Renormalizable models of the electromagnetic and weak interactions[1)]

A. I. Vaĭnshteĭn and I. B. Khriplovich

Nuclear Physics Institute, Siberian Division, USSR Academy of Sciences
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This review examines the renormalizability of various theories of vector fields. A discussion is given of how renormalizable theories in which the mass of a vector field arises from spontaneous symmetry breaking are used to construct models of the weak, electromagnetic, and strong interactions. Certain specific schemes and the experimental bounds on the parameters of these schemes are analyzed.

CONTENTS

1. INTRODUCTION

In 1934 Fermi [1] proposed a description of the β decay of the neutron in terms of a point interaction of two vector currents $(\bar{p}\gamma_\mu h)(\bar{e}\gamma_\mu \nu)$. The analogy of this model with quantum electrodynamics is obvious. The current local variant of weak-interaction theory differs from the original version mainly in that it allows for parity non-conservation, i.e., it makes use of a linear combination of vector and axial-vector currents [2,3].

Moreover, the current theory describes not only β decay, but also a large number of other weak processes, an important circumstance being the universality of the weak interaction. The effective coupling constant turned out to be precisely the same for such diverse weak reactions like β decay, muon decay and muon capture by a proton. The simplest way of explaining universality is to assume that the weak interactions are mediated by a charged vector boson which has the same coupling constant with all the currents, in the same way that the electric charge—the coupling constant of the photon with the electric currents of different particles—turns out to be the same. Although the intermediate vector boson, unlike the photon, must be massive (indeed, long-range weak interactions do not exist), there is no doubt that the theory involving an intermediate vector boson is similar to electrodynamics. In this respect, it seems very natural to endeavor to construct a unified theory of the weak and electromagnetic (WEM) interactions [4-6].

Weinberg [7] and Salam [8] proposed a scheme in which one starts with a Lagrangian for massless vector fields of the Yang-Mills type. The masses of the vector particles are due to their interaction with scalar fields having non-zero vacuum averages. Such a mechanism for the origin of the mass was first considered by Higgs [9] (see also [10,11]).

There has recently been a rapid proliferation in the number of models of this type which describe the WEM interactions [12-24]. The development of this approach was stimulated by the success of the theory of gauge fields and, in particular, by the work of G.'t Hooft [25], who proved that such models are renormalizable.

The essence of the matter is that, in the ordinary four-fermion theory of the weak interaction, the amplitudes in the Born approximation grow as a power of the energy, resulting in a violation of the unitarity condition at energies exceeding a few hundred GeV (in the c.m.s.). Moreover, the fact that the Born amplitudes grow with energy makes it impossible to calculate the radiative corrections, even for low-energy processes. The higher the order of perturbation theory, the higher the degree of divergence; in other words, the theory is nonrenormalizable (see the review [26]). The introduction of the intermediate vector boson does not in itself render the theory renormalizable. Although it is possible that the difficulties in nonrenormalizable theories are merely a reflection of our inability to go beyond the framework of perturbation theory, renormalizable theories nevertheless seem preferable.

There are now numerous renormalizable models of the WEM interactions of leptons and hadrons. With these models, one can calculate higher-order effects (of course, without allowance for the strong interactions) by making an expansion in the small constant α/π, as in ordinary quantum electrodynamics.

The present review is devoted to the study of these models. The renormalizability of various vector theories is discussed in Sec. 2. The criterion for renormalizability which we use is that the Born amplitudes in the asymptotic energy region must not exceed the unitarity limit [27]. It is shown that the theory of a massive Yang-Mills field is nonrenormalizable if the mass is introduced as input and is renormalizable if the mass results from spontaneous symmetry breaking. It is explained why an additional scalar particle must be introduced to render the theory renormalizable. Variants of the renormalizable electrodynamics of vector bosons are also considered here.

In Sec. 3 we describe models of the WEM interactions of leptons. This section begins with a discussion of what symmetry such schemes must possess and what additional particles must be introduced in them.

We then discuss Weinberg's model [7], which involves neutral weak currents, and the model of Georgi and Glashow [15], in which such currents are absent. We give the constraints on the parameters of these theories which follow from the experimental data.

In Sec. 4 of the review we discuss the incorporation of hadrons in the renormalizable schemes. Strong experimental constraints on the neutral strangeness-changing hadronic currents imply that the symmetry of the strong interactions must be higher than SU(3). We discuss a generalization of Weinberg's model to the case of hadrons [12–14], when these requirements can be satisfied at the cost of introducing a fourth quark [28]. At the present time, this model is at the verge of being incompatible with the experimental data on the neutral strangeness-conserving hadronic currents. Neutral weak currents are generally absent in schemes for the WEM interactions of hadrons [15,19,20] which generalize the Georgi-Glashow model. At the end of the section, we give a brief discussion of the utilization of renormalizable vector theories in giving a unified description of the strong, electromagnetic and weak interactions of hadrons [22].

The Appendix contains a prescription for obtaining the Feynman rules for the Lagrangians of the vector fields considered in the review.

This review is not intended to provide a comprehensive exposition. In particular, we do not consider the complications connected with anomalies in the divergence of the axial-vector current [29]. Such anomalies occur when allowance is made for the diagrams with fermion loops, and they generally lead to the nonrenormalizability of the theory. However, in many models[13,15–22,30–32] there are cancellations among the anomalous contributions of different fermions. Moreover, for certain physical processes, these complications appear only in high orders of perturbation theory. It is mainly because of this last circumstance that we do not discuss the foregoing set of problems.

Another interesting problem which is not discussed here is the possibility of calculating relations among the masses of elementary particles in renormalizable theories [23,33–36].

In addition, we do not consider the theoretical papers [37–44] devoted to various proofs of renormalizability, the derivation of Ward identities, and the construction of gauge-invariant methods of regularization for vector fields.

To conclude the introduction, we call the reader's attention to the rather detailed review [45], which contains, in particular, information about the problems that are not considered here (see also [46,47]).

2. RENORMALIZABLE VECTOR THEORIES

a) Our criterion for renormalizability is the fulfillment of the unitarity condition for the asymptotic behavior of the Born amplitudes for real processes in the energy region bounded by the condition $(\alpha/\pi)\ln(E/m) \lesssim 1$. It is easy to see that this criterion is equivalent to the usual one. In fact, from the Born amplitudes one can reconstruct the Lagrangian in terms of the fields that describe the physical degrees of freedom. In the case of "good" Born amplitudes, this Lagrangian is renormalizable, as can be seen from a dimensional analysis.

The radiative corrections can be expressed directly in terms of integrals of the Born amplitudes by using the conditions of analyticity and unitarity for the closed loops. A good asymptotic behavior of the Born amplitudes then guarantees that the number of subtraction constants is finite.

Vector theories are characterized by the existence of relations that follow from gauge invariance. The requirement that these relations are satisfied in higher orders leads to the problem of gauge-invariant regularization in the Lagrangian approach. This problem does not arise in the approach based on unitarity and analyticity, since the Born amplitudes are known to be determined unambiguously. In this case, however, it is assumed that the gauge-invariant tensor structures can describe not only the imaginary part of the amplitude (which is obvious in the case of diagrams with one closed loop), but also its real part. The latter turns out to be impossible for fermion diagrams that give anomalies in the divergence of the axial-vector current [48]. The anomalies violate gauge invariance and, when they appear, this leads to the nonrenormalizability of theories of vector fields with spontaneous mass generation (see, e.g., [13]).

We shall first consider the renormalizability of massless vector fields of the Yang-Mills type [49]. We recall that conserved currents provide the sources of the vector fields in such theories. The corresponding charges are the generators of the symmetry group of the Lagrangian.

In particular, the group SU(2), which has three generators, corresponds to a triplet of vector fields $\mathbf{b}_\mu$, whose Lagrangian has the form[2)]

$$L = -\frac{1}{4}\mathbf{b}_{\mu\nu}\mathbf{b}_{\mu\nu},\quad \mathbf{b}_{\mu\nu} = \partial_\mu \mathbf{b}_\nu - \partial_\nu \mathbf{b}_\mu + 2g\,[\mathbf{b}_\mu \mathbf{b}_\nu]. \tag{1}$$

The fields describing particles with definite charges are related to the hermitian fields $\mathbf{b}_\mu$ as follows:

$$b_\mu^\pm = \frac{1}{\sqrt{2}}(b_\mu^1 \mp i b_\mu^2),\quad b_\mu^0 = b_\mu^3. \tag{2}$$

Let us check that the criterion for renormalizability is satisfied for the elastic scattering amplitude for vector particles. In the case of the process $b^+b^- \to b^+b^-$, this amplitude is described by the diagrams shown in Fig. 1. It is easy to obtain from the Lagrangian (1) the following expressions for the vertices:

$$\Gamma_{\mu\nu\lambda}^{+-0}(k_+, k_-, k_0) = 2g\,[-\delta_{\mu\nu}(k_+ - k_-)_\lambda - \delta_{\nu\lambda}(k_- - k_0)_\mu - \delta_{\lambda\mu}(k_0 - k_+)_\nu], \tag{3a}$$

$$\Gamma_{\mu\nu\varkappa\lambda}^{-++-} = 4g^2(2\delta_{\mu\lambda}\delta_{\nu\varkappa} - \delta_{\mu\nu}\delta_{\varkappa\lambda} - \delta_{\mu\varkappa}\delta_{\nu\lambda}). \tag{3b}$$

In writing Eqs. (3a) and (3b), we regard all the particles as outgoing. As to the propagator of the vector field, its longitudinal part cannot be determined without imposing supplementary conditions, as in the case of the photon propagator in quantum electrodynamics. However, this part drops out of the expression for the Born amplitudes if the external particles are real. In fact, it follows from (3a) that

$$k_{0\lambda}\Gamma_{\mu\nu\lambda}^{+-0}(k_+, k_-, k_0)\,\varepsilon_\mu(k_+)\,\varepsilon_\nu(k_-) = 2g\,[(k_+^2 - k_-^2)\,\delta_{\mu\nu} - k_{+\mu}k_{+\nu} + k_{-\mu}k_{-\nu}]\,\varepsilon_\mu(k_+)\,\varepsilon_\nu(k_-) = 0, \tag{4}$$

provided that

$$k_+^2 = k_-^2,\quad k_{+\mu}\varepsilon_\mu^*(k_+) = k_{-\nu}\varepsilon_\nu(k_-) = 0. \tag{4a}$$

The conditions (4a) are known to be satisfied for real states.

