Geometry and physics of the microcosmos

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In this article a critical analysis is given of basic geometric concepts as applied to the physics of the microcosmos. The article begins with an outline of geometry in macroscopic physics. This part is based on the classical papers of H. Poincare, A. Einstein, and A. A. Fridman. An indication of possible limitations on the concept of a point event associated with limiting densities of matter represents new material. Subsequently localization of elementary particles is discussed and limits of possible accuracy are indicated. In the article it is emphasized that the logical structure of local quantum field theory presupposes the existence of particles of arbitrarily large mass. The existence of "maximons"—particles of a limiting, but finite mass—strong gravitation (collapse of particles), instability of particles with respect to decay due to the weak interaction (this interaction can become strong for very heavy particles and can lead to lead to total instability of such a particle). In the conclusion of the article two features of a nonlocal field theory are considered. In this theory the coordinates of a point event are operators, while momentum space remains a numerical space (either curved or flat). The first variant presupposes commutation conditions for the coordinates of a point (the Snyder-Kadyshevskiĭ theory), while the second variant presupposes anticommutation conditions (the author's theory).

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The present article is a description of problems arising in transferring geometrical concepts from classical physics into the world of elementary particles.

1. ORDERING OF EVENTS IN SPACE

Theoretical physics begins with the ordering of events. This is the foundation of all foundations. To order events a set of four numbers $(x) = x_0, x_1, x_2, x_3$—the coordinates of an event—are associated with each point event $\mathcal{P}$; if a single set of four numbers is insufficient, then the event is not a point event. In what follows we shall call this operation the arithmetization of events. The arithmetization of events presupposes a definite physical method of realizing it. This method contains an essential element of convention[1].

Current convention is based: a) on the principle of universal constancy of the velocity of light and b) on the assumption of the existence of "standard" clocks[2].

The arithmetization based on these accepted conventions leads to the Minkowski space with an indefinite metric which we write in the usual notation:

$$ds^2 = c^2\,dt^2 - d\mathbf{x}^2. \quad (1)$$

Two observers $\bar{\Sigma}$ and Σ moving with respect to each other and carrying out the arithmetization by the same method will nevertheless ascribe different coordinates $(\bar{x})$ and (x) to the same event $\mathcal{P}$.

It is assumed that there exists a mutually unique correspondence between the coordinates $(\bar{x})$ and (x) and that the metric (1) is universal. Then the transformation which connects the coordinates $(\bar{x})$ and (x) in the Cartesian system of coordinates is the Poincare-Lorentz transformation

$$\bar{x} = \Lambda(u)\,x + a, \quad (2)$$

where the parameters of the transformation u and a represent the relative velocity of the coordinate systems $\bar{\Sigma}$ and Σ and the relative displacement of the origin of coordinates; $\Lambda(u)$ is the transformation matrix.

We emphasize that the mutual uniqueness of the relation between the coordinates $(\bar{x})$ and (x) of the same event $\mathcal{P}$ is an assumption. Indeed, the parameters of the transformation (2) u and a could be random variables or even operators. In this case the transformation (2) must be complemented by specifying the probability $dw(u, a) \geq 0$ or correspondingly the state vector Ψ upon which the operators $\hat{u}$ and $\hat{a}$ can act (cf., ch. 3).

For the problems considered below it is important that the metric (1) is indefinite. From the indefiniteness of this metric it follows that the concept of the closeness of two events $\mathcal{P}$ and $\mathcal{P}'$ in the space $\mathcal{R}_4(x)$ is not an invariant concept and can be formulated only with respect to a given reference system Σ.

Such in its most important features is the geometry or, using a better expression, the chronogeometry (cf.,[6-8]) which corresponds to the content of the special theory of relativity.

Naturally, one could choose a different physical basis for the arithmetization of events, in analogy with the fact that one can choose different units. In such a case we would arrive at a different geometry and at a different method of describing physical events.

The fundamental advantage of the method which underlies the basis of A. Einstein's relativity theory consists of the fact that specifically in applying this method of arithmetization of events one brings out the invariance of the fundamental laws of physics. A law expressed by the relation

$$F(A, B, x, \ldots) = 0 \tag{3}$$

in the reference system Σ, is expressed in the system $\overline{\Sigma}$ by the relation

$$F(\overline{A}, \overline{B}, x, \ldots) = 0, \tag{3'}$$

where the quantities $\overline{A}$, $\overline{B}$, ... are scalars, spinors, vectors or tensors.

Therefore not every method of arithmetization of events is acceptable. The method of arithmetization, first of all, must be physically realizable (at least in an ideal experiment) and, secondly, it must be maximally universal; this means that it must be based on a set of phenomena which is the most comprehensive[3)].

With this we can conclude the description of the method of arithmetization of events adopted in classical relativistic physics. Possible limitations of this method are discussed below. The discussion of these limitations, as will be seen from what follows, is useful for the understanding of the more complex situation in the world of elementary particles.

2. POSSIBLE LIMITATIONS OF THE ADOPTED ARITHMETIZATION

In classical physics the concept of a point event corresponds well to the concept of a material point—an object of finite mass $m_0 \neq 0$ and of arbitrarily small size $a \to 0$.

In virtue of the assumed continuity of space one can construct at each of its points the space of tangent vectors of infinitely small displacements and the covariant momentum space $\mathcal{R}_4(p)$. The metric of this space is also indefinite and has the form

$$dp^2 = dp_0^2 - d\mathbf{p}^2, \tag{4}$$

where $d\mathbf{p}^2 = dp_1^2 + dp_2^2 + dp_3^2$. This form is determined by the metric adopted in the space $\mathcal{R}_4(x)$. Thus, the structures of the spaces $\mathcal{R}_4(x)$ and $\mathcal{R}_4(p)$ are not independent.

