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CLOSE BINARY STARS AND THEIR SIGNIFICANCE FOR THE THEORY OF STELLAR EVOLUTION

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CONTENT

1. INTRODUCTION

UNTIL recently, the theory of stellar evolution, like the theory of the internal structure of stars, had been developed for single stars. At the same time, there are fewer single stars[1] than there are double, triple, and multiple stars in general. True, the evolution of a "multiple" star like the sun is no different from the evolution of a single star because the planets of the solar system are comparatively far from the sun and have negligible masses. The same can also be said of the majority of the so-called visual binary stars, in which the components[1)] have comparable masses but are separated from one another by distances hundreds of times greater than their diameters.

Close binary stars are another matter: their components have dimensions comparable with the distances separating them, so that various kinds of interactions take place between them—reciprocal tidal effects, strong reciprocal irradiation, exchange of matter between their outer layers, etc. The orbital motions of the components have velocities in the tens and hundreds of kilometers per second that are easily observed and measured through the Doppler shift of the spectral lines, which varies periodically with the period of the orbital revolution. Binary stars that have been identified on the basis of this criterion have come to be known as spectral binaries (SB). When the orbital plane of an SB forms a small angle to the line of sight, the observer may witness periodically recurring eclipses of one component by the other, something that is easily established with the aid of photometric measurements. Such binaries are known as eclipsing binaries (EB). Two minima—primary and secondary—may be observed during one orbital period of an EB (Fig. 1a), but the secondary minimum often escapes the observer, as when the companion has a much lower surface brightness than the primary component or is much fainter to begin with. In this case, not even the spectral lines of the companion are observed, and the periodic line shifts of the primary component alone, which reflect its motion along its orbit, determine the dimensions only of this orbit, while the orbit of the companion remains unknown.

Every EB is at the same time an SB, but the converse is not true. On the other hand, other instrumental aids equal, stars that are incomparably fainter than the spectral binaries are accessible to photometric observations. For this reason, there are many more known EB than SB. Thus, more than 4000 EB[2] and fewer than 800 SB[3] had been identified by 1970.

Analysis of spectral observations—the radial-velocity variations of the components and photometric measurements taken from the plotted brightness curve (see Fig. 1a)—yield much information on the physical characteristics of the components: their masses $\mathfrak{M}_1$ and $\mathfrak{M}_2$, radii R_1 and R_2, mean densities ρ_1 and ρ_2, the inclina-

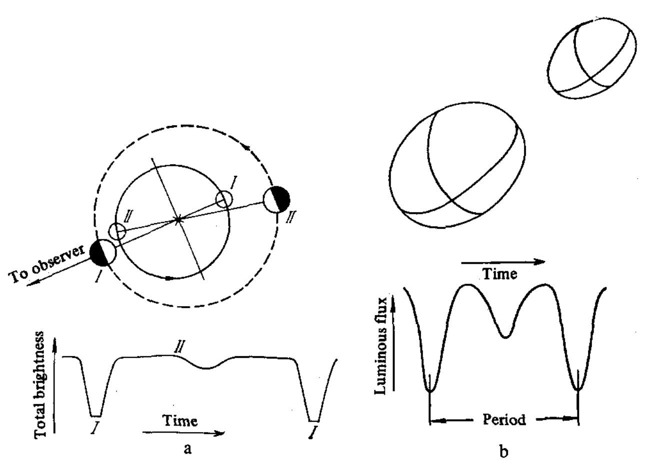

FIG. 1. a) Diagram of eclipsing binary with spherical components in elliptical orbit (showing two positions of the components in their orbits about the center of mass (X) and (at the bottom) the points corresponding to these positions on the brightness curve); b) diagram of eclipsing binary β Lyrae with ellipsoidal components, and the corresponding brightness curve (bottom).

[1)]That is, the stars composing the particular pair.

tion of the orbit to the line of sight, and the relative luminosities L_1 and L_2 of the components (assuming $L_1 + L_2 = 1$)[4,5], from which it is easy to convert to the absolute values of L in fractions of the sun's luminosity $L_\odot$ if the distance is known. On the other hand, if the components are strongly deformed by tidal effects (Fig. 1b), the degree of the deformation is determined. When the brightness curve is known with very high accuracy, the distribution of brightness over the disk of one or both of the components can be determined[5]. All of these quantities are determined without supplementary hypotheses, i.e., they may be regarded as observed quantities. But when the lines of only one component of an SB are observed, the masses and dimensions of the components cannot be determined without additional hypotheses. In this case, the mass function $f(\mathfrak{M})$, which has the sense[4]

$$f(\mathfrak{M}) = \mathfrak{M}_2^3 \sin^3 i/(\mathfrak{M}_1 + \mathfrak{M}_2)^2, \qquad (1)$$

can be derived from spectrogram measurements; here, i is the complement of the angle between the line of sight and the orbital plane, which approaches 90° for EB and is determined from observations. If we make a hypothesis concerning the mass ratio $\alpha = \mathfrak{M}_2 : \mathfrak{M}_1$ (where knowledge of L_1 and L_2 sometimes helps), formula (1), rewritten in the form

$$f(\mathfrak{M}) = \mathfrak{M}_2 \sin^3 i/(1 + \alpha^{-1})^2, \qquad (1')$$

enables us to determine the mass $\mathfrak{M}_2$ and then also $\mathfrak{M}_1 = \alpha^{-1}\mathfrak{M}_2$. We shall find a use for these arguments later on (see Sec. 7).

One of the major achievements of Twentieth Century astrophysics has been the establishment of a relation between the mass of a star and its luminosity, i.e., the total amount of energy radiated by the star per unit of time. The luminosities of stars are usually expressed in units of the sun's luminosity $L_\odot \approx 4 \cdot 10^{33}$ erg/sec. In view of the tremendously wide range of stellar luminosities (from 10^{-6} to 10^5 $L_\odot$), they are represented in logarithmic form as absolute stellar magnitudes M:

$$M = 2.5 \lg L + \text{const},$$

where the constant is so selected that the sun has $M \approx 5^m$.[2)] Statistically averaged, luminosity is proportional to the cube of the mass of the star, or, more precisely[4]

$$L \backsim \mathfrak{M}^{3+p},$$

where $p \leq 1$, but certain groups of stars may have $p < 0$. Hence it is understandable that massive stars, e.g., stars with masses of 15–30 $\mathfrak{M}_\odot$, radiate energies incomparably larger than that of the sun.

Another fundamental advance of Twentieth Century astronomy has been the construction of the Hertzsprung-Russell (H–R) diagram, which determines the positions of stars on the absolute magnitude vs. spectrum or luminosity vs. effective temperature plane.[3)] As we see from Fig. 2, the stars do not array

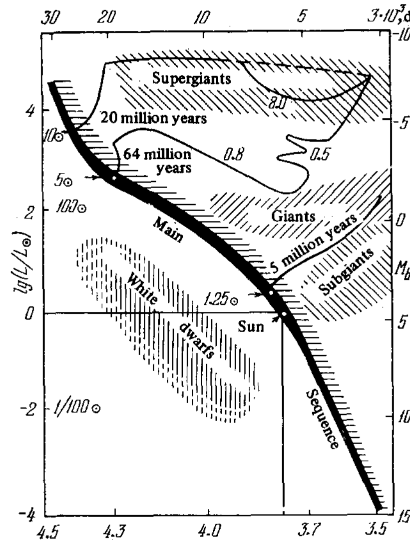

FIG. 2. Hertzsprung-Russell diagram indicating the evolutionary tracks of stars with various masses. The times (in millions of years) spent by stars on the various path segments are indicated beside the tracks. (The abscissa is $-\log T_{eff}$, with T_{eff} at the top of the figure.)

themselves uniformly on the $L - T_{eff}$ diagram, but form distinct groups or sequences.

Most of them lie approximately along the diagonal of the sequence that has come to be known as the Main Sequence (MS). The so-called dwarfs are represented in the lower part of the MS, and the hot stars at the top; the latter are known in larger numbers because, owing to their high luminosities, we can observe them and study them in detail even if they are distant from us. However, we do not call them giants, since this name is reserved for a definite group of stars that are cold but radiate 50–100 times more strongly than the sun (obviously in virtue of their very large dimensions). Below the giants we have the subgiants, which can also be regarded as excessively luminous yellow and red MS stars. The fact that their masses are generally not large inclines us to this interpretation.

The very top of the H–R diagram is occupied by the supergiants, which radiate colossal amounts of energy and have correspondingly large masses. Conversely, below the Main Sequence at the bottom of the H–R diagram, we have the white dwarfs, which have comparatively high effective temperatures—the basis for differentiating them from the ordinary Main Sequence dwarfs (red dwarfs).

Because of their low luminosities, white dwarfs are observed only in the immediate vicinity of the sun and are known in comparatively small numbers, although they may in fact constitute a highly numerous group of stars. Between the hot MS stars and the white dwarfs, there is a small number of stars that are sometimes referred to as the white-blue sequence. Still another group, the subdwarfs, is observed below the middle of the MS.

Theories of stellar evolution are tested by establishing links between the various groups of stars on the H–R diagram, with sparse population of a given region on the diagram taken as signifying a rapidly unfolding phase in the development of the star. Needless to say, it is necessary to take account of observational selec-

2) The superior m stands for stellar magnitude.

3) The effective temperature is obtained on the assumption that the star radiates as a black body, i.e., the luminosity of the star expressed in watts is $L = 4\pi R^2 \sigma T_{eff}^4$ where R is the radius of the star.

tion here. Thus, although many giants are known to us, their spatial density is extremely low. On the other hand, the white dwarfs represent some extremely long phase in stellar evolution, and the transition from the hot stars to the white dwarfs (assuming that it does occur) takes place very rapidly, like that from the MS stars to the giants. The evolution of a star can be described in very cursory fashion as follows.

It is assumed that a star forms from a condensation that has formed in a dense cosmic gas-and-dust cloud. During the compression that follows, the condensation is heated at the expense of a decrease in its gravitational potential energy. This process unfolds comparatively rapidly, and is accelerated by certain elementary thermonuclear reactions that do not require high temperatures. The result is a star; when its center has reached a temperature on the order of 10^7 °K, which supports a sufficiently effective "hydrogen-burning" reaction in which four protons are fused to form an α particle, the star enters a phase in which it exists for a long time, its "maturity." At this time, the star is represented on the H–R diagram by a point on the latter's left boundary, which is known as the initial MS. It radiates by the reactions ${}_1H^1 + {}_1H^1 + {}_1H^1 \rightarrow {}_2He^3$, ${}_2He^3 + {}_2He^3 \rightarrow {}_2He^4 + 2{}_1H^1$ (proton-proton reaction) or by the fusion of four protons to ${}_2He^4$ with participation of C^{12} and the intermediaries ${}_7N^{13}$, ${}_6C^{13}$, ${}_7N^{14}$, ${}_8O^{15}$, ${}_7N^{15}$ (Bethe carbon-nitrogen cycle) ([4], Chap. IV; [6], Chap. 1).

