nonequilibrium states, Boltzmann's expression for the entropy takes partial account of the contribution

of the correlations since, according to (2.6), the function f_1 does not coincide in general with the distribution function for an ideal gas.

In the equilibrium state we have $f_1 = f_1^0$, and expression (1.10) determines the entropy of an ideal gas.

However, in the equilibrium state, even in the approximation of the first collisions, the contribution of the particle interaction to the expression for the entropy is not equal to zero. This shows that in the pair-collision approximation the Boltzmann expression (1.10) does not take full account of the particle interaction.

We denote the unaccounted-for part by ΔS and write down an expression for the entropy of the gas in the form

$$S = S_B + \Delta S. \quad (3.1)$$

It is shown in^[21] that ΔS is given by¹⁾

$$\Delta S = - \frac{\kappa n^2}{2} \int f_2 \ln \frac{f_2}{f_1 f_1} \frac{dx_1 dx_2}{V}. \quad (3.2)$$

We consider this expression for the equilibrium state. In this case we can carry out in (3.2) integration over the momenta. As a result

$$\Delta S = - \frac{\kappa n^2}{2} \int f_2(r_1, r_2) \ln f_2(r_1, r_2) \frac{dr_1 dr_2}{V}.$$

In the equilibrium state we have $f_2 = C \exp(-\Phi_{12}/\kappa T)$, therefore

$$\Delta S = - \frac{\kappa n^2}{2} \int \left[\frac{\Phi_{12}}{\kappa T} e^{-\Phi_{12}/\kappa T} - (e^{-\Phi_{12}/\kappa T} - 1) \right] dr, \quad r = r_1 - r_2. \quad (3.3)$$

A similar expression follows also from the Gibbs canonical distribution. The first term in (3.3) is then determined by the contribution of the internal energy Δu (see (2.11)), and the second term by the contribution of the free energy

$$\Delta F = - \kappa T \frac{n^2}{2} \int (e^{-\Phi_{12}/\kappa T} - 1) dr.$$

Thus, in the equilibrium state we have

$$\Delta S = \frac{\Delta u - \Delta F}{T}.$$

We note that in the non-equilibrium state expression (3.2) can be represented in the form of a sum of two parts, one of which, on going to the equilibrium state, is determined by the contribution of the internal energy, and the second by the contribution of the free energy. To this end we represent the functions f_1 and f_2 in the form

$$f_1 = \left(\int F_1 \frac{dx_1}{V} \right)^{-1} F_1, \quad f_2 = \left(\int F_2 \frac{dx_1 dx_2}{V^2} \right)^{-1} F_2, \\ F_2 = F_1 F_1 + G_2.$$

In the equilibrium state we have $F_2 = \exp(-\Phi_{12}/\kappa T)$, $G_2 = F_2 - 1$.

Expression (3.2) can be rewritten, using the functions F_1 , F_2 , and G_2 , in the form^[21]

$$\Delta S = - \frac{\kappa n^2}{2} \int \left(\ln \frac{F_2}{F_1 F_1} - G_2 \right) \frac{dx_1 dx_2}{V}.$$

The second term in this expression determines the contribution to ΔS corresponding to the contribution from $-\Delta F/T$.

A more complete account of the interaction leads to a change of the Boltzmann H-function. It is now determined by the expression^[21]

$$-H = S' = - \kappa n \int f_1 \ln f_1 \frac{dx_1}{V} - \frac{\kappa n^2}{2} \int f_2 \ln \frac{f_2}{f_1 f_1} \frac{dx_1 dx_2}{V}.$$

The additional term coincides with (3.2).

4. KINETIC EQUATION FOR A NONIDEAL GAS WITH ALLOWANCE FOR TRIPLE COLLISIONS

To obtain a kinetic equation with allowance for triple collisions, it is necessary to use, in addition to Eqs. (1.3) and (1.4) for the functions f_1 and f_2 , also one more equation of the chain for the function f_3

$$\left(\frac{\partial}{\partial t} + H^{(3)}\right) f_3 = n \int (\theta_{14} + \theta_{24} + \theta_{34}) f_4 dx_4 \equiv J_3. \quad (4.1)$$

We have used here the following notation for the operators:

$$\theta_{ij} = \frac{\partial \Phi_{ij}}{\partial r_i} \frac{\partial}{\partial p_i} + \frac{\partial \Phi_{ij}}{\partial r_j} \frac{\partial}{\partial p_j}, \quad (4.2)$$

$$H^{(n)} = \sum_{1 \leq i \leq n} v_i \frac{\partial}{\partial r_i} - \sum_{1 \leq i < j \leq n} \theta_{ij}.$$

In the triple-collision approximation, the expression for J_3 can be represented in the form^[22]

$$J_3 = \left(\frac{\partial}{\partial t} + v_1 \frac{\partial}{\partial r_1} + v_2 \frac{\partial}{\partial r_2} + v_3 \frac{\partial}{\partial r_3}\right) f_1 f_2 f_3. \quad (4.3)$$

As a result we obtain a closed system of equations for the functions f_1 , f_2 , and f_3 .

In the case of a spatially-homogeneous distribution of the gas particles, we obtain from (4.1) and (4.2), in first order in τ_0/τ_{rel} , and expression for f_3 in the triple-collision approximation

$$f_3^{(tr)}(x_1, x_2, x_3, t) = S_{-7}^{(3)} f_1(p_1, t) f_2(p_2, t) f_3(p_3, t) - \frac{\partial}{\partial t} \int_0^{\infty} \tau \frac{d}{d\tau} S_{-7}^{(3)} f_1 f_2 f_3 d\tau; \quad (4.4)$$

$S_{-7}^{(3)}$ is the shift operator for a system of three particles.

This expression is analogous to expression (2.2) for f_2 , used in the pair-collision approximation. Taking the triple collisions into account, the expression for f_2 takes the form^[22]

$$f_2^{(tr)} = S_{-2}^{(2)} f_1 f_2 + n \int_0^{\infty} d\tau \int dx_3 S_{-2}^{(2)}(1, 2) \left(1 - \tau \frac{\partial}{\partial t}\right) \times [(\theta_{13} + \theta_{23}) f_3^{(tr)} - S_{-2}^{(2)}(1, 2) (\theta_{13} f_2^{(p)}(1, 3) f_1 + \theta_{23} f_2^{(p)}(2, 3) f_1)]. \quad (4.5)$$

The functions $f_2^{(tr)}$ and $f_2^{(p)}$ are defined by expressions (4.4) and (2.2).

To obtain the kinetic equation for a nonideal gas with allowance for triple collisions, it is necessary to substitute (4.5) into the right-hand side of (1.3).

This equation coincides with the Cho-Uhlenbeck equation, if we neglect the terms containing $\tau \partial/\partial t$ in (4.5), (4.4), and (2.2).

When triple collisions are taken into account, the expression for the internal-energy density becomes

$$U = n \int \frac{p^2}{2m} f_1 \frac{dx_1}{V} + \frac{n^2}{2} \int \Phi_{12} f_2^{(tr)} \frac{dx_1 dx_2}{V}. \quad (4.6)$$

When (4.5) is substituted here, we can leave out the terms containing $\tau \partial/\partial t$. In the equilibrium state, (4.6) coincides with the three terms of the virial expansion.

In the Cho-Uhlenbeck approximation of the kinetic equation, the expression for U contains the function $f_2^{(p)}$ instead of $f_2^{(tr)}$. This inconsistency in the approximations for the dissipative and nondissipative characteristics of the gas is the consequence of the fact that the particle interaction is incompletely taken into account in the Cho-Uhlenbeck kinetic equation (in the triple collision approximation).

The incompleteness consists in the fact that the dissipative processes are described in the triple-collision

approximation, and the non-dissipative processes in the pair-collision approximation.