Thus, the amplitude for the process $b^+(k_1) + b^-(k_2) \to b^+(k_3) + b^-(k_4)$ can be represented in the form

$$M = \varepsilon_\mu(k_1)\,\varepsilon_\nu(k_2)\,\varepsilon_\varkappa(k_3)\,\varepsilon_\lambda(k_4)\Big[\Gamma_{\nu\mu\rho}^{+-0}(-k_2, -k_1, k_1+k_2)\frac{1}{(k_1+k_2)^2} \times \Gamma_{\varkappa\lambda\rho}^{+-0}(k_3, k_4, -k_3-k_4) + \Gamma_{\varkappa\mu\rho}^{+-0}(k_3, -k_1, k_1-k_3)\frac{1}{(k_1-k_3)^2} \times \Gamma_{\nu\lambda\rho}^{+-0}(-k_2, k_4, k_2-k_4) + \Gamma_{\mu\nu\varkappa\lambda}^{-++-}\Big]. \tag{5}$$

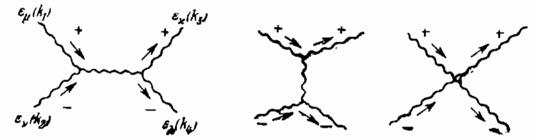

FIG. 1

Since the three-dimensional transverse polarization vectors of massless particles do not grow with energy, the asymptotic behavior of the amplitude M corresponds to a renormalizable theory. We note that the partial amplitudes in this case contain a divergence due to the exchange of a massless particle, but this divergence has no bearing on the high-energy behavior.

It is easy to see that all the Born amplitudes have a good high-energy behavior (the same behavior as in the electrodynamics of scalar particles, for example), so that the theory is renormalizable. The renormalizability is preserved when allowance is made for an interaction with fields of spin 0 and $1/2$, provided that a minimal coupling is introduced as an interaction with the isospin current.

A massive vector boson differs from a massless one by the presence of a state with zero helicity. The corresponding polarization vector has the form

$$\varepsilon_\nu = \left(\frac{|\mathbf{k}|}{\mu},\ \frac{\mathbf{k}}{|\mathbf{k}|}\frac{\omega}{\mu}\right) = \frac{k_\nu}{\mu}\frac{\omega}{|\mathbf{k}|} - n_\nu\frac{\mu}{|\mathbf{k}|},\ k_\nu\varepsilon_\nu = 0, \tag{6}$$

where $n_\nu = (1, 0, 0, 0)$. The growth of ϵ_ν with energy in the general case can lead to a growth of the Born amplitudes that is inadmissible for renormalizable theories.

In the case of a neutral vector field, the term k_ν/μ in Eq. (6) for ϵ_ν does not contribute to the amplitudes for physical processes if the current is conserved. In the case of mutually interacting vector fields, it is also natural to expect that the high-energy behavior of the amplitudes is best in theories of the Yang-Mills type, in which the currents are conserved. However, if a mass is introduced in the Lagrangian (1) in the usual way by adding to it a term $(1/2)\mu^2\mathbf{b}_\mu\cdot\mathbf{b}_\mu$, then current conservation, while giving a partial reduction in the rate of growth of the amplitude, does not guarantee renormalizability [50-52].

To prove nonrenormalizability, it is sufficient to consider the elastic scattering amplitude of charged quanta of zero helicity (see Fig. 1). The propagator of a massive vector particle has the form $-i[\delta_{\mu\nu} - (k_\mu k_\nu/\mu^2)](k^2-\mu^2)^{-1}$. However, the terms proportional to $k_\mu k_\nu$ do not contribute, as in the massless case, by virtue of the relations (4) and (4a). Therefore the expression for the Born amplitude can be obtained from (5) by making the substitution $1/k^2 \to 1/(k^2-\mu^2)$. Using Eq. (6) for the polarization vectors, we find that in the asymptotic region the Born amplitude grows quadratically with energy (in the c.m.s.):

$$M \approx \frac{2g^2}{\mu^2}(k_1k_2 - k_1k_3). \tag{7}$$

If the interaction were not of the Yang-Mills form, this amplitude would grow like the fourth power of the energy, but in our case there is a cancellation among such contributions from different diagrams. However, a quadratic growth is sufficient for the renormalizability of the theory.

b) Since the usual introduction of mass in the Lagrangian of a Yang-Mills field leads to nonrenormalizability, it is natural to turn to theories [9,11] in which the mass of a vector field arises from spontaneous symmetry breaking.

Let us consider a gauge-invariant Lagrangian describing the interaction of a Yang-Mills triplet $\mathbf{b}_\mu$ with a doublet

$$\varphi = \begin{pmatrix}\varphi^+\\ \varphi^0\end{pmatrix}$$

of scalar fields:

$$L = -\frac{1}{4}\mathbf{b}_{\mu\nu}\mathbf{b}_{\mu\nu} + (D_\mu\varphi)^+(D_\mu\varphi) + m^2\varphi^+\varphi - f^2(\varphi^+\varphi)^2; \tag{8}$$

in this equation

$$D_\mu\varphi = \partial_\mu\varphi - ig(\tau\mathbf{b}_\mu)\varphi, \tag{8a}$$

and τ are the Pauli matrices. The Lagrangian (8) is an invariant of the following gauge transformations [49]:

$$b_\mu = S^{-1}\left[b'_\mu + \frac{i}{g}\partial_\mu S\cdot S^{-1}\right]S,\ \varphi = S^{-1}\varphi', \tag{9}$$

where S is a unitary unimodular matrix depending on the coordinates, and $b_\mu = \tau\cdot\mathbf{b}_\mu$.

We note that the mass term of the field φ has the opposite sign to the usual one. Owing to the "wrong" sign, the solution $\varphi(x) \equiv 0$ does not correspond to the minimum of energy when the fields are regarded as classical fields. This minimum is given by x-independent solutions for which $\varphi_0^+\varphi_0 = m^2/2f^2$. The orientation of the axes in isotopic space can always be chosen so that

$$\varphi_0 = \frac{m}{\sqrt{2}f}\begin{pmatrix}0\\1\end{pmatrix}. \tag{10}$$

We shall take the "true" field to be the difference $\varphi(x) - \varphi_0$, which can be represented in the form

$$\varphi(x) - \varphi_0 = \frac{1}{\sqrt{2}}(\sigma + i\tau\psi)\begin{pmatrix}0\\1\end{pmatrix}, \tag{11}$$

where σ and ψ are hermitian fields.

We note that all considerations relating to the field $\varphi(x)$ may be omitted and that one can avoid attaching any physical significance to this field, but instead regard as fundamental the Lagrangian of the fields $\mathbf{b}_\mu$, σ and ψ that results when (11) is substituted in (8). This Lagrangian is, as before, gauge invariant. Gauge transformations of the fields σ and ψ can be obtained by substituting (11) in (9). To ascertain what particles are described by the theory, it is convenient to choose a gauge in which $\psi(x) \equiv 0$. In this case, the Lagrangian has the form

$$L = -\frac{1}{4}\mathbf{b}_{\mu\nu}\mathbf{b}_{\mu\nu} + \frac{1}{2}g^2\left(\frac{m}{f} + \sigma\right)^2\mathbf{b}_\mu\mathbf{b}_\mu + \frac{1}{2}\partial_\mu\sigma\partial_\mu\sigma - m^2\sigma^2 - fm\sigma^3 - \frac{f^2}{4}\sigma^4. \tag{12}$$

This Lagrangian describes a triplet of vector fields $\mathbf{b}_\mu$ with a mass $\mu = gm/f$ (in the ordinary Proca formalism [53]) and a scalar field σ with a mass $m\sqrt{2}$. The above-mentioned mechanism of generating the mass of a vector field as a result of its interaction with a scalar field having a non-zero vacuum average is known as the Higgs phenomenon.

What would spontaneous symmetry violation give if the interaction with the vector field $\mathbf{b}_\mu$ is switched off? Substituting (11) in (8) and putting $\mathbf{b}_\mu \equiv 0$, we find the well-known Lagrangian of the σ model [54],

$$L=\frac{1}{2}(\partial_\mu\sigma\partial_\mu\sigma+\partial_\mu\boldsymbol{\psi}\partial_\mu\boldsymbol{\psi})-m^2\sigma^2-fm\sigma(\sigma^2+\boldsymbol{\psi}^2)-\frac{f^2}{4}(\sigma^2+\boldsymbol{\psi}^2)^2. \quad (13)$$

This Lagrangian describes a field σ with mass $m\sqrt{2}$ and a triplet of zero-mass fields $\boldsymbol{\phi}$. The statement that spontaneous symmetry breaking leads to the appearance of massless particles is the essence of the Goldstone theorem [55,56].

With spontaneous symmetry breaking in the presence of a massless vector field, the fields $\boldsymbol{\phi}$ may be interpreted as the zero helicities of a vector field which acquires a mass, so that no massless particles remain in the theory. This is a result of the Higgs phenomenon.

We shall now discuss the renormalizability of the foregoing theory of vector fields. Let us examine the high-energy behavior of the elastic scattering amplitude of two charged zero-helicity quanta in this theory. For this process, the Lagrangian (12) gives, in addition to the diagrams of Fig. 1, new diagrams (Fig. 2) connected with the introduction of σ particles (indicated by dashed lines). It is easy to derive the following expression for the matrix element corresponding to the diagrams of Fig. 2:

$$\Delta M=-4g^2\mu^2\left[\frac{(\varepsilon_1\varepsilon_2)(\varepsilon_3\varepsilon_4)}{(k_1+k_2)^2-2m^2}+\frac{(\varepsilon_1\varepsilon_3)(\varepsilon_2\varepsilon_4)}{(k_1-k_3)^2-2m^2}\right]. \quad (14)$$

Substituting ϵ_i in the form (6), we see that there is a cancellation between the part of M which grows with energy (see (7)) and ΔM, so that the amplitude $M+\Delta M$ does not exceed the unitarity limit.

For the general discussion of processes involving zero-helicity quanta [57], it is convenient to go over to the Coulomb gauge, in which the field b_μ satisfies the three-dimensional transversality condition $\partial_m b_m = 0$ and describes quanta with helicities ±1, while zero-helicity quanta are described by the fields $\boldsymbol{\phi}$. A standard analysis of the Lagrangian that results when $\varphi(x)$ from (11) is substituted in (8) indicates that all the Born amplitudes have a good asymptotic behavior (the same behavior as in scalar electrodynamics, for example), so that the theory is renormalizable.

