Weak interactions at short distances

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The article reviews the present status of the theory of weak interactions of elementary particles at high energies and large momentum transfers; in the spatial picture this corresponds to short distances. The usual four-fermion *V-A* theory of weak interactions and the theory with intermediate *W* boson are considered. Special attention is paid to virtual processes, particularly hadron-lepton and nonlepton weak interactions (weak neutral currents, K_L^0 and K_S^0 meson mass difference, hadronic processes with strangeness and parity change). It is shown that a theoretical investigation of these processes in the comparison of its results with experiment leads to serious contradictions in the theory. Different ways out of the difficulties existing in the theory are discussed.

1. INTRODUCTION. THE ESSENCE OF THE PROBLEM

The weak interactions become stronger as the energy increases. This circumstance, which was pointed out by Heisenberg[1] as early as 1936, is related to the fact that the coupling constant G of the four-fermion theory of weak interactions is dimensional: $G = 10^{-5}/m^2$ (here m is the nucleon mass, $\hbar = c = 1$). From this simple fact and dimensional considerations it follows that in the four-fermion theory the cross sections for two-particle processes, such as, for instance, elastic neutrino-electron scattering, must behave (in first approximation of perturbation theory) at high energies like $\sigma \sim G^2 s$, where $s = 4E^2$, E is the energy of each of the particles in the center-of-mass system. Experimentally the increase of the cross sections of weak interactions as a function of energy is observed[2] in the scattering of neutrinos on nucleons, where one finds approximately

$$\sigma^{\text{tot}}_{\nu_\mu N} \approx \frac{G_s^2}{4\pi}, \quad s \approx 2mE_\nu; \tag{1.1}$$

here $\sigma^{\text{tot}}_{\nu_\mu N}$ is the total interaction cross section of muonic neutrinos with nucleons and E_ν is the neutrino energy in the laboratory system. (The relation (1.1) has been tested up to $E_\nu \approx 10$ GeV.) It also follows from dimensional considerations that at high energies the cross sections of inelastic weak processes involving the production of n lepton pairs, in first nonvanishing order of perturbation theory will be of the order $\sigma_n \sim G^2 s (Gs)^{2n}$. Thus (and this also was already known to Heisenberg[1]), for energies $s \sim 1/G$ the cross sections of weak inelastic processes turn out to be of the same order as the cross sections for elastic processes, i.e., the weak interaction becomes effectively strong, although the cross sections of the weak interactions are still small compared to the cross sections of the strong interaction processes. More precisely, one should say that the weak interactions become strong at short distances $r \lesssim G^{1/2} \approx 6 \times 10^{-17}$ cm, since large values of the cross sections for inelastic processes appear at high energies on account of the large momentum transfers $q \sim s^{1/2}$, corresponding to short distances $r \sim 1/q$.

Such a situation (albeit for somewhat different reasons) arises also in the theory of weak interactions involving an intermediate vector boson W.

The growth of the cross sections of inelastic processes with the energy is related to the so-called non-renormalizability of the theory of weak interactions, i.e., to the fact that the higher orders of perturbation theory in the weak coupling constant turn out to be divergent, and the degree of divergence increases with the order of the approximation; they are of order $(G\Lambda^2)^n$, where Λ^2 is an effective squared mass of the virtual particles (Λ^2 corresponds to the quantity s for real processes). It would be natural to expect a picture where the weak interaction amplitudes increase according to perturbation theory up to values $\Lambda^2 \sim 1/G$ where the weak interaction begins to become strong, and then the weak interaction cuts itself off. Such a behavior of the amplitudes would mean at the same time that the higher order corrections with respect to the weak interactions to the amplitudes of low-energy processes will, in general, be of the order of unity.

We are thus led to the following problem. On the one hand, the experimental data (ν_μN-scattering) and the theory tell us that the amplitudes of weak processes grow with energy in agreement with perturbation theory, so that corrections of higher order in the weak interaction to processes at low energies (compared to $1/G^{1/2}$) must be essential. On the other hand, the weak interaction does not conserve parity, strangeness and isospin, whereas it is well known from experiments that in the strong and electromagnetic interactions at low energies (compared to $G^{-1/2}$) parity and strangeness are conserved to a high degree of accuracy—of the order 10^{-6}—and isospin is conserved to order $\alpha \sim 10^{-2}$. In addition, the higher approximations in the weak interaction should lead to weak hadron-lepton processes with the emission of neutral leptonic systems, which to date have not been observed experimentally, in spite of the high accuracy of the experiments that were carried out.

Thus, the contribution of virtual weak interactions to low energy processes turns out experimentally to be very small, which points to an inconsistency (or maybe even to a contradiction) in the present-day treatment of weak interaction theory.

A way out from these contradictions might consist in the fact that, for some reason unknown to us now, the weak interaction is cut off at distances considerably larger than $G^{1/2}$. It is quite likely that such a cut-off can have a fundamental physical significance[1)].

The main purpose of the present review article is to analyze the higher order corrections of perturbation theory (with respect to the weak interactions) to the amplitudes of low energy processes, with the purpose of finding out where such contradictions have the sharpest character, i.e., to find the "hot spots" of the theory; this will make it possible to construct a new theory of weak interactions which is free from the indicated difficulties.

We shall first consider the behavior of the cross sections of real weak elastic and inelastic processes at high energies. The exposition of these parts of the review does not claim to be complete, its purpose is to give the reader a physical picture. After that we discuss consistently (and as completely as possible) the contribution of virtual weak interaction at high energies to the amplitudes of various low-energy processes. Finally, at the end of the review, we indicate new theoretical schemes for weak interaction theory, proposed recently with the purpose of overcoming the difficulties which appear.

2. THE FORM OF THE WEAK INTERACTION

We start with the universal V – A theory of the weak interactions[3,4] and consider two versions of the theory: the four-fermion interaction, where the weak interaction Hamiltonian is of the form[2)]:

$$\mathscr{H}_w=\frac{G}{\sqrt{2}}\,j_\lambda^+(x),\ j_\lambda^-(x), \tag{2.1}$$

and the theory with intermediate W bosons, with the interaction Hamiltonian

$$\mathscr{H}_w=gj_\lambda^+(x)\,W_\lambda(x)+gj_\lambda^-(x)\,W_\lambda^+(x); \tag{2.2}$$

here $j_\lambda^\pm(x)$ is the weak current, which is the sum of the leptonic and hardonic parts:

$$j_\lambda^-(x)=j_\lambda^{-l}(x)+j_\lambda^{-h}(x),\quad j_\lambda^+=(j_\lambda^-)^+, \tag{2.3}$$

the leptonic current having the form

$$j_\lambda^{-l}=j_\lambda^{-\mu}+j_\lambda^{-e},$$
$$j_\lambda^{-\mu}=\bar\psi_\mu\gamma_\lambda(1+\gamma_5)\,\psi_{\nu_\mu},\quad j_\lambda^{-e}=\bar\psi_e\gamma_\lambda(1+\gamma_5)\,\psi_{\nu_e}, \tag{2.4}$$

and the hadronic current can be written in the form

$$j_\lambda^{-h}=(V_\lambda-A_\lambda)_2^1\cos\theta_C+(V_\lambda-A_\lambda)_3^1\sin\theta_C, \tag{2.5}$$

where θ_C is the Cabbibo angle, $V_{\lambda k}^i$ and $A_{\lambda k}^i$ are the vector and axial vector currents, i, k are SU(3) indices (we use the notations of the review article by Berestetskiĭ[5]) The normalization of the hadronic current is such that, for example, in the quark model we have

$$j_\lambda^{-h}=\bar n^1\gamma_\lambda(1+\gamma_5)\,p^1\cos\theta_C+\bar\Lambda^1\gamma_\lambda(1+\gamma_5)\,p^1\sin\theta_C. \tag{2.6}$$

The coupling constant g of the W-boson is related to the Fermi coupling constant G by $4\pi g^2/m_W^2=2^{-1/2}G$. We shall not make any assumptions on the value of the mass m_W of the W boson, and thus on the magnitude of the semiweak coupling constant g. We shall neglect the effect of CP-violation.

3. THE REAL PROCESSES AT HIGH ENERGIES

3.1. Processes in first order of G. a) Elastic scattering of leptons. We consider the process of elastic scattering of electronic neutrinos or antineutrinos on unpolarized electrons. In the first approximation in G the differential cross sections of these processes, computed in the four-fermion theory, according to the usual Feynman rules have the form

$$\frac{d\sigma_{\nu_e e}}{d\Omega}=\frac{G^2}{(2\pi)^2}\,s, \tag{3.1}$$

$$\frac{d\sigma_{\bar\nu_e e}}{d\Omega}=\frac{G^2}{(2\pi)^2}\,s\cdot\frac{1}{4}\,(1+\cos\theta)^2, \tag{3.2}$$

where $s=4E^2$, E and θ are the energy and the scattering angle in the c.m.s. (In (3.1) and (3.2) the electron mass has been neglected compared to E, the same approximation will be made in the sequel.) The total cross sections are respectively

$$\sigma_{\nu_e e}=\frac{G^2s}{\pi}, \tag{3.3}$$

$$\sigma_{\bar\nu_e e}=\frac{G^2s}{3\pi}. \tag{3.4}$$

The main contribution to the total cross sections (3.3) and (3.4) comes from the region of large momentum transfers $-q^2=4E^2\sin^2\theta/2\sim s$, which in the space-time picture corresponds to a main contribution of small impact parameters $r\sim 1/(-q^2)^{1/2}\sim s^{-1/2}$.

The differential and total cross sections for the process $\nu_\mu+e\to\mu+\nu_e$ is described by the same equations (3.1) and (3.3) as the cross sections for elastic ν_ee scattering, and the cross section for the $\bar\nu_\mu$e interaction vanishes in the approximation under discussion. One partial wave contributes to cross sections (3.1) and (3.2) which rise linearly with s. But the contribution of a single partial wave to the total cross section cannot exceed a magnitude of the order $\pi\lambda\!\!\!^-{}^2=4\pi/s$, which restricts the growth of the cross sections (3.1), (3.2) due to a single partial wave. (This circumstance was first pointed out by Blokhintsev[6]; earlier a similar consideration was made by Landau[48] for the problem of photon scattering by a spin-1 particle.)

It is worthwhile considering this question somewhat more in detail. In the weak interaction Hamiltonian (2.1) the lepton wave functions enter only by two components; e, ν and μ always exhibiting only left-handed helicity, whereas e^+, $\bar\nu$ and μ^+ are always right-handed. At high energies, when the lepton masses are negligible, helicity is conserved in weak interaction processes, so that in ν_ee scattering $\lambda_{\nu_e}=\lambda_e=-1/2$, and in $\bar\nu_e$e scattering $\lambda_{\bar\nu_e}=1/2$, $\lambda_e=-1/2$. Owing to the pointlike structure of the weak interaction Hamiltonian (2.1) (i.e., due to the absence of coordinate derivatives in (2.1)) it follows that the only partial wave contributing to ν_ee scattering has j = 0, and the only partial wave contributing to $\bar\nu_e$e scattering has j = 1.

As is well known[7], the amplitude of an arbitrary two-particle reaction a + b → c + d with given values of the helicities, $f_{\lambda_c\lambda_d;\lambda_a\lambda_b}$, has the following partial-wave expansion:

$$f_{\lambda_c\lambda_d;\,\lambda_a\lambda_b} = \frac{1}{2p}\sum_j (2j+1)\, f^j_{\lambda_c\lambda_d;\,\lambda_a\lambda_b}\, e^{i(\lambda-\mu)\varphi}\, d^j_{\lambda\mu}(\theta); \qquad (3.5)$$

here $\lambda = \lambda_a - \lambda_b$, $\mu = \lambda_c - \lambda_d$, p is the c.m.s. momentum of the colliding particles, $d^j_{\lambda\mu}(\theta)$ are the functions defined by Jacob and Wick[7], $f^j_{\lambda_c\lambda_d;\lambda_a\lambda_b}$ are the partial wave helicity amplitudes related to the scattering phase-shifts $\delta^j_{\lambda_c\lambda_d;\lambda_a\lambda_b}$ by means of the following relations

$$f^j_{\lambda_c\lambda_d;\,\lambda_a\lambda_b} = -i\,[\exp(2i\delta^j_{\lambda_c\lambda_d;\,\lambda_a\lambda_b}) - \delta_{\lambda_a\lambda_c}\delta_{\lambda_b\lambda_d}] \qquad (3.6)$$

for the case of elastic scattering, and by

$$f^j_{\lambda_c\lambda_d;\,\lambda_a\lambda_b} = -i\exp(2i\delta^j_{\lambda_c\lambda_d;\,\lambda_a\lambda_b}) \qquad (3.7)$$

for the case of inelastic scattering. The differential cross section of the process a + b → c + d has the following expression in terms of the amplitude $f_{\lambda_c\lambda_d;\lambda_a\lambda_b}$:

$$\frac{d\sigma_{\lambda_c\lambda_d;\,\lambda_a\lambda_b}}{d\Omega} = |f_{\lambda_c\lambda_d;\,\lambda_a\lambda_b}(\theta,\varphi)|^2. \qquad (3.8)$$

Application of Eqs. (3.5)—(3.7) to ν_ee and $\bar{\nu}_e$e scattering in the first approximation with respect to G, and utilizing the equalities $d^0_{00} = 1$, $d^1_{11}(\theta) = \frac{1}{2}(1 + \cos\theta)$ yields[8]

$$f_{\nu e e} = \frac{1}{E} f^0_{\nu e e}, \qquad (3.9)$$

$$f_{\bar{\nu} e e} = \frac{3}{2E} f^1_{\bar{\nu} e e}(1+\cos\theta), \qquad (3.10)$$

$$f^j = \frac{1}{2i}(e^{2i\delta_j} - 1), \qquad (3.11)$$

where f^j and δ_j are respectively the partial-wave amplitude and phase shift for angular momentum j. On the other hand, it follows from (3.1) and (3.2) that

$$f_{\nu e e} = -\frac{G\sqrt{s}}{\sqrt{2}\pi}, \qquad (3.12)$$

$$f_{\bar{\nu} e e} = -\frac{G\sqrt{s}\,(1+\cos\theta)}{2\sqrt{2}\pi}. \qquad (3.13)$$

(The sign in (3.12) and (3.13) has been chosen in agreement with the usual definition of the amplitude.) It follows from a comparison of (3.9), (3.10) with (3.12), (3.13) that the partial wave amplitudes in the approximation under discussion are

$$f^0_{\nu e e} = -\frac{Gs}{2\pi\sqrt{2}}, \qquad (3.14)$$

$$f^1_{\bar{\nu} e e} = -\frac{Gs}{6\pi\sqrt{2}}. \qquad (3.15)$$

The unitarity condition contained in (3.11) sets the following bounds on the quantities $|f^j|$ and $|\mathrm{Re}\, f^j|$:

$$|f^j| \leqslant 1, \qquad (3.16)$$

$$|\mathrm{Re}\, f^j| \leqslant 1/2, \qquad (3.17)$$

i.e., it restricts the maximally possible values of $s = s_{max}$ in (3.14), (3.15). If one uses the stronger restriction (3.17), one obtains for s_{max}[9]

$$\nu_e e\text{ — scattering: } s_{max} = \frac{\pi\sqrt{2}}{G}, \quad 2E_{max} = 620\ \text{GeV}, \qquad (3.18)$$

$$\bar{\nu}_e e\text{ — scattering: } s_{max} = \frac{3\pi\sqrt{2}}{G}, \quad 2E_{max} = 1080\ \text{GeV}. \qquad (3.19)$$

In the sequel the values E_{max} (or s_{max}) of the energy where the growth of the weak interaction cross section produced by a single partial wave ceases will be called the unitarity limit. It should be stressed that the bounds (3.18) and (3.19) characterize the unitarity limit only as an order of magnitude. Indeed, for s_{max} defined according to (3.18) and (3.19), Re $f^j(s_{max}) = 1/2$, but then, according to (3.11), Im $f_j(s_{max}) = 1/2$, which contradicts (3.14) and (3.15). Thus, the behavior of the partial-wave amplitudes $f^0_{\nu_e e}(s)$ and $f^1_{\bar{\nu}_e e}(s)$ must begin to differ from (3.14), (3.15) on account of the unitarity condition already for s considerably smaller than s_{max}. (We do not discuss here the problem of artificial unitarization of the partial-wave amplitudes, which is easily achieved by setting for all values of s: $\delta_0^{\nu_e e} = -Gs/2\pi 2^{1/2}$, $\delta^{\bar{\nu}_e e} = -Gs/6\pi 2^{1/2}$, since for s ~ s_{max} the contribution of diagrams that do not reduce to simple unitarization becomes important.

For energies of the order of the unitarity limit the cross sections for ν_ee and $\bar{\nu}_e$e scattering, defined by (3.3) and (3.4) (as well as the cross sections of other weak processes), are still quite small, considerably smaller than the cross sections that are characteristic for strong interaction processes (e.g., $\sigma_{\nu_e e}(s_{max}) = (G^2/\pi)s_{max} = 2^{1/2}G = 6 \times 10^{-33}$ cm²).

We now consider these same processes in a weak interaction theory with intermediate vector bosons. The corresponding Feynman diagrams are represented in Figs. 1 and 2, and the differential cross sections have the form

$$\frac{d\sigma_{\nu e}}{d\Omega} = \frac{G^2 s}{(2\pi)^2}\left\{\frac{m_W^2}{m_W^2 + [s(1-\cos\theta)/2]}\right\}^2, \qquad (3.20)$$

$$\frac{d\sigma_{\bar{\nu} e}}{d\Omega} = \frac{G^2 s}{(2\pi)^2}\cdot\frac{1}{4}(1+\cos\theta)^2\left(\frac{m_W^2}{m_W^2 - s}\right)^2. \qquad (3.21)$$

for $s \gg m_W^2$ the total cross sections become

$$\sigma_{\nu e e} = \frac{G^2 m_W^2}{\pi}, \qquad (3.22)$$

$$\sigma_{\bar{\nu} e e} = \frac{G^2 m_W^2}{3\pi}\,\frac{m_W^2}{s} \qquad (3.23)$$

and, in distinction from the four-fermion interaction, they tend to a constant limit, or even decrease as s → ∞. Such a behavior of the cross sections is however atypical for processes of first order in G in theories with intermediate vector bosons: the cross sections of other processes increase linearly with s.

One can give a very simple explanation to the growth of the interactions of a vector boson with energy. A vector boson, having spin 1, is characterized by three independent polarization unit vectors ϵ^i (i = 1, 2, 3), which in the rest system of the boson can be directed along the three coordinate axes. Then, for a boson moving along the z axis with momentum k, the Lorentz boost will leave unchanged the transverse polarizations ϵ^T (T = x, y) and the longitudinal polarization vector will take on the form $\epsilon^L = \{k/m_W, 0, 0, k_0/m_W\}$ where k_0 is the boson energy, or approximately for $k \gg m_W$

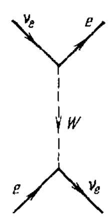

FIG. 1

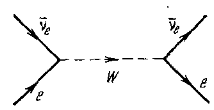

FIG. 2

$$\epsilon_\mu^L \approx \frac{k_\mu}{m_W}, \qquad (3.24)$$

i.e., the four-vector ϵ_μ^L will increase with k. This circumstance leads to a growth with energy of the interaction amplitudes of longitudinally polarized vector bosons as long as the vertices where these bosons are emitted do not fall off sufficiently fast with the increase in energy. The simplest case where a vector boson interaction does not lead to an increase of the amplitude with energy is the well-known interaction of the boson with a conserved current $j_\mu(x)$, i.e., where $\partial_\mu j_\mu(x) = 0$, and consequently, as $k \to \infty$ the expression $\epsilon_\mu^L(k) j_\mu(k)$ does not contain terms which are linear in k. (Such a situation occurs, e.g., in quantum electrodynamics, where the longitudinal photons do not interact at all.) In the weak interaction theory described by the Hamiltonian (2.2) the current $j_\mu^\pm(x)$ is not conserved, and therefore, at least in perturbation theory, the amplitudes for the interactions of longitudinal W bosons increase with the energy.

What was said above allows one to understand easily why the lepton scattering cross sections in a W-boson theory do not increase with energy in the lowest order of G. Indeed, an increase with energy could be caused by longitudinally polarized virtual W bosons in the diagrams of Figs. 1 and 2. However, neglecting the lepton mass, their contribution to the amplitudes vanishes, since the free leptonic current is conserved.