The motion of a material point (or of a system of points) can be formulated in the most general manner in terms of the geometry of Finsler[9]. The Finsler geometry is a generalization of Riemann geometry in the sense that an element of length ds in this geometry in the general case depends not only on the point in space, but also on the direction of the ray towards a neighboring point. In particular

$$ds = L(x, dx), \tag{5}$$

where L is a homogeneous function of the first degree of the displacements dx. In accordance with what was stated above this function can depend in an arbitrary manner on the ratios dx_k/dx_i, and in particular in such a manner that ds would be a relativistic invariant.

If we now consider ds as the differential of a Lagrangian function, then the principle of least action turns out to be identical to the condition that the material point moves along a geodesic in Finsler space: $\delta \int ds = 0$.

No logical contradictions exist between Einstein's method of arithmetization and the mechanics of material points within the framework of the special theory of relativity. Therefore material points and special theory of relativity can be regarded as objects which physically realize the point event $\mathcal{P}(x)$. Limitations come from gravitation. We shall regard a material point as a material particle of finite dimensions a. Let m_0 be its rest mass. Then, if the gravitational radius of this particle a_g

$$a_g = \frac{2km_0}{c^2} \tag{6}$$

(here $k = 6.7 \times 10^{-8}$ cm^3/g sec^2 is Newton's gravitational constant), is greater than its size a, then the metric relationships "inside" the particle are altered in an essential manner. The metric becomes nonstationary and the phenomenon of collapse occurs[10,11]. In this case no signal from the region $r < a_g$ can reach an external observer, and, therefore, no information is possible concerning the ordering of events inside the collapsing particle. From (6) it can be seen that it is useful to have as objects marking the points of space-time, material particles with the least possible mass ($m_0 \to 0$). But as $a \to a_g$ a critical density occurs

$$\rho_g = \frac{3}{4\pi}\left(\frac{c^2}{2k}\right)^3 \frac{1}{m_0^2} \tag{7}$$

This density for $m_0 \to 0$ can exceed the limits known to us from the physics of elementary particles. It is of interest to note that the critical density ρ_g does not exceed the density of elementary particles ρ_0 if $m_0 > M_g = 0.52 \times 10^{-5}$ g, i.e., smaller than the mass of a gravitational "maximon" (cf., Ch. 4).

In this connection a curious question arises: would it not be possible for the method of arithmetization of events adopted in the theory of relativity to lose its validity before the condition $\rho = \rho_g$ is reached. Indeed, if at a certain density of matter $\rho_k < \rho_g$ not a single light signal nor even a neutrino signal can be propagated in the medium because of the exceptionally strong extinction, the ordering of events in such a medium with the aid of light or neutrino waves becomes impossible. Under such conditions a sound signal might turn out to be a more appropriate method for ordering events. The speed of such a signal v can even be greater than the velocity of light in vacuo c, nevertheless no contradiction with the principle of causality arises, since the v-signal and not the c-signal is utilized for ordering events (cf., details in[6]).

Limitations of a different kind on the applicability of the standard method of ordering events arise from stochastic gravitational fields. Fields produced by a turbulent motion of matter inevitably lead to the fact that the metric tensor $g_{\mu\nu}(x)$ becomes a stochastic quantity $\hat{g}_{\mu\nu}(x)$. The interval between events:

$$d\hat{s}^2 = \hat{g}_{\mu\nu}\, dx^\mu\, dx^\nu. \tag{8}$$

also acquires the same nature of a random variable. If the fluctuations $\hat{h}_{\mu\nu}(x)$ of the metric tensor $\hat{g}_{\mu\nu}(x)$ are not great compared to the average values $\langle \hat{g}_{\mu\nu}(x)\rangle = \bar{g}_{\mu\nu}(x)$, then it is useful to represent this tensor in the form

$$\hat{g}_{\mu\nu}(x) = \bar{g}_{\mu\nu}(x) + \hat{h}_{\mu\nu}(x). \tag{9}$$

In this case the ordering of events can be based on the metric defined by the principal part of the metric tensor (cf., [6, 12]). But if the fluctuations are not small, then the ordering of events in $\mathscr{M}_4(x)$ becomes essentially stochastic. Spaces with a stochastic metric have been discussed from an axiomatic point of view in references[13, 14]. But the axiomatic basis for these spaces was restricted to a positive-definite metric. The extension of the axiomatic basis to stochastic spaces of the Minkowski type presents still another problem. The problems relating to this and the paths towards their solutions are described in the monograph[6] and in the essay[15].

The basic question, which is raised more likely by a physicist than by a mathematician, is related to indicating the method of arithmetization of events.

Do we not come here right up to the very boundary of applicability of the concept of ordering events?

Problems associated with the metric in the case of large fluctuations and in the case of extremely high densities of matter will probably become of the highest importance in analyzing the early stages of the "big bang". We know that at present definite laws operate and definite symmetries exist, but there is no basis for asserting that these forms of existence of matter have been prescribed for eternity. The possibility is not excluded that the present day vacuum and the world of elementary particles which is known to us represent only one of the possible paths of evolution of the Universe selected as the result of competition of different possibilities. However at the present stage of development of our knowledge we do not have sufficient data in order to discuss this aspect of the subject in greater detail.

3. A POINT EVENT IN THE MICROCOSMOS

We now turn to the world of elementary particles. At the basis of modern quantum field theory with the aid of which we describe the behavior of elementary particles there stands the condition of locality

$$[\hat{\varphi}(x),\ \hat{\varphi}(y)] = D(x-y), \tag{10}$$

$$D(x-y) = 0 \quad \text{for} \quad (x-y)^2 < 0; \tag{10$'$}$$

here $\hat{\varphi}(x)$ is the operator for the field taken at the point (x), $\hat{\varphi}(y)$ is the operator for the same field taken at the point (y), $[\hat{A}, \hat{B}]$ denotes the commutator of the operators $\hat{A}$ and $\hat{B}$[4)]. Condition (10) is an expression of the principle of causality and denotes the independence of fields if the points (x) and (y) are separated by a space-like interval $(x-y)^2 < 0$. In other words, an arbitrary variation of the field at the point (x) can not affect the field at the point (y) since a signal propagated with the speed $v \leq c$ can not in this case reach the point (y) (and conversely).