From our point of view, a major feature of the scheme is the fact that, owing to the non-zero vacuum average of the field φ, the scalar fields $\boldsymbol{\phi}$ may be interpreted as the zero helicities of the vector fields. The renormalizability of the theory itself is obvious. The point is that neither the masses of the particles nor the shifts in the fields that are proportional to these masses show up in the high-energy behavior of the Born amplitudes corresponding to the Lagrangian (8). In other words, the amplitudes behave asymptotically in the same way as in a renormalizable theory of a massless Yang-Mills field interacting with a doublet of scalar particles.

Although the Coulomb gauge enables us in principle to ascertain the renormalizability of the theory, it is inconvenient for concrete calculations, owing to its lack of covariance. The Proca gauge, in spite of its

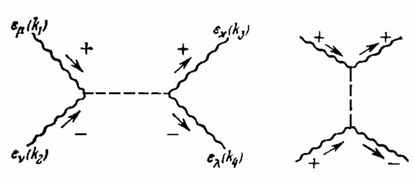

FIG. 2

covariance, is also inconvenient for calculations of the radiative corrections, as the contributions of the individual diagrams in this gauge are highly divergent. The Feynman rules for vector fields of the Yang-Mills type in "good" covariant gauges have been constructed by De Witt [58], Faddeev and Popov [59] and Fradkin and Tyutin [60] using a functional integration method. We give a prescription for obtaining these rules in the Appendix at the end of this paper.

c) We would like to indicate, from a somewhat different point of view [61], why it has been found to be necessary to introduce an additional scalar particle in order to obtain a renormalizable theory of massive vector fields[3)].

Vector theories are characterized by the presence of a symmetry group related to the local gauge transformations. In order to study the consequences of this symmetry, it is convenient to transform to the Coulomb gauge by means of the substitution

$$b_\mu=U^{-1}\left(b_\mu^c+\frac{i}{g}\partial_\mu U\cdot U^{-1}\right)U,\quad b_\mu=\boldsymbol{\tau}\mathbf{b}_\mu, \quad (15)$$

where $\mathbf{b}_\mu^c$ satisfies the condition $\partial_m \mathbf{b}_m^c = 0$ ($m = 1, 2, 3$) and describes quanta with helicities ±1, and U is a unitary unimodular matrix determined by three fields $\boldsymbol{\zeta}(x)$ (e.g., $U(x)=[1-(ig/2\mu)\boldsymbol{\tau}\cdot\boldsymbol{\zeta}(x)][1+(ig/2\mu)\boldsymbol{\tau}\cdot\boldsymbol{\zeta}(x)]^{-1}$). The fields $\boldsymbol{\zeta}(x)$ correspond to zero-helicity quanta. Gauge invariance for the fields $\mathbf{b}_\mu^c(x)$ and $\zeta(x)$ leads to invariance with respect to coordinate-independent transformations

$$b_\mu^c=S^{-1}b_\mu^{c\prime}S,\quad U(\zeta)=S^{-1}U(\zeta'). \quad (16)$$

For the fields $\boldsymbol{\zeta}$, these transformations are obviously not the same as the isotopic transformations. As a result, the full symmetry group of the theory in question is $SU(2)\otimes SU(2)$, and the fields $\boldsymbol{\zeta}$ form a nonlinear realization of the $(1/2, 1/2)$ representation of this group, in analogy with pions in chiral $SU(2)\otimes SU(2)$ symmetry. For this reason, when mass is introduced in the usual way in a vector theory, the Lagrangian of the zero-helicity fields has the form [52]

$$L=\frac{1}{2}\frac{\partial_\mu\zeta\partial_\mu\zeta}{[1+(g^2/4\mu^2)\zeta^2]^2}, \quad (17)$$

in agreement with the well-known Lagrangian for pions [62]. The essential nonlinearity of Eq. (17) leads to the nonrenormalizability of the theory.

However, if there is an additional scalar particle in the theory, it is possible to have a linear realization of the $(1/2, 1/2)$ representation of the group $SU(2)\otimes SU(2)$. In this case, it is no longer necessary for the Lagrangian to be essentially nonlinear.

The renormalizable and $SU(2)\otimes SU(2)$ invariant Lagrangian of four hermitian fields $\tilde{\sigma}$ and $\boldsymbol{\phi}$ has a uniquely determined form [51]:

$$L=\frac{1}{2}\mathrm{Sp}\,[\partial_\mu\Phi^+\partial_\mu\Phi+m^2\Phi^+\Phi-f^2(\Phi^+\Phi)^2]; \quad (18)$$

here the matrix Φ is given by

$$\Phi=\frac{1}{\sqrt{2}}(\tilde{\sigma}+i\boldsymbol{\psi}\boldsymbol{\tau}). \quad (18a)$$

This quantity behaves like a spinor in each of the indices individually, in accordance with the $SU(2)\otimes SU(2)$ symmetry of the Lagrangian. It is easy to see that the Lagrangians (18) and (13) coincide if we take into account the relation $\tilde{\sigma}=(m/f)+\sigma$.

We note that the Lagrangian (12) of the physical fields b_μ and σ possesses only isotopic symmetry. These fields are singlets with respect to the spontaneously broken subgroup of the symmetry. However, at large momenta, when all the masses can be neglected, we recover the original $SU(2) \otimes SU(2)$ symmetry: the field σ and the zero helicities of the fields b_μ are again grouped into a quadruplet. The theory in question can therefore serve as an example of the realization of an asymptotic symmetry.

d) Bearing in mind the description of the electromagnetic interactions, let us indicate how to incorporate a massless vector field in the scheme under consideration. We must introduce in the Lagrangian (8) a singlet a_μ, which interacts with the hypercharge current of the field φ. To do this, we must add to (8) the Lagrangian of the free field a_μ and define $D_\mu\varphi$ as follows:

$$D_\mu\varphi = \partial_\mu\varphi - ig\,(\boldsymbol{\tau}\mathbf{b}_\mu)\,\varphi + ig' a_\mu\,\varphi. \qquad (19)$$

As a result, the Lagrangian of the vector and scalar particles has the form

$$L = -\frac{1}{4}\,\mathbf{b}_{\mu\nu}\mathbf{b}_{\mu\nu} - \frac{1}{4}\,a_{\mu\nu}a_{\mu\nu} + (D_\mu\varphi)^+(D_\mu\varphi) + m^2\varphi^+\varphi - f^2(\varphi^+\varphi)^2; \qquad (20)$$

here $a_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$.

It is easy to see that, owing to the spontaneous symmetry breaking (the non-zero vacuum average of the field φ), the linear combination of neutral vector fields

$$Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}\,(g b_\mu^3 + g' a_\mu) \qquad (21a)$$

acquires a mass $\mu_Z = (\sqrt{g^2+g'^2}/f)m$, while the field

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}\,(-g' b_\mu^3 + g a_\mu) \qquad (21b)$$

remains massless.

On the other hand, one may construct a theory in which there is only a triplet of vector fields and a mass occurs only for the charged particles. To construct such a scheme, one must use, instead of the doublet φ, a Hermitian triplet of scalar fields, whose neutral component has a non-zero vacuum average. The remaining components of the triplet describe the zero helicities of the charged vector fields. The explicit form of the corresponding Lagrangian is given below (see Sec. 3c).

These schemes may be regarded as renormalizable models of the electrodynamics of vector bosons.

3. MODELS OF THE WEAK AND ELECTROMAGNETIC INTERACTIONS OF LEPTONS

a) In the models under consideration, the weak interactions are mediated by massive vector bosons. We know that some of them must be charged in order to describe the known weak processes. In order to make the electromagnetic interactions of vector bosons renormalizable, it is necessary, as we have seen, to introduce a neutral scalar particle.

Now the interaction of the vector bosons with leptons must be introduced in such a way that the currents—the sources of the vector fields—are conserved, as before, otherwise the renormalizability will be violated. The charges corresponding to these currents are the generators of the group SU(2) in the case of three vector fields and the group $SU(2) \otimes U(1)$ in the case of four vector

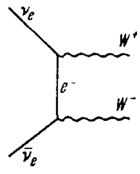

FIG. 3

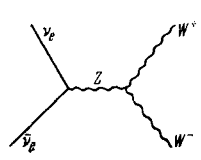

FIG. 4

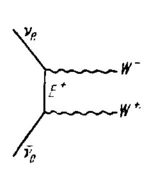

FIG. 5

fields. The leptons must therefore be combined into multiplets of the group SU(2), which we shall call isotopic multiplets (in analogy with the hadrons).

If the model involves only the three vector fields W_μ^+, W_μ^- and the electromagnetic field A_μ, they interact with the isospin current of the leptons [15]. In this case, the electric charge coincides with T_3, the third projection of the isospin, so that the average charge of the leptonic multiplets must be equal to zero. If the conservation of muonic charge is taken into account, then the known leptons (e, ν_e and μ, ν_μ) can be grouped only into singlets and doublets, and the average electric charge of the doublets is non-zero. To construct multiplets with zero average charge, we must introduce two new charged leptons—partners to the e, ν_e and to the μ, ν_μ. Moreover, to describe parity non-conservation, we must introduce two more neutral heavy leptons. We shall return to the discussion of this model below (Sec. 3c).

If, on the other hand, we do not introduce any new leptons, we must attribute hypercharge to the leptons in order to obtain the correct relation between the electric charge and the isospin projection. An additional isoscalar vector field interacts with the hypercharge current (see Sec. 2d).

The need to introduce an additional neutral vector boson or new leptons in a renormalizable theory may be illustrated by the following example [45,63]. Let us consider the process $\nu_e\bar{\nu}_e \to W^+W^-$. In the ordinary weak-interaction theory involving an intermediate vector boson, this process is described by a single diagram (Fig. 3). In the case when zero-helicity quanta are produced, the corresponding amplitude grows quadratically with the energy. When additional particles are incorporated in the theory, this growth is compensated by the contribution of the diagram involving a neutral vector boson in the s-channel (Fig. 4) or by the contribution of the diagram involving a fermion pole in the u-channel, corresponding to a new charged lepton (Fig. 5).

b) In this subsection we shall discuss the model of the WEM interactions of leptons proposed by Weinberg [7]. In this model, there are no new leptons, while the known leptons are grouped into a doublet

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

and a singlet $R = e_R$, where $l_{L,R} = [(1 \pm \gamma_5)/2]l$. The muon and the muonic neutrino are incorporated in the theory in analogy with the e and ν_e. Therefore we do not write out the corresponding expressions explicitly in what follows.