In order to illustrate the behavior of other cross sections in a W-boson theory, we consider the hypothetical process $W^+ + e^- \to W^+ + e^-$ at high energies $s \gg m_W^2$. We shall assume that both the initial and the final W bosons are longitudinally polarized. The matrix element for the process described by the diagram in Fig. 3 is of the form

$$M_{\mu\nu} = 4\pi g^2 \bar{u}(p') \gamma_\mu (1+\gamma_5)(\hat{p}+\hat{k})^{-1} \gamma_\nu (1+\gamma_5) u(p), \qquad (3.25)$$

so that

$$|M_{\mu\nu}\epsilon_\mu^L(k')\epsilon_\nu^L(k)|^2 = \frac{64}{m_W^4}(4\pi g^2)^2 (pk)(pk'), \qquad (3.26)$$

and the differential cross section turns out to be

$$d\sigma = \frac{G^2 s}{32\pi^2}(1+\cos\theta)\, d\Omega, \qquad (3.27)$$

i.e., increases linearly with s. One partial wave with $j = 1/2$ contributes to the cross section (3.27). It is easy to determine the unitarity limit for this process, namely $s_{max} = 4\sqrt{2}\pi/G$.

b) Neutrino-nucleon interactions. The experimental data on the total cross sections of neutrino-nucleon interactions are at present the only empirical proof of the point structure of the weak interaction at short distances. Figure 4 shows the total cross section of the process $\nu_\mu + N \to \mu^- +$ hadrons as a function of energy, from the CERN neutrino experiment[2]. As can be seen from the figure, up to neutrino laboratory energies $E_{lab} \approx 10$ GeV (i.e., $s^{1/2} \approx (2mE_{lab})^{1/2} \approx 4.5$ GeV) $\sigma_{\nu_\mu N}$ increases linearly with the energy: $\sigma_{\nu_\mu N} = (0.8 \pm 0.2) \times 10^{-38} E_{GeV}$ cm^2. Here the "elastic" cross section (i.e., the cross

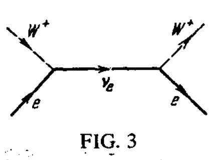

FIG. 3

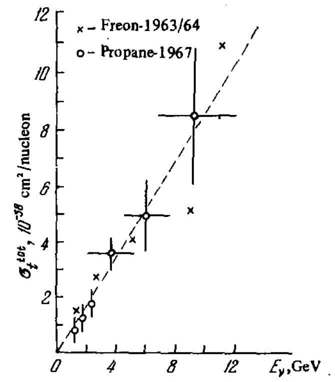

FIG. 4. The total cross section for neutrino-nucleon scattering as a function of the neutrino energy E_ν in the laboratory system, from CERN bubble chamber data. The freon-chamber data have been multiplied by 1.35 to normalize them with those of the propane chamber; all errors are statistical.

section of the process $\nu_\mu + n \to \mu^- + p$) remains approximately constant in the energy interval 1–3 GeV, and is considerably lower than the total cross section: σ_{el} $(0.5-0.7) \times 10^{-38}$ cm^2[10,11]. For the case of $\bar{\nu}_\mu N$ scattering the total cross section also increases linearly with the energy and considerably exceeds the elastic cross section[11].

Since the linear growth with respect to s of the cross section of $\nu_\mu N$ scattering is due to the region of large momentum transfers to the hadrons, $|q^2| \sim s$, and in the space-time picture large $|q^2|$ correspond to small transverse distances (with respect to the vector **q**) $\rho \lesssim |q^2|^{-1/2}$, this experimental result means that the weak interaction is a point interaction up to distances $r \sim s^{-1/2} \lesssim 10^{-14}$ cm. One should stress the distinction between the deep-inelastic process of $\nu_\mu N$ interaction, in which hadron states with large mass $W \gg m$ are produced, from the elastic scattering processes (or the production of low-lying resonances). In the latter the presence of the form factors has the effect that values $|q^2| \lesssim m^2$ play a fundamental role, the cross sections do not increase with energy and the process is characterized by finite transverse distances $\rho \sim 1/m$ and longitudinal distances which increase with energy[12,13].

In order to give more precise formulations, we first discuss the kinematics of the problem. We consider the process $\nu(\bar{\nu}) + N \to l(\bar{l}) +$ hadrons under the conditions when one measures the energies E of the incident neutrino (antineutrino), E′ of the scattered lepton (antilepton) and the scattering angle θ of the lepton (antilepton) (all quantities are measured in the laboratory system), and a summation is carried out over the hadronic states. The matrix element of the process is described by the diagram of Fig. 5. We denote by p the 4-momentum of the incident nucleon; $q = p_\nu - p_l$ (or $q = p_{\bar{\nu}} - p_{\bar{l}}$) is the 4-momentum transfer from the leptons to hadrons. Then

$$q^2 = -4EE' \sin^2\frac{\theta}{2}, \qquad (3.28)$$

$$\nu \equiv pq = m(E - E'). \qquad (3.29)$$

In the four-fermion theory the differential cross section for the scattering of a neutrino (antineutrino) by an unpolarized nucleon is of the form[14,15]

$$\frac{d\sigma^{\nu(\bar{\nu})}}{dq^2\, d\nu} = \frac{G^2}{2\pi m}\frac{E'}{E}$$
$$\times \left[\cos^2\frac{\theta}{2} w_2^\pm(\nu, q^2) + 2\sin^2\frac{\theta}{2} w_1^\pm(\nu, q^2) \mp \frac{E+E'}{m}\sin^2\frac{\theta}{2} w_3^\pm(\nu, q^2)\right], \qquad (3.30)$$

where the upper sign corresponds to neutrino scattering, the lower one corresponds to antineutrino scattering, and

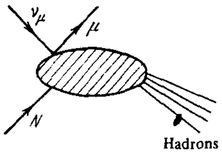

FIG. 5

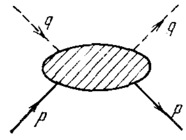

FIG. 6

$w_i^{\pm}(\nu, q^2)$ (i = 1, 2, 3) are functions of the two Lorentz invariants ν and q^2 which determine the kinematics of the problem. (Terms proportional to the lepton mass have been neglected.)

Owing to isospin invariance (and neglecting strangeness-changing terms which are proportional to $\sin^2\theta_C$)

$$w_i^{\nu p} = w_i^{\bar{\nu} n}, \quad w_i^{\bar{\nu} p} = w_i^{\nu n}. \tag{3.31}$$

One can use the kinematic inequality restricting the mass W of the resulting hadron state

$$W^2 = (p+q)^2 \geqslant m^2, \text{ i.e., } \nu \geqslant \frac{|q^2|}{2}, \tag{3.32}$$

the expression for the total cross section $\sigma^{\nu(\bar{\nu})}$ becomes ($Q^2 = -q^2$)

$$\sigma^{\nu(\bar{\nu})}(E) = \frac{G^2}{2m\pi E}\int_0^{mE} d\nu \int_0^{\nu/2} dQ^2 \left(E - \frac{\nu}{m}\right)\left\{\left[1 - \frac{Q^2}{4E\,(E-(\nu/m))}\right] w_2^{\mp}(\nu, Q^2)\right.$$
$$\left. + \frac{Q^2}{2E\,[E-(\nu/m)]} w_1^{\mp}(\nu, Q^2) \mp \frac{2E-(\nu/m)}{m}\,\frac{Q^2}{4E\,[E-(\nu/m)]} w_3^{\mp}(\nu, Q^2)\right\}. \tag{3.33}$$

Introducing the notation

$$x = \frac{Q}{2\nu} \quad 0 \leqslant x \leqslant 1, \quad y = \frac{\nu}{mE}, \quad 0 \leqslant y \leqslant 1 \tag{3.34}$$

and neglecting terms of higher order of smallness in m/E one can rewrite (3.33) in the form

$$\sigma^{\nu(\bar{\nu})}(E) = \frac{G^2}{\pi} Em^2 \int_0^1 y\,dy \int_0^1 dx \left[\frac{E}{m}(1-y)\,w_2^{\mp}(Emy,\, x)\right.$$
$$\left. + xy w_1^{\mp}(Emy,\, x) \mp \frac{E}{m}\left(1 - \frac{y}{2}\right) xy w_3^{\mp}(Emy,\, x)\right]. \tag{3.35}$$

The term with $w_3^{\pm}(Emy, x)$ must be smaller than the sum of the two other terms in (3.35), since according to (3.31) and (3.35) it has the same magnitude, but different signs for νp and $\bar{\nu} n$ scattering. Therefore a linear growth of $\sigma^{\nu(\bar{\nu})}(E)$ appears only in the case when for large ν one of the following asymptotic relations holds[3]:

$$\lim_{\substack{\nu\to\infty,\\ x=\text{const}\neq 0}} w_1^{\pm}(\nu,\, x) = \frac{1}{m} F_1^{\pm}(x), \tag{3.36}$$

$$\lim_{\substack{\nu\to\infty,\\ x=\text{const}\neq 0}} \nu w_2^{\pm}(\nu,\, x) = m F_2^{\pm}(x). \tag{3.37}$$

It is natural to assume that both asymptotic equalities are realized simultaneously, as well as the relation

$$\lim_{\nu\to\infty} \nu w_3^{\pm}(\nu,\, x) = m F_3^{\pm}(x), \tag{3.38}$$

which guarantees a contribution of w_3 to the total cross section of the same order as that of w_1 and w_2. (Although the sequel remains valid also in the case when $F_3^{\pm}$ and (or) one of the functions $F_1^{\pm}$, $F_2^{\pm}$ vanishes.) The relations (3.36)–(3.38) have been called scaling relations and were first introduced by Bjorken[16]. They follow from the assumption that for large ν and $|q^2|$ no external dimensional parameter enters into the theory, so that the dependence of physical quantities[4)] on ν and $|q^2|$ is of the form $\nu^{d/2}F(x)$, where d is the dimension of the quantity under discussion (in mass units) and F(x) is a function of the dimensionless parameter $x = |q^2|/2\nu$. (Hence the name "scale invariance" or scaling.) If for large ν the relations (3.36)–(3.38) are valid, then[16]

$$\sigma^{\nu(\bar{\nu})}(E) = \frac{G^2}{2\pi} Em \int_0^1 dx\left[F_2^{\pm}(x) + \frac{2}{3}F_1^{\pm}(x) \mp \frac{2}{3} xF_3^{\pm}(x)\right]. \tag{3.39}$$

Thus, in order to ensure a linear growth of $\sigma^{\nu(\bar{\nu})}(E)$ it is necessary that finite values of x, of the order of unity, play an essential role in the integral (3.35); for these values $|q^2|$ must be of the order of ν and consequently, $|q^2| \sim mE \sim s$.

The relations (3.6)–(3.38) have also been directly verified experimentally by measuring $d\sigma^{\nu}/dq^2 d\nu$ for large ν and $|q^2|$ and agreement was found between the predictions of scaling theory and experiment[2,17] (although so far not to very good accuracy).

Finally, another argument in favor of the scaling relations (3.36)–(3.38) is the fact that similar scaling relations have been verified to a high degree of accuracy in processes of deep-inelastic electroproduction on nucleons[18-20].

In order to relate the behavior of the functions $w_i(\nu, q^2)$ to a space-time picture it is convenient to consider in place of the total cross section the absorptive part of the matrix element $M^{\pm}$ for forward scattering of an intermediate boson of momentum q on an unpolarized nucleon of momentum p, this absorptive part being proportional to the total cross section[5)].

The corresponding Feynman diagram is represented in Fig. 6. The absorptive part of the amplitude, Abs $M^{\pm}_{\mu\lambda}$, can be represented in the form

$$\text{Abs}\, M^{\pm}_{\mu\lambda} = \pi \sum_n \langle p | j_\mu^{\mp h}(0) | n\rangle \langle n | j_\lambda^{\pm h}(0) | p\rangle (2\pi)^3 \delta^4(p+q-p_n), \tag{3.40}$$

where the summation is over the hadronic states with four-momentum p_n. It can also be expressed in terms of the current commutator:

$$\text{Abs}\, M^{\pm}_{\mu\lambda} = \frac{1}{2}\int d^4x\, e^{iqx} \langle p | [j_\mu^{\mp h}(x),\ j_\lambda^{\pm h}(0)] | \rangle. \tag{3.41}$$

Abs $M^{\pm}_{\mu\lambda}$ can be expressed in terms of the invariant functions $w_i^{\pm}(\nu, q^2)$ in the following manner:

$$\frac{1}{\pi}\,\text{Abs}\, M^{\pm}_{\mu\lambda} = -\left(\delta_{\mu\lambda} - \frac{q_\mu q_\lambda}{q^2}\right) w_1^{\pm}(\nu,\, q^2)$$
$$+ \frac{1}{m^2}\left(p_\mu - \frac{\nu q_\mu}{q^2}\right)\left(p_\lambda - \frac{\nu q_\lambda}{q^2}\right) w_2^{\pm}(\nu,\, q^2) - i\varepsilon_{\mu\lambda\rho\sigma} p_\rho q_\sigma\, \frac{1}{2m^2} w_3^{\pm}(\nu,\, q^2). \tag{3.42}$$

(In (3.42) we have omitted terms containing q_μ or q_λ which after multiplication by the leptonic part of the matrix element will be proportional to the lepton mass.)

Let us consider the expression (3.41). For $|q^2| \gg m^2$, (3.32) implies the inequality $q_0^2 \geq (q^2)^2/4m^2 \gg q^2$, and the exponent of the exponential function can be represented

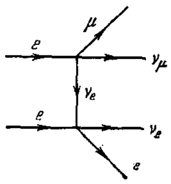

FIG. 7

in the form (the **z** axis is along the vector **q**)

$$qx = q_0 t - \sqrt{q_0^2 - q^2}\, z \approx q_0 (t - z) + \frac{q^2}{2q_0} z = \frac{\nu}{m}(t - z) + \frac{q^2 m}{2\nu} z. \quad (3.43)$$

Substituting (3.43) into (3.41) it is easy to see that for large ν and q^2, small values of $t - z \lesssim m/\nu$ and values of the transverse distance $z \approx t \lesssim \nu/|q^2|m$ play a role in (3.41). Since the current commutator vanishes outside the light cone according to the causality condition $x^2 = (t - z)(t + z) - \rho^2 \geq 0$, these estimates yield a restriction[13] on the magnitude of transverse distances $\rho^2 \leq 1/|q^2|$, as well as on the magnitude of the interval $x^2 \lesssim 1/|q^2|$.

We thus reach the conclusion that small transverse distances $\rho \lesssim 1/s^{1/2}$ are indeed responsible for the linear growth with s of the total cross sections for neutrino-nucleon interactions, and the scattering process occurs in a region close to the light cone $x^2 \lesssim 1/s$.

Up to now in the present section we have considered the four-fermion theory of weak interactions. If intermediate vector bosons exist then in the expression of the differential cross section, (3.30), there appears an additional factor $[m_W^2/(m_W^2 - q^2)]^2$, and under the integral sign in (3.33) there appears the factor $[1 + (2xyEm)/m_W^2]^{-2}$. As a result of this the formula for the total cross section for $s \gg m_W^2$ takes the form[21]

$$\sigma^{\nu(\bar\nu)}(s) = \frac{G^2}{2\pi} s \int_0^1 dx \frac{1}{1 + (xs/m_W^2)} F_2^{\pm}(x) \approx \frac{G^2}{2\pi} m_W^2 \ln\left(\frac{s}{m_W^2}\right) F_2^{\pm}(0), \quad (3.44)$$

i.e., the linear growth of the cross section as a function of s is replaced by a logarithmic one (as long as $F_2^{\pm}(0) \neq 0$).

In addition to the problems discussed above, the investigation of high energy neutrino-nucleon interaction leads to other interesting questions, which we shall not discuss here, referring the interested reader to the excellent review of Pais (1971).

3.2. Processes of order higher than one in G.

a) Inelastic lepton scattering. In this section we shall consider inelastic lepton scattering processes which appear in second order in G in the four-fermion theory and in the order g^3 in the theory involving a W boson; we shall deal only with processes for which the diagrams do not require an integration over the momenta of the virtual particles. The purpose of this discussion is to illustrate on concrete examples the general assertions made in Sec. 3.1 on the behavior of the cross sections of inelastic processes, as well as to obtain concrete estimates on the cross sections of inelastic processes.

As a first example we consider the process

$$e^- e^- \to e^- \mu^- \nu_e \bar\nu_\mu. \quad (3.45)$$

The matrix element of this process is described by the diagram of Fig. 7, and by the diagram obtained from it by antisymmetrization in the incident electrons. For large s the total cross section is[8]

$$\sigma = \frac{G^4 s^3}{(2\pi)^5 45} \frac{1 + 10 - \pi^2}{2}. \quad (3.46)$$

The relation (3.46) literally confirms (if one does not pay attention to numerical factors) the assertions made in Sec. (3.1) to the effect that near the unitarity limit the cross sections of inelastic weak processes are of the same order of magnitude as the elastic ones. In (3.46) there is however a small numerical coefficient $\sim 10^{-6}$. Owing to this small multiplier, at the unitarity limit the total cross section of the process (3.45) (estimated in accordance with (3.18), (3.19)) turns out to be 3–4 orders of magnitude smaller than the cross section for elastic $\nu_e e$ scattering.

For all other inelastic processes of the same order

$$e^+e^- \to e^+\mu^-\nu_e\bar\nu_\mu, \quad e^+e^- \to \mu^+\mu^-\nu_\mu\bar\nu_\mu, \quad e^+e^- \to \mu^+\mu^-\nu_e\bar\nu_e,$$

analogous small numerical coefficients appear[8].

Thus, for energies of the order of the unitarity limit inelastic leptonic processes still contribute a relatively small contribution to the total cross section of lepton-lepton scattering, so that the cross sections of elastic and inelastic processes can become comparable only at energies several times larger than s_{max} (of course, if one can attribute significance to numerical estimates in the region of the unitarity limit, particularly to estimates based on perturbation theory).

As an example of inelastic processes which occur in a theory with intermediate W bosons we consider the reaction

$$\nu_\mu e^- \to \mu^- e^- W^+. \quad (3.47)$$

The Feynman diagram for this process is represented in Fig. 8, and its matrix element has the form

$$M_\sigma = (4\pi g^2)^{3/2} \bar u(p_\mu) \gamma_\lambda (1 + \gamma_5) u(p_\nu)$$
$$\times \bar u(p_e') \gamma_\sigma (1 + \gamma_5)(\hat p_e + \hat k)^{-1} \gamma_\lambda (1 + \gamma_5) u(p_e) \frac{1}{q^2 - m_W^2}, \quad (3.48)$$

where p_ν, p_μ, k, p_e, p_e' are respectively the momenta of the neutrino, the muon, the W-boson, the initial and final electrons; $q = p_\nu - p_\mu$. For $s \gg m_W^2$ the differential cross section for the production of a longitudinally polarized W boson on an unpolarized electron is of the form

$$d\sigma = \frac{32 g^6}{\pi^2 m_W^2 E_\mu E_e' E_W} \frac{p_\mu p_e'}{(q^2 - m_W^2)} d\mathbf{p}_\mu\, d\mathbf{p}_e'\, d\mathbf{k}\, \delta^4 (p_e + p_\nu - p_\mu - p_e' - k),$$

and the total cross section equals

$$\sigma = \frac{3}{16\pi^3 \sqrt{2}} G^3 m_W^2 s. \quad (3.49)$$

A comparison of (3.49) and (3.22) shows that near the unitarity limit the cross sections for elastic and inelastic processes are of the same order of magnitude.

b) A lower bound on the cross section for elastic e^+e^- scattering. Use of the unitarity condition allows one to derive a lower bound on the cross section for elastic e^+e^- scattering produced by the weak interaction[9]. In per-

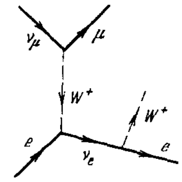

FIG. 8

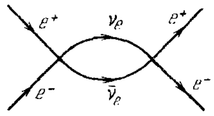

FIG. 9

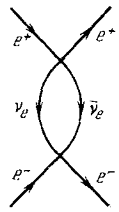

FIG. 10

turbation theory the amplitude of this process is proportional to G^2 and described by the diagrams of Figs. 9 and 10 (we consider the four-fermion theory of weak interactions). A lower bound for the magnitude of the cross section of e^+e^- scattering is obtained if one takes into account only the imaginary part of the scattering amplitude (due to the diagram in Fig. 9), which by virtue of the unitarity condition can be expressed in terms of the transition amplitudes into real physical states.

In the case of elastic e^+e^- scattering, the particle helicities in the initial and final states are equal to $\lambda_e = -1/2$ and $\lambda_{e^+} = 1/2$, respectively, and in the approximation under consideration the total angular momentum is $j = 1$. Therefore, according to (3.5), the general expression for the amplitude has the form

$$f_{e^+e^-}(E, \theta) = A(E)\, d^{1}_{11}(\theta) = A(E) \cdot \frac{1}{2}(1 + \cos\theta), \tag{3.50}$$

where $A(E)$ is a function only of E, the particle energy in the c.m.s. The imaginary part of $f_{e^+e^-}(E, \theta)$ can be determined considering the scattering amplitude in the forward direction and utilizing the optical theorem:

$$\operatorname{Im} f_{e^+e^-}(E, 0) = \operatorname{Im} A(E) = E\sigma^{\text{tot}}_{e^+e^-}(E) \cdot \frac{1}{4\pi}, \tag{3.51}$$

where $\sigma^{\text{tot}}_{e^+e^-}$ is the total cross section of e^+e^- annihilation due to the weak interaction. In the G^2 approximation $\sigma^{\text{tot}}_{e^+e^-}$ is the cross section of the process $e^+e^- \to \nu_e\bar{\nu}_e$ and equals

$$\sigma^{\text{tot}}_{e^+e^-} = \frac{2G^2 s}{3\pi}. \tag{3.52}$$

(We recall that in this reasoning e^+ and e^- are considered polarized, with $\lambda_{e^-} = -1/2$, $\lambda_{e^+} = 1/2$; therefore (3.52) differs by a factor of 2 from (3.4).) Substituting (3.52) into (3.51) and (3.49) we obtain

$$\operatorname{Im} f_{e^+e^-}(E, \theta) = \frac{G^2 s}{12\pi^2} E(1 + \cos\theta), \tag{3.53}$$

so that the contribution of the imaginary part of the amplitude to the cross section for the scattering of unpolarized e^+e^- equals [9]

$$\frac{d\sigma^{\text{Im}}(E, \theta)}{d\Omega} = \frac{G^4 s^3}{2^8 3^2 \pi^4}(1 + \cos\theta)^2. \tag{3.54}$$

As was shown in [9] (cf. also [22]), at an energy $s^{1/2} \approx 500$ GeV and scattering angles $\theta \approx 90°$ the differential cross section $d\sigma^{\text{Im}}/d\Omega$ (3.54) exceeds the cross section for elastic e^+e^- scattering produced by the electromagnetic interaction. Integrating (3.54) over the angles yields a lower bound on the total cross section for weak e^+e^- scattering

$$\sigma^{\text{Im}} = \frac{G^4 s^3}{2^4 \cdot 3^3 \pi^3}.$$

c) The behavior of the total cross sections of weak interactions of leptons at energies beyond the unitarity limit. On the basis of all the preceding one can now make more precise the assertion in the introduction (Sec. 1) that for energies beyond the unitarity limit the weak interaction becomes effectively strong. In making this assertion one has in fact two circumstances in mind: 1) that at least in some elastic (or two-particle) processes the scattering phase shift in one of the partial wave becomes of the order of unity; 2) that the cross sections of inelastic (many-particle) weak processes become of the same order as those for elastic processes. However, in the region of the unitarity limit, the cross sections of weak processes are still quite small: $\sigma \sim 2\pi/G \sim 10^{-32}$ cm^2, since the effective range of the interaction $r \sim G^{1/2}$ is small. There arises the question: can the weak cross sections become of the same order as the cross sections of strong interactions $\sigma_{\text{str}} \gtrsim 1/m^2$ (i.e., can the effective range become of the order $1/m$) and if yes, for what energies? The answer to this question was given by Pomeranchuk [23] (cf. also the discussion of this problem in [24]). We reproduce below the basic ideas and conclusions of [23].