If the gas particles are not uniformly distributed in space, then corresponding additional terms appear in the expressions for $f_2^{(tr)}$ and $J^{(tr)}$. On going over to the gasdynamics equations, the expressions for the viscosity and thermal-conductivity coefficients coincide with those obtained in the paper of Cho and Uhlenbeck. The expressions for the internal energy and pressure contain each the first three terms of the corresponding virial expansions.

The Cho-Uhlenbeck papers^[5, 48] contain a derivation of the gasdynamics equations with allowance for the contribution of the triple collisions in the expressions for the viscosity and thermal-conductivity coefficients. This corresponds to taking terms of order $\epsilon \epsilon_g$ into account.

5. KINETIC EQUATIONS FOR A NONIDEAL PLASMA

In the derivation of the kinetic equations for a nonideal plasma it is convenient to start with a system of equations for the phase densities in the space of the coordinates and momenta of the individual plasma components

$$N_a(x, t) = \sum_{1 \leq i \leq N} \delta(x - x_i(t)), \quad x = (r, p)$$

and of the microscopic field intensities^[37, 39]. The collision integrals J_a are expressed in this case in terms of the correlation of the fluctuations of the phase densities and of the field intensities.

For a Coulomb plasma we have

$$J_a = \sum_b n_b \int \frac{\partial \Phi_{ab}}{\partial r_a} \frac{\partial}{\partial p_b} g_{ab} dx_b = -\frac{e_a}{n_a} \frac{\partial \delta N_a \delta E}{\partial p_a}. \quad (5.1)$$

In first order in the plasma parameter μ (in the polarization approximation), the equations for the fluctuations δN_a and δE can be represented in the form^[39]

$$\left(\frac{\partial}{\partial t} + v_a \frac{\partial}{\partial r_a} + e_a E \frac{\partial}{\partial p_a}\right) (\delta N_a - \delta N_a^{(s)}) + e_a n_a \delta E \frac{\partial f_a}{\partial p_a} = 0, \quad (5.2)$$

$$\text{div } \delta E = 4\pi \sum_a e_a \int \delta N_a dp_a. \quad (5.3)$$

It follows from (5.2) that the fluctuations δN_a can be represented in the form of a sum of an induced part (proportional to δE) and a source $\delta N_a^{(s)}$. The fluctuations of the phase density of the source satisfy the relation

$$\left(\frac{\partial}{\partial t} + v \frac{\partial}{\partial r} + e_a E \frac{\partial}{\partial p}\right) \delta N_a^{(s)} = 0. \quad (5.4)$$

This leads to an equation for the two-time correlation $(\delta N_a \delta N_b)_{rr'pp'tt'}$, which is solved subject to the initial condition

$$\overline{(\delta N_a \delta N_b)_{rr'pp'tt'}^{(s)}} = \delta_{ab} \delta(r-r') \delta(p-p') f_a(r', p', t). \quad (5.5)$$

In^[39], a system of kinetic equations was obtained on the basis of (5.2)–(5.4) for a nonideal spatially-homogeneous plasma. One of the consequences of these equations is a conservation law for the total kinetic and potential energy of the plasma

$$U = \sum_a n_a \int \frac{p^2}{2m_a} f_a dp + \frac{1}{8\pi} \int (\delta E \delta E)_k \frac{dk}{(2\pi)^3}, \quad (5.6)$$

where

$$(\delta E \delta E)_k = \sum_a \frac{(4\pi)^2 e_a^2 n_a}{k^2} \int \frac{f_a}{|\varepsilon(kv, k)|^2} dp \quad (5.7)$$

is the nonequilibrium spectral function of the field.

In the equilibrium state we have $(\delta\mathbf{E} \cdot \delta\mathbf{E})_{\mathbf{k}} = 4\pi\kappa T / (1 + r_D^2 k^2)$. This expression corresponds to the Debye correlation function

$$g_{ab}(r) = -\frac{e_a e_b}{\kappa T r} e^{-r/r_D}. \quad (5.8)$$

The kinetic equation obtained in^[39] is a generalization of the Balescu-Lenard equation for a non-ideal plasma. Two particular cases of this equation are of interest. One corresponds to the Landau approximation. The collision integral is then

$$J_a = \sum_b \frac{2}{\pi} e_a^2 e_b^2 n_b \frac{\partial}{\partial p_i} \operatorname{Re} \int_0^\infty \int \frac{k_i k_j}{k^4} e^{-\Delta\tau - i(\mathbf{k}\mathbf{v} - \mathbf{k}\mathbf{v}')\tau} \times \left(\frac{\partial}{\partial p_j} - \frac{\partial}{\partial p'_j} \right) \left(1 - \tau \frac{\partial}{\partial t} \right) f_a(\mathbf{p}, t) f_b(\mathbf{p}', t) d\tau d\mathbf{p}' d\mathbf{k}.$$

The region of integration with respect to \mathbf{k} is bounded by the condition $1/r_{\min} \geq k \geq 1/r_D$.

Neglecting delay, Eq. (5.9) coincides with the Landau collision integral.

Expression (5.9) takes into account the contribution of only the short-wave part of the spectrum, with wave numbers $k \geq 1/r_D$. In the other limiting case, the contribution of only the long-wave excitations (plasmons) is taken into account, and the collision integral becomes^[39, 37]

$$J_a = \frac{e_a^2}{(2\pi)^4} \frac{\partial}{\partial \mathbf{p}} \operatorname{Re} \int_0^\infty \int \frac{\mathbf{k}}{k^2} \left\{ \left(1 - \tau \frac{\partial}{\partial t} \right) (\delta\mathbf{E}\delta\mathbf{E})_{\omega, \mathbf{k}, i\mathbf{k}} \frac{\partial f_a(\mathbf{p}, t - \tau)}{\partial \mathbf{p}} + \frac{8\pi^2}{\omega} \operatorname{sign} \frac{\partial \mathbf{E}'}{\partial \omega} \delta(\mathbf{E}'(\omega, \mathbf{k})) f_a(\mathbf{p}, t) \right\} e^{-\Delta\tau + i(\omega - \mathbf{k}\mathbf{v})\tau} d\tau d\omega d\mathbf{k}. \quad (5.10)$$

The system of equations for the functions f_a is in this case not closed. It must be supplemented by an equation for the spectral function of the plasmons $(\delta\mathbf{E} \cdot \delta\mathbf{E})_{\omega, \mathbf{k}, t}$ ^[37]. The integrals (5.9) and (5.10) take into account contributions from different regions of the spectrum over \mathbf{k} . The total integral can be approximately represented as a sum of the expressions (5.9) and (5.10).

6. BOLTZMANN'S EQUATION FOR A NONIDEAL PLASMA WITH ALLOWANCE FOR THE AVERAGED DYNAMIC POLARIZATION

In the polarization approximation (the first approximation in μ), the interaction of the charged particles of the plasma is not taken into account accurately. One of the consequences of this fact is a logarithmic divergence in the collision integral at small distances. To the contrary, in the pair-collision approximation for a plasma, the interaction at short distances is taken into account correctly, but the corresponding collision integral contains a logarithmic divergence at large distances.

In the analysis of processes occurring in a nonideal plasma, it is necessary to have a kinetic equation in which the contributions of the pair and collective interactions are simultaneously taken into account. This problem can be solved by introducing an effective potential $\tilde{\Phi}_{ab}$ of the particle interaction, in which the averaged dynamic polarization is taken into account^[39]. The form of the effective potential is obtained in the following manner.