Depending on its original mass, the MS phase of the star lasts from 10^{10} years for stars of small mass to 10^7 years for the most massive stars. In this phase, the star becomes slightly hotter at its center, its dimensions and luminosity increase slightly, and its T_{eff} decreases, but the star does not leave the basic MS strip. In the absence of mixing of matter in the star's interior, at its very center, its hydrogen is gradually exhausted, and a helium core with no energy sources (except for the gravitational source) is formed; it will be compressed and heated, while the remainder of the star expands in the same way as a heated gas, and the "burning" of hydrogen in a layer next to the isothermal helium core comes to be the basic reaction sustaining the radiant emission of the star and the expansion of its envelope. This process, which is attended by a substantial (by tens and hundreds of times) increase in radius, takes place very quickly—during a time 1–2 orders of magnitude shorter than the MS stage. Naturally, the effective temperature T_{eff} drops off sharply with the increase in radius, and the star enters the giant phase. And since compression of the core continues, its temperature reaches ~ 200 million degrees near the center. This gives free rein to the reaction in which three α particles combine to form a carbon nucleus: $3{}_2He^4 \rightarrow {}_6C^{12}$; the reaction takes place so violently that there is not sufficient time for all of the energy to escape, the core is heated still more strongly and expands, and the layered burning of hydrogen subsides somewhat owing to the decrease in density, so that the luminosity of the star decreases after the onset of the helium flash. This is followed by compression of the envelope and an increase in the temperature T_{eff}. The combined release of energy in the layer and in the helium core heats the star to a supergiant state with moderate temperature ([6], Chap. 6; [7a]). However, this stage is reached only by stars with large masses.

The subsequent development of the star is not quite clearly understood, but since other nuclear reactions with carbon and heavier elements are not sufficiently effective, the star goes over into a short-lived phase of radiant emission at the expense of gravitational energy, shrinks rapidly, and, as the theory goes, enters the white-dwarf state, i.e., that of a stellar configuration of very (!) small size, most of whose volume is occupied by a degenerate core.[4)] Now its density reaches values around 10^7 g/cm^3, so that the degeneracy may be relativistic. It is supplanted by ordinary degeneracy in the regions next to the core, where the densities are 1–2 orders of magnitude lower; only the layers at the very surface of the white dwarf are free of degeneracy.

The shrinkage of the star—its collapse—occurs because the gas pressure is no longer capable of resisting gravitation. Theory indicates that a white-dwarf configuration with a mass larger than $1.2\mathfrak{M}_\odot$ is unstable. Such white dwarfs cannot exist. If a massive star goes over to the white-dwarf state, it must at some point eject its excess mass. It is generally believed that this ejection occurs catastrophically and is observed as a supernova outburst[8b].

2. BINARY STARS

Representatives of all groups of stars except perhaps the pulsars are encountered among the components of binaries (it is possible that two x-ray pulsars, Cen X-3 and Cyg X-1, are components of binary systems). Further, combinations of components that we regard as having widely differing ages are encountered among the star pairs, e.g., the close pair BD + 16° 516, where a K0V Main Sequence star is paired with a hot white dwarf (see Table). The red dwarf is, of course, by no means young, but the white dwarf is much older. How could they have been paired? The combination of a relatively young star of spectral class A2V with a white dwarf in the Sirius visual binary system is even more paradoxical. It differs from the first case in that the components of the Sirius system are very far apart ($P \approx 50$ years). But all of the many cases in which an MS star is combined in a pair with a subgiant, the latter having a mass smaller than that of the MS star, appear no less paradoxical: do not stars with smaller masses evolve more slowly, and does not the subgiant represent an object that has already left the Main Sequence?

The physical characteristics of the most typical representatives of the various classes of stars encountered in close binaries are assembled in the table.

Spectral observations of EB have shown that various signs of the existence of hot gaseous masses in the space between the components are frequently observed in such systems.[5)] This is most frequently manifested in the form of emission lines indicating either a gaseous ring revolving around the main component or gas streams in motion from one star of the pair to the other, sometimes at very high velocity (in the hundreds of km/sec). The case that we have in mind here is not the

[4)] The reference here is to a degenerate electron gas ([7b], Chap. 5; [8a]), whose pressure at $T = 10^7 - 10^8$ °K is considerably greater than the ion and radiation pressures.

[5)] See the excellent review of observational data in Batten [9], with its extensive bibliography.

Physical characteristics of the components of typical close binary systems

Types of stars in combination	Name of system	Period, days	Spectral classes of components	M_V	T_{eff}	Mass in units of $\mathfrak{M}/\mathfrak{M}_\odot$	Radius in units of $R/R_\odot$	Diameter of orbit, units of $A/R_\odot$
Both stars MS	Y Cygni	2.996	B0 B0	$-2^m.5$ -2.5	22 000° 22 000	17.7 17.2	5.9 5.9	28.5
Same	TT Aurigae	1.333	B3 B7	-1.4 -1.2	16 000 13 000	6.7 5.3	3.7 3.4	11.7
» »	YY Geminorium	0.814	M1 M1	9.3 9.5	3 500 3 500	0.64 0.64	0.62 0.62	1.98
Same, but contact system	W Ursae Majoris	0.334	F8 F8	4.1 4.7	6 200 6 200	1.3 0.6	1.1 0.6	2.5
MS star and subgiant	U Cephei	2.493	B8 G8III	-0.6 2.3	12 000 4 000	4.8 1.9	2.7 4.6	14.7
MS star and subgiant	ζ Aurigae	972.1	K4II B6V	-2.5 -1.1	3 700 14 000	8.3 5.3	160 2.3	960
Two giants	α Aurigae	104.0	G5III G0III	-0.3 $+0.1$	4 650 5 300	3.09 2.95	14 8	169
MS star and white dwarf	BD +16°516	0.521	K0V B	+6.3 +11	4 900 15 000	0.8 0.6	0.6 0.012	3
MS star and Wolf-Rayet star	V 44 Cygni	4.212	O6 WN6	-5 -3.5	30 000 13 000?	35 (27?) 19.5 (11?)	12.5 18	42
Supergiant and?	ϵ Aurigae	9890	F2Ia ?	-8 —	7 000 ?	35 23	290	7500
Hot MS and?	β Lyrae	12.9	B8p A7?	-3.4 —	12 000 7 700?	10 20	15	68

phenomenon of the so-called Wolf-Rayet stars (see also p. 794), where a permanent gaseous envelope that surrounds one star of the pair predominates, but that of assymetrically disposed, rather dense streams that vary in time. These streams are especially conspicuous in β Lyrae, RZ Scuti, and others[4,5].

The existence of gaseous streams in close binaries can be explained from the premises of celestial mechanics, mainly with the aid of the classical limited three-body problem[10,11].

This problem examines the motion of a point of vanishingly small mass in the gravitational field of two likewise point masses $\mathfrak{M}_1$ and $\mathfrak{M}_2$, which are in motion on circular orbits about a center of mass (CM) at the Keplerian angular velocity ω_K. The equation of motion of this point in an (x, y, z) coordinate frame in rotation at velocity ω_K will be

$$\frac{d^2\mathbf{r}}{dt^2}+2\left(\omega_K\frac{d\mathbf{r}}{dt}\right)=\operatorname{grad}\Psi, \tag{2}$$

where Ψ is the potential function (taken with the minus sign). This equation has the Jacobi's integral

$$(1/2)\,(d\mathbf{r}/dt)^2=\Psi-C,$$

where C is a constant determined by the initial conditions and the equipotential surface $\Psi = C$ is the limiting surface for motions of this kind, since we have $\Psi < C$ on more distant surfaces and the velocity would be imaginary. Hence the surface

$$\Psi=C$$

is known as the zero-velocity surface.

The initial conditions determining the constant C are as follows: if the point has the coordinates (x_0, y_0, z_0) and the velocity-vector components $[dx/dt]_0\ldots$, and its distances from $\mathfrak{M}_1$ and $\mathfrak{M}_2$ are r_{10} and r_{20}, respectively, then

$$C=\frac{1-\mu}{r_{10}}+\frac{\mu}{r_{20}}-\mu x_0+\frac{1}{2}\left[\left(\frac{dx}{dt}\right)_0^2+\left(\frac{dy}{dt}\right)_0^2+\left(\frac{dz}{dt}\right)_0^2\right]+\frac{1}{2}(x_0^2+y_0^2);$$

here $\mu = \mathfrak{M}_2/(\mathfrak{M}_1 + \mathfrak{M}_2)$.

The values of the function Ψ are large for small distances from $\mathfrak{M}_1$ and $\mathfrak{M}_2$, and the surfaces Ψ = const are isolated spheres around each of the masses. With decreasing C, these spheres are drawn out toward one another and ultimately merge into a single hourglass-shaped surface (Fig. 3) with a figure-of-eight shape in the cross section cut by the orbital plane. This surface is the limiting Roche surface, and the point at which the two cavities meet is the first Lagrangian point. With a further decrease in the value of C, the surfaces envelope both points M_1 and M_2, and they cease to be closed surfaces at the second Lagrangian point L_2.[6)]

As long as the limiting Roche surface remains the region on which motions are possible, a material point belongs to either one of the attracting masses $\mathfrak{M}_1$ or $\mathfrak{M}_2$, but if it arrives at point L_1 with zero velocity, it must remain there, since not only the velocity, but also the acceleration vanishes at L_1. Thus, matter may accumulate at the first Lagrangian point.

If a material point reaches the Roche surface with a certain velocity, it will cross this surface, and our point will be found inside the surface enclosing the entire system, i.e., it will belong to neither of the masses $\mathfrak{M}_1$ or $\mathfrak{M}_2$. This may be either a short-lived or a long-lived phenomenon, and matter may be transferred from the cavity of $\mathfrak{M}_1$ to the cavity of $\mathfrak{M}_2$ or in the reverse direction. Such transfer will be easiest through point L_1, because grad $\Psi = 0$ at this point and the transition is accomplished without performance of work. In analogous fashion, it is easiest for matter to escape the system through point L_2.

The solutions of the limited three-body problem can be applied to the motions of gas streams within a binary star system if it can be assumed that the mass of each component is concentrated at its center (Roche model).

6)We consider here only some of the solutions to the limited three-body problem.

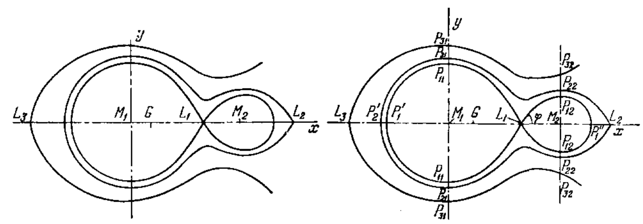

FIG. 3. Cross section of two zero-velocity surfaces in the equatorial plane for the mass ratio 1:0.216. Points L_1 and L_2 are the first and second Lagrangian points.

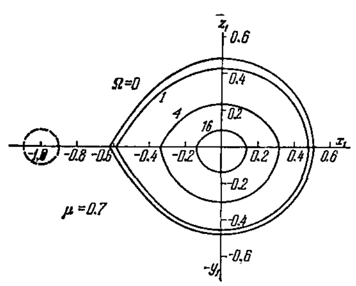

FIG. 4. Equatorial and polar sections through limiting Roche surfaces for component rotating nonsynchronously with angular velocities $\Omega = 0, 1, 2, 4$, and $16\omega_K$ for $\alpha = 0.43$.

