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POLARIZATION EFFECTS IN HOLOGRAPHY

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The review considers the recording and reproduction of complete as well as partial information concerning the vector characteristics of a field. The principal methods and features of producing a complete recording of the wave polarization are indicated, namely the use of two reference beams with orthogonal polarizations, coding the reference beam with random wave fronts, and the use of three-dimensional holograms. Different aspects of the use of holographic recording of partial information concerning field polarization are considered. In such a procedure, the information on the polarization characteristics of the object wave is coded by means of other field characteristics, the amplitude and the phase. In this connection, in analogy with the phase-contrast method, the concept of polarization contrast is introduced and defined as the transformation of the changes of the state of polarization over the object into variation of the intensity of the image reconstructed from the hologram. The production of holograms by this procedure is described consistently with the aid of the correlation-matrix formalism, and also with Stokes parameters. The intensity of the virtual image is expressed in terms of the coherence matrix of the object and reference waves, or else with the aid of corresponding Stokes parameters. It is shown that the polarization state of the reference wave serves as the analyzer of the vector characteristics of the object field. The method of double exposure using orthogonally linearly polarized waves for different exposures is compared with the method of single exposure with circularly polarized fields. Certain limitations imposed on the objects and conditions of the experiment, under which both methods coincide, are indicated. Experiments with a very simple object, illustrating the procedure, are described.

CONTENTS

1. INTRODUCTION

THE capabilities of a "complete" holography experiment, including the recording of information on the amplitude, phase, and state of polarization of the object wave, was first discussed in[1]. It was proposed for this purpose to use two reference sources with orthogonal polarizations not only to record but also to reconstruct the vector characteristics of the field scattered by the object. This procedure was subsequently improved many times[2,3], but remains difficult to realize experimentally to this day, and has therefore not found extensive practical applications. On the other hand, it is clear from the general concepts concerning the interference of wave fields[4] that a linearly polarized reference wave serves as an analyzer when holograms are recorded, and thus, together with the amplitude and phase characteristics, it makes it possible to record also polarization information in the form of amplitude changes of the interference pattern[5,6]. Such a method of observing the state of polarization of the field is analogous to a definite degree to the visualization of phase contrast in ordinary holography of scalar fields[7]. Similar phenomena at low intensities of the interfering fluxes were considered in[8,9] on the basis of the concept of quantum coherent packets.

It should be noted, however, that at present there is still no complete and consistent analysis of polarization phenomena in the holographic process. In this connection, we attempt in Chap. 2 of the present paper to summarize the results of the research on these questions, published through the end of 1971. In Chap. 3 we give a detailed theoretical analysis of polarization experiments of another type, which in our opinion are the most promising in holography. Finally, in Chap. 4 we give the results of original experiments that illustrate well the indicated procedure.

2. RECORDING AND RECONSTRUCTION OF VECTOR WAVE FIELDS

As already noted, the ordinary scalar hologram is not a complete recording of the field scattered by the object, since the electric field **E** in the plane of the hologram consists of two polarization components, each of which has its own amplitude and phase. The idea of recording and reconstructing the vector wave fronts was first advanced in 1965 in[1]. It was proposed to use for this purpose two references waves with linear orthogonal polarizations. When recorded, each reference wave, interfering with the component of the object wave of similar polarization, produces on the hologram a separate interference picture. These two simultaneously recorded diffraction gratings are incoherently superimposed on the hologram. The reconstruction process follows the same scheme as the recording, and for a correct reproduction of the polarization it is necessary to preserve not only the amplitudes of the two reference beams, but also the ratio of their phase shifts. Indeed, by resolving the vector $\mathbf{E}(r, t)$ of the object wave into two orthogonal components corresponding to the polarizations of the reference waves:

$$E(r, t) = \mathbf{e}_1 A_1 e^{i(\omega t - kr + \varphi_1)} + \mathbf{e}_2 A_2 e^{i(\omega t - kr + \varphi_2)}, \tag{1}$$

where $\mathbf{e}_i$, A_i, and φ_i ($i = 1, 2$) are the unit vector, amplitude, and initial phase shift of each component, we see that the Jones vector $\mathscr{E}$, which determines the character of the polarization, depends on the ratio of the amplitudes $A_2/A_1 = \tan R$ and on the phase difference $\gamma = \varphi_2 - \varphi_1$:

$$\mathscr{E} = \begin{pmatrix} \cos R \cdot e^{-i\gamma/2} \\ \sin R \cdot e^{i\gamma/2} \end{pmatrix}. \tag{2}$$

Another feature of this scheme is that the reconstruction produces both in the virtual plane and in the real plane false images obtained as a result of the diffraction of each reference wave by the other grating, which are superimposed on the true virtual and real images of the recording geometry is incorrectly chosen. This leads to uncertainties in the reconstruction of the polarization of the object wave, since sections of the object having one state of polarization in the false image have a form that is orthogonal to it. Therefore a unique reconstruction of the object wave with one hologram using two reference beams is possible only if the geometry of the recording is correctly chosen, such that the spatial spectra of two components do not overlap. An experimental realization of such a recording of the polarization is similar to the multiple-holography procedure used by Leith and Upatnieks in[10].