The correlation function that determines the form of the collision integral in the polarization approximation is $\sim \tilde{\Phi}_{ab}(\mathbf{k}) / |\epsilon(\mathbf{k}\mathbf{v}, \mathbf{k})|^2$. It follows therefore that it is possible to determine the effective potential, with al-

lowance for the dynamic polarization averaged over the velocities:

$$\tilde{\Phi}_{ab}(\mathbf{k}) = \frac{\Phi_{ab}(\mathbf{k})}{\sum_c e_c^2 n_c} \sum_c e_c^2 n_c \int \frac{f_c}{|\epsilon(\mathbf{k}\mathbf{v}, \mathbf{k})|^2} d\mathbf{p} = \frac{e_a e_b}{\sum_c e_c^2 n_c} \frac{(\delta\mathbf{E}\delta\mathbf{E})_{\mathbf{k}}}{4\pi}. \quad (6.1)$$

We have used here (5.7).

In the local-equilibrium approximation it follows from (6.1) that

$$\tilde{\Phi}_{ab}(\mathbf{k}) = \frac{\Phi_{ab}(\mathbf{k})}{1 + r_D^2 k^2} \text{ and } \tilde{\Phi}_{ab}(r) = \frac{e_a e_b}{r} e^{-r/r_D}, \quad (6.2)$$

i.e., the effective potential coincides with the Debye potential.

This model does not reduce to a Boltzmann gas in which the particle-pair interaction is determined by expression (6.2). The point is that the effective potential enters only in the equations for the correlation functions g_{ab} , and in the expression (5.1) for the collision integral the potential Φ_{ab} remains a Coulomb potential as before.

To determine the collision integral J_a it is necessary to solve equations for f_{ab} in the pair-collision approximation with an interaction potential Φ_{ab} . The solution can be represented in a form analogous to (2.2) if \mathbf{P} is replaced by $\tilde{\mathbf{P}}$, i.e., by the initial momenta of the particles interacting in accordance with the law $\tilde{\Phi}_{ab}$.

The internal energy is given by an expression analogous to (2.5). Its form in the equilibrium state is

$$U = \sum_a n_a \frac{3}{2} \kappa T + \frac{1}{2} \sum_{ab} n_a n_b \int \Phi_{ab}(r) f_{ab}(r) dr, \quad (6.3)$$

where

$$f_{ab}(r) = \exp\left(-\frac{e_a e_b}{\kappa T r} e^{-r/r_D}\right) \quad (6.4)$$

is the radial distribution function of a nonideal plasma.

At short distances this function goes over into the distribution function of a Boltzmann gas, and at large distances it coincides with the Debye distribution function, corresponding to the first-order approximation in the plasma parameter μ . The corresponding limiting properties are possessed also by the thermodynamic functions.

Allowance for the average dynamic polarization and for the nonideality effects leads to a change in the kinetic coefficients.

It is shown in^[39] that the first of these causes leads to a change in the form of the Coulomb logarithm. The effective electric field \mathbf{E}_{eff} in a non-ideal plasma was calculated in^[14]. In the calculation of the electric conductivity σ , both effects decrease the value of σ . It is possible that this explains why the experimental data obtained by measuring the electric conductivity are lower than those calculated by Spitzer's theory^[46-48].

7. QUANTUM KINETIC EQUATIONS FOR A NONDEGENERATE NONIDEAL GAS

The quantum collision integral can be expressed in the form^[14]

$$J_a = \frac{i}{\hbar} \sum_b N_b \int [(\Phi_{ab}(|\mathbf{r}_a - \mathbf{r}_b|) - \Phi_{ab}(|\mathbf{r}_a' - \mathbf{r}_b|))] \rho_{ab}(\mathbf{r}_a, \mathbf{r}_a', \mathbf{r}_b, \mathbf{r}_b, t) e^{-i(\mathbf{r}_a' - \mathbf{r}_a) \cdot \mathbf{p}_a / \hbar} \frac{d\mathbf{r}_a' d\mathbf{r}_a d\mathbf{r}_b}{V}. \quad (7.1)$$

Here ρ_{ab} is the density matrix of the two particles. The

normalization conditions for the functions f_a and ρ_{ab} are

$$\frac{V}{(2\pi\hbar)^3} \int f_a d\mathbf{p} = 1, \quad \int \rho_{ab}(\mathbf{r}_a, \mathbf{r}_a, \mathbf{r}_b, \mathbf{r}_b, t) \frac{d\mathbf{r}_a d\mathbf{r}_b}{V^2} = 1.$$

The equations for the functions f_a and ρ_{ab} in the pair-collision approximation are analogous to Eqs. (1.3) and (1.6).

The collision integral J_a for a spatially-homogeneous gas can be represented in first order in $\tau_0 \partial/\partial t$ in the form of a sum of two parts. The right-hand part, analogous to the classical expression (1.8), can be written in the form^[14]

$$J_a(\mathbf{p}_a, t) = \frac{i}{\hbar} \frac{V^2}{(2\pi\hbar)^6 \hbar^3} \sum_b N_b \int \left[\Phi_{ab} \left(\left| \mathbf{r}_a - \mathbf{r}_b - \frac{1}{2} \hbar \mathbf{v}_a \right| \right) - \Phi_{ab} \left(\left| \mathbf{r}_a - \mathbf{r}_b + \frac{1}{2} \hbar \mathbf{v}_a \right| \right) \right] e^{-i\mathbf{v}_a \mathbf{p}_a} \Psi_{\mathbf{p}_a \mathbf{p}_b}(\mathbf{r}_a + \frac{1}{2} \hbar \mathbf{v}_a, \mathbf{r}_b) \times \Psi_{\mathbf{p}_a \mathbf{p}_b}^*(\mathbf{r}_a - \frac{1}{2} \hbar \mathbf{v}_a, \mathbf{r}_b) f_a(\mathbf{p}_a, t) f_b(\mathbf{p}_b, t) d\mathbf{v}_a d\mathbf{p}_a d\mathbf{r}_b d\mathbf{p}_b. \quad (7.2)$$

Here $\Psi_{\mathbf{p}_a \mathbf{p}_b}(\mathbf{r}_a, \mathbf{r}_b)$ are the eigenfunctions of the Hamiltonian of the two gas particles.

This expression coincides with that obtained by Green^[10]. It is shown in^[49] that expression (7.2) can be reduced to the usual form of the Boltzmann collision integral with a quantum cross section.

In the first order in $\tau_0 \partial/\partial t$, we obtain an additional contribution, analogous to (2.4), to the collision integral.^[14] This part of the collision integral takes into account the nonideality effects.

The collision integral in the quantum kinetic equation has the same properties as in the classical case. In particular, the expression for the internal energy of the gas is determined by a sum of the kinetic and potential energies

$$U = \sum_a n_a \int \frac{p^2}{2m_a} f_a \frac{V}{(2\pi\hbar)^3} d\mathbf{p} + \sum_{ab} \frac{n_a n_b}{2} \int \Phi_{ab} \rho_{ab}(\mathbf{r}_a, \mathbf{r}_a, \mathbf{r}_b, \mathbf{r}_b) d\mathbf{r}_{ab}. \quad (7.3)$$

The function $\rho_{ab}(\mathbf{r}_a, \mathbf{r}_a, \mathbf{r}_b, \mathbf{r}_b, t)$ which enters in this expression is the distribution function of the coordinates of the two particles. In the zeroth order in the delay, it is given by the expression

$$\rho_{ab} = \frac{V^4}{(2\pi\hbar)^6} \int |\Psi_{\mathbf{p}_a \mathbf{p}_b}(\mathbf{r}_a, \mathbf{r}_b)|^2 f_a(\mathbf{p}_a, t) f_b(\mathbf{p}_b, t) d\mathbf{p}_a d\mathbf{p}_b. \quad (7.4)$$

Just as in the classical case, the first quantum distribution function is expanded in terms of the density (see (2.6)). The difference from the classical case lies in the fact that the term proportional to the density does not vanish even in the equilibrium state. It vanishes only as $\hbar \rightarrow 0$. Accurate to \hbar^2 we have

$$f_a^{\text{qu}}(\mathbf{p}_a) = f_a^0 + \sum_b n_b \frac{\hbar^2}{24\kappa T} \int \left(\frac{\partial}{\partial \mathbf{r}_a} \frac{\partial}{\partial \mathbf{p}_a} \right)^2 (\Phi_{ab} f_a^0) g_{ab}^0(\mathbf{r}_a, \mathbf{r}_b) \frac{d\mathbf{r}_a d\mathbf{r}_b}{V}; \quad (7.5)$$

f_a^0 is the Maxwellian distribution, and $g_{ab}^0 = \exp - (\Phi_{ab}/\kappa T) - 1$ is the classical correlation function of the Boltzmann gas.