Under the conditions of locality (10) the coordinates of the points (x), (y) are assumed to be defined with arbitrarily high accuracy. Such an assumption is equivalent to assuming the existence of point events $\mathscr{P}(x)$, $\mathscr{P}(y)$, and we are going to investigate how noncontradictory is this assumption within the framework of the same local theory.

The natural candidates for the role of representatives of point events are the elementary particles themselves—the analogues of material points of classical physics. But this analogy turns out to be not too far reaching because of a number of peculiarities dictated by the laws of quantum physics.

First of all, all the particles of rest mass $m_0 = 0$ must be excluded from the analogy since they are nonlocalizable in the space $\mathscr{R}_4(x)$. They can be localized only in the tangent space $\mathscr{M}_4(p)$.

But particles of rest mass $m_0 \neq 0$ also present difficulties.

Bosons of rest mass $m_0 \neq 0$ can not be localized in the space $\mathscr{R}_4(x)$ with an accuracy greater than within the limits $\Delta(x) \approx \hbar/m_0c$.

Indeed, the density $\rho(t, \mathbf{x})$ of the meson field $\varphi(t, \mathbf{x})$ which obeys a conservation law is at $t = 0$ equal to

$$\rho(0,\ \mathbf{x}) = \varphi^*(0,\ \mathbf{x})\,\hat{\Omega}\varphi(0,\ \mathbf{x}) \tag{11}$$

(where $\hat{\Omega} = (m_0^2 - \nabla^2)^{1/2}$ is the operator for the frequency, while ∇ is the gradient operator). It is positive-definite only in the region $|\nabla| \ll m_0$, i.e., in the nonrelativistic region. In this case the quantity

$$\rho(0,\ \mathbf{x}) \approx |\varphi(0,\ \mathbf{x})|^2 \geqslant 0 \tag{12}$$

and can be interpreted as the probability density for finding a boson at the point $\mathbf{x}$ at the instant $t = 0$. But for $|\nabla| \ll m_0$ the density $\rho(0, \mathbf{x})$ is distributed in space in the region $|\Delta\mathbf{x}| \gg \hbar/m_0c$.

For spinor particles obeying the Dirac equation there exists a positive-definite probability density

$$\rho(0,\ \mathbf{x}) = \bar{\psi}(0,\ \mathbf{x})\,\psi(0,\ \mathbf{x}) \geqslant 0, \tag{13}$$

where $\psi(0, \mathbf{x})$ is the wave function for a single-particle state. There exists a belief that for a single-particle state $\overline{\Delta x^2} > (\hbar/m_0c)^2$. In actual fact, for a single-particle state the usual indeterminacy relation holds $\overline{\Delta x^2} > \hbar^2/4\overline{\Delta p^2}$ (cf., [6, 16]). However, it is necessary to take into account the exchange of states between the particle under consideration and the particles of the vacuum. This exchange as a result of the Pauli principle leads to the polarization of the vacuum in a region of the order of $\hbar/m_0c$ (cf., [43]). Because of this the position of the initial particle also becomes indefinite within the same region. One must keep in mind that the construction also of a wave packet of size $\Delta x \lesssim \hbar/m_0c$ with the aid of an external field, even when it is switched on adiabatically, will lead to the creation of pairs of particles, so that it is impossible to realize a single-particle state with such a narrow distribution.[5)] Therefore, the exact localization of spinor particles also turns out to be illusory.

We see that in the microcosmos there are no objects which could be a model of a point event $\mathscr{P}(x)$, since elementary particles can not be localized with greater precision than[6)]

$$\Delta x > \frac{\hbar}{m_0 c} \tag{14}$$

In classical physics not only can one consider material points as realizations of a point event, but they can also be selected as a reference object (Bezugskörper), which fixes the reference system. In the world of elementary particles this turns out to be impossible.

If for a reference object one takes an elementary particle of rest mass m_0 then in the Lorentz transformation (2) u will be the four-dimensional velocity of the particle $u = p/m_0c$ (here p is the momentum of the particle) while the spatial components of the displacement a_1, a_2, a_3 will be its coordinates at time t = 0.

From the uncertainty relation

$$[p_i,\ a_k] = i\hbar\delta_{ik} \tag{15}$$

it follows that the parameters of the transformation (2) become operators. Therefore the coordinates $(\bar{x})$ measured with respect to such a reference object also become operators. In particular, from (2) and (15) it is not difficult to evaluate the commutator of $\bar{x}$ and $\bar{t}$

$$[\bar{x},\ \bar{t}] = i\frac{\hbar}{m_0 c}(x - vt), \tag{16}$$

where v is the operator for the three-dimensional velocity of the particle.

Thus, elementary particles of finite rest mass can be utilized neither as objects with the aid of which one can denote points in the space $\mathscr{R}_4(x)$, nor as reference objects.

On the other hand, experimental facts indicate that predictions of a local field theory based on the condition of microscopicity (10) are valid down to a scale of the order of 10^{-15} cm (cf., [18]).

Therefore one should assume that there exist elementary particles of a mass significantly greater than the nucleon mass m_p for which

$$\Delta x \approx \frac{\hbar}{m_0 c} = 2\cdot 10^{-14}\,\text{cm}.$$

From the preceding it follows that a local theory implicitly presupposes the existence of arbitrarily heavy elementary particles ($m_0 \to \infty$). Under this assumption the contradiction between utilizing the concept of arbitrarily exact coordinates of a point in the space $\mathscr{R}_4(x)$ and the absence of objects suitable to play the role of point events would be removed.

The existence of an upper bound on the masses of particles in the form of a certain limit $m_0 = M$ ("maximon") would denote a limitation in principle of the applicability of a local theory to dimensions of the order $\Delta x \sim \hbar/Mc$.

The requirements of an ideal experiment on specifying a point in space-time turn out to be directly opposite in classical physics and in quantum physics. We shall later discuss the possible reasons for the existence of an upper limit on the mass of an elementary particle.

4. GRAVITATION IN THE MICROCOSMOS

Limitations on the mass of elementary particles may arise, just as in macroscopic physics, from considerations of gravitation.