Defining the hypercharge Y by the relation $Q = T_3 + (Y/2)$, we find that $Y = -1$ for the doublet and $Y = -2$ for the singlet.

The Lagrangian for the interaction of the leptons with the vector bosons is constructed from the free Lagrangian

$$L_l^0 = i\bar{L}\gamma_\mu\partial_\mu L + i\bar{R}\gamma_\mu\partial_\mu R, \quad (22)$$

as in electrodynamics by extending the derivatives

$$\partial_\mu \to \partial_\mu - 2ig\mathbf{T}\mathbf{b}_\mu + ig'Ya_\mu, \quad (23)$$

where T are the isospin matrices, and Y is the hypercharge of a particle. Specifically,

$$\begin{aligned}\partial_\mu L &\to (\partial_\mu - ig\boldsymbol{\tau}\mathbf{b}_\mu - ig'a_\mu) L,\\ \partial_\mu R &\to (\partial_\mu - 2ig'a_\mu) R\end{aligned} \quad (23a)$$

As to the electron mass, its conventional inclusion in the Lagrangian would violate the symmetry and hence the renormalizability of the theory. However, if we introduce a renormalizable interaction of the type

$$-h\,(\bar{L}\varphi R + \bar{R}\varphi^+ L), \quad (24)$$

then the electron mass appears as a result of the c-number part of φ, the doublet of scalar fields. At the same time, there is also an interaction of the field σ with the electron.

This mechanism of producing the electron mass, like that of producing the mass of a vector field, represents a spontaneous symmetry breaking. The symmetry is recovered in the region of large momenta.

We have already considered the form of the interactions of vector particles with each other and with the scalar field σ (see (20)). As we pointed out in Sec. 2d, there is a definite mass for the fields

$$\left.\begin{aligned}W_\mu^\pm &= b_\mu^\pm = \frac{b_\mu^1 \mp ib_\mu^3}{\sqrt{2}}, \quad \mu_W = \frac{g}{f}m,\\ Z_\mu &= \frac{1}{\sqrt{g^2+g'^2}}(gb_\mu^3 + g'a_\mu), \quad \mu_Z = \frac{\sqrt{g^2+g'^2}}{f}m,\\ A_\mu &= \frac{1}{\sqrt{g^2+g'^2}}(-g'b_\mu^3 + ga_\mu), \quad \mu_A = 0.\end{aligned}\right\} \quad (25)$$

The leptonic part of the Lagrangian (see (22) and (23a)), expressed in terms of these fields, can be written in the form

$$\begin{aligned}L_l &= \bar{e}i\gamma_\mu\partial_\mu e + \bar{\nu}_e i\gamma_\mu\frac{1+\gamma_5}{2}\partial_\mu\nu_e - m_e\left(1+\frac{g}{\mu_W}\sigma\right)\bar{e}e\\ &+ \frac{2gg'}{\sqrt{g^2+g'^2}}A_\mu\bar{e}\gamma_\mu e + \frac{g}{\sqrt{2}}\left[W_\mu^+\bar{\nu}_e\gamma_\mu(1+\gamma_5)e + W_\mu^-\bar{e}\gamma_\mu(1+\gamma_5)\nu_e\right]\\ &+ \frac{1}{2}\sqrt{g^2+g'^2}\,Z_\mu\left[\bar{e}\gamma_\mu\left(\frac{3g'^2-g^2}{g'^2+g^2}-\gamma_5\right)e + \bar{\nu}_e\gamma_\mu(1+\gamma_5)\nu_e\right]\\ &+ (e\to\mu,\ \nu_e\to\nu_\mu).\end{aligned} \quad (26)$$

It follows from this expression that the electric charge e and the weak-interaction constant G are given by

$$\begin{aligned}e &= \frac{2gg'}{\sqrt{g^2+g'^2}} = 2g\sin\theta,\\ \frac{G}{\sqrt{2}} &= \frac{f^2}{2m^2} = \frac{g^2}{2\mu_W^2}.\end{aligned} \quad (27)$$

As a result, we have the following bounds:

$$\begin{aligned}\mu_W &= \frac{1}{\sin\theta}\sqrt{\frac{\pi\alpha}{G\sqrt{2}}} \geqslant 37.3\ \text{GeV},\\ \mu_Z &= \frac{2}{\sin 2\theta}\sqrt{\frac{\pi\alpha}{G\sqrt{2}}} \geqslant 74.6\ \text{GeV}.\end{aligned} \quad (28)$$

The existence of neutral weak currents is characteristic of the model. We note that they do not have the V-A structure for electrons and muons.

The neutral currents as well as the ordinary charged currents contribute to the scattering of ν_e and $\bar{\nu}_e$ by the electron. At the present time, there exists the experimental bound [64] (with 90% confidence)

$$\sigma(\bar{\nu}_e e \to \bar{\nu}_e e) < 3{,}0\,\sigma_{V-A}(\bar{\nu}_e e \to \bar{\nu}_e e), \quad (29)$$

where σ_{V-A} is determined by the standard V-A theory of the weak interactions [2], in which there are no neutral currents. It follows from the bound (29) that [65]

$$\sin^2\theta = \frac{g'^2}{g^2+g'^2} < 0.32. \quad (30)$$

Other leptonic processes that are crucial for testing Weinberg's model are the reactions of scattering of ν_μ and $\bar{\nu}_\mu$ by the electron, which proceed purely as a result of the neutral currents [65,66]. The existing experimental data lead to the following bounds on the cross sections and the mixing angle[67,68]:

$$\left.\begin{aligned}\sigma(\nu_\mu e \to \nu_\mu e) &< 0.16\sigma_{V-A}(\nu_e e \to \nu_e e),\\ \sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e) &< 0.55\sigma_{V-A}(\nu_e e \to \nu_e e),\\ 0.1 < \sin^2\theta &< 0.6;\end{aligned}\right\} \quad (31)$$

here $\sigma_{V-A}(\nu_e e \to \nu_e e) = 1.6\times10^{-41}\,(E_\nu/\text{GeV})\ \text{cm}^2$.

As to the interaction of the scalar field σ with the leptons, the corresponding constant

$$\frac{h}{\sqrt{2}} = m_e\sqrt{G\sqrt{2}} \quad (32)$$

is very small (2×10^{-6} for the e and 4×10^{-4} for the μ).

We note that the weak-interaction contribution to the g-factor of the muon in the scheme under consideration is of order [69]

$$\frac{Gm_\mu^2}{\sqrt{2}\,8\pi^2} \sim 10^{-9},$$

while the existing experimental accuracy is $\sim 3\times10^{-7}$ (see [70]). Although the radiative corrections may be $\sim\alpha/\pi$ in the general case (when $\mu_Z \sim \mu_W \sim m$), the contribution to the g-factor contains the square of the lepton mass. The point is that the original interactions conserve helicity in the limit of zero lepton mass. Consequently, the helicity-flip weak-interaction contribution to the anomalous magnetic moment is equal to zero in this limit, so that

$$\Delta\mu \sim \frac{g^2}{32\pi^2}\frac{m_\mu}{\mu_W^2}.$$

c) We now turn to the discussion of the model [15] in which the only neutral current is the electromagnetic current and the vector fields form a triplet b_μ:

$$\left(\frac{b_\mu^1 - ib_\mu^2}{\sqrt{2}},\ b_\mu^3,\ \frac{b_\mu^1 + ib_\mu^2}{\sqrt{2}}\right) = (W_\mu^+,\ A_\mu,\ W_\mu^-).$$

As we pointed out in Sec. 2d, the charged fields $W_\mu^\pm$ acquire a mass as a result of the interaction with the triplet of scalar fields $\boldsymbol{\varphi}$. The corresponding Lagrangian has the form [25]

$$L_V = -\frac{1}{4}\mathbf{b}_{\mu\nu}\mathbf{b}_{\mu\nu} + \frac{1}{2}(\partial_\mu\boldsymbol{\varphi} - e\mathbf{b}_\mu\times\boldsymbol{\varphi})(\partial_\mu\boldsymbol{\varphi} - e\mathbf{b}_\mu\times\boldsymbol{\varphi}) + \frac{m^2}{2}\boldsymbol{\varphi}^2 - \frac{f^2}{4}\boldsymbol{\varphi}^4; \quad (33)$$

here $\mathbf{b}_{\mu\nu}$ is defined by Eq. (1) with the substitution $2g \to -e$. As in the model discussed in Sec. 2b, the neutral component of $\boldsymbol{\varphi}$ has a non-zero vacuum average, $\varphi^3 = m/f + \sigma$. As to the fields $\varphi^\pm$, they can be eliminated by a choice of the gauge. The mass of the fields $W_\mu^\pm$ is equal to $\mu_W = (e/f)m$; of course, the electromagnetic field A_μ remains massless.

In this model, the leptons must be grouped into a multiplet with an average electric charge equal to zero (see Sec. 3a). The number of new particles is then minimal if the leptons are combined into a triplet E:

$$\left(\frac{E^1 - iE^2}{\sqrt{2}},\ E^3,\ \frac{E^1 + iE^2}{\sqrt{2}}\right) = (X^+,\ \nu',\ e^-); \quad (34)$$

here X^+, ν' and e^- are four-component spinors, e^- describes the electron, and X^+ is a new charged lepton, whose mass must be sufficiently large to avoid a contradiction with experiment. As to ν', this spinor cannot describe only the electronic neutrino, since it has four components. The two "extra" components describe a right-handed state, so that the theory contains a new neutral lepton X^0. If the mass of the X^0 were equal to zero, this lepton would simply be a right-handed neutrino with exactly the same interaction as the ordinary left-handed one. In other words, parity would be conserved. However, this was to be expected, since the weak currents and the parity-conserving electromagnetic current appear in a single multiplet. Thus, parity is not conserved only because of the non-zero mass of the X^0.