This is not, of course, the case, but calculations[12] made for stars constructed in accordance with polytropic models with the matter concentrated to various degrees indicate that the pattern of the motions remains qualitatively unchanged, and that the dimensions of the limiting Roche surface and the position of the Lagrangian points are affected very little.

The applicability of the limited three-body problem to the motion of particles in gas streams of high density is another problem. In this case, it is necessary to take account of the gas pressure p in the equation of motion (2) (specifically, to introduce the term $-\rho^{-1}$ grad p into its right member), concerning which the celestial-mechanical theory set forth above tells us nothing. The real—gasdynamic—problem has only been stated in application to our case, and then only with very strong simplifications[13,14]. Rotation of one or both components of the pair introduces certain changes into the problem if they rotate with angular velocities Ω that differ from the orbital angular velocity ω_K[15-17]. In this variant of the limited three-body problem, it is necessary to include in the potential expression an expression for the centrifugal-force potential in a coordinate system (ξ, η, ζ) that rotates together with the star, i.e., the term $(1/2)(1 + f)^2(\xi^2 + \eta^2)$, where $1 + f = \Omega/\omega_K$, and this has as a consequence that the limiting Roche surfaces are significantly reduced in size at large f, as is shown by way of example in Fig. 4. In this case, a strong instability is created along the entire equator of the rapidly rotating star.

The relative dimensions of the cavities in the Roche surface depend only on the mass ratio $\mathfrak{M}_1 : \mathfrak{M}_2$. Detailed tables have been compiled for quantitative description of the zero-velocity surfaces[18].

Since both the mass ratios and the sizes of the components are known for many EB, an inference can be drawn as how far the surfaces of the components are from the critical Roche surface of the particular system. An analysis of this problem led Kopal[19] in 1955 to the conclusion that close binary stars are distributed along three groups: a) detached systems, in which the components have dimensions much smaller than their Roche cavities, b) semidetached systems, in which the surface of one component coincides with its Roche cavity, and c) contact systems, in which both components fill their Roche cavities. Referring to our list in the table, for example, the first three systems would be classified as detached, the fifth as semidetached, together with the first EB to be discovered—β Persei or Algol. Finally, the contact systems are represented by the fourth star on our list, W Ursae Majoris. While the detached systems are accurately described by the diagram in Fig. 1a, the contact systems are more similar to Fig. 1b, except that the components should be brought even closer together.

3. ORBIT VARIATIONS OF A CLOSE PAIR

If a star in a close pair fills its Roche-surface cavity, then even the simple thermal motion of the gas particles of its atmosphere will be sufficient for their escape either into the space around the star or through the point L_1 to the companion star. We have seen above that stellar evolution brings an MS star to the giant stage, in which its radius increases by tens and hundreds of times, and at a rather rapid rate. But then this star, as part of a close pair, must fill its Roche cavity and "overflow" continuously across the rim of that cavity or, more correctly, through the "spout" at point L_1. If this process is sufficiently stable, one star of the pair, namely the more massive one, will anticipate the evolution of its companion and begin to lose mass rapidly, transferring it to the companion, which retains approximately the same size but increases steadily in mass. As a result of such evolution, the components of the pair exchange roles and the erstwhile companion becomes the Primary star. Precisely this "role-changing" mechanism was proposed by Crawford[20] in 1955 to explain the aforementioned paradox in which an MS star is paired with a subgiant of lesser mass. This idea has been exceedingly fruitful and has been developed in numerous later studies.

Application of this mechanism to real stars requires, first of all, work on the problem of orbital-diameter variation in double stars, since the absolute dimensions of the Roche cavity depend both on the mass ratio of the components and on the absolute dimensions of the orbit. In fact, the orbital angular momentum $\mathscr{K}$ of a binary system is determined by the equation (see, for example,[15])

$$\mathscr{K}^2 = G\mathfrak{M}_1^2\mathfrak{M}_2^2(\mathfrak{M}_1+\mathfrak{M}_2)^{-1}a(1-e^2),$$

where G is the gravitational constant and a is the semimajor axis of the orbit. If no matter escapes the system, this quantity should be a constant. We then have for a circular orbit ($e = 0$) with the notation $\mathfrak{M} = \mathfrak{M}_1 + \mathfrak{M}_2$

$$a = \frac{\mathscr{K}^2(\mathfrak{M}_1+\mathfrak{M}_2)}{G}\frac{1}{\mathfrak{M}_1^2\mathfrak{M}_2^2} = \frac{\text{const}}{\mathfrak{M}_1^2(\mathfrak{M}-\mathfrak{M}_1)^2}. \tag{3}$$

This function will have its minimum value when $\mathfrak{M}_1 = \mathfrak{M}_2$.

However, if there is isotropic escape of matter from the system, the condition[19,21]

$$a(\mathfrak{M}_1+\mathfrak{M}_2) = \text{const}, \tag{4}$$

is satisfied, but only for slow mass variations. And a significant correction may be applied to condition (3) when transfer of matter from one component to the other is also accompanied by transfer of part of the rotational ("spin") momentum of the star that loses mass. Needless to say, this process is effective only if all rotations in this system are out of synchronism.

A detailed analysis shows[21,22] that when mass transfer takes place from the more massive to the less massive component, the dimensions of the orbit (and the period of revolution) diminish. The opposite effect takes place in the reverse process. We note that the masses $\mathfrak{M}_1$ and $\mathfrak{M}_2$ appear symmetrically in formula (3). If the orbit becomes smaller on $\mathfrak{M}_1 \rightarrow \mathfrak{M}_2$ mass transfer, then as $\mathfrak{M}_1$ decreases further after having become equal to $\mathfrak{M}_2$, the dimensions of the orbit should be restored before the new $\mathfrak{M}_2 : \mathfrak{M}_1$ becomes equal to the original $\mathfrak{M}_1 : \mathfrak{M}_2$. However, this does not occur if the matter transferred from $\mathfrak{M}_1$ to $\mathfrak{M}_2$ forms a rotating ring around $\mathfrak{M}_2$, since it is into this ring that the transferred angular momentum goes[23]. The interaction between the rotational and orbital angular momenta is extremely complex. Particles are either transferred to the other component or returned depending on the velocity and direction of the sample. The circumstances of transfer will also be different depending on whether the axial rotation of the star is faster or slower than its orbital motion. For particles returned to the star, momentum will be transferred from the axial rotation to the orbital revolution in the former case and in the opposite direction in the latter. But for particles transferred to the second component, as we noted above, it is important whether transfer occurs from the less massive or the more massive star. Thus, the period of a close binary star will be subject to variations in accordance with the predominance of one process or the other[15,21]. In addition, if matter leaves the system altogether, the semimajor axis of the orbit will increase in accordance with (4), and the period will increase steadily.

Observations of the changes in the periods of EB, which can be made with high accuracy, are one of the means available for quantitative verification of the arguments advanced here. These observations have shown that most EB have inconstant periods that vary both jumpwise and continuously[24,25], with alternating increases and decreases in the period. There are as many cases in which the period is increasing as there are in which it is decreasing[26].

Another tool consists in spectrographic observations of EB, primarily of their emission lines, which, as a rule, do not indicate the same radial velocities as the absorption lines, which belong to the photospheres of the stars. In addition, even the absorption lines of certain pairs (U Cephei, RW Tauri) are found to be distorted as a result of absorption by cold gas masses that mask the star to the observer. But it is the higher strength of the emission lines and the distortions of the absorption lines that indicate a substantial optical thickness for the gas streams and a rather high density of the matter in them, and, as we noted above, the pure celestial-mechanics approach to the mass-transfer problem is inadequate for quantitative analysis in such cases. However, if we look beyond the details, the population of EB with emission lines agrees statistically with calculations performed to find stable periodic orbits about a component that has captured matter from the other component[27]. Direct observations of the eclipsing binary RW Tauri during a total eclipse of the primary component indicate that when a vanishing fraction of light from the companion arrives from the system, otherwise invisible emission lines emerge distinctly in the spectrum, indicating gaseous masses in motion around the primary component with enormous velocities (up to 350 km/sec)[28]. However, this effect was found to be transitory. It did not recur at all in later observations, in accordance with the theoretical analysis, which indicated the absence of stable periodic orbits under the given concrete conditions[29].

Another factor that causes the period and, with it, the orbital dimensions of the close binary system to vary is tidal friction. It is absent when the orbital revolution (ω_K) and the axial rotation (Ω) are synchronous. If they are not synchronous, a force couple makes its appearance and tends to reconcile the two angular velocities through the viscosity of the star's matter: in its presence, the tidal elongation does not coincide with the line of centers of the two stars of the pair, and the elongation itself shifts around the body of the star. Depending on the sign of the difference $\omega_K - \Omega$, this will cause acceleration or retardation of the star's rotation as rotational angular momentum is borrowed from or converted to orbital angular momentum. The detailed theory of this problem in[30], which is hindered by the uncertainty of viscosity estimates for stellar matter, indicates that for a star with a convective shell (these are stars with small masses, e.g., 1.0–0.6 $\mathfrak{M}_\odot$), even the total absence of synchronism between rotation and revolution is eliminated after a few thousand years, and if desynchronization arises at $\omega_K = \Omega$, it is eliminated within a few hundred orbital revolutions. If, on the other hand, the star has a convective core with radiant transfer of energy in the envelope (this is a property of massive stars), the tidal effects are much weaker because of the small dimensions of the core and the time to synchronization is longer—on the order of tens of millions of years for a star with $\mathfrak{M} = 2.5\ \mathfrak{M}_\odot$ and hundreds of thousands of years at $\mathfrak{M} = 10\ \mathfrak{M}_\odot$, but in either case no more than 10% of the time spent by such a star on the MS. The tidal effect on the radiating layers of the star is incomparably weaker, and a convection-free core may retain its original rapid rotation long after the convective envelope has arrived at synchronism.

All of the above pertains to the interaction of stars with constant mass. But if matter is transferred from one component to the other, there is also transfer of angular momentum, and the synchronism that has been attained is disturbed. And even though it is restored quickly, continuation of this process will result in a continuously operative exchange of momenta between rotation and revolution in the system. The possibility of this mechanism was noted more than a quarter of a century ago[31a], but it has still not been developed quantitatively.

Finally, in very short-period systems with periods of 12 hours or less, orbital angular momentum may be substantially exhausted on the radiation of gravity waves without affecting the masses of the components[32] (see Sec. 6 on this subject).

4. EVOLUTION OF STARS IN CLOSE BINARY SYSTEMS

Thus, we shall take as our basis the evolutionary path of a single star, as developed on the basis of theoretical conceptions as to its internal structure and its energy sources and from comparisons of the observed integral characteristics with those calculated theoretically. What changes are produced in this picture when such a star enters into a star pair? As for the internal structure of the star and its energy balance, the rotational and tidal phenomena will evidently introduce no substantial changes into this picture, at least if the rotation and revolution are synchronous[33]. But, beginning at a certain phase, the evolution of the star will proceed along a significantly different path—one on which mass is rapidly lost from the outer regions of the star, which are richest in hydrogen.