We consider the scattering of leptons in the four-fermion theory. The idea of the analysis is to use the analyticity properties of scattering amplitudes, expressed as dispersion relations which relate the behavior of the amplitudes at large and small energies. The main results are the following. The use of dispersion relations for the forward scattering amplitude leads the result that the value $s = s_1$, for which the amplitude of lepton-lepton scattering attains the value $\sigma(s_1) = \sigma_1 \gg 2\pi/G$, satisfies the inequality

$$s_1 \gg \frac{\sigma_1}{G^2}. \tag{3.55}$$

Use of dispersion relations for the derivative with respect to the momentum transfer t of the scattering amplitude $f(s, t)$ at $t = 0$ yields the stronger inequality

$$s_1 \gg \frac{\sigma_1^2}{32\pi G^3}. \tag{3.56}$$

In order to prove the inequalities (3.55) and (3.56) we consider the processes of $\nu_e e$ and $\bar{\nu}_e e$ scattering by a polarized electron ($\lambda_e = -1/2$) and introduce the invariant crossing-symmetric amplitude

$$F(s, t) = \frac{1}{2}[F_{\nu_e e}(s, t) + F_{\bar{\nu}_e e}(s, t)]; \tag{3.57}$$

here t is the square of the momentum transfer, $t = 1/2\, s(1 - \cos\theta)$, and the amplitudes $F_{\nu_e e}$ and $F_{\bar{\nu}_e e}$ differ from the amplitudes $f_{\nu_e e}$ and $f_{\bar{\nu}_e e}$ introduced in Sec. 3.1a) by the factor $8\pi s^{1/2}$. According to the optical theorem

$$\operatorname{Im} F(s, 0) = s\sigma(s), \quad \sigma(s) = \frac{1}{2}[\sigma_{\nu_e e}(s) + \sigma_{\bar{\nu}_e e}(s)]. \tag{3.58}$$

For $F(s, t)$ we write the once-subtracted dispersion relation in s:

$$F(s, t) = F(0, t) + \frac{s}{\pi}\int_{s_0}^{\infty} ds' \operatorname{Im} F(s', t)\left[\frac{1}{s'(s' - s)} - \frac{1}{(s' + t)(s' + s + t)}\right] \tag{3.59}$$

and set $t = 0$. For $t = 0$ the dispersion integral converges if

$$\int^{\infty} \frac{\sigma(s')}{s'^2} ds' < \infty.$$

Assume that for $s_1 \gg 1/G$, $\sigma(s_1) \equiv \sigma_1$ and for $s > s_1$, $\sigma(s) > \sigma_1$. For $s \ll s_1$, (3.59) implies the inequality

$$F(s, 0) - F(0, 0) \geqslant \frac{2s^2}{\pi}\int_{s_0}^{s_1} ds' \frac{\sigma(s')}{s'^2 - s^2} + \frac{2\sigma_1 s^2}{\pi s_1}. \tag{3.60}$$

We consider in (3.60) values $s \sim 1/G$. According to (3.12) and (3.13) the left-hand side of (3.60) will then be of the order of unity. In the right-hand side of (3.60) the integral over the region $s_0 < s' \lesssim s$ will also be of the order of unity ($\sigma(s') \sim G$) and the positive integral over the region $s' > s$ may be neglected, which strengthens the inequality (3.60). Consequently

$$1 \geqslant \frac{2\sigma_1}{\pi s_1 G^2},$$

i.e., the inequality (3.55) is valid. It is easy to see that the analyticity conditions and the low-energy behavior of the forward scattering amplitude cannot exclude the equal sign in (3.55). Indeed, consider the amplitude

$$\tilde{F}(s,\ 0) = -\frac{4G^2}{3\pi^2} s^2 \ln(e^{-\pi i} s^2) + P(s) \qquad (3.61)$$

where P(s) is a polynomial. This amplitude satisfies the analyticity requirements contained in (3.59), and for an appropriate choice of P(s) it has a low-energy behavior compatible with perturbation theory (in the G^2-approximation, according to (3.3), (3.4), $\sigma(s) = 4G^2 s/3\pi$). Together with this, (3.61) implies $\sigma(s) \sim G^2 s$, which corresponds to the equal sign in (3.55).

The use of dispersion relations for the function $(\partial F(s, t)/\partial t)_{t=0}$ allows one to strengthen the inequality. Here the idea is that for $s \lesssim 1/G$ the quantity $\partial F(s, t)/\partial t$ is small,

$$\frac{\partial F(s,\ t)}{\partial t} = 2\sqrt{2} G \qquad (3.62)$$

(cf. (3.12) and (3.13), and an important role is played by $|t_{eff}| \sim s \sim 1/G$, whereas for $s \approx s_1$, where $\sigma \sim \sigma_{str}$ values $|t_{eff}| \sim m^2$ must play a dominant role. Such a radical change of the t-dependence of F(s, t) requires an extension of the s interval where this change takes place, thus strengthening the inequality.

One can write the order-of-magnitude relation

$$\sigma_{el} \sim \left(\frac{d\sigma_{el}}{dt}\right)_{t=0} |t_{eff}| = \frac{|t_{eff}|}{16\pi} \left| \frac{F(s,\ 0)}{s} \right|^2 \geqslant \frac{|t_{eff}|}{16\pi} \sigma^2, \qquad (3.63)$$

hence

$$|t_{eff}| < \frac{16\pi}{\sigma}. \qquad (3.64)$$

We differentiate (3.59) with respect to t and set t = 0. This yields

$$\frac{\partial F(s,\ t)}{\partial t}\bigg|_{t=0} = \frac{\partial F(0,\ t)}{\partial t}\bigg|_{t=0} + \frac{2s^2}{\pi} \int_{s_0}^{\infty} \frac{ds'}{s'(s'^2 - s^2)} \frac{\partial}{\partial t} \operatorname{Im} F(s',\ t)\bigg|_{t=0} + \frac{1}{\pi} \int_{s_0}^{\infty} ds' \left[\frac{1}{s'^2} - \frac{1}{(s'+s)^2}\right] \operatorname{Im} F(s',\ 0). \qquad (3.65)$$

Assume that in (3.65) $s \sim 1/G$. Then the left-hand side of (3.65), the subtraction term and the contribution to the integrals from the region $s' \leq s$ are all of the order G^{-1}. For $s' > s$ the integrand in (3.65) is positive, since $(\partial/\partial t)(\operatorname{Im} F(s, t))_{t=0} > 0$. Omitting the positive second integral in (3.65) and retaining in the first integral only the integration region $s' \geq s_1 \gg G^{-1}$, we obtain the inequality

$$G \geqslant s^2 \int_{s_1}^{\infty} \frac{ds'}{s'^3} \left| \left(\frac{\partial}{\partial t} F(s',\ t)\right)_{t=0} \right|^2. \qquad (3.66)$$

According to (3.63)

$$\left| \left(\frac{\partial F(s',\ t)}{\partial t}\right)_{t=0} \right| \sim \frac{1}{|t_{eff}|} s' \sigma(s) \geqslant \frac{s' \sigma^2(s')}{16\pi}. \qquad (3.67)$$

Assuming that $\sigma(s') = \text{const}$ for $s' > s_1$, substituting (3.67) into (3.66), and setting $s \sim 1/G$, we arrive at an inequality equivalent to (3.56) (up to a factor $1/2$, which is obtained from a more rigorous derivation of the relation (3.67)).

As was mentioned above, the growth of the cross section of the weak interaction beyond the unitarity limit is related to a growth of the effective radius of the interaction.

Use of this connection allows one to obtain restrictions on the asymptotic behavior of the lepton-lepton scattering cross sections as $s \to \infty$[25] (cf. also[26]). Since in the case of weak lepton-lepton scattering the exchange of massless particles is possible, the scattering amplitude has a singularity at t = 0, and the Froissart theorem (cf., e.g.,[27]), which establishes bounds on cross sections for $s \to \infty$ ($\sigma \leq \sigma_0 \ln^2 s$), is not applicable. For the sake of concreteness we consider $\nu_e e$ and $\bar{\nu}_e e$ scattering and assume that $\sigma(s)$, (3.58), behaves as $s \to \infty$ like $\sigma = \sigma_0 (s/\tilde{s})^\alpha$, $0 < \alpha < 1$. A power-law behavior of $\sigma(s)$ leads to a growth of the effective range, i.e., to a shrinking of the diffraction cone. It is therefore reasonable to assume[25] that for $s \to \infty$ and $t < 0$

$$\operatorname{Im} F(s,\ t) \to s\sigma_0 \left(\frac{s}{\tilde{s}}\right)^\alpha e^{at(s/\tilde{s})^\beta}, \quad a = \text{const}, \qquad (3.68)$$

and it follows from (3.67) that

$$\sigma > \sigma_{el} \sim \int |F(s,\ t)|^2 \frac{1}{s^2} dt, \ \beta > \alpha.$$

The requirement of analyticity of F(s, t) in s for fixed $t < 0$ and the relation (3.68) allows one to determine the nature of the singularity of F(s, t) for t = 0. We write down a dispersion relation with one subtraction for $\partial F(s, t)/\partial t$ at small t:

$$\frac{\partial F(s,\ t)}{\partial t} - \frac{\partial F(0,\ t)}{\partial t} = \frac{2s^2}{\pi} \int_{s_0}^{\infty} \frac{ds'}{s'(s'^2 - s^2)} \frac{\partial}{\partial t} \operatorname{Im} F(s',\ t). \qquad (3.69)$$

(It is easy to see that the second integral in (3.65) is negligible.) Substituting (3.68) into (3.69) and considering s small we obtain

$$\frac{\partial F(s,\ t)}{\partial t} - \frac{\partial F(0,\ t)}{\partial t} = \frac{2s^2}{\pi} \sigma_0 a \frac{1}{\tilde{s}} \int_{s_0}^{\infty} \frac{ds'}{\tilde{s}} \left(\frac{s'}{\tilde{s}}\right)^{\alpha+\beta-2} e^{at(s/\tilde{s})^\beta} \qquad (3.70)$$

Assume $\beta > \alpha \geq 1/2$. Then in the integral (3.70) an important role is played by values $s/\tilde{s} \sim |t|^{-\beta}$ and for small t the right-hand side of (3.70) will be of the order $|t|^{(1-\alpha)/\beta - 1}$, and consequently F(s, t) will have a singularity of the form $t^{(1-\alpha)/\beta}$ (or t ln t, if $(1-\alpha)/\beta$ is an integer) as $t \to 0$. One can show that this result remains in force also for $0 < \alpha < 1/2$ or $\alpha > 1$. In addition, the derivation shows that the result can be reformulated for other types of dependence of Im F(s, t), which differ from (3.68) (but still correspond to a cross section growing like s^α and a shrinking cone).

Making various assumptions on the character of the singularity of F(s, t) at t = 0, one can derive various restrictions on the behavior of the cross section for $s \to \infty$[25]:

1) if F(s, t) is finite for $t \to 0$, ($\alpha < 1$), then $\sigma(s) < s$;

2) if $\partial F(s, t)/\partial t$ is finite for $t \to 0$, then $\sigma(s) < s^{1/2}$;

3) if F(s, t) has a singularity of the form $t^2 \ln t$ as $t \to 0$ (a singularity of this type appears in the simplest perturbation theory diagrams), then $\sigma(s) < s^{1/3}$.

Here it is important to make one crucial remark. As will be shown in Chap. 4, a consideration of virtual processes implies that the weak hadron-lepton interaction cannot conserve its structure up to the unitarity limit, but must somehow change it. Therefore the results obtained in the present section cannot be applied to hadron-lepton interactions, and their applicability to lepton-lepton interactions will be justified only if under that change the four-fermion form of the weak leptonic interactions remains the same.

4. VIRTUAL PROCESSES

In this chapter of our review we shall investigate the influence of weak virtual interactions which take place at high energies (or, more precisely, at small distances) on processes occurring at low energies.

Qualitatively the situation that arises can be explained on the example of the already discussed e^+e^- scattering in the G^2 approximation (cf. Figs. 9 and 10). The diagrams of Figs. 9 and 10 diverge, when integrated over the momenta of the virtual neutrinos. A simple count of powers shows that the degree of divergence according to the Feynman rules is quadratic. Cutting off the momenta of the virtual particles at a value Λ, the real part of the matrix element for e^+e^- scattering will be proportional to

$$\frac{G^2\Lambda^2}{\pi^2}\bar{u}(p'_+)\gamma_\mu(1+\gamma_5)u(p_+)\cdot\bar{u}(p'_-)\gamma_\mu(1+\gamma_5)u(p_-), \qquad (4.1)$$

where p_-, p'_-, p_+, p'_+ are the initial and final momenta of the electrons and positrons (the spinor structure of (4.1) follows uniquely from the helicities of e^+ and e^-). The same result can also be obtained otherwise, by means of the dispersion-relations method. We denote by $\Phi(s, t)$ the invariant amplitude in front of the spinor structure of (4.1). (The amplitude $\Phi(s, t)$ is proportional to $f_{e^+e^-}/E$.) As follows from (3.52), Im $\Phi(s, 0) \sim G^2 s$. If one assumes that for $s > \Lambda^2$, Im $\Phi(s, 0)$ decreases sufficiently rapidly, the contribution of the right-hand cut to $\Phi(s, 0)$ can be written by means of the unsubtracted dispersion relation:

$$\Phi(s, 0) = \frac{1}{\pi}\int\frac{\operatorname{Im}\Phi(s', 0)}{s'-s}\,ds'. \qquad (4.2)$$

Substituting into (4.2) Im $\Phi(s', 0) \sim G^2 s'$ and integrating with respect to Λ^2, we obtain $\Phi(s, 0) \sim G^2\Lambda^2$, i.e., the result (4.1). Similar reasoning can be applied to the diagram of Fig. 10, considering the imaginary part of $\Phi(s, t)$ in the t-channel.

For a value of Λ of the order of the unitarity limit ($\Lambda^2 \sim 2\pi/G$) the effective coupling constant of the e^+e^- interaction in (4.1) turns out of the order of G, i.e., there appears an interaction of neutral weak currents which were absent in first order of G, having a coupling constant of the same order of magnitude as the coupling of charged currents. It should be stressed that the e^+e^--scattering amplitude for $\Lambda^2 \sim 2\pi/G$ and $s \sim 2\pi/G$ still remains small, $f_{e^+e^-} \sim G^{1/2}$.

The same cannot be said, however, about the corrections to the self-energy. Consider the simplest diagram of this type in the four-fermion theory, namely the correction to the mass operator M(p) of the electron (Fig. 11). A count of the degree of divergence shows that the matrix element corresponding to this diagram must diverge like the fourth power of the momentum (the integration over the angles diminishes the degree of divergence of the diagram by one unit), so that to order G^2

$$M(\hat{p}) \sim \frac{G^2\Lambda^4}{\pi^4}\hat{p}(1+\gamma_5) \qquad (4.3)$$

(terms proportional to the electron mass vanish in (4.3) owing to the two-component character of the weak interaction). It can be seen from (4.3) that for $\Lambda^2 \sim 2\pi/G$ the corrections to the mass operator are of the order of 1[8)].

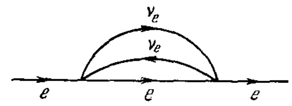

FIG. 11

As is well known, the coefficient of p in the mass operator determines the Green's function renormalization constant Z_2 of the electron (or the ψ-function renormalization $\psi = Z_2^{1/2}\psi_R$), constant which in the case of nonconservation of parity is in fact a matrix $Z_2 = A + B\gamma_5$ (A and B are numbers). Since the renormalization constant Z_2 enters in the definition of the physical charge of various interactions, interactions which conserve parity will no longer conserve it when this renormalization constant is taken into account (the violation being of the order of 1 for $\Lambda^2 \sim 2\pi/G$). For leptons the only parity-conserving interaction is the electromagnetic interaction[7)], for which it can be shown[30] that owing to gauge invariance the contribution of the parity-nonconserving terms in Z_2 is completely compensated by the renormalization of the vertex part, so that no parity-nonconserving terms $\sim G^2\Lambda^4$ (or in general $\sim (G\Lambda^2)^{n+1}$) survive.

In strong interactions the effects of parity-nonconservation which stem from the factor Z_2 (and also from the corrections to the vertex parts and amplitudes of various processes) could be essential if the integrals over the momenta of the virtual particles would not be cut off by the strong interactions. In this case, in addition to the nonconservation of parity, there would appear large strangeness nonconservation effects, of the order of unity for $\Lambda^2 \sim 2\pi/G$ (e.g., the amplitude of the transition $\Lambda \to N + \pi$ would be of order 1 due to the diagram of the transition $\Lambda \to n$ analogous to Fig. 11, with subsequent emission of a pion). However, until the problem whether the strong interactions do cut off the integrals with respect to the momenta of the virtual particles is solved, one cannot draw any conclusions. Section 4.2 will be devoted to a discussion of this important question. We now go over to a concrete investigation of various observable effects which appear on account of virtual weak interactions.

In order to be able to compare the results obtained in investigating various processes the following approach[31,32,29] will be used: we will consider a theory with cutoff, i.e., the integration over the momenta of the virtual leptons, W bosons and photons (but not hadrons!) will be extended to values of the order Λ^2 of the square of the four-momentum, and then the values of Λ^2 will be estimated by comparing the theoretical predictions with experimental data. A "natural" value of "self-cutoff" should be $\Lambda \sim (2\pi/G)^{1/2} \sim 600$ GeV, and values of $\Lambda \ll 600$ GeV will indicate inconsistencies in the theory of weak interactions.

A similar method of estimating the domain of applicability of a theory, by comparing the calculated radiative corrections (calculated with a cutoff) with experimental data has been in widespread use in quantum electrodynamics. In particular, the latest measurements of the anomalous magnetic moment of the muon, when compared to the theoretical calculations[33a] yield as range of applicability of quantum electrodynamics $\Lambda_{QED} > 5$ GeV[33b]. (An investigation of the real process of ee^--scattering yields a close value $\Lambda_{QED} > 6$ GeV[34].)

It is important to note that whereas in quantum electrodynamics the investigation of real and virtual processes makes it possible to obtain only lower limits on the cutoff parameters, in the weak interactions the virtual processes yield upper limits on Λ, and real processes yield lower limits.

In view of the fact that all the difficulties of the theory which will be discussed below appear already in first order of the virtual weak interaction, we will essentially consider only corrections of order $G\Lambda^2$ and $(G\Lambda^2)^2$ (keeping in mind, in particular, that for $\Lambda \ll 1/G$, $G\Lambda^2 \ll 1$).

Before starting a discussion of concrete processes, we have to make one remark of a general nature. In considering the higher orders of weak (or any other) interactions two points of view are possible, in principle. According to the first, the weak interaction Hamiltonian (2.1) or (2.2) expressed in terms of the "bare" (non-interacting) particles, has a certain symmetry in terms of the "bare" fields (equal coefficients in front of the V and A interactions, absence of neutral currents, absence of a $\nu_\mu e$ interaction, etc.). Taking into account the strong interactions and also the higher orders of the weak interactions, leads to a violation of this symmetry, and according to this point of view, the symmetry can be maintained only if there are physical laws requiring the conservation of symmetry as the interaction is turned on. One of the most important tasks of elementary particle physics is to establish these laws.

