SOVIET PHYSICS USPEKHI VOLUME 15, NUMBER 6 MAY-JUNE 1973

533.951

ANOMALOUS NONLINEAR DISSIPATION OF HIGH-FREQUENCY RADIO WAVES IN PLASMA

V. P. SILIN

P. N. Lebedev Physics Institute, USSR Academy of Sciences

Usp. Fiz. Nauk 108, 625–654 (December, 1972)

The review covers theoretical and experimental studies of the parametric effect of high-power electromagnetic radiation on a plasma. Under the action of such a field, parametric instabilities develop in a plasma, leading to appearance of a turbulent state with an increased level of fluctuations of the internal field in the plasma. Under the action of these fluctuations the particle distribution in the plasma changes and, in particular, their energy increases, which corresponds to anomalously fast transfer of the energy of the radiation field to the plasma particles or, in other words, corresponds to an anomalous increase of the high-frequency conductivity of a parametrically unstable plasma. This theoretically predicted pattern find confirmation in a number of experimental investigations, also discussed in the review. Experiments demonstrate the appearance, under the action of high-power radiation, of an increased level of plasma fluctuations, a large high-frequency resistance, an increase in the rate of plasma heating, and, finally, of fast particles. All of these effects are consistent with the theoretical ideas and permit us to discuss qualitatively a new group of physical phenomena arising in the action of high-power radiation on a plasma.

CONTENTS

1. INTRODUCTION

THE action of electromagnetic radiation on a plasma is interesting from the point of view of our need to understand a number of physical phenomena. Among these are the propagation and absorption of electromagnetic waves in the ionosphere and in the plasma of outer space, plasma heating in devices proposed as a means of solving the problem of controlled nuclear fusion and which use electromagnetic radiation of radio or laser frequencies, and also phenomena in which the possibility appears of using plasma for transformation of various waves. Many investigators have studied rather thoroughly the effects of weak waves on plasma, where these phenomena correspond to linear electrodynamics. Action of high-power electromagnetic radiation on a plasma has been less studied and is being intensively studied at the present time; here the possibility arises of a number of different nonlinear phenomena. The present review is devoted to one of these groups of nonlinear phenomena in plasma, characterized by the appearance of an anomalous high-frequency resistance due to resonance parametric action of high-power radiation leading to development of perturbations in the plasma and to a turbulent state.

Before going to the main part of the review, we will briefly discuss here the ideas on which the ordinary theory of linear action of radiation on a plasma is based, and also the ideas of nonlinear theory, which take into account among the possible dissipative processes only Coulomb collisions of the plasma particles. Both here and everywhere below we will discuss only the properties of a completely ionized plasma.

In the theory of the linear action of radiation on a plasma, the complex dielectric constant is usually used,

$$\varepsilon = \varepsilon' + i\varepsilon'',$$

which in the absence of a magnetic field is a scalar quantity. Here the real part

$$\varepsilon'(\omega_0) = 1 - (\omega_{Le}^2/\omega_0^2) \quad (\omega_{Le}^2 = 4\pi e^2 n_e/m_e \approx 3\cdot 10^9 n_e)$$

permits us, in particular, to discuss the opacity of the plasma for radio waves with frequency ω_0 less than the electronic Langmuir frequency ω_{Le}. This expression for ϵ' has meaning for high-frequency waves where ω_0 is appreciably greater than the electron-ion collision frequency ν_{ei} characterizing dissipative processes in a completely ionized plasma:

$$\nu_{ei} = 4(2\pi)^{1/2} e^2 e_i^2 n_i \Lambda / 3 m_e^2 v_{Te}^3 \text{ (where } v_{Te}^2 = \varkappa T_e/m_e).$$

Correspondingly, for the imaginary part of the dielectric constant, which determines the linear absorption of electromagnetic waves by the plasma, we have

$$\varepsilon'' = \omega_{Le}^2 \nu_{ei}/\omega_0^3 \equiv 4\pi\sigma/\omega_0 \qquad (\sigma = \omega_{Le}^2 \nu_{ei}/4\pi\omega_0^2),$$

where σ is the high-frequency conductivity of the plasma. For example, for a plasma with an electron concentration $n_e \sim 10^{11}\ \mathrm{cm}^{-3}$ and a temperature $\kappa T_e \approx 1$ eV we have $\Lambda \sim 10$ and $\omega_{Le} \sim 2\times 10^{10}\ \mathrm{sec}^{-1}$, $\nu_{ei} \sim 2\times 10^{6}\ \mathrm{sec}^{-1}$. Therefore for frequencies $\omega_0 \sim \omega_{Le}$ we have $\epsilon'' \sim 10^{-4}$, which corresponds to relatively weak absorption of the electromagnetic wave by the plasma.