As in investigation of the evolutionary path of a single star, the problem is solved by integrating the general equations of equilibrium, state, and energy transfer, but with an additional condition of mass loss, the rate of which is determined basically by the rate at which the star leaves the limits of its Roche surface. Since the dimensions of this surface decrease progressively, the loss of mass proceeds at accelerating rates. This last effect constitutes the difference between the contemporary approach to the problem and its formulation prior to Crawford's suggestion in the 1920's[34a] and later[34b], which considered that mass was lost either with radiant energy or by corpuscular radiation. Both of these processes result in the loss of an insignificant fraction of the star's mass.

While the star is on the MS, i.e., has moderate dimensions (in detached systems), it evolves like a single star, but when the hydrogen in its core has been exhausted, its dimensions increase ([6], Chap. 6;[35]) in a manner that varies in accordance with the mass of the star (cf. Fig. 2), so that the surfaces of the stars cross the Roche surfaces at different stages on their stellar evolutionary path.

Combining an approximate expression for the mean radius r_1 of the Roche cavity[18]

$$\lg (r_1/a) = -0.4221 - 0.2084 \lg (\mathfrak{M}_2/\mathfrak{M}_1)$$

with Kepler's third law in the form (see[4], formula (12.28))

$$(R_1/R_\odot)^3 = 74.4\, (r_1/a)^3\, P^2\, (\mathfrak{M}_1 + \mathfrak{M}_2),$$

we can find the orbital period P at which component I fills the Roche cavity for any values of $\mathfrak{M}_1$ and $\mathfrak{M}_2$. In particular, Paczynski[36] obtains for the values $\mathfrak{M}_1 = 5\,\mathfrak{M}_\odot$ and $\mathfrak{M}_1 : \mathfrak{M}_2 = 2$

$$\lg P_{\text{days}} = 1.5 \lg (R_1/R_\odot) - 0.84.$$

Figure 5a shows values of $R_1/R_\odot$ and the corresponding phases in the nuclear evolution of a star for this case. The time scale begins at the point at which component I leaves the Main Sequence (on which it has spent about 10^8 years). A star with a mass of $1.2\,\mathfrak{M}_\odot$ shows the same pattern of variations[37], but the process takes five to ten times longer. Naturally, the corresponding processes unfold much more rapidly for more massive stars.

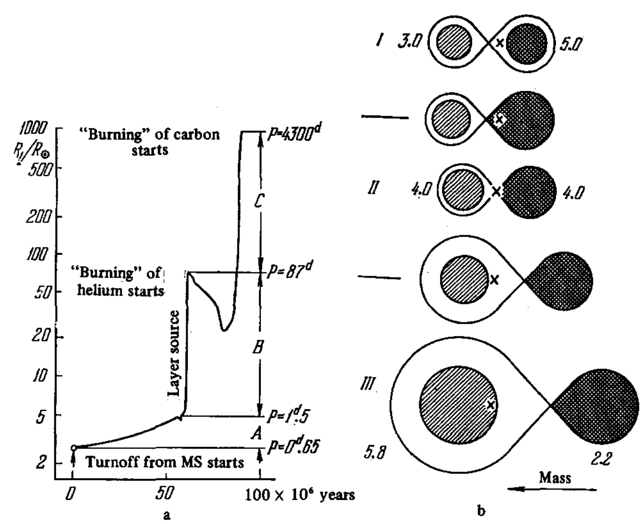

FIG. 5. a) Radius variations of star with mass $5\mathfrak{M}_\odot$ during its evolution as part of a close binary system with a secondary component of $2.5\,\mathfrak{M}_\odot$ (showing the changes in the radius of the primary component if it has reached the limiting Roche surface, and the corresponding values of the orbital period P; A, B, and C are possible mass-exchange stages) (after [36]); b) evolution of components in a close binary system with masses of 5 and $3\mathfrak{M}_\odot$ (model of the U Cephei system) and evolution of the Roche surface (the numbers indicate the masses of the components at the given stage) (after [51]).

Depending on the orbital period in the particular binary system and on the mass ratio, a given component may fill its Roche cavity while in the phase of slow radial growth (stage A in Fig. 5a), rapid growth with heating of the core followed by the 3α process (stage B), or renewed heating with activation of "carbon-burning" processes (stage C). Only massive stars with $\mathfrak{M} > 10\mathfrak{M}_\odot$ can reach this last stage, and then only in long-period systems (as, for example, VV Cephei, P = 20 years, ϵ Aurigae, P = 27 years, etc.), since they would otherwise be overtaken by mass loss at an earlier stage.

Over the past decade, the processes of stellar development with mass loss at the various stages (A, B, and C) have been treated in numerous papers, in which a wide variety of initial-mass values were assumed. As we noted above, this problem is solved by modelling in a purely computational procedure. The exposition of the results is usually prefaced by a description of the method, but independent expositions are also encountered[38].

Since Morton's first calculations in 1960[39], the greatest progress in the study of stellar evolution in close binary systems has been recorded in the papers of Warsaw[40-57], Gottingen[58-70], and Prague (Ondrejov)[71-87] scientists. Scientists of other countries[88-96] have also made appreciable contributions at various stages in these investigations. It is not possible to expound on all of these studies, which detail the development of various component combinations in close binary systems. We shall therefore confine ourselves to a description of certain typical results. But first a general remark on the process suggested by Crawford is in order.

Having lost a certain fraction of its mass, a star will tend to recover its disturbed hydrostatic equilibrium, something that is possible only when it recedes into its Roche cavity. This will take place quickly, during a time on the order of 10^4 sec. But there the star will be at variance with its internal structure and energy release, which require that it leave the confines of its Roche surface, which it does, with a new loss of mass as a result. This process is, of course, continuous, and it stops only when thermal equilibrium has finally been established in the star, i.e., in the course of the Kelvin time scale ([4], p. 217;[97]):

$$\tau = G\mathfrak{M}^2/RL = 3\cdot 10^7\,(\mathfrak{M}/\mathfrak{M}_\odot)^2\,(R_\odot/R)\,(L_\odot/L).$$

This is a very short time compared to the characteristic time of the star's nuclear evolution, even for massive stars ($\mathfrak{M} > 10\mathfrak{M}_\odot$).

Let us now examine a few particular cases. The evolution of a not particularly massive star in a close pair is among the simpler problems when the star reaches the dimensions of the Roche cavity in stage A—that of progressive exhaustion of the hydrogen in the core. By way of example, we take a system that consists originally of two MS stars with masses of 3 and $5\mathfrak{M}_\odot$[78,96].

Figure 5b shows the relative positions of the components and the Roche surface enveloping them. The orbital period in the system is 1.23 days. After 50 million years on the MS, the more massive component nears exhaustion of its core hydrogen, increases in size, and gradually fills its cavity of the Roche surface (stabe b), after which mass loss begins, slowly at first. Maintaining equilibrium, the star contracts somewhat, but its Roche cavity contracts more rapidly and the transfer of mass from the massive star to its compansion star proceeds at accelerating rates (see the argument in support of this in[94]). The dimensions of the orbit shrink progressively and the period of revolution on it also decreases; when the masses of the two components have been equalized (stage c) at four sun masses, the period reaches its minimum value of 0.9 day. Transfer of mass continues, but matter is now moving from the less massive to the more massive component, resulting in an increase in the size of the orbit and, soon thereafter, an increase in the absolute dimensions of the Roche cavity for the component under consideration, which has become the secondary component, while the erstwhile companion has become the primary body in the system (stage d). Since the size of the orbit is increasing, the mismatch between the growth rate of the companion star and its cavity is reduced and, finally, they have been equalized at stage e. The masses of the components are now $5.8 + 2.2\,\mathfrak{M}_\odot$, approaching the contemporary component-mass values in the U Cephei system—a typical EB in the group of semi-detached pairs. But the subgiant companion in this real system is much colder—it belongs to spectral class G2 or G5, and not F1III as in Fig. 6a, which illustrates the described evolution through the H—R diagram.

The evolution of the path of the component with initial mass $5\,\mathfrak{M}_\odot$ in Fig. 6a should be compared with the evolutionary path of a single star of the same mass,

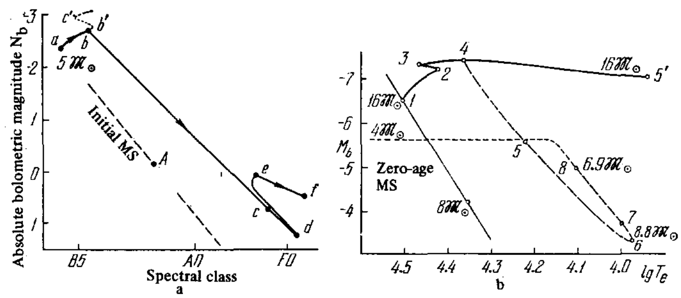

FIG. 6. a) Evolution of star (U Cephei) with mass of $5\mathfrak{M}_\odot$ as part of a close binary system with a secondary component of $3\mathfrak{M}_\odot$(A) (this scheme must be considered together with Fig. 5b and Fig. 2) [98]; b) evolution of massive star in close pair with $16 + 8\mathfrak{M}_\odot$(after [45]). The $16\mathfrak{M}_\odot$ star passes through stages 1-4 in 7 million years and stages 4-8 in 4000 years. In stage 6-7, the star loses $0.003\mathfrak{M}_\odot$ per year. Hydrogen has been exhausted from the core at point 3, the masses of the components are equalized at point 5, and the "burning" of helium begins at the knee of the dashed curve.

which is shown in Fig. 2. The difference between them is striking. It becomes especially conspicuous when we consider the rates of evolution: our star covers the distance from b to c in 70 000 years, losing about 10^{28} g ($10^{-5}\,\mathfrak{M}_\odot$) per year, but then the process slows down and 900 000 years are required for the path from c to e. And instead of entering the supergiant region (see Fig. 2), our star is found in the group of giants or even subgiants with absolute magnitudes $M_b \sim 0^m - 1^m$. This is because the star continues to lose mass in spite of the equilibrium reached in the system. The central temperature of the star (which is proportional to $\mathfrak{M}/R$), decreased during the rapid decrease in mass, the core contracted, but the 4H → He transformation continued, a as on the MS, but now with a smaller mass. The star expands, but this time slowly, losing approximately $10^{-8}\,\mathfrak{M}_\odot$ per year, so that 40 million years (from e to f) are required for a further decrease in its mass by $0.4\,\mathfrak{M}_\odot$. The mass of the star reaches $1.8\,\mathfrak{M}_\odot$, and the primary component of the pair now has a mass of $6.2\,\mathfrak{M}_\odot$.