An argument in favor of this point of view are the experimental facts of equality of the muon decay coupling constant G_μ and the vector coupling constant G_V in beta decay and the difference between G_μ and the axial vector coupling constant G_A in beta decay. Indeed, as was shown by Gershteĭn and Zel'dovich[35] and by Gell-Mann and Feynman[3], the conservation of the vector current in strong interactions implies that the coupling constant G_V is not subject to renormalization on account of the strong interactions, i.e., that the strong interactions do not violate the equality $G_V = G_\mu$. The axial vector current is not conserved in strong interactions so that the strong interactions lead to the difference between G_A and G_μ. Moreover, the relatively small difference between G_A and G_V ($\sim 20\%$) also finds its explanation in the framework of this theory, on the basis of partial conservation of the axial current (PCAC). Another argument in favor of this point of view is the approximate validity of isospin invariance in strong interactions and its violation by quantities of order α.

According to the same point of view, if a symmetry-breaking occurs in virtual processes, then the effective coupling constants of real processes with the corresponding symmetry-breaking must be of the same order of magnitude as the effective coupling constants (amplitudes) of virtual processes (if again, there are no special regions for their "vanishing"). A confirmation of this circumstance is all that we know of the physics of strong interactions, where, as a rule, the effective coupling constants are always of the order of unity. (With the exception of those cases where the smallness of the corresponding quantities, e.g., the pion-nucleon scattering length, is the result of an approximate conservation law.)

Another point of view (formulated by Kirzhnits[36]) starts from the idea that it is meaningless to talk about symmetry in terms of "bare" particles and all relations should refer to physical particles after the interaction is switched on. In this approach the effective coupling constants of real processes are introduced into the theory from the outside, i.e., from experiment, and they have no relation to the amplitudes of virtual processes. From this point of view one cannot pose in the present theory the problem of estimating corrections due to higher orders of weak interactions to low-energy amplitudes, so that, e.g., the problem of the accuracy to which strangeness or parity are conserved in strong interactions is, in general, not part of the theory. (The same refers to the coupling constants of the weak neutral currents.) Thus, from this point of view, the problem under discussion does not even exist.

In view of the arguments brought forward above, and since the first point of view seems to be of higher heuristic value, the remainder of the exposition will be based only on it.

4.1. Leptonic Interactions

In processes where only leptons and photons participate the virtual weak processes can only contribute to the lepton masses and to the amplitudes of various weak processes. Since the masses (and mass differences) of the leptons cannot be calculated in the present state of the theory, there is no sense in separating the contributions to them of the weak interactions.

4.1.1. Electromagnetic interactions of leptons. We consider first the contributions of the weak interactions to the magnetic moment of the muon (or electron). To order G^2 in the four-fermion theory this contribution is determined by the diagram of Fig. 12. A power-count of the degree of divergence of the diagram (taking into account that the contribution to the magnetic moment is proportional to the momentum q of the external field) leads to the following estimate of the magnitude of the magnetic moment of the muon due to weak interactions: $(g-2)/2 \sim G^2\Lambda^2 m_\mu^2$. For a definite choice of the cutoff factor one obtains[37]

$$\frac{g-2}{2} \approx -G^2\Lambda^2 m_\mu^2 \frac{\ln 2}{3\pi^4}, \qquad (4.4)$$

which for $\Lambda^2 \sim 2\pi/G$ yields $(g-2)/2 \sim 10^{-9}$. The accuracy of experiments measuring the anomalous magnetic moment of the muon is at present 10^{-7}, therefore the correction (4.4) is so far beyond the capabilities of experiments. It is easy to show that in higher orders of G^2 the following estimate is valid:

$$\frac{g-2}{2} \sim \frac{1}{\pi^2}\left(\frac{G\Lambda^2}{\pi^2}\right)^n G m_\mu^2 \leqslant 10^{-8}-10^{-9},$$

which is also beyond present experimental capability.

In a theory with intermediate W bosons, to first order in G, the contribution to the anomalous magnetic moment of the muon is determined by the diagram of Fig. 13. If the W boson does not have an anomalous magnetic moment this diagram diverges only logarithmically (the Feynman rules for W boson electrodynamics can be found, e.g., in[38]). The contribution of this diagram has been calculated in several papers[39–47], and is (taking

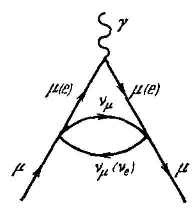

FIG. 12

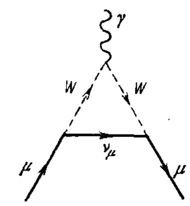

FIG. 13

into account only logarithmic terms)

$$\frac{g-2}{2} = -\frac{Gm_\mu^2}{4\sqrt{2}\pi^2} \ln \frac{\Lambda^2}{m_W^2} \approx -2 \cdot 10^{-8} \qquad (4.5)$$

for $\Lambda^2 \sim \Lambda^2_{weak} \approx 2\pi/G$, $m_W \approx 5$ GeV. In higher orders (up to logarithmic terms) we get the estimate $(g-2)/2 \sim (G\Lambda^2/\pi^2)^n Gm_\mu^2/\pi^2 \sim 10^{-8}$. In considering the electromagnetic interactions of the intermediate bosons one must keep in mind that vector-boson electrodynamics is also a nonrenormalizable theory, that the electromagnetic interactions of W-bosons grow with the energy (cf., e.g.,[38]). The effective parameter that determines the energies where the electromagnetic interactions of the W boson become strong (in the same sense as was discussed earlier for the weak interactions) has the order of magnitude[48-50] $\Lambda^2_{em} \sim m_W^2 (\pi/\alpha)$, so that $\Lambda^2_{em} < < \Lambda^2_{weak}$ for not too large values of m_W (up to several tens of GeV). For this reason one might think that a more correct estimate of the cutoff parameter in (4.6), for not too large values of m_W would be $\Lambda^2 \sim \Lambda^2_{em} \sim m_W^2(\pi/\alpha)$. (The order of magnitude of (4.5) is not changed by this.)

One can study in more detail the problem of the contribution of virtual electromagnetic interactions of the W bosons to the magnitude of the muon's anomalous magnetic moment by considering the contribution to $g-2$ of terms of the order e^2G[51]. Terms of that order correspond, e.g., to the diagrams in Figs. 14 and 15 (in all, to order e^2G there are 22 diagrams). A count of the degree of divergence of these diagrams show that they diverge quadratically, i.e., that the contribution to $(g-2)/2$ will be (up to a logarithm)

$$\frac{g-2}{2} \sim \alpha G \Lambda^2 m_\mu^2/m_W^2. \qquad (4.6)$$

Here it is very essential which of the Λ^2 one substitutes into (4.6): Λ^2_{weak} or Λ^2_{em}, since for $\Lambda^2 \sim \Lambda^2_{weak}$ the contribution to the anomalous magnetic moment of the muon turns out to be sufficiently large $(g-2)/2 \sim \alpha(m_\mu/m_W)^2$. In order to clarify the problem one can proceed as follows. We divide the integration region in the diagrams of Figs. 14, 15 and similar, into two regions: 1) where the momenta of the virtual W bosons are much larger than the momenta of the virtual photons; 2) where they are of the same order. Obviously, the integration over the first region can have a cutoff only on account of the weak interactions, so that for this region $\Lambda^2 \sim \Lambda^2_{weak} \sim 2\pi/G$. For the second region we can expect that $\Lambda^2 \sim \Lambda^2_{em} \sim m_W^2 (\pi/\alpha)$. Starting from the Ward identity one can show[51] that the contribution of the first region of integration vanishes exactly and consequently the cutoff limit in (4.6) will be of order $\Lambda^2 \sim \Lambda^2_{em} \sim m_W^2 (\pi/\alpha)^{8)}$. Then one obtains for the contribution of terms of the order e^2G to $g-2$ the estimate

$$\frac{g-2}{2} \sim \frac{\alpha}{(4\pi)^3} \frac{G\Lambda^2 m_\mu^2}{m_W^2} \ln \frac{\Lambda^2}{m_W^2} \sim \frac{\pi}{(4\pi)^3} Gm_\mu^2 \ln \frac{\pi}{\alpha} \sim 3 \cdot 10^{-9}. \qquad (4.7)$$

Another effect that results from the virtual weak interactions is the interaction of neutrinos with electro-

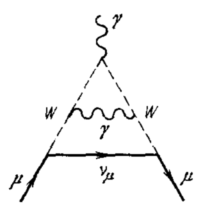

FIG. 14

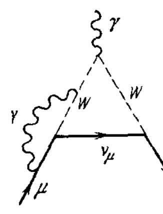

FIG. 15

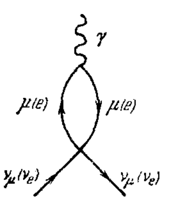

FIG. 16

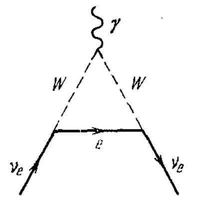

FIG. 17

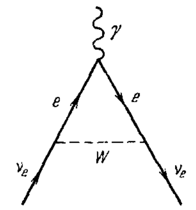

FIG. 18

magnetic fields[52,45,53]. The general expression for the interaction vertex of the neutrino with electromagnetic field has the form

$$\Gamma_\lambda(p', p; q) = \Pi_{\lambda\sigma}(q) \bar{u}(p') \gamma_\sigma (1+\gamma_5) u(p), \qquad (4.8)$$

where $u(p)$ and $u(p')$ are the spinors corresponding to the initial and final states of the neutrino, $q = p' - p$.

From the conservation of the electric current it follows that $q_\lambda \Pi_{\lambda\sigma}(q) = 0$, whence $\Pi_{\lambda\sigma}(q) = -(\delta_{\lambda\sigma} q^2 - q_\lambda q_\sigma) F(q^2)$. Substituting this expression into (4.8) we find

$$\Gamma_\lambda(p', p; q) = -q^2 F(q^2) \bar{u}(p') \gamma_\lambda (1+\gamma_5) u(p). \qquad (4.9)$$

In the four-fermion theory, to first order in G, the electromagnetic interaction of neutrinos arises on account of the diagram of Fig. 16. A calculation of this diagram leads to the following expression for $F(q^2)$[52]:

$$F(q^2) = -\frac{G}{12\sqrt{2}\pi^2} \ln \frac{\Lambda^2}{m_l^2} \qquad (4.10)$$

for $|q^2| \ll m_l^2$ (m_l is the lepton mass). For $|q^2| \gg m_l^2$ the quantity m_l^2 in (4.10) gets replaced by $|q^2|$. In the W-boson theory the process is described by the diagrams in Figs. 17, 18 and for $|q^2| \ll m_l^2$[52,45,53]

$$F(q^2) = \frac{G}{12\sqrt{2}\pi^2} \left(\frac{5}{4} \ln \frac{\Lambda^2}{m_W^2} - \ln \frac{m_W^2}{m_l^2} \right). \qquad (4.11)$$

The dependence of $F(q^2)$ on q^2 for $|q^2| \ll m_l^2$ is given in[53].

The electromagnetic vertex of charged fermions acquires parity-nonconserving terms[54] on account of the virtual weak interactions, so that in addition to the usual electric and magnetic form factors of the electron and muon there appears a new form factor $F_a(q^2)$ (which was named "anapolar form factor" by Zel'dovich[54]), corresponding to the additional term in the electromagnetic interaction vertex of the electron or of the muon of the form

$$-q^2 F_a(q^2) \bar{u}(p') \left(\gamma_\mu - \frac{\hat{q} q_\mu}{q^2} \right) \gamma_5 u(p). \qquad (4.12)$$

In the W-boson theory to first order in G the quantity $F_a(q^2)$ is determined by the diagram of Fig. 13. The calculations yield the following result[45]:

$$F_a(0) = \frac{5G}{48\sqrt{2}\pi^2} \ln \frac{\Lambda^2}{m_W^2}.$$

An experimental investigation of the electromagnetic interactions of the neutrinos will probably become possible when neutrino beams with energies of several hundred GeV become available. (For $E_\nu \sim 200$ GeV we have $\sigma(\nu + p \to \nu + \text{hadrons}) \sim 10^{-39}$ cm^2.)

4.1.2. Weak interactions of leptons. For energies much smaller than the unitarity limit the amplitudes of the weak two-particle scattering interactions of leptons (or the amplitude of muon decay) are determined uniquely, up to numerical factors, by the requirements of Lorentz invariance and the two-component character of all leptons participating in the interaction. Therefore

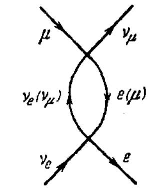

FIG. 19

higher-order corrections with respect to the weak interaction can only modify the numerical factors in front of the various amplitudes, without modifying their form[55]. We consider consecutively the lowest order corrections in G^2 to the coupling constants of various processes.

a) Muon decay. In the four-fermion theory, in first approximation in G, the correction to the muon decay coupling constant G_μ is determined by the diagram of Fig. 19. In the intermediate state of this diagram there are a lepton and an antilepton.

In order to investigate diagrams where a lepton and antilepton always enter in the same vertex it is convenient to introduce external currents I_λ, adding to the interaction Hamiltonian the term

$$\mathscr{H}' = \frac{G}{\sqrt{2}} \left(I_\lambda^+ (j_\lambda^{-\mu} + j_\lambda^{-e}) + (j_\lambda^{+\mu} + j_\lambda^{+e}) I_\lambda\right). \tag{4.13}$$

The matrix element corresponding to diagrams that have μ and ν_μ at one vertex and e and ν_e at the other can be written in the form

$$\bar{u}_{\nu_\mu}(p_{\nu_\mu}) \gamma_\sigma (1+\gamma_5) u_\mu (p_\mu) \cdot \bar{u}_e (p_e) \gamma_\lambda (1+\gamma_5) u_{\nu_e} (p_{\nu_e}) \Pi_{\lambda\sigma} (p_e - p_{\nu_e}), \tag{4.14}$$

$$\Pi_{\lambda\sigma}(q) = \int e^{iqx} d^4x \left[\frac{\delta^2 S_{00}}{\delta I_\lambda (x)\, \delta I_\sigma^+ (0)}\right]_{+,\, I_\lambda = 0}, \tag{4.15}$$

where S_{00} is the vacuum expectation value of the S matrix, considered as a functional of the external currents I (cf., e.g.,[56]). We take the current in the form $I_\lambda(x) = \partial\varphi(x)/\partial x_\lambda$, which corresponds to a calculation of the longitudinal part of $\Pi_{\lambda\sigma}$, i.e., $q_\lambda \Pi_{\lambda\sigma} q_\sigma$. Transforming in the total lepton Lagrangian (with lepton mass terms neglected) the wave functions of the left-handed leptons

$$\psi_{e,\mu}^L = \exp\left(-i \frac{G}{2\sqrt{2}} (\tau^+ \varphi + \tau \varphi^+)\right) \psi_{e,\mu}'^L, \quad \tau = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \psi_e = \begin{pmatrix} e \\ \nu_e \end{pmatrix}, \quad \psi_\mu = \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix},$$

it is easy to show that up to terms which are quadratic in φ or φ^+ the part of the interaction Hamiltonian which depends on the external currents becomes

$$\mathscr{H}' = -i \frac{G^2}{8} \sum_{k,\, i = e,\, \mu} \Big\{ \bar{\psi}_i' \tau_3 \gamma_\lambda \psi_k \left(\frac{\partial \varphi^+}{\partial x_\lambda} \varphi - \varphi^+ \frac{\partial \varphi}{\partial x_\lambda}\right) \delta_{ik} + 2 (\bar{\psi}_i' \gamma_\lambda \tau_3 \psi_i' \bar{\psi}_k' \gamma_\lambda \tau^+ \psi_k' \varphi - \bar{\psi}_i' \gamma_\lambda \tau \psi_i' \bar{\psi}_k' \gamma_\lambda \tau_3 \psi_k' \varphi^+) \Big\}. \tag{4.13'}$$

Terms proportional to $\sim G^2$ in (4.15) can appear only because of the first term in the right-hand side of (4.13′), however, its contribution to order G^2 vanishes on account of antisymmetry upon the substitutions $e \to \nu_e$ and $\mu \to \nu_\mu$. Therefore up to terms proportional to G^4 we have $q_\lambda \Pi_{\lambda\sigma} q_\sigma = 0$. This implies that to the same accuracy $\Pi_{\lambda\sigma} \sim \delta_{\lambda\sigma} q^2 - q_\lambda q_\sigma$.

Thus the term $\delta_{\lambda\sigma} G^2 \Lambda^2$ is absent from the diagram of Fig. 19[31]. Similarly one can prove the absence of a correction of order $G\Lambda^2$ to the muon-decay coupling constant due to virtual hadrons.

The reason for the vanishing of these corrections of order $G\Lambda^2$ is quite simple: the current j_μ is conserved in that approximation (if we disregard higher corrections

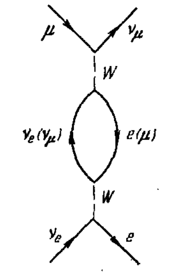

FIG. 20

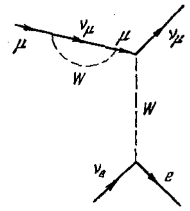

FIG. 21

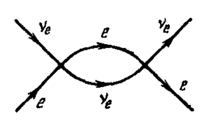

FIG. 22

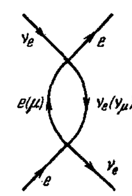

FIG. 23

It is necessary to make one stipulation. Taking into account corrections of the order $G\Lambda^2$ and neglecting corrections of higher orders $(G\Lambda^2)^n$, $n > 1$ is justified (in addition to order of magnitude estimates) if as a result of this it turns out that $\Lambda^2 \ll 2\pi/G$, i.e., the cutoff of the weak interactions takes place on account of "something new." But then the requirement that terms $\sim G\Lambda^2$ vanish in the diagram of Fig. 19 will be legitimate only if this "something new" does not violate the conservation of the current j_μ. Therefore in the sequel we shall also consider the possible nonvanishing of these terms.

In the theory with intermediate bosons, the diagram of Fig. 20 and the four diagrams of the type of Fig. 21 corresponding to the wave-function renormalizations of the initial and final leptons (Z_2-factors) contribute to the renormalization of the muon decay constant to first order in G. All that was said above about the diagram of Fig. 19 in the case of the four-fermion theory also refers to the diagram of Fig. 20. Diagrams of the type of Fig. 21 contribute a term proportional to $G\Lambda^2$ to the renormalization of the muon decay constant.

b) Elastic ν_ee scattering. In the four-fermion theory we have only two types of diagrams: those in Fig. 22 and Fig. 23. There is no reason to require the vanishing of the quadratically divergent terms for the diagram of Fig. 22 with lepton number 2 in the intermediate state. The diagram of Fig. 23 gives a contribution to the ν_ee-scattering constant $G_{\nu_e e}$, which is equal to the contribution of the diagram in Fig. 19 to the muon decay constant G (independent of whether or not it vanishes). Therefore the ratio $G_{\nu_e e}/G_\mu$ is determined by the diagram of Fig. 22 and equals[31]

$$\frac{G_{\nu_e e}}{G_\mu} = 1 + \frac{G\Lambda^2}{\sqrt{2}\pi^2} \tag{4.16}$$

For Λ of the order of the unitarity limit $\Lambda^2 \sim 2\pi/G$ the quantity $G\Lambda^2/2^{1/2}\pi^2$ is of the order of unity, so that in this case one could expect a significant difference between the constant of ν_ee-scattering and the muon decay constant.

In the W-boson theory there is an analogous situation, with the only difference that to order $G\Lambda^2$, in addition to the diagram corresponding to Fig. 22, there are also contributions from diagrams with renormalized external lines.

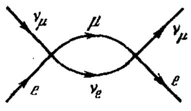

FIG. 24

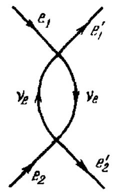

FIG. 25

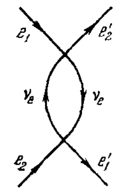

FIG. 26

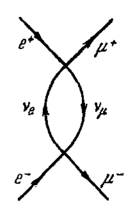

FIG. 27

According to the latest experimental data[57] $G_{\nu_e e}/G < 2$, i.e., the accuracy of the experiment is insufficient to draw any definite conclusions.

c) Elastic $\nu_\mu e$ scattering. In the usual theory of the universal weak interaction this process does not exist to first order in G, and a nonvanishing amplitude for this process can appear only as a consequence of higher orders in the weak interaction (or on account of the electromagnetic formfactor of the neutrino—cf. the diagram of Fig. 16). In the four-fermion theory to order G^2 the amplitude of the process is described by the diagram of Fig. 24 and the coupling constant of the $\nu_\mu e$ interaction turns out to be

$$G^F_{\nu_\mu e} = G\frac{G\Lambda^2}{\sqrt{2}\,\pi^2}. \tag{4.17}$$

In the W-boson theory we have a similar result

$$G^W_{\nu_\mu e} = G\frac{G\Lambda^2}{\sqrt{2}\,(4\pi)^2}. \tag{4.18}$$

At present the experimental restrictions on the coupling constant of $\nu_\mu e$ scattering are rather weak[58]: $G_{\nu_\mu e}/G < 0.6$.

d) Elastic ee scattering and the reaction $e^+e^- \to \mu^+\mu^-$. The weak elastic ee scattering to order G^2 is described in the four-fermion theory by the diagrams of Figs. 25 and 26, and the reaction $e^+e^- \to \mu^+\mu^-$, by the diagram of Fig. 27. In all these diagrams the lepton and the antilepton enter into the same vertex, so that the reasoning of the subsection 4.2.2a) is applicable, thus proving the vanishing of the corrections of order $\sim G^2\Lambda^2$. (According to the remark at the end of subsection 4.2.2a) it is necessary that conservation of the currents $(\bar{\nu}_e\nu_e)$ and $(\bar{\nu}_e\nu_\mu)$ not be violated when the form of the weak interactions is changed, otherwise the terms $\sim G^2\Lambda^2$ will not vanish.)