The recording and reconstruction principle described above was first realized experimentally in[11]. The experimental setup is shown in Fig. 1. Two plane reference waves with identical amplitudes and with different slopes (20 and 30°) relative to the object beam were used. The polarization plane of one of the reference waves was rotated through 90° with the aid of a $\lambda/2$ plate. The object, which consists of polarization plates with intersecting direction, was exposed to a circularly-polarized wave. In the reconstruction, the polarization of the virtual image of the object coincided with the polarization of the object itself, while in the real image, as expected, it gave way to orthogonal polarization. For a correct reproduction of the object, the real image must thus be viewed through a $\lambda/2$ plate. The same paper indicates the possibility of recording and reconstructing an object wave with the aid of two circularly polarized waves rotating in opposite directions. To this end, the $\lambda/2$ plate must be replaced by a $\lambda/4$ plate in each reference beam, so that their fast axes are rotated through 45 and $-45°$ relative to the initial direction of the polarization plane. Such a scheme, obviously, is suitable only for objects that emit circularly-polarized waves.

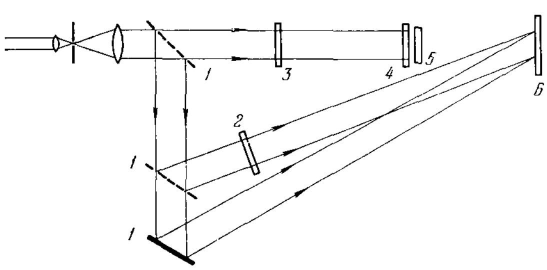

FIG. 1. Schematic diagram of setup for holographic recording and reconstruction of the vector characteristics of a field. 1—Mirrors; 2—$\lambda/2$ plate; 3—$\lambda/4$ plate; 4—scatterer; 5—object; 6—hologram.

As already noted, for a correct reproduction of the state of polarization of the object wave it is necessary to preserve the phase relations between the reference beams. This condition imposes stringent requirements on the accuracy of the setting of the hologram relative to the reconstructing waves[12], for a shift of $\lambda/2$ in the reconstruction corresponds to a change of $\pi/2$ in the phase angle γ (see (1) and (2)). In[12] they investigated the possibility of recording and reconstructing a wave transmitted through a transversely-compressed birefringent disk. The references were two spherical waves with orthogonal linear polarizations. For comparison, the virtual images of the object obtained from the holograms and the object itself were viewed through a linear analyzer. It turned out that only when the hologram is situated at the exposure location is it possible to reconstruct the initial polarization of the object, and even in this case it is necessary to compensate for the shrinkage of the emulsion by raising the temperature of the latter. For other cases when the hologram was positioned with accuracy not worse than 0.5 mm, no exact reproduction of the polarization of the object wave was observed, owing to the violation of the phase relations between the reference waves.

If the reference waves are not spherical but plane, it is possible to relax somewhat the requirements concerning the accuracy of the setting of the hologram, for in this case the longitudinal position of the hologram is less critical in the reproduction[13].

Another method interesting of recording and reconstructing the polarization of the object wave was proposed in[2]. Although the principle remains the same (the presence of two orthogonal polarizations), it makes use of one reference wave, the orthogonally-polarized components of which are coded in the hologram by different random wave fronts. The experimental setup is shown in Fig. 2a. Two coherent waves are made to expand with the aid of micro-objectives 1. One of them illuminates through scatterer 2 the object 4, which consists of birefringent ribbons with different orientations, and forms the object wave. The other wave is a reference wave and after going through a depolarizating diffuser 3 it strikes the photographic plate 5. An important feature of this method is that the linearly polarized reference beam is scattered by the diffuser and becomes depolarized, forming two orthogonal polarizations with random wave fronts, making it possible to record the polarization of the object wave and at the same time to get rid of false images during the reconstruction. The absence of false images was verified experimentally in accordance with the scheme of Fig. 2a, in which the polarizer (analyzer) 6 was removed, and a polarizer in an immersion liquid 7 was placed between the depolarizer 3 and hologram 5 during the time of exposure.

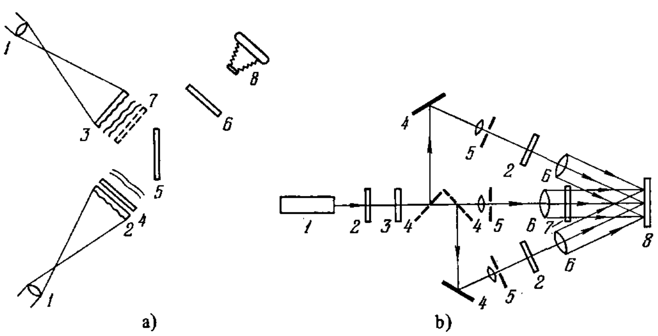

FIG. 2. Holographic schemes in which false polarization states can be eliminated by using: a) a depolarizing diffuser (1–micro-objectives, 2–scatterer, 3–diffuser; 4–object, 5–photographic plate, 6–analyzer, 7–polarizer, 8–photographic camera); b) thick emulsions (1–HeNe laser, 2–polarizers, 3–λ/4 plate, 4–mirrors, 5–micro-objectives with pinhole diaphragms, 6–lens systems, 7–object, 8–hologram).

Reconstruction with a reference beam having the same polarization made visible the corresponding polarization part of the object wave. However, when the polarizer in the immersion liquid was rotated through 90°, the reconstructed image vanished, i.e., there were actually no false images. The immersion liquid was necessary to prevent changes in the phase of the reference beam when the polarizer was rotated. The described procedure is suitable for the recording and reconstruction of the polarization of an object wave. It does, however, transform the false image into an undesirable noise[14], which usually is inconvenient. This scheme is furthermore very sensitive to the position of the diffuser and of the other optical elements.