From (7.5) follows an expression for the average kinetic energy^[50]

$$\int \frac{p^2}{2m_a} f_a^{\text{qu}} d\mathbf{p} = \frac{3}{2} \kappa T + \frac{\hbar^2}{24m_a \kappa T} \sum_b n_b \int \frac{\partial^2 \Phi_{ab}}{\partial \mathbf{r}_a^2} g_{ab}^0 \frac{d\mathbf{r}_a d\mathbf{r}_b}{V}.$$

Inasmuch as the contribution of the correlations to the expression for U enters both via the potential energy and via the kinetic energy even in the equilibrium state, it is convenient to represent the expression for U in the form

$$U = U_{\text{id}} + U_{\text{cor}}. \quad (7.6)$$

Here U_{id} is the internal energy of the ideal gas, and U_{cor} is the correlation part of the internal energy. It is expressed in the following manner in terms of the potential energy U_{pot} ^[14, 38]:

$$U_{\text{cor}} = \int_0^1 \frac{d\lambda}{\lambda} \frac{\partial}{\partial \beta} (\beta U_{\text{pot}}(\lambda, \beta)), \quad \beta = 1/\kappa T. \quad (7.7)$$

This expression determines the second virial coefficient of the expansion of the internal energy of a quantum gas in terms of the density.

8. QUANTUM KINETIC EQUATION FOR A NONIDEAL PLASMA

The quantum kinetic equations for a plasma are usually obtained in the polarization approximation^[8, 38]. Taking the dynamic polarization into account ensures screening at large distances. At short distances, the particle interaction in the collision integral is described in the Born approximation.

In a nonideal plasma, the strong interactions of the charge particles at short distances play an important role, so that we must have kinetic equations in which the strong interaction at short distances, as well as polarization effects, are taken into account.

An exact account of the dynamic polarization, just as in the classical case (Chap. 5), leads to exceedingly complicated expressions. As a result, it is advantageous also in the quantum case to use an approximation in which the averaged dynamic plasma polarization is taken into account. Just as in Chap. 5, we can introduce a quantum effective potential^[14]

$$\tilde{\Phi}_{ab}(\mathbf{k}) = \frac{\Phi_{ab}(\mathbf{k})}{\sum_c e_c^2 n_c} \sum_d e_d^2 n_d \frac{1}{2} \int \frac{f_d(\mathbf{p} + \frac{1}{2} \hbar \mathbf{k}) + f_d(\mathbf{p} - \frac{1}{2} \hbar \mathbf{k})}{|\varepsilon(\mathbf{k}\mathbf{v}, \mathbf{k})|^2} \frac{V d\mathbf{p}}{(2\pi\hbar)^3}, \quad (8.1)$$

where

$$\varepsilon(\mathbf{k}\mathbf{v}, \mathbf{k}) = 1 + \sum_a \frac{4\pi e_a^2 n_a}{k^2} \int \frac{f_a(\mathbf{p} + \frac{1}{2} \hbar \mathbf{k}) - f_a(\mathbf{p} - \frac{1}{2} \hbar \mathbf{k})}{\omega - \mathbf{k}\mathbf{v} + i\Delta} \frac{V d\mathbf{p}}{(2\pi\hbar)^3}$$

is the quantum dielectric constant of the plasma.

As $\hbar \rightarrow 0$, expression (8.1) goes over into classical expression (6.1). We note that even in the quantum case the effective potential can be expressed in terms of the corresponding spectral density $(\delta \mathbf{E} \cdot \delta \mathbf{E})_{\mathbf{k}}$.

To obtain an equation for the density matrix $\tilde{\rho}_{ab}$, it is necessary to use an equation for the function ρ_{ab} in the pair-collision approximation^[14] and replace Φ_{ab} in it by $\tilde{\Phi}_{ab}$. Accordingly, the eigenfunctions $\Psi_{\mathbf{p}_a \mathbf{p}_b}$ of the particle-pair Hamiltonian are replaced by the eigenfunctions $\tilde{\Psi}_{\mathbf{p}_a \mathbf{p}_b}$ of the particle pair with potential $\tilde{\Phi}_{ab}$.

This means that after the substitution $\Psi \rightarrow \tilde{\Psi}$ the expression (7.2) will determine the quantum collision integral for the plasma in the averaged dynamic polarization approximation. The additional term to the collision integral, which takes the nonideality effects into account, is determined in a corresponding manner.

It appears that an exact solution of the wave equation for the function $\tilde{\Psi}_{\mathbf{p}_a \mathbf{p}_b}$ is at present unknown even for the simplest effective potential. Consequently, in calculations of, say, thermodynamic functions the results can be represented only in the form of series in the Born parameter $\xi_{ab} = -e_a e_b / \kappa T \lambda_{ab}$ (λ_{ab} is the de Broglie wavelength).

Expressions for the thermodynamic functions of a nonisothermal plasma were obtained in^[14]. It is inter-

esting that although the theory is constructed for a fully ionized plasma, the thermodynamic functions take into account the contribution of the bound states close to the boundary of the continuous spectrum.

In^[14] we considered also the quantum distribution functions for the positions of two atoms $\tilde{\rho}_{ab}(\mathbf{r}_a, \mathbf{r}_a, \mathbf{r}_b, \mathbf{r}_b)$ for a nonisothermal plasma. This function is finite for all distances $r = |\mathbf{r}_a - \mathbf{r}_b|$. The decrease at large distances is due to allowance for screening. The finite character at short distances is due to quantum effects.

Thus, for example, in the case of a Debye effective potential the value of this distribution function at zero is determined for a nonisothermal plasma by the expression

$$\tilde{\rho}_{ab}(r=0) = 1 - \sqrt{\pi} \frac{m_a + m_b}{m_a \lambda_{ab} + m_b \lambda_{ab}} \frac{e_a e_b}{\lambda_{ab}} \exp\left(\frac{\lambda_{ab}}{2r_D}\right)^2 \left[1 - \Phi\left(\frac{\lambda_{ab}}{2r_D}\right)\right]; \quad (8.2)$$

Φ is the error function, and ξ_{ab} and λ_{ab} are determined for a nonisothermal plasma by the expressions

$$\xi_{ab} = -\frac{e_a e_b}{\lambda_{ab}}, \quad \lambda_{ab} = \frac{m_a + m_b}{m_a m_b} \left(2 \frac{\lambda_{ab}^2}{m_a} + 2 \frac{\lambda_{ab}^2}{m_b}\right)^{-1/2}.$$

Klimontovich and Kraft^[70] have shown that when exchange effects are taken into account, an additional factor $(1 - \delta_{ab}/2)$ appears in (8.2), so that at $a = b$ the result is decreased by one-half.