According to the basic idea of A. Einstein the curvature of space-time $R_{\mu\nu}$ and its metric R are determined by the motion of matter. The basic equations of the theory state:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi k}{c^2} T_{\mu\nu}(x). \tag{17}$$

We recall that here $R_{\mu\nu}$ is the curvature tensor, $T_{\mu\nu}$ is the energy-momentum tensor, k is the gravitational constant.

In the detailed description of the motion of matter quantum phenomena necessarily appear on the scene. Consequently, the tensor $T_{\mu\nu}$ must be regarded as a stochastic quantity represented by the operator $\hat{T}_{\mu\nu}$. At the same time the quantities on the left hand side of equation (17) also become operators. In other words, as soon as the motion of matter is treated only with an accuracy up to quantum phenomena, the gravitational field becomes a quantum field[7)].

An entirely different question is one as to what might be the role of gravitational phenomena in the quantum domain.

For example, the ratio of the gravitational field to which zero-point oscillations of a solid body give rise to the field produced by the fundamental mass of its atoms is determined by the fraction $\hbar\omega_0/m_0c^2$, where ω_0 is the Debye frequency, while m_0 is the mass of an atom (or a molecule). In order of magnitude this fraction is equal to $10^{-11}\,A^{-1}$ (A is the atomic weight of the atoms).

We now decompose the energy-momentum tensor $T_{\mu\nu}$ into two parts:

$$T_{\mu\nu}(x) = \overline{T}_{\mu\nu}(x) + \hat{t}_{\mu\nu}(x), \tag{18}$$

where the part $\overline{T}_{\mu\nu}(x)$ is determined by the average motion of matter, while the part $\hat{t}_{\mu\nu}(x)$ is determined by the fluctuations of this motion. We represent the metric tensor $g_{\mu\nu}(x)$ in the form indicated in (9). Then the Einstein equation (17) assumes the form

$$A^{\rho\sigma}_{\mu\nu}\hat{h}_{\rho\sigma} + B^{\rho\sigma\alpha}_{\mu\nu}\frac{\partial\hat{h}_{\rho\sigma}}{\partial x_\alpha} + C^{\rho\sigma\alpha\beta}_{\mu\nu}\frac{\partial^2\hat{h}_{\rho\sigma}}{\partial x_\alpha\,\partial x_\beta} = \frac{8\pi k}{c^2}\hat{t}_{\mu\nu}(x), \tag{19}$$

where the tensors A, B, C depend only on the average tensor $\bar{g}_{\mu\nu}$ and on its derivatives.

If the masses m which determine the average metric are of dimensions of the order of a, then the curvature of space R in order of magnitude is equal to

$$R = \frac{1}{l^2} \approx \frac{1}{a^2}\frac{a_g}{a}, \tag{20}$$

this following directly from the Einstein equation (17); here a_g is the gravitational radius of the body (cf., (6)), while l is a length characterizing the curvature of space. If the characteristic mass of the fluctuation is given by Δm, and its characteristic dimension is b, then Eq. (19) can be schematically written in the form

$$\frac{\alpha}{l^2}\hat{h} + \frac{\beta}{ll'}\hat{h} + \frac{\gamma}{l'^2}\hat{h} \approx \frac{4\pi b_g}{b^3}, \tag{21}$$

where

$$b_g = \frac{2k\Delta m}{c^2} \tag{22}$$

is the gravitational radius of the fluctuation, l' is a length characterizing the gradient of the stochastic field $\hat{h}$; α, β, γ are numerical coefficients.

In accordance with the meaning of equation (14) $l' \ll l$. Further, from the fact that the equation is linear it follows that the length determining the gradient of the tensor $\hat{h}$ and the length determining the gradient of the matter tensor must be comparable. From this it follows that $l' \approx b$. Thus, from (21) we obtain

$$\hat{h} \approx \frac{b_g}{b}. \qquad (23)$$

This equation determines the order of magnitude of the fluctuations of the gravitational field $\hat{h}$ in terms of quantities characterizing the fluctuations of matter.

From this it is not difficult to determine the fluctuations of the metric tensor due to the zero-point oscillations of some quantum field. For example, for a scalar field $\varphi(x)$ the zero-point oscillations of a mass having a scale exceeding b amount to

$$\Delta m(b) = \frac{\Delta E(b)}{c^2} = \frac{b^3}{c^2}\int_0^{1/b} \frac{\hbar\omega_k}{2}\, d^3k = \frac{\hbar}{cb}, \qquad (24)$$

where $\hbar\omega_k/2$ is the zero-point energy of the k-th oscillation.

Substituting (24) into (23) we obtain

$$\hat{h} \approx \frac{8\pi k}{c^2}\frac{\Delta m(b)}{b} = \frac{\Lambda_g^2}{b^2}, \qquad (25)$$

where

$$\Lambda_g = \sqrt{\frac{8\pi k\hbar}{c^3}} = 0.82\cdot 10^{-32}\ \text{cm} \qquad (26)$$

is a certain length which involves both the gravitational constant and the Planck constant.

It can be seen from (25) that this length determines the magnitude of the fluctuations of the metric tensor produced by the quantum fluctuations of the material field. These fluctuations are small over the whole range of frequencies for which

$$b \gg \Lambda_g. \qquad (27)$$

We now turn to the collapsed particle discussed in chapter 2, and take into account quantum effects. A particle of mass m has an effective size of the order of $a = \hbar/mc$; we assume that this particle has reached a particle mass so that its gravitational radius $a_g = a$. Then from the condition $8\pi km/c^2 \approx \hbar/mc$ it follows that

$$a = \Lambda_g, \quad m = M_g = \frac{\hbar}{\Lambda_g c} = 0.52\cdot 10^{-5}\ \text{g}. \qquad (28)$$

Thus, the length Λ_g determines the maximum mass which can be attained by a particle obeying the laws of quantum theory. In this case in accordance with (7) the density of matter attains a limiting value. Such a particle was given by Markov the name of "maximon"[19,20].