The possibility of registering the polarization state of waves with the aid of three-dimensional holograms is described in[3]. By using the angular selectivity properties of three-dimensional holograms it is possible to suppress the false images almost completely without introducing additional noise[15]. The experimental setup is illustrated in Fig. 2b. Two plane reference waves with orthogonal polarization are used. The object wave is produced by passage of an elliptically polarized beam through a compressed V-shaped Plexiglas plate. The described setup hardly differs from that in[1], but the angles between the reference and object waves should be larger. After processing, the hologram is returned to its initial position in a special micrometric holder, and is then irradiated by the initial reference beams. An analysis of the reconstructed image with the aid of a rotating linear analyzer shows that the state of polarization of the object wave is fully reproduced in such a recording scheme if rigid control is exercised to maintain the phase relations between the reference waves.

This holographic method, like the preceding one, demonstrates the possibility of recording and reproducing the amplitude, phase, and polarization of the object wave. Such a complete recording of the vector wave field is very attractive, since the reconstructed field can be used to perform all known scalar and vector operations, such as focusing, spatial filtering, interferometry, etc., and this is of great significance in the study of short-duration birefringent processes etc. However, the realization of such methods, as shown above, entails many experimental difficulties.

We note also that since none of the described schemes can make the directions of propagation of the two reference beams identical, a rigorous analysis calls for the introduction of a three-dimensional description of the corresponding vectors **E**. As shown in[16], this leads in general to distortions of the reconstructed image; these distortions are connected with the change of R calculated from formula (2) in the hologram plane.

A method of determining the Stokes parameters in holographic experiments was proposed in[17] and yields the polarization states of the field and the Muller matrices of the object with the aid of 4 and 16 holograms, respectively.

In many practical situations, however, there is no need for a complete reproduction of the polarization state of the object wave, and only a determination of the polarization contrast is necessary, i.e., a determination of the distribution of a certain polarization state over the object. As already noted, such information can be obtained also in ordinary holography with a single reference beam. In the reconstruction there is no need to retain the same polarization of the reference beam, since the polarization of the reconstructed wave can be arbitrary, and the required polarization characteristics of the object are expressed in the form of changes of the brightness in the corresponding sections of the image. The possibilities of using this idea were discussed in[5]. The holographic method was used to investigate a glass sample under pressure, and produced the typical polarization figures corresponding to parallel and crossed polaroids in the ordinary polarization-optics scheme. Rotation of the polarization plane of the reference wave through $\pi/2$ in the recording leads to the appearance of an additional picture in the reconstructed image. These questions are discussed in greater detail in[6] as applied to birefringent crystals. A mathematical model is presented, and it is experimentally confirmed that the linearly polarized reference beam serves as an analyzer. It is emphasized that by rotating in the polarization plane of the reference beam it is easy to separate a definite linearly polarized component from the series of polarization components of the object. The possibility of using this phenomenon in the study of the characteristics of transparent photoelastic materials is mentioned in[18]. A holographic method was used in[19] to study the distribution of stresses in transparent models. The hologram yielded the isochromes (the curves along which the difference between the principal stresses are constant)[20a]. The isochromes obtained by this method hardly differ from the isochromes obtained in the usual photoelasticity methods. This method of investigating photoelasticity was subsequently used in[20b].

It should be noted that the image quality can be improved by using a non-depolarizing diffuser in the object beam[21].

An experiment in accordance with the described procedure, but with circularly polarized waves, was first performed in[22]. The photographic plate recorded in this case two holograms, one of which was formed by a component parallel to the incidence plane, and the other by a component perpendicular to this plane. If the object has no polarization properties, then the two holograms coincide, otherwise they differ and two images that interfere with each other are produced upon reconstruction. The polarization properties of the object can be assessed from the picture of this interference. It is stated that this scheme is perfectly analogous to the holographic two-exposure method. As will be shown below, this statement is not quite correct, and holds only for definite experimental situations.

3. METHOD OF POLARIZATION CONTRAST

In analogy with the phase-contrast method, i.e., the conversion of spatial phase modulation into spatial intensity modulation, we use the term polarization-contrast method to define the transformation of the changes of the state of polarization over the object into changes of the intensity of the image reconstructed from the hologram. Such a procedure does not differ technically in any way from ordinary holography, and is much simpler than the recording and reconstruction of vector wave fields. At the same time, it retains all the advantages of holography and makes it possible to observe the spatial distribution of the polarization centers. To demonstrate this, we consider a self-luminous point located in the (ξ, η) plane at a distance z_0 from the hologram (the x, y) plane:

$$\mathbf{E}(\xi, \eta) = \mathbf{D}\delta(\xi - \xi_0;\ \eta - \eta_0), \tag{3}$$

where $\mathbf{D}$ is a vector denoting the amplitude and the state of polarization of the field radiated by the object, and $\delta(\xi - \xi_0;\ \eta - \eta_0)$ is a two-dimensional δ function. In the Fresnel approximation, the amplitude of the object wave in the plane of the hologram is given by the expression

$$\mathbf{Q}(x, y) = \frac{e^{ikz_0}}{i\lambda z_0}\int\int_{-\infty}^{\infty} \mathbf{E}(\xi, \eta)\, e^{i(k/2z_0)[(x-\xi)^2+(y-\eta)^2]}\, d\xi\, d\eta = \frac{\mathbf{D}e^{ikz_0}}{i\lambda z_0} e^{i(k/2z_0)[(x-\xi_0)^2+(y-\eta_0)^2]}. \tag{4}$$

The hologram records the interference between the object and reference waves $\mathbf{Q}(x, y)$ and $\mathbf{B}\exp(i\alpha x)$, where $\alpha = k\sin\theta$ is the spatial frequency of the reference beam. The amplitude transmission T of the processed hologram is given by

$$T \sim I = \frac{|\mathbf{D}|^2}{\lambda^2 z_0^2} + |\mathbf{B}|^2 + \frac{(\mathbf{DB})\, e^{ikz_0}}{i\lambda z_0} e^{i(k/2z_0)[(x-\xi_0)^2+(y+\eta_0)^2]-i\alpha x} - \frac{(\mathbf{DB})\, e^{-ikz_0}}{i\lambda z_0} e^{-i(k/2z_0)[(x-\xi_0)^2+(y-\eta_0)^2]+i\alpha x}. \tag{5}$$

