Magnetophonon resonance in semiconductors

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Usp. Fiz. Nauk **112**, 3–36 (January 1974)

An account is given of the theory of magnetophonon resonance—an effect arising from the inelastic character of the scattering of electrons by optical phonons in strong magnetic fields. The electron-phonon collision frequency depends nonmonotonically on the magnetic field: It increases sharply when the optical-phonon energy $\hbar\omega_0$ becomes a multiple of the cyclotron energy $\hbar\Omega$. Therefore, all kinetic coefficients of a dissipative nature should have an oscillatory dependence on the magnetic field. The experimental study of the magnetophonon oscillations makes it possible to determine the cyclotron mass of the electrons, and also its dependence on temperature, pressure, etc. Thanks to development in technique for high magnetic fields, magnetophonon resonance has become both one of the most important methods for determining the band-structure parameters of semiconductors and a means for studying the interaction of electrons with optical phonons.

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INTRODUCTION

Strong magnetic fields are a very important tool for investigating the electron system of solids, and, in particular, the energy spectrum of the electrons. By strong magnetic fields, we mean fields satisfying the condition

$$\Omega\tau \gg 1, \tag{1}$$

where $\Omega = eH/mc$ is the cyclotron frequency and τ is the relaxation time.

When the condition (1) is fulfilled, effects associated with scattering are suppressed and the characteristic features of the electron dynamics are most clearly manifested. This opens up the possibility of studying the structure of the equal-energy surfaces of the electrons by studying different kinetic and resonance phenomena in a strong magnetic field (magnetoresistance, cyclotron resonance, ultrasonic absorption, etc.). In recent years, the effective masses of electrons in semiconductors and the shapes of the Fermi surfaces of many metals and semimetals have been determined in precisely this way.

The quantum-mechanical problem of the motion of an electron in a magnetic field $H = (0, 0, H)$ in the case of a quadratic isotropic dispersion law was first solved by Landau[1]. With the choice of gauge in which the vector potential is equal to $A = (0, Hx, 0)$, the eigenvalues ϵ_ν of the Hamiltonian $\mathscr{H}_0$ and the wavefunctions ψ_ν have the form (neglecting the spin)

$$\hat{\mathscr{H}}_0 = \frac{1}{2m}(\hat{p}_x^2 + \hat{p}_z^2) + \frac{1}{2m}(\hat{p}_y + m\Omega x)^2, \tag{2}$$

$$\varepsilon_\nu = \left(N + \frac{1}{2}\right)\hbar\Omega + \frac{\hbar^2 k_z^2}{2m} \quad (N = 0, 1, 2, \ldots), \tag{3}$$

$$\psi_\nu = \exp(ik_y y + ik_z z)\,\varphi_N\left(\frac{x - X_\nu}{L}\right); \tag{3a}$$

here, $\nu = (N, k_y, k_z)$ is a complete set of quantum numbers; $\hbar k_y = p_y$ and $hk_z = p_z$ are the components of the quasi-momentum of the electron along the y and z axes; $X_\nu = -p_y/m\Omega$ is the x-coordinate of the center of the Larmor orbit; $L = (ch/eH)^{1/2}$ is the magnetic length; φ_N is a normalized oscillator wavefunction. From the form of the Hamiltonian (2), it can be seen that the energy of the electron in the magnetic field is quantized and the spectrum is a set of N Landau sub-bands. Quantization of the electron energy occurs not only for the simple dispersion law considered, but also in the more general case when the electrons undergo a finite motion over a closed trajectory in a plane perpendicular to the magnetic field. Since, according to (3), the electron energy depends only on the two quantum numbers N and k_z, each Landau sub-band is degenerate, the degeneracy being proportional to the magnetic field. The magnetic field, as it were, collects states distributed uniformly over the band into discrete sub-bands. As a consequence of this, the density of states is also changed substantially:

$$g(\varepsilon) = \left(\frac{2m}{\hbar^2}\right)^{1/2}\frac{1}{2\pi L^2}\sum_N(\varepsilon - \varepsilon_N)^{-1/2}, \quad \varepsilon_N = \left(N + \frac{1}{2}\right)\hbar\Omega. \tag{4}$$

The density of states becomes infinite at the bottom of each Landau sub-band, i.e., for $p_z = 0$. For brevity, energy levels with $p_z = 0$ will be called, simply, Landau levels. The condition (1), which can be rewritten in the form

$$\hbar\Omega \gg \frac{\hbar}{\tau}, \quad (1a)$$

means that in a strong magnetic field the spacing between Landau levels is much greater than the broadening of the levels that arises from collisions. It is obvious that the fulfillment of this condition is necessary for the observation of all the effects associated with the quantization of the electron spectrum. Allowance for the smearing of the Landau levels leads to smoothing of the singularities in the density of states $g(\epsilon)$[2].

The nonmonotonic dependence of $g(\epsilon)$ is the principal reason for the oscillations of the various thermodynamic quantities (de Haas-van Alphen oscillations) and kinetic coefficients (Shubnikov-de Haas oscillations) which have been observed in metals and semiconductors with a degenerate electron gas at low temperatures. A large number of experimental and theoretical papers (see the review[3]) have been devoted to quantum oscillations in semiconductors.

No less important is another consequence of the quantization of the electron energy—the change in the character of the scattering. Investigation of the field and temperature dependences of the magnetoresistance in the quantum limit $\hbar\Omega > \bar{\epsilon}$ ($\bar{\epsilon}$ is the characteristic electron energy, equal to k_0T in the case of classical statistics, or the Fermi energy ζ in the case of degeneracy) makes it possible to obtain information on the dominant scattering mechanisms[4]. The influence of strong magnetic fields on the character of the elastic scattering is also demonstrated by the negative longitudinal magnetoresistance effect[5,6]. Thus, experimental investigations in strong magnetic fields not only turn out to be fruitful in the study of the electron spectrum, but also give information about the interaction of the electrons with phonons, impurities, etc. One of the effects due to the electron-phonon interaction is the new type of oscillations of the kinetic coefficients first predicted by Gurevich and Firsov[7] and Klinger[8]. This effect has been given the name of "magnetophonon resonance" (MPR), since it is due to inelastic resonance scattering of electrons by phonons of a definite frequency—in particular, by optical phonons, whose dispersion can be neglected. The average scattering probability increases when the optical-phonon energy $h\omega_0$ coincides with the spacing between any two Landau levels, and this leads to a nonmonotonic dependence of the kinetic coefficients on the magnetic field.

Magnetophonon resonance is the first example known to science of an internal resonance in a solid, i.e., of a resonance in which internal vibrations of the solid, e.g., optical phonons, are the perturbing agent. MPR differs in this way from external resonances (cyclotron, paramagnetic, etc.) in which the perturbing agent is an external oscillating electromagnetic field.

The first brief communication on the observation of the new resonance effect in n-InSb is due to Prui and Geballe[9]. A detailed experimental study of the features of manifestations of MPR in the transverse and longitudinal magnetoresistance was carried out in[10,11], again on n-type indium antimonide, which has been found to be the (MP) oscillations. Because of the large mobility of the electrons, the criterion (1), or the equivalent criterion $uH/c \gg 1$, is well fulfilled even in magnetic fields $\sim 10^4$ Oe. At the present time, because of developments in technique for stationary and pulsed magnetic fields, the range of fields $\sim 10^5 - 10^6$ Oe is fully accessible, and extensive experimental data on MPR in different semiconductors have been obtained. It may be said that MPR is becoming one of the most important methods for studying both the energy spectrum of the electrons, and their interaction with the phonon system.

MAGNETOPHONON OSCILLATIONS OF THE TRANSVERSE MAGNETO-RESISTANCE

a) Conductivity in a transverse magnetic field. In the experimental study of galvanomagnetic effects, one usually measures components of the magnetoresistance tensor $\rho_{ik}(H)$. In the theory, however, it is more convenient to calculate the conductivity tensor $\sigma_{ik}(H)$ or the current density **j**:

$$j_i = \sigma_{ik}(H) E_k; \quad (5)$$

E is the electric-field intensity; i, k = x, y, z.

We choose the z axis to be along the direction of the magnetic field. Then, in the case of an isotropic electron dispersion law, the tensor $\sigma_{ik}(H)$ has the form

$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ -\sigma_{xy} & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \sigma_{xx} = \sigma_{yy}. \quad (6)$$

The tensor σ has the same form, irrespective of the electron dispersion law, in the more general case when the z axis in a cubic crystal is a three-, four-, or six-fold symmetry axis. The non-zero components of the magnetoresistance tensor ρ_{ik} are related to the components of the tensor σ_{ik} in the following way:

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} = \rho_{yy}, \quad \rho_{xy} = -\frac{\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2}, \quad \rho_{zz} = \frac{1}{\sigma_{zz}}. \quad (7)$$

If the electron isoenergetic surface does not contain open orbits, then, in magnetic fields satisfying the condition (1), the components σ_{ik} have, as is well known, the following asymptotic forms[1]:

$$\sigma_{xx} \sim \sigma_0 (\Omega\tau)^{-2}, \quad \sigma_{xy} \sim \sigma_0 (\Omega\tau)^{-1}, \quad \sigma_{zz} \sim \sigma_0, \quad (8)$$

where σ_0 is the conductivity when H = 0. Thus, the ratio $\sigma_{xx}/\sigma_{xy} \sim (\Omega\tau)^{-1} \ll 1$ is a small parameter, and in the lowest approximation in this parameter we obtain from (7)

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xy}^2}, \quad \rho_{xy} = -\frac{1}{\sigma_{xy}}. \quad (7a)$$

It is important that, for an arbitrary dispersion law in this approximation, the Hall conductivity σ_{xy} does not depend on the scattering:

$$\sigma_{xy} = \frac{enc}{H}; \quad (9)$$

n is the electron concentration. At the same time, the conductivity σ_{xx} is proportional to, and σ_{zz} inversely proportional to the scattering probability. This is connected with the different character of the motion of the electron parallel and perpendicular to the magnetic field. This is most simply understood in the case of a quadratic isotropic dispersion law, when the Hamiltonian has the form (2). The matrix elements of the velocity operator $\hat{\mathbf{v}}$ are equal to

$$v^x_{NN'} = i\sqrt{\frac{\hbar\Omega}{2m}} \left(\sqrt{N}\,\delta_{N',N-1} - \sqrt{N+1}\,\delta_{N',N+1}\right) \delta_{k'_y k_y} \delta_{k'_z k_z},$$
$$v^y_{NN'} = \sqrt{\frac{\hbar\Omega}{2m}} \left(\sqrt{N}\,\delta_{N',N-1} - \sqrt{N+1}\,\delta_{N',N+1}\right) \delta_{k'_y k_y} \delta_{k'_z k_z}, \quad (10a)$$

$$v^z_{NN'} = \frac{\hbar k_z}{m}\delta_{NN'}\delta_{k'_y k_y}\delta_{k'_z k_z}. \qquad (10b)$$

Therefore, to calculate the macroscopic average values of the current density j_x along the electric field $\mathbf{E} = (E, 0, 0)$ and of the Hall-current density j_y, knowledge of the off-diagonal elements of the density matrix $\rho_{\mu\nu}$ is necessary:

$$j_\alpha = \mathrm{Sp}(\hat\rho, \hat{j_\alpha}) = \sum_{\mu,\nu} \rho_{\mu\nu}(j_\alpha)_{\nu\mu} \qquad (\alpha = x, y). \qquad (11)$$

Thus, the problem of calculating the transverse σ_{xx} and Hall σ_{xy} components of the conductivity tensor reduces to finding the density-matrix operator from its equation of motion:

$$i\hbar\frac{\partial\hat\rho}{\partial t} = [\hat{\mathscr{H}}, \hat\rho]. \qquad (12)$$

In this equation, $\hat{\mathscr{H}}$ is the total Hamiltonian, including the electric field and the interaction of the electron with scatterers. This problem was solved in a paper by Adams and Holstein[4] in the lowest approximation in the parameter $(\Omega\tau)^{-1}$ for elastic scattering. By an analogous method, Argyres and Roth[12] obtained a formula for σ_{xx} in the case of inelastic scattering.