There was an insignificant decrease in core hydrogen content during stages b—d, which our star passed through quickly. Its brightness dropped sharply, since a substantial part of the nuclear energy was expended on the mechanical work of expanding the star. A more substantial depletion of hydrogen in the core takes place during the comparatively long stage d—e, hydrogen is nearly exhausted during stage e—f, and the layer source is activated, leading to growth of the helium core. The degeneration of the electron gas in the core protects it from rapid contraction, but when the mass of the helium core reaches 0.4—0.5 $\mathfrak{M}_\odot$ ($M_b \approx -1^m$), the degeneration stops (as a result of the increase in T_c), the helium flash begins, and the star contracts rapidly, detaching itself from its Roche cavity, and the semi-detached system becomes a detached one. But this stage in the evolution of the star is the same as for a single star. It is another matter if the orbital period of the pair is so large that its components develop without mass transfer all the way to the stage of the strong layer source, when its radius increases rapidly (see Fig. 5a) during stage b and brings the surface of the star to the limiting Roche surface[41,58].

In particular, a star with mass greater than 10 $\mathfrak{M}_\odot$ will pass through the stage in which its Roche cavity is filled in approximately 10 000 years. The chances of happening upon a star pair in this phase of its development are very poor. Here the helium core continues to develop almost independently of the envelope on the time scale corresponding to nuclear evolution. The envelope is transferred almost entirely to the other component, the core contracts and is heated, and the surface of the star is also heated, so that the star crosses to the left of the Main Sequence on the H–R diagram until the vigorous course of the reaction $3\alpha \rightarrow C^{12}$ stops the expansion of the star and the transfer of mass (Fig. 6b). It is precisely for this reason that during the evolution described above, the star loses a large fraction of its mass from its outer layers, nearly down to the layer source, where a substantial part of the hydrogen has been transformed to helium; it becomes a helium star in the spectroscopic sense. In this case, determination of the helium and hydrogen contents from the spectral lines of these elements indicates that helium predominates by a considerable margin. Thus, the He : H ratio is 2.25 for β Lyrae instead of the usual 1 : 3–1 : 4[99]. At the same time, this pair can be observed as a close double one of whose components is a Wolf-Rayet star.[7)] The star spends a time on the order of 10^5 years to the left of the MS[43,62].

If the stage-B mass transfer overtakes a star of smaller mass (3–5 $\mathfrak{M}_\odot$), the helium core that is formed becomes degenerate without reaching the critical size 0.4–0.5 $\mathfrak{M}_\odot$, and this will prevent its further contraction and expansion. Nevertheless, the layer source will maintain the size of the star and its slow growth, so that although mass will be lost, it will be lost slowly, and the star will spend a long time in the subgiant stage. But we shall observe this pair as a semidetached pair of the Algol type, such as are frequently encountered among close EB. Ultimately, the core of this star will also grow to critical dimensions, the helium flash will occur, and the dimensions of the star will decrease, this time for good. The system remains semidetached, but its subgiant is "undersized," with highly excessive luminosity (up to 10^m) for its mass and an almost pure helium composition. This also applies to stars with original masses greater than 10 $\mathfrak{M}_\odot$. The difference between the two consists chiefly in the diameters of the orbits and, as a consequence, in the periods of the orbital revolution. Semidetached systems with helium subgiants have periods that are longer the larger their masses. But if the original mass was small (for example, $\mathfrak{M}_1 = 2\ \mathfrak{M}_2$), rapid transfer of matter to the other component with $\mathfrak{M}_2 = \mathfrak{M}_\odot$ will leave the first component with a mass of only 0.26 $\mathfrak{M}_\odot$[59]. The central temperature of such a star cannot rise high enough to initiate burning of helium, and, having exhausted its nuclear energy sources, the star will contract to a white dwarf. At the same time, the second component, which has become the principal one, evolves with a mass of 2.74 $\mathfrak{M}_\odot$ and may reach the yellow- or red-giant stage. In Sec. 2 we described the system of BD + 16°516B, which consists of red and white dwarfs[100,101] (see Table on p. 789). This pair is as yet the only one of its kind known to us, but there are certainly many such systems: they are merely difficult to detect owing to the small amplitude and rapidity of the brightness changes. Without mass transfer during its past history, such a combination of stars would appear totally unnatural, since, assuming normal evolution, their ages differ by at least an order of magnitude. The age discrepancy in a pair with (0.26 + 2.74) $\mathfrak{M}_\odot$, $\sim 10^{10}-3 \times 10^8$ years, is even larger.

Mass transfer can occur in the $3\alpha \rightarrow C^{12}$ stage only if a massive pair has escaped rapid mass transfer during the earlier stages of its existence. This will occur with a sufficiently long period (see Fig. 5a); in stars with small masses, however, the carbon-burning reaction cannot be initiated at all[102]. The evolution of a star during this stage is examined in[64,65,88].

An example of a pair that has experienced or is experiencing burning of carbon may be the aforementioned eclipsing system VV Cephei, which has a period of 20 years and consists of a red supergiant of spectral class M2 ($R = 1.3 \cdot 10^3\ R_\odot$) and a hot supergiant of spectral class Be; their masses are 18.3 and 19.8 $\mathfrak{M}_\odot$, respectively. Spectral observations indicate substantial gaseous streams within the system[103].

There is one more possible variant of the evolution of the close pair; it is intermediate between A and B, and in it mass transfer occurs even while hydrogen is being burned in the core[54-57].[8)]

The most common systems among EB—the contact systems—are, unfortunately, not accounted for either by the circle of ideas developed here or anywhere else. Particularly numerous among these stars are the so-called W Ursae Majoris stars, which are distinguished by continuous brightness variation much as described in Fig. 1b but with a secondary minimum of approximately the same depth as the primary minimum. The masses of the components differ little from one another—the statistically averaged mass ratio is 2 :1. This is partly due to the similarity between the sizes of the two Roche cavities, which are filled by the component stars. The spectral classes of the components vary within a comparatively narrow range, F-G-K (T_{eff} from 8000 to 4000°). The nature of the brightness variations of EB of this class and the variation of the spectral-line widths during the orbital cycle (which often appear very broad, but do not split at the nodes[9)]) suggest insistently

[7)]These are stars with extensive outer envelopes and an abundance of bright bands in the spectrum, which indicate rapid expansion of the envelope and the presence of substantial amounts of carbon and nitrogen in the atmosphere.

[8)]See the survey by Plavec [104] for a more detailed nontechnical exposition of all of the variants of mass transfer that have been studied for close systems.

[9)]The nodes of binary systems are the points at which a component, in motion on its orbit, crosses the picture plane drawn through the center of mass of the system.

that the two components of contact systems have a common envelope. Equilibrium considerations indicate that this envelope should be convective[105]. A considerable fraction of the energy produced in the massive star is transferred through this envelope to the other component. But there is no basis for transfer of matter from one component to the other, since both of them fill their Roche cavities. It is no accident that among all EB, we observe very short periods precisely in contact binaries of the W Ursae Majoris type. Mass exchange according to scheme A might occur here from the very genesis of the system, but it is difficult to say whether it has already taken place or is about to take place, because it is unclear whether the components of the pair are still moving toward the zero line of the MS or whether at least one of them is leaving the MS[105-107]. It may be that one of the components (the more massive one) has already advanced so far that the reaction of the CN cycle is in progress throughout its entire volume, while the less productive proton-proton reaction may have only begun in the fainter star.

In general, these systems exhibit much that is unique. Thus, the luminosities of the components are related as the first powers of their masses (instead of the usual exponent 3—4), and this is because the more massive component in a contact pair has deficient luminosity and, at the same time, a lowered surface brightness, so that it is the secondary rather than the primary minimum that is observed when it is eclipsed.

Tidal phenomena should play an important if not decisive role in the evolution of such a binary system, but unfortunately the literature devotes no attention to this problem.

5. SUCCESSES AND FAILURES OF THEORY

It is clear from the entire preceding exposition that the theory of transfer of matter within close binary systems has explained a number of the observed phenomena and factors. Some of them, as, for example, the observed spectral criteria for the existence of gas streams within close pairs, testify directly to movements of matter from one component to the other. Others suggest the same thing, but indirectly. Foremost among these are the previously described "impossible" component-age combinations, which are now described by the theory not only qualitatively, but quantitatively. Further, we should note the fact that there are no subgiants among the primary components in EB systems[94]. At the same time, it is precisely the subgiants in EB that exhibit a wide variety of masses and luminosities, something that is not the case with the single subgiants. It appears that the lower mass limit that exists in the Wolf-Rayet stars is also in harmony with the scheme according to which these stars were formed as a result of evolution of massive stars with mass loss at stage B before the helium flash, when a convective zone appears at the surfaces of helium stars at $\mathfrak{M} > 8\ \mathfrak{M}_\odot$[45,108].

The aforementioned variability of the periods of many EB inevitably points to the conclusion that mass transfer occurs[24,109]. In some pairs, we observe a secular decrease in the period (SV Centauri, $P = 1^d.66$), and in others an increase (β Lyrae, $P = 12^d.9$; W Serpentis, $P = 14^d.16$), sometimes occurring stepwise (RU Monocerotis, $P = 3^d.58$) and often changing sign (XZ Andromedae, $P = 1^d.36$). Period variation is frequently observed in systems that are well separated. True, all of these changes indicate only the ejection of mass from one or both of the components, but tell us nothing concerning the later fate of the ejecta. A systematic increase in the period might, for example, also suggest transfer of mass from a light component to a heavier component, or the escape of matter beyond the limits of the binary system (see (4)).

The statistics on masses in close systems offer some indication as to the nature of mass exchange[98]. In detached subgiants and in contact systems, the masses M_2 decrease statistically and the M_1 increase with decreasing mass ratio $\alpha = M_2/M_1$ from unity to zero, indicating acquisition of mass by the principal body. On the other hand, this effect is not observed in detached and semidetached systems—the M_2 decrease and the M_1 remain unchanged as α decreases; consequently, if evolution with loss of mass by the second component does occur here, the primary component gains nothing as a result, and the lost mass leaves the system. These conclusions are confirmed by statistics on the orbital angular momenta of many EB[110,111].

A more radical point of view[112] that proceeds from statistical comparison of the orbits and orbital momenta in detached and semidetached systems is also encountered: in systems with moderate masses ($\sum_i \mathfrak{M}_i < 6\ \mathfrak{M}_\odot$) when the primary component loses 84% of its mass, 2/3 of this loss escapes the system altogether. In more massive pairs ($8 < \sum_i \mathfrak{M}_i < 12$), the companion receives only 10% as the primary star loses 75% of its mass.

But if this is true, the constructions of the preceding section are to a substantial degree rendered valueless, since they are based on the hypothesis that the total mass and total angular momentum of the system remain unchanged. Since the mass ratio $q = \mathfrak{M}_1/\mathfrak{M}_2$ decrease much more slowly when the system as a whole loses mass, while the dimensions of the system increase, the loss of mass by the primary star will take place slowly and perhaps only during stage B, which includes the helium flash, during which the dimension of the star increase sharply and rapidly. This variant has not yet been treated mathematically.

Another problem is the evolution of the companion that acquires mass from the primary star and itself becomes the primary component of the binary system. Evolution in the opposite direction when the companion quickly picks up a substantial mass can be assumed purely qualitatively; its normal evolution as an MS star with the transformation 4H → He is accelerated manyfold and results in a size increase in which its surface moves out beyond the limits of its Roche cavity. This results in formation of a contacting system, but one of a singular kind: its components possess considerable masses.

