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V. A. Moskalenko. The Theory of Superconductors with Overlapping Energy Bands. Energy bands overlap near the Fermi level in various metals and alloys. This overlap must be taken into account in superconductivity theory for more accurate determination of the parameters of real superconductors.

A model that takes account of the possible formation of bound electron pairs with opposite momenta and spins within each band was proposed in[1] for the simplest case in which two energy bands overlap. The model assumes the existence of two spherical cavities of the Fermi surface in the neighborhood of which—in energy intervals of width $2\omega_n$ ($n = 1, 2$)—conduction electrons participate in the formation of the superconductive state. Since the momenta of these cavities are assumed to be different, the formation of bound electron pairs from electrons belonging to different bands is not considered.

Interband interaction of electrons results in an additional indirect interaction of electrons within each band. As a result of this interaction superconductivity intervenes simultaneously in both bands and has a single critical temperature. A review of the basic results obtained in the theory of pure two-band superconductors will be found in[2]. Below we present the basic results obtained on the basis of the model in[1] for superconductors with nonmagnetic and paramagnetic impurities.

The impurity-atom concentration is assumed to be small, so that the relaxation time of the band electrons on the impurity is substantially larger than the reciprocal Fermi energy and, consequently, the individuality of the Bloch electrons of the individual bands is preserved.

As a result of interband scattering of electrons by a nonmagnetic impurity with relaxatinn times τ_{12} and τ_{21}, where 1 and 2 are the band numbers, this impurity has the effects of weakening the correlation between the bound-pair electrons, with a resulting decrease in the critical temperature T_c[3] of the two-band semiconductor, reducing its ordering parameters Γ_n[4], giving rise to a single energy gap in the elementary excitation spectrum, and strongly smearing the electron-state densities of both bands near the corresponding ordering parameters[4].

With the condition that the frequencies ω_n are approximately the same and equal to the Debye frequency, the critical temperature of a two-band superconductor with a nonmagnetic impurity is determined from the equation

$$a\xi_c^2 - b\xi_c + c = 0, \tag{1}$$

where

$$\xi_c = \ln\frac{2\gamma\beta_c\hbar\omega_D}{\pi} \quad \left(\beta_c = \frac{1}{k_B T_c}\right), \quad a = N_1 N_2 (V_{11}V_{22} - V_{12}^2),$$
$$b = b_0 + aI, \quad c = 1 + eI;$$

here

$$b_0 = N_1 V_{11} + N_2 V_{22}, \quad I = \psi\left(\frac{1 + (\beta_c\rho/\pi)}{2}\right) - \psi\left(\frac{1}{2}\right)$$
$$\left(\rho = \frac{\hbar}{2}\left(\frac{1}{\tau_{12}} + \frac{1}{\tau_{21}}\right)\right), \quad e = b_0 - \tau \quad \left(\tau = \frac{N_1^2 V_{11} + N_2^2 V_{22} + 2N_1 N_2 V_{12}}{N_1 + N_2}\right).$$

At a low impurity concentration ($\beta_c\rho \ll 1$), the value of T_c decreases linearly as the concentration rises[3]. At the higher impurity concentration ($\beta_c\rho \gg 1$), ξ_c is determined from the equation

$$\xi_c \approx \frac{1 + e\eta}{\tau + a\eta}, \quad \eta = \ln\frac{\rho}{\hbar\omega_0} + \frac{\pi^2}{6(\beta_c\rho)^2}. \tag{2}$$

In the weak-coupling limit, T_c tends to a nonzero limit with rising impurity concentration:

$$T_c \to \frac{2\gamma}{\pi}\hbar\omega_D e^{-1/\tau}. \tag{3}$$

Vanishing of T_c under the influence of a nonmagnetic impurity is impossible, even with the special relation $a = \epsilon\tau$ between the constants N_n and V_{nm} of the model under consideration. In a two-band superconductor with a paramagnetic impurity, interband exchange scattering of electrons by the impurity with relaxation times τ_{nn}^{s} produces an additional mechanism lowering T_c and the ordering parameters, over and above the effect of the nonmagnetic impurity[5,6]. At the paramagnetic-impurity concentration determined by the condition

$$(1 - \beta_{11} - \beta_{12} + \alpha_1)(1 - \beta_{22} - \beta_{21} + \alpha_2) = \alpha_1\alpha_2, \tag{4}$$

where

$$\beta_{nm} = \frac{\hbar}{\Gamma_n \tau_{nm}^s}, \quad \alpha_1 = \frac{\hbar}{2\tau_{12}\Gamma_1}, \quad \alpha_2 = \frac{\hbar}{2\tau_{21}\Gamma_2},$$

the gap ω_g in the elementary-excitation spectrum vanishes, and then the superconductivity disappears when the critical impurity concentration is reached[5]. Unlike the single-band system with paramagnetic impurity[7], the two-band case has a critical concentration determined not only by T_{c0}, but also by the parameters N_n and V_{nm} of the system.