Summarizing the discussion of the weak interactions of leptons we would like to stress that at present we know very little from experiment about the structure of these interactions. At the same time such information would be extremely important for the theory. In particular, it would be very interesting to determine (or find bounds on) the coupling constant for $\nu_\mu e$ scattering, which according to (4.17), (4.18) would make it possible to estimate higher-order weak corrections in leptonic interactions.*

4.2. Hadron-Lepton interactions

The fundamental problem which appears in considering corrections due to virtual weak interactions in hadron-lepton processes is whether the strong interactions cut off the growth with energy of the virtual weak interactions. For several years, no progress had been made in the solution of this problem. It was usually assumed (cf., e.g., the discussion of this problem in[31,32]) that the strong interactions lead to the appearance of formfactors $F(q^2)$ for the hadrons, formfactors which decrease rapidly as q^2 (the square of the momentum transfer) increases for $|q^2| \gg m^2$, and therefore the integration over the momenta of the virtual hadrons is effectively cut off at values $|q^2| \sim m^2$. These assumptions rested on the experimentally observed behavior of the form factors of elastic and inelastic ep scattering processes, and by analogy were extended to many-particle processes (although doubts were also expressed regarding the legitimacy of such an extrapolation[29]). However, the many-particle processes remained essentially a terra incognita.

An essential breakthrough in this question was realized only relatively recently, as a result of the development of current algebra methods. The use of current algebra made it possible to show that in some cases the strong interactions do not cut off the momentum integrals of virtual weakly interacting particles[9]. In the sequel we restrict our attention just to these most interesting cases.

We shall use the following notations for the weak hadronic current densities

$$j^{(L)i}_{\mu k} = V^i_{\mu k} - A^i_{\mu k}, \quad j_{\mu k}{}^{Ri} = V^i_{\mu k} + A^i_{\mu k}. \tag{4.19}$$

(Wherever there is no risk of confusion the index L will be omitted.)

We shall assume that the charge operators

$$Q^{(L,R)i}_k(x_0) \equiv Q^{Vi}_k \mp Q^{Ai}_k = \int d\mathbf{x}\, j^{(L,R)i}_{0k}(x) \tag{4.20}$$

and the currents $j^{(L,R)i}_{\mu k}$ satisfy the SU(3) × SU(3) algebra proposed by Gell-Mann[60] (cf. also the book by Adler and Dashen), i.e., that the following equal-time commutation relations are valid

$$[Q^{(L,R)i}_k(x_0),\ j^{(L,R)l}_{\mu m}(x)] = 2\,(\delta^i_m j^{(L,R)l}_{\mu}(x) - \delta^l_k j^{(L,R)i}_{\mu m}(x)), \tag{4.21}$$

$$[Q^R, j^L] = [Q^L, j^R] = 0, \quad (i, k, l, m = 1, 2, 3). \tag{4.22}$$

In a number of cases (and these are the most interesting), weaker assumptions will be sufficient, namely the validity of the SU(2) × SU(2) or SU(3) algebra for the vector currents. The validity of the SU(2) × SU(2) algebra is confirmed by all the results which follow from the PCAC hypothesis (cf. the review article[61]). The predictions of SU(3)-symmetry regarding weak decays are also in agreement with experiment[62]. Therefore the use of the SU(3) × SU(3) algebra in the form (4.21) and (4.22) seems to be well justified.

As will become clear from the sequel, the results described below remain valid also if the symmetry of the strong interactions, SU(2) × SU(2) or SU(3), is broken to a significant degree. It is only important that the weak interaction currents of the hadrons have the form (2.5) and are subject to the commutation relations (4.21), (4.22).

4.2.1. Weak neutral currents. a) The decay $K^0_2 \to \mu^+\mu^-$ (W-boson theory). As follows from the form of the Hamiltonian of the weak interactions [(2.1), (2.2)], in orders higher than one in the weak interaction hadronic transitions can occur with the emission of leptonic $l\bar{l}$-pairs with zero total charge, transitions which are absent in first order of G. To a high degree of accuracy such transitions have not been seen experimentally. Following[63], we consider the decay $K^0_L \to \mu^+\mu^-$, where the strictest experimental bound on the magnitude of neutral currents has been obtained[64]

$$\left.\frac{w(K_L^0\to\mu^+\mu^-)}{w(K_L^0\to \text{all})}\right|_{\text{exp}} < 1.8\cdot 10^{-9}. \qquad (4.23)$$

In the theory with intermediate vector bosons the matrix element for the decay mode $K^0 \to \mu^+\mu^-$ is described by the diagram of Fig. 28 and can be represented in the form

$$M = (4\pi g^2)^2 \sin\theta_C \int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m_W^2)^2}\left(\delta_{\lambda\alpha}-\frac{k_\lambda k_\alpha}{m_W^2}\right)\left(\delta_{\beta\sigma}-\frac{k_\beta k_\sigma}{m_W^2}\right) \qquad (4.24)$$
$$\times\, \bar u(p'_\mu)\gamma_\lambda(1+\gamma_5)\hat k^{-1}\gamma_\sigma(1+\gamma_5)u(p_\mu)M_{\alpha\beta}(k).$$
$$M_{\alpha\beta}(k) = i\int d^4x e^{ikx}\langle 0|T\{j_{\alpha1}^{(L)2}(x),\ j_{\beta3}^{(L)1}(0)\}|K^0\rangle. \qquad (4.24')$$

In (4.24) the momenta of the external particles have been neglected in comparison with k. In order to calculate the leading divergence in the integration with respect to k in (4.24) it suffices to consider only the longitudinal part, proportional to $k_\mu k_\nu/m_W^2$, in the Green's functions of the W bosons. Introducing k_α under the integral with respect to x, replacing $k_\alpha \exp(ikx)$ by $-i(\partial/\partial x_\alpha)\exp(ikx)$, integrating by parts and setting $\partial_\alpha V^2_{\alpha1}(x) = 0$, we find

$$k_\alpha M_{\alpha\beta}(k) = -\int d^4x e^{ikx}\langle 0|[j_{01}^{(L)2}(x),\ j_{\beta3}^{(L)1}(0)]\delta(x_0)$$
$$-T\{\partial_\alpha A^2_{\alpha1}(x),\ j^{(L)1}_{\beta3}(0)\}|K^0\rangle. \qquad (4.25)$$

The second term in (4.25) is proportional to the divergence of the axial-vector current without strangeness change and on the basis of the PCAC hypothesis it contains at least the small parameter $(\mu_\pi/m_0)^2 \sim 0.1$. (For large k the smallness parameter can be even smaller.) Therefore one may neglect in (4.25) the term which is proportional to $\partial_\alpha A^2_{\alpha1}$.

In order to find the equal-time commutators in (4.25) we assume that these commutators contain only terms which are proportional to $\delta(\mathbf{x})$ and $\partial_k\delta(\mathbf{x})$. Terms containing the first derivatives of $\delta(\mathbf{x})$ will not contribute to (4.24). In view of the fact that the $K^0 \to$ vacuum transition changes parity, one can rewrite the right-hand side of (4.25) in the form

$$\langle 0|[Q_1^{V2}(0),\ A^1_{\beta3}(0)] + [Q_1^{A2}(0),\ V^1_{\beta3}(0)]|K^0\rangle. \qquad (4.26)$$

We now assume (and this is the essential point of the whole reasoning) that the current-charge commutators are subject to the SU(2) × SU(2) algebra, i.e.,

$$[Q_1^{V2}(0),\ A^1_{\beta3}(0)] = [Q_1^{A2}(0),\ V^1_{\beta3}(0)] = -A^2_{\beta3}(0). \qquad (4.27)$$

It should be stressed that there is no need to require the SU(3) × SU(3) symmetry in order to obtain the result, but it suffices to assume SU(2) × SU(2) symmetry. (Thus, in place of calculating the commutators of charges with currents according to (4.27) one could act with the operators Q_1^{V2} and Q_1^{A2} on the state vectors between which matrix elements are taken, and utilize the PCAC results.) Substituting (4.27) into (4.26) and (4.26) into (4.24) we obtain after an integration with respect to k

$$M = -g^4\frac{\Lambda^2}{m_W^4}\langle 0|A^2_{\beta3}(0)|K^0\rangle\,\bar u(p'_\mu)\gamma_\beta(1+\gamma_5)u(p_\mu)\sin\theta_C. \qquad (4.28)$$

In view of the isospin invariance the matrix element $\langle 0|A^2_{\beta3}(0)|K^0\rangle$ equals the matrix element of the $K^+ \to \mu^+\nu$ decay

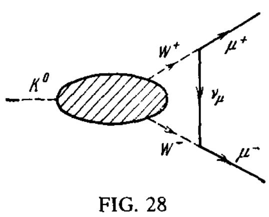

FIG. 28

$$\langle 0|A^2_{\beta3}(0)|K^0\rangle = \langle 0|A^1_{\beta3}(0)|K^+\rangle = f_K q_\beta, \qquad (4.29)$$

where q is the K-meson momentum. Making use of (4.29) and (4.28) it is easy to find the ratio of the rates for the $K_L^0 \to \mu^+\mu^-$ and $K^+ \to \mu^+\nu$ decay modes:

$$\frac{w(K_L^0\to\mu^+\mu^-)}{w(K^+\to\mu^+\nu)} = 4\left(\frac{g^2\Lambda^2}{4\pi m_W^2}\right)^2 = 2\left[\frac{G\Lambda^2}{(4\pi)^2}\right]^2. \qquad (4.30)$$

Here it is appropriate to make some remarks regarding the assumptions made in this derivation. Since the result (4.30) is proportional to the fourth power of the cutoff parameter Λ one can hardly expect more than order-of-magnitude accuracy from this result. Therefore the terms omitted from (4.25), terms which are proportional to the divergence of the axial vector current, could be essential only in the case when they completely cancel the contribution of the fundamental term, which seems highly improbable in view of the approximate conservation of the axial vector current. If we assume further that the SU(3) symmetry is better satisfied than the SU(2) × SU(2) symmetry, one may use the fact that a nonvanishing contribution to (4.24) comes only from products of the vector current and the axial vector current to reformulate the proof so that the omitted terms contain only the divergence of the strangeness-changing vector current, $\partial_\beta V^1_{\beta3}(x)$, and utilizing the commutation relations for the group SU(3). Finally, in (4.24) we have not taken into account diagrams with the emission of two W bosons from the same point. One can prove[63] that the contribution of these diagrams leads to a different spinor structure of the leptonic part of the matrix element, therefore these diagrams cannot compensate the contribution of the main term (the compensation of (4.30) does not exceed 20%).

A comparison of the result (4.30) with experiment is at present difficult since the experimental result (4.23) contradicts the inequality [65]

$$\frac{w(K_L^0\to\mu^+\mu^-)}{w(K_L^0\to \text{all})} > 6\cdot 10^{-9},$$

which follows from the unitarity condition (taking into account only the intermediate 2γ state) and the reason for this discrepancy is not clear at the present time.* Therefore, for the sake of caution, we adopt in place of (4.23) a weaker inequality

$$\frac{w(K_L^0\to\mu^+\mu^-)}{w(K_L^0\to \text{all})} < 10^{-8}, \qquad (4.31)$$

which is also based on experimental data[66]. It follows from (4.30) and (4.31) that

$$\Lambda < 25\ \text{GeV}, \qquad (4.32)$$

i.e., that the cutoff limit is rather low.

b) The decay mode $K_L^0 \to \mu^+\mu^-$ (four-fermion theory). In the four-fermion theory the matrix element for the decay $K^0 \to \mu^+\mu^-$ (Fig. 29) takes the form

$$M = \frac{G^2}{2}\sin\theta_C\int\frac{d^4k}{(2\pi)^4}\bar u(p'_\mu)\gamma_\alpha(1+\gamma_5) \times \hat k^{-1}\gamma_\beta(1+\gamma_5)u(p_\mu)M_{\alpha\beta}(k), \qquad (4.33)$$

where $M_{\alpha\beta}(k)$ is defined according to (4.24′). The leptonic part of (4.33) can be written as follows:

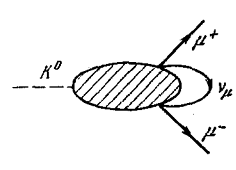

FIG. 29

$$2\bar{u}(p'_\mu)\,[k_\alpha\gamma_\beta + k_\beta\gamma_\alpha - \delta_{\alpha\beta}\hat{k} - i\varepsilon_{\alpha\beta\lambda\sigma}k_\lambda\gamma_\sigma] \times (1+\gamma_5)\,u(p_\mu)\,k^{-2}. \qquad (4.34)$$

Under the assumption of SU(3) × SU(3) symmetry and neglecting the current divergences, the contribution of the first two terms in the square brackets in (4.34) to the matrix element (4.33) can be calculated in the same manner as in the W-boson theory, yielding for the matrix element the expression (4.28), but with a numerical coefficient eight times larger. The contribution of the terms $-\delta_{\alpha\beta}\hat{k} - i\epsilon_{\alpha\beta\lambda\sigma}k_\lambda\gamma_\sigma$ can be calculated[67] if one considers the Bjorken assumption[68], that for $k_0 \to \infty$ and $\mathbf{k}$ = const

$$\lim k_0 M_{\alpha\beta}(k) = -\int dx e^{-ikx}\langle 0|[j^{(L)2}_{\alpha_1}(x),\ j^{(L)}_{\beta 3}(0)]|K^0\rangle \qquad (4.35)$$

and utilizes the commutation relations between the space components of the currents. These relations are model-dependent. In the quark model consideration of the terms $-\delta_{\alpha\beta}\hat{k} - i\epsilon_{\alpha\beta\lambda\sigma}k_\lambda\gamma_\sigma$ leads to the result that the numerical coefficient in (4.28) is reduced by a factor of two compared to the value it had when only the terms $k_\alpha\gamma_\beta + k_\beta\gamma_\alpha$ are taken into account. Thus, as a result, in the quark model one obtains the expression (4.28) with the additional factor 4, i.e., (4.31) implies the following bound on Λ:

$$\Lambda < 12.5\ \text{GeV} \qquad (4.36)$$

c) Other hadron-lepton interactions. The method exposed above can also be applied[63] to the discussion of other hadron-lepton interactions involving neutral currents, such as, for example, the elastic scattering of neutrinos on nucleons or the decay modes $K^+ \to \pi^+e^+e^-$, $K^+ \to \pi^+\bar{\nu}\nu$. In the first case (in the W-boson theory) the effective coupling constant of neutral leptonic currents to nucleons is

$$G_{\text{neutr}} = G\frac{G\Lambda^2}{(4\pi)^2}, \qquad (4.37)$$

whereas experimentally $G_{\text{neutr}} < 0.3G$[69]. For the ratio of the rates of the decays $K^+ \to \pi^+e^+e^-$ and $K^+ \to \pi^0e^+\nu$ one obtains in the W-boson theory the expression

$$\frac{w(K^+\to\pi^+e^+e^-)}{w(K^+\to\pi^0e^+\nu)} = \left[\frac{G\Lambda^2}{(4\pi)^2}\right]^2. \qquad (4.38)$$

A comparison with the experimental bound[70]

$$\frac{w(K^+\to\pi^+e^+e^-)}{w(K^+\to \text{all})} < 4\cdot 10^{-7} \qquad (4.38')$$

yields

$$\Lambda < 200\ \text{GeV}.$$

In the four-fermion theory (with the quark model) the same data yield

$$\Lambda < 100\ \text{GeV}$$

The ratio of the decay rates for the modes $K^+ \to \pi^+\bar{\nu}_e\nu_e$ and $K^+ \to \pi^0e^+\nu$ in the W-boson theory is equal to (4.38). The experimental bound[71] on the rate of the mode $K^+ \to \pi^+\bar{\nu}\nu$ is $w(K^+ \to \pi^+\bar{\nu}\nu)/w(K^+ \to \text{all}) < 1.2 \times 10^{-6}$, and leads to the inequality $\Lambda < 220$ GeV. It should be noted that from a theoretical point of view an investigation of the decay mode $K^+ \to \pi^+\nu^-$ has definite advantages over the decay $K^0_L \to \mu^+\mu^-$ or $K^+ \to \pi^+e^+e^-$, since transitions due to both weak and electromagnetic interaction are possible in the latter, according to the chain $K^0_L \to 2\gamma \to \mu^+\mu^-$ or $K^+ \to \pi^+\gamma \to \pi^+e^+e^-$ these, by virtue of the unitarity condition, determine the lower bound of the decay rates. In the process $K^+ \to \pi^+\bar{\nu}\nu$ such transitions are absent, which, in principle, allows one to obtain stronger bounds on Λ.

The values $\Lambda < 25$ GeV in the W-boson theory, and $\Lambda < 12.5$ GeV in the four-fermion theory, derived from the consideration of the $K^0_L \to \mu^+\mu^-$ decays, are at present the strongest and most accurate restrictions on the domain of applicability of the theory of weak interactions in its present form. These values of Λ are considerably lower than the unitarity limit, indicating the need for changing the theory of weak interactions (or at least part of it) for momentum transfers larger or equal to several tens of GeV, i.e., for distances smaller than 10^{-15} cm.

4.2.2. The ratio of the beta and muondecay vector coupling constants As is well known, in the V – A theory of weak interactions the vector coupling constant in beta decay is not renormalized due to the strong interactions, so that to first order in the weak interaction the beta decay vector coupling constant (divided by cos θ_C) must be equal to the muon decay coupling constant. The experiments confirm this equality within an accuracy of 1% (a certain indeterminacy arises on account of electromagnetic radiative corrections, which introduce an uncertainty of the order 1% into the ratio G_β/G_μ).

In higher orders of perturbation theory the weak interactions lead to a violation of this equality. In the four-fermion theory the corrections to the muon decay coupling constant of order $G\Lambda^2$ due to weak interactions are determined by the diagram of Fig. 19, and the corrections of the order $G\Lambda^2$ to the beta decay coupling are described by the diagram of Fig. 30. The contributions from the diagrams of Fig. 19 and Fig. 30 are identical. Therefore (if one ignores the corrections due to nonleptonic weak interactions which are possibly cut off by the strong interactions) in the four-fermion theory the equality $G^V/\cos\theta_C = G$ remains in force when terms of the order $G\Lambda^2$ are taken into account.

In the terms of order $(G\Lambda^2)^2$ the equality $G^V_\beta/\cos\theta_C = G_\mu$ is violated. A consideration of these terms does not lead however to strong restrictions on Λ[72].

In the W-boson theory the diagram of Fig. 20 contributes to the renormalization of the muon decay constant, as well as the four diagrams of Fig. 21. A diagram corresponding to that of Fig. 20 exists for the beta decay, so that in the calculation of the ratio G^V_β/G_μ the contribution of these diagrams may be omitted. The same is true for diagrams where the W boson is emitted and absorbed by the electron or the electronic neutrino. We consider the contribution to the renormalization of G_β of the other diagrams of order $G\Lambda^2$ in the W-boson theory (Fig. 31). The matrix element for the correction of order g^2 to the vertex part describing the emission of the W in the $n \to p$ transition has the form

$$\Gamma'_\sigma = 4\pi g^2\int\frac{d^4k}{(2\pi)^4}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{m^2_W}\right)\frac{1}{k^2 - m^2_W}M_{\mu\nu\sigma}(k), \qquad (4.39)$$

$$M_{\mu\nu\sigma}(k) = \frac{i}{2}\int d^4x\,d^4y\,e^{ik(x-y)}\langle p|T[j^+_\mu(x), j^-_\nu(y), j^+_\sigma(0) + j^-_\mu(x), j^+_\nu(y), j^+_\sigma(0)]|n\rangle. \qquad (4.40)$$

The most divergent expressions in the integration with

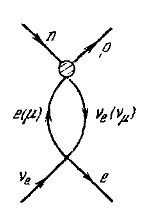

FIG. 30

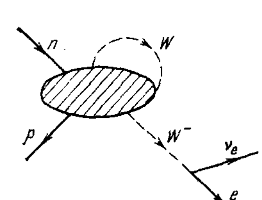

FIG. 31

respect to k in (4.39) appear due to the term $-k_\mu k_\nu/m_W^2$ in the W-boson Green's function. The quadratically divergent terms in (4.39) can be brought to the form

$$\Gamma'_\sigma = -\frac{i}{2}\frac{4\pi g^2}{(2\pi)^4}\frac{1}{m_W^2}\int\frac{d^4k}{k^2}\int d^4x\, d^4y\, e^{ik(x-y)}$$
$$\times\langle p\,|\,[j_0^+(x),\ [j_0^-(y),\ j_\sigma^+(0)]]\,\delta(x_0)\,\delta(y_0) + 2T\,\{[j_0^+(x),\ \partial_\nu j_\nu^-(y)]\,\delta(x_0-y_0),\ j_\sigma^+(0)\}$$
$$+T\,\{\partial_\nu j_\nu^+(y),\ [j_0^-(x),\ j_\sigma^+(0)]\,\delta(x_0)\} + T\,\{\partial_\mu j_\mu^+(x),\ [j_0^-(y),\ j_\sigma^+(0)]\,\delta(y_0)\} +$$
$$+T\,\{\partial_\mu j_\mu^+(x),\ \partial_\nu j_\nu^-(y),\ j_\sigma^+(0) + \partial_\mu j_\mu^-(x),\ \partial_\nu j_\nu^+(y),\ j_\sigma^+(0)\}\,|\,n\rangle. \tag{4.41}$$

All terms, except the first, in the right-hand side of (4.41) are proportional to the divergence of the weak current, i.e., they contain the small parameter $(\mu_\pi/m_0)^2$ or $\sin^2\theta_C$. In addition, all terms, except the first and second, depend on k, and one might think that they decrease with increasing k. Therefore it is reasonable to consider only the first term in the right-hand side of (4.41). In the calculation of the equal-time commutator, like in the discussion of the decay $K^0\to\mu^+\mu^-$, it suffices to use the charge algebra $SU(2)\times SU(2)$ in place of the current algebra. Neglecting $\sin^2\theta_C$ we obtain

$$\int d^4x\, d^4y\, e^{ik(x-y)}\langle p\,|\,[j_0^+(x),\ [j_0^-(y),\ j_\sigma^+(0)]]\,\delta(x_0)\,\delta(y_0)\,|\,n\rangle \tag{4.42}$$
$$=\langle p\,|\,[Q^{(L)+}(0),\ [Q^{(L)-}(0),\ j_\sigma^+(0)]]\,|\,n\rangle = 8\langle p\,|\,j_\sigma^+(0)\,|\,n\rangle,$$

$$\Gamma'_\sigma = -\frac{g^2\Lambda^2}{m_W^2}\langle p\,|\,j_\sigma^+(0)\,|\,n\rangle. \tag{4.43}$$

The correction of order g^2 to the vertex where the W boson is emitted has been expressed in terms of the matrix element of the current. Since the vector part of the matrix element is not changed on account of the strong interactions, this correction has the same form as if the strong interactions did not exist. But in the absence of the strong interactions the renormalization of the beta decay constant to order g^2 is due only to the renormalization of the external nucleon lines (except for the diagram analogous to Fig. 20). The quadratically divergent parts of these diagrams are obviously the same as for the corresponding diagrams of muon decay. (One can check directly that the two diagrams in Fig. 21 contribute to the renormalization of g a term equal to (4.43) by calculating the Z_2 factor for the muon in the g^2 approximation taking into account that in view of parity nonconservation and the two-component character of the interaction, the relation between the "bare" and renormalized coupling constants in the g^2-approximation has the form[30] $g = g_0 Z_2/(2 - Z_2)$.) Therefore in the W-boson theory the ratio between the vector coupling constants of the beta and muon decays does not change when corrections of the order $G\Lambda^2$ are taken into account.