In the zeroth approximation in the scattering potential,

$$j_y = \frac{enc}{H}E, \qquad (13)$$

which agrees with the classical expression (9) for σ_{xy}. The Hall current has a nondissipative character and is a consequence of the electron drift, with velocity $v_{dr} = cE/H$, in the crossed electric and magnetic fields. In this approximation, the current along the electric field, $j_x = 0$. Because an electron interacts with crystal imperfections, transitions of the electron between different states, with change of the quantum number X_ν, become possible. The number of transitions per unit time against the electric field turns out to be greater than in the direction of the field, and this leads to an electric current j_x proportional to the transition probability. Allowance for scattering in the Born approximation leads to the following expression for the transverse conductivity:

$$\sigma_{xx} = \frac{e^2}{k_0 T}\sum_{\mu,\nu} f^0_\mu (1 - f^0_\nu)\, w_{\mu\nu}\frac{(X_\mu - X_\nu)^2}{2}, \qquad (14)$$

where $f^0_\mu = f_0(\epsilon_\mu)$ is the equilibrium electron distribution function and $w_{\mu\nu}$ is the probability of transition of an electron from the state μ to the state ν. For the electron-phonon interaction,

$$w_{\mu\nu} = \frac{2\pi}{\hbar}\sum_{\mathbf{q}} |C_q|^2 |\langle\mu| e^{i\mathbf{qr}} |\nu\rangle|^2 [(N_q + 1)\,\delta(\varepsilon_\mu - \varepsilon_\nu - \hbar\omega_q) + N_q\delta(\varepsilon_\mu - \varepsilon_\nu + \hbar\omega_q)], \qquad (15)$$

where C_q is the Fourier transform of the interaction potential, $N_q = [\exp(\hbar\omega_q/k_0 T) - 1]^{-1}$ is the Planck function, and

$$\langle\mu| e^{i\mathbf{qr}} |\nu\rangle = I_{NN'}\delta(k'_y - k_y - q_y)\,\delta(k'_z - k_z - q_z),$$
$$I_{NN'} = \int \varphi_{N'} e^{iq_x x}\varphi_N\, dx. \qquad (16)$$

Going over to integration over the energy, we can represent the expression (14) in the following form:

$$\sigma_{xx} = \frac{e^2 N_0}{(2\pi)^4\hbar^2\Omega k_0 T}\int d\varepsilon \sum_{N,N'} G_{NN'}(\varepsilon)\frac{f_0(\varepsilon)[1 - f_0(\varepsilon + \hbar\omega_0)]}{\sqrt{\varepsilon - \varepsilon_N}\sqrt{\varepsilon - \varepsilon_{N'} + \hbar\omega_0}}; \qquad (17)$$

here, $G_{NN'}(\epsilon)$ is a certain smooth function of the energy and of the quantum numbers N and N':

$$G_{NN'}(\varepsilon) = \int dq_x\, dq_y\, |C_q|^2 q_y^2 |I_{NN'}|^2. \qquad (18)$$

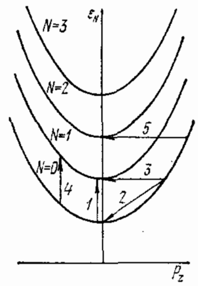


FIG. 1. Scheme of electrons between Landau sub-bands.

The dispersion of the optical phonons can be neglected, and therefore $\hbar\omega_0$ and N_0 do not depend on the wave vector $\mathbf{q}$.

The integrand in formula (17) has singularities at the points $\epsilon = \epsilon_N$ and $\epsilon = \epsilon_{N'} - \hbar\omega_0$, which reflect singularities in the densities of initial and final electron states. Each of these singularities is of the integrable type, and consequently, for an arbitrary value of the magnetic field, the integral over the energy, generally speaking, has no singularities. An exception is constituted by those values of H for which the singularities of the initial and final states coincide, i.e.,

$$M\Omega = \omega_0 \qquad (M = 1, 2, 3, \ldots). \qquad (19)$$

In this case, the integral over the energy diverges logarithmically and, consequently, the transverse magnetoresistance increases sharply.

Similar behavior of $\rho_{xx}(H)$ near the resonance values of H determined by formula (19) leads to oscillations that are periodic in the inverse magnetic field, with period

$$\Delta\left(\frac{1}{H}\right) = \frac{e}{m\omega_0 c}. \qquad (20)$$

The period of the oscillations depends on the effective electron mass m and on the limiting frequency ω_0 of the optical phonons. These oscillations are called magnetophonon (MP) oscillations. Physically, the MP oscillations are due to the discontinuous character of the density of states (4) in its energy dependence. The oscillation maxima of $\rho_{xx}(H)$ should occur when electron transitions between two Landau levels with absorption or emission of an optical phonon $\hbar\omega_0$ are possible.

For the first resonance (M = 1) the possible transitions are shown in Fig. 1 by the arrow 1. In general, as follows from an analysis of expression (17), for the oscillatory effect to appear a nonmonotonic variation of some of the quantities characterizing both the initial and final states is necessary. Therefore, transitions of, e.g., the type 2 in Fig. 1 make no contribution to the oscillating part of $\sigma_{xx}(H)$, since for these only the density of final states has a singularity. Such transitions, and also transitions of the type 4, produce the nonoscillating "background" of the function $\sigma_{xx}(H)$. In addition, horizontal transitions of the types 3 and 5, associated with elastic scattering by acoustic phonons and impurities, make a contribution to the nonoscillating background of the magnetoresistance[2)].

Since, in the lowest approximation in the interaction, the transverse conductivity σ_{xx} is proportional to the scattering probability, the different scattering mechanisms do not interfere, i.e., they make an additive contribution to σ_{xx}:

$$\sigma_{xx} = \sigma^{\mathrm{opt}}_{xx} + \sigma^{\mathrm{ela}}_{xx}, \qquad (21)$$

where σ_{xx}^{opt} is the nonmonotonic part (17) due to scattering of electrons by optical phonons, and σ_{xx}^{ela} is a certain smooth function of the magnetic field, associated with the elastic scattering. The general formula (14) is valid for σ_{xx}^{ela}, if we substitute into it the appropriate expression for the transition probability $w_{\mu\nu}$. In the quantum limit $\hbar\Omega > \zeta$ or $\hbar\Omega \gg k_0T$, the transverse conductivity σ_{xx}^{ela} is a monotonic function of the magnetic field[4].

b) Characteristic features of the MP oscillations. We shall consider those characteristic features of MP oscillations which distinguish them from Shubnikov-de Haas (SH) and de Haas-van Alphen quantum oscillations. First of all, we note that MP resonance is associated with electron scattering and therefore cannot be manifested in thermodynamic-equilibrium effects, i.e., effects of a nondissipative nature. As follows from (20), unlike the period of SH oscillations the magnitude of the period $\Delta(1/H)$ does not depend on the electron concentration. The simple dependence of $\Delta(1/H)$ on the effective mass m and on the limiting frequency ω_0 makes it possible to determine one of these quantities experimentally if the other is known.

It follows from formula (17) that the nonmonotonic dependence of $\sigma_{xx}(H)$ is not connected with the form of the distribution function $f_0(\epsilon)$. This means that MPR should occur irrespective of the statistics of the electrons. The case of Fermi statistics was investigated by Éfros[13], who showed that the MP oscillations remain present in the quantum limit $\hbar\Omega > \zeta$, where SH oscillations are impossible.

The amplitude of the MP oscillations of ρ_{xx} also has a distinctive temperature dependence. At low temperatures $T \ll \theta_0$ ($\theta_0 = \hbar\omega_0/k_0$ is the characteristic excitation temperature of the optical phonons), the quantity σ_{xx}^{opt} (17) is proportional to $\exp(-\theta_0/T)$ and the main contribution to the conductivity is made by the elastic scattering processes. An increase in temperature leads to an increase in the role of scattering of electrons by optical phonons, as a result of which the amplitude of the MP oscillations also increases. However, at temperature comparable with the temperature θ_0, thermal broadening of the Landau levels, i.e., thermal spread of the electrons over the Landau sub-band, becomes important. In this case, in the region of magnetic fields $\Omega \lesssim \omega_0$ in which MP oscillations are possible, the average thermal electron energy $k_0T \gtrsim \hbar\Omega$. As shown in[7], for $\hbar\Omega < k_0T$ the oscillating part of σ_{xx}^{opt} is a small correction of order $\hbar\Omega/k_0T$ to the classical value of σ_{xx}^{opt} in the absence of a magnetic field. Thus, the temperature dependence of the amplitude of the oscillations is nonmonotonic. There exists a certain optimal temperature at which the amplitude of the MP oscillations is a maximum. This temperature, which is less than θ_0, depends on the contribution of the elastic scattering processes to the total conductivity. It should be emphasized that the amplitude of SH oscillations increases as the temperature is lowered, down to those temperatures at which the broadening of the Landau levels as a result of collisions begins to play the principal role in "killing off" the oscillation peaks.

The logarithmic increase of $\sigma_{xx}(H)$ near the resonance values of H does not depend on the form of the function $G_{NN'}(\epsilon)$ (18) and is due only to the specific behavior of the density of states. As is clear from (18), the form of the function $G_{NN'}(\epsilon)$ is determined by the character of the dependence of C_q on the phonon wave vector q. In turn, the dependence of C_q on q reflects the character of the interaction of the electrons with the optical phonons. Thus, in crystals with ionic bonding, in which electrons interact with polarization vibrations,

$$|C_q|^2 = \frac{A}{q^2}, \quad A = 2\pi\hbar\omega_0 e^2\left(\frac{1}{\varkappa_\infty} - \frac{1}{\varkappa_0}\right); \tag{22}$$

κ_0 and κ_∞ are the static and dynamic dielectric constants. In crystals with covalent bonding, in which the interaction of the electrons with the optical phonons has a nonpolar character, C_q is a constant, independent of the wave vector q:

$$C_q = C_0. \tag{23}$$

The explicit dependence $\sigma_{xx}(H)$ near the resonances for the case of interaction of the electrons with polarization phonons was obtained in[7] (nondegenerate electron gas) and in[13] (degenerate gas). We introduce the quantity δ, which serves as a measure of the deviation from resonance:

$$\delta = \left|M - \frac{\omega_0}{\Omega}\right| \quad (M = 1, 2, \ldots). \tag{24}$$

Then, for $\delta\hbar\Omega/k_0T \ll 1$ in the case of Boltzmann statistics,

$$\sigma_{xx} = \sigma_{xx}^B\left[1 + \frac{3}{4}\sqrt{\frac{\Omega}{\omega_0}}\sqrt{\frac{\hbar\Omega}{k_0T}}\ln\left(\frac{2k_0T}{\delta\hbar\Omega}\right)\right], \tag{25}$$

where σ_{xx}^B is the monotonic part of the conductivity:

$$\sigma_{xx}^B = \frac{4\alpha ne^2}{3m}\left(\frac{\omega_0}{\Omega}\right)^2\frac{\hbar}{k_0T}\exp\left(-\frac{\hbar\omega_0}{k_0T}\right). \tag{25a}$$

In the case of Fermi statistics,

$$\sigma_{xx} = \sigma_{xx}^F\left(1 + \frac{3}{8}\frac{\hbar\Omega}{\zeta}\ln\frac{1}{\delta}\right), \tag{26}$$

$$\sigma_{xx}^F = \frac{4\alpha e^2}{3\pi^2\hbar}\frac{\zeta}{k_0T}\left(\frac{\omega_0}{\Omega}\right)^2\sqrt{\frac{2m\omega_0}{\hbar}}\exp\left(-\frac{\hbar\omega_0}{k_0T}\right). \tag{26a}$$

In these formulas, α is the dimensionless electron-optical phonon coupling constant:

$$\alpha = \frac{e^2}{\hbar}\sqrt{\frac{m}{2\hbar\omega_0}}\left(\frac{1}{\varkappa_\infty} - \frac{1}{\varkappa_0}\right). \tag{27}$$

c) Amplitudes of the MP oscillations of σ_{xx}. The expressions (25) and (26) lead to infinite values of the oscillation maxima. However, for sufficiently small δ the conductivity σ_{xx} ceases to increase proportionally to $\ln(1/\delta)$. From a physical point of view, it is obvious that there always exists some suppression mechanism limiting the height of the oscillation peak. We introduce the quantity δ_0 (the minimum value of δ) by defining it in such a way that $\ln(1/\delta)$ in (25) determines the maximum value of σ_{xx} at resonance, calculated with allowance for the suppression. In practice, this approach, which does not take into account the MP resonance line shape, can give only a rather crude estimate of the amplitude of the oscillations. Among the possible mechanisms that remove the divergence in (17), we note the following[14]:

1) Broadening of the Landau levels as a result of elastic collisions, which leads to smoothing of the singularities in the density of states. In this case,

$$\delta_0 \approx (\Omega\tau)^{-1}. \tag{28}$$

2) Broadening due to electron-electron interaction. The role of the Coulomb interaction of the electrons appears in the renormalization of the electron-phonon scattering potential. As shown in[14], the screening of the electron-phonon interaction also limits the amplitude

of the resonance peak, and the corresponding value of δ_0 is equal to

$$\delta_0=\frac{3\sqrt{\pi^3e_0^C}}{e_0^2}\frac{ne^2}{\varkappa_0 m\omega_0^2}, \qquad (29)$$

where e_0 is the base of the natural logarithms, and $C = 0.577$ is Euler's constant.

3) The dispersion of the optical phonons. Let the optical-phonon frequency be

$$\omega_q=\omega_0(1-a^2q^2),$$

where a is a quantity of the order of the lattice constant. Then for δ_0 we can obtain

$$\delta_0=e_0\left(\frac{a}{L}\right)^2. \qquad (30)$$

A comparison of the quantities δ_0 given by formulas (28)–(30) shows that the phonon dispersion can practically never play an important role. Collisional broadening of the Landau levels is the decisive factor in conditions when elastic scattering is dominant. The role of the Coulomb interaction can become noticeable only at sufficiently high electron concentrations.

For a correct calculation of the line shape and amplitude of the MP resonance it is necessary to take more consistent account of the interaction of the electrons with the optical phonons, and not confine ourselves to lowest order of perturbation theory. Sufficiently strong interaction of an electron with polarization vibrations leads to the formation of a polaron. For $\alpha \ll 1$, the energy of the polaron is equal to[15]

$$\varepsilon=\frac{\hbar^2k^2}{2m}\left(1-\frac{\alpha}{6}\right)-\alpha\hbar\omega_0. \qquad (31)$$

It can be seen from (31) that the polaron effect leads to an increase of the effective electron mass and to a lowering of the energy by an amount $\alpha h\omega_0$. In weak magnetic fields, $\Omega \ll \omega_0$, exact allowance for the electron-phonon interaction also leads only to a renormalization of the cyclotron mass[16]. The situation is radically changed in the region of magnetic fields $\Omega \approx \omega_0$[17,18]. Because of the sharp amplification of the electron-phonon interaction in magnetic fields satisfying the MP resonance condition (19), a rearrangement of the energy spectrum of the electron-phonon system occurs. In particular, for $\Omega = \omega_0$, the energy of the phononless state in which an electron is situated in the first Landau level ($N = 1$) differs from the energy of the state in which the electron belongs to the zeroth Landau level and in which there is one optical phonon with energy $\hbar\omega_0$. The energy level of the phononless state is found to be strongly broadened as a result of resonance emission of optical phonons. These features of the energy spectrum are manifested in the study of the cyclotron resonance[19], interband magneto-absorption[18] and impurity combined resonance in InSb[20].