The spinor ν' can be represented as follows:

$$\nu' = \nu_L \sin\beta + X_L^0 \cos\beta + X_R^0, \qquad (35)$$

where $X^0_{L,R} = (1/2)(1 \pm \gamma_5)X^0$, and β is the mixing angle of the X^0_L and ν_L. The combination that is orthogonal to ν'_L is

$$s_L = -\nu_L \cos\beta + X_L^0 \sin\beta \qquad (36)$$

and is a singlet of the group. The invariant Lagrangian of the leptons can be written

$$L_l = \bar{\mathbf{E}} i\gamma_\mu (\partial_\mu \mathbf{E} - e\,[\mathbf{b}_\mu, \mathbf{E}]) + \bar{s}_L i\gamma_\mu \partial_\mu s_L - m_0 \bar{\mathbf{E}}\mathbf{E} + ih_1\varphi\,[\bar{\mathbf{E}}, \mathbf{E}] - h_2\varphi\,(\bar{\mathbf{E}} s_L + \bar{s}_L \mathbf{E}). \qquad (37)$$

The μ, ν_μ and the two heavy leptons Y^+, Y^0 corresponding to them are incorporated in the theory in precisely the same way, so that all our considerations refer equally to the muonic multiplets. We note only that μ-e universality is not a necessary consequence of the scheme under consideration and is achieved only with identical mixing angles in the electronic and muonic multiplets.

Unlike Weinberg's model, the symmetry group in this scheme does not contain parity-changing transformations. This allows us to introduce an "input" lepton mass in the Lagrangian (the term $-m_0\bar{\mathbf{E}}\cdot\mathbf{E}$ in (37)). The last two terms in (37) split the lepton masses, as a result of the non-zero vacuum average of the field $\langle\boldsymbol{\varphi}\rangle_0 = (0, m/f, 0)$. For the mass terms of the leptons to be diagonal, it is necessary that

$$2m_{X^0}\cos\beta = m_{X^+} + m_e. \qquad (38)$$

We now give the explicit form for the interactions of the leptons with the vector fields:

$$L^{\rm int} = -eW_\mu^+ \Big\{ \tfrac{1}{2}\sin\beta\,[\bar{\nu}\gamma_\mu(1+\gamma_5)e - \bar{X}^+\gamma_\mu(1+\gamma_5)\nu] + \bar{X}^0\gamma_\mu\Big(\cos^2\frac{\beta}{2} - \gamma_5\sin^2\frac{\beta}{2}\Big)e^- - \bar{X}^+\gamma_\mu\Big(\cos^2\frac{\beta}{2} - \gamma_5\sin^2\frac{\beta}{2}\Big)X^0\Big\} + \text{h.c.} + eA_\mu(\bar{e}^-\gamma_\mu e^- - \bar{X}^+\gamma_\mu X^+) + (e\to\mu,\ \nu_e\to\nu_\mu,\ X^{+,0}\to Y^{+,0}). \qquad (39)$$

The weak interaction constant G in this scheme is given by

$$\frac{G}{\sqrt{2}} = \frac{f^2\sin^2\beta}{4m^2} = \frac{e^2\sin^2\beta}{4\mu_W^2}, \qquad (40)$$

from which we obtain for μ_W

$$\mu_W = \sqrt{\frac{\sqrt{2}\pi\alpha}{G}}\sin\beta \leqslant 52.8\ \text{GeV}. \qquad (41)$$

In contrast with Weinberg's model, the interaction of the field σ with the leptons may be appreciable. The appropriate constant for electrons is equal to $e(m_{X^+} - m_e)/2\mu_W$.

Moreover, the effect of the weak interactions on the g-factor of the muon turns out to be conspicuously large. The corresponding contribution is given by [71-73]

$$\frac{\Delta g_\mu}{2} = -\frac{\alpha m_\mu m_{Y^+}}{2\pi\mu_W^2} \qquad (42)$$

We have given the result under the assumption that $m_{Y^0} \approx m_{Y^+}/2\cos\beta \ll \mu_W \sim m_\sigma$. Experimentally [70], $-3\times10^{-7} < \Delta g_\mu/2 < 9\times10^{-7}$ (with 95% confidence). Taking into account the experimental bound [74] $m_{Y^+} > 2.4$ GeV and the inequality (41), we find that

$$2.4\ \text{GeV} < m_{Y^+} < 8\ \text{GeV},\quad 28\ \text{GeV} < \mu_W < 53\ \text{GeV}. \qquad (43)$$

It is easy to estimate the lifetimes of heavy leptons. The widths of their leptonic decays are found from the Lagrangian (39), for example $\Gamma(X^+ \to \text{leptons}) = 10^{11}(m_{X^+}/\text{GeV})^2\ \text{sec}^{-1}$. If the total probability of hadronic decays is of the same order of magnitude, then the lifetime is $\tau_X \lesssim 10^{-11}$ sec for $m_X \gtrsim 1$ GeV. A detailed theoretical discussion of the properties of heavy leptons can be found in [75], and the experimental situation on the search for such particles is described in the review [76].

With this we conclude the description of the WEM interactions of leptons. The scope of the discussion has excluded the models of [16,17], which incorporate both a neutral boson and heavy leptons, without the term $\nu\gamma_\mu(1+\gamma_5)$ in the neutral current. We have also not considered models [23,24] involving a larger number (>4) of vector bosons.

4. DESCRIPTION OF THE WEAK AND ELECTROMAGNETIC INTERACTIONS OF HADRONS

a) We turn now to the problem of including hadrons in the foregoing schemes. Like the leptons, the hadrons must realize a representation of the symmetry group of the weak and electromagnetic interactions (SU(2) in the case of three vector fields, and $SU(2)_L \otimes U(1)$ in the case of four vector fields). Consequently, the full symmetry group of the strong interactions must include the abovementioned symmetry as a subgroup. To preserve the renormalizability, this subgroup of the symmetry need not be violated by the strong interactions, but may be violated only spontaneously by the electromagnetic and weak interactions. Since strangeness is not conserved in the weak interactions, this subgroup of the hadron symmetry is not the same as the isotopic group. The strong interactions must therefore possess a very high symmetry. The question of what structure it has is closely related to the problem of neutral currents.

This problem is significantly more acute for the hadrons than for the purely leptonic interactions. In particular, there are experimental indications that there are no strangeness-changing neutral currents. The strongest bound follows from the data [77] on the $K_L \to \mu^+\mu^-$ decay probability,

$$\frac{\Gamma(K_L\to\mu^+\mu^-)}{\Gamma(K^+\to\mu^+\nu)} = (4^{-3.6}_{-1.8})\cdot10^{-9}. \qquad (44)$$

However, in a renormalizable theory it is possible to have a neutral current with $\Delta S = 1$, while preserving the usual structure of the charged weak currents based on the SU(3) symmetry of the hadrons. In fact, by virtue of the SU(2) symmetry of the WEM interactions, the neutral current is related to the charged current by the equation

$$j_\mu^3(x) = \Big[\int d\mathbf{y}\, j_0^+(y),\ j_\mu^-(x)\Big]. \qquad (45)$$

The current j^3_μ must coincide with the electromagnetic current of the hadrons in a model with three vector fields, and with a linear combination of the electromagnetic current and the neutral weak current in models involving four vector fields. The usual SU(3) structure of the charged weak current (for clarity, we use the p, n and λ quarks)

$$j^+_\mu = \bar p\gamma_\mu(1+\gamma_5)(n\cos\theta+\lambda\sin\theta) \tag{46}$$

leads to

$$j^3_\mu = \bar p\gamma_\mu(1+\gamma_5)p - (\bar n\cos\theta+\bar\lambda\sin\theta)\gamma_\mu(1+\gamma_5)(n\cos\theta+\lambda\sin\theta), \tag{47}$$

so that the strangeness-changing neutral current is of the same order as the charged current.

In order to overcome this difficulty, we must enlarge the symmetry of the strong interactions [28], which corresponds to an increase in the number of quarks. This enables us to modify the structure of $j^\pm_\mu$ and hence that of j^3_μ. In the following subsections we shall discuss how this is done concretely.

Further conditions on the structure of the weak currents $j^\pm_\mu$ appear when one considers the higher-order contributions of perturbation theory. The point is that, owing to these contributions, the ratio (44) is generally of order $(\alpha/2\pi)^2 \sim 10^{-6}$, even in the absence of neutral currents with $|\Delta S| = 1$, which is a clear contradiction with experiment. The structure of $j^\pm_\mu$ must therefore ensure that an additional higher-order contribution is added.

We note that the structure of the weak currents must also be modified in nonrenormalizable theories [28] in order to suppress the higher-order contributions to the $K_L \to 2\mu$ decay amplitude. Otherwise, the limiting momentum below which the theory is applicable turns out to be very low [78].

b) In a model with four vector fields, the neutral current with $|\Delta S| = 1$ can be eliminated by introducing, for example, a supercharged fourth quark p′ [12-14]. The quarks are grouped according to the $SU(2)_L$ subgroup of the WEM interactions into the doublets

$$L_1 = \begin{pmatrix} p_L \\ \tilde n_L \end{pmatrix}, \quad L_2 = \begin{pmatrix} p'_L \\ \tilde\lambda_L \end{pmatrix} \tag{48}$$

and the singlets p_R, $\tilde n_R$, p'_R and $\tilde\lambda_R$; here

$$\tilde n = n\cos\theta + \lambda\sin\theta, \quad \tilde\lambda = -n\sin\theta + \lambda\cos\theta.$$

The electric charges of the p and p′ quarks are equal to each other and exceed the charge of the n and λ quarks by unity, but otherwise they may be chosen arbitrarily. The Lagrangian of the WEM interactions is constructed from the free Lagrangian in the same way as in the case of leptons (see (23)).

Owing to the introduction of the p′ quark, the interaction of fermions with vector bosons is invariant under the transformation $p \leftrightarrow p'$, $\tilde n \leftrightarrow \tilde\lambda$, i.e.,

$$p \leftrightarrow p', \quad n \leftrightarrow \lambda, \quad \theta \leftrightarrow -\theta.$$

Therefore the n and λ quarks appear in the neutral currents in the combination

$$\bar{\tilde n}\tilde n + \bar{\tilde\lambda}\tilde\lambda = \bar n n + \bar\lambda\lambda,$$

so that the transition with $|\Delta S| = 1$ does not occur.

What is required of the strong interactions in this model? To preserve the renormalizability of the theory, they must have the symmetry group $SU(4)_L \otimes SU(4)_R$. In fact, the ordinary isotopic symmetry of the strong interactions allows us to mix the p and n quarks, and the transformations of the group $SU(2)_L$ take p_L into $n_L\cos\theta + \lambda_L\sin\theta$ and p'_L into $-n_L\sin\theta + \lambda_L\cos\theta$. With a combination of these transformations, allowing for parity conservation, we may mix all four quarks.