4.3. Nonleptonic interactions

a) The $K_L^0 - K_S^0$ mass difference. The $K_L^0 - K_S^0$ mass difference $\Delta m = m_{K_S} - m_{K_L}$ is of particular interest, since experimentally $\Delta m \sim G^2 m^5$ (more precisely, $\Delta m_{exp} = -0.8\times 10^{-4} G^2 m_N^4 m_K$, where m_N, m_K are respectively the masses of the nucleon and the K^0-meson), i.e., the quantity Δm is so far the only experimentally observable quantity which is of second order in the weak interaction. If one estimates, however, the magnitude of Δm by means of perturbation theory without taking into account the strong interactions[59,73], e.g., by considering the transition $K^0\to\bar K^0$ via a virtual nucleon-antinucleon pair, one obtains for Δm the considerably larger value $\Delta m \sim G^2\Lambda^2 m^3$ (i.e., the experiment would require that $\Lambda \sim m$). Since the strong interactions can lead to a cutoff of the integrals at momenta $\Lambda \sim m$, it is very important to study this problem in a manner which takes the strong interactions into account correctly.

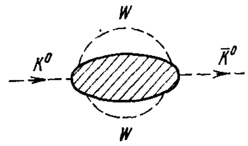

FIG. 32

We consider this problem following[63] (cf. also[74]). The mass difference between the K_L^0 and K_S^0 mesons is determined by the matrix element $M_{K^0\to\bar K^0}$ of the $K^0\to\bar K^0$ transition according to

$$\Delta m = m_{K_L} - m_{K_S} = -2M_{K^0\to\bar K^0}. \tag{4.44}$$

In the theory with intermediate W bosons the matrix element $M_{K^0-\bar K^0}$ is described by the diagram of Fig. 32 and has the form

$$M_{K^0\to\bar K^0} = \frac{(4\pi g^2)^2}{(2\pi)^8}\frac{1}{2m_K}\int\frac{d^4k_1}{k_1^2-m_W^2}\frac{d^4k_2}{k_2^2-m_W^2}\left(\delta_{\mu\lambda}-\frac{k_{1\mu}k_{1\lambda}}{m_W^2}\right)$$
$$\times\left(\delta_{\nu\sigma}-\frac{k_{2\nu}k_{2\sigma}}{m_W^2}\right)M_{\mu\nu\lambda\sigma}(p,\ k_1,\ k_2), \tag{4.45}$$

where

$$(2\pi)^4\,\delta^4(p_{K^0}-p_{\bar K^0})\,M_{\mu\nu\lambda\sigma}(p,\ k_1,\ k_2)$$
$$= -i\int d^4x\, d^4x'\, d^4y\, d^4y'\, e^{-ik_1(x-x')+ik_2(y-y')}$$
$$\times\langle\bar K^0\,|\,T\,\{j_\mu^+(x),\ j_\nu^-(y),\ j_\lambda^-(x'),\ j_\sigma^+(y')\}\,|\,K^0\rangle \tag{4.46}$$

and the state $|K^0\rangle$ is normalized so that $||K^0\rangle|^2 = 1$.

Considering in (4.45) only the most divergent terms, i.e., the terms proportional to $k_{1\mu}k_{1\lambda}k_{2\nu}k_{2\sigma}M_{\mu\nu\lambda\sigma}$, utilizing the $SU(3)\times SU(3)$ current algebra, and neglecting the divergences of currents, we obtain from (4.45), (4.46) after some transformations

$$M_{K^0\to\bar K^0} = \frac{(4\pi g^2)^2}{(2\pi)^8}\frac{4\sin^2\theta_C\cos^2\theta_C}{m_W^4\cdot 2m_K}\int\frac{d^4K\,d^4k\,K_\nu K_\lambda}{[K^2+(k^2/4)-m_W^2]^2-(Kk)^2}M_{\lambda\nu}(p,\ k), \tag{4.47}$$

where

$$K = \frac{k_1+k_2}{2},\quad k = k_1-k_2, \tag{4.48}$$

$$M_{\lambda\nu}(p,\ k) = i\int d^4x\, e^{ikx}\langle\bar K^0\,|\,T\,\{j_{\lambda 3}^2(x),\ j_{\nu 3}^2(0)\}\,|\,K^0\rangle. \tag{4.49}$$

The integral over K in (4.47) diverges quadratically. In order to evaluate it we assume that $M_{\lambda\nu}$ is a function that decreases sufficiently rapidly as k increases, so that the integral with respect to k converges. Then one can neglect k compared to K. Integrating with respect to K and k in (4.47) and making use of (4.44), we obtain

$$\Delta m = -\frac{G^2\Lambda^2\sin^2\theta_C\cos^2\theta_C}{(4\pi)^2\cdot 2m_K}\mathrm{Re}\,\langle\bar K^0\,|\,j_{\mu 3}^2(0)\,j_{\mu 3}^2(0)\,|\,K^0\rangle. \tag{4.50}$$

We note that the reasoning above can be carried over without modification to the discussion of the amplitude of any nonleptonic process with $\Delta S = 2$, and (up to the normalization of the initial and final states) this amplitude will be equal to the right-hand side of the relation (4.50).

Introducing in (4.50) a summation over a complete set of intermediate states $|n\rangle$, we write Δm in the form

$$\Delta m = \frac{G^2\Lambda^2\sin^2\theta_C\cos^2\theta_C}{(4\pi)^2\cdot 2m_K}\sum_n C_n\{|\langle n\,|\,V_{\mu 3}^2(0)\,|\,K^0\rangle|^2 - |\langle n\,|\,A_{\mu 3}^2(0)\,|\,K^0\rangle|^2\}, \tag{4.51}$$

where C_n is the charge-conjugation parity of the state $|n\rangle$. In order to estimate the order of magnitude of Δm we consider the contribution to the sum over n in (4.50) of several of the lowest intermediate states. The lowest intermediate state in (4.51) is the vacuum state. In this case the matrix element with the axial-vector current defined in (4.29) comes into play. Substituting (4.29) into (4.51), we obtain

$$(\Delta m)_{\mathrm{vac}} = -\frac{G^2\Lambda^2}{(4\pi)^2}\,\frac{1}{2}\sin^2\theta_C\cos^2\theta_C f_K^2 m_K,\quad f_K \approx m_\pi. \tag{4.52}$$

Comparing (4.52) with the experimental value of Δm we get the inequality

$$\Lambda \leqslant 5\ \mathrm{GeV} \tag{4.53}$$

Such a low value of the cutoff would seem to imply that the weak interaction changes its form at relatively modest energies. In reality, however, one cannot draw such a conclusion from a consideration of the $K_L - K_S$ mass difference. In order to see this we consider the contribution to (4.51) of the next-higher state, namely the one-pion intermediate state. In this case the matrix element of the vector current does not vanish, and equals the matrix element for K_{e3} decay:

$$\langle \pi^0 | V^3_{\mu_3}(0) | K^0\rangle = \langle \pi^0 | V^1_{\mu_3}(0) | K^+\rangle. \tag{4.54}$$

Substituting for the latter the expression that results from SU(3)-symmetry

$$\langle \pi^0 | V^1_{\mu 3}(0) | K^+\rangle = \frac{1}{\sqrt{2}}(p_\mu + q_\mu) F(\varkappa^2),\quad \varkappa^2 = (p-q)^2, \tag{4.55}$$

where p and q are the momenta of the K and π, respectively, and $F(\kappa^2)$ is the form factor of K_{e3} decay, we obtain after simple manipulations

$$(\Delta m)_{\pi^0} = \frac{G^2\Lambda^2}{(4\pi)^4}\sin^2\theta_C\cos^2\theta_C \int\limits_0^\infty q_0\,dq_0\,(m_K + 2q_0) F^2(m_K^2 - 2mq_0). \tag{4.56}$$

The integral in (4.56) contains values of $F(\kappa^2)$ in the unphysical region $\kappa^2 < 0$ for K_{e3} decay, and the integral converges if $F(\kappa^2)$ decreases in that region faster than $(-\kappa^2)^{3/2}$. If one assumes that this is indeed so, i.e., that for instance $F(\kappa^2) \sim [1 - (\kappa^2/m_0^2)]^{-2}$ for $-\kappa^2 \gtrsim m_0^2$, one obtains the following estimate for the contribution of the one-pion state to (4.51) (for $m_0 \gg m_K$)

$$(\Delta m)_{\pi^0} = \frac{G^2\Lambda^2}{(4\pi)^4}\,\frac{\sin^2\theta_C\cos^2\theta_C}{12}\,\frac{m_0^6}{m_K^3} \tag{4.57}$$

and for $m_0 \approx m_{K^*}\,\Delta m_{\pi^0} \approx -0.5\,\Delta m_{\mathrm{vac}}$. From here it can be seen that a very strong compensation of the contributions of various intermediate states is possible for the quantity Δm defined according to (4.51) (it is not even excluded that a complete compensation occurs, i.e., that the sum over n in (4.51) vanishes). Thus, one can at best attribute only a qualitative meaning to the estimate (4.53) for the cutoff, obtained by taking into account only the vacuum as an intermediate state.

A similar consideration with the same conclusions can be carried out in the four-fermion theory.[74] However, additional assumptions must be made in order to derive the result.

b) Hadronic processes involving change of strangeness, parity or isospin. One of the most serious difficulties of the theory appears in the discussion of hadronic processes with $|\Delta S| = 1$ or $P = -1$. Experimentally, these processes are possible due to the weak interactions and their (dimensionless) amplitudes are of the order Gm_0^2 (m_0 is some characteristic mass $\lesssim 1$ GeV)[10)], whereas theoretically one could expect that their amplitudes will be of the order $G\Lambda^2$, i.e., even of order 1 for $\Lambda^2 \sim G^{-1}$.

Let us consider the situation which appears in the W-boson theory[76]. (In the four-fermion theory the situation is less well-defined.) The matrix element for the transition between the hadronic states a and b, schematically represented in Fig. 33, can be written in the form:

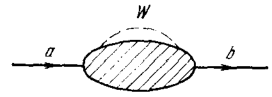

FIG. 33

$$M(2\pi)^4\delta^4(p_a - p_b) = \frac{4\pi g^2}{(2\pi)^4}\, i\int \frac{d^4k}{k^2 - m_W^2}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{m_W^2}\right) \times \int d^4x\, d^4y\, e^{ik(x-y)}\langle b | T\{j^+_\mu(x),\ j^-_\nu(y)\} | a\rangle. \tag{4.58}$$

Strictly speaking the expression (4.58) must be symmetrized with respect to the currents j^+ and j^-, but since a rigorous discussion which takes into account the symmetrization leads to the same result as when one forgets to symmetrize, we shall not list here the appropriate formulas.

The quadratically divergent term in (4.58) has the form

$$\begin{aligned} M_{\mathrm{div}}(2\pi)^4\delta^4(p_a - p_b) &= \frac{g^2}{4\pi^3 m_W^2}\, i\int\frac{d^4k}{k^2}\int d^4x\, d^4y\, e^{ik(x-y)}\,\delta(x_0 - y_0)\langle b | [\partial_\mu j^+_\mu(x),\ j^-_0(y)] | a\rangle \\ &= \frac{g^2\Lambda^2}{4\pi m_W^2}\int d^4x\,\langle b | [\partial_\mu j^+_\mu(x),\ Q^{(L)-}(x_0)] | a\rangle, \end{aligned} \tag{4.59}$$

In going from (4.58) to (4.59) we have assumed that $\int d^4x \langle b | T\{\partial_\mu j^+_\mu(x),\ \partial_\nu j^-_\nu(0)\} | a\rangle$ decreases as $k \to \infty$, and that the Schwinger terms are c-numbers. In addition, in the transition to the last of the equations (4.59) it was assumed that the equal-time commutator $[\partial_\mu j^+_\mu(x),\ j^-_0(y)]$ contains only terms proportional to $\delta(\mathbf{x} - \mathbf{y})$ and $\delta'(\mathbf{x} - \mathbf{y})$.

We first apply the equation we have obtained to a $|\Delta S| = 1$ transition, and for concreteness we assume that $S_a - S_b = 1$. Then a nonvanishing contribution to (4.59) comes only from terms with $\Delta S = 0$ from $\partial_\mu j^+_\mu(x)$ and from terms with $|\Delta S| = 1$, involving $Q^-(x_0)$. The main contribution to the divergence of the strangeness-conserving current comes from the axial vector current, so that $\partial_\mu j^+(x)$ is determined by a small parameter which determines the partial conservation of the axial current (PCAC), namely $(\mu_\pi/m_0)^2 \sim 0.1$. Consequently we obtain for the appropriate dimensionless matrix element of the $|\Delta S| = 1$ transition the estimate[76b]

$$M^{|\Delta S|=1}_{\mathrm{div}} \sim \frac{G\Lambda^2}{(4\pi)^2\sqrt{2}}\sin\theta_C\left(\frac{\mu_\pi}{m_0}\right)^2. \tag{4.60}$$

Comparing (4.60) with the experimentally observed order of magnitude of the matrix element for the transition with $|\Delta S| = 1$, $M^{|\Delta S|}_{\mathrm{exp}} = 1 \sim Gm_0^2\sin^2\theta_C$ we obtain for the cutoff limit the following estimate

$$\Lambda \leqslant 25\ \mathrm{GeV}. \tag{4.61}$$

We now consider $\Delta S = 0$ transitions which change the parity, $P = -1$. In this case one of the weak currents in (4.58) must be a vector current and the second one an axial vector current, and one can write the quadratically divergent part of (4.59) in such a form that it contains the divergence of the vector current, $\partial_\mu V^+_\mu$. If both currents do not change the strangeness, the smallness of $\partial_\mu V^+_\mu$ is determined by the violation of isospin invariance, i.e., by the parameter e^2/π and the following estimate holds for M_{div}[76b]

$$M^{P=-1}_{\mathrm{div}} \sim \frac{G\Lambda^2}{(4\pi)^2\sqrt{2}}\,\frac{e^2}{\pi}. \tag{4.62}$$

If both currents are strangeness-changing the smallness of $\partial_\mu V^+_\mu$ will be determined by the violation of the SU(3)-

symmetry (i.e., by the parameter $\lambda \sim 1/5$) and M_{div} will be of the order

$$M_{\rm div}^{P=-1} \sim \frac{G\Lambda^2}{(4\pi)^2\sqrt{2}} \sin^2\theta_C \cdot \lambda. \tag{4.63}$$

Comparing (4.62), (4.63) with $M_{exp}^{P=-1} \sim GM_0^2$, we obtain

$$\Lambda \leqslant 100 - 150\,\text{GeV}. \tag{4.64}$$

Transitions involving a change of isospin $|\Delta T| = 1$, $\Delta S = 0$, $P = +1$, are not that critical for the theory, since such transitions could take place on account of virtual electromagnetic interactions, and the experimentally observable matrix element would be of the order e^2/π. A theoretical estimate for the matrix element of the weak transition follows from (4.59), the largest contribution coming from the case when both currents j^+ and j^- are axial-vector with $\Delta S = 0$. Then

$$M_{\rm div}^{|\Delta T|=1} \sim \frac{G\Lambda^2}{(4\pi)^2\sqrt{2}} \left(\frac{\mu_\pi}{m_0}\right)^2. \tag{4.65}$$

A comparison of (4.65) with $M_{exp}^{|\Delta T|=1} \sim e^2/\pi$ yields

$$\Lambda \leqslant 700\ \text{GeV}. \tag{4.66}$$

Estimates of $|\Delta S| = 1$ (and also $P = -1$) hadronic processes lead to a rather low cutoff Λ, of the same order as that obtained from the $K_L^0 \to \mu^+\mu^-$ decay. The consideration of hadronic processes, however, differs from the consideration of hadron-lepton processes in one important aspect. The divergent part of the matrix element (4.59) is proportional to the x-integral of the equal-time commutator $[\partial_\mu j_\mu^+(x), Q^{(L)-}(x_0)]$ and thus depends on the structure of the strong interactions, since the divergence of the current can be expressed in terms of the strong interaction Hamiltonian $\mathscr{H}_S(t)$ in the following way:

$$\int dx \partial_\mu j_\mu^+(x) = \int dx \partial_0 j_0^+(x) = i\,[\mathscr{H}_S(x_0), Q^{(L)+}(x_0)]. \tag{4.67}$$

Therefore, in principle, there is a possibility of making the matrix elements in the right-hand side of (4.59) vanish, by means of a special choice of the strong interaction Hamiltonian.

This possibility was indicated by Bouchiat, Illiopoulos and Prentki[77], who have assumed that the strong interaction Hamiltonian consists only of terms of two types: an $SU(3) \times SU(3)$-symmetric $\mathscr{H}_0$ and a term $\mathscr{H}_1$ which violates the $SU(3) \times SU(3)$-symmetry, and transforms according to the $(\overline{3}, 3) + (3, \overline{3})$ representation:

$$\mathscr{H}_S = \mathscr{H}_0 + \mathscr{H}_1, \quad \mathscr{H}_1 = \sum_{i=1}^{3} \varepsilon_i L^i R_i + \text{h.c.}, \tag{4.68}$$

where ϵ_i are constants, and L^i and R^i are operators transforming according to the triplet representations of the group $SU(3)$, with the generators $Q_k^{(L)i}$ and $Q_k^{(R)i}$, respectively. (The Hamiltonian (4.68) for the strong interactions had been proposed earlier by Gell-Mann[60].) In the simplest quark model the mass-term corresponding to the Hamiltonian $\mathscr{H}_1$ in (4.68) has the form

$$\sum_{i=1}^{3} m_i \bar{\psi}_i \psi_i = \frac{1}{2} \sum_{i=1}^{3} m_i\,[\bar{\psi}_i (1+\gamma_5)\psi_i + \bar{\psi}_i (1-\gamma_5)\psi_i], \tag{4.69}$$

so that the operators L and R are the ψ-functions of the quarks with left and right helicity, respectively.

The requirement of isospin invariance leads to the equality $\epsilon_1 = \epsilon_2$. With the aid of the Hamiltonian (4.68) it is not hard to calculate the expression for the divergence of the current (4.67) and the equal-time commutator in (4.59), making use of the commutation relations of $SU(3)$:

$$[L_i, Q_m^{(L)e}] = \delta_m^l L_i - \delta_i^l L_m, \quad [L^i, Q_m^{(L)l}] = \delta_m^i L^l - \delta_m^l L^i,$$
$$[R_i, Q_m^{(L)e}] = [R^i, Q_m^{(L)e}] = 0. \tag{4.70}$$

For $|\Delta S| = 1$ transitions one obtains

$$\int d^4x\,[\partial_\mu j_\mu^+(x), Q^{(L)-}(x_0)]_2^3 = 2\cos\theta_C \sin\theta_C \int d^4x \partial_\mu F_{\mu_2}^3(x), \tag{4.71}$$

where

$$F_{\mu_2}^3(x) = \frac{\gamma}{2}\,[j_{\mu 2}^{(L)3}(x) + j_{\mu 2}^{(R)3}(x)] + \frac{1}{2\gamma}\,[j_{\mu 2}^{(L)3}(x) - j_{\mu 2}^{(R)3}(x)], \quad \gamma = \frac{\varepsilon_2 + \varepsilon_3}{\varepsilon_2 - \varepsilon_3}. \tag{4.72}$$

Substituting (4.71) into (4.59) yields $M_{div}^{|\Delta S|=1} = 0$, since

$$\int d^4x \langle b \mid \partial_\mu F_\mu(x) \mid a\rangle =$$
$$= i\,(2\pi)^4 \delta^4(p_b - p_a)(p_b - p_a)_\mu \langle b \mid F_\mu(0) \mid a\rangle = 0, \tag{4.73}$$

on account of four-momentum conservation in the $a \to b$ transition.