This process emerges in somewhat different form when it is recognized that the descending masses of gas are in motion at supersonic velocity[113,114] and are heated in a shock wave. They are heated even further during the subsequent contraction. The secondary star becomes much hotter and brighter, its radius increases, and it comes to exceed the dimensions of its Roche cavity after having acquired a quite insignificant fraction of the mass of the primary star. If this mechanism really exists, it can result in the formation of a contacting system of moderate mass even if a substantial part of the mass escapes the system.

In any event, when substantial amounts of matter and rotational momentum are transferred rapidly from one star to the other, the latter may not be capable of accepting so much. Either the transfer is broken off at a comparatively early stage and material is ejected out of the system, in the manner just described, or a dense disk containing a substantial fraction of the system's mass and angular momentum is formed around the companion. Then the companion continues to develop normally[23]. We spoke of this earlier.

If it were proven possible for such a disk to exist and to place the other component in deep eclipse, a number of difficulties would be eliminated in the interpretation of certain EB and, in particular, one of the most typical and celebrated systems—β Lyrae, where the more massive component makes no imprint at all spectroscopically but causes deep eclipsing of the companion, which dominates in the spectrum (Fig. 7). In the model proposed in[115,116], the disk is assigned a mass equal to the mass of the star that it surrounds, on the order of 10 $\mathfrak{M}_\odot$. The stability of such a disk requires further investigation, but in spite of the doubts as to its stability, the disk hypothesis finds a confirmation in polarization observations of β Lyrae in[117-119].

6. NOVAE AND BINARY STARS

Novae and their "softened" variant, the nova-like stars (or recurrent novae) and dwarf novae occupy a special position in the problem of stellar evolution. All have outbursts—one or two, or even three, and many in the case of the dwarf novae. During an outburst, the brightness of the star increases by a factor of 10^4 for a nova and 50—100 for a dwarf nova. Spectroscopically, this is associated with ejection of a mass that is small by comparison with the mass of the star—on the order of 10^{-3} $\mathfrak{M}_\odot$ or smaller.

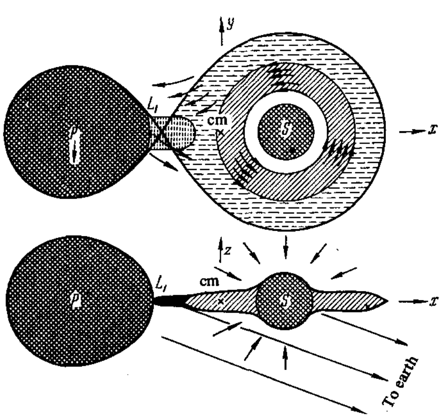

FIG. 7. Diagram of the β Lyrae system. The system is shown in two projections: at the top, it is given "in plan," the picture plane coinciding with the plane of the orbit; at the bottom, it is shown "in cross section." The symmetry plane of the disk surrounding the secondary component coincides with the plane of the orbit. Also shown are the gas streams, which can be observed spectroscopically [144a].

A remarkable factor that has been established during the last 15 years is that in all cases in which a star of one of the above types has been subjected to thorough photometric or spectroscopic analysis, it has proven to be a close binary with an ultrashort period: with two exceptions (T Coronae Borealis and GK Persei), they have periods of a few hours, or as short as 81.5 minutes in the case of one recurrent nova (WZ Sagittae).

In several cases, it has not been possible to prove that the star is a binary, but it has not been possible to exclude the possibility either, and this result has urged us to the conclusion that the phenomenon of the nova and its variations must arise out of the fact that the nova is a binary star.

We enumerate the cases of binarism that have been reliably established during recent years: four of the novae with recent outbursts[120]—N 1891 ≡ T Aurigae, N 1901 ≡ GK Persei, N 1918 ≡ V 603 Aquilae, and N 1934 Herculis[122] 10), two presumptive former novae—UX Ursae Majoris and RW Trianguli, two recurrent novae—T Coronae Borealis and WZ Sagittae[123], and thred dwarf novae—U Geminorum[124,125], SS Cygni[126], and RU Pegasi[127].

In all of these pairs, a star of late spectral class is combined with a hot star surrounded by a dense gaseous envelope or a strongly flattened disk[14]. It is the hot component that exhibits explosive activity. On the H-R diagram, this star is to the left of and below the MS, but somewhat higher than the white dwarfs (see Fig. 2), so that it is regarded as a subdwarf. However, it may be a genuine white dwarf in some cases.

The gaseous envelope or disk possesses substantial optical (and geometric) thickness, and to some degree it screens the hot star, which excites the gaseous masses to emit spectroscopically manifest light. On the other hand, the photometric pattern during and between eclipses in the systems of DQ Herculis[122], UX Ursae Majoris, and U Geminorum[124,125] is easily interpreted by positing a powerful gaseous stream issuing from the cold component, encountering the gaseous disk at supersonic velocity, and causing strong heating of the gaseous masses (in a cross section only 30 000 km in diameter). Thus, there existed or there exists in the biographies of such stars an evolutionary stage with loss of transfer of mass within the system, and this process caused ejection of particularly large amounts of mass in the novae proper.

Theoretical analysis[128,129] has shown that the accretion of hydrogen-rich matter by the surface of a subdwarf or white dwarf may cause vigorous release of energy in a thin layer source at the boundary with the degenerate 11) hot core, which contains nearly the entire mass of such stars, the only exception being a thin surface layer. Pulsating instability, which results in detachment of external masses from the star, may occur as one of the possible variants[120].

10) Systematic screening of explosive stars for binarism began with the discovery of eclipses of this star by Walker [121] in 1954.

11) The conditions of degeneracy are important here, since pressure is nearly independent of temperature when degeneracy is pronounced, and therefore the star will not dissipate its heat on expansion and configuration change. The fact that the hot core of a white dwarf is not far beneath its surface is also important.

Unfortunately, none of the authors of model calculations of this kind have applied them quantitatively to real cases, because the process of change in the white dwarf during "burning" of hydrogen has been regarded as a series of static states, while development proceeds so rapidly in the final stage that it is necessary to take dynamic effects into consideration.

Sufficiently intensive accretion of matter from the interstellar medium by a single star is, of course, improbable. Inclusion of a subdwarf or white dwarf into a binary system makes this process much easier and quite probable. But there is one difficulty inherent in the pattern described above: the very short period in the binary system in which the outburst occurs.[12)] If the component that flares up has undergone accelerated evolution with mass loss at stages b or c (to get to the degenerate stage), this occurred in a wide long-period pair (period in the hundreds of days), such as we find only in one nova—T Coronae Boraelis ($P = 227^d$). But there is no evident mechanism that would close up such a pair, leaving an insignificant mass to the component that has developed precociously. As a rule, the masses of the hot components in the systems of novae and nova-like stars are insignificant (from 1 to $0.1\mathfrak{M}_\odot$, except for T Coronae Borealis, where it reaches $2.6\,\mathfrak{M}_\odot$). Moreover, if it has reached the white-dwarf state, it cannot have a mass larger than $1.2\,\mathfrak{M}_\odot$[4], and if it had this mass earlier, it should have ejected it in some kind of catastrophic process similar to a supernova outburst[8b], but this would be ejection to the outside of the binary system, whereupon the dimensions of the orbit and the orbital period would increase.

Thus, only systems with very short periods and small masses could constitute the stage preceding the binary state with nova-like outbursts of the hot component. This again suggests contact systems[130]. A strong argument in their favor is found in the identical spatial distribution and kinematics of star systems of the W Ursae Majoris and U Geminorum types[130]. But it has not yet been possible to show how a contact system consisting of two nonmassive stars of later spectral classes is transformed into a semidetached system in which one of the components is very hot.

In those ultrashort-period systems in which the times of the minima have been recorded over a sufficiently long span of time, we observe an increase in the period, which, if it is interpreted as a consequence of mass loss by the system as a whole, indicates the following mass-loss rates (according to (4)): $2.03 \times 10^{-8}\,\mathfrak{M}_\odot$/year for DQ Herculis[122], $3.1 \times 10^{-7}\,\mathfrak{M}_\odot$/year for U Geminorum[125], and $1.8 \times 10^{-7}\,\mathfrak{M}_\odot$/year for SS Cygni[126]. The mass loss of the cold companion would be of the same order for the same change in the period under the hypothesis in which matter is transferred to the hot star. But as soon as lengthening of the period is observed, it is necessary that the companion be the less massive body in the system (see p. 791). In fact, this is the case in some systems (GK Persei, U Geminorum, WZ Sagittae), while the converse is true in others (DQ Herculis, RU Pegasi, SS Cygni). No great importance should be attached to this subdivision, since the mass determinations were made unreliably in all of the cases described (except for GK Persei and RU Pegasi).

In addition, there exists a mechanism that makes shortening of the period possible even when $\mathfrak{M}_2 < \mathfrak{M}_1$. This is the gravitational radiation[32] that we mentioned earlier, which becomes appreciable when the period is sufficiently short. According to[131], the loss of energy with gravity waves in a system of masses in rotation on circular orbits is given by

$$\frac{dE}{dt} = -\frac{32}{5}\frac{G}{c^5}\left(\frac{\mathfrak{M}_1\mathfrak{M}_2}{\mathfrak{M}_1+\mathfrak{M}_2}\right)^2 a^4\omega^6$$

($\omega = 2\pi/P$), from which we can derive the following expression for the variation of the period[32]:

$$\frac{dP}{dt} = -1.169\cdot 10^{-61}\frac{\mathfrak{M}_1\mathfrak{M}_2}{(\mathfrak{M}_1+\mathfrak{M}_2)^{1/3}}P^{-5/3}. \qquad (5)$$

At values of $P = 10^3-10^4$ sec, this results in a rapid approach of the components to one another, with the result that the companion star may prove to be larger than its Roche cavity even at very small mass values ($\mathfrak{M}_2 \approx 0.1-0.3\,\mathfrak{M}_\odot$). Transfer of matter to the hot component begins and will be self-sustaining if $\mathfrak{M}_2 > \mathfrak{M}_1$, and result in an increase in the period in spite of (5) if $\mathfrak{M}_2 < \mathfrak{M}_1$. Quantitatively, this process depends on the chemical composition of the companion[32,132,133]. If the process were constant, it would result in the formation of a system composed of a star and a satellite of planetary dimensions. If no mass transfer occurred, gravitational radiation would result in merging of the two components. However, it is well to remember that the existence of gravitational waves has not yet been confirmed experimentally.