Reconstructing the image by the wave $\mathbf{C}\exp(-i\alpha_1 x)$ and taking into account in (5) only the last term, which describes the real image of the object, we obtain in the case of Fresnel diffraction

$$\mathbf{E}'(\xi', \eta') = \frac{\mathbf{C}(\mathbf{DB})\, e^{-ikz_0} e^{ik_1 z}}{(-i\lambda z_0)\, i\lambda_1 z}\int\int_{-\infty}^{\infty} e^{-i(k/2z_0)[(x-\xi_0)^2+(y-\eta_0)^2]+i\alpha x - i\alpha_1 x} \times e^{i(k_1/2z_0)[(\xi'-x)^2+(\eta'-y)^2]}\, dx\, dy, \tag{6}$$

where $k_1 = 2\pi/\lambda_1$ is the wave vector of the reconstructing wave. Imposing on (6) the focusing condition

$$k/z_0 = k_1/z$$

and putting $k = k_1$, we get from (6)

$$\mathbf{E}'(\xi', \eta') = \mathbf{C}(\mathbf{DB})\,\delta(\xi_0 - \xi';\ \eta_0 - \eta')\, e^{i(k/2z_0)[\xi'^2-\xi_0^2+\eta'^2-\eta_0^2]}. \tag{7}$$

Expression (7) describes the real image of the initial object located in the (ξ', η') plane at a distance z_0 behind the hologram. The state of the polarization in the image is determined by the reconstructing wave, and the amplitude is proportional to the scalar product of the object and reference waves, corresponding to a dependence of the intensity of the image on the polarization states of these fields. Thus, the object itself is an aggregate of luminous points with coordinates $\mathbf{r}_i$, the reconstructed picture preserves their spatial distribution, but the intensity of each of the image point is proportional to $\mathbf{D}_i \cdot \mathbf{B}$, i.e., the contrast of the polarizations is indeed converted into visible brightness contrast.

A theoretical analysis of the method of polarization contrast in holography is best carried out with the aid of the Jones vectors (see[6,23]) $\mathscr{E}_1$ and $\mathscr{E}_2$ of the object and reference waves:

$$\mathscr{E}_i = \begin{pmatrix} E_1^{(i)} \\ E_2^{(i)} \end{pmatrix}, \quad i = 1, 2. \tag{8}$$

The superscript denotes the number of the wave, and tne subscript the corresponding projection of the field intensity. The transformation of the source wave $\mathscr{E}_0$ by the object is described by the expression

$$\mathscr{E}_1 = L\mathscr{E}_0, \tag{9}$$

where L is an operator that takes into account the amplitude, phase, and polarization characteristics of the objects. In the case of a pure phase and polarization object, the operator L is unitary, i.e., $L^+ = L^{-1}$. We shall henceforth assume that the average optical path lengths l_1 and l_2 of the object and reference rays are equal after the amplitude splitting of the source wave, i.e., the following relations hold

$$\frac{l_1}{c} \approx \frac{l_2}{c} \quad \text{or} \quad \frac{l_1}{c} - \frac{l_2}{c} \ll \tau_c, \tag{10}$$

where τ_c is the coherence time and c is the speed of light. This enables us to assume the interfering fields on the hologram to be fully coherent, and to time-average only factors of the type $\exp(i\omega t)$, assuming the amplitudes and the initial phases to be constant.

The field intensity in the plane of the hologram can be written in the form

$$I = \mathrm{Sp}\,[\langle(\mathscr{E}_1 + \mathscr{E}_2) \times (\mathscr{E}_1 + \mathscr{E}_2)^+\rangle], \tag{11}$$

where the symbol $\times$ denotes the direct (Kronecker) product of the matrices, and the angle brackets denote averaging over the time. It is convenient to transform (11) in the following manner:

$$I = \mathrm{Sp}\,[\langle\mathscr{E}_1 \times \mathscr{E}_1^+\rangle + \langle\mathscr{E}_2 \times \mathscr{E}_2^+\rangle + \langle\mathscr{E}_1 \times \mathscr{E}_2^+\rangle + \langle\mathscr{E}_2 \times \mathscr{E}_1^+\rangle] = \mathrm{Sp}\,[J_{11} + J_{22} + \Gamma_{12} + \Gamma_{21}], \tag{12}$$

where J_{11} and J_{22} are the coherence matrices of the object and reference waves:

$$J_{ii} = \langle\mathscr{E}_i \times \mathscr{E}_i^+\rangle = \begin{pmatrix} G_{ii}(x_1, x_1) & G_{ii}(x_1, x_2) \\ G_{ii}(x_2, x_1) & G_{ii}(x_2, x_2) \end{pmatrix}, \quad i = 1, 2, \tag{13a}$$

Γ_{12} and Γ_{21} are the matrices of mutual correlation of the object and reference rays:

$$\Gamma_{12} = \langle\mathscr{E}_1 \times \mathscr{E}_2^+\rangle = \begin{pmatrix} G_{12}(x_1, x_1) & G_{12}(x_1, x_2) \\ G_{12}(x_2, x_1) & G_{12}(x_2, x_2) \end{pmatrix}; \quad \Gamma_{12}^+ = \Gamma_{21}, \tag{13b}$$