Similarly, and with the same result $M_{div}^{P=-1} = 0$, one can also discuss transitions with a change in strangeness.

For transitions involving a change in isospin the commutator $[\partial_\mu j_\mu^+(x), Q^{(L)-}(x_0)]$ does not reduce to the divergence of some operator, so that for these transitions the proof that M_{div} vanishes is not valid, and M_{div} turns out, in general, to be different from zero. However, as was shown (cf. (4.66)), the experimental data do not lead to strong restrictions on the cutoff parameter Λ in such transitions and consequently the non-vanishing of the quadratically divergent terms in the matrix elements of transitions with $|\Delta T| = 1$, $\Delta S = 0$, $P = 1$ presents no difficulty for the theory.

The result obtained above can be given a simple explanation[11], by noting that in the theory with the Hamiltonian (4.68) (or in the quark model (4.69)) the current divergences have the forms of mass terms (in general, nondiagonal). A computation of the commutator of such a term with the operator $Q^{(L)-}$ corresponds to a rotation in unitary space which also transforms it into a mass term, but with different unitary indices. In view of what was said before, this mass term is proportional to the divergence of some vector current, as long as this divergence does not vanish identically. It thus becomes clear why the quantity M_{div} defined according to (4.59) is zero for transitions with $|\Delta S| = 1$ or $P = -1$, to which correspond currents with nonvanishing divergence, whereas M_{div} does not vanish for transitions with $|\Delta T| = 1$, to which corresponds a conserved current, on account of isospin invariance.

One could attempt to find out whether the conclusions derived from the hypothesis about the strong interaction Hamiltonian (4.68) remain valid for corrections of arbitrary order $(G\Lambda^2)^n$ to hadronic processes. This question was investigated in[78,79], and it was shown that whereas for transitions with $|\Delta S| = 1$, $M_{div}^{|\Delta S|=1} = 0$ to all orders, for transitions with $P = -1$, $M_{div}^{P=-1} \neq 0$ in higher orders.

Finishing the discussion of the hypothesis of Bouchiat, Iliopoulos and Prentki[77] it should be noted that for its success it is necessary that the strong interaction Hamiltonian have the form (4.68), without any correction terms transforming according to other representations of the group $SU(3) \times SU(3)$, since the presence of even relatively small terms of this kind (say, of order 10^{-2} of the fundamental ones) would lead for $\Lambda^2 \sim G^{-1}$ to very large values of the transition amplitudes with $|\Delta S| = 1$ or $P = -1$, in contradiction with the experimental data. On the other hand, a rigorous absence of such correction terms appears to be quite strange. There is, however, an escape, consisting in the fact that for the vanishing of M_{div} one must not necessarily require that the Hamiltonian of the strong interactions have the form (4.68), but it suffices that the equal-time commutator in (4.59) have the same form as for the Hamiltonian (4.68) (i.e., the form (4.71) for $|\Delta S| = 1$ transitions), i.e., that the structure of the singularity of the commutator $[\partial_\mu j_\mu(x), Q^{(L)-}(y)]$ for small $x - y$ remain the same. (An analysis of the structure of the singularity of current commutators was proposed in [80].)

Although, as was already explained, transitions with $|\Delta T| = 1$ present a difficulty for the theory, it is interesting to investigate whether one cannot generalize the form of the Hamiltonian (4.68) in such a manner that one obtains a vanishing of the quadratically divergent terms also in the amplitude for these transitions. Such an attempt was undertaken in [81,82].

In [81] it was assumed[12)] that in the Hamiltonian (4.68) there are present terms which violate the isospin invariance, i.e., $\epsilon_1 \neq \epsilon_2$. Then it is convenient to expand $\mathscr{H}_1$ in terms of representations of the group SU(3), writing

$$\mathscr{H}_1 = \lambda_0 u_0 + \lambda_3 u_3 + \lambda_8 u_8, \tag{4.74}$$

where u_0 is a unitary scalar, and u_3 and u_8 enter in the octet representation of SU(3). Substituting (4.74) and the expression for $Q^{(L)-}$ into (4.67) and (4.59) it is easy to calculate the integral $\int d\mathbf{x}[\partial_\mu j^+_\mu(x), Q^{(L)-}(x_0)]$. The amplitudes for the transitions with $|\Delta S| = 1$ and $P = -1$ vanish as before, and the requirement that the coefficients of the terms which transform like u_3 and u_8 vanish leads to two conditions. (In the case of the Hamiltonian (4.74) the expressions which transform like u_3 and u_8 cannot be expressed in terms of divergences of currents, so that one must require the vanishing of the coefficients in front of these expressions.) These conditions have the form [81]

$$\mathrm{tg}^2\,\theta_C = \frac{\rho}{3+\rho} + \sqrt{1+\left(\frac{\rho}{3+\rho}\right)^2} - 1 \approx \frac{\rho}{3}, \tag{4.75}$$

$$\frac{\sqrt{3}\,\lambda_3}{\lambda_8} = \frac{\rho^2}{3+\rho+\sqrt{\rho^2+(3+\rho)^2}} \approx \frac{\rho^2}{6}, \tag{4.76}$$

where

$$\rho = -\left(1+\sqrt{2}\,\frac{\lambda_0}{\lambda_8}\right). \tag{4.77}$$

The relations (4.75)—(4.77) are extremely interesting, since they express a connection between the Cabibbo angle and the parameters of the violation of the SU(3) × SU(3)-symmetry.

The SU(2) × SU(2) symmetry leads to vanishing of ρ, so that ρ is small, of the order $(\mu_\pi/m_0)^2 \sim 0.1$. The magnitude of λ_3 must be of the order of the violation of isospin invariance and λ_8 must be of the order of the violation of SU(3) symmetry, i.e., $\lambda_3 \sim \alpha/\pi$, $\lambda_8 \sim 1/5$. Considering ρ small and eliminating it from (4.75)—(4.76) we obtain the relation

$$\frac{\lambda_3}{\lambda_8} = \frac{\sqrt{3}}{2}\,\mathrm{tg}^4\,\theta_C. \tag{4.78}$$

Numerically, for $\tan\theta_C = 0.25$ the right-hand side of (4.78) equals $3.4 \times 10^{-3} = 1.5(\alpha/\pi)$, and the left-hand side is of the order $5(\alpha/\pi)$, i.e., the order-of-magnitude agreement is not bad.

Since the violation of isospin invariance due to the electromagnetic interactions does not obviously reduce only to the effective Hamiltonian (4.74), one cannot take the relations (4.75)—(4.77) too seriously. However, the order-of-magnitude agreement of these relations with experiment may possibly indicate that taking into account isospin violations leads to some compensations in the divergent parts of the transition amplitudes with $|\Delta T| = 1$. Another more consistent approach will be discussed in the following section.

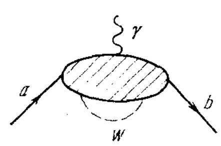

FIG. 34

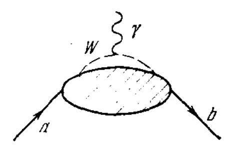

FIG. 35

c) Hadronic processes involving the emission of photons. As in the preceding section we restrict our attention to the theory with W bosons. In the case of a general strong interaction Hamiltonian the analysis of processes involving the emission of photons does not yield anything new compared to the analysis of weak corrections to hadronic processes carried out in subsection 4.3b), and leads to the same restrictions on the cutoff parameters, (4.61) and (4.64). It is however interesting to investigate the situation of weak corrections to the amplitudes of hadronic processes in the case when the strong interaction Hamiltonian has the form (4.68). This investigation is quite important, since if in the amplitudes of hadronic processes with $|\Delta S| = 1$ or $P = -1$, and emission of photons in the model with the Hamiltonian (4.68), the quadratic divergences remain and the effective coupling constants of such processes with the emission of n photons would be of the order $G\Lambda^2 e^n$, it would follow that the cutoff parameter Λ is subject to a strong inequality, and this fact would completely disrupt the scheme proposed in [77].

The amplitude of a hadronic process with the emission of a single photon is determined by the two diagrams in Fig. 34 and Fig. 35, which correspond to the emission of the photon by hadrons and by the virtual W boson.

A computation of the quadratically divergent terms on the matrix element of the transition $a \to b + \gamma$ due to the diagram of Fig. 34 leads to the following result [83]:

$$M_{1\lambda} = -4\pi g^2 \frac{\sqrt{4\pi}\,e}{(2\pi)^4}\cdot\frac{1}{m_W^2}\, i \int \frac{d^4k}{k^2 - m_W^2} \{\langle b|[Q^{(L)-}(0), [Q^{(L)+}(0), j_\lambda^{EM}(0)]]|a\rangle + \int d^4x \langle b|T\{[Q^{(L)-}(x_0), \partial_\mu j^+_\mu(x)], j_\lambda^{EM}(0)\}|a\rangle\}, \tag{4.79}$$

where j_λ^{EM} is the hadronic electromagnetic current, which in the SU(3) scheme has the following expression in terms of the currents j_k^i which form the unitary octet:

$$j^{EM} = j_1^1 = \frac{1}{2}(j^{(L)1}_1 + j^{(R)1}_1). \tag{4.80}$$

Making use of the Ward identity one can show [83] that the contribution of the quadratically divergent terms to the diagram in Fig. 35 describing the matrix element of the transition $a \to b + \gamma$ is equal in magnitude (but has opposite sign) to the first term in curly brackets in (4.79), so that the total matrix element is determined by the second term in (4.79). If the strong interaction Hamiltonian has the form (4.68), one can use Eq. (4.71) for the calculation of the second term in (4.79) for transitions with $|\Delta S| = 1$, yielding

$$\begin{aligned}\int d^4x \langle b|T\{[Q^{(L)-}(x_0), \partial_\mu j^+_\mu(x)], j_\lambda^{EM}(0)\}|a\rangle_2^3 \\ = -2\cos\theta_c \sin\theta_c \int d^4x \langle b|T\{\partial_\mu F^3_{\mu 2}(x), j_\lambda^{EM}(0)\}|a\rangle \\ = 2\cos\theta_c\sin\theta_c \int d^4x\,\delta(x_0)\langle b|[F^3_{c2}(x), j_\lambda^{EM}(0)]|a\rangle = 0,\end{aligned} \tag{4.81}$$

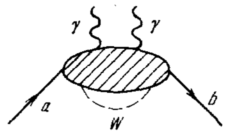

FIG. 36

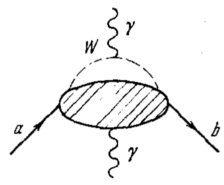

FIG. 37

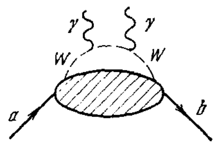

FIG. 38

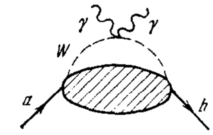

FIG. 39

since the charges $Q_2^{(L)_3}$ and $Q_2^{(R)_3}$, through which $\int dx\, F_{02}^3(x)$ is expressed, commute with j_λ^{EM}, according to (4.80). Thus, in the case of a strong interaction Hamiltonian of the form (4.68) the quadratically divergent terms in the amplitudes of hadronic processes with $|\Delta S| = 1$ and emission of photons vanish. A similar assertion can be proved for parity-changing transitions.

One can treat similarly the amplitudes of hadronic processes with emission of two photons. Here one must take into account four types of diagrams, Figs. 36–39. A consideration of these diagrams shows that: 1) the sum of the diagrams of Fig. 36 and Fig. 39 does not contain quadratically divergent terms, independently of the form of the strong interaction Hamiltonian; 2) in the sum of the diagrams of Fig. 38 and Fig. 37 only quadratically divergent terms survive which vanish for $|\Delta S| = 1$ for $P = -1$ transitions, if the Hamiltonian has the form (4.68), i.e., in this case too the hypothesis of Bouchiat, Iliopoulos and Prentki leads to the vanishing of quadratically divergent terms.

Knowing the amplitudes of hadronic processes involving the emission of photons one can attempt to investigate the question of corrections of order Ge^2 to the amplitudes of hadronic processes (the corresponding diagrams are obtained from those in Figs. 36–39 by closing the photon lines into one another). Such an approach is obviously more consistent than the approach discussed at the end of the preceding section, based on the Hamiltonian (4.74), which effectively takes into account the violation of isospin invariance due to electromagnetic interactions.

A study of this question (I do not dwell on the details, referring the reader to[83]) shows that in diagrams in which the photon is absorbed and emitted by hadrons (Fig. 38) the quadratic divergences survive, so that the method of[81,82], in which only these diagrams are effectively taken into account and where it turns out to be possible to realize a complete cancellation of these divergences, is not completely correct. However, in the sum of all diagrams such divergences cancel, and the result—the presence or absence of quadratically divergent terms—turns out to be dependent on the behavior of the forward scattering amplitude of virtual W bosons and photons, i.e., the amplitudes of the reactions $W + a \to W + b$, $\gamma + a \to \gamma + b$, for large squared mass q^2 of the W or the γ, with the result that quadratic divergences survive only when these amplitudes decrease as $1/q^2$ for $q^2 \to \infty$. The last assertion is in no way related to the divergence of the currents, and therefore quadratic divergences, if they exist, cannot be liquidated by means of relations of the type (4.75)–(4.77). It is very likely, however, that for processes involving a change of strangeness or parity (and maybe even of isospin) they are simply absent, since there are reasons to think that the amplitudes under considerations fall off faster than $1/q^2$.

In the whole discussion of this section it was assumed that the W boson does not have an anomalous magnetic moment. If this is not so, the situation changes drastically, and quadratically divergent terms proportional to the anomalous magnetic moment of the W boson appear in the amplitudes of hadronic processes with photon emission according to the diagram in Fig. 35. The matrix element then takes the form[83]

$$M = -\frac{g^2 e}{4\sqrt{\pi}} \gamma \frac{\Lambda^2}{m_W^4} q^2 \langle b | I_\lambda(0) | a \rangle, \tag{4.82}$$

where q is the photon momentum and γ is the anomalous magnetic moment,

$$I = \cos^2\theta_C (j_1^1 - j_2^2) + \sin^2\theta_C (j_1^1 - j_3^3) - \cos\theta_C \sin\theta_C (j_3^2 + j_2^3). \tag{4.83}$$

Since M is proportional to q^2 it is necessary to consider processes with the emission of a virtual photon in order to obtain from (4.82) restrictions on Λ^2. The most interesting among these processes is the decay mode $K^+ \to \pi^+ e^+ e^-$. For the branching ratio of this process, Eqs. (4.82) and (4.83) yield

$$\frac{w(K^+ \to \pi^+ e^+ e^-)}{w(K^+ \to \pi^0 e^+ \nu)} = \frac{\alpha^2 \gamma^2}{4(4\pi)^2} \left(\frac{\Lambda}{m_W}\right)^4. \tag{4.84}$$

A comparison with the experimental limit (4.38′) gives

$$\Lambda < 3 m_W \gamma^{-1/2}. \tag{4.85}$$

This result is an argument against the existence of W-bosons with an anomalous magnetic moment $\gamma \gtrsim 1$.

5. ATTEMPTS AT CONSTRUCTING A NEW THEORY OF WEAK INTERACTIONS

The contradictions which have appeared in the theory of weak interactions make it necessary to construct a new theory of weak interactions. The main requirement on the new theory must insist, obviously, that at large distances (i.e., for low energies and small momentum transfers) the new theory goes over into the usual one, but that the growth of the weak interaction stop for $r \sim 10^{-15}$ cm ($E \sim 20$–50 GeV), at least in those processes where at present we encounter the strongest contradictions.

A series of attempts were made in this direction. We shall describe below some of the approaches to a new theory of weak interactions which have acquired wide publicity. We shall not go in detail into the merits and deficiencies of these approaches, which would enlarge this review article too much, but indicate only the formulation of the fundamental ideas and the basic literature, from where the reader can obtain all the information that interests him in regard to this problem.

5.1. A theory with compensating intermediate boson fields

(Gell-Mann, Goldberger, Kroll, Low)[84]. Several intermediate vector and/or scalar mesons with vector coupling are introduced, with different coupling constants to the hadron and lepton currents. The interaction between the currents $j_\mu^i(x)$ and $j_\nu^k(x')$ (the superscripts i, k denote the $(\bar{e}\nu_e)$- and $(\bar{\mu}\nu_\mu)$-currents with $\Delta S = 0$, $|\Delta S| = 1$ hadronic currents) is described by the propagations

$$\Delta_{\mu\nu}^{ik}(x - x') = i \sum_r g_r^i g_r^k \int d^4x\, e^{iq(x-x')} \langle 0 | T\{\chi_\mu^r(x), \chi_\nu^{r+}(x')\} | 0 \rangle, \tag{5.1}$$

where $\chi^r_\mu(x) = W^r_\mu(x)$ in the case of vector fields and $\chi^r_\mu(x) = \partial\varphi^r/\partial x_\mu$ in the case of scalar fields, and g^i_r are the appropriate coupling constant. For the interactions of currents of the same kind (diagonal interactions, $i = k$), according to the Lehmann theorem the behavior of $\Delta^{ii}_{\mu\nu}(x)$ cannot be more singular than the behavior of $\Delta_{\mu\nu}(x)$ for a single vector field, i.e., by this method one cannot achieve a less singular behavior than in the usual theory for small x. However, (and this is the basic idea of this approach) by an appropriate choice of the coupling constants g^i_r one can realize a less singular behavior of $\Delta^{ik}_{\mu\nu}(x)$ for $x \to 0$, e.g., such as in the theory mediated by scalar bosons (in momentum-space, for $k^2 \to \infty$, $\Delta^{ik}_{\mu\nu}(k) \sim \delta_{\mu\nu}/(k^2)$ or $k_\mu k_\nu/k^4$). One of the simplest realizations of the scheme under discussion is a theory with one vector meson of mass μ and two scalar mesons (with masses μ_1 and μ_2) with vector coupling and propagators of the form

$$\Delta^{ik}_{\substack{\mu\nu\\ i\neq k}} \sim \frac{\delta_{\mu\nu}}{k^2-\mu^2} - \frac{k_\mu k_\nu}{\mu^2}\left(\frac{1}{k^2-\mu^2} + \frac{1-\alpha}{\alpha}\frac{1}{k^2-\mu_1^2} - \frac{1}{\alpha}\frac{1}{k^2-\mu_2^2}\right). \quad (5.2)$$

The contradictions discussed above between theory and experiment on account of virtual processes appeared for nondiagonal currents $j^i j^k$, $i \neq k$. In the theory under consideration these discrepancies disappear, since the effective propagator $\Delta^{ik}(k)$ ($i \neq k$) behaves like a propagator of a renormalizable theory for $k \to \infty$ (one can even make it decrease faster). For the diagonal interactions, where there is no cutoff of the growth of the interaction with energy, the corrections due to virtual weak interactions remain the same as in the usual theory, i.e., of order $G\Lambda^2$, which is large. Therefore in the scheme of [84] one should expect a difference between the constant of $\nu_e e$ scattering and the coupling in muon decay.

A difficulty in the scheme of Gell-Mann et al. appears in the discussion of strangeness-changing hadronic interactions. In order to consider this interaction nondiagonal it is necessary to introduce different coupling constants for the V and A hadronic currents, i.e., one must give up the two-component character of the weak hadronic current, which is esthetically undesirable. If this is not done, odd-P hadronic interactions will appear on account of the diagonal terms and turn out to be large, contradicting experiment. One can avoid this difficulty by combining the scheme [84] with the assumption of a definite structure of the strong interactions, discussed in subsection 4.3b).

5.2. A theory with strong interactions of the intermediate vector bosons [85,89] The basic idea here is the assumption of a strong interaction of the intermediate W bosons with the hadrons (1-st version) or with each other (2-nd version), which ensures the cutoff of all integrals with respect to the momenta of the W bosons at values characteristic for the strong interactions. In the first version a quadratic strong interaction of the W bosons with the hadrons of the form $(\bar{N}N)W^+W$ is introduced, and definite isospin and SU(3) quantum numbers are ascribed to the W bosons. Thus, in [85] the W bosons are classified as a triplet of the SU(3) group, forming an isodoublet (W^+, W^0) and an isosinglet $W^{0\prime}$, and have nonvanishing triality $t = 1$, so that their charge is $Q = T_Z + (Y/2) + (t/3)$, where T_Z is the isospin projection and Y is the hypercharge. Conservation of these quantum numbers allows one to forbid a series of unobserved processes (e.g., $K^0_L \to \mu^+\mu^-$ owing to the strong and semiweak interaction with an amplitude of the order g^2), however, for other unobserved processes (the reaction $\nu_\mu + N \to \mu^- + W +$ hadrons with an amplitude $\sim g$, the reaction $\nu_\mu + N \to \nu_\mu +$ hadrons with an amplitude $\sim g^2$) the smallness of the cross sections can only be accounted for by assuming sufficiently large masses for the W. For an arbitrary (but SU(3)- or SU(2)-invariant) form of the $(\bar{N}N)(W^+W)$ interaction, the beta decay vector coupling is renormalized on account of the strong interactions. One can achieve its nonrenormalization by introducing [87] a gradient coupling, but in this case it is not clear whether one can obtain the required magnitude of the "weak magnetism."