From (27) and (28) follow quite different conclusions concerning the role played by gravitation in the world of elementary particles depending on what values of the frequency $\Omega_0 = c/a$ limit in actual fact the spectrum of vacuum fluctuations. According to modern theory it is distributed almost uniformly in frequency, and the high frequencies should give indefinitely great contributions to gravitation. If later it should turn out that for one reason or another the possible frequencies in the microcosmos are limited by an "elementary" length $a \gg \Lambda_g$, then the gravitational effects will not be significant.

In the opposite case they will play a fundamental role in the microcosmos, but in their quantum aspect[8].

Predictions and hopes based on classical calculations of gravitation will be swamped by quantum effects.

At the same time the "classical" average metric will lose its determining significance and the situation will arise noted in ch. 2: the concept of an interval between events, and at the same time the very idea of the possibility of ordering events in $\mathcal{R}_4(x)$ becomes more than doubtful. We approach here the edge of an "abyss" into which, possibly, it is yet too early to peer.

In what follows we shall consider other possibilities for limiting a local theory. Among the competitors of gravitation in this role we shall find the "weak" interaction.

5. "A WEAK MAXIMON"

At the present time we distinguish three types of interactions: the strong interaction, the electromagnetic and the weak one. We compare their behavior at high energies utilizing a criterion proposed in reference[22]. According to this criterion an interaction is said to be strong if in the process of interaction the density of the kinetic energy of the particles ϵ_k is considerably smaller than the absolute value of the density of the energy of their interaction:

$$\varepsilon_k \ll |W|. \qquad (29)$$

We discuss from the point of view of this criterion first the collision of a nucleon (N) and a pion (π). The density of the total energy in this case is equal to

$$H = \hbar c\bar{\psi}\,\partial\psi + Mc^2\bar{\psi}\psi + \frac{1}{2}(\Box\varphi^2 + m^2\varphi^2) + g\bar{\psi}\gamma_5\tau\varphi\psi, \qquad (30)$$

where ψ is the nucleon field, φ is the meson field, M is the nucleon mass, m is the meson mass, $\partial = \gamma^\mu\partial/\partial x_\mu$, g is the interaction constant. Let l be the length determining the value of the gradient in the c.m.s. ($l \approx \hbar/p = \lambda$, p is the momentum of the particles). Then the density of the kinetic energy of the nucleon is of the order of magnitude of

$$\varepsilon_N \approx \frac{\hbar c}{l}\bar{\psi}\psi \qquad (31)$$

(since $\partial \sim 1/l$), the density of the meson kinetic energy is equal to

$$\varepsilon_\pi \approx \frac{\varphi^2}{l^2} \qquad (32)$$

(since $\Box^2 \sim l^{-2}$). From this it follows that

$$|W| \approx g\bar{\psi}\psi\varphi \approx \frac{g}{\hbar c l^2 \varepsilon_N \sqrt{\varepsilon_\pi}}. \qquad (33)$$

From condition (29), having in mind that $\epsilon_k = \epsilon_N + \epsilon_\pi$, we obtain

$$1 \ll \frac{g}{\hbar c}\frac{\varepsilon_N}{\varepsilon_N + \varepsilon_\pi}\sqrt{\varepsilon_\pi}\, l^2. \qquad (34)$$

Further we have

$$\varepsilon_\pi \approx \frac{pc}{l^3} \approx \frac{\hbar c}{l^4}, \qquad (35)$$

$$\frac{\varepsilon_N}{\varepsilon_N + \varepsilon_\pi} \approx \frac{1}{2}. \qquad (36)$$

As a result of this we obtain

$$\frac{g^2}{\hbar c} \gg 1, \qquad (37)$$

from which it follows that the strong interaction in accordance with our criterion is strong under all conditions (since the inequality (37) is always satisfied).

We now apply the same criterion to the interaction of the electromagnetic field with a charge spinor particle. In this case we have

$$W = e\bar{\psi}A\psi, \tag{38}$$

where $A = \gamma^{\mu}A_{\mu}$, A_{μ} is the vector-potential, e is the charge of the particles. Following the same procedure we obtain

$$\frac{e^2}{\hbar c} \gg 1. \tag{39}$$

This inequality is not satisfied. Consequently, in accordance with our criterion the electromagnetic interactions do not belong to the number of strong interactions[9)].

We now turn to the case of interest to us of a weak interaction. The total energy density now has the form

$$H = \hbar c\bar{\psi}\gamma\psi + Mc^2\bar{\psi}\psi + \hbar c\bar{\varphi}\,\partial\varphi + mc^2\bar{\varphi}\varphi + g_F\bar{\psi}O_\alpha\psi\bar{\varphi}O^\alpha\varphi, \tag{40}$$

where ψ is the nucleon field, φ is the lepton field, M and m are the masses of these particles. G_F is the Fermi constant, O_α is a spinor operator. It can easily be seen that in this case the energy of the interaction W is of the order of magnitude of

$$|W| \approx g_F \frac{\varepsilon_N l}{\hbar c}\frac{\varepsilon_l l}{\hbar c}, \tag{41}$$

where ϵ_N is the density of the kinetic energy of the nucleons, while ϵ_l is the density of the kinetic energy of the leptons. From condition (29), having in mind that $\epsilon_N \approx \epsilon_l \approx pc/l^3 = \hbar c/l^4$, we obtain

$$\frac{\Lambda_F^2}{l^2} \gg 1, \tag{42}$$

where $\Lambda_F = \sqrt{g_F/\hbar c} = 0.66 \times 10^{-16}$ cm, $l \approx \lambda = \hbar/p$. From this it follows that a weak interaction becomes strong when the energy of the particles is given by $E \sim \hbar c/\Lambda_F \sim 300$ GeV (cf., also[23,24]).