To preserve the renormalizability, the $SU(4)_L \otimes SU(4)_R$ symmetry of the strong interactions must be violated only as a result of the interaction with a scalar field φ having a non-zero vacuum average. In particular, the quark masses appear as a result of an interaction of the following form:

$$\begin{aligned}-\frac{g\sqrt{2}}{\mu_W}\{&m_n[(\bar L_1\varphi)\cos\theta - (\bar L_2\varphi)\sin\theta](\tilde n_R\cos\theta - \tilde\lambda_R\sin\theta)\\ &+ m_\lambda[(\bar L_1\varphi)\sin\theta + (\bar L_2\varphi)\cos\theta](\tilde n_R\sin\theta + \tilde\lambda_R\cos\theta)\\ &+ m_p(\bar L_1\varphi^c)p_R + m_{p'}(\bar L_2\varphi^c)p'_R + \text{h.c.}\},\end{aligned} \tag{49}$$

where $\varphi^c = i\tau_2\varphi^*$. We recall that

$$\varphi_0 = \frac{\mu_W}{g\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}.$$

It should be noted that there exist as yet no experimental indications of SU(4) symmetry of the strong interactions. In particular, supercharged partners of the known hadrons are not observed, although the masses of these partners need not be very large, as is shown by considering the higher-order contributions to the $K_L \to 2\mu$ decay amplitude and to the K_L–K_S mass difference.

Let us consider this point in greater detail. As we have already mentioned, the higher-order contribution to the $K_L \to 2\mu$ decay amplitude must be further suppressed. To estimate this decay amplitude, let us consider the process $n\bar\lambda \to \mu^+\mu^-$ without allowance for the strong interactions. The suppression occurs because of a mutual cancellation of the contributions of the p and p′ quarks, which is clear from the example of the diagrams shown in Fig. 6.

The complete calculation [79], allowing also for diagrams involving the Z boson that are not shown in Fig. 6, yields the following expression for the matrix element for the process (under the assumption that $m_p \ll m_{p'} \ll \mu_W$):

$$M(n\bar\lambda \to \mu^+\mu^-) = -\frac{G^2 m^2_{p'}\cos\theta\sin\theta}{4\pi^2}\bar\lambda\gamma_\mu(1+\gamma_5)n\bar\mu\gamma_\mu\gamma_5\mu. \tag{50}$$

We retained here only the axial-vector current of the muons, since only this current contributes to the decay $K_L \to \mu^+\mu^-$. Comparing Eq. (50) with the amplitude for the allowed process $p\bar\lambda \to \mu^+\nu_\mu$

$$M(p\bar\lambda \to \mu^+\nu_\mu) = -\frac{G\sin\theta}{\sqrt{2}}\bar\lambda\gamma_\mu(1+\gamma_5)p\bar\nu\gamma_\mu(1+\gamma_5)\mu \tag{51}$$

and taking into account the fact that

$$\langle 0|\bar\lambda\gamma_\mu(1+\gamma_5)n|K^0\rangle = \langle 0|\bar\lambda\gamma_\mu(1+\gamma_5)p|K^+\rangle = f_K k_\mu, \tag{52}$$

we find

$$\frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K^+ \to \mu^+\nu_\mu)} = \frac{G^2 m^4_{p'}\cos^2\theta}{2\pi^4}. \tag{53}$$

As a result, it follows from (53) and (44) that

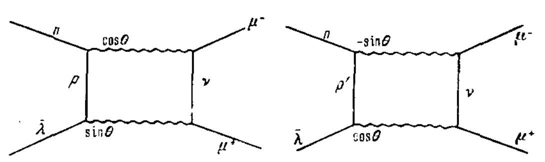

FIG. 6

$$m_{p'} \leqslant 10\,\text{GeV}. \qquad (54)$$

The free-quark approximation used for this estimate can be justified if it is assumed that the strong interactions are unimportant for momenta of the virtual W bosons $q \gg m_p$.

Transitions with $|\Delta S| = 2$ can be estimated in a similar way (by considering the process $n\bar{\lambda} \to \lambda\bar{n}$). In particular, such an estimate gives for the $K_L - K_S$ mass difference

$$m_L - m_S = \frac{2(m_{p'} - m_p)^2}{m_\mu^2}\,\Gamma(K^+ \to \mu^+\nu_\mu). \qquad (55)$$

This gives

$$m_{p'} - m_p \sim 1\,\text{GeV}. \qquad (56)$$

This estimate is less reliable than (54), since it does not allow for the contributions of the intermediate states $W^+ + W^-$ + hadrons.

Let us turn to the discussion of weak non-leptonic processes with $\Delta S = 0, \pm 1$. The danger is that the corresponding amplitudes in a renormalizable theory might be of order α/π, in sharp conflict with experiment. However, if the Feynman integral that defines the relevant amplitude M converges, it is easy to obtain for it one of the following estimates, depending on the rate of convergence:

$$M \sim \frac{\alpha}{\pi}\frac{E^2}{\mu_W^2} \sim \frac{GE^2}{4\pi^2} \quad \text{or} \quad M \sim \frac{GE^2}{4\pi^2}\ln\frac{\mu_W^2}{E^2};$$

here E is the energy of the process or a characteristic hadron mass, and it is assumed that $E \ll \mu_W$.

If the above-mentioned integral diverges, however, then the Lagrangian must contain a counter-term which cancels this infinity. Strictly speaking, the magnitude of the effect remains indeterminate in this case, although it is natural to assume that it is of order α/π. The possible types of counter-terms are determined by the symmetry of the theory. In particular, transitions with $|\Delta S| = 1$ are forbidden in the limit of zero quark masses in the scheme under consideration, since the order of magnitude of the counter-terms and hence of the effect is $\sim(\alpha/\pi)m^2/\mu_W^2 \sim Gm^2/4\pi^2$.

As to transitions with $\Delta S = 0$, they are distinguished from electromagnetic processes only by the effects of parity non-conservation. Whether these effects can be of order α/π (owing to the counter-terms) depends on the form of the strong interactions. In particular, in a model in which the strong interactions are mediated by a neutral vector field whose source is the baryon charge, there are no parity non-conservation effects of order $\sim\alpha/\pi$.

What is most crucial for this scheme is a comparison of its predictions with the experimental data on neutral currents with $\Delta S = 0$. As can be seen from the accompanying table (taken from the review [45]), this scheme is, at the present time, on the verge of being in conflict with experiment (but perhaps also on the verge of experimental confirmation).

c) If we interpret the experimental data as an indication that there are no weak neutral currents (not only with $|\Delta S| = 1$, but also with $\Delta S = 0$), it is natural to turn to models such as the Georgi-Glashow scheme, in which the only neutral current is the electromagnetic current.

	Experiment	Theory
$\frac{\sigma(\nu p \to \nu p)}{\sigma(\nu n \to \mu^- p)}$	0.12 ± 0.06 [80]	$0.15 \div 0.25$ [12]
$\frac{\sigma(\nu p \to \nu n\pi^+)}{\sigma(\nu p \to \mu^- p\pi^+)}$	0.08 ± 0.04 [80]	> 0.03 [81]; 0.11 [12]
$\frac{\sigma(\nu p \to \nu p\pi^0) + \sigma(\nu n \to \nu n\pi^0)}{2\sigma(\nu n \to \mu^- p\pi^0)}$	$\leqslant 0.14$ [82]	$\geqslant 0.6$ [83]; $\geqslant 0.4$ [84]; $\geqslant 0.19$ [81]

However, effective neutral currents appear in such models as a result of higher approximations [85,86] and are of the order $(G/\sqrt{2})(\alpha/\pi)$ in the amplitude. This is, of course, compatible with the experimental data for processes with $\Delta S = 0$. However, to suppress the $K_L \to 2\mu$ decay amplitude, as in the model with four vector fields, we must introduce additional quarks, so that the total number of quarks must be at least eight. By the same arguments as for Weinberg's model, the mass differences between the ordinary and supercharged quarks must be of the order of several GeV [73].

A relatively elegant scheme for the strong interactions that contains such an abundance of quarks is the $SU(3)' \otimes SU(3)''$ symmetric model of Han and Nambu [87]. This model is based on three triplets of quarks with integral charges. Such a scheme is employed for the construction of models of the WEM interactions of hadrons in [19,20,88,89]. However, the model of [19] does not ensure that the higher-order contribution to the $K_L \to 2\mu$ decay amplitude is suppressed.

We recall that the weak currents in schemes like the Georgi-Glashow model do not have the V-A structure. Consequently, such schemes generally lack the predictions for the $K \to 2\pi$ and 3π decay amplitudes (see the review [90]) which follow from partial conservation of the axial-vector current and the assumption that the weak-interaction hamiltonian has a V-A structure.

We note also that the exchange of scalar σ particles can be important in such models for non-leptonic decays. In particular, the assumption that this exchange dominates over the exchange of W bosons is used in [91] to explain the $\Delta T = 1/2$ rule. After a Fierz transformation, this mechanism would lead to a V-A structure for the interactions of ordinary particles if they were much lighter than the supercharged particles. However, it is difficult to reconcile this last assumption with the above-mentioned bounds on the mass differences.

To conclude this subsection, let us briefly consider the magnitude of the radiative corrections to the experimentally observed processes. It is of the greatest interest to calculate the renormalization of the ratio G_β^V/G_μ^V of the vector constants for β and μ decay, since this ratio is not renormalized by the strong interactions and is determined experimentally with good accuracy. We recall that the electromagnetic correction to G_β^V/G_μ^V diverges logarithmically in the local four-fermion theory (see, e.g., [92]). The introduction of an intermediate vector boson renders this quantity finite, even in the framework of the ordinary nonrenormalizable theory. In this case, the cut-off parameter is replaced by the mass of the W boson [92]. It can be shown [93] that the transition to renormalizable theories does not affect the terms $\sim(\alpha/\pi)\ln(\mu_W/m)$, which, as before, are given by the electromagnetic corrections. Allowance for a neutral vector boson leads to corrections $\sim\alpha/\pi$. But the contribution of a neutral scalar field is $\sim(\alpha/\pi)m_p^2/\mu_W^2$ and is negligibly small. (We have put $\mu_W \sim \mu_Z \sim m_\sigma \gg m_p$ in the foregoing estimates.) However, the uncertainty in the calculation due to the presence of strong interactions is of order α/π. It is therefore clear that, in going over to renormalizable theories, the situation regarding the calculation of radiative corrections to the ratio G_β^V/G_μ^V is actually the same as in the ordinary theory involving an intermediate vector boson.

d) In this approach to the problem of including the WEM interactions of hadrons, we do not, in essence, consider the strong interactions; it is assumed only that they have a very high symmetry.