If the deBroglie length is much smaller than the Debye radius and the plasma is isothermal, then in the zeroth approximation in the parameter λ_{ab}/r_D we obtain from (8.2)

$$\tilde{\rho}_{ab}(r=0) = \left(1 - \sqrt{\pi} \frac{e_a e_b}{\lambda_{ab}}\right) \left(1 - \frac{\delta_{ab}}{2}\right).$$

This result was obtained by Kelbg^[51] and by Trubnikov and Elesin^[52].

It was already noted in Chap. 6 that the deviation of the electric conductivity from that calculated by Spitzer's formula is due to two causes. The first is the influence of the screening, and the second is the difference between the effective (acting) field $\mathbf{E}_a \text{ eff}$ and the average field \mathbf{E} . The field $\mathbf{E}_a \text{ eff} = \mathbf{E} + (1/e_a) \int p \mathbf{J} V (2\pi\hbar)^3 d\mathbf{p}$. Both effects are of importance when it comes to calculating the electric conductivity of a nonideal plasma.

For the effective potential (6.2), in the first approximation in the Born parameter, the effective field in a nonisothermal plasma is given by

$$\mathbf{E}_a \text{ eff} = \mathbf{E} \left\{ 1 - \frac{4\pi e_a}{3} r_D \sum_b e_b^2 n_b \frac{(m_a + m_b)(e_a m_b + e_b m_a)}{(\lambda_{ab}^2 m_b + \lambda_{ba}^2 m_a)^2} K\left(\frac{\lambda_{ab}}{r_D}\right) \right\}, \quad (8.3)$$

$$K(\eta) = \frac{\sqrt{\pi}}{2\eta} \left\{ 1 + \frac{1}{\sqrt{\pi}} \eta - \left(1 + \frac{\eta^2}{4}\right) e^{\eta^2/4} \left[1 - \Phi\left(\frac{\eta}{2}\right)\right] \right\}.$$

It follows therefore that the increment to the external field depends on two parameters, the plasma parameter and the parameter $\eta = \lambda_{ab}/r_D$.

In the classical limit ($\eta = 0$, $K = 1$), formula (8.3) for the effective field acting on the electrons coincides with the result of Kadomtsev^[53].

9. KINETIC EQUATIONS FOR DENSE GASES

In Chaps. 2 and 4 we considered the kinetic equations for a nonideal gas with allowance for pair and triple collisions. It is impossible to construct in accordance with this scheme kinetic equations for denser gases, when more complicated collisions must be taken into

account, since the corresponding collision integrals diverge (see the Introduction).

We have seen that the kinetic equation is derived using the condition of the damping of the fast (small-scale) fluctuations. The boundary between the fast and slow fluctuations depends on the considered approximation. It is determined by the dimensions of the corresponding physically infinitesimally small intervals τ_{ph} and l_{ph} (see p. 515 of the Introduction). In the pair-collision approximation we have $\tau_{ph}^{(2)} \sim \sqrt{\epsilon} \tau_{col}$ and $l_{ph}^{(2)} \sim \sqrt{\epsilon} l$, for triple collisions we have $\tau_{ph}^{(3)} \sim (\epsilon)^{1/4} \tau_{col}$ and $l_{ph}^{(3)} \sim (\epsilon)^{1/4} l$. For quadrupole collisions we have $\tau_{ph}^{(4)} \sim \tau_{col}$ and $l_{ph}^{(4)} \sim l$. It follows therefore that the collision integral determined by the collisions of four particles cannot be constructed by perturbation theory in terms of the density, since the $\tau_{ph}^{(4)} \sim \tau_{col}$, $l_{ph}^{(4)} \sim l$, and these quantities can therefore not be regarded as negligibly small. Consequently, the collision integral depends explicitly on τ_{col} (or l). This introduces an additional dependence on the density (see p. 515).

In other words, in the construction of the collision integrals that take into account collisions of four and more particles, it is necessary to construct the theory in such a way that the collision integrals take explicit account of the contributions of the pair collisions. This can be done in the following manner:

We use the Liouville equation averaged over the physically infinitesimally small volume $(l_{ph}^{(2)})^3$, which is determined in the pair-collision approximation^[24]. From (1.13) we have $(l_{ph}^{(2)})^3 \sim 1/\sqrt{\epsilon} n$ (we recall that $l_{ph}^{(2)} \sim \sqrt{\epsilon} l \ll l$). We denote the corresponding averaged distribution function by \tilde{f}_N ; then, according to^[24] we have

$$\left(\frac{\partial}{\partial t} + H^{(N)}\right) \tilde{f}_N - \sum_{1 \leq i < j \leq N} \overline{\theta_{ij}^{(i)}} = 0, \quad (9.1)$$

where (for a spatially-homogeneous gas)

$$\tilde{f}_N^{(i)}(x_1, \dots, x_N, t) = \tilde{f}_N(x_1, \dots, \mathbf{r}_i \mathbf{P}_i(-\infty), \dots, \mathbf{r}_j \mathbf{P}_j(-\infty), \dots, x_N, t). \quad (9.2)$$

We use here also the notation introduced on pp. 514 and 520. The tilde over the last term in (9.1) denotes that it is necessary to average over the volume $(l_{ph}^{(2)})^3$.

On the basis of (9.1) we can construct a chain of equations for the functions $\tilde{f}_1 \equiv f_1, \tilde{f}_2, \tilde{f}_3, \dots$. It differs from the chain of the Bogolyubov equation in the fact that all the equations of the chain contain dissipative terms due to pair collisions.

The first equation of the chain for a spatially-homogeneous gas takes the form

$$\frac{\partial f_1}{\partial t} + n \int \theta_{12} (f_2^{(1,2)} + \tilde{f}_2) dx_2 = J. \quad (9.3)$$

We see that the collision integral consists of two parts. One, containing the function $f_1^{(1,2)}$, is determined by the small-scale fluctuations ($l_{cor} \leq l_{ph} \sim \sqrt{\epsilon} l$), and the second is determined by large-scale fluctuations. The function $f_2^{(1,2)}$ can be represented in the form $f_2^{(1,2)} = (f_1 f_1)^{(1,2)} + g_2^{(1,2)}$, where $g_2^{(1,2)}$ is the corresponding correlation function. We neglect the correlation at small scales ($g_2^{(1,2)} = 0$), then expression (9.3) takes the form

$$\frac{\partial f_1}{\partial t} + n \int \theta_{12} [f_1(\mathbf{P}_1(-\infty), t) f_1(\mathbf{P}_2(-\infty), t) + \tilde{f}_2] dx_2 = J_B + \Delta J. \quad (9.4)$$

The first term in (9.4) coincides with the Boltzmann collision integral (1.8). The second term (ΔJ) is determined by the contribution of the large-scale correlations. It is just through the function f_2 in the kinetic equation (9.4) that the contribution of the more complicated particle interactions than pair collisions will be taken into account.

We write down for \tilde{f}_2 an equation that follows from (9.1). We make here the following assumptions: we represent the functions $f_3^{(1,3)}$ and $f_3^{(2,3)}$, which enter in the equation for \tilde{f}_2 , in a form analogous to (1.5), and consider in it the second-correlation approximation ($g_3^{(1,3)} = 0$, $g_3^{(2,3)} = 0$). The equation obtained in this approximation for f_2 , with allowance for Eq. (9.3) (or (9.2)), can be written in the form

$$\left(\frac{\partial}{\partial t} + H^{(2)} + \delta J_{x_1} + \delta J_{x_2}\right) (\tilde{f}_2 - f_1 f_1) = \theta_{12} f_1 f_1 + \overline{\theta_2 (f_1 f_1)^{(1,2)}} + n \int [\theta_{13} (\tilde{f}_3 - \tilde{f}_2(1, 3)) f_1(2) + \theta_{23} (\tilde{f}_3 - \tilde{f}_2(2, 3)) f_1(1)] dx_3. \quad (9.5)$$

We have introduced here symbols for the linearized pair-collision integrals δJ_{x_1} and δJ_{x_2} (for the Bogolyubov form). For example,

$$\delta J_{x_1}(\tilde{g}_2) = -n \int \theta_{13} f_1(r_1, P_1(-\infty)) \tilde{g}_2(x_2, r_3, P_3(-\infty)) + \tilde{g}_2(r_1, P_1(-\infty), x_2) f_1(r_3, P_3(-\infty)) dx_3. \quad (9.6)$$

The minus sign has been introduced in the definition (9.6) for the sake of convenience.