We now consider the decay of a heavy hadron of mass M determined by the weak interaction: $M \to m + l + \tilde{\nu}$; here m is the nucleon mass, l is the lepton, $\tilde{\nu}$ is the antineutrino. The decay constant Γ for a process of the type indicated above with $M \gg m$ is equal to[25]

$$\frac{\Gamma}{M} = \frac{1}{4\pi^3} G_F^2 M^4 N, \tag{43}$$

where $G_F = (g_F/\hbar c) \times 10^{-5}/m^2$, N is the number of channels of different decays which need not be small. From this formula it can be seen that with a hadron mass

$$M > m_F = \frac{\hbar}{\Lambda_F c} \tag{44}$$

the decay constant Γ becomes comparable with the hadron mass M and the hadron ceases to exist as an elementary particle since it can not be ascribed any definite mass. It is useful to give the name of a weak maximon to such a conventional particle of mass M_F. This nomenclature is all the more justified since the weak interaction at distances of the order $R \lesssim \Lambda_F$ leads to a mass defect D equal to the mass of the maximon. This result follows from the calculation of pair (lepton) forces which was first carried out in[47]. In accordance with this calculation the potential for such an interaction V is equal to $V = -(2\pi)^{-3}(\Lambda_F/R)^5 M_F c^2$. Therefore for $R < \Lambda_F$ the mass defect $D \sim V/c^2 \approx M_F$, so that $M_F + M_F = D \sim M_F$. This limitation on the mass of the particles, as can be seen from (44) and (26), occurs before the limitation dictated by gravitation, since $M_F \ll M_g$. At the same time the assumed limitation of the local theory in this case must occur considerably earlier than would follow from an assumption of the existence of a gravitational maximon M_g.

6. "BLACKNESS" OF PARTICLES AND LOCALITY

An elementary particle represents a certain medium described by the creation and annihilation of virtual particles.

It is natural to pose the question of the conditions of propagation of a metric signal in such a peculiar medium. If one uses perturbation theory then the answer to this question is given by the Green's function which, being based on a local theory, guarantees the propagation of an interaction with the velocity of light.

However, the situation is changed if the interaction becomes strong. In this case there arise, firstly, nonlinear phenomena and, secondly, a strong absorption as a result of inelastic processes.

The first group of phenomena occurs in the domain of strong fields and small field gradients. In references[26,27] it was shown on the examples of a scalar field and the electromagnetic field that the law for the propagation of these fields is essentially altered including even the disappearance of any possibility of propagation: the characteristics of nonlinear equations become imaginary and the equation is converted from a hyperbolic type into one of elliptic type. The situation arising in this case has been called a "blob" of events. A more modern terminology would be "light collapse"[6,22].

In the domain of high gradients inelastic processes appear. Reference[29] called attention to a possible limitation of the space-time description of the structure of elementary particles arising from the fact that the cross section of an inelastic process does not diminish with increasing energy, but tends to a constant limit or even increases slowly.

At the same time elastic scattering assumes the nature of diffraction scattering by a "black" sphere of dimension a. In particular, on the basis of the first articles on the scattering of pions by nucleons it was noted[28,29] that the "effective" potential for such scattering is purely imaginary and is well represented by the formula[10)]

$$\tilde{V}(q) = iA(E)\,e^{-a^2q^2}; \tag{45}$$

here q is the transferred momentum, A(E) is a certain function of the energy E determined from the principal diffraction maximum.

In the case when the role of inelastic processes becomes predominant the information refers not so much to the space-time structure, as to the creation of new particles.

The resulting "blackness" of the particle prevents one from using elastic scattering for studying the space-time distribution of matter. The example quoted above involving mesons is a very particular one, and therefore does not have significance as a matter of principle.

For the problem studied in the present paper only such a situation would be of interest in which "blackness" would arise for the most universal metric signal. The most universal ones are the weak interactions. If we admit the growth of a weak interaction up to values dictated by the unitary limit, then the possible limitation of a local theory by the conditions for the propagation of a signal inside an elementary particle coincides with the condition arising from the existence of a "weak maximon".

7. THE MOMENTUM SPACE $\mathscr{R}_4(p)$

The classical theory operates simultaneously with the space $\mathscr{R}_4(x)$ and the contravariant tangent space $\mathscr{R}_4(p)$. A different situation exists in the domain of quantum phenomena. In quantum motion the trajectory of a material point is nondifferentiable (cf., [31]), and the spaces $\mathscr{R}_4(x)$ and $\mathscr{R}_4(p)$ are mutually complementary. They refer to two different incompatible classes of measurement.

Both these spaces are theoretically equivalent, since the transition from a description in one of them to a description in the other is accomplished with the aid of a unitary transformation of state vectors Ψ and the corresponding transformation of the operators $\hat{L}$, representing physical quantities.

However, these two descriptions are nonequivalent in a physical experiment. The space $\mathscr{R}_4(x)$ occurs in an experiment in its macroscopic aspect. The microscopic ordering of events does not manifest itself directly in an experiment since the causality observed in an experiment is macroscopic.

Indeed, in order that an event A situated in a space-time region $\mathscr{G}_A(x)$ can be regarded as the cause of an event B situated in the domain $\mathscr{G}_B(y)$ it is necessary to be certain that when A occurred a quantum was emitted of energy $\epsilon = \hbar\omega \geq 0$ and of momentum $\mathbf{p} = \hbar\mathbf{k}$, which later was absorbed in the domain $\mathscr{G}_B(y)$ generating thereby the event B.

In this description of a causal connection we utilize both spaces $\mathscr{R}_4(x)$ and $\mathscr{R}_4(p)$; the former in order to note the mutual position of events A and B and the latter in order to indicate the direction of the transmission of energy and momentum [6]. The simultaneous utilization of the mutually complementary spaces $\mathscr{R}_4(x)$ and $\mathscr{R}_4(p)$ leads us into the domain of classical, i.e., macroscopic physics.

Consequently, the space-time description is realized with an accuracy far from sufficient to make limitations of type (14) noticeable.