Attempts to describe the strong interactions within the framework of renormalizable vector theories seem attractive. An example [25] is provided by the Lagrangian (12), in which the field $\mathbf{b}_\mu$ is identified with the ρ-meson field, while the field σ describes a neutral scalar meson. The interaction of the ρ mesons with the other hadrons can be introduced with a minimal coupling (see (23)) as an interaction with the isospin current.

A renormalizable strong-interaction scheme with broken $U(3)_L \otimes U(3)_R$ symmetry was considered in [94]. This scheme contains nonets of vector (ρ, K^*, ω, φ), pseudovector, scalar and pseudoscalar (π, K, η, η') mesons, as well as the nonets of scalar and pseudoscalar particles that are required for renormalizability (the analogues of the σ field).

We shall illustrate the inclusion of the electromagnetic and weak interactions in such an approach by means of the simple example [25] of the electrodynamics of π and ρ mesons. Let us construct a strong-interaction Lagrangian by adding to (8) a pion part:

$$\frac{1}{2}(\partial_\mu\boldsymbol{\pi}+2g\boldsymbol{\rho}_\mu\times\boldsymbol{\pi})(\partial_\mu\boldsymbol{\pi}+2g\boldsymbol{\rho}_\mu\times\boldsymbol{\pi})-\frac{1}{2}m_\pi^2\boldsymbol{\pi}^2.$$

We introduce the electromagnetic field a_μ as an isoscalar field interacting with the hypercharge, which is equal to zero for $\boldsymbol{\rho}_\mu$ and $\boldsymbol{\pi}$ and unity for φ. The hypercharge for the electron is $Y=-1$. Then

$$\begin{aligned}L = -\frac{1}{4}\boldsymbol{\rho}_{\mu\nu}\boldsymbol{\rho}_{\mu\nu}+(D_\mu\varphi)^+(D_\mu\varphi)+m^2\varphi^+\varphi-f^2(\varphi^+\varphi)^2\\ +\frac{1}{2}(\partial_\mu\boldsymbol{\pi}+2g\boldsymbol{\rho}_\mu\times\boldsymbol{\pi})(\partial_\mu\boldsymbol{\pi}+2g\boldsymbol{\rho}_\mu\times\boldsymbol{\pi})-\frac{1}{2}m_\pi^2\boldsymbol{\pi}^2\\ -\frac{1}{4}a_{\mu\nu}a_{\mu\nu}+\bar{e}\,[i\gamma_\mu(\partial_\mu-iea_\mu)-m_e]\,e,\end{aligned} \tag{57}$$

where

$$D_\mu\varphi = \partial_\mu\varphi - ig\,(\boldsymbol{\tau}\boldsymbol{\rho}_\mu)\,\varphi + iea_\mu\varphi.$$

The electromagnetic current of the hadrons in this model, j_μ^{el}, is given by

$$j_\mu^{el} = i\,[(D_\mu\varphi)^+\varphi-\varphi^+D_\mu\varphi]. \tag{58}$$

In the gauge in which

$$\boldsymbol{\psi}=0,\quad \varphi=\frac{1}{\sqrt{2}}\left(\frac{m_\rho}{g}+\sigma\right)\binom{0}{1}, \tag{58a}$$

we obtain

$$j_\mu^{el} = -\frac{m_\rho^2}{g}\left(\rho_\mu^3-\frac{e}{g}a_\mu\right)\left(1+\frac{g}{m_\rho}\sigma\right)^2. \tag{59}$$

Strictly speaking, it is not the fields ρ_μ^3 and a_μ, but linear combinations of them, that have definite masses. In this case, both linear combinations interact with the leptons. In writing the current in the form (59), it is assumed that allowance is made for the mixing of ρ_μ^3 and a_μ according to perturbation theory.

Although the expression (59) is very reminiscent of the electromagnetic current in the vector dominance model [95], the dependence of j_μ^{el} on the field σ leads to commutators which correspond more to the algebra of currents than to the algebra of fields, with all the resulting experimental consequences.

An interesting scheme which includes the strong and WEM interactions of hadrons in such an approach was proposed in [22]. The authors started from the abovementioned model of the strong interactions [94]. Neutral currents with $|\Delta S|=1$ are eliminated here at the cost of introducing additional scalar particles, and not by increasing the number of quarks.

5. CONCLUSIONS

Let us summarize the situation. The renormalizability of the models that we have discussed leads to small values for the radiative corrections in the amplitudes for weak processes. However, in order to satisfy the bounds on the effective neutral currents with $|\Delta S|=1$ and the transitions with $|\Delta S|=2$ that follow from the experimental data, we must introduce a special mechanism to suppress the corresponding amplitudes. These same processes have also been a basic stumbling block for nonrenormalizable theories, and it is in the framework of these theories that the foregoing mechanism was first proposed [28]. Thus, in this respect there is no advantage in going over to renormalizable theories. Nothing essentially new in comparison with the ordinary theory involving the W boson appears in calculating the radiative corrections to the known processes and, in particular, in calculating the renormalization of the ratio of the vector constants for β and μ decay.

From a general point of view, however, the discovery that it is possible to unify the weak and electromagnetic, and perhaps also the strong, interactions within the framework of a renormalizable theory seems extremely attractive, although it is difficult to call the concrete models elegant or economic. Nevertheless, it is by virtue of their ineconomy that these models lead to an appreciable number of experimental consequences.

Let us enumerate the experiments that are crucial for the models in question.

a) Searches for new particles. 1) A feature common to all the models is that they involve charged vector W bosons and a neutral scalar σ particle. Bounds on the W-boson masses are given by Eqs. (28), (30) and (43). In Weinberg's model there is another neutral vector boson with a mass that is bounded by the conditions (28) and (30).

2) The schemes of [15-17] predict the existence of both charged and neutral heavy leptons. In the Georgi-Glashow model the mass of the charged lepton is bounded by the condition (43). Its lifetime does not exceed 10^{-11} sec.

Stronger constraints on the mass of the heavy lepton may result if the magnetic moment of the muon is measured more accurately.

3) All known schemes that guarantee a small $K_L \to 2\mu$ decay amplitude (whether or not they are renormalizable) predict the existence of supercharged hadrons which decay purely as a result of the weak interactions. The masses of these particles apparently need not exceed several GeV (see (53) and (54)).

This bound does not hold in the model of [22], in which only scalar supercharged particles are required. The absence of a bound is connected with the fact that, in contrast with the other models, the masses of the non-supercharged partners of these hadrons are unknown here.

b) Searches for neutral weak currents.

1) $\nu_e e$ and $\bar{\nu}_e e$ scattering;

2) $\nu_\mu e$ and $\bar{\nu}_\mu e$ scattering;

3) elastic and inelastic neutrino-nucleon scattering: $\nu(\bar{\nu}) + N \to \nu(\bar{\nu}) + \text{hadrons}$.

The processes 2) and 3), as well as a deviation of the cross sections for the processes 1) from the predictions of the ordinary V-A theory, occur only in Weinberg's model. A comparison of the predictions of the model with the existing data (see Secs. 3b and 4b) indicates that it would be extremely crucial for this model to obtain experimental data that are several times more accurate.

In conclusion, we stress that, although the main virtue of renormalizable models of the WEM interactions is the possibility of correctly calculating the radiative corrections, the study of higher-order effects is not likely to enable us to discriminate between the various schemes. It seems more realistic to suppose that such a choice will be made on the basis of an experimental test of the predictions concerning new interactions and new particles.

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APPENDIX

We shall outline here a method of constructing the Feynman rules for Lagrangians of the Yang-Mills type in covariant gauges. These rules have been derived and rigorously established by a functional integration method [58-60,96]. We shall not give this derivation here, but confine ourselves to convincing, although non-rigorous, arguments formulated in the language of the ordinary Lagrangian formalism.

One of the covariant gauges for a massive vector field is the Proca gauge considered in Sec. 2b. However, it is inconvenient to use this gauge, since the corresponding propagator of a vector field does not fall off at large momenta, and the increasing divergences cancel only in the sum of the diagrams. We shall describe below the construction of the Feynman gauge, in which the propagator of a vector field has the form $-i\delta_{\mu\nu}(k^2-\mu^2)^{-1}$.

For definiteness, let us consider the Lagrangian (33), which describes interacting triplets of vector and scalar fields b_μ and φ:

$$L = -\frac{1}{4}\,\mathbf{b}_{\mu\nu}\mathbf{b}_{\mu\nu} + \frac{1}{2}(D_\mu\boldsymbol{\varphi})(D_\mu\boldsymbol{\varphi}) + \frac{m^2}{2}\boldsymbol{\varphi}^2 - \frac{f^2}{4}\boldsymbol{\varphi}^4; \qquad (A.1)$$

here

$$D_\mu\boldsymbol{\varphi} = \partial_\mu\boldsymbol{\varphi} - e\mathbf{b}_\mu \times \boldsymbol{\varphi}, \qquad (A.1a)$$

$$\boldsymbol{\varphi} = \left(\varphi^+,\ \frac{\mu}{e} + \sigma,\ \varphi^-\right)$$

and μ is the mass of the vector field, $\mu = (e/f)m$. In the quadratic approximation in the fields, the fields $b_\mu^\pm$ and $\varphi^\pm$ appear in the Lagrangian only in the combination

$$\tilde{b}_\mu^\pm = b_\mu^\pm \mp i\,\frac{\partial_\mu\varphi^\pm}{\mu}. \qquad (A.2)$$

The propagator of the field $\tilde{b}_\mu^\pm$ is therefore independent of the gauge. To determine its form, we make use of the Proca gauge, in which $\varphi^\pm = 0$ and the field $\tilde{b}_\mu^\pm$ coincides with $b_\mu^\pm$. It is then clear that

$$\tilde{D}_{\mu\nu}(k) = \overline{\tilde{b}_\mu^+\tilde{b}_\nu^-} = \int dx\,e^{ikx}\langle 0|T\tilde{b}_\mu^+(x)\tilde{b}_\nu^-(0)|0\rangle = i\,\frac{-\delta_{\mu\nu} + (k_\mu k_\nu/\mu^2)}{k^2-\mu^2}. \qquad (A.3)$$