The concepts of the linear theory are inapplicable under conditions of high intensity of the electric field of the waves. We note that conditions were obtained in the laboratory long ago under which the plasma pressure was less than the pressure of the microwave electric field:

$$n_e \kappa T_e < E_0^2/4\pi.$$

For radio frequencies close to ω_{Le} this inequality corresponds to the thermal velocity v_{Te} of the plasma electrons being small in comparison with the velocity $v_E = |e| E_0/m_e\omega_0$ of the electron oscillations in the electric field of the wave. Thus, for example, for $\kappa T_e \sim 1$ eV and $\omega_0 \sim 2\times 10^{10}\ \mathrm{sec}^{-1}$ it turns out that $v_{Te} = v_E$ for $E_0 = 300$ V/cm.

We will make here an observation on the action of the field of the electromagnetic wave on the plasma particles which is important for the entire following discussion. First of all we must note that under the conditions $\omega_0 \sim \omega_{Le}$ the magnetic-field intensity of the wave has an order no greater than the electric-field intensity. This means that under nonrelativistic conditions, to which we will confine ourselves below, when $v_{Te} \ll c$ and $v_E \ll c$ (c is the velocity of light), the contribution of the magnetic field of the wave to the Lorentz force $e_a(\mathbf{E} + c^{-1}\mathbf{v}_a \times \mathbf{B})$ can be assumed small. Therefore in the action of the wave field on the plasma particles the principal effect is that of the electric field. This permits us to discuss the plasma properties in a strong oscillating electric field $\mathbf{E}(t) = \mathbf{E}_0 \sin \omega_0 t$.

The effect of the electric field appears comparatively simply in effects due to the ordinary mechanism of Coulomb collisions. Here, in the first place, for comparatively weak fields, as the result of ohmic heating of the electrons by the electric field of the wave, the electron temperature increases and consequently the electron-ion collision frequency[1] decreases, and in the second place, in a strong field where $v_E \gg v_{Te}$ the effective collision frequency drops rapidly according to a v_E^{-3} law (see ref. 2). Thus, we can state that the concepts of the ordinary electromagnetic-field dissipation mechanism lead to the conclusion that the dissipative effects are reduced as the power is increased. This conclusion makes particularly interesting the predictions of the theory of parametric action of an electromagnetic wave on a plasma, since they give a picture which is qualitatively different from the usual picture.

In discussing the parametric action of high-power radiation on a plasma, we have in mind the following physical picture. The parameters which determine the state of the plasma change with time—oscillate—under the influence of the electromagnetic field. Thus, as a result of the difference in the signs of the charges and in the masses of the electrons and ions in the field of the wave, there is a relative oscillatory motion of the plasma particles, as a result of which the electric current density in the plasma oscillates. The variation of the plasma parameters with time, for sufficiently large amplitude, as in the well understood case of mechanical closed oscillatory systems, leads to the possibility of parametric resonance. A large amplitude of the plasma-parameter oscillations occurs under the action of sufficiently powerful electromagnetic radiation on the plasma. Here the appearance of parametric resonance or, more generally, parametric instability in the plasma will lead to appearance of perturbations of the plasma oscillations, accompanied by an increase in the fluctuations of the internal fields. The state of the plasma with developed field fluctuations is a turbulent state. It is already widely known at the present time that turbulent plasma arises in a large number of experimental situations, and its properties turn out to be qualitatively different from those of a laminar plasma in which there are only thermal fluctuations of the internal fields.

The problem of the present review is to explain the results of studies of the parametric action of high-power radiation on plasma which have been obtained in recent years by a large number of workers who have investigated this field of plasma physics. Here we will limit ourselves to those results which are rather complete. This remark is necessitated by the fact that the pattern of appearance of parametric instability and the corresponding turbulent state turns out to be quite peculiar and comparatively complex as a result of the fact that the spectrum of the internal plasma-field fluctuations which build up lies in a region of frequencies comparable with the frequency of the high-power external field of the pumping radiation or in a lower frequency region. However, in spite of this complication, we can state that the ideas formulated by the theory of parametric action of high-power radiation on a plasma are based on a rather large set of theoretical results. At the same time they have been the starting point for formulation of experimental investigations which have led, in particular, to the experimental discovery of the phenomenon, predicted by the theory, of anomalously rapid transfer of the energy of the electromagnetic field to the plasma or, in other words, the phenomenon of anomalous high-frequency resistance of the plasma.