In contrast to the space-time description, the energy-momentum description in the space $\mathscr{R}_4(p)$ is realized experimentally with an accuracy which appears to be unlimited. In this description the microscopic causality expressed by the condition of local commutativity (10) manifests itself only indirectly in the behavior of the amplitudes $T_{if}(p)$ predicted on the basis of a local theory (here the letter i indicates the initial state, while f indicates the final state of different physical processes). In particular, microcausality finds its expression in the analytic properties of the amplitude $T_{if}(p)$ in the complex plane of the variable p[11]. In the space $\mathscr{R}_4(p)$ the state of free stable particles is described by points on the hyperboloid

$$p^2 \equiv p_0^2 - \mathbf{p}^2 = m_0^2, \tag{46}$$

where m_0 is the particle mass. Each such hyperboloid is a Lobachevsky space $R_3(\mathbf{p})$ of curvature $R = -1/m_0^2$ [33].

In the space $\mathscr{R}_4(p)$ due to the indefiniteness of its metric (4) there exists no invariant concept of a large or a small momentum. Because of this there also do not exist any invariant limitations on the frequency ω or on the propagation vector $|\mathbf{k}|$. Such a restriction would necessarily pick out some one reference system. This statement is a supplementary one to the assertion concerning the absence of an invariant measure of nearness of events in the space $\mathscr{R}_4(x)$.

The amplitudes $T_{if}(p)$ describing physical processes are matrix element of the scattering matrix S:

$$S_{if} = \delta_{if} + iT_{if}. \tag{47}$$

As is well known, this matrix determines the state of the particles at the "instant" of time $t_f = +\infty$, if it is given at the "instant" of time $t_i = -\infty$. From a geometrical point of view the S-matrix transforms the state of the particles given in a certain direct product of the Logachevsky spaces $R_3(\mathbf{p}_1) \times R_3(\mathbf{p}_2) \ldots R_3(\mathbf{p}_i)$ into a new state given, generally speaking, in another product of such spaces $R_3(\mathbf{p}_1') \times R_3(\mathbf{p}_2') \ldots R_3(\mathbf{p}_f)$.

Since the momenta of the particles are given, the coordinates of the particles are indeterminate, and also the "instants" of time $t = \pm\infty$ are indeterminate. Therefore the ordering of events in $\mathscr{R}_4(x)$, attained with the aid of the S-matrix, is minimal.

Contrary to a widespread opinion a description of the phenomena of the microcosmos with the aid of an S-matrix is incomplete. By means of an S-matrix it is impossible to describe the behavior of unstable particles since the establishment of initial conditions in this case can not refer to the instant of time $t = -\infty$[12].

A situation arising in the case of K^0-mesons can serve as an illustration of this assertion when it is necessary to trace the evolution of the state:

$$\bar{K}^0 = \frac{1}{\sqrt{2}}\left[K_S^0(t) - K_L^0(t)\right], \tag{48}$$

where $\bar{K}^0$ are the states of the antimeson, while K_S^0, K_L^0 are the states of the short- and the long-lived mesons, and t is the time. Old fashioned methods of description appear here to be inevitable, since an ordering of events in time is needed with an accuracy $\Delta t \ll \tau_S$—the lifetime of the shortlived K^0-meson.

These remarks referring to the S-matrix do not restrict the possibilities of a description in the space $\mathscr{R}_4(p)$ which may be extended into the domain of complex values of p. Moreover such an extension appears to be necessary for the description of the behavior of unstable particles.

Therefore, in spite of the formal equivalence of the description of phenomena in $\mathscr{R}_4(x)$ and $\mathscr{R}_4(p)$, a description in the latter space is less vulnerable to that criticism which is addressed against a local theory operating in the space-time $\mathscr{R}_4(x)$.

Apparently it is just in this connection that already in the 1940's Snyder [34] published an attractive idea in accordance with which the metric of the momentum space $\mathscr{R}_4(p)$ can be more complicated than the Minkowski metric (4); in particular, instead of (4) a Riemann metric is proposed

$$dp^2 = g_{\mu\nu}\, dp_\mu\, dp_\nu, \tag{49}$$

where the metric tensor $g_{\mu\nu}$ is a function of the momentum p:

$$g_{\mu\nu} = g_{\mu\nu}(p,\ p_\alpha), \tag{50}$$

and of the parameter

$$p_\alpha = \frac{\hbar}{a}; \tag{51}$$

here a is a certain "elementary length" while p_α is a momentum which determines the scale of the curvature of the momentum space.

The relation of the space $\mathscr{R}_4(x)$ to the space $\mathscr{R}_4(p)$ is based on the assumption that the curved space is a space of constant curvature[13]. This restriction enables one to treat the coordinates x_0, x_1, x_2, x_3 as displacement operators in space

$$x_\mu \to \hat{x}_\mu \equiv \left(i \frac{\partial}{\partial p_\mu} + A_\mu^\nu \frac{\partial}{\partial p_\nu} \right). \qquad (52)$$

For a = 0 $A_\nu^\mu(p) = 0$, so that (52) goes over into a representation of the coordinate operators characteristic of a local theory. But also in this variant the space $\mathscr{R}_4(x)$ ceases to exist as a four-dimensional set of points representing coordinates of point events; the point is that the operators x_μ do not commute among themselves:

$$[\hat{x}_\mu, \hat{x}_\nu] = iL_{\mu\nu} \neq 0. \qquad (53)$$

As a result of this the four-dimensional space $\mathscr{R}_4(x)$ decomposes into one-dimensional rays directed from or consisting of points ("quantization" of space-time).

The geometry of a space of constant curvature imposes a limitation either on the magnitude of a timelike momentum, or on the magnitude of a spacelike momentum. In the former case a limitation is imposed on the mass of the particles:

$$p_0^2 - \mathbf{p}^2 = m_0^2 < \frac{\hbar^2}{a^2} \equiv M_0^2. \qquad (54)$$

This possibility is in agreement with the concept developed in the present paper in accordance with which the real limit to the applicability of a local theory arises in the case if for one reason or another there exists a limiting value of the mass of the particles ("maximon"). The mass $M_0 = \hbar/a_0c$ has the meaning of a "geometrical maximon"[14].