Broadening of the Landau level $N = 1$ also leads to the suppression of the MP maximum in the transverse conductivity. With allowance for the broadening, Dworin[21] calculated the quantity σ_{xx} at the point $\Omega = \omega_0$. Rigorous allowance for the interaction of the electrons with the optical phonons gives a finite value for the conductivity at the resonance point:

$$\sigma_{xx}=\frac{\alpha ne^2\hbar}{mk_0T}\exp\left(-\frac{\hbar\omega_0}{k_0T}\right)\left[1+\frac{2}{\sqrt{\pi}}\left(\frac{\hbar\omega_0}{k_0T}\right)^{1/2}F\left(\alpha^{2/3}\frac{\hbar\omega_0}{k_0T}\right)\right], \qquad (32)$$

where F is a certain monotonic function, calculated in[21]. For $\alpha^{2/3}\hbar\omega_0/k_0T \lesssim 1$, this function is close to unity. The appearance in the argument of F of the coupling constant α to the power 2/3 reflects the fact that the expression (32) cannot be obtained in the framework of perturbation theory. Using the method of Green functions, Nakayama[22] calculated the spectrum and density of states of the electrons under conditions of resonance interaction with the optical phonons. A calculation of the transverse conductivity in the region of magnetic fields $\Omega \approx \omega_0$ shows that, together with the suppression of the MPR peak, there occurs a small shift in the positions of the MP maxima in the direction of higher fields relative to the resonance values (the polaron shift). This shift can be interpreted as a renormalization of the electron mass as a consequence of the resonance electron-phonon interaction. The renormalized mass for $\Omega \approx \omega_0$ is greater than the polaron mass $m_{pol} = m[1 + (\alpha/6)]$ determined by formula (31).

In order to estimate the role of the polaron effect in the suppression of the magnetophonon peak, we shall consider the case of n-InSb ($\alpha = 0.02$, $u = 5 \times 10^5$ cm^2/sec) at T = 100°K. In a magnetic field H = 34 kOe, corresponding to the first MP maximum, the value of σ_{xx} given by expression (32) is approximately seven times smaller than that obtained from (25) and (28). Thus, resonance emission of optical phonons is the principal effect determining the amplitude of the MP oscillations. This is all the more so for semiconductors with a lower mobility and larger electron coupling constant than for n-InSb.

Together with the broadening due to resonance emission of optical phonons, the Landau-level broadening associated with multiple scattering of slow electrons with $p_z \approx 0$ by impurity centers[22] has been considered by Barker[23]. A formula was obtained for the oscillating part of the transverse magnetoresistance:

$$\frac{\Delta\rho_{xx}^{osc}}{\rho_0}=\sum_{r=1}\frac{1}{r}\exp\left(-\frac{r\Gamma\omega_0}{\Omega}\right)\cos\left(2\pi r\frac{\omega_0}{\Omega}\right). \qquad (33)$$

The quantity Γ, which determines the amplitude of the MP peak, depends on the coupling constant α and on the scattering amplitude at the impurity center, and does not depend on the magnetic field. According to[23], the broadening due to interaction of the electrons with impurity centers is the determining factor for n-GaAs.

Formulas (25), (26) and (32) give the possibility of an order-of-magnitude estimate of the ratio of the oscillating part of σ_{xx} to the background only in the case when scattering by the optical phonons is dominant. If, however, elastic scattering mechanisms dominate, then, as can be seen from (21), the background is determined by the additive contribution σ_{xx}^{ela}. In the quantum limit $\hbar\Omega > \zeta, k_0T$, the dependence of σ_{xx}^{ela} on H has a monotonic character. Gurevich et al.[24] give the following estimates for the ratio of σ_{xx}^{osc} to the magnitude of the conductivity σ_0^{ela} in the semiclassical limit $\hbar\Omega \ll k_0T$. In the case of Boltzmann statistics,

$$\frac{\sigma_{xx}^{osc}}{\sigma_0}\approx\alpha\exp\left(-\frac{\hbar\omega_0}{k_0T}\right)\left(\frac{\hbar\omega_0}{k_0T}\right)^{3/2}\frac{uH}{c}\left(\frac{k_0T}{\hbar\Omega}\right)^n. \qquad (34)$$

The power n is determined by the scattering mechanism (n = 0 for scattering by impurity ions, n = 1 for scattering by piezo-acoustic vibrations, and n = 2 for scattering by the deformation potential). In the case of Fermi statistics

$$\frac{\sigma_{xx}^{osc}}{\sigma_0}\approx\alpha\exp\left(-\frac{\hbar\omega_0}{k_0T}\right)\frac{\hbar\omega_0}{k_0T}\frac{uH}{c}\left(\frac{\hbar\omega_0}{\zeta}\right)^{3/2}. \qquad (35)$$

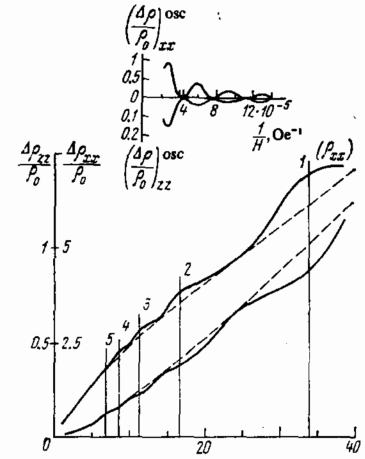
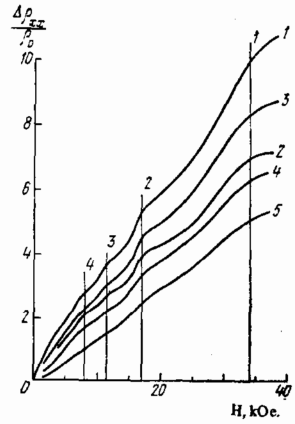


FIG. 2. Dependence of the transverse ($\Delta\rho_{xx}/\rho_0$) and longitudinal ($\Delta\rho_{zz}/\rho_0$) magnetoresistances on the magnetic field (kOe) for InSb at T = 90° K. Sample 1: n = 6.5 × 10[13] cm[-3], u = 6.7×10[5] cm²/V.sec, Sample 2: n = 4.1×10[13] cm[-3], u = 5.5×10[5] cm²/V.sec[11]. (The dashed curves represent the monotonic background on which the oscillations of a resonance nature are superimposed. In the upper part of the Figure, the oscillating part of the magnetoresistance is given as a function of the inverse magnetic field. The vertical straight lines correspond to resonance fields with M = 1, 2, 3 . . . according to (19).)

FIG. 3. Transverse magnetoresistance as a function of magnetic field at T = 90°K for samples of n-InSb with different electron concentrations. n(cm^{-3}) = 5.2×10^{13} (13), 2.4×10^{14} (4), and 1.3×10^{15} (5).



It follows from these estimates that, for the observation of MP oscillations, semiconductors with a high electron mobility should be preferable, and the range of temperatures should not be too low compared with θ_0 in order that the optical branch of the phonon spectrum be excited to a sufficient degree. In the Appendix, values of the limiting frequencies ω_0 of the longitudinal optical phonons and the coupling constants α are given for a number of semiconducting compounds.

d) Experimental results for MPR in σ_{xx}. As already noted, n-InSb is an extremely convenient material for studying MPR, since the high electron mobilities make it possible to use easily attainable stationary magnetic fields[10,11]. The relatively simple structure of the conduction band of InSb gives rise also to a simple experimental pattern of oscillations. The characteristic temperature θ_0 = 275°K for InSb, and therefore the optical branch of the phonon spectrum is excited to a sufficient degree at T ~ 100°K.

The experimental data from the study of MPR in n-InSb have confirmed the following principal conclusions of the theory:

1) The experimental dependence of the transverse magnetoresistance for a pure sample of n-InSb (n = 6.5 × 10^{13} cm^{-3}, u = 6.7 × 10^{5} cm^2/V.sec), shown in Fig. 2, shows that the system of maxima of ρ_{xx} (H_{max} = 34, 17, 11.3 and 8.5 kOe) is periodic in the inverse field with period $\Delta(1/H)$ = (3.0 ± 0.2) × 10^{-5} Oe. The positions of the maxima correspond to the MPR condition (19) for transitions of electrons from the Landau level N = 0 to the levels N = 1, 2, 3, 4.

From the position of the first maximum (from the side of high magnetic fields) H_1 = 34 kOe, with the known value ω_0 = 3.64 × 10^{13} sec^{-1}, we can determine the magnitude of the effective electron mass m. Since the conduction band of InSb is nonparabolic, and in resonance scattering the electron energy is changed appreciably (by $\hbar\omega_0$), the calculated value of the effective electron mass m = 0.016m$_0$ is found to be greater than the value m_n = 0.014m$_0$ for the mass at the bottom of the conduction band (m$_0$ is the free-electron mass). The magnitude of the period of the oscillations (calculated from formula (20) for m = 0.016m$_0$) $\Delta(1/H)$ = 2.96 × 10^{-5} Oe^{-1} agrees well with the experimental value $\Delta(1/H)$ = 3 × 10^{-5} Oe^{-1}.

2) As has been shown, an optimal temperature should exist at which the oscillating part of the magnetoresistance is a maximum. From the experimental curves given in[10] for ρ_{xx}(H), measured on one sample of n-InSb in the temperature range 63–195°K, it follows that the height of the oscillation peaks is a maximum at T = 104°K and amounts to ~ 15% of the background. The period and phase of the oscillations in the transverse magnetoresistance do not depend on the temperature.

3) As follows from the theory, the positions of the maxima of the transverse magnetoresistance and the period of the MP oscillations should not depend on the electron concentration. The series of experimental curves shown in Fig. 3, obtained for n-InSb samples with different electron concentrations (from 5.2 × 10^{13} to 1.3 × 10^{15} cm^{-3}) at T = 90°K, show clearly the difference between the MPR effect and SH oscillations. The amplitude of the MP oscillations decreases as the mobility decreases with increasing alloying of the sample.

The use of strong pulsed magnetic fields has made it possible to display the MP oscillations of the transverse magnetoresistance in n-InAs[25], in which the effective electron mass is greater and the frequency of the longitudinal optical phonons higher than in InSb. It must be noted that, because of the large monotonic component of the magnetoresistance, the authors of[26] were able to observe the MP oscillations of ρ_{xx}(H) only as a result of a substantial raising of the resolving power of the apparatus, using a double differentiation technique. For n-InAs at T = 300°K, two maxima of ρ_{xx}(H) were detected, corresponding to resonance transitions of electrons between the zeroth and first (H_1 = 76 kOe) and between the zeroth and second (H_2 = 33 kOe) Landau levels. Analogous results for the MP oscillations of the transverse magnetoresistance in n-InAs were obtained in the paper[27]. Using formula (19) and putting ω_0 = 4.6 × 10^{13} sec^{-1}, we obtain for the effective electron mass the value m = 0.025m$_0$, which, because the band is nonparabolic, differs from the value of the mass at the bottom of the band, m_n = 0.023m$_0$. The effect of the nonparabolicity on the position of the MP extrema is considered in more detail in Sec. 4.

MP oscillations in the transverse magnetoresistance have also been studied in n-GaAs[26,28,29], n-Ge[30,31],

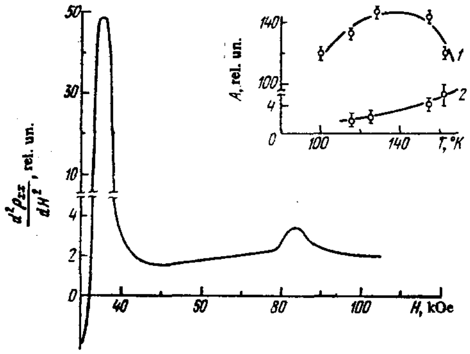


FIG. 4. Dependence of $d^2\rho_{xx}/dH^2$ on H at T = 163°K. (In the upper right inset, the dependence of the amplitude A on temperature is depicted for the maxima at H = 34 kOe (curve 1) and H = 82 kOe (curve 2) [40].)

n-InP[32,33], n-CdTe[34], n-CdSe[35], p-Te[36-38], p-Ge[31], p-InSb and p-GaAs[39]. Investigations of MPR in substances with a relatively large coupling constant α (n-CdTe, n-CdSe, p-Te) have shown that, under resonance conditions $\Omega = \omega_0$, the renormalization of the effective mass can be described by the empirical formula

$$\tilde{m} = m\left(1 + \frac{\alpha}{2}\right). \tag{36}$$

The authors of[40], studying the temperature dependence of the amplitudes of the MP peaks of $\rho_{xx}(H)$ in n-InSb, observed that, for T > 100°K, along with the principal maxima an additional maximum at H = 82 kOe appears (Fig. 4). The anomalous temperature dependence of its amplitude, which increases when T > 120°K when the amplitudes of the ordinary MP maxima decrease, makes it possible to assume that this peak is caused by inelastic scattering with participation of two optical phonons. The probability of a two-phonon absorption process is proportional to $\exp(-2\theta_0/T)$, whereas the probability of a one-phonon process is proportional to $\exp(-\theta_0/T)$. Therefore, the temperature at which the two-phonon peak should attain its greatest magnitude should be higher than the optimal temperature for the one-phonon peaks. The MPR condition for two-phonon processes has the form

$$\varepsilon_{N'} - \varepsilon_N = 2\hbar\omega_0. \tag{37}$$

From condition (37), with allowance for the nonparabolicity of the conduction band of InSb (cf. Sec. 4), the value H = 83.5 kOe is obtained, in good agreement with experiment.