$G_{ij}(x_k, x_l) = \langle E_c^{(i)} E_l^{(j)*}\rangle$ (i, j, k, l = 1, 2) is the correlation function of the light-beam components. If the photographic material is developed to a contrast $\gamma = -2$, then the amplitude transmission of the hologram is given by expression (12). The amplitude of the field behind the hologram following the reconstruction is

$$\mathbf{F} = \mathbf{E}_r \mathrm{Sp}\,(J_{11} + J_{22} + \Gamma_{12} + \Gamma_{21}), \tag{14}$$

where $\mathbf{E}_r$ is the amplitude of the reconstructing wave. In analogy with the case of scalar holography, it is easy to verify that the terms J_{11} and J_{22} in (14) describe the zeroth order of the diffraction, and Γ_{12} and Γ_{21} describe the virtual and real images of the object, respectively. We confine ourselves subsequently to an investigation of the term Γ_{12} responsible for the virtual image, the intensity of which is

$$I_v = (\mathbf{E}_r \cdot \mathbf{E}_r^*)\,\mathrm{Sp}\,\Gamma_{12}\,(\mathrm{Sp}\,\Gamma_{12})^* = I_r\,\mathrm{Sp}\,(\Gamma_{12} \times \Gamma_{21}). \tag{15}$$

We have used here the relation SpA SpB = Sp(A × B). It should be noted that the correlation functions in (15) are of the equal-time type, although, if account is taken of (10), we can put

$$G_{ij}(x_k, x_l;\ t,\ t+\tau) \approx e^{i\omega\tau} G_{ij}^0(x_k, x_l;\ t,\ t), \tag{16}$$

as a result of which the product of correlation functions of the type $G_{ij}(x_k, x_l)G_{pq}(x_m, x_n)$ can be represented as follows:

$$G_{ij}(x_k, x_l)\,G_{pq}(x_m, x_n) = G_{iq}(x_k, x_n)\,G_{pj}(x_m, x_l) = G_{pj}(x_m, x_l)\,G_{iq}(x_k, x_n). \tag{17}$$

Using (16), we have

$$I_v = I_r\,\mathrm{Sp}\,(\Gamma_{12} \times \Gamma_{21}) = I_r\,\mathrm{Sp}\,(J_{11} J_{22}). \tag{18}$$

The last expression coincides with the expression describing the intensity of the wave (represented by the coherence matrix J_{11}) passing through an analyzer oriented along the direction of the vector $\mathscr{E}_2$ of the reference wave. Indeed, noting that in the latter case the coherence matrix J_{11} is transformed into

$$J'_{11} = P J_{11} P, \tag{19}$$

where P is the operator of projection on the vector $\mathscr{E}_2$, $PJ_{22}P = J_{22}$, we get

$$\mathrm{Sp}\,(J'_{11} J_{22}) = \mathrm{Sp}\,(J_{11} J_{22}). \tag{20}$$

We thus verify that the polarization state of the reference beam serves as an analyzer in the usual polarization-optical scheme.

Expression (18) contains complete information concerning the object, since the expression for the intensity of the virtual image

$$I_v = I_r\,\mathrm{Sp}\,(L J_0 L^+ J_{22}) \tag{21}$$

contains explicitly the operator L that depends in turn not only on the amplitude and phase characteristics of the object, but also on the polarization characteristics. For sufficiently complicated objects, the determination of L from (21) entails considerable mathematical difficulties. This problem, however, can be simplified by using in the experiment different states of the polarization J_0 and J_{22}. We note that the projection operator contained in (19) is generally speaking an operator of more general type than the operator $P(\theta)$ of projection on a fixed axis[24]. In particular, it can determine the projection on the direction of a vector **E** that varies arbitrarily in time and in space. In analogy with the case of a linear analyzer, we define as the projection of $\mathscr{E}$ on an arbitrary polarization state determined by the Jones vector **A** to be the following vector:

$$\mathscr{E}_A = P\mathscr{E}, \tag{22}$$

where P is the operator for which A is an eigenvector with an eigenvalue $\lambda = 1$:

$$PA = A. \tag{23}$$

It can be shown that the explicit form of the operator P is determined by the expression

$$P = \frac{1}{|A_1|^2 + |A_2|^2}\begin{pmatrix} A_1A_1^* & A_1A_2^* \\ A_2A_1^* & A_2A_2^* \end{pmatrix}. \tag{24}$$

The matrix in the right-hand side of (24) is the normalized coherence matrix for a field in the state A. If the latter has a complicated space-time dependence, then the practical determination of the projection $\mathscr{E}_A$ can be effected only by holographic methods, since the construction of an optical device realizing the transformation (22) is hardly feasible. An analysis of the experiments described in the next chapter for a relatively simple object reduces in final analysis to a calculation of the projections $\mathscr{E}_A$ on states with linear and circular polarizations.

The appearance of coherence matrices in expressions (11) and (18) enables us to express the intensity of the virtual image of the object in Stokes space. Indeed, expanding the coherence matrices J_{11} and J_{22} in Pauli spin matrices (see[24])

$$J_{11} = (1/2)\sum_{i=0}^{3} S_{1i}\sigma_i, \qquad J_{22} = (1/2)\sum_{j=0}^{3} S_{2j}\sigma_j, \tag{25}$$

(where S_{1j} and S_{2j} are the Stokes parameters for the object and reference waves, respectively) and substituting (25) in (18), we have

$$I_v = (1/4)\sum_{i,\,j=0}^{3} \mathrm{Sp}\,(S_{1i}\sigma_i S_{2j}\sigma_j) = (1/4)\sum_{i,\,j=0}^{3} S_{1i}S_{2j}\,\mathrm{Sp}\,(\sigma_i\sigma_j). \tag{26}$$

Using the relations $\mathrm{Sp}(\sigma_i\sigma_j) = 2\delta_{ij}$ (i, j = 0, 1, 2, 3), where δ_{ij} is the Kronecker symbol, we obtain from (26)

$$I_v = (1/2)\sum_{i=0}^{3} S_{1i}S_{2i} = (1/2)\,\mathbf{S}_1\mathbf{S}_2. \tag{27}$$

Expression (27) is the scalar product of the Stokes vectors of the object and reference waves.