The applicability of the ideas of parametric action of radiation on a plasma is now clear, both in the microwave region and in the optical (laser) region. Since in the current level of development of technology the possibilities of appearance of parametric effects are broader in the microwave region, and the experimental data are also substantially more numerous, we will present, in our specific evaluations below, data referring to the microwave region.

2. RESULTS OF THE THEORY OF PARAMETRIC INSTABILITY OF A PLASMA IN THE FIELD OF AN ELECTROMAGNETIC WAVE

The theory of the natural oscillations and stability of a plasma in strong electromagnetic field, which has predicted a large number of specific phenomena in a plasma in a high-frequency field, is now a new and very extensive division of plasma physics, and in order to provide any detailed account of this theory it is necessary to write a separate review (cf., for example, our 1968 review[3]). Therefore we will concentrate our attention below on the qualitative aspects and the final results of this theory.

The theory, which is based on the systematic kinetic description of a plasma, has received substantial development in recent years. We will discuss below the consequences of this theory for instabilities with perturbation wavelengths much less than the size of plasma nonuniformities and for the case in which the external electromagnetic field can be considered spatially uniform, $\mathbf{E}_0(t) = \mathbf{E}_0 \sin \omega_0 t$. Then the kinetic theory developed by us previously[4] leads to the dispersion equation

$$1=\frac{\delta\varepsilon_i(\omega,\mathbf{k})}{1+\delta\varepsilon_i(\omega,\mathbf{k})}\sum_{n=-\infty}^{+\infty} J_n^2(\mathbf{kr}_E)\frac{\delta\varepsilon_e(n\omega_0+\omega,\mathbf{k})}{1+\delta\varepsilon_e(n\omega_0+\omega,\mathbf{k})};$$

here $\delta\epsilon_e(\omega, \mathbf{k})$ and $\delta\epsilon_i(\omega, \mathbf{k})$ are respectively the electronic and ionic contributions to the ordinary longitudinal dielectric permittivity, which depends on the frequency ω and wave vector $\mathbf{k}$; $J_n(a)$ is a Bessel function of the first kind, $\mathbf{r}_E$ is the amplitude of the relative oscillations of the plasma particles; for example, in the absence of a constant magnetic field $\mathbf{r}_E = e\mathbf{E}_0/m_e\omega_0^2$.

In a previous article[4] devoted to the theory of parametric resonance in a plasma located in a strong electric field from a high-frequency wave, where $v_E \gg v_{Te}$, we showed that in the region of external-field frequencies $\omega_0 \sim \omega_{Le}$ and $\omega_0 \lesssim \omega_{Le}$ an instability associated with the increase in perturbations of the longitudinal electric field arises in the plasma. Here, for example, in the vicinity of $\omega_0 \sim \omega_{Le}$ as the frequency of the external field approaches the electron Langmuir frequency from the high-frequency side, oscillations arise in the plasma with frequencies close to harmonics of the external field, and with a low frequency. In the same region but on the low-frequency side $\omega_0 \lesssim \omega_{Le}$, among the amplitudes of the perturbations which build up there is an amplitude building up asperiodically, and here there are no low frequency oscillations. The theory predicted that in the region $n\omega_0 \approx \omega_{Le}$ (n is an integer) the perturbations of the longitudinal field build up with a maximal increment

$$\gamma \sim \omega_{Le}(m_e/m_i)^{1/3}, \tag{2.1}$$

and for $\omega_0 < \omega_{Le}$ outside the region $|n\omega_0 - \omega_{Le}| \sim \omega_{Le}(m_e/m_i)^{1/3}$

$$\gamma \sim \omega_{Le}(m_e/m_i)^{1/2}.$$

These values of the maximal increment occur for perturbations with a wavelength comparable with the electron oscillation amplitude in the electric field of the wave:

$$|\mathbf{kr}_E| \sim 1.$$

In a certain sense this plasma instability can become similar to a beam instability, since it is also due to the motion of electrons relative to ions.