The contribution of multi-phonon scattering processes to the kinetic coefficients is not great for $T < \theta_0$, and it is not possible to exhibit it in practice. It is only the resonance character of the scattering in MPR conditions that makes it possible to detect this contribution clearly.

The intensity of the peaks caused by two-phonon processes is not great. The amplitude of the maximum in ρ_{xx} at H = 82 kOe amounts to about 20% of the most intense maximum (H_1 = 34 kOe) in the principal series, or about 1% of the entire magnitude of ρ_{xx}. Some of the two-phonon extrema are positioned close to much more intense one-phonon extrema, and it is probable, therefore, that it has not been possible to detect them.

Resonance peaks associated with multi-phonon processes have also been detected in the study of MPR in n-GaAs[41].

3. MAGNETOPHONON OSCILLATIONS OF THE LONGITUDINAL MAGNETORESISTANCE.

a) Conductivity in a longitudinal magnetic field. As follows from (10b), only the diagonal elements $(j_z)_{NN}$ of the operator of the current density in the direction of the magnetic field are non-zero. Therefore, to describe longitudinal effects, only the diagonal elements of the density matrix are necessary. The equation of motion for the diagonal elements of the density matrix, i.e., for the distribution function, in the linear approximation in the electric field $E = E_z$ has the form of the usual Boltzmann kinetic equation:

$$eE\,\frac{\hbar k_z}{m}\,\frac{df_\mu^0}{d\varepsilon_\mu} = \sum_\nu \left[w_{\mu\nu} f_\mu (1 - f_\nu) - w_{\nu\mu} f_\nu (1 - f_\mu)\right]; \tag{38}$$

here $f_\mu = \rho_{\mu\mu}$ is the nonequilibrium and $f_\mu^0 = f_0(\epsilon_\mu)$ the equilibrium distribution function, and $w_{\mu\nu}$ is the transition probability (cf. (15)).

From the expression for the longitudinal-current density

$$j_z = -\frac{e\hbar}{m} \sum_\mu k_z f_\mu \tag{39}$$

it follows that the non-zero current is due to the part of the distribution function f_μ that is odd in k_z. We shall seek the solution of Eq. (38) in the usual form:

$$f_\mu = f_\mu^0 + \chi_\mu \frac{df_\mu^0}{d\varepsilon_\mu} eE, \tag{40}$$

where χ_μ is a certain odd function of k_z. Then from (38) the following equation for χ_μ is obtained:

$$\frac{\hbar k_z}{m}(1 - f_\mu^0) = \sum_\nu w_{\mu\nu}(1 - f_\nu^0)(\chi_\mu - \chi_\nu). \tag{41}$$

If the electron gas is not degenerate, we can neglect f_μ^0 and f_ν^0 in comparison with unity in (41).

As a rule, to solve Eq. (41) is an extremely complicated problem. There are, however, cases in which we can obtain an exact solution. We turn to expression (15) for the transition probability. If $C_q = C_0$ and $\omega_q = \omega_0$ do not depend on q, then the summation over q in (15) is simply performed if we make use of the relation

$$\int dq_x\,dq_y\,|\langle\mu|\,e^{iqr}\,|\nu\rangle|^2 = \frac{2\pi}{L^2},$$

which is a consequence of the normalization of the wavefunctions ψ_ν (3a). Then it is not difficult to show that $w_{\mu\nu}$ depends on k_z' only through the energy $\epsilon_\nu = (N' + 1/2)\hbar\Omega + (\hbar^2 k_z^2/2m)$ and is thus an even function of k_z'. Therefore, the "incoming" terms in Eq. (41), i.e., the terms containing χ_ν, give zero in the sum, and the function χ_μ has the following form:

$$\chi_\mu = \frac{\hbar k_z}{m}\tau_\mu, \tag{42}$$

where τ_μ is the relaxation time:

$$\tau_\mu^{-1} = \frac{2\pi}{\hbar} C_0^2 \left[N_0 g(\varepsilon_\mu + \hbar\omega_0) + (N_0 + 1)\,\theta(\varepsilon_\mu - \hbar\omega_0)\,g(\varepsilon_\mu - \hbar\omega_0)\right]. \tag{43}$$

Here we have introduced the discontinuous function $\theta(x)$:

$$\theta(x) = \begin{cases} 0, & x < 0, \\ 1, & x \geqslant 0. \end{cases} \tag{44}$$

In an analogous way, we can find an exact solution for χ_μ in the case of elastic scattering of electrons by acoustic phonons and by impurities with a δ-function potential[3].

The physical reason for the magnetophonon oscillations is the inelastic character of the scattering of the electrons by the optical phonons and the nonmonotonic quasi-periodic energy dependence of the density of states in a quantizing magnetic field. The character of the interaction of the electrons with the phonons, which determines the form of the dependence of C_q on q, plays a secondary role. Therefore, in order to elucidate the important features of MPR of the longitudinal conductivity, it is convenient to consider the case of nonpolar interaction of the electrons with the optical phonons, $C_q = C_0$. The longitudinal magnetoresistance in the case of interaction of the electrons with polarization vibrations is considered in[42,43]. The collision frequency τ_μ^{-1} has the same features as in the case under consideration $C_q = C_0$. The longitudinal conductivity is simply expressed in terms of the relaxation time:

$$\sigma_{zz}=\left(\frac{e\hbar}{m}\right)^2\sum_\mu k_z^2\tau_\mu\frac{df_\mu^0}{d\epsilon_\mu}. \qquad (45)$$

It can be seen from (43) that the collision frequency τ_μ^{-1} becomes infinite for $\epsilon_\mu \pm\hbar\omega_0 = \epsilon_N$, when, as a consequence of absorption or emission of an optical phonon, the electron falls into a final state at the bottom of the Landau sub-band. This singularity of τ_μ^{-1} appears as a result of the summation over the final states in (41) and does not depend on the form of the function C_q. Here, it is not important which initial state the electron undergoes the transition from, i.e., whether the initial state belongs to one of the Landau levels. If we make use of the analogy with optical interband transitions, we can say that only "indirect" transitions with change of k_z make a contribution to the relaxation of the longitudinal momentum $\hbar k_z$, i.e., to the quantity τ_μ, whereas direct transitions (transitions of the type 1 in Fig. 1) between the Landau levels make the main contribution to the transverse conductivity. This difference predetermines the specific features of the MP oscillations of ρ_{zz}.

Going over in formula (45) to an integration over the energy, we obtain

$$\frac{\sigma_{zz}}{\sigma_0}=\frac{1-e^{-\gamma}}{\gamma}\frac{I(\gamma)}{I_0}, \qquad (46)$$

$$I(\gamma)=\int_0^\infty\frac{p_0(x)e^{-x}dx}{p_1(x+\beta)+e^{\beta}\theta(x-\beta)p_1(x-\beta)}, \quad p_m(x=)\sum_N(x-N\gamma)^{(1/2)-m}, \qquad (47)$$

$$I_0=\frac{1}{3}\int_0^\infty\frac{x^{3/2}e^{-x}dx}{(x+\beta)^{1/2}+e^{\beta}\theta(x-\beta)(x-\beta)^{1/2}}; \qquad (48)$$

here σ_0 is the conductivity for H = 0; $\gamma = \hbar\Omega/k_0T$; $\beta = \hbar\omega_0/k_0T$. The first factor in (46) is a monotonic function of the magnetic field, which describes the behavior of σ_{zz} in the quantum limit $\gamma > \beta \gg 1$ and determines a certain monotonic background in the oscillation region $\gamma \lesssim \beta$. The integral $I(\gamma)$ for $\gamma \lesssim \beta$ is a nonmonotonic function and describes the MP oscillations of the conductivity. In the low-temperature region, $e^\beta \gg 1$, we can omit the second terms in the denominators of formulas (47) and (48), since the main contribution to the conductivity is made by electrons with energy $\epsilon < \hbar\omega_0$, which are scattered only with absorption of an optical phonon. The contribution of the remaining electrons is proportional to the small parameter $e^{-2\beta}$. Transforming the functions $p_0(x)$ and $p_1(x+\beta)$ by means of the Poisson summation formula and separating out from $I(\gamma)$ the oscillating contribution $\Delta I = I(\gamma) - I_0$, we obtain for $\gamma < \beta$ ($\Omega < \omega_0$)

$$\frac{\Delta\sigma_{zz}}{\sigma_0}=-\sqrt{\frac{\gamma}{3+2\beta}}\sum_{r=1}A_r\cos\left(2\pi r\frac{\omega_0}{\Omega}+\varphi_r\right), \qquad (49)$$

$$A_r=\frac{1}{\sqrt{r}\,[1+(2\pi rk_0T/\hbar\Omega)^2]^{5/4}}, \qquad (50)$$

$$\varphi_r=-\frac{\pi}{4}+\frac{5}{2}\operatorname{arctg}\frac{2\pi rk_0T}{\hbar\Omega}. \qquad (51)$$

The expression (49) is completely analogous to the formula (cf., e.g., (71) in[3]) which describes the SH oscillations for elastic scattering of the electrons. However, the phases φ_r (51) depend essentially on the temperature. This means that the positions of the MP extrema of $\rho_{zz}(H)$ should depend on the temperature. A direct investigation of this effect starting from (49) is difficult, since the amplitudes A_r fall off rather slowly with the index r and a large number of terms must be taken into account in the sum. Therefore, we shall consider in detail the behavior of $I(\gamma)$ near the resonance point $\gamma = \beta$[44].

b) Temperature shift of the MP extrema. In the region of low temperatures, $\beta \gg 1$, in expressions (15) and (16) we can confine ourselves to integrating up to $x = \beta$. We shall study the behavior of $I(\gamma)$ in the region $\gamma \approx \beta$ ($\Omega \approx \omega_0$). For $\gamma \le \beta$, we have

$$I(\gamma)=\int_0^{2\gamma-\beta}\frac{e^{-x}x^{1/2}dx}{(x+\beta)^{-1/2}+(x+\beta-\gamma)^{-1/2}}+\int_{2\gamma-\beta}^{\beta}\frac{e^{-x}x^{1/2}dx}{(x+\beta)^{-1/2}+(x+\beta-\gamma)^{-1/2}+(x+\beta-2\gamma)^{1/2}}+\int_\gamma^\beta\frac{e^{-x}(x-\gamma)^{1/2}dx}{(x+\beta)^{-1/2}+(x+\beta-\gamma)^{-1/2}+(x+\beta-2\gamma)^{-1/2}}. \qquad (52)$$

The first term in (52) describes the contribution to the conductivity from electrons of the zeroth Landau sub-band with energies $\epsilon \le 2\hbar\Omega - \hbar\omega_0$, which on absorption of an optical phonon undergo transitions to the zeroth or first Landau sub-band (these transitions are depicted by arrows 1 and 2 in Fig. 5a). The second term takes into account, in addition, the resonance transitions to the second Landau sub-band (transitions of the type 3 in Fig. 5a). Finally, the third term corresponds to the contribution to the conductivity from electrons of the first Landau sub-band.

The derivative $dI/d\gamma$ with respect to the magnetic field at the point $\gamma = \beta-$, i.e., the derivative from the left, is equal to

$$\left(\frac{dI}{d\gamma}\right)_{\beta-}=\frac{2\beta e^{-\beta}}{1+\sqrt{2}}-\frac{1}{2}\int_0^\beta e^{-x}\left[1+\left(\frac{x}{x+\beta}\right)^{1/2}\right]^{-2}dx. \qquad (53)$$

The positive term in (53) corresponds to an increase of conductivity with magnetic field, arising from the increase of the contribution from electrons for which resonance scattering with a transition into the second Landau sub-band, i.e., with a transition of the type 3, is impossible. The negative term denotes the decrease in the conductivity as a consequence of the increase in the probability of a transition to the first sub-band. This

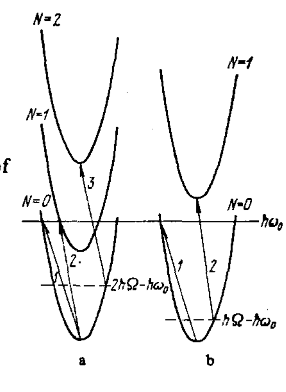


FIG. 5. Possible types of transitions of electrons with absorption of an optical phonon. a) $\Omega < \omega_0$; b) $\Omega > \omega_0$

probability, which is proportional to k_z^{-1}, increases when the level $\hbar\Omega$ approaches $\hbar\omega_0$, and transitions of the type 2 occur in the immediate vicinity of the point $k_z = 0$. For $\beta > 2$, the integral in (53) is approximately equal to

$$\int_0^\beta e^{-x}\left[1+\left(\frac{x}{x+\beta}\right)^{1/2}\right]^{-2} dx \approx [1+(1+\beta)^{-1/2}]^{-2}.$$

The accuracy of this estimate is higher the greater is β. Hence, it can be seen that at very low temperatures, $\beta \gg 1$, the derivative $(dI/d\gamma)_{\beta-}$ is negative, i.e., to the left of the resonance point $\gamma = \beta$ the conductivity falls off. The absolute value $|dI/d\gamma|$ decreases with rise of temperature, and at certain value $2 < \beta_0 < 3$ this derivative becomes positive.