In matrix notation, Eq. (27) takes the form

$$I_v = (1/2)\,S_1^+ S_2. \tag{28}$$

The Stokes vector of the object wave S_1 can be represented as follows[24]:

$$S_1 = [T\,(L \times L^*)\,T^{-1}]\,S_0 = MS_0, \tag{29}$$

where S_0 is the Stokes vector of the source illuminating the object, T is a certain matrix, M is the Muller matrix describing the characteristics of the object, and L is the object operator introduced earlier. When (29) is taken into account, we can represent (28) in the form

$$I_v = (1/2)\,S_0^+ M^+ S_2. \tag{30}$$

Since the operator L is in principle a Jones matrix, and expression (30) follows from (18), we can choose in each concrete case the most convenient variant for the analysis of the polarization characteristics of the object.

In conclusion, let us stop to compare the holography of polarization objects by circularly-polarized waves, on the one hand, and the method of two exposures for linear orthogonal polarizations, on the other. The amplitude transmission T of the hologram in double exposure, in analogy with (12), can be written in the form

$$T = \mathrm{Sp}\,(J_{11} + J_{22} + \Gamma_{12} + \Gamma_{21} + J_{33} + J_{44} + \Gamma_{34} + \Gamma_{43}), \tag{31}$$

where the subscripts 3 and 4 correspond to the subscripts 1 and 2, but for the second exposure. We consider again only the terms Γ_{12} and Γ_{34}, which describe in the reconstruction the virtual image of the object whose intensity (in double exposure)

$$I_{\text{v.d.e.}} = I_r \mathrm{Sp}\,(\Gamma_{12} + \Gamma_{34})\,\mathrm{Sp}\,(\Gamma_{21} + \Gamma_{43}). \tag{32}$$

Using (17), we rewrite (32) in the following manner:

$$I_{\text{v.d.e}} = I_r \mathrm{Sp}\,(J_{11}J_{22} + J_{33}J_{44} + e^{i\varphi}J_{13}J_{42} + e^{-i\varphi}J_{31}J_{24}), \tag{33}$$

where J_{13} and J_{31} are the matrices of the mutual correlation of the object waves

$$J_{13} = \langle \mathscr{E}_1 \times \mathscr{E}_3^+ \rangle = \begin{pmatrix} G_{13}(x_1, x_1) & G_{13}(x_1, x_2) \\ G_{13}(x_2, x_1) & G_{13}(x_2, x_2) \end{pmatrix};$$
$$J_{13}^+ = J_{31},$$

J_{23} and J_{42} are the correlation matrices of the reference waves

$$J_{24} = \langle \mathscr{E}_2 \times \mathscr{E}_4^+ \rangle = \begin{pmatrix} G_{24}(x_1, x_2) & G_{24}(x_1, x_2) \\ G_{24}(x_2, x_1) & G_{24}(x_2, x_2) \end{pmatrix},$$
$$J_{24}^+ = J_{42}.$$

The factors $\exp(\pm i\varphi)$ are the result of the relative change of the optical length in the first and second exposures. The first two terms in (33) describe the images of the object at different exposures, while the last two describe their interference in the case of reconstruction of the hologram. A similar procedure, in which orthogonal linearly polarized waves are used for different exposures, is not the complete analog of a single exposure using circularly polarized beams. Indeed, if the object is given by the operators

$$L = \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix},$$

and the reference and object-illuminating fields are described by the Jones vectors $\mathscr{E}_0^{\mathrm{I}}$ and $\mathscr{E}_0^{\mathrm{II}}$ for the first and second exposures, respectively:

$$\mathscr{E}_0^{\mathrm{I}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \mathscr{E}_0^{\mathrm{II}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{34}$$

then it follows from (33) that

$$I_{\text{v.d.e}} = I_r [\,|L_{11}|^2 + |L_{22}|^2 + 2\,\mathrm{Re}\,(L_{11}L_{22}^* e^{i\varphi})]. \tag{35}$$

At the same time, for a single exposure using a circularly polarized wave, whose amplitudes are $\sqrt{2}$ times larger than in the case (34), for equalization of the intensity of the reconstructed image we have

$$\mathscr{E}_0 = \begin{pmatrix} 1 \\ -i \end{pmatrix}. \tag{36}$$

From (21) we obtain

$$I_v = I_r \left\{ |L_{11} + L_{22}|^2 + |L_{12} - L_{21}|^2 \left[1 - \mathrm{Im}\left(\frac{L_{11} + L_{22}}{L_{12} - L_{21}} \right) \right] \right\}.$$

Thus, both methods give identical results if the following equation is satisfied:

$$2\,\mathrm{Re}\,[L_{11}L_{22}^*(1 - e^{i\varphi})] + |L_{12} - L_{21}|^2 \left[1 - \mathrm{Im}\left(\frac{L_{11} + L_{22}}{L_{12} - L_{21}} \right) \right] = 0. \tag{37}$$

In particular, this takes place for objects having $L_{12} = L_{21}$ at $\varphi = 0$. The latter denotes equality of the optical paths in the successive exposures, which imposes stringent requirements on the experimental conditions. We note also that expression (33) is in variant against the transformation $1 \rightleftarrows 3$, $2 \rightleftarrows 4$, $\varphi \to -\varphi$. This means that when the sequence of the exposures is reversed, the results remain unchanged only if a phase delay is introduced in an experiment of the same type. Such a condition is significant only for polarization objects, for otherwise $L_{11} = L_{22}$ and, as follows from (35), $I_{\text{v.d.e}}$ is an even function of φ.