In the 2-nd version [85,88] one assumes a strong cubic self-interaction of the W bosons. In this case one can introduce in a natural manner into the theory a CP-violation, assuming that the weak interaction has CP = −1. (The presence of the W^3 interactions does not allow one to make the transformation $W \to iW$.) On the basis of the Lehmann theorem the W3 interaction alone does not guarantee a cutoff of the integrals in the amplitudes of the nonleptonic processes (the diagram of Fig. 33), so that this model must be combined with the hypothesis advanced in [77] which we have discussed above.

5.3. The hypothesis was advanced that in the theory with intermediate vector bosons the growth of the weak interactions at small distances may be cut off by the electromagnetic interactions of the W bosons, which also increase at short distances [90]. The electromagnetic interactions of W bosons become strong at distances r of the order of $1/m_W\alpha^{1/2}$. Therefore, according to this hypothesis the cutoff of the integrals in virtual weak processes should occur for $\Lambda^2_{EM} \sim m^2_W/\alpha$, so that the effective expansion parameter here is $G\Lambda^2_{EM} \sim 10^{-3}$ $(m_W/m)^2$, which is a small quantity if $m_W \sim 10$ GeV. An analysis of this hypothesis within perturbation theory has shown [91] that when only longitudinal W bosons participate in the weak virtual processes (this corresponds to the most essential corrections $\sim (G\Lambda^2)^n$, the electromagnetic interaction of the W bosons introduces a relatively small contribution and cannot guarantee a cutoff of the weak interactions; consequently, the possibility that they cut off the weak interactions appears only when at least one of the virtual W bosons is transverse. This means that the electromagnetic interaction of the W bosons is capable of cutting off the growth of the weak interaction only in combination with another mechanism, which guarantees the vanishing of terms of order $(G\Lambda^2)^n$.

5.4. There exists the point of view that all the difficulties of the theory of weak interactions are due to the use of perturbation theory, which in those cases when there appear contradictions does not even give a qualitatively correct answer and that in the correct theory all these problems will go away. Attempts at a nonperturbative approach to the equations were made made both in the framework of the usual formalism [92] and outside this framework [93]. Although not very much was accomplished, one cannot consider this possibility as ruled out at the present time.

5.5. T. D. Lee and G. C. Wick [94] have attempted to solve the problem by introducing an indefinite metric into the W-boson theory. In their approach the propagator of the W-boson has the form $\delta_{\mu\nu}/(k^2 - m_{1W}) - (k_\mu k_\nu/m^2_{1W})$ $\times[(k^2 - m^2_{1W})^{-1} - (k^2 - m^2_{2W})^{-1}]$ (or the form of a difference of two propagators), in place of the usual form $(\delta_{\mu\nu} - k_\mu k_\nu/m^2_W)(k^2 - m^2_W)^{-1}$, i.e., an additional state of spin zero and negative norm is introduced into the theory.

It is obvious that for this form of the propagator the weak interactions will not grow at short distances and the contribution of the virtual processes remains small. However, theories with indefinite metric have their own very serious difficulties, which until now have not been overcome, in spite of the intensive activity in this direction (cf., e.g.,[95] and a discussion of the difficulties of the Lee-Wick theory in[96]).

The main assertion of these authors (which was proved for simple models) is that the unitarity difficulties occurring in such theories will disappear if the negative-norm state (or W boson) is unstable. It was shown in[97] that in the theory proposed by Lee and Wick there appear at high energies signals propagating faster than light, i.e., a violation of the causality principle.

An attempt was made to reformulate the theory of weak interactions as a nonlocal theory: in[98] second order weak corrections to muon decay were discussed within the framework of the nonlocal theory developed by one of the authors[99].

5.6. A theory with scalar baroleptons [100,101]. It was assumed that the weak interaction is described by the Hamiltonian

$$\mathscr{H} = g_1\bar{p}(1-\gamma_5)e^C B_1 + \tilde{g}_1\bar{n}(1-\gamma_5)\nu_e^C B_1 + g_2\bar{p}(1-\gamma_5)\mu^C B_2 + \tilde{g}_2\bar{n}(1-\gamma_5)\nu_\mu^C B_2 + g_3\bar{\nu}_\mu(1-\gamma_5)e^C B_3 + \tilde{g}_3\bar{\mu}(1-\gamma_5)\nu_e^C B_3 + g_4\bar{\Lambda}(1-\gamma_5)\nu_e^C B_1 + g_5\bar{\Lambda}(1-\gamma_5)\nu_\mu^C B_2, \tag{5.3}$$

where B_1 and B_2 are scalar neutral bosons carrying baryon and lepton numbers equal to 1 (baroleptons), and B_3 is a charged scalar boson; the superscript "C" denotes charge conjugation. If the mass of the bosons is sufficiently large, then after the exclusion of the virtual boson fields from (5.3) an effective four-fermion interaction appears in order g^2, corresponding to hadron-lepton and lepton-lepton processes. The properties of the Fierz transformation implies that the interaction expressed in terms of the fields e, ν_e, μ, ν_μ, acquires the form of the usual V – A interaction[100]. The theory with the Hamiltonian (5.3) is obviously renormalizable and there is no growth of the weak interactions at small distances (for $r < 1/m_B$).

However, some of the achievements of weak interaction physics are lost in this theory: the universality of the interaction has a fortuitous character, there is no reason for conservation of the vector currents and the description of nonleptonic processes requires the introduction of new bosons. In addition, in the form (5.3), the theory leads to a large difference between the cross sections for e^+p and e^-p scattering[102], in disagreement with experiment.

5.7. A theory in which the four-fermion interaction arises as the low-energy limit of a fourth-order effect in the interaction $g\psi_a(A + B\gamma_5)\psi_b\varphi_c$, where φ_c is a spin-0 boson field and A and B are constants[103-105].

Several charged and neutral heavy bosons are introduced into the theory, as well as two heavy neutral leptons (with the quantum numbers of the muon and of the electron) and at least one heavy baron. An example of such a theory is described by the Hamiltonian

$$\mathscr{H} = g_1[\bar{\mu}(1-\gamma_5)l_\mu^0 + \bar{F}(1+\gamma_5)p + \bar{e}(1-\gamma_5)l_e^0]\bar{B} + g_2[\bar{\nu}_\mu(1-\gamma_5)l_\mu^0 + \bar{n}(1-\gamma_5)F + \bar{\nu}_e(1-\gamma_5)l_e^0]B^0 + g_3\bar{\Lambda}(1-\gamma_5)FB^0 + \text{h.c.} \tag{5.4}$$

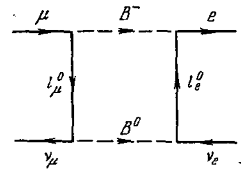

FIG. 40

In the theory with the Hamiltonian (5.4) the matrix element of muon decay is described by the diagram of Fig. 40, which, if one neglects the momenta and the masses of the leptons, has the form

$$M = \frac{4g_1^2g_2^2}{\pi^2}\, i\int d^4k\, \frac{k_\alpha k_\beta}{k^4(k^2-M_B^2)^2}\,\bar{u}_{\nu\mu}\gamma_\alpha(1+\gamma_5)u_\mu\cdot\bar{u}_e\gamma_\beta(1+\gamma_5)u_{\nu e}. \tag{5.5}$$

(the boson masses are set equal to $M_B = M_{B0}$). Calculation of the integral in (5.5) yields

$$M = \frac{1}{2}\,\frac{g_1^2g_2^2}{M_B^2}\,\bar{u}_{\nu_\mu}\gamma_\lambda(1+\gamma_5)u_\mu\cdot\bar{u}_e\gamma_\lambda(1+\gamma_5)u_{\nu e}, \tag{5.6}$$

i.e., there appears an effective muon decay interaction with the coupling constant $G/2^{1/2} = (1/2)g_1^2g_2^2/M_B^2$. One can carry out a similar consideration for the beta decay interaction. If one adopts the Bjorken assumption[68] regarding the asymptotic behavior of the matrix element M(k) of scalar fields for $k_0 \to \infty$:

$$M(k) = -i\int e^{ikx}d^4x\,\langle p|T\{\bar{p}(x)(1+\gamma_5)n(x),\ \bar{F}(0)(1+\gamma_5)n(0)\}|n\rangle \to \frac{1}{k_0}\int d\mathbf{x}e^{-i\mathbf{kx}}\langle p|[\bar{p}(x)(1+\gamma_5)n(x),\ \bar{F}(0)(1+\gamma_5)n(0)]|n\rangle_{x_0=0} \tag{5.7}$$

(cf. the critique of this assumption for the case of nonconserved currents in[10,6]), then one can show[105] that the beta decay interaction is of the V – A form, and the vector coupling constant is not subject to renormalizations due to the strong interactions.

The theory with the Hamiltonian (5.4) is obviously both renormalizable and universal. The difficulties of this theory are the nonleptonic processes with $|\Delta S| = 1$ or $P = -1$, which according to (5.4) occur in order g^2 and are characterized by a dimensionless constant g^2 which is considerably larger than Gm^2 (m is the nucleon mass), and thus is in contradiction with the experimental data. Moreover, the coupling constants of the neutral currents $\bar{\nu}\nu$ are of the order G, which is also hard to reconcile with experiment. In order to avoid these difficulties, various modifications of the theory have been proposed[104,105], introducing additional intermediate baryons, leptons and bosons; the hadronic part of the weak interaction Lagrangian has been assumed strangeness-conserving (or strangeness- and parity-conserving) and the effects of strangeness (or strangeness and parity) nonconservation in nonleptonic processes were due to virtual leptonic pairs. This allowed the authors to get rid of part of the difficulties. However, at the present time we are not convinced that by going this way one can construct a self-consistent theory of weak interactions, agreeing with experiments.

5.8. A theory of weak interactions based on an assumed SU(4) symmetry of the hadrons[107] (cf. also the earlier work[108]). It is assumed that, at least at short distances, the strong interactions exhibit the SU(4) symmetry. Since in this symmetry there appears a new quantum number, called hypercharge, there exist hypercharged particles. In quark language this means that in addition to the three quarks p, n, Λ, which form an SU(3) triplet, there exist a fourth quark, p′, which is assigned the same electric charge as the p quark (but in distinction from the p, n, Λ it has nonvanishing hypercharge). The basic idea consists in selecting a weak current of the form

$$j_\mu^{-h} = \cos\theta_C \bar{n}\gamma_\mu (1+\gamma_5) p + \sin\theta_C \bar{\Lambda}\gamma_\mu (1+\gamma_5) p - \sin\theta_C \bar{n}\gamma_\mu (1+\gamma_5) p' + \cos\theta_C \bar{\Lambda}\gamma_\mu (1+\gamma_5) p', \tag{5.8}$$

interacting with the field of charged W-bosons, and using the SU(4) × SU(4) algebra of currents. This form of the hadronic current leads to vanishing of terms of order $G^2\Lambda^2$ (and also of order $G(G\Lambda^2)^n$ in the matrix elements of the hadron interactions with neutral leptonic currents, for transitions with $|\Delta S| = 1$, i.e., one of the basic difficulties of the theory, namely the absence of neutral currents in the leptonic decays of strange particles (e.g., $K_L^0 \to \mu^+\mu^-$) is removed in this approach. Terms of order $G^2\Lambda^2$ remain in the matrix elements of the interactions of hadrons with neutral leptonic currents for transitions with $\Delta S = 0$, the effective coupling constant satisfying the equality (4.37). In this theory there are no contributions of order $G^2\Lambda^2 m^3$ (and also of order $G(G\Lambda^2)^n m^3$) to the $K_L - K_S$ mass difference. In order to achieve the vanishing of terms proportional to $G\Lambda^2$ in matrix elements of nonleptonic processes with S = 1 or P = −1 this theory has to assume, by analogy to [77], that the strong interaction Hamiltonian consists of SU(4) × SU(4)-invariant terms, and of terms which transform according to the representation $(\bar{4}, 4) + (4, \bar{4})$ of the group SU(4) × SU(4). The basic problem of this theory is, of course, the problem of existence of hypercharged particles, which in the present scheme cannot be too heavy, and must be pair-produced in strong interactions and decay into hadrons and leptons due to their weak interactions.

5.9. Theories with intermediate Yang-Mills vector bosons. This approach, which has shown remarkable successes in recent months is possibly one of the most promising. As is well known, the theory of massless Yang-Mills gauge bosons [109] is renormalizable [110], and there is no increase of the interactions at short distances. Up to now, the application of this theory to weak interactions was hampered by the fact that the Yang-Mills gauge bosons were supposed to have a vanishing mass; if the mass is nonzero the theory remains unrenormalizable [111], and in this sense there is little difference from the usual intermediate vector boson theory (at least as long as one remains within perturbation theory). In a recent paper 't Hooft [112]*** has shown that in a gauge theory of a special form, where the gauge fields interact with a scalar field with nonvanishing vacuum expectation value, the gauge fields acquire a mass and the renormalizability of the theory remains in force. On the basis of this result it was possible to prove the renormalizability of various schemes for the theory of weak interactions with Yang-Mills gauge bosons having mass (and also including the photon as a massless gauge field), proposed earlier [113], or newly invented [114]. The proposed theory is still far from completion: some aspects of the hadron-lepton and hadron-hadron weak interactions still await their explanation. Nevertheless the idea of this approach is very beautiful and its direction is quite promising.****

6. CONCLUSION

An investigation of higher-order corrections in the weak interaction in the framework of the usual theory shows that there is a contradiction between the concept of a natural cutoff of the growth of the weak interactions with energy at energies of the order $\Lambda \sim G^{-1/2} \sim 10^3$ GeV and the experimental data, which require a cutoff at substantially lower energies, of the order of several tens of GeV. Such a contradiction appears both in the W-boson theory and in the four-fermion theory of weak interactions. The contradiction manifests itself in its strongest form in the leptonic decays of strange particles with emission of lepton pairs of total charge zero (the decay $K_L \to \mu^+\mu^-$) and in strangeness- or parity-changing nonleptonic processes. In the latter case, however, the discrepancy can be removed by assuming a definite symmetry of the strong interaction Hamiltonian (or at least, assuming a definite symmetry of the strong interactions at short distances. For purely leptonic processes one cannot make any definite statements at the present time, in view of the insufficient accuracy of the experimental data.

The experimental consequences of what was said above consist, first, in the necessity of measuring lepton-lepton interaction, particularly the $\nu_\mu e$ and $\nu_e e$ scattering. Another conclusion for experimentalists (in addition to the obvious ones, like searching for W bosons) is the necessity to find out up to what energies the neutrino-nucleon scattering cross section continues its increase, and whether such an increase exists when the strangeness of the hadrons produced in the process is different from zero. Further searches for neutral leptonic currents in hadron-lepton processes without change of strangeness are needed, by means of an investigation of the reaction $\nu + N \to \nu$ + hadrons in neutrino experiments (cf. in this connection Translator's footnote**).

For the theorist, the conclusion is, first of all, that time is ripe for the construction of a new theory of weak interactions free from the difficulties described above, but containing all the advantages of the old theory. In the construction of such a theory one must remove at first all the difficulties related to the problem of neutral currents (in particular, for strangeness-changing processes). Finally, both theoretical and experimental clarification is required for the problem of the structure of strong interactions (more precisely, of the current commutator) at short distances, problem which is important for understanding the nonleptonic processes.

I wish to express my gratitude to Ya. B. Zel'dovich and L. B. Okun' who have induced me to write this review article, have carefully read the manuscript and have given me some valuable indications.

[1)] The situation in the theory of weak interactions is somewhat reminiscent of what happened in classical field theory. Starting from that theory itself, it was established that classical electrodynamics is not applicable (becomes self-contradictory) at distances of the order of the classical radius of the electron $r_0 = e^2/mc^2$. However, experiment has shown that owing to the existence of the quantum of action ħ, the validity of classical electrodynamics breaks down considerably earlier, at distances of the order of $\hbar/mc = r_0/\alpha$. At the present time one cannot exclude the possibility that the "cutoff" of the weak interactions is connected with a reason as fundamental as the existence of the quantum of action. (This remark belongs to S. S. Gershteĭn.)

[2)] In this review we use the following notation: the components of the metric tensor $\delta_{\mu\nu}$ are $\delta_{00} = -\delta_{11} = -\delta_{22} = -\delta_{33} = 1$, $a_\mu b_\mu = a_0 b_0 - \mathbf{a}\cdot\mathbf{b}$, $\partial_\mu = \partial/\partial x_\mu = (\partial/\partial t, -\partial/\partial \mathbf{x})$, the Dirac matrices are

$$\gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \boldsymbol{\gamma} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}, \quad \gamma_5 = -\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

$\hat{p} = p_\mu \gamma_\mu$, and the spinors u and $\bar{u}$ are normalized by the condition $u\bar{u} = 2m$.

[3)] By large ν we understand here $\nu \gg m^2$, but at the same time $\nu \ll G^{-1}$, so that the strong interaction asymptotic behavior has already started, but one can still use the first approximation in G.

[4]It is understood that the definition of physical quantities does not involve external dimensional parameters. The functions w_1, w_2, w_3 are not such quantities, since in this section, in agreement with generally adopted notations, a normalization of the spinors describing the initial state of the nucleons is used such that $u\bar{u} = 1$. The transition to the normalization $u\bar{u} = 2m$, corresponding to invariant normalization of states (cf. infra) in (3.40)–(3.42), $p'p = 2p_0\delta^3(p-p')$, and taking into account the external dimensional parameters $1/m^2$ which enter the defintion of w_2 and w_3 in (3.42), leads to agreement of the equations (3.36) and (3.38) with the requirements of dimension theory.

[5]In order to avoid misunderstandings we note that in this part of this section we consider the four-fermion theory, and not the W-boson theory, and that the term "matrix element for the forward scattering of the virtual intermediate boson" is used only for the convenience of describing the quantities $M_{\mu\lambda}$.

[6]The assertions made above can also be reformulated in the language of the "prerenormalization era." In that language the electronneutrino field produces a potential which acts between the nucleons [28] (cf. also [29]) having the form $V(r) \sim G^2/r^5$. The minimal distances r_0, for which this potential makes sense are $r_0 \sim G^{1/2}$. To the potential V(r) corresponds a scattering amplitude $s^{-1/2} f \sim \int e^{iqr} V(r)\, dr \sim G^2 r_0^{-2} \sim G$, and a level-shift $\Delta E \sim \int V(r)|\psi(r)|^2 dr$, which attains values ~m (or maybe even $G^{-1/2}$, due to the noncovariant way of writing) if the ψ-function is concentrated in regions of the size r_0. (This case corresponds to the non-cutting-off of weak interactions by strong interactions.)

[7]We do not consider here the gravitational interaction, for which a similar situation holds.

[8]This is also an argument in favor of electromagnetic cutoff in (4.5).

[9]Investigation of deep-inelastic electromagnetic and weak processes (cf. Sec. 3.1b)) has also shown the absence of rapidly decreasing form-factors.

[10]Thus, for instance, the experimental values of the coupling constants of $\Lambda N\pi$ and $K_1^0\pi\pi$ interactions, corresponding to the Hamiltonians $g\bar{\psi}_\Lambda(1 + \lambda\gamma_5)\psi_N\varphi_\pi$ and $f\varphi_K\varphi_\pi^+\varphi_\pi$ are the following: $g \approx (1/5)Gm_K^2 \sin\theta_C$, $f \approx 3Gm_K^3 \sin\theta_C$ and the effective dimensionless coupling constant of the parity-nonconserving four-nucleon interaction equals [75] $F \approx (2-4) \times 10^{-7}$.

[11]This explanation belongs to V. I. Zakharov.

[12]The approach of [82] differs somewhat from that in ref. [81], although the ideas of both papers are close to each other.

Translator's Notes

*Recent evidence (talk by H. Wachsmuth (CERN) at Berkeley APS Meeting, August 14, 1973; NAL Preprint, Sept. 1973) shows that neutral currents are present (in agreement with the Weinberg model, cf. infra); in particular one event of $\nu_\mu e$ scattering has been reported.

**More recent experimental evidence shows less (or no) discrepancy.

***The basic idea of a gauge theory of weak and electromagnetic interactions goes back to Schwinger (Ann. Phys. (N.Y.) 11, 1 (1957), cf. also M. E. Mayer, Physik. Verhandl. 9, 57 (1958). However, the use of the Higgs mechanism [113] to give the vector bosons mass is due independently to 't Hooft and Weinberg (Phys. Rev. Lett. 29, 338 (1972), where earlier references can be found), based on a model invented by Weinberg in 1967.

****Quite recently some very promising progress has been made in using nonabelian gauge theories in strong interactions too; cf. D. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973), H. D. Politzer, ibid., 1346 (1973); G. 't Hooft, to be published, S. Weinberg, Phys. Rev. Lett. 31, 494 (1973). For an early attempt to use gauge vector bosons to mediate both weak and strong interactions, cf, also M. E. Mayer, Nuovo Cimento 17, 802 (1960).

*****Some of Blokhintsev's results were obtained in collaboration with the Translator.

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