We consider the right-hand side of (9.5). Its third term takes into account the influence of triple and more complicated particle interactions. The first and second terms of the right-hand side are determined by the first distribution functions, and can therefore be regarded as the source in the equation for the correlation function $\tilde{g}_2 = \tilde{f}_2 - f_1 f_1$. We shall show in Chap. 10 that they determine the spectrum of the large-scale fluctuations. At the present we consider only the contribution of the third term in the right-hand side of (9.5) in the kinetic equation for the dense gases.

We compare Eq. (9.5) (without allowance for the source) with the second equation of the Bogolyubov-equation chain, namely Eq. (1.4).

For the spatially-homogeneous distribution of the gas particles, Eq. (1.4) (with allowance for Eq. (1.3)) can be written in the form

$$\left(\frac{\partial}{\partial t} + H^{(2)}\right) (f_2 - f_1 f_1) = \theta_{12} f_1 f_1 + n \int [\theta_{13} (f_3 - f_2(1, 3)) f_1(2) + \theta_{23} (f_3 - f_2(2, 3)) f_1(1)] dx_3. \quad (9.7)$$

In the pair-collision approximation, the second term of the right-hand side of this equation vanishes and the equation coincides with (1.6) ($f = 0$ in (9.7) because of the spatial homogeneity). Thus, the contribution of the triple and more complicated interactions of the gas particles is determined by the second term of the right-hand side of (9.7). It coincides with the second term of the right-hand side of (9.5).

It was already noted in the introduction that the use of perturbation theory in terms of the density to solve the chain of Bogolyubov equations leads in the kinetic equation to the appearance of diverging integrals. The situation, however, changes if perturbation theory with respect to density is applied when solving the equations for the distribution functions $\tilde{f}_3, \tilde{f}_4, \dots$. Since the right-hand sides in Eqs. (9.5) and (9.7), which take into account the contributions of the triple, quadrupole, and

higher interactions, coincide, it follows that the main difference will lie in the replacement of the operator $(\partial/\partial t + H^{(2)})$ in (9.7) by $(\partial/\partial t + H^{(2)} + \delta J_{x_1} + \delta J_{x_2})$ in Eq. (9.5).

When solving (9.5) we can disregard the solution of the homogeneous equation, since the initial correlations attenuate within a time on the order of $\tau_{COL}^{(2)}$, and the characteristic times of the integrals of the triple and quadrupole collisions are of respective order $\tau_{COL}^{(2)}/\epsilon$, $\tau_{COL}^{(2)}/\epsilon^2, \dots$. When the inhomogeneous solution of Eq. (9.5) is used, owing to the presence of the dissipative term $(\delta J_{x_1} + \delta J_{x_2}) \tilde{g}_2 \sim \tilde{g}_2/\tau_{eff}$, the collision integrals are cut off at times $t \sim \tau_{eff}$, where τ_{eff} is the effective free-path time in pair collisions.

For example, when triple collisions are taken into account, the following change takes place in (4.5):

$$\int_0^\infty d\tau S_{\tau}^{\Omega_{\tau}}(1, 2) \dots \rightarrow \int_0^\infty d\tau e^{-\frac{\tau}{\tau_{eff}}} S_{\tau}^{\Omega_{\tau}}(1, 2) \dots$$

Thus, the divergence in the collision integrals, which take into account the contributions of triple and more complicated collisions, is eliminated because of the allowance for the pair-collision dissipative contribution to the equations for the functions \tilde{f}_2, f_3, \dots . A similar result can be obtained with the aid of a diagram technique, by summing the diagrams that diverge most strongly when expanded in the density. The result of such a summation corresponds to the approximation of the second correlations for small-scale correlations in the equation for the function f_2 .

10. KINETIC THEORY OF FLUCTUATIONS IN A GAS AND A PLASMA

We consider again a gas rarefied to such an extent that we can confine ourselves to the pair-collision approximation. We recall (see the Introduction) that in the derivation of the Boltzmann kinetic equation one uses the condition of complete weakening of the initial correlations. Actually, as already noted on p. 516, a weakening of the small-scale correlations with $\tau_{COR} \gtrsim \sqrt{\epsilon} \tau_{COL}$ and $r_{COR} \leq \sqrt{\epsilon} l$ takes place. Thus, in the derivation of the Boltzmann kinetic equation it is actually assumed that the large-scale fluctuations (with $\tau_{COR} \gtrsim \sqrt{\epsilon} \tau_{COL}$, $r_{COR} \gtrsim \sqrt{\epsilon} l$) do not play any role in the kinetic theory.

It is only by paying this price that we can obtain the kinetic equation, a closed equation for the distribution functions f_1 .

The large-scale fluctuations in general case do not have time to attenuate within the relaxation time of the function f_1 . As a result, the chain of Bogolyubov equations (or the Liouville equation) does not lead, when account is taken of the large-scale fluctuations, to the Boltzmann kinetic equation, and we obtain only a system of equations for the function f_1 and the correlation functions of the large-scale fluctuations $\tilde{g}_2 \dots \tilde{g}_3 \dots$. This system of equations can be replaced approximately by the Langevin equation for random functions f_1 —the Boltzmann equation for a random source $y(x_1, t)$. This, naturally, raises the problem of determining the statistical characteristics of the random source. For gas states close to equilibrium, this problem was first solved by Kadomtsev^[53]. In this paper the correlation function of a random source y is calculated by a method similar to that first used to derive the Boltzmann kinetic equation itself.

Gor'kov, Dzyaloshinskiĭ, and Pitaevskiĭ^[35] used the method of Rytov^[54] and of Landau and Lifshitz^[50,55,56] to construct a kinetic theory of fluctuations in the equilibrium state.

Kogan and Shul'man^[36] determined the correlation function of a random source in Boltzmann's equation for stationary states that deviate strongly from the equilibrium state.

This problem was solved independently by Gantsevich, Gurevich, and Katilyus^[57] on the basis of equations they derived for large-scale correlation functions. In^[24], to describe the large-scale fluctuations in a gas and a plasma, a chain of equations that follows from (9.1) was used for the functions $f_1, \tilde{f}_2, \tilde{f}_3$ (see Chap. 9). The fluctuations were calculated in^[24] in the polarization approximation. The additional contribution made to the kinetic equation (9.4) by the large-scale fluctuations was found in the same paper.

We consider first the results of the calculation for the case of local equilibrium.

We turn to Eq. (9.5) for the correlation function $\tilde{g}_2 = \tilde{f}_2 - f_1 f_1$. The last term in the right-hand side for a rarefied gas (in the pair-collision approximation) can be omitted, since it takes into account the contribution of the triple and more complicated interactions (see Chap. 9). The second term in the right-hand side of $\overline{\theta_{12}(f_1 f_1)^{(1,2)}}$, is equal to zero in the local-equilibrium approximation. Thus, in the right-hand side of (9.5) there remains only one term $\theta_{12} f_1^{(0)} f_1^{(0)}$, where $f_1^{(0)}$ is the local Maxwellian distribution. Recognizing that $\delta J_X \{f_1^{(0)}\} = 0$, we obtain in the considered approximation the solution of Eq. (9.5)

$$\tilde{g}_2(x_1, x_2) = (e^{-\frac{\Phi_{12}}{\kappa T}} - 1) f_1^{(0)} f_1^{(0)} \approx -\frac{\Phi_{12}}{\kappa T} f_1^{(0)} f_1^{(0)}, \quad (10.1)$$

since $r \gtrsim \sqrt{\epsilon} l \sim r_0 / \sqrt{\epsilon} \gg r_0$.