We now consider the behavior of $I(\gamma)$ in the region $\gamma \geq \beta$:

$$I(\gamma) = \int_0^{\gamma-\beta} e^{-x} x^{1/2} (x+\beta)^{1/2} dx + \int_{\gamma-\beta}^{\beta} \frac{e^{-x} x^{1/2} dx}{(x+\beta)^{-1/2} + (x+\beta-\gamma)^{-1/2}}. \quad (54)$$

The first integral describes the contribution of electrons which on scattering undergo transitions within the first Landau sub-band (transitions of the type 1 in Fig. 5b). This term increases with magnetic field. The second integral takes into account transitions to the first Landau band (transitions of the type 2). It decreases with magnetic field. As can be seen from (52) and (54), the function $I(\gamma)$ is continuous at the point $\gamma = \beta$. The derivative $dI/d\gamma$ for $\gamma > \beta$ is equal to

$$\frac{dI}{d\gamma} = e^{\beta-\gamma} (\gamma-\beta)^{1/2} \gamma^{1/2} - \frac{1}{2} e^{\gamma-\beta} \int_0^{2\beta-\gamma} \frac{e^{-x}(x+\gamma-\beta)^{1/2} dx}{x^{1/2}\{1+[x/(x+\gamma)]^{1/2}\}^2}. \quad (55)$$

At the point $\gamma = \beta_+$,

$$\left(\frac{dI}{d\gamma}\right)_{\beta+} = -\frac{1}{2}\int_0^\beta e^{-x}\left[1+\left(\frac{x}{x+\beta}\right)^{1/2}\right]^{-2} dx, \quad (56)$$

i.e., ρ_{zz} increases with magnetic field for all β. We shall find the position of the first MP maximum by equating $dI/d\gamma$ in (55) to zero. The corresponding equation is easily solved in the limiting case $\beta \gg 1$:

$$\frac{\Omega_1}{\omega_0} = 1 + \frac{1}{4}\left(\frac{k_0 T}{\hbar\omega_0}\right)^2. \quad (57)$$

For $\Omega > \Omega_1$ the quantity ρ_{zz} falls off, since the number of electrons for which transitions to the first Landau sub-band are possible decreases. It can be seen from (57) that the MP maximum of $\rho_{zz}(H)$ is shifted toward higher fields with rise of temperature. The reason for this shift is that the electrons with $k_z = 0$ at the bottom of the first Landau sub-band make no contribution to the longitudinal conductivity, since the velocity along the field $v_z = \hbar k_z/m = 0$. The large density of states at $k_z \approx 0$ is compensated by the small value of the electron velocity along the field, so that electrons with $\epsilon_z = \hbar^2 k_z^2/2m \sim k_0 T$ play the main role. A rise of temperature leads to an increase of the mean energy ϵ_z and causes a shift of the MP maxima toward higher fields.

Comparing (53) and (56), we see that the derivative $dI/d\gamma$ displays a discontinuity at the point $\gamma = \beta$. This violation of continuity is due to the discontinuous character of the relaxation time in inelastic scattering by optical phonons (the factor $\theta(x-\beta)$ in (43)) and to the discontinuities in the density of states at $\epsilon = \epsilon_N$. Allowance for the broadening of the Landau levels removes the discontinuities in the density of states and in the derivative $dI/d\gamma$. However, in a small region of $\Delta\Omega = |\Omega - \omega_0|$, $dI/d\gamma$ changes from (53) to (56).

It is important to note that the derivative $(dI/d\gamma)_{\beta+}$ to the right of the resonance is always negative, i.e., $\rho_{zz}(H)$ increases. Therefore, the minimum at the point $\gamma = \beta$ is manifested only in conditions when, to the left of the resonance, $(dI/d\gamma)_{\beta-} \geq 0$. But this means that the character of the oscillations of ρ_{zz} depends essentially on the temperature. If $\beta \gg 1$, there should be a maximum of $\rho_{zz}(H)$ for $\Omega = \omega_0$. With increase of temperature this maximum shifts toward higher fields and its amplitude decreases. Finally, at a certain temperature T_0 in the interval

$$\frac{1}{3}\theta_0 < T_0 < \frac{1}{2}\theta_0 \quad (58)$$

there should be a minimum of $\rho_{zz}(H)$ at the point $\Omega = \omega_0$. This minimum is not displaced on further increase of the temperature. The next MP extrema at $\Omega = \omega_0/M$ ($M = 2, 3, \ldots$) can be treated analogously.

c) Pseudo-resonances in the longitudinal magnetoresistance. Up to this point, we have not taken into account the contribution to σ_{zz} from electrons with energies $\epsilon > \hbar\omega_0$. The corresponding correction to $I(\gamma)$ (52) can be represented in the following form:

$$I_1(\gamma) = e^{-2\beta}\int_0^\infty \frac{e^{-x} p_0(x+\beta)\, dx}{e^\beta p_1(x+2\beta) + p_1(x)}. \quad (59)$$

The function $I_1(\gamma)$ is finite and continuous for all γ. However, by analyzing the behavior of the derivative $dI_1/d\gamma$ (in a manner analogous to the way in which we investigated $dI/d\gamma$), we can show that $dI_1/d\gamma$ has finite discontinuities at those points at which the singularities of $p_1(x+2\beta)$ and $p_1(x)$ coincide i.e., for $N\gamma - 2\beta = N'\gamma$, or for

$$M\Omega = 2\omega_0. \quad (60)$$

The discontinuities of the derivative $dI_1/d\gamma$ lead to breaks in the function $I_1(\gamma)$ at the corresponding points. The nature of the breaks is such that $\rho_{zz}(H)$ increases to the right and falls off to the left of the values of H determined by the equality (60). Thus, for sufficiently high temperatures, when $e^{-2\beta}$ is not too small compared with unity, in addition to the ordinary MP minima, a new series of minima with

$$(2M+1)\Omega = 2\omega_0 \quad (M = 0, 1, 2, \ldots). \quad (61)$$

should appear in $\rho_{zz}(H)$. The depth of these minima increases with increasing temperature, and, as shown by the numerical calculations of Peterson[45], are comparable at $T \sim \theta_0$ with the amplitude of the MP extrema. This is illustrated by Fig. 6 of[45]. It can also be seen from this figure that with increasing temperature the

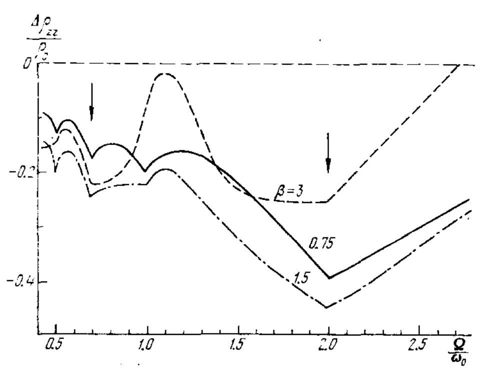


FIG. 6. Dependence of the longitudinal magnetoresistance on the magnetic field, calculated for three values of the temperature ($\beta = \hbar\omega_0/k_0T = 3$, 1.5 and 0.75). (Pseudo-resonances are indicated by the arrows [45].)

positions of the MP maxima are shifted toward higher fields and minima are formed in the resonance magnetic fields.

The positions of the additional minima of $\rho_{ZZ}(H)$, which Peterson called pseudo-resonances, coincide with the positions of the two-phonon MP extrema (37). The amplitude of both is proportional to $\exp(-2\theta_0/T)$. It must be emphasized, however, that the pseudo-resonances are due to one-phonon processes. In magnetic fields satisfying the conditions (60 and (61), electrons from a certain state ν can undergo a transition either to one of the lower Landau levels with emission of one phonon, or to one of the upper levels with absorption of a phonon. Since the contribution of one-phonon processes to the resistance is proportional to the square of the coupling constant of the electron-phonon interaction, while the contribution of multi-phonon processes contains higher powers of the coupling constant, it may be thought that the experimentally observed[26,32,46-48] additional minima of $\rho_{ZZ}(H)$ are more likely to be due to pseudo-resonances than to two-phonon processes. Nonetheless, the contribution of resonance multi-phonon processes to the magnetoresistance may be observed explicitly not only in ρ_{XX}[40,41], in which pseudo-resonances are absent, but also in ρ_{ZZ}. Investigating the longitudinal magnetoresistance in n-InSb, the authors of[49] observed, at T = 150°K, a minimum corresponding to a resonance three-phonon transition.

The theoretical study of MPR in the longitudinal magnetoresistance has also been the subject of papers by Peterson[50], who considered the cases of interaction with nonpolar[50a] and polar[50b] phonons. In calculating the longitudinal current j_Z, Peterson used a nonequilibrium distribution function in the following form:

$$f_\mu = R\exp\left[-\frac{\varepsilon_N}{k_0T}-\frac{(p_z-m\bar{v})^2}{2mk_0T}\right], \tag{62}$$

where the electron drift velocity $\bar{v}$ along the field is determined from the momentum balance equation. This approach is equivalent to the variational method of[51a] in the most crude approximation when only one trial function, proportional to the momentum p_Z, is chosen. This approximation is often used for the calculation of the conductivity for elastic scattering of the electrons; it is clearly inadequate, however, when inelastic scattering is considered. In the latter case, to obtain a correct value for the conductivity at H = 0 it is necessary to use a set of at least two trial functions[48], one of which depends on the electron energy. The same should be true in a quantizing magnetic field. (In[51b], it is shown that for inelastic scattering one must include functions with a root singularity in the set of trial functions). It is not surprising, therefore, that as a result of the numerical calculations Peterson found neither the temperature shift of the MP maxima of ρ_{ZZ}, in contradiction with the results of[43,44], nor the appearance of MP minima of ρ_{ZZ} in resonance magnetic fields[44] (in[43,44], the nonequilibrium distribution function was determined by solution of the kinetic equation).

<u>d) MP oscillations in degenerate semiconductors.</u> We now consider the MP oscillations of ρ_{ZZ} for a degenerate electron gas[52]. Again, Eq. (41) can be solved exactly only for $C_q = C_0$, and for the electron-phonon collision frequency we obtain

$$\tau_\mu^{-1} = \frac{m^{1/2}C_0^2}{\sqrt{2}\,\pi\hbar^2L^2}B(\varepsilon_\mu),\qquad B(\varepsilon_\mu)=\sum_{N'}\left[\frac{N_0+f_0(\varepsilon+\hbar\omega_0)}{\sqrt{\varepsilon_\mu-\varepsilon_{N'}+\hbar\omega_0}}+\frac{N_0+1-f_0(\varepsilon_\mu-\hbar\omega_0)}{\sqrt{\varepsilon_\mu-\varepsilon_{N'}-\hbar\omega_0}}\right]. \tag{63}$$

The collision frequency, as in the case of a nondegenerate electron gas, becomes infinite if the electron, as a consequence of absorption or emission of a phonon, can fall into a Landau level. Now, however, only those initial states which belong to the Fermi level are important. In the limit of strong degeneracy, we can replace $df_\mu^0/d\epsilon_\mu$ in (45) by a δ-function:

$$\frac{df_0}{d\varepsilon_\mu} = -\delta(\varepsilon_\mu-\zeta). \tag{64}$$

Then the following expression is obtained for ρ_{ZZ}:

$$\rho_{zz} = \frac{\pi C_0^2 m}{e^2\hbar}\,\mathrm{cth}\,\beta\,\frac{\sum_N[(\zeta+\hbar\omega_0-\varepsilon_N)^{-1/2}+(\zeta-\hbar\omega_0-\varepsilon_N)^{-1/2}]}{\sum_N(\zeta-\varepsilon_N)^{1/2}}. \tag{65}$$

Hence it can be seen that ρ_{ZZ} increase without limit in magnetic fields satisfying the condition

$$\zeta(H)\pm\hbar\omega_0 = \hbar\Omega\left(N+\tfrac{1}{2}\right); \tag{66}$$

the signs ± correspond to transitions with absorption or emission of a phonon. For $\zeta > \hbar\omega_0$ both processes are possible, and they make the same contribution to ρ_{ZZ}. In fact, the probability of transition of an electron with emission of a phonon is proportional to $[1-f_0(\epsilon-\hbar\omega_0)](N_0+1)$, and the probability of transition with absorption of a phonon is $[1-f_0(\epsilon+\hbar\omega_0)]N_0$, and these quantities are equal for electrons at the Fermi level $\epsilon = \zeta$.

If $\zeta < \hbar\omega_0$, which is possible for not very high electron concentrations, only transitions with absorption of a phonon are important.

The resonance condition (66) shows that MP oscillations of a completely new type can occur in the case of a degenerate gas. As has been shown in Sec. 2, the positions of the MP maxima of ρ_{XX} are determined by the condition (19) irrespective of the electron statistics. The value of the Fermi energy ζ does not appear in (19), since in the calculation of the transverse conductivity σ_{XX} the whole energy interval $\zeta-\hbar\omega_0 \le \epsilon \le \zeta$ is important, and not only the energy $\epsilon = \zeta$.

The values of the magnetic fields determined by (66) depend on the concentration of electrons (through the quantity ζ), and also on their effective mass and on the phonon frequency ω_0. The oscillations considered have features in common both with the ordinary MP oscillations and with SH oscillations. In essence, they are Shubnikov oscillations under conditions of inelastic electron scattering. For $\zeta \gg \hbar\omega_0$, the term $\pm\hbar\omega_0$ in (66), associated with the inelasticity of the scattering, can be omitted and we then obtain the well-known condition for SH oscillations. Therefore, the positions of the MP maxima can differ appreciably from the positions of the SH maxima only for not too large values of the Fermi level.