Systematic consideration of the thermal motion of the plasma particles has shown[5] that parametric plasma-instability effects involving the buildup of oscillations of the potential field are possible when the external-field frequency is appreciably greater than the value corresponding to the electron Langmuir frequency, provided that the electron temperature T_e is significantly greater than the ion temperature T_i. Specifically, over a wide range of frequencies

$$\omega_{Le}^2 \leqslant \omega_0^2 < \omega_{Le}^2 T_e [T_i \ln (T_e^3 m_i / T_i^3 m_e)]^{-1} \tag{2.2}$$

instability can arise; here, if ω_0 is not close to ω_{Le}, the oscillations which build up have a frequency $\sim\omega_{Li}$, which is the limiting value of the ion-accoustic frequency for wavelengths less than the electron Debye radius r_{De}.

As we have shown previously[5], the development of instability resulting in buildup of potential oscillations of ion-accoustic waves in a collisionless plasma is due to the Cerenkov effect on the electrons. Specifically, electrons oscillating under the action of the wave field, for sufficiently high electric-field strength, radiate waves primarily through the Cerenkov effect, while in the absence of a pumping field the preferential absorption of the waves by electrons leads to collisionless Landau damping. Under these conditions the increment of the perturbations which build up is small in comparison with the frequency and, for example, for $v_E \gtrsim v_{Te}$ we have ($kr_{De} \gg 1$)

$$\gamma \sim \omega_{Li}^2\omega_{Le}^2/k^3v_E^3. \tag{2.3}$$

For an instability of this type we will show below the possibility of occurrence of anomalous plasma resistance. After the publication of our earlier articles[4,5], a number of investigators were occupied with study of parametric instabilities of plasma in a strong high-frequency field against development of potential perturbations[6-8]. We can now state that the results of these earlier studies[4,5] have been confirmed and that accepted criteria exist for plasma instability in a strong high-frequency field (see also refs. 9 and 10).

As the frequency ω_0 of the external field of the pumping wave approaches the electron Langmuir frequency ω_{Le} (and also for corresponding harmonic resonances $n\omega_0$) the threshold value of electric-field intensity at which parametric plasma instability becomes possible decreases. At the same time the wavelength of the perturbations which build up in the plasma increases. At the present time a particularly detailed theoretical study has been made of the near-threshold region of parametric plasma instability in the vicinity of $\omega_0 = \omega_{Le}$, as reported in a number of articles[11-20]. This near-threshold region is of interest first of all because of the comparative simplicity of the phenomena arising here. We will briefly describe the theoretical results obtained for this region in the articles cited.

As in the case already discussed of a strong field, in the near-threshold region development is possible of both aperiodic and almost periodic instabilities. The possibilities of occurrence of these instabilities are determined by requirements on the difference of the external-field frequency and the frequency of the ordinary longitudinal plasma wave

$$\Delta\omega_0 = \omega_0 - (\omega_{Le}^2 + \omega_{Li}^2 + 3k^2r_{De}^2\omega_{Le}^2)^{1/2} \approx (2\omega_0)^{-1}(\omega_0^2 - \omega_{Le}^2 - \omega_{Li}^2 - 3k^2r_{De}^2\omega_{Le}^2). \tag{2.4}$$

Let us turn first to consideration of aperiodic instability, which is characterized by presence in the perturbation field of a harmonic building up aperiodically with time according to an exponential law, and also of harmonics with the external-field frequency. In this case for appearance of an aperiodic instability the condition $\Delta\omega_0 < 0$ must be satisfied. The boundary of such instability, corresponding to the values of the wave vectors of the perturbations building up for a given electric-field intensity of the pumping wave, is determined by the relation[15,19]

$$\frac{(\mathbf{k}r_E)^2_{\lim}}{k^2(r_{De}^2+r_{Di}^2)} = \frac{(\mathbf{k}\mathbf{E}_0)^2_{\lim}}{4\pi k^2(n_e\varkappa T_e+n_i\varkappa T_i)} = 4\frac{(\Delta\omega_0)^2+\tilde{\gamma}^2}{\omega_{Le}^2+|\Delta\omega_0|},$$

where

$$\tilde{\gamma} = \sqrt{\frac{\pi}{8}}\frac{\omega_{Le}}{k^3r_{De}^3}\exp\left(-\frac{\omega_0^2}{2k^2v_{Te}^2}\right)+\frac{1}{2}\nu_{ei}. \tag{2.5}$$

In Eq. (2.5) the first term characterizes the Cerenkov interaction of the electrons with a high-frequency harmonic of the longitudinal plasma perturbations, and the second term—damping due to collisions of electrons with ions. We will designate by k_{st} the wave vector for which the two terms in the right-hand side of (2.5) are equal ($k_{st}^2 r_{De}^2 \approx \ln^{-1}(\omega_{Le}^2/\nu_{ei}^2)$).