We use the well known formula relating the correlation function g_2 with the equal-time correlation of the fluctuations of the phase density in coordinate and momentum space $N(x, t) = \sum_i \delta(x - x_i(t))$, where $x = (r, p)$:

$$\overline{(\delta N \delta N)_{xx't'}} = \left(\delta(x-x') n f_1 - \frac{n}{V} f_1(x, t) f_1(x', t) \right) + n^2 g_2(x, x', t). \quad (10.2)$$

Here n is the average concentration, and $\delta N(x, t) = N - \bar{N}$. The average phase velocity is connected with the distribution function f_1 by the relation $\bar{N}(x, t) = n f_1(x, t)$.

From (10.1) and (10.2) follows an expression for the large-scale part of the equal-time correlation of the fluctuations δN in the state of local equilibrium

$$\overline{(\delta N \delta N)_{xx't'}} = n \left(\delta(x-x') f_1^{(0)} - \frac{1}{V} f_1^{(0)} f_1^{(0)} \right) - n^2 \frac{\Phi_{12}}{\kappa T} f_1^{(0)} f_1^{(0)}. \quad (10.3)$$

The infinitesimally small length element in the $\delta(r - r')$ function of (10.3) is l_{ph} .

To find the spectral density $(\delta N \delta N)_{\omega k p p'}$ of the large-scale fluctuations, we use the solution for the two-time correlation fluctuations δN . It is obtained by different methods in^[57,24] and is given by

$$\left(\frac{\partial}{\partial t} + v \frac{\partial}{\partial r} + F \frac{\partial}{\partial p} + \delta J_p \right) (\delta N \delta N)_{xx'tt'} = 0. \quad (10.4)$$

It is necessary to add to this equation the initial condition, namely the value of the function $\overline{(\delta N \delta N)_{xx'tt'}}$ at $t = t'$. For the local-equilibrium approximation we obtain from (10.4) an initial condition (10.3) the follow-

ing expression for the spectral function

$$(\delta N \delta N)_{\omega k p p'} = \frac{(\delta J_p + \delta J_{p'}) \left(n \delta(p-p') f_1^{(0)} - \frac{n^2 \Phi(k)}{\kappa T} f_1^{(0)} f_1^{(0)} \right)}{(-i(\omega - kv) + \delta J_p)(i(\omega - kv') + \delta J_{p'})}. \quad (10.5)$$

Using this expression, we can find the spectral function of a random source in the Boltzmann equation, if the latter is regarded as a Langevin equation. It is determined by the numerator of (10.5), i.e.,

$$n^2 (yy)_{\omega k p p'} = (\delta J_p + \delta J_{p'}) \left(n \delta(p-p') f_1^{(0)} - \frac{n^2 \Phi(k)}{\kappa T} f_1^{(0)} f_1^{(0)} \right). \quad (10.6)$$

The first term in this expression determines the spectral function of the source of large-scale fluctuations for an ideal gas, and coincides with the expression obtained by Kadomtsev^[34]. The second term in (10.6) takes into account the nonideal character of the gas (in the pair-collision approximation).

The foregoing results can be generalized to include the case when the gas is not in the equilibrium state^[36,57,24]. The deviation from equilibrium is due to the action of the constant field. The result follows again from Eq. (9.5), from which we omit the last term of the right-hand side. However, the second term of the right-hand side does not vanish now. This leads to the appearance of an additional term in the expression for the spectral functions $(\delta N \delta N)_{\omega k p p'}$ and $(yy)_{\omega k p p'}$.

In^[57,36], the following expression was obtained for the function $(yy)_{\omega k p p'}$ at small k ($kl \ll 1$):

$$n^2 (yy)_{\omega k p_1 p_2} = (I_{p_1} + I_{p_2}) n \delta(p_1 - p_2) f_1 + n^2 \int \theta_{12} f_1(p_1(-\infty)) f_1(p_2(-\infty)) dr_2. \quad (10.7)$$

Here $I_p = F(\partial/\partial p) + \delta J_p$.

The second term in (10.7) is determined by the particle interaction. It differs from the collision integral and Boltzmann's equation (1.8) in that in (1.8) there still remains integration with respect to p_2 .

In the equilibrium state, the first term of (10.7) coincides with the first term of (10.6), and the second term of (10.7) vanishes. The interaction of the particles in (10.7) has not been fully taken into account, since it is lacking a term that coincides with the second term of (10.6) in the equilibrium state.

The spectral functions of the large-scale fluctuations are used, for example, to calculate the fluctuations in semiconductor devices,^[36,57] and to investigate the scattering and transformation of waves by a non-equilibrium plasma in semiconductors^[58].

Let us consider the additional contribution ΔJ made by the large-scale fluctuations to the collision integral in the pair-collision approximation. To this end it is necessary to determine from (9.5) the non-equilibrium solution of Eq. (9.5). (This can be done in the perturbation-theory approximation) and to substitute the obtained expression for \tilde{g}_2 in the second term of the right-hand side of (9.4). As a result we obtain

$$\Delta J = n \frac{\partial}{\partial p_i} \int \Phi^2(k) \text{Im} \frac{1}{(kv_1 - kv_2) - i(\delta J_{p_1} + \delta J_{p_2})} k_i k_j \times \left(\frac{\partial}{\partial p_{1j}} - \frac{\partial}{\partial p_{2j}} \right) f_1 f_1 dp_2 \frac{dk}{(2\pi)^3}. \quad (10.8)$$

Let us compare this expression with the Landau approximation for the Boltzmann collision integral J_B in Eq. (9.4). It follows from (10.8) if we assume the damping time of the fluctuations to be infinitesimally small, i.e.,

$$\text{Im} \frac{1}{(kv_1 - kv_2) - i(\delta J_{p_1} + \delta J_{p_2})} \rightarrow \pi \delta(kv_1 - kv_2). \quad (10.9)$$

It follows from the foregoing that this is possible only for small-scale fluctuations, for which $\tau_{\text{cor}} \lesssim \sqrt{\epsilon} \tau_{\text{col}} \ll \tau_{\text{col}}$.

To take into account the nonideality effects in the kinetic equation (9.4), it is necessary to add to J_{B} the expression (2.4), and to express ΔJ in a form analogous to (5.9), in which a substitution inverse to (10.9) is made and $4\pi e^2/k^2 \rightarrow \Phi(k)$.

The kinetic theory of fluctuations in a plasma has been developed in^[24], where the spectral functions of the large-scale fluctuations were determined in the polarization approximation and the corresponding expression was obtained for the additional contribution to the collision integrals of charge particles. It is shown there that the boundary separating the small- and large-scale fluctuations is determined by the Debye radius.

11. GENERALIZED KINETIC EQUATION

It was already noted in the introduction that recently there appeared many papers in which attempts were made to obtain, by different methods, the most general kinetic equation that takes direct account of correlations of any order. This problem simply and brilliantly in a recent paper by Zubarev and Novikov^[33], the gist of which is the following.

We express the collision integral in terms of the distribution function of all N particles $F_N = V^N f_N (\int F_N dx_1 \dots dx_N V^{-N} = 1)$. For the spatially homogeneous distribution of the particles it follows from (1.3) that

$$\frac{\partial f_1}{\partial t} = J(p_1, t) = n \int \frac{\partial \Phi_{12}}{\partial r_1} \frac{\partial F_N}{\partial p_1} \frac{dx_2 \dots dx_N}{V^{N-1}}. \quad (11.1)$$

For the function F_N , the Liouville equation is used in^[33]. Solving the equation for F_N under the condition of attenuation of all the initial correlations, we can express the function F_N in terms of the first distribution functions. Substituting this expression in (11.1), we obtain a closed equation for the function F_1 —the generalized kinetic equation.