The infinite amplitude of the peaks of ρ_{ZZ} in (65) is a consequence of the approximation (64). If we take into account the incomplete degeneracy of the electron gas, we can obtain the following expression for the nonmonotonic correction $\Delta\rho_{ZZ}$[52b]:

$$\frac{\Delta\rho_{zz}}{\rho_0} = \frac{\pi^2}{\gamma}\frac{\sqrt{2\hbar\Omega}}{\sqrt{\zeta_+}+\sqrt{\zeta_-}} \times\sum_{r=1}(-1)^r\frac{\sqrt{r}}{\mathrm{sh}(2\pi^2r/\gamma)}\left[\cos\left(\frac{2\pi r\zeta}{\hbar\Omega}-\frac{\pi}{4}\right)+\cos\left(\frac{2\pi r\zeta}{\hbar\Omega}-\frac{\pi}{4}\right)\right] \tag{67}$$

($\zeta_\pm = \zeta \pm \hbar\omega_0$). The oscillating part of $\Delta\rho_{ZZ}$ consists of two sets of harmonics, periodic in the inverse magnetic field. At very low temperatures, the amplitude of the oscillations will be determined, clearly, not by the thermal broadening of the Landau levels but by their broadening as a consequence, principally, of non-Born scattering of the electrons by impurities.

It is understandable that the experimental observation of MP oscillations in conductors with a degenerate electron gas presents considerable difficulties. To fulfil the condition for sufficiently strong degeneracy, low temperatures and high concentrations of electrons and, consequently, of the alloying impurity are required. In such a case, the main contribution to the resistance is made by elastic scattering by the impurity ions, and the principal effect in quantizing fields will be SH oscillations. The MP oscillations should probably appear as small distortions of the SH oscillations. However, with increasing temperature the amplitude of the latter decreases, whereas the contribution of inelastic scattering by optical phonons increases, and in a certain temperature range the amplitude of the MP oscillations should increase. Oscillations of precisely this type were observed by Ponomarev and Tsidil'kovskiĭ in an investigation of ρ_{zz} in degenerate samples of n-GaSb and n-HgTe[52].

e) MP oscillations for mixed scattering. If, in addition to the inelastic scattering by optical phonons, the electrons are also scattered elastically, the total collision frequency is additive:

$$\tau^{-1} = \tau^{-1}_{\text{опт}} + \tau^{-1}_{\text{упр}}. \qquad (68)$$

Suppose that the elastic scattering is due to the interaction of the electrons with acoustic phonons. Then the quantity τ^{-1}_{ac} is proportional to the density of states:

$$\tau^{-1}_{\text{ак}} = \frac{2\pi}{\hbar}\frac{E_1^2 k_0 T}{w^2 d}, \qquad (69)$$

where E_1 is the deformation-potential constant, d is the density of the crystal, and w is the sound velocity.

Since the different scattering mechanisms do not make an additive contribution to ρ_{zz} (unlike ρ_{xx}) even in the Born approximation, the theoretical study of MPR in the longitudinal magnetoresistance is more complicated. MPR in conditions of mixed scattering of electrons by optical and acoustic phonons was first considered by Gurevich and Firsov[42]. As shown in[42], when the contribution of elastic scattering to the collision frequency is sufficiently large, the MP maxima of $\rho_{zz}(H)$ should be replaced by minima. However, the criterion obtained in[42] is too crude and is in contradiction with the numerous experimental data. A more rigorous treatment of the MP oscillations of $\rho_{zz}(H)$ was carried out in[44].

For mixed scattering of electrons, the expression for σ_{zz} has its previous form (46), but now the integral $I(\gamma)$ is equal to

$$I(\gamma) = \int_0^\infty \frac{p_0(x)\, e^{-x}\, dx}{p_1(x) + \lambda N_0 p_1(x+\beta) + \lambda (N_0+1)\,\theta(x-\beta)\, p_1(x-\beta)}; \qquad (70)$$

here $\lambda = C_0^2 w^2 d / E_1^2 k_0 T$ is the parameter characterizing the contribution of inelastic scattering to the collision frequency. It is clear that at low temperatures, $e^{\beta} \gg 1$, the oscillating part of ρ_{zz} is proportional to $\lambda e^{-\beta}$.

It is possible to establish, analogously to the way this was done in Sec. 3(c), that the derivative $dI/d\gamma$ is discontinuous at the points corresponding to the MPR condition (19). To the right of the resonance, this derivative is negative for all values of β and λ, i.e., $\rho_{zz}(H)$ increases independently of the temperature and for arbitrary contribution from the elastic scattering. The maxima of $\rho_{zz}(H)$ are shifted toward higher fields relative to the resonance values of H. The sign of the derivative $dI/d\gamma$ to the left of the resonance depends on the values of the parameters β and λ. In particular, for $\beta \gg 1$ and $\lambda \to \infty$ (low temperatures and a small contribution from elastic scattering) this derivative is negative and the discontinuity of the function $I(\gamma)$ is exponentially small. The discontinuity in $I(\gamma)$ increases with decrease of the parameters β and λ, and when the derivative $(dI/d\gamma)_{\beta-}$ becomes positive an MP minimum is formed at the resonance point. The condition that the derivative to the left of the resonance be equal to zero is a fairly complicated equation for λ, the solution of which depends on the parameter β. We denote this solution by $\lambda_0(\beta)$. Then, for $\lambda < \lambda_0$, a minimum of $\rho_{zz}(H)$ corresponds to resonance magnetic fields. But if $\lambda > \lambda_0$, the minimum of $\rho_{zz}(H)$ is attained at lower values of H, while the maxima are shifted toward higher fields. In this case, the resonance points are not extrema for the function $\rho_{zz}(H)$ at all.

The quantity λ_0 can be found easily for $\beta \gg 1$ and $\lambda_0 N_0 \ll 1$[44]:

$$\lambda_0 = 4\beta. \qquad (71)$$

This expression is sufficiently accurate for $\beta \gtrsim 6$. On the other hand, for a certain $\beta < 3$, as follows from (58), λ_0 should increase to infinity, since even for purely inelastic scattering there should be minima of $\rho_{zz}(H)$ in the resonance fields. Numerical estimates lead to the values $\lambda_0 \approx 30$ for $\beta = 4$ and $\lambda_0 \approx 40$ for $\beta = 3$. For comparison, we cite the estimate of the authors of[42], who obtained $\lambda_0 \approx 2$ independently of the temperature.

Thus, when the contribution of elastic scattering is sufficiently large, $\rho_{zz}(H)$ has a minimum in resonance fields at all temperatures. Qualitatively, a decrease of the parameter λ leads to the same consequences as an increase of temperature under conditions of purely inelastic scattering.

We note that if several elastic scattering mechanisms are important, the general parameter λ is determined by the formula

$$\lambda^{-1} = \sum_i \lambda_i^{-1}. \qquad (72)$$

Expressions for λ_{ac} and λ_{ion} for elastic scattering by acoustic phonons and impurity ions are given in[44].

MP oscillations have been investigated experimentally by many authors in different semiconductors: n-InSb[11,26,46], n-InAs[25,26,47], n-GaAs[26,53], n-Ge[54], n-InP[32,48], n-CdTe[34], n-CdSe[35], p-InSb[55], and p-Te[36,38]. In all cases, the extrema are periodic in the inverse magnetic field with period (20). In semiconducting compounds with small coupling constant $\alpha \lesssim 0.1$, and also in n-Ge and p-Te, the minima of ρ_{zz} in the resonance fields are to be found in the whole temperature range in which MPR is observed. In accordance with the theory of[44], the parameter λ in these materials is found to be smaller than the value λ_0, i.e., the contribution of elastic scattering leads to the formation of MP minima at resonance, even at low temperatures $T \ll \theta_0$. In materials with a sufficiently large coupling constant α (n-CdTe, n-CdSe), certain intermediate phases of the curves of $\rho_{zz}(H)$ are to be found in resonance fields for $T \ll \theta_0$: the maxima are shifted toward higher fields, and the minima toward lower fields.

4. MAGNETOPHONON RESONANCE IN SEMICONDUCTORS WITH NON-STANDARD BAND SHAPES

a) Nonquadratic isotropic dispersion law. For an arbitrary dispersion law, the MPR condition can be written, starting from the law of conservation of energy, in the form

$$\varepsilon_{Ns}-\varepsilon_{N's'}=\hbar\omega_0, \tag{73}$$

where ϵ_{Ns} is the energy of the N-th Landau level with spin quantum number $s = \pm 1/2$. In the case of an isotropic nonquadratic dispersion law, which is found, e.g., for the electrons of a number of III-V compounds, we can write down an explicit expression for ϵ_{Ns}. The nonparabolicity of the conduction band near the edge arises in this case as a result of the interaction between the conduction band and the valence bands at k = 0. The energy levels in a magnetic field for the conduction band, the light-hole band, and the band split off as a consequence of the spin-orbit interaction are given, when terms containing the free-electron mass m_0 (which is usually much smaller than the effective mass m_n at the band edge) are neglected, by a cubic equation analogous to Kane's equation in the absence of a magnetic field[56]:

$$\varepsilon_\mu(\varepsilon_\mu+\varepsilon_g)(\varepsilon_\mu+\varepsilon_g+\Delta)-P^2\left[k_z^2+(2N+1)\frac{1}{L^2}\right]\left(\varepsilon_\mu+\varepsilon_g+\frac{2}{3}\Delta\right)+\frac{2s}{3}\frac{P^2\Delta}{L^2}=0, \tag{74}$$

where ϵ_g is the band gap at k = 0, Δ is the value of the spin-orbit splitting of the valence band, and P is the interband matrix element of the momentum operator. In (74), the bottom of the conduction band is chosen as the zero of energy. For H = 0, Eq. (74) goes over into Kane's equation. The spin-dependent terms are proportional to L^{-2} and give a spin splitting of the levels, which is proportional to the spectroscopic splitting factor g.

With the assumption $\epsilon_\mu \ll \epsilon_g + (2\Delta/3)$, which is valid in a wide range of energies for all the semiconductors studied, the cubic equation (74) reduces to a quadratic equation, the solution of which for the conduction band is of the form

$$\varepsilon_\mu=-\frac{\varepsilon_g}{2}+\frac{1}{2}\sqrt{\varepsilon_g^2+4\varepsilon_g\left[\left(N+\frac{1}{2}\right)\hbar\Omega_n+\frac{\hbar^2k_z^2}{2m_n}+sg_n\mu_BH\right]}, \tag{75}$$

where $\Omega = eH/m_nc$, $\mu_B = e\hbar/2m_0c$, $g_n = -(2m_0/m_n)\Delta/(2\Delta + 3\epsilon_g)$ is the g-factor at the bottom of the band, and

$$\frac{1}{m_n}=\frac{2P^2}{3\hbar^2}\left(\frac{2}{\varepsilon_g}+\frac{1}{\varepsilon_g+\Delta}\right). \tag{76}$$

For the situation when $\hbar\omega_0 \ll \epsilon_g$, the MPR condition, as follows from (73) and (75), takes the form[57]

$$\Omega_n=\frac{\omega_0}{N'-N}\left(1+\frac{N'+N+t+1}{N'-N}\frac{\hbar\omega_0}{\varepsilon_g}\right), \tag{73a}$$

where $t = g_n m_n/2m_0 = -\Delta/(2\Delta + 3\epsilon_g)$.

Using the expression (75), from the position of the magnetoresistance peaks and with the aid of the resonance condition (73) for s = s′ we can determine the effective electron mass m_n at the bottom of the band, if ω_0, ϵ_g and Δ are known. For n-InSb, e.g.,[46] the values of m_n calculated with allowance for the non-parabolicity of the conduction band coincide with the cyclotron resonance data and differ markedly from the values obtained for a simple parabolic band (for 120°K, the relation $\Omega_n = \omega_0$ gives $m_n/m_0 = 0.016$ (cf. Sec. 2), while the true value is $m_n = 0.014m_0$).

Allowance for the nonparabolicity of the conduction band, i.e., use of formula (73a), has made it possible to interpret the results of MPR measurements correctly and to determine the dependences of the effective mass m_n (76) on pressure for InSb[58] and on temperature for InSb, InAs and GaAs[29].

As follows from (75), the Landau levels ϵ_{Ns} are inequivalent for a nonparabolic band. In connection with this, the values of the resonance magnetic fields depend not only on the difference N′ − N, but also on the labels N and N′ of the levels (cf. (73a)). This should lead to a specific asymmetry in the MPR lineshape (a broadening of the lines on the high-field side), which should increase with increasing temperature. Such features have been observed in InSb[29]. The contribution of transitions between high Landau levels, when these transitions cannot be resolved, leads to a shift of the MP extremum toward higher fields. Allowance for such transitions in InAs at room and higher temperatures (θ_0 = 340°K)[29,59] leads to good agreement between the calculated (76 kOe) and experimental (76 ± 4 kOe) resonance magnetic field values corresponding to a transition between the zeroth and first Landau levels (if we disregard the contribution of transitions between levels 1 → 2 and 2 → 3, the calculated value of H_{res} = 71 kOe).

b) Anisotropic quadratic dispersion law. In many-valley semiconductors of the N-Ge type, the MPR can be of two types. One of these, which has been discussed above, is associated with the scattering of electrons by optical phonons, the electron remaining within a given energy valley. The second type of MPR is due to electron-phonon scattering with a transition from one energy valley to another[60]. In intervalley scattering, the electron absorbs (emits) an "intervalley" phonon with quasimomentum $\hbar q_{12}$ equal to the distance between the centers of the valleys 1 and 2, and with energy $\hbar\omega_q$. The fundamental conditions for the appearance of MPR in the case of intervalley transitions remain the same as for intravalley transitions: it is necessary that the density of electron states have singularities and that it be possible to neglect the dispersion of the phonons.