The threshold field value, at which aperiodic instability becomes possible, and the corresponding value of the wave vector depend on the magnitude of the detuning, by which we mean the difference in the external-field frequency ω_0 and the plasma frequency $\omega_p = (\omega_{Le}^2+\omega_{Li}^2)^{1/2}$. Here, if

$$-(1/2)\nu_{ei} < \omega_0-\omega_p < (3/2)\omega_p k_{st}^2 r_{De}^2, \tag{2.6}$$

we have for the threshold value of the wave vector and the electric-field intensity of the pumping wave[15]

$$\begin{gathered} k_{\rm thr}^2 r_{De}^2 = [\nu_{ei}+2(\omega_0-\omega_p)]/3\omega_p, \\ \frac{r_{E,\,\rm thr}^2}{r_{De}^2+r_{Di}^2} = \frac{E_{\rm thr}^2}{4\pi(n_e\varkappa T_e+n_i\varkappa T_i)} = \frac{4\nu_{ei}}{\omega_{Le}}. \end{gathered} \tag{2.7}$$

For a small excess of the field over the threshold value, when the increment is small in comparison with $\tilde{\gamma}$, the high-frequency part of the electron charge-density oscillations which build up has the form

$$\gamma_{\max} \approx (\omega_{Le}/8)(r_E^2-r_{E,\,\rm thr}^2)/(r_{De}^2+r_{Di}^2)$$

From this it follows that the rapid dependence of the perturbations on time has a 45° phase shift relative to the phase of the electric field of the high-frequency pumping wave.

For a small excess above threshold, the increment also reaches its maximum value

$$e^{i\mathbf{kr}+\gamma t}\sin[\omega_0 t+(\pi/4)].$$

for the following wave vector value:

$$k_{\max}^2 = \left[\frac{2}{3}\frac{\omega_0-\omega_p}{\omega_p}+\frac{r_E^2}{12(r_{De}^2+r_{Di}^2)}\right]\frac{1}{r_{De}^2}.$$

For a further increase of the pumping-wave electric-field strength the maximum increment increases linearly with field strength:

$$\gamma_{\max} \approx (\omega_{Li}/\sqrt{6})(r_E/r_{De}).$$

For detunings greater than (2.6), instability arises for perturbations in which $k > k_{st}$. This occurs for

$$\omega_0-\omega_p \gg (3/2)\omega_p k_{st}^2 r_{De}^2.$$

Then

$$\begin{gathered} \frac{E_{\rm thr}^2}{4\pi(n_e\varkappa T_e+n_i\varkappa T_i)} = \frac{r_{E,\,\rm thr}^2}{r_{De}^2+r_{Di}^2} = \frac{\pi}{12(k_{\rm thr}r_{De})^{10}}\exp\left(-\frac{\omega_0^2}{2k_{\rm thr}^2v_{Te}^2}\right), \\ k_{\rm thr}^2 r_{De}^2 = 2(\omega_0-\omega_p)/3\omega_p. \end{gathered} \tag{2.8}$$

It follows from this that with increasing detuning the threshold electric-field intensity increases exponentially.

For a small excess over the threshold field (2.8), the high-frequency part of the electron-density oscillations has the form

$$e^{i\mathbf{kr}+\gamma t}\cos\omega_0 t,$$

which corresponds to a phase shift of these oscillations in time by 90° relative to the external-field phase.