7. "BLACK HOLES"

The theory of stellar evolution considers it possible for a star to take a path such that when its nuclear energy sources are exhausted, the star collapses catastrophically[134] to an extremely small size, smaller than the Schwarzschild gravitational radius (G is the gravitational constant):

$$R_g = 2G\mathfrak{M}/c^2.$$

The force of gravitation becomes infinite on the surface of a sphere of radius R_g (Schwarzschild sphere), and processes unfold infinitely slowly to an outside observer[135]. Thus, a star that has withdrawn inside its Schwarzschild sphere no longer has any communication with the external universe other than gravitation. Evolving toward the white-dwarf state, a star will avoid this state as soon as its mass is below the limit established by Chandrasekhar[136]:

$$\mathfrak{M}_{\rm lim} = 1.44\,\mathfrak{M}_\odot.$$

Cold spherical bodies at hydrostatic equilibrium have no equilibrium configurations if their masses exceed this limit. Incidentally, consideration of other effects lowers this limit even further, to $1.2\,\mathfrak{M}_\odot$ (the corresponding radius is $R = 250$ km). If the star had a mass

[12)] The 1934 nova in Hercules was a binary, and had the same 4 hour 34 minute period before the 1934 outburst as after it.

larger than this, it would avoid total collapse by ejecting the mass excess "in time," perhaps by the process that is manifested as a supernova outburst. If for some reason this does not happen, it becomes a "collapsar" or "frozen star"[7b,135]. The term "frozen" proceeds from the phenomenon described above in which time slows down near $R = R_g$. The process in which the star withdraws inside the Schwarzschild sphere and all phenomena on it are, as it were, frozen for an outside observer.

Otherwise, if a small mass excess (on the order of (0.5–0.8) $\mathfrak{M}_\odot$) has not been eliminated, the star will not withdraw inside the Schwarzschild sphere, but will acquire such a high density ($10^{14}-10^{15}$ g/cm^3) that the bulk of its matter is transformed into neutrons. The upper mass limit for neutron stars has not yet been accurately established (it is probably near $2\mathfrak{M}_\odot$). The recently discovered pulsars are apparently nothing other than such neutron stars. It is probable that the rotation of a star is of no small importance in determining its fate—whether it becomes a neutron star or retracts within the gravitational radius. Only a very few stars could be collapsars[7b,135,137].

It would be extremely interesting to find among the stars one for which densities greater than $10^{14}-10^{15}$ g/cm^3 were established from observation. They should be looked for among the components of binary stars—stars with large masses and negligible dimensions, a state that would be manifested in negligible or even zero luminosity coupled with substantial mass. These collapsed stars are referred to fancifully as "black holes."

Cases are encountered among the SB in which the value of the mass function (see formulas (1) and (1′)) is large and the companion is in no way in evidence. Thus, for example, in the eclipsing system β Lyrae $f(\mathfrak{M}) = 8.5\,\mathfrak{M}_\odot$; varying $\alpha^{-1} = \mathfrak{M}_{pr} : \mathfrak{M}_{co}$ from 0 to 2, we obtain (with $i = 90°$) values from 8.5 to 76.5 $\mathfrak{M}_\odot$ for $\mathfrak{M}_{co}$ and from 0 to 153 $\mathfrak{M}_\odot$ for $\mathfrak{M}_{pr}$. A reasonable choice can be made on the basis of indirect considerations. Needless to say, the extremes of the ranges given here are naturally discarded, since the visible star of the pair, the primary, has an absolute visual magnitude $M_V = -3^m.4$, which requires a substantial but by no means exceptional mass. At the same time, when $\alpha^{-1} = 1$, $\mathfrak{M}_{pr} = \mathfrak{M}_{co} = 34\,\mathfrak{M}_\odot$, and the masses of the components are even larger when $\alpha^{-1} > 1$, which is possible only if the primary component is very deficient in luminosity. A simpler hypothesis is that $\alpha^{-1} < 1$,[13)] e.g., with $\alpha^{-1} = 1/2$ we find $\mathfrak{M}_{co} = 20\,\mathfrak{M}_\odot$ and $\mathfrak{M}_{pr} = 10\,\mathfrak{M}_\odot$. Now an even more conspicuous luminosity deficiency appears for the companion. Nevertheless, we can concur with this version, since several other arguments support it[139].

The experience of spectroscopists has shown that the fainter component in a binary system cannot, as a rule, be observed in the spectrum even when the component luminosities are related as $L_1 : L_2 < 1:3-1:4$, since its spectral lines have too little contrast against the background of the primary's brighter spectrum. The eclipses that occur in the β Lyrae system enable us to establish a ratio $L_1 : L_2 = 1 : 5$. But we have just assumed that $\alpha = 2$. In normal stars, the luminosity ratio should be of the order of 8 with a mass ratio of 20 : 10. This means that the luminosity of the companion in the β Lyrae system is deficient for its mass by a factor of 40 or by 4^m. In fainter stars (e.g., R Canis Majoris), we observe an even greater discrepancy between mass and luminosity in the other direction. Generally speaking, therefore, we should not conclude from a large value of the mass function for an SB with one spectrum that the companion is a "black hole" or, more generally, a collapsar. Such SB have been discussed in the literature[134,140], but in no case has there been even the slightest conviction that there is a collapsar in any of them. Statistical considerations[141–143] have also been invoked, and also unsuccessfully.

The same uncertainty would still prevail for the β Lyrae system if it were only an SB. But the eclipses observed in this system have left the collapsar hypothesis as good as refuted, since the eclipsing of the primary means that the companion has nonzero size, and the secondary brightness minimum suggests that the companion has nonzero luminosity. Moreover, a spectral class has been arrived at for it from photometric observations: from A7 to F2 (i.e., $T_{eff} \approx 7000-8000$ degrees).

We have already said that difficulties in interpreting the brightness curve of β Lyrae have made it necessary to depart significantly from the classical scheme of Fig. 1b and to assume that the companion is surrounded by a large gaseous disk that is strongly flattened toward the plane of the orbit. We have noted that the possibility of the existence of such a disk has been neither proven nor refuted dynamically[144b]. But while a disk with a mass equal to the mass of the star immersed in it[115] had previously appeared highly improbable, the hypothesis of the collapsar, in which almost the entire mass of the companion is concentrated, enables us to assume the possible existence of such a disk. There is no limit on the stationary gravitational field of the collapsar. Its great mass tends to favor the "black hole" rather than the neutron star.

On accretion of particles to a collapsar in a binary system, the particles acquire nearly relativistic velocities (from the very strong gravitational field), and strong x-ray emission[7b,135,145] appears in combination with weak optical emission in the resulting shock wave with its temperatures on the order of 10^7-10^8 °K: L $L \approx 10^{29}-10^{31}$ erg/sec. The same thing might result from synchrotron radiation of a magnetized plasma compressed by accretion. In one way or another, an x-ray and optical aureole forms around the collapsar. It goes unnoticed in the light of the bright primary component. In a binary star, a collapsar behaves as an x-ray source—the only one in the system, since the other component, a normal star, does not emit x-radiation. The theory indicates the possibility of x-ray flares on such stars, and several flaring x-ray sources—Cen X-2, Cen X-4, and Cyg X-1—are known to us.

[13)] It is curious that the first investigator of β Lyrae as an SB, A. A. Belopol'skiy, made precisely this assumption [138] and found $\mathfrak{M}_{pr} = 8.4\mathfrak{M}_\odot$ and $M_{co} = 19.0\mathfrak{M}_\odot$.

But the β Lyrae system itself has not been identified as an x-ray source, although its distance from us is small (260 parsecs $\approx 8 \times 10^{20}$ cm). The collapsar hypothesis fails in this case. Nor have other stars closely similar to β Lyrae[146] been identified as x-ray sources.

The well-known x-ray source Cygnus X-1, from which radio emission at frequencies of 1495 and 2695 MHz was recently detected[147], is a more encouraging prospect in the search for collapsars. Its exact position in the radio band has been determined (to within a few seconds of arc), placing it close to the star HD 226868 $\equiv$ BD + 34°3815. Its magnitude is $8^m.89$ (V), and its spectrum is that of a hot B0Ib supergiant[148]. In 1971, this star was subjected to thorough and exhaustive study[149], which failed to bring out any significant deviations from the norm, but it was found to have a periodic radial motion and was therefore classified as an SB with one observable spectrum[150]. Its period of revolution $P = 5^d.60$, $f(\mathfrak{M}) = 0.12\,\mathfrak{M}_\odot$, and the semi-major axis of the orbit of the bright star $a_B \sin i = 6.6\, R_\odot$, so that the orbit is smaller than the star itself. The previously observed variations in the flux of x-radiation from the Cygnus X-1 source are also subject to a period of $5^d.60$, but the sharp decreases in the flux occur at phases in which the supergiant is behind the invisible companion, so that the x-ray source is eclipsed by that companion or, more accurately, the source is above the surface of the companion on the side toward the primary star. This is consistent with our conception in which a stream of matter is transferred from the supergiant, which in this case has overgrown the limits of its Roche cavity. The authors of[150] find from their ingenious calculations that the B0Ib supergiant has a radius $R_B \geq 10\, R_\odot$ and a mass in the range $10\,\mathfrak{M}_\odot \leq \mathfrak{M}_B \leq 30\,\mathfrak{M}_\odot$, giving a mass in the range 2.5–6.0 $\mathfrak{M}_\odot$ for the invisible companion. It may, of course, be a normal B0V or B8I star, but the fact that it emits x-radiation justifies regarding it as a collapsar that has withdrawn into its Schwarzschild gravitational radius. Needless to say, everything that we have said here holds provided that the identification of the Cygnus X-1 source with HD 226868 is correct.

Our refusal to grant collapsar status to the companion of β Lyrae is not categorical. New facts[151a] and arguments[151b] that have recently become known incline us in this direction. Photometric measurements in six segments of its ultraviolet spectrum from 3300 to 1380 Å have indicated[151a] that the secondary minimum in this system becomes deeper the shorter the wavelength and would be as deep as the primary minimum somewhere around 1200 Å. This should not happen if both stars radiated as black bodies (at any temperature, *a hotter body radiates more from 1 cm² of surface* than a colder body at any wavelength). We may therefore suppose that the hot, massive companion in this system emits a great wealth of short-wave radiation that is transformed in the surrounding gaseous disk. However, the companion cannot have a high luminosity, since the light pressure would make the existence of the disk impossible.

In actuality, as we have seen above, x-ray radiation arises during accretion near and around a collapsar, in the dense parts of the disk. This radiation is transformed in the disk to ultraviolet and visible radiation with a power of 10^{35}–10^{36} erg/sec or more (which we indeed observe). However, *the x-radiation finds egress* in directions nearly normal to the disk, where the optical thickness is lowest. Only in these directions can a binary system of this kind be observed as an x-ray source. But as we observe an eclipsing system, we are always at a certain angular elevation above the plane of the orbit (and the disk).

We encounter one more collapsar candidate in the ϵ Aurigae system. This is the binary system with the longest period among EB—its period is about 27 years ($\sim 10^4$ days). Difficulties arose in its interpretation due to the fact that when its primary component, an F2Ia supergiant, was in total eclipse, its spectrum remained visible. One of the latest models of this system[152] supposes that the primary minimum occurs when the disk of dust around the companion eclipses the primary star. Support of the disk would require a massive body (with $\mathfrak{M} \geq 23\,\mathfrak{M}_\odot$), and if this body is not manifested in the spectrum, we might suppose it to be a collapsar[153].

All of the systems considered here require intensified study, since we still cannot state with confidence that collapsars have been discovered among the eclipsing binaries.