The condition (54) should not apply to macroscopic systems whose mass can be arbitrarily large. Therefore a problem arises: how can we restrict the applicability of these conditions to the world of elementary particles? In this connection reference[39] is of interest which develops a variant of Snyder's theory in which an extension of momentum space beyond the limits of the mass shell (54) forms a de Sitter space. This space can be regarded as a four-dimensional surface on a five-dimensional hyperboloid:

$$p_0^2 - p_1^2 - p_2^2 - p_3^2 - p_4^2 = M_0^2. \qquad (55)$$

In this theory the total momentum P of a system of particles remains in the flat space $\mathscr{R}_4(P)$, while the internal momenta of the system belong to a de Sitter space. The theory is attractive not only because in it there is contained from the outset an assumption of the existence of a maximon, but also because it, most probably, can be developed in an axiomatic form. Another variant of a nonlocal theory also developed in axiomatic form is based on a nonlocal field $\Psi(x)$ for which a generalization can be given of the condition of local causality given in the form of a T-product[40]:

$$T(\Psi(x)\Psi(y)) = \mathscr{D}_c(x - y, a), \qquad (56)$$

where $\mathscr{D}_c(x - y, a)$ is a nonlocal causal function. Its properties can be seen most clearly from the Fourier-representation

$$\mathscr{D}_c(x, a) = \int \widetilde{K}(p, a)\, e^{ipx} d^4p, \qquad (57)$$

where

$$\widetilde{K}(p, a) = \frac{\widetilde{V}(p, a)}{p^2 - m^2 + i\varepsilon}, \qquad (58)$$

with $\widetilde{V}(m, a) = 1$, while the other function $\widetilde{K}(p, a)$ is an integral function which vanishes as $p^2 \to -\infty$ and whose increase with $|\mathbf{p}|$ is of order $\rho \geq 1/2$[41].

At first glance this theory, which operates, just as a local theory, with exact values of coordinates in the space $\mathscr{R}_4(x)$, has no relation to any kind of a modification of geometry. But in reference[42] it is shown that the nonlocal field $\Psi(x)$ can be regarded as an average of the field $\Psi(x)$ defined in the stochastic space $\Gamma_4(\hat{x})$ in which the operators for the coordinates $\hat{x}_\mu$ are equal to

$$\hat{x}_\mu = x_\mu + a\gamma_\mu,$$

where γ_μ are the Dirac matrices and a is a certain length. It is assumed that the average of $\hat{x}_\mu$ is equal to x_μ. The averaging is carried out over the distribution dw(a) of the length a concentrated near the maximon mass M_0, i.e., near $a \approx a_0 = \hbar/M_0c$.

Both variants of the nonlocal theory considered above are based on the assumption of a new metric of space.

As is well known, in the general theory of relativity the metric is not prescribed externally, but is formed by the self-consistent motion of matter.

One can assume that also in the case of the microcosmos the metric of space-time can be dictated by the field of the elementary particles.

The weak interaction which, in all probability, is sufficiently universal could claim to exert an influence on the metric in the domain of extremely small dimensions.

This may be so or not, but it is clear that not only a quantization of space in the spirit of the conditions (53) or (59), but any dependence of the metric on the motion of microparticles inescapably leads us into the domain of stochastic spaces. A common feature of spaces of this kind is the probabilistic ordering of point events.

The method by means of which a new probabilistic aspect enters in such a case into the theory of the microcosmos differs in principle from the one which is introduced by the quantum-mechanical description of the fields.

Statistics in this case extends not only to kinematics and dynamics, but also to the ordering of point events in space-time.

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1) We note that we are here speaking particularly about conventions which have been emphasized already in the first papers of H. Poincare [1] and A. Einstein [2]. Cf., also [3] and particularly the brilliant book of A. A. Fridman [4]. Lately this question was the subject of discussion in the pages of Usp. Fiz. Nauk [5]. These questions were also discussed in the monograph [6].

2) In principle the function of such a clock can be served by a "light" clock consisting of a light pulse periodically reflected between two closely situated mirrors. Then assumption b) is equivalent to the assumption of the existence of an invariant standard of length—the distance between the mirrors. At the present time the wavelength of one of the krypton lines has been accepted as such a standard. For details concerning the choice of a clock cf., the thesis of R. Martske [7] (cf., also [8]).

3) Thus, for example, a convention founded on the velocity of sound u in place of the velocity of light c would introduce into a discussion of all physical events the quite peculiar features of sound phenomena. Similarly, the measurement of lengths by means of a spring dynamometer would introduce into the discussion of all phenomena the very specific properties of a spring (cf., in this connection [4,6].) For the same reason the choice of the coordinate system, which in principle is arbitrary, in actual fact must correspond in the best possible manner

to the nature of the problem being investigated so as not to obscure the essence of the phenomena.

4) Or the anticommutator if the field is a spinor field. We have explicitly written out the condition for the scalar field $\varphi(x)$.

5) For example, in a compound nucleus formed when two nuclei with charges Z_1, Z_2 approach each other closely, under the condition $Z_1 + Z_2 > 137$ an electron orbit appears of radius $a_0 \approx h/m_0c$. However, e^+, e^- pairs will be created adiabatically in this case, and thus the phenomenon is not a single-particle one (cf., in this connection [17]).

6) Attention was paid to the possible significance in principle of such an inaccuracy already in the early stages of the development of quantum field theory (cf., [44-46]).

7) Other opinions also exist concerning the question of the possibility of extending the Einstein equations into the domain of quantum phenomena. The approach described above is the most natural development of Einstein's idea.

8) This direction is being developed already for many years by Wheeler and co-workers (cf., [21]).

9) This conclusion is based on the interaction (38). Vector mesons are not taken into account in the estimate given above.

10) A potential of this kind is at present being successfully used as a first approximation in the theory of the "quasipotential" [30].

11) The "dispersion relations" which are important for the analysis of experimental data [32] are based on these properties.

12) With the exception of certain special cases when, for example, an unstable particle can be regarded as a resonance.

13) Snyder's idea has been developed further in the papers by Yu. A. Gel'fand [35], V. G. Kadyshevskiĭ [36] and I. E. Tamm [37].

14) We do not investigate the second possibility.

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