4. EXPERIMENTAL INVESTIGATION OF THE POLARIZATION-CONTRAST METHOD

Visual observation of polarization contrast by the method described in the preceding chapter does not differ in principle in any way from the ordinary holographic experiments. We used for such investigations a setup whose diagram is shown in Fig. 3. The source of the coherence waves was an LG-36 He-Ne laser ($\lambda = 0.6328\ \mu$), operating in a one-mode regime. The laser beam was converted with the aid of $\lambda/4$ plate 2 into a right-hand circularly polarized beam. The semitransparent mirror 3 served as an amplitude splitter of the source field, forming the reference and object beams, whose average optical paths were equalized. The beams were broadened by means of micro-objectives 6. The necessary linear polarizations were produced by inserting polaroids 5 in the beam paths. A nondepolarizing diffuse scatterer 7 was placed in front of the object.

The hologram was recorded on "Mikrat VR-L" photographic plates, which were mounted in a special micrometric holder that make it possible to obtain the required number of holograms on a single photographic plate. By way of an object that illustrates well the relations given in the preceding chapter, and is at the same time simple enough to obtain the quantitative characteristics, we choose a wedge-shaped quartz crystal cut parallel to the optical axis. It was oriented in such a way that the operator L, which describes the given object, took the following form[25]:

$$L = \begin{pmatrix} e^{i\delta(x)/2} & 0 \\ 0 & e^{-i\delta(x)/2} \end{pmatrix}, \tag{38}$$

where $\delta(x) = k(n_e - n_0)x \tan\psi$, n_e and n_0 are the refractive indices of quartz for the extraordinary and ordinary

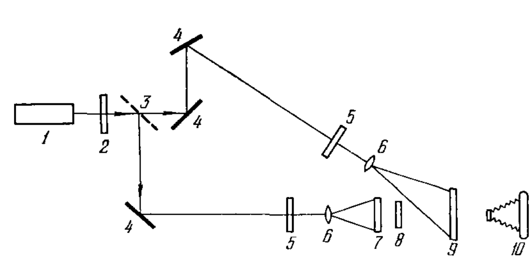

FIG. 3. Experimental setup for visual observation of polarization contrast. 1—He-Ne laser; 2—$\lambda/4$ plate; 3—semitransparent mirrors; 4—100% mirrors; 5—polarizers; 6—micro-objectives; 7—scatterer; 8—object; 9—photographic plate; 10—photographic camera.

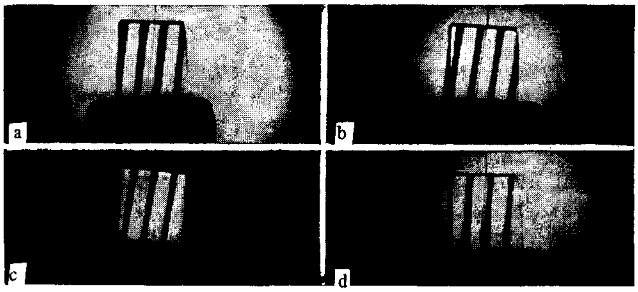

FIG. 4. Photograph of the images of a wedge-like quartz crystal, reconstructed from holograms obtained with different states of polarization of the reference and object-illuminating beams (the position of the cross hair shows the displacement of the fringes).

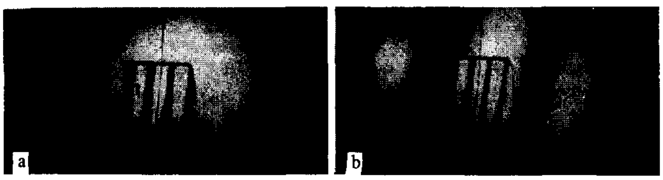

FIG. 5. Photograph of the images of a wedge-like quartz crystal, reconstructed from holograms obtained in circularly polarized light (a) and by the double exposure method (b) ($\varphi \neq 0$).

rays, x is the running coordinate along the broad leg of the wedge cross section, and ψ (1.5°) is the wedge angle. Experiments of two types were performed: 1) by the method of single exposure using linearly and circularly polarized waves, corresponding to the determination of the projections (22) of the wave scattered by the object on these states, and 2) by the method of double exposure with orthogonal linearly polarized fields.

4.1. We write down the Jones vectors $\mathscr{E}_2$ and $\mathscr{E}_0$ for the reference wave and for the object-illuminating wave, respectively:

$$\mathscr{E}_0 = \begin{pmatrix} \cos R_0 \cdot e^{-i\gamma_0/2} \\ \sin R_0 \cdot e^{i\gamma_0/2} \end{pmatrix}, \qquad (39)$$

$$\mathscr{E}_2 = \begin{pmatrix} \cos R_2 \cdot e^{-i\gamma_2/2} \\ \sin R_2 \cdot e^{i\gamma_2/2} \end{pmatrix}.$$

Substituting (39) in (21) and taking (38) into account, we obtain

$$I_v = (1/2)\, I_r \{1 + \cos 2(R_0 + R_2) + \sin 2R_0 \cdot \sin 2R_2 [1 + \cos(\delta(x) + \gamma_2 - \gamma_0)]\}.$$