8. USE OF CLOSE BINARIES TO TEST THEORIES OF THE INTERNAL STRUCTURE OF STARS

Existing theories of the internal structure of stars are tested by comparing the calculated and observed characteristics of the stars, such as mass, luminosity, and radius. The two latter quantities are related to one another by the effective temperature T_{eff}, which determines the total energy flux leaving the star:

$$L = 4\pi R^2 \sigma T_{eff}^4,$$

on the (quite reasonable) assumption that the star radiates as a black body. The mass of a star is also connected with its luminosity by a relationship that is different for different groups of stars. Nor is the position of a star on the H-R diagram ($f(L, T_e)$) arbitrary; *it is determined by the group to which the star belongs.* In modelling stars, however, the theoretician invokes definite relationships pertaining to the mode of energy generation in the star and the mode of radiant-energy propagation from its center toward its periphery. No assumption is sacrosanct in this plan, and this combination of properties of the star—L, $\mathfrak{M}$, and R—can be arrived at by different paths, i.e., with different density, temperature, and pressure variations with movement from the surface toward the center of the star. The values of these quantities in the interior of the star cannot be observed. Only in the case of the sun does the neutrino radiation produced in its central regions in the thermonuclear reactions that determine the generation of energy in it become an object of observation. The stars are too remote for detection of their neutrino radiation.

However, the dependence of the density ρ on the distance r from the center of the star, i.e.,

$$\rho = f(r),$$

determines the moment of inertia of the star quite exactly, and for a rotating star that is part of a binary system, the rotational flattening and tidal deformation are definite functions of the moments of inertia about the axes, and of the velocity of rotation as a function of the mass of the perturbing body and its distance from the star in question.

Thus, the deviation of the figure of a binary component from sphericity depends on its internal structure. Unfortunately, the nonsphericities of the components can be determined only very roughly for EB and can serve only for rough qualitative estimates of $\rho(r)$.

But there is another quantity that can be derived more reliably from observations. Newton himself knew that the attraction of an ellipsoid of revolution decreases along the equatorial plane more rapidly than in accordance with the law of inverse squares. Then the orbit of another body in rotation near this ellipsoid will not be a closed ellipse, but an ellipse that rotates in its own plane, so that its major axis—the line of apsides—rotates uniformly forward in the direction of the orbital motion at a velocity that depends on the moments of inertia, masses, and relative dimensions of the components[31,154,155].

The theory of rotation of the line of apsides in binary systems has now been elaborated most completely in the papers of Cowling[156] and Sterne[157] under the following assumptions: the periods of rotation and revolution of the stars are the same (synchronism of rotation); the components rotate as though they were solid bodies (rigid rotation). Then the ratio of the orbital period P to the period of revolution U of the line of apsides is determined as the sum of terms of the fifth, seventh, etc. powers of the relative sizes (sizes referred to the sizes of the orbits) r_1 and r_2 of the components. If we write only the first few terms, the expression will be

$$\frac{P}{U} = k_{21} r_1^5 \left\{ \frac{\mathfrak{M}_2}{\mathfrak{M}_1} [15 f_2(e) + g_2(e)] + g_1(e) \right\} + k_{22} r_2^5 \left\{ \frac{\mathfrak{M}_1}{\mathfrak{M}_2} [15 f_2(e) + g_2(e)] + g_2(e) \right\} \ldots; \tag{6}$$

here the functions f and g are pure functions of orbital eccentricity, for which tables have been compiled[158]. The terms k_{21} and k_{22} reflect the inhomogeneity of the component stars and are determined by the $\rho(r)$ law. These coefficients (and coefficients for higher-order terms) have been evaluated in the theory of rotationally and tidally deformed polytropic gaseous spheres, which has been elaborated to high perfection by Chandrasekhar[159]. For the homogeneous model, $k_2 = 3/4$, $k_3 = 3/8$, $k = 1/4$, and for the Roche model they all vanish.

The rotation of the line of apsides can be obtained from observations of the radial velocities of SB, since they also yield the eccentricity and the position of periastron[14)] relative to the line of sight (the so-called longitude of periastron $\tilde{\omega}$). If the observations are repeated after a sufficiently long interval of time, the change in $\tilde{\omega}$ will give P/U. Unfortunately, $\tilde{\omega}$ is not determined accurately, especially when e is small, and the variations of $\tilde{\omega}$ are very slow: the smallest values of U are of the order of 30 years, and they usually range into the hundreds of years.

14) This is the name given the point on the orbit where the distance between the components is shortest, i.e., to one of the vertices of the orbital ellipse.

Much more accurate results can be obtained from observation of the epochs of the minima of EB, since the times of the primary and secondary minima as the line of apsides rotates are shifted periodically in antiphase. An example of these periodic shifts is shown in Fig. 8, where we have assembled the brightness curves of RU Monocerotis at five epochs from the beginning of this century to 1971 and the positions of the orbital ellipse implied by these curves. For greater ease of inspection, the relative displacements of the secondary minimum between successive primary minima are also indicated.

The rotation of the line of apsides has now been proven conclusively for around 15 systems, and the values of P/U have been reliably determined[160]. To obtain the parameters k from these results, it is necessary to know r_1 and r_2, and since they appear in higher powers, beginning with the fifth, they must be known accurately. Study of the rotation of the line of apsides requires thorough photometric analysis of the EB system.

Strictly speaking, Eq. (3) is inadequate for a determination of k, since two unknowns, k_{21} and k_{22}, appear in it. They are usually assumed to be equal, and this is justified if the two components have similar physical characteristics, as they do in most cases. The value found for k is compared with the theoretical values, and in particular with those calculated for various classes of polytropism (see above). The values of k usually correspond to class 3 to 4 polytropism, indicating that the ratio ρ_c/ρ_m of central to mean density ranges from 54 to 623. But if the stars are not constructed in accordance with the polytropic model, the ratio ρ_c/ρ_m may be quite different.

Let us consider the example of the so-called generalized Roche model, which can be constructed from a real star by transferring all matter to the center, so as to form a homogeneous core of radius r_0 with a density equal to the central density ρ_c, while an atmosphere of vanishing mass surrounds the core out to the radius R of the real star. The model has the same ratio $D = \rho_c/\rho_m = (R/r_0)^3$ as the real star, and the smallest of all possible values of the moment of inertia. A star constructed after such a model must possess the highest possible homogeneity, i.e., the smallest value of D, so as to produce the motion of the line of apsides that

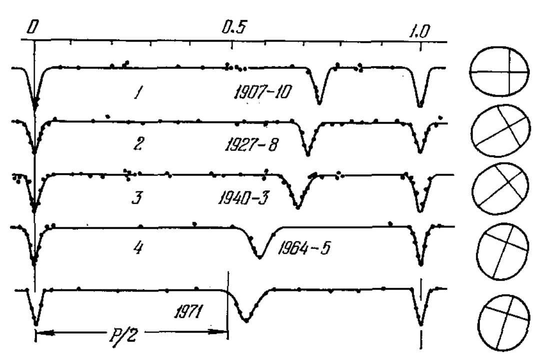

FIG. 8. Progressive changes in brightness curve of RU Monocerotis due to rotation of the orbit. The diagrams on the right indicate the positions of the orbit with respect ot the observer (who is assumed to be below it). The first curve was derived from Wendell's observations, and the others from those of the author [31b].

is actually observed. As was shown by Sterne[157], $k_2 = (3/4)D^{-5/3}$ for this model, but all systems with observable motion of the line apsides give values of D_{min} from 10 to 30, i.e., a substantially lower degree of concentration of the matter.

In fact, the internal structure of stars deviates strongly from polytropic models, especially in chemically inhomogeneous configurations; for this reason, an effective polytropic exponent, i.e., one that yielded the same values of k_2, k_3, and k_4 as the model being tested, was calculated when it became the practice to test a constructed stellar model against the rotation of the line of apsides[154,161]. However, it would be more reasonable to test models of real stars in systems with apsidal motion by stating the (calculated) quantities k_2, k_3, and k_4 themselves and comparing them with observations. This has been done in a series of papers[162-168] by a progressively improving method.

The problem reduces to finding the solution of the generalized Radau geodetic equation (see, for example,[169]):

$$r\frac{d\eta_n}{dr}+\eta_n^2-\eta_n(n+1)n_1^1+\frac{6\rho}{\rho_m}(\eta_n+1)=0, \qquad (7)$$

which is solved with the initial values $\eta_n = 0, 1, 2$ for $n = 2, 3, 4$ when $r = 0$. For $n = 2$, η has a simple meaning: $\eta_2 = d \ln \epsilon/d \ln r$, and $r = R$ on the outer surface of the star (ϵ is the rotational flattening of the star). Equation (7) is solved numerically for a given distribution $\rho = f(r)$. This is followed by determination of

$$k_{2.1} = [3-\eta_2(R_1)]/[4+2\eta_2(R_1)],$$

and then

$$k_{3\cdot 1}=\frac{4-\eta_3(R_1)}{6+2\eta_3(R_1)}, \qquad k_{4\cdot 1}=\frac{5-\eta_4(R_1)}{8+2\eta_4(R_1)},$$

and similarly for $k_{2,2}$, $k_{3,2}$, and $k_{4,2}$.

The same method has also been used in attempts to fit the chemical composition of the star[166] or the thickness of the outer convective layer for subgiants[168] to the observed values of k. However, little should be expected from such comparisons of theory with observations, firstly because of the large number of possible theoretical variants and secondly because it is necessary to have more thoroughly studied cases of apsidal motion with accurately determined physical characteristics of the components.

Difficulties of theoretical analysis are associated on the one hand with the depth of the convective zone, which increases during the evolution of a star of insignificant mass ($(1.0-1.5)\mathfrak{M}_\odot$), in which case energy transfer is accomplished by both radiation and convection. In this case, the values of k_2 may increase sharply (by an order of magnitude), and if we inspect k_2 as a function of time, we may also observe a deep minimum of this quantity and a substantial increase in its value for the component at the initial subgiant stage, while the fraction of hydrogen in the stellar matter $x \geq 0.57$[167]. However, when it is recognized that r_1 and r_2 in (6) can be in error by no more than 10%, the theoretical value of k_2 in this model may nearly double.

On the other hand, complications arise when we consider the stage of rapid evolution with transfer of mass from one component to the other. The theoretical value of the coefficient k_2 may decrease by one or two orders of magnitude as compared with the value obtained for single stars at the same stage but without the rapid evolution[168].

Thus, although the velocity of rotation of the line of apsides is a function of the density distribution inside the star, it cannot be used without circumspection for selection of the $\rho = f(r)$ law in the model, but only for selection of the model—one of the many different admissible or possible models; this, of course, is important but not compulsory for the theory of the internal structure of stars.

Nor may we forget the constraints used in solving the problem of rotation of the line of apsides (see above). Attempts to inject synchronism of the orbital revolution and axial rotation into the analysis have indeed been made[170], but it is still a long way to successful analysis of nonrigid rotation of the components. The contribution of rotational flattening to the motion of the line of apsides (the separate term $g_2(e)$ in formula (7)) is much smaller than the contribution of tidal deformation. As a result, the nonsynchronism of revolution and rotation has only a slight influence on the rotation of the line of apsides.

We have deliberately avoided problems of the formation of binary stars in our exposition. This is a special and extremely difficult problem whose solution is of universal significance for astrophysics, since double and multiple stellar systems are common phenomena in the starry universe.

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