Let us now discuss the results of the theory of parametric instability in the near-threshold region which relate to development of almost periodic oscillations. This instability, as in the case of strong fields, occurs in the region of external field frequencies ω_0 exceeding the plasma frequency ω_p. For a slowly varying perturbation harmonic frequency ω in the near-threshold region we have the following equation[18]:

$$\begin{aligned} &(\omega^2-\omega_s^2)\{[(\Delta\omega_0)^2-\omega^2+\tilde{\gamma}^2]^2+4\omega^2\tilde{\gamma}^2\} \\ &\quad = (\mathbf{k}r_E/2kr_{De})^2(\omega_s)^2[(\Delta\omega_0)^2-\omega^2+\tilde{\gamma}^2]\omega_0\Delta\omega_0; \end{aligned} \tag{2.9}$$

here $\omega_s = \omega_{Li}kr_{De}$ is the frequency of ion-acoustic waves of a plasma in which the electron temperature appreciably exceeds the ion temperature. Correspondingly we have for the increment

$$\begin{aligned} \gamma = &\left\{\left(\frac{\mathbf{k}r_E}{2kr_{De}}\right)^2\frac{\omega_s^2\omega_0\tilde{\gamma}\Delta\omega_0}{[(\Delta\omega_0)^2-\omega^2+\tilde{\gamma}^2]^2+4\omega^2\tilde{\gamma}^2}-\gamma_s\right\}\times \\ &\times\left[1-\left(\frac{\mathbf{k}r_E}{2kr_{De}}\right)^2\frac{\omega_s^2\omega_0\Delta\omega_0\,\{[(\Delta\omega_0)^2-\omega^2+\tilde{\gamma}^2]^2-4\tilde{\gamma}^2[(\Delta\omega_0^2)+\tilde{\gamma}^2]\}}{\{[(\Delta\omega_0)^2-\omega^2+\tilde{\gamma}^2]^2+4\omega^2\tilde{\gamma}^2\}^2}\right]^{-1}, \end{aligned} \tag{2.10}$$

where

$$\gamma_s = \sqrt{\frac{\pi}{8}}\frac{\omega_{Li}}{\omega_{Le}}\omega_s+\sqrt{\frac{\pi}{8}}\frac{\omega^4 r_{De}^3}{\omega_s^3 r_{Di}^3}\exp\left(-\frac{\omega^2}{2k^2v_{Ti}^2}\right)+\frac{4}{5}\frac{k^2v_{Ti}^2\nu_{ii}}{\omega^2} \tag{2.11}$$

is the low-frequency damping decrement, in which the last term is determined by the frequency of collision of ions with ions

$$\nu_{ii} = 4\pi^{1/2}e_i^4 n_i\Lambda/3(\varkappa T_i)^{3/2}m_i^{1/2}.$$

If we neglect ion-ion collisions and assume that $\Delta\omega_0 = \omega_s \gg \tilde{\gamma}$ we have the result of Du Bois and Goldman[11]. The limit $\tilde{\gamma} \gg \omega_s$ corresponds to that obtained by Nichikawa[15].

We emphasize that the case $\Delta\omega_0 = \omega_s$, or in explicit form

$$\omega_0 = (\omega_{Le}^2+\omega_{Li}^2+3k^2r_{De}^2\omega_{Le}^2)^{1/2}+\omega_s,$$

represents the interaction of this type of three waves: the external high-frequency wave, the longitudinal high-frequency Langmuir plasma wave, and the ion-acoustic wave; this process can be called wave decay (or, what amounts to the same thing, combination scattering). We note that the plasma wave decay instability discussed by Oraevskii and Sagdeev[21] can be important for the subsequent fate of the longitudinal parametric excitations in the plasma.

A general investigation of the near-threshold region of buildup of almost periodic perturbations has been reported by Andreev et al.[19] We will concern ourselves here only with a strongly anisothermal plasma ($T_e \gg T_i$) with comparatively infrequency collisions, when

$$\nu_{ei}/\omega_{Le} \gg (m_e T_i/m_i T_e)^{1/2}\,[\ln(\omega_{Le}/\omega_{Li})]^{-1/2}.$$

Then the minimum threshold for parametric instability occurs for $\Delta\omega_0 = \omega_S$ and is determined by the formula

$$E_0^2/4\pi n_e \varkappa T_e = (8\pi)^{1/2}\,\nu_{ei}\omega_{Li}/\omega_{Le}^2. \tag{2.12}$$

This threshold lies in the region of small detunings

$$\omega_0 - \omega_p \ll (3/2)\,\omega_p k_{st}^2 r_{De}^2 \sim \omega_p \ln^{-1}(\omega_{Le}^2/\nu_{ei}^2),$$

where the dissipation is determined by collisions, since the perturbations arising are long-wavelength.