The resonance condition in a transition between equivalent valleys, i.e., valleys in which the cyclotron masses are equal and which, consequently, are not shifted in energy in a magnetic field, have the same form as (19):

$$\omega_q = M\Omega \quad (M = 1, 2, 3, \ldots), \tag{77}$$

but the frequency ω_0 is replaced by ω_q. In n-Ge, e.g., only longitudinal optical and acoustic phonons[61], for which the characteristic temperature $\theta_q = \hbar\omega_q/k_0$ = 315°K[62], can take part in intervalley scattering. For transitions between inequivalent valleys, displaced relative to each other by $\hbar(\Omega_1 - \Omega_2)/2$, the resonance condition has the form

$$\omega_q=\frac{\Omega_1-\Omega_2}{2}+N\Omega_1-N'\Omega_2 \quad (N, N'=0, 1, 2, \ldots). \tag{78}$$

Since the electron scattering probability increases sharply at resonance, the dependence of ρ_{xx} on H should have maxima when the resonance conditions (77) or (78) are fulfilled. The longitudinal magnetoresistance, most probably, should have a minimum at resonance, since usually the contribution of intervalley scattering is small compared with that of elastic scattering[63].

MP oscillations due to intervalley scattering in n-Ge have been observed in the investigation of ρ_{xx}[30,31] and ρ_{zz}[54,64].

The ratio of the oscillating part of the kinetic coefficients to the monotonic background should be considerably smaller for n-Ge than for, say, n-InSb, since the contribution of scattering by optical phonons in InSb is important while resonance scattering mechanisms make only an insignificant contribution to the total scattering in n-Ge.

The MP oscillations of ρ_{xx} in Ge are so small that, in the temperature range 30–340°K, it has been possible to observe them[30,31] only by using the technique of double time-differentiation of the signal from the potential probes. In the curves $\rho_{zz}(H)$ for the longitudinal effects, the minima correspond to the resonance conditions[54,64]. Most of the extrema of ρ_{xx} and ρ_{zz} correspond to intravalley transitions with $\theta_0 = 430$°K. At the same time, in the curves of $\rho_{xx}(H)$[30] and $\rho_{zz}(H)$[64] for samples with concentrations 10^{14}–10^{15} cm^{-3} for $90 < T < 180$°K, peaks (at 155 and 80 kOe) that can be attributed only to intervalley transitions with $\theta_q = 315$°K have been reliably observed. That these peaks are due to intervalley transitions can be confirmed by the small amplitude compared with the other peaks (it is approximately an order of magnitude smaller than the amplitudes of the other peaks).

In samples with concentrations $n < 10^{14}$ cm^{-3} and $n > 10^{15}$ cm^{-3}, the MP oscillations of ρ_{zz} have been hindered from being clearly exhibited by the presence of oscillations of unknown origin (it is possible that they are associated with effects in the contact layers), the frequency and amplitude of which are greater than those of the MP oscillations. The reason for the absence of MP oscillations of ρ_{zz} in samples with $n = 2 \times 10^{12}$ cm^{-3}[31] is unclear.

MP oscillations of ρ_{xx} have also been investigated in the p-type semiconductors p-Ge[31], p-InSb and p-GaAs[39]. Peaks for light and heavy holes have been observed. The shape of the isoenergetic surfaces of the latter is not spherical. Therefore, the MPR period depends on the orientation of the magnetic field, and the effective masses found are anisotropic (Fig. 7). The complicated band-shape for the heavy holes leads to the appearance of fine structure in the peaks[31].

c) Anisotropic nonquadratic dispersion law. An anisotropic nonquadratic dispersion law holds, e.g., for holes in tellurium, in which MPR has been observed[36-38]. Tellurium, a crystal with a sharply anisotropic structure, belongs to the trigonal system and contains three atoms in the unit cell. The latter fact and also the high atomic polarizability (high atomic number) lead to the result that, despite the dominance of homopolar bonding in

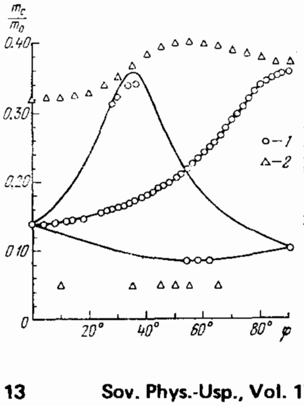


FIG. 7. Dependence of the cyclotron mass m_c of the electrons (1) and holes (2) in germanium on the angle φ between the magnetic field and the axis [100] in the (110) plane, found from measurements of MPR of ρ_{xx}[31].

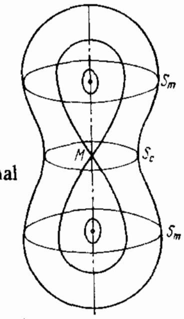


FIG. 8. Isoenergetic surfaces of the holes in tellurium for difference energies, and the types of extremal cross sections.

tellurium, the long-wave optical vibrations can be accompanied by the appearance of dipole moments. The existence of polar optical vibrations in atomic semiconductors with two atoms per unit cell (e.g., Ge) is forbidden by symmetry[65]. The polar optical vibrations in tellurium make a sufficiently effective contribution to the scattering of the charge carriers to enable MPR to be observed. As shown in[37], the greatest contribution to the scattering of holes is made by phonons with energy $\hbar\omega_0 = 13.2$ meV.

In the impurity regime, the conduction in tellurium is p-type. The minima of the valence band[3)] of tellurium are situated on neighboring edges of the six-faced prism representing the Brillouin zone, near a vertex of the prism. The dispersion law for the holes in the vicinity of the band minimum is described by the expression

$$\varepsilon = Ak_z^2 + Bk_\perp^2 - \sqrt{\lambda^2 + Ck_z^2} + D, \qquad (79)$$

where k_z and $k_\perp = \sqrt{k_x^2 + k_y^2}$ are the components of the wave-vector in the direction of the symmetry axis of the Brillouin zone and perpendicular to it, and the constants A, B, C, λ and $D = (\lambda^2 A/C^2) + (C^2/4A)$ have been determined from experimental data[66].

At energies below 2.3 meV, the constant-energy surface is an ellipsoid of revolution with axis parallel to the symmetry axis of the Brillouin zone Δ. For a hole energy of $\epsilon_0 = 2.3$ meV, two ellipsoids near one vertex of the prism merge, forming a solid of revolution resembling a dumbbell (Fig. 8).

The energy levels of the holes in a magnetic field were found in[37]. For $H \parallel c_3$ (c_3 is the threefold axis), two levels, arising from the maximum (S_m) and minimum (S_c) cross sections of the dumbbell (Fig. 8), correspond to each quantum number N. In the case $H \perp c_3$, levels with $\epsilon < \epsilon_0$ corresponding to a pair of cross sections of the ellipsoids are degenerate. For $\epsilon = \epsilon_0$, the trajectory of a hole in momentum space becomes a curve which intersects itself, and the singular point M appears on it. In this case (for $\epsilon \approx \epsilon_0$), two types of trajectory can exist simultaneously: an elliptical one and a dumbbell-shaped one of twice the area, i.e., the degeneracy of the levels is lifted as a consequence of the interaction of the orbits through the energy barrier (magnetic breakdown).

Figure 9 shows the experimental dependences of the magnetoresistance of tellurium for different orientations of the magnetic field and of the current relative to the c_3 axis. The position of the oscillation peaks was found to depend on the orientation of the magnetic field with respect to the c_3 axis. The positions of the peaks in the range 20–250°K does not depend on the temperature, but the amplitudes are greatest near 80–100°K. For the orientation $H \parallel c_3 \perp j$ and $H \parallel c_3 \perp j$, measurements on

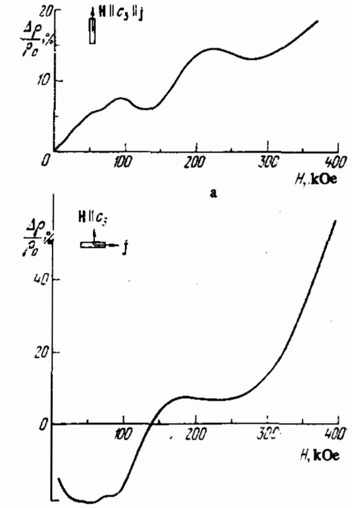


FIG. 9. Dependence on magnetic field of the longitudinal (a) and transverse (b) magnetoresistance of tellurium at 77°K. (The hole concentration $p = 1 \times 10^{15}$ cm^{-3} [37].)

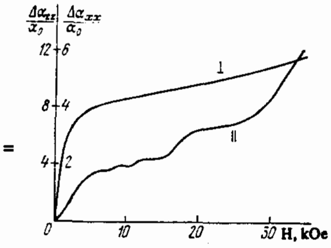


FIG. 10. Longitudinal (||) and transverse (⊥) thermoelectric power of n-InSb ($n = 2.6 \times 10^{14}$ cm^{-3} at T = 130°K[69].

samples with hole concentrations from 2×10^{14} to 4×10^{16} cm^{-3} at 77°K have shown that the positions of the oscillation extrema do not depend on the concentration. Both these facts indicate that the cause of the oscillations is MPR. It can be seen from a comparison of Figs. 9a and 9b that the minima of the longitudinal magnetoresistance correspond to maxima of the transverse magnetoresistance. This means that at 77°K the contribution of the scattering of holes by optical vibrations is small compared with the contribution of scattering by acoustic vibrations. For $H \parallel c_3 \parallel j$, an additional minimum, the nature of which is still unclear, is observed at H = 280 kOe. The rest of the pattern of MP oscillations in tellurium is in satisfactory agreement with the theoretical analysis carried out by Bresler and Mashovets[37].

Recently, Miura et al.[38] investigating MPR in tellurium, discovered that for $H \perp c_3$ the positions of the MP peaks in the transverse magnetoresistance depend on the direction of the current. Inasmuch as the cyclotron mass depends only on the direction of H, the displacement of the peaks can be caused by the participation of phonons of more than one optical mode in the scattering.

5. MAGNETOPHONON OSCILLATIONS OF OTHER KINETIC COEFFICIENTS.

a) Thermomagnetic effects and the Hall effect. A nonmonotonic dependence of kinetic coefficients on H, due to resonance interaction of electrons with optical phonons, can be manifested, obviously, not only in the magnetoresistance but also in other effects of a dissipative nature. We shall consider, e.g., the electron thermoelectric power α^e, which is due to the departure of the electrons from thermodynamic equilibrium as a consequence of a temperature gradient. The transverse thermoelectric power $\alpha^e_{xx}(H \perp \nabla_x T)$ in the lowest approximation in $(\Omega\tau)^{-1}$ does not depend on the scattering and, as was shown by Obraztsov[67], is simply related to the entropy of the electron gas:

$$\alpha^e_{xx} = \frac{S}{en}. \tag{80}$$

At the same time, the longitudinal thermoelectric power α^e_{zz} can be expressed directly in terms of the electron relaxation time[68]. Therefore, MP oscillations of α^e_{zz} are easily observed experimentally[69–71]. Figure 10[69] shows experimental curves of $\Delta\alpha_{zz}/\alpha_0$ and $\Delta\alpha_{xx}/\alpha_0$ as a function of H for n-InSb, and these clearly illustrate the difference in behavior of the dissipative and nondissipative effects. A study of the temperature dependence of the MPR of α_{zz} in n-InSb[69] shows that the amplitude of the oscillations is a maximum at T ~ 200°K. Since the monotonic part of the thermoelectric power depends weakly (logarithmically) on H[68,70], the MP oscillations are manifested better in α^e_{zz} than in the magnetoresistance. In favorable conditions, their amplitude can be of the order of magnitude of the monotonic background[69].

The positions of the extrema in the curve of $\alpha^e_{zz}(H)$ depend, as in the case of $\rho_{zz}(H)$, on the contribution of the elastic scattering mechanisms and on the temperature. Pavlov and Firsov[72] showed that, in the case when elastic scattering dominates, the minima of α^e_{zz} should correspond to the resonance fields, but reliable quantitative criteria were not obtained. From the experimental curves of[69,70], the MP maxima are shifted toward higher fields, and the minima toward lower fields.

In quantizing magnetic fields, a significant (and sometimes the main) contribution to the thermoelectric power is made by the departure of the long-wave acoustic phonons from equilibrium[70,73]. This drag thermopower α^p is proportional to the electron-phonon collision frequency. Although the optical phonons, which have a small group velocity, do not take part in the creation of the drag current, the quantity α^p_{zz}, which is proportional to the longitudinal magnetoresistance, can display MP oscillations if the contribution to ρ_{zz} from inelastic scattering is important. MP oscillations of α_{zz} in n-Ge, due to both intervalley and intravalley electron transitions, were observed in the work of[74].

MP oscillations have also been observed in the study of the Nernst-Ettingshausen (NE) effect in n-InSb[71]. The amplitude of the oscillations is considerably smaller than in the thermoelectric power. This is connected, apparently, with the effect of micro-inhomogeneities in the distribution of impurities, which in strong magnetic fields ($\Omega\tau \gg 1$) lead to a sharp increase in the monotonic NE effect but do not appreciably influence the thermoelectric power[75].