The ratios $n_n(\omega)$ of the electron-state densities in the superconductive $N_n(\omega)$ and normal N_n states,

$$n_n(\omega) = \frac{N_n(\omega)}{N_n} = \pm\,\mathrm{Im}\left(\frac{u_n(z)}{\sqrt{1 - u_n^2(z)}}\right)_{z=\omega\pm i0+}, \tag{5}$$

and the related functions are determined on the basis of solution of the equations

$$\begin{aligned} \frac{z}{\Gamma_1} &= u_1(z) - \frac{\beta_{11}u_1(z)}{\sqrt{1 - u_1^2(z)}} + \frac{\alpha_1(u_1(z) - u_2(z))}{\sqrt{1 - u_2^2(z)}} - \frac{\beta_{12}u_1(z)}{\sqrt{1 - u_2^2(z)}}, \\ \frac{z}{\Gamma_2} &= u_2(z) - \frac{\beta_{22}u_2(z)}{\sqrt{1 - n_2^2(z)}} + \frac{\alpha_2(u_2(z) - u_1(z))}{\sqrt{1 - u_1^2(z)}} - \frac{\beta_{21}u_2(z)}{\sqrt{1 - u_1^2(z)}}; \end{aligned} \tag{6}$$

the ordering parameters Γ_n of the system are found from the equation system

$$\Gamma_n = \sum_m V_{nm} N_m \frac{\pi}{\beta} \sum_\Omega \frac{1}{\sqrt{\tilde{u}_m^2(\Omega)+1}}, \qquad (7)$$

where

$$\Omega = (2r+1)\frac{\pi}{\beta} \quad (r = 0, \pm 1, \ldots), \quad \tilde{u}_n(\Omega) = -iu_n \quad (s = i\Omega).$$

On the basis of (6) and (7), expressions were obtained in[4,6,8-12] for the energy gap ω_g and the state densities $n_n(\omega)$ of a two-band superconductor with a nonmagnetic impurity ($\beta_{nm} \neq 0$). It was possible in the extreme cases of large and small parameters α_n to obtain explicit expressions for ω_g and the function $n_n(\omega)$ over the entire frequency range[8-12]. N. I. Botoshan and M. I. Vladimir carried out computer calculations for intermediate values of the parameters of the theory. It was shown that for a system with a nonmagnetic impurity, ω_g is always larger than the smaller and smaller than the larger of the two system ordering parameters Γ_n. As the concentration of this impurity increases with $\Gamma_1 < \Gamma_2$, the value of ω_g/Γ_1 increases with it. At high impurity concentrations, when the inequality $\alpha_n \gg \hbar\omega_g/\Gamma_n$ is satisfied, we obtain for ω_g on the assumption of weak coupling

$$\omega_g \approx \frac{\Gamma_1\Gamma_2(\alpha_1+\alpha_2)}{\Gamma_1\alpha_1+\Gamma_2\alpha_2} \approx 2\omega_D e^{-1/\tau}. \qquad (8)$$

The relation of the single-band theory of Bardeen, Cooper, and Schrieffer[13] and Bogolyubov[14] holds between ω_g and T_c in this extreme case, thus demonstrating the single-band nature of the superconductive properties of the two-band model with a high nonmagnetic-impurity concentration.

Investigation of the electromagnetic properties of the two-band model with impurity[15] also confirms this conclusion.

Values of $n_n(\omega)$ were calculated on the basis of a method of analysis developed in[8,11] and based on separation of the nonanalytic dependence of these functions on the impurity parameters in Eq. (6) in the frequency ranges near the energy gap and near the larger of the two ordering parameters. At frequencies far from these values, the computation method was based on expansion of the functions $n_n(\omega)$ in powers of the small parameters of the theory. It was shown that at small concentrations of the nonmagnetic impurity, the state densities have sharp maxima near the respective ordering parameters. The heights of the maxima increase with decreasing impurity concentration, their halfwidths decrease, and the frequencies of the maxima approach the values for the energy gaps of the pure substance.

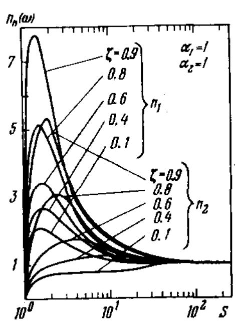

FIG. 1

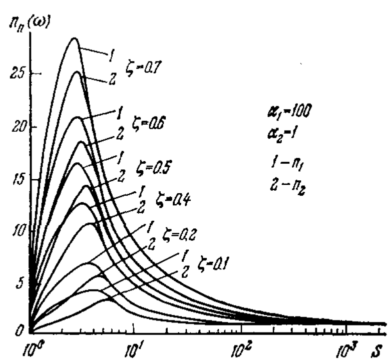

FIG. 2

Figures 1 and 2 present computer-generated curves[12] of $n_n(\omega)$ as functions of

$$\frac{\omega}{\Gamma_1} = \frac{\omega_g}{\Gamma_1} + \frac{Q}{3}(s-1),$$

where

$$Q = \frac{(\alpha_1\alpha_2)^{2/3}\omega_0^{1/3}(1-\zeta)^{1/3}(\omega_0-1)(1+\zeta\omega_0)}{\zeta\alpha_1^2+(\omega_0-1)(1+\zeta\omega_0)^2} \quad \left(\omega_0 = \frac{\omega_g}{\Gamma_1}\right)$$

at various values of the parameters α_n and $\zeta = \Gamma_1/\Gamma_2$.

The entropy and thermal conductivity of a two-band superconductor with impurity[16] and its absorption of light and ultrasound[17] were computed on the basis of the method developed for investigation of the functions n_n and functions related to them.

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Yu. E. Perlin and B. S. Tsukerblat. Dichroism Effects in Systems with Dynamic Jahn-Teller Coupling. Insurmountable mathematical difficulties are encountered in attempts to calculate the shape of the optical bands that arise on transitions between Jahn-Teller electron vibrational states of a crystal impurity