For large detuning, collisions make a small contribution in comparison with the Cerenkov effect. In particular, in the detuning region

$$k_{st}^2 r_{De}^2 < 2(\omega_0 - \omega_p)/3\omega_p < k_2^2 r_{De}^2 \tag{2.13}$$

(where k_2 is the wave vector for which $\widetilde{\gamma} = \omega_S$) the spectrum of low-frequency perturbations building up coincides with the ion-acoustic spectrum ($\omega = \omega_S$) if the pumping-wave field intensity satisfies the inequality

$$E_0^2/64\pi n_e \varkappa T_e \ll k v_s/\omega_0. \tag{2.14}$$

Here we have for the increment

$$\gamma = -\gamma_s + \left(\frac{k r_E}{2 k r_{De}}\right)^2 \frac{\omega_s^2 \omega_0 \Delta\omega_0 \widetilde{\gamma}}{[(\Delta\omega_0)^2 - k^2 v_s^2]^2 + 4k^2 v_s^2 \widetilde{\gamma}^2}, \tag{2.15}$$

where γ_S and $\widetilde{\gamma}$ are defined by Eqs. (2.11) and (2.5), in which we can omit the collision contributions. The increment (2.15) becomes positive if the electric-field strength E_0 exceeds the threshold value determined by the relation

$$E_{\rm thr}^2/4\pi n_e \varkappa T_e = 16\gamma_s(k_0)\,\widetilde{\gamma}(k_0)/\omega_0\omega_s(k_0), \tag{2.16}$$

where the value of the wave vector k_0 corresponds to the decay value:

$$\omega_s(k_0) \equiv k_0 v_s = \Delta\omega_0(k_0) \equiv \omega_0 - \omega_p\,[1 + (3/2)\,k^2 r_{De}^2].$$

The fact that under these conditions the dissipation is due to the Cerenkov effect, as we will show below, permits theoretical results characterizing the anomalous plasma resistance to be obtained for the detuning (2.13).

Under real conditions a plasma is always to a certain degree spatially nonuniform. The existence of this nonuniformity leads to the possibility of parametric resonance at the natural frequency of surface waves[22] and to the possibility of demultiplication parametric resonance in the vicinity of twice the plasma frequency[23,24].

Another effect of spatial nonuniformity can appear in the influence which the plasma gradients exert on the parametric-instability thresholds in the vicinity of $\omega_0 = \omega_{Le}$, which we have discussed above in the case of a spatially uniform plasma. Here, for example, in limited structures whose dimensions are small in comparison with the electron free path, the role of the mean time between collisions in the formula for the threshold can be played by the time of flight of an electron through such a plasma structure. Under conditions where the thresholds are determined by the Cerenkov mechanism of dissipation, the role of the corresponding wave vector, for sufficiently small plasma dimensions, can be played by the reciprocal magnitude of this dimension. We note that Perkins and Flick[25] have given results on the increase of parametric-instability thresholds in a nonuniform plasma.

3. QUASILINEAR AND NONLINEAR THEORY OF ANOMALOUS HIGH-FREQUENCY CONDUCTIVITY OF A PARAMETRICALLY UNSTABLE PLASMA

a) The development of parametric instability in a plasma leads to an increase in the intensity of longitudinal-field perturbations, which in turn can change the distributions of the plasma particles. Although the appearance itself of such an instability is a nonlinear effect with respect to the pumping-wave field, the effects arising in a turbulent plasma with a high intensity of perturbations arising in it require formulation of a theory which is nonlinear with respect to such perturbations. The corresponding theory has been formulated in refs. 4, 26–29.

A comparatively simple discussion permits us to obtain a quasilinear system of equations for a plasma in a strong high-frequency electric field. Here, if we have in mind a monochromatic dependence of the external field on time ($\mathbf{E} = \mathbf{E}_0 \sin\omega_0 t$), it is convenient to represent the perturbation potential of the longitudinal field in the plasma in the form of a harmonic expansion

$$\varphi_{\mathbf{k}}(t) = \sum_{n=-\infty}^{+\infty} e^{-in\omega_0 t}\,\varphi_n(\mathbf{k}, t),$$

where the amplitudes φ_n do not change greatly during the period of oscillation of the external electric field. For the distribution functions we have respectively

$$f_\alpha(\mathbf{p}, t) = F_\alpha\left(\mathbf{p} + \left(\frac{e_\alpha}{\omega_0}\right)\mathbf{E}_0 \cos\omega_0 t,\ t\right),$$

$$F_\alpha(\mathbf{p}, t) = \sum_{n=-\infty}^{+\infty} e^{-in\omega_0 t} F_\alpha^{(n)}(\mathbf{p}, t).$$