As was shown in Sec. 2, the Hall current j_y (13) is nondissipative in the zeroth approximation in the parameter $(\Omega\tau)^{-1}$. Oscillations in the Hall effect appear only in the second order in $(\Omega\tau)^{-1}$. It can easily be seen from (7) that both the small quantity σ^2_{xx} and the terms of order $(\Omega\tau)^{-2}$ from σ_{xy} make a scattering-dependent contribution to ρ_{xy}. It is clear, therefore, that the MP oscillations in the Hall effect should be extremely small. In fact, the oscillations of ρ_{xy} observed in n-InSb[76,77] and n-InP[77], with minima in the resonance fields, have amplitude two orders of magnitude smaller than the oscillations of ρ_{xx}. At the same time, in n-GaAs the ratio of the oscillating part of ρ_{xy} to the monotonic background is considerably greater[77], and the maxima correspond to the resonance fields.

b) Hot-electron MPR. At sufficiently low temperatures $T \ll \theta_0$, MP oscillations in effects that are linear in the electric field disappear, since the contribution of scattering by optical phonons to the electron collision frequency is exponentially small. However, in the region of strong electric fields the emission of phonons by the nonequilibrium system of electrons can be the principal mechanism of energy relaxation in a number of cases. In quantizing magnetic fields, the resonance character of the relaxation of the electron energy should lead to nonmonotonic dependences of the kinetic coefficients. This effect is treated theoretically in the electron-temperature approximation in[78-80]. The approximation of an electron temperature T_e may turn out to be inadequate if the electron-electron collision frequency is less than the electron-phonon collision frequency. However, it enables us to elucidate the principal qualitative features of the physical phenomena in the hot-electron regime. In the review by Zlobin and Zyryanov[81], the limits of applicability of the electron-temperature concept are investigated, and cases are treated in which a direct solution of the equation for the nonequilibrium electron distribution function is possible. In the electron-temperature approximation, the nonequilibrium distribution function is postulated to be of the form

$$f_\mu = R \exp\left(-\frac{\varepsilon_\mu}{k_0 T_e}\right), \qquad (81)$$

where R is a normalization constant and T_e is determined from the energy-balance equation

$$\mathbf{jE} = \mathscr{P}(T_e); \qquad (82)$$

here $\mathbf{j} \cdot \mathbf{E} = \sigma_{ik} E_i E_k$ is the Joule power and $\mathscr{P}(T_e)$ is the power transferred to the phonons by the system of nonequilibrium electrons:

$$\mathscr{P}(T_e) = \frac{2\pi}{\hbar} \sum_{\mu, \nu, q} (\varepsilon_\mu - \varepsilon_\nu) |C_q|^2 |\langle\mu| e^{i\mathbf{qr}} |\nu\rangle|^2 \times [(N_0 + 1) f_\mu - N_0 f_\nu]\, \delta(\varepsilon_\mu - \varepsilon_\nu - \hbar\omega_0). \qquad (83)$$

An analysis of the expression (83) shows that the power loss $\mathscr{P}(T_e)$ at a fixed value of $T_e > T$ is an oscillating function of the magnetic field. When the MPR condition (20) is fulfilled, $\mathscr{P}(T_e)$ diverges logarithmically. If the resistance is due to elastic scattering of the electrons, which is always the case at low temperatures, the Joule power $\mathbf{j} \cdot \mathbf{E}$ is a smooth function of H. Consequently, the electron temperature $T_e(H)$ defined by Eq. (82) must be a nonmonotonic function. In particular, for $M\Omega = \omega_0$, T_e becomes equal to the lattice temperature T. As a consequence of the resonance increase in the frequency of emission of optical phonons, a sharp cooling of the electron gas occurs.

Since, in magnetic fields far from the resonance fields, T_e increases with increasing electric field, the amplitude of the oscillations is determined by the electric field. Allowance for the broadening of the Landau levels leads to the removal of the divergence of $\mathscr{P}(T_e)$, and therefore T_e will be somewhat larger than T in MPR conditions. Allowance for the nonequilibrium character of the optical phonons also limits the power $\mathscr{P}(T_e)$ at resonance[82]. The nonmonotonic character of the dependence $T_e(H)$ can lead to MP oscillations of the magnetoresistance even when the relaxation of the momentum occurs as a result of elastic oollisions, and "ordinary" MP oscillations (in the ohmic region of electric fields) are impossible. MPR has not been observed in any of the materials investigated in weak electric fields $E < 50$ mV/cm in the temperature range $T < 40°$K. However, in sufficiently strong electric fields, when T_e appreciably exceeds T, MP oscillations have been observed even at liquid-helium temperatures in n-InSb[83].

Since hot-electron MPR is associated with relaxation of energy, and not of momentum, the oscillations of ρ_{xx} and ρ_{zz} should be similar both in amplitude and in phase. Since ρ_{xx} and ρ_{zz} are decreasing functions of T_e for elastic scattering by acoustic phonons or impurities, they should attain their maximum values in the resonance fields. This conclusion is confirmed by the numerical calculations of Peterson[79,80]. However, minima of ρ_{xx} and ρ_{zz} have been observed experimentally in resonance fields in n-GaAs[84] and n-InSb[85]. It is possible that this fact shows that the electron-temperature approximation is not justified in the conditions of the experiments of[84,85], i.e., the electron distribution function does not have the simple form (81). Attempts have been made[86,87] to determine the form of the nonequilibrium distribution function in strong electric fields directly from solution of the kinetic equation. It follows from the results of[86,87] that there should be resistance minima at resonance, but it is difficult to assess the validity of the approximations made (cf. also[81]). Recently, Stradling[39] has reported results from the observation of MPR in the resistance for hot electrons in n-SnSb, n-InAs, n-GaAs, n-InP and n-CdTe in the temperature range 10–20°K. In all the cases studied, the MP extrema were shifted toward lower fields relative to the resonance fields. The magnitude of the shift correlates with the ionization energy of the donors. Stradling advanced the hypothesis that, in the process of emission of an optical phonon, the electron undergoes a resonance transition from an upper Landau level to an impurity level. It is not clear, however, under what conditions this will be more probable than a resonance transition to the zeroth Landau level.

The authors of[88] observed MPR in n-InSb at T = 77°K in the weakly non-ohmic region. A special sensitive technique made it possible to observe oscillations of the "non-ohmicity" coefficient $\beta = (u(E) - u_0)/E^2$. It is interesting that the extremely small oscillations of the coefficient β ($\Delta u \sim 10^{-6} u_0$) were observed against the background of the ordinary MP oscillations of σ_{xx}, the amplitude of which amounts to several per cent of the monotonic part.

While studying the acousto-electric current in n-InSb, Colat and Bray[89] detected MP oscillations that could be interpreted using the electron-temperature approximation[80]. The acousto-electric current is inversely proportional to T_e, and therefore the effect has maxima at resonance.

c) MP oscillations of the photomagnetic effect and of the photoconductivity. In addition to SH oscillations, MP oscillations have been detected in the study of the photomagnetic effect (PME) and the photoconductivity in n-InSb at liquid-helium temperatures[90,91]. In[90], the dependence of the odd-PME voltage V_{PM} on the magnetic field was investigated for samples with $n = 2.2 \times 10^{14} - 1.1 \times 10^{17}$ cm^{-3}. With decreasing electron concentration, the SH

TABLE

Material	T, °K	$\omega_0 \times 10^{-13}$ sec^{-1}	α *)	Material	T, °K	$\omega_0 \times 10^{-13}$ sec^{-1}	α *)
InSb	4	3.72 [97]	0.02	PbSe	4	2.5 [100]	
	100	3.64 } [98]		PbS	4	3.99 [100]	
	300	3.59 } [98]		CdTe	20	3.23 [102,103]	0.3
InAs	4	4.59 [97]	0.05		300	3.22 [104]	
InP	4	6.59 } [99]	0.11	CdSe	20	4.09 [103]	0.5
	300	6.5 } [99]		(wurtzite)	300	3.98 [105]	
GaSb	4	4.53 [97]	0.025	CdS	77	5.75 [106]	0.6
GaAs	4	5.59 } [99]	0.07	(wurtzite)	300		
	300	5.5 } [99]		HgTe	10	2.17 [107]	0.1
GaP	4	7.6 [99]	0.2	Ge	300	5.65 [62]	
	300			Te	300	1) 1.81 } [108] **)	0.04 } [108]
AlSb	4	6.49 } [99]	0.13			2) 2.0 } [108] **)	0.13 } [108]
	300	6.41 } [99]				3) 2.73 } [108] **)	0.01 } [108]
PbTe	4	2.07 [100,101]					

*The coupling constants α are calculated from formula (27). For κ_0 and κ_∞ in the III-V compounds, the values given in the review [109] are taken. The values of κ_0 and κ_∞ in CdTe, CdSe and CdS are taken from [110], and in HgTe from [107]. The values of the effective masses m are tken from [111].

**Three effective longitudinal optical frequencies from data on reflection in the infrared region, with light polarized parallel to c_3(1) and perpendicular to c_3 (2 and 3), are presented. There is a misprint in the paper [108] for the coupling constant α in case (1).

oscillations are displaced toward weaker magnetic fields and, in the region of the quantum limit $\hbar\Omega > \zeta$, an additional series of peaks, the positions of which do not depend on the concentration, are detected. The additional series of oscillations is periodic in the inverse magnetic field, with a period $\Delta(1/H) = 3 \times 10^{-5}$ Oe^{-1} that coincides with the period of the MP oscillations of the kinetic coefficients in n-InSb. The minima of $V_{PM}(H)$ correspond to the resonance values of the fields for the MP oscillations. When the temperature is raised to 20°K, the MP oscillations of the PME disappear. It should be noted that 1) in the conditions of the experiment, the principal carriers in n-InSb (the electrons) play the decisive role, whereas, according to the diffusion theory, the holes should make the main contribution to the PME, and 2) the MP oscillations are detected at very low temperatures. These facts cannot be explained from the standpoint of the ordinary diffusion theory of the PME. As the authors of [92-94] have shown, to understand the observed features of the PME it is necessary to take into account the heating of the electrons by the light. In samples with sufficiently high electron concentrations, the frequency of collisions of photoelectrons with equilibrium electrons is higher than the frequency of emission of optical phonons. Therefore, the main part of the excess energy of the photoelectrons is redistributed between all the electrons. As a result, the electron distribution function has the Fermi form with an effective temperature T_e and chemical potential ζ_e somewhat different from the equilibrium values T and ζ, and T_e and ζ_e depend on the coordinates. In the photodiffusion current, there appears, together with the usual term proportional to the gradient of the electron concentration, a term proportional to ∇T_e. On application of a magnetic field, this term gives the main contribution to the short-circuiting photomagnetic current, and the PME is thus, in essence, a NE effect with an electron-temperature gradient. The appearance of MP oscillations of V_{PM} can then be understood in the following way. The probability of emission of an optical phonon by a photoelectron situated in a Landau level increases sharply when the condition (19) is fulfilled. At the same time, the electron-electron collision frequency is a smooth function of H. Hence, it follows that near a resonance the amount of energy transferred by the photoelectrons to the electron system decreases. This corresponds to a decrease of T_e. Consequently, the PME voltage, which is related to ∇T_e, should have minima when $M\Omega = \omega_0$. For sufficiently low electron concentrations, the electron-electron interaction is weak and all the photoelectrons have time to emit optical phonons. This leads to a decrease of the PME signal and to the disappearance of the MP oscillations, and this is observed experimentally. MP oscillations of the photoconductivity on heating of the electrons by the light have also been observed in CdS[95].

6. CONCLUSION

In the decade that has passed since the first observation of MPR[9], this effect has become a new and powerful means for studying the band-structure parameters of charge carriers in semiconductors. In many cases, the effective masses and their dependence on the temperature and pressure have been determined with high accuracy, comparable with the accuracy of optical and magneto-optical (cyclotron resonance) methods. However, the role of MPR in the study of semiconductors is considerably wider, and it may be hoped that other potentialities of MPR, which make it possible to study complicated details of the electron-phonon interaction, will be realized in the future Conjecturally, we can point to such effects as the polaron interaction, multiphonon processes, and broadening of the Landau levels, which all determine the lineshapes and amplitudes of the MP oscillations. Both experimentally and theoretically, these effects have not yet been sufficiently studied. Another group of little-studied problems is associated with the MP oscillations in the hot-electron regime—

these problems are the mechanisms of the energy relaxation and the form of the nonequilibrium distribution function in MPR conditions. Finally, the study of spin-magnetophonon resonance, in which electron scattering with a spin-flip occurs, could be promising. This effect, which is treated theoretically in papers by Pavlov and Firsov[96] and has been detected experimentally in n-InAs[59], can give information about the g-factor, the magnitude of the spin-orbit interaction energies and the spin-lattice relaxation time in semiconductors with strong spin-orbit interaction.

APPENDIX

Values of the limiting optical-phonon frequencies ω_0 and coupling constants α of the charge carriers with the optical phonons for certain semiconducting materials.

1)We do not consider the case of two types of charge carrier of opposite sign and equal concentrations, e.g., pure semimetals or intrinsic semiconductors. For an equal number of electrons and holes in strong magnetic fields, $\sigma_{xy} = 0$ and $\rho_{xx} = 1/\sigma_{xx}$.

2)The inelastic nature of the scattering of electrons by acoustic phonons can only be important in the region of very low temperatures or in ultrahigh magnetic fields. However, this inelasticity cannot lead to the appearance of oscillations similar to MPR, since the acoustic phonons have strong dispersion $\omega_q \sim q$.

3)The energy of the holes is assumed to increase on moving away from the boundary – the minimum – into the interior of the band.

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Translated by P. J. Shepherd