The use of the Liouville equation as the equation for F_N leads, however, to certain difficulties on changing over to the ordinary (Boltzmann) form of the collision integral and in the establishment of the conservation laws. These difficulties can be avoided by using as the equation for f_N

$$\left(\frac{\partial}{\partial t} + H^{(N)} \right) F_N = \left(\frac{\partial}{\partial t} + \sum_{1 \leq i \leq N} \left(v_i \frac{\partial}{\partial r_i} + F \frac{\partial}{\partial p_i} \right) \right) \prod_i F_i(x_i, t). \quad (11.2)$$

This equation has the same structure as Eq. (1.6) for f_2 in the pair-collision approximation, or Eq. (4.1) with the right-hand side of (4.3) in the triple-collision approximation.

It is important that Eq. (11.2) is not a Liouville equation. This is the N -th equation of the chain of Bogolyubov equations in the approximation in which the collisions of the particle $N+1$ are neglected. We assume here that $N \gg 1$.

In the case of complete weakening of the initial correlations, the solution of Eq. (11.2) for a spatially-homogeneous gas can be written in the form

$$F_N = \prod_{1 \leq i \leq N} F_i(x_i, t) + \int_0^{\infty} \frac{dS^{(N)}}{d\tau} \prod_{1 \leq i \leq N} F_i(x_i, t-\tau) d\tau. \quad (11.3)$$

Substituting (11.3) in (11.1), we obtain the collision integral of the generalized kinetic equation.

The kinetic equations considered above in Chap. 2 and 4 follow from the generalized kinetic equation, if one uses the virial expansions for the nonequilibrium distribution functions. These expansions were obtained by Cohen^[20] and by others.

The collision integral obtained in this manner has the properties (1.9) at $\varphi = 1$ or p_1 . At $\varphi = p_1^2/2m$, it follows from (11.1) and (11.3) that

$$n \int \frac{p_1^2}{2m} J(x_1, t) dx_1 = -\frac{n^2}{2} \frac{\partial}{\partial t} \int \Phi_{12} F_N \frac{dx_1 \dots dx_N}{V^{N-1}}. \quad (11.4)$$

Accordingly, the expression for the density of the internal energy is

$$U = n \int \frac{p_1^2}{2m} f_1 \frac{dx_1}{V} + \frac{n^2}{2} \int \Phi_{12} F_N \frac{dx_1 \dots dx_2}{V^{N-1}}. \quad (11.5)$$

In the equilibrium state, expression (11.3) for f_N coincides with the Gibbs distribution for the system of N particles.

The generalized kinetic equations have not yet been used to solve particular problems.

12. CONCLUSION

Let us examine briefly certain problems of the kinetic theory of nonideal systems.

In Chaps. 2 and 4 we have obtained the kinetic equations for a nonideal gas with allowance for pair and triple collisions. It is possible, as we have seen (Chap. 9), to construct kinetic equations also for denser gases, when collisions of a larger number of particles become important. This method of successive approximations for the construction of kinetic equations for dense gases, however, is not very effective. A different approach is needed here. It is analogous to that used in the calculation of thermodynamic functions in dense gases and liquids.

For thermodynamic functions there exists a virial expansion. In practice, however, one uses only the first few terms of the expansion. The calculation of the thermodynamic functions in dense gases is carried out either by numerical method (the method of molecular dynamics, the Monte Carlo method) or by solving model integral equations for the pair correlation function of the gas molecules.

At the present time, three equations of this type are used. These are the Kirkwood-Bogolyubov-Born-Green equation, in which the Kirkwood superposition approximation is used for the triple distribution function, the Percus-Yevick equation, and the so-called hypernetted chain equation. Information on these equations can be found in the review papers^[62, 63]. In^[64] there is considered one other example of such integral equations for a gas and for a plasma. All these integral equations for the correlation functions take exact account of the contributions of the first three terms of the virial expansion, and approximate account of the contributions of the more complicated interactions.

A similar approach is possible also for the construction of kinetic equations for dense gases and a plasma. For example, for a gas it is necessary to use in place of the kinetic equation, which is a closed equation for the first distribution function, a system of two equations for the functions f_1 and g_2 . The equation for g_2 is a

nonlinear integral equation. It can be chosen such that it leads to expression (4.5) in the triple-collision approximation. Then the system of equations for the functions f_1 and g_2 can be replaced by one equation for the function f_1 , which was considered in Chap. 4. In the equilibrium state, the equation for g_2 can coincide with one of the equations listed above.

This article dealt with kinetic equations for a non-ideal gas and for a fully ionized nonideal plasma. These are the two limiting cases of a partly ionized plasma. In the first case, the degree of ionization is equal to zero, and in the second case it is equal to 100%.

The kinetic theory of a partly ionized plasma is much more complicated. This is due primarily to the need for taking into account intramolecular motions, and also ionization and recombination processes that lead to a change in the concentrations of the free and bound charge particles.

At the present time, only a kinetic theory for an ideal partly ionized plasma has been developed^[65]. On the basis of the microscopic equations, a system of three kinetic equations was constructed for the distribution functions of the electrons, ions, and atoms. The collision integrals were obtained in the polarization approximation (the Born approximation with allowance for the polarization effects). It is also possible to take into account nonideality effects in this approximation.

A more complete development of the kinetic theory was obtained (without the use of perturbation theory in the interaction) for polyatomic gases. The nonideality effects due to molecule interaction were not taken into account. Papers have recently appeared aimed at deriving kinetic equations for reacting gases^[66,67].

This article did not touch upon questions of the kinetic theory of turbulent states of gases and plasma. Naturally, in a turbulent plasma the role of nonideality effects can be appreciable. At the present time, only the theory of weak turbulence has been sufficiently fully developed (see the latest review^[68]).

Problems analogous to those considered above are encountered also in solid-state theory. We cannot, however, consider them here.

We can point to many examples where nonideality effects appear in experimental investigations of processes in gases and in plasma. One of them is the experimentally observed decrease of the electric conductivity of a nonideal plasma in comparison with the conductivity of an ideal plasma (Chaps. 5 and 8). Nonideality effects appear in a plasma also in the presence of a high-frequency field. They determine the additional contributions to the polarization of the medium. The shift of the atomic levels under the influence of electromagnetic radiation is also the consequence of nonideality of the system (atoms and electromagnetic radiation). Its magnitude is determined by the correlation of the fluctuations of the density matrix of the atoms and the fluctuations of the field^[69]. In sufficiently dense gases, the correlation due to the interaction of gas particles influences also the width of the spectral lines, and therefore the analysis of the emission spectrum of dense gases in which fast processes occur is possible only if nonideality effects are taken into account in the kinetic equations.

In conclusion we wish to note once more that the

kinetic theory of nonideal systems began to develop relatively recently, so that many of its problems have not yet been fully solved, and many have not even been formulated.

Note added in proof. To calculate the thermodynamic functions of a nonideal plasma it is useful to use the so-called pseudopotentials (for a review of work on this question see [70]). In [70] there is proposed for the pseudopotential an expression that has a finite value (owing to the quantum effects) at $r = 0$. In the intermediate region, where $a_0 \ll r \ll r_D$ (a_0 is the Bohr radius), the pseudopotential is close to the Coulomb potential, and at large distances it coincides with the Debye potential.

¹⁾It appears that expression (3.2) was first derived by Green [40]. In deriving (3.2), he used the Gibbs expression for the entropy of the entire system.

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