Expression (40) describes in the general case a grating with a period $T_x = \lambda/(n_e - n_0) \tan\psi$, the contrast in the position of the fringes of which depends on the polarization state of the reference and the object-illuminating waves. Figures 4 and 5a show the virtual images of the object, magnified two times, obtained for the following beam parameters:

a) the reference and object-illuminating waves $\mathscr{E}_2$ and $\mathscr{E}_0$ are linearly polarized at an angle 45°, $R_0 = R_2 = 45°$, $\gamma_0 = \gamma_2 = 0$ (Fig. 4a):

$$I_v = I_r [1 + \cos\delta(x)]/2; \qquad (41)$$

b) the wave $\mathscr{E}_0$ is circularly polarized, $R_0 = 45°$, $\gamma_0 = 90°$, and $\mathscr{E}_2$ is linearly polarized at an angle 45°, $R_2 = 45°$, $\gamma_2 = 0$ (Fig. 4b):

$$I_v = (1/2)\, I_r [1 + \cos(\delta(x) - \pi/2)]; \qquad (42)$$

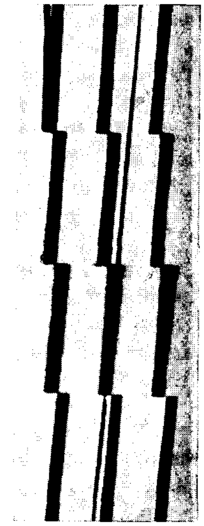

FIG. 6. Shift of fringes on a quartz crystal, due to different viector characteristics of the beams interfering on the hologram.

c) the waves $\mathscr{E}_0$ and $\mathscr{E}_2$ are linearly polarized but are orthogonal, $R_0 = R_2 = 45°$, $\gamma_0 = 0$, $\gamma_2 = \pm 180°$ (Fig. 4c):

$$I_v = (1/2)\, I_r [1 + \cos(\delta(x) - \pi)]; \qquad (43)$$

d) the wave $\mathscr{E}_0$ is circularly polarized, $R_0 = 45°$, $\gamma_0 = 90°$, $\mathscr{E}_2$ is linearly polarized at an angle −45°, $R_2 = 45°$, $\gamma = \pm 180°$ (Fig. 4d):

$$I_v = (1/2)\, I_r [1 + \cos(\delta(x) - 3\pi/2)]; \qquad (44)$$

e) the waves $\mathscr{E}_0$ and $\mathscr{E}_2$ are circularly polarized, $R_0 = R_2 = 45°$, $\gamma_0 = \gamma_2 = 90°$ (Fig. 5a):

$$I_v = I_r [1 + \cos\delta(x)]/2. \qquad (45)$$

As seen from (41)—(45), the pictures are complimentary for cases (a) and (c), while for (b) and (d) they are shifted by $\pm T_x/4 \approx \pm 0.7$ mm, while cases (a) and (e) coincide. These displacements were determined experimentally relative to a cross hair aligned with the object and are shown in Fig. 6. Quantitatively, the results agree well with the calculations. It follows from a comparison of (a) with (e) and from an analysis of (40) that for arbitrary but equal states of polarization of the waves $\mathscr{E}_0$ and $\mathscr{E}_2$ there are no fringe displacements, and all that changes is the contrast, which depends on the ratio of the field components $\tan R_i = E_2^{(i)}/E_1^{(i)}$ (i = 0, 2). If the polarizations of the field are not identical, one observes not only a change in the contrast but also a shift of the fringes, determined by the difference $\gamma_2 - \gamma_0$.

4.2. Let us examine further the double exposure method for a similar object. Using relations (35) and (38) we obtain

$$I_{v.d.e} = I_r [1 + \cos(\delta(x) + \varphi)]/2. \qquad (46)$$

The operator L from (38) satisfies the equality (37), so that at $\varphi = 0$ the pictures described by (45) and (46) coincide (we assume that the amplitudes become equalized during the recording in accordance with (36)). Figure 5b shows the virtual image of the object obtained by the double exposure method at $\varphi \neq 0$. The phase delay was introduced by means of a wedge-shaped polarizer with a very small angle, producing a periodic lattice on the diffuse scatterer behind the object, which is clearly seen in Fig. 5b.

Putting

$$\varphi = \varphi(x) + \varphi_0,$$

where $\varphi(x) = x/\theta_x$, and θ_x is the lattice period produced by the polarizer, we obtain

$$I_{v.d.e} = I_r[1 + \cos(\delta(x) + \varphi(x) + \varphi_0)]/2. \quad (47)$$

Relation (47) describes a lattice whose displacement relative to the picture for the case (e) is determined by the quantity φ_0, and the resultant period T_Σ connects T_x and θ_x by the relation

$$T_\Sigma^{-1} = T_x^{-1} + \theta_x^{-1}. \quad (48)$$

The values of T_Σ and θ_x measured from Fig. 5b agree well with expression (48). Since the phase delay was introduced by a polarizer and is thus connected with the polarization state, a reversal of the order of the exposures, as expected from (33), does not change the observed picture.

The polarization-contrast method considered above can find wide use in investigations of objects for which the spatial distribution of the polarization properties is important. We note certain fields of optical research for which such a method, in our opinion, is the most promising. This pertains first of all to an investigation of the local polarization centers in different media, including crystalline media. In the latter case, in conjunction with the double exposure technique (the "polarization" method of double exposure), it can serve as a convenient tool for the analysis of crystal structure. In the study of the photoelasticity phenomenon, the method of polarization contrast makes it possible to calculate the position and intensity of holographic interference fringes obtained by the double exposure method. Finally, this method can be used to investigate reflecting and scattering objects having a complicated structure, in the optics of turbid media, for plasma research, etc.

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