We wish to transform to the Feynman gauge, in which the propagator of the field b_μ is given by

$$D_{\mu\nu}(k) = \overline{b_\mu^+ b_\nu^-} = \frac{-i\delta_{\mu\nu}}{k^2-\mu^2}. \qquad (A.4)$$

Since the Green's function of $\tilde{b}_\mu^\pm$ is given by Eq. (A.3), the fields $\varphi^\pm$ must be non-zero in this gauge. If we assume that $\varphi^\pm$ and $b_\mu^\pm$ are independent, so that

$$\overline{b_\mu^+\varphi^-} = 0, \qquad (A.5)$$

then it follows from (A.2)–(A.5) that

$$\overline{\varphi^+\varphi^-} = \frac{i}{k^2-\mu^2}. \qquad (A.6)$$

The Lagrangian which leads to the Green's functions (A.4)–(A.6) is obtained from (A.1) by adding to it ΔL:

$$\Delta L = -\frac{1}{2}(\partial_\mu b_\mu^3)^2 - (\partial_\mu b_\mu^+ + i\mu\varphi^+)(\partial_\mu b_\mu^- - i\mu\varphi^-) = -\frac{1}{2}(\partial_\mu\mathbf{b}_\mu - e\boldsymbol{\varphi}\times\boldsymbol{\varphi}_0)^2, \qquad (A.7)$$

where $\boldsymbol{\varphi}_0 = (0, \mu/e, 0)$. The term $-(1/2)(\partial_\mu b_\mu^3)^2$ guarantees the Feynman form for the propagator of the massless field b_μ^3:

$$\overline{b_\mu^3 b_\nu^3} = \frac{-i\delta_{\mu\nu}}{k^2}. \qquad (A.8)$$

If the fields

$$\boldsymbol{\zeta} = \partial_\mu\mathbf{b}_\mu - e\boldsymbol{\varphi}\times\boldsymbol{\varphi}_0 \qquad (A.9)$$

satisfied the free equations

$$(\Box-\mu^2)\zeta^\pm = 0, \qquad \Box\zeta^3 = 0, \qquad (A.10)$$

then the Lagrangian $L + \Delta L$ would be equivalent to L. This is the situation which occurs in electrodynamics and in the theory of a neutral vector field when the mass is included in the usual way.

Let us see what equations follow from $L + \Delta L$ for $\boldsymbol{\zeta}$. Variations in $\mathbf{b}_\mu$ and $\boldsymbol{\varphi}$ lead to

$$D_\mu\mathbf{b}_{\mu\nu} - e\boldsymbol{\varphi}\times D_\nu\boldsymbol{\varphi} + \partial_\nu\boldsymbol{\zeta} = 0, \qquad (A.11)$$

$$-D_\mu D_\mu\boldsymbol{\varphi} + m^2\boldsymbol{\varphi} - f^2\boldsymbol{\varphi}(\boldsymbol{\varphi}^2) + e\boldsymbol{\varphi}_0\times\boldsymbol{\zeta} = 0. \qquad (A.12)$$

By applying the operator D_ν (see (A.1a)) to Eq. (A.11) and making use of (A.12), we obtain

$$\partial_\nu D_\nu\boldsymbol{\zeta} - e^2\boldsymbol{\varphi}_0\times[\boldsymbol{\varphi}\times\boldsymbol{\zeta}] = 0. \qquad (A.13)$$

In deriving (A.13), it is useful to make use of the relation

$$(D_\mu D_\nu - D_\nu D_\mu)\ldots = -e\mathbf{b}_{\mu\nu}\times\ldots \qquad (A.14)$$

It follows from (A.13) that the fields $\boldsymbol{\zeta}$ interact with the fields $\mathbf{b}_\mu$ and $\boldsymbol{\varphi}$, so that the Lagrangian $L + \Delta L$ is not equivalent to the original one. In other words, the Lagrangian $L + \Delta L$ leads to the emission of pairs of unphysical particles and hence to a violation of the unitarity condition.

Correct results can be obtained in using $L + \Delta L$, provided that we subtract this unphysical contribution from the cross sections for the processes. To avoid carrying out this operation "by hand," we introduce in the theory fictitious scalar particles [58-60,97] described by the fields $\boldsymbol{\eta}$. Since the probability of emitting pairs of the fictitious $\boldsymbol{\eta}$ particles must be negative, the fields η must be assumed to be non-Hermitian and must be quantized according to Fermi statistics. Clearly, the field η_L must satisfy the same equation as the field ζ, Eq. (A.13). This equation is obviously given by the following Lagrangian:

$$L_\eta = \partial_\mu \eta^* D_\mu \eta - e^2 \varphi_0 \times [\eta^* \times [\varphi \times \eta]] = \partial_\mu \eta^* \partial_\mu \eta - \mu^2 (\eta_+^* \eta_- + \eta_-^* \eta_+) + e\mathbf{b}_\mu \times [\partial_\mu \eta^* \times \eta] - e\mu\sigma (\eta_+^* \eta_- + \eta_-^* \eta_+) + e\mu\eta_3 (\eta_+^* \varphi_- + \eta_-^* \varphi^+). \tag{A.15}$$

The full Lagrangian of the system in the Feynman gauge is given by $L + \Delta L + L_\eta$. It describes the fields $\mathbf{b}_\mu$, σ, $\varphi^\pm$, η and η^*, of which the physical degrees of freedom correspond to the four-dimensional transverse part of $b_\mu^\pm$, the three-dimensional transverse part of the massless field b_μ^3 and the field σ; the remaining fields are auxiliary fields.

The propagators of the fields are determined by Eqs. (A.4)–(A.6), (A.8) and

$$\overline{\sigma\sigma} = \frac{i}{k^2 - 2m^2}, \tag{A.16}$$

$$\overline{\eta_\pm^* \eta_\mp} = \frac{i}{k^2 - \mu^2}, \tag{A.17}$$

$$\overline{\eta_3^* \eta_3} = \frac{i}{k^2}. \tag{A.18}$$

The form of the vertices is found from the Lagrangian in the usual way. Some of the vertices were given earlier (see Eq. (3) in the main text). Since the field η is quantized as a fermion field, we must include a factor $(-1)^n$ in the amplitude, where n is the number of closed loops involving η particles.

We note that, instead of summing over the physical polarizations of the vector particles, we can use the formula

$$\sum_i \varepsilon_\mu^{(i)} \varepsilon_\nu^{(i)} = -\delta_{\mu\nu}. \tag{A.19}$$

In this case, however, we must add to the cross section the cross sections for the processes in which the vector quanta are replaced by the $\eta^\pm$ and η^3 particles, their antiparticles, and the $\varphi^\pm$ particles (for each quantum, we must take into account all the replacements that are allowed by electric charge). We stress that the η particle is produced in pairs, each such pair contributing a factor (–1). In addition, owing to the non-hermiticity of L_η, we must use the matrix element for the process in which the η particles are replaced by their corresponding antiparticles instead of taking the complex conjugate matrix element. These remarks must also be borne in mind when employing the unitarity condition.

The Lagrangian (A.1) considered above describes bosons in the Georgi-Glashow model. To describe the interaction of bosons with leptons in the Feynman gauge, we must add to $L + \Delta L + L_\eta$ the Lagrangian L_1 (see (37)), in which the fields $\mathbf{b}_\mu$, $\varphi^\pm$ and σ appear together with the leptons. As to the fields η, they do not interact directly with the leptons.

We now quote the expressions for ΔL and L_η which must be added to the Lagrangian of Weinberg's model to transform to the Feynman gauge. We recall that this Lagrangian is given by Eq. (20) (taking into account (11) and (21)). It depends on the fields $W_\mu^\pm$, Z_μ, A_μ, σ and ϕ:

$$\Delta L = -(\partial_\mu W_\mu^+ + \mu_W \psi^+)(\partial_\mu W_\mu^- + \mu_W \psi^-) - \frac{1}{2}(\partial_\mu Z_\mu + \mu_Z \psi^3)^2 - \frac{1}{2}(\partial_\mu A_\mu)^2,$$

$$L_\eta = \partial_\mu \eta_+^* \partial_\mu \eta_- + \partial_\mu \eta_-^* \partial_\mu \eta_+ - \mu_W^2 (\eta_+^* \eta_- - \eta_-^* \eta_+) + \partial_\mu \eta_Z^* \partial_\mu \eta_Z - \mu_Z^2 \eta_Z^* \eta_Z + \partial_\mu \eta_A^* \partial_\mu \eta_A - g\mu_W \sigma (\eta_+^* \eta_- + \eta_-^* \eta_+) - \sqrt{g^2 + g'^2}\, \mu_Z \sigma \eta_Z^* \eta_Z \tag{A.20}$$

$$+ \frac{ig'\mu_W}{\sqrt{g^2 + g'^2}} [(g'\eta_Z + g\eta_A)(\psi_- \eta_+^* - \psi_+ \eta_-^*) - \text{h.c.}] - 2g\mathbf{b}_\mu [\partial_\mu \eta^* \times \eta] + 2g\mu_W \boldsymbol{\psi} [\eta^* \times \eta]; \tag{A.21}$$

for brevity, we have introduced here the isovectors

$$\mathbf{b}_\mu = \left(W_\mu^+, \frac{gZ_\mu - g'A_\mu}{\sqrt{g^2 + g'^2}}, W_\mu^-\right), \qquad \eta = \left(\eta_+, \frac{g\eta_Z - g'\eta_A}{\sqrt{g^2 + g'^2}}, \eta_-\right). \tag{A.22}$$

The expressions for the propagators and vertices can be derived from the Lagrangian in the usual way.

A more detailed description of the Feynman rules in various models and gauges can be found in [72].

Note added in proof. There have recently appeared experimental data on the neutrino-nucleon interaction which indicate that neutral currents exist (see [98]).

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1) An expanded text of a review presented at the scientific session of the Nuclear Physics Division, USSR Academy of Sciences (October 1972).

2) We note that one often uses a definition of the constant g which is obtained from ours by the substitution $2g \to g$.

3) The reader may omit this subsection without loss of understanding of the following material.

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