The zero harmonic $F_\alpha^{(0)}$ of the distribution function obeys the Fokker-Planck equation which describes diffusion in momentum space:

$$\frac{\partial F_\alpha^{(0)}}{\partial t} = \frac{\partial}{\partial p_i}\left[D_{ij}^\alpha(\mathbf{p}, t)\,\frac{\partial F_\alpha^{(0)}(\mathbf{p}, t)}{\partial p_j}\right], \tag{3.1}$$

and for the zero harmonic of the longitudinal-field potential we have

$$\frac{d\varphi_0(\omega, \mathbf{k}, t)}{dt} - \gamma(\omega, \mathbf{k}, t)\,\varphi_0(\omega, \mathbf{k}, t) = 0,$$

where γ is the increment.

The higher harmonics of the potential are determined by the relation

$$\varphi_n \approx \varphi_0 \frac{\delta\varepsilon_i(0)}{1 + \delta\varepsilon_i(0)} \sum_{l=-\infty}^{+\infty} J_l(a)\,J_{l-n}(a)\,\frac{\delta\varepsilon_e(l)}{1 + \delta\varepsilon_e(l)},$$

where we have used the designations

$$a = \mathbf{K}\mathbf{r}_E, \quad \delta\varepsilon_\alpha(\omega + n\omega_0, \mathbf{k}) \equiv \delta\varepsilon_\alpha(n).$$

For the higher harmonics of the electron distribution function, if they are relatively small, we have

$$F_e^{(n)}(\mathbf{p}, t) = \frac{i}{n\omega_0}\frac{\partial}{\partial p_i}\left[D_{ij}(\mathbf{p}, t;\ n)\,\frac{\partial F_e^{(0)}(\mathbf{p}, t)}{\partial p_j}\right], \tag{3.2}$$

where

$$D_{ij}(\mathbf{p}, t;\ n) = e^2 \int \frac{d\mathbf{k}}{(2\pi)^3} k_i k_j \sum_u |\varphi_0(\omega_u(\mathbf{k}), \mathbf{k}, t)|^2 \sum_{l=-\infty}^{+\infty} J_l(a)\,J_{l-n}(a)$$

$$\times \frac{|\delta\varepsilon_i(0)|^2}{[1+\delta\varepsilon_e(l)][1+\delta\varepsilon_e(l-n)]}\left[\frac{i}{L(l)+i0}-\gamma\frac{\partial}{\partial\omega_u}\frac{1}{L(l)+i0}\right];\quad (3.3)$$
$$L(l) = l\omega_0+\omega-\mathbf{kv},$$

and the summation is carried out over all roots ω_u of the dispersion equation for plasma oscillations in a high-frequency electric field.

The diffusion coefficient of Eq. (3.1) for electrons and ions, respectively, has the form

$$D_{ij}^{(e)}(\mathbf{p},t) = e^2\int\frac{d\mathbf{k}}{(2\pi)^3}k_ik_j\sum_u|\varphi_0(\omega_u,\mathbf{k},t)|^2\sum_{l=-\infty}^{+\infty}J_l^2(a) \quad (3.4)$$
$$\times\left|\frac{\delta\varepsilon_i(0)}{1+\delta\varepsilon_e(l)}\right|^2\left\{\pi\delta(L(l))-\gamma\left(\frac{\partial}{\partial\omega_u}\right)\left[\frac{P}{L(l)}\right]\right\},$$
$$D_{rj}^{(i)}(\mathbf{p},t) = e_i^2\int\frac{d\mathbf{k}}{(2\pi)^3}k_rk_j\sum_u|\varphi_0(\omega_u,\mathbf{k},t)|^2\sum_{l=-\infty}^{+\infty}\left|\frac{\delta\varepsilon_i(0)}{1+\delta\varepsilon_i(l)}\right|^2$$
$$\times\left|\sum_{m=-\infty}^{+\infty}J_m(a)J_{m-l}(a)\frac{\delta\varepsilon_e(m)}{1+\delta\varepsilon_e(m)}\right|^2\left\{\pi\delta(L(l))-\gamma\frac{\partial}{\partial\omega_u}\frac{P}{L(l)}\right\}$$

where P designates the Cauchy principal value. The relations written down here permit many properties of a parametrically unstable turbulent plasma to be characterized. Here, for example, for the rate of heating of the plasma the necessary expression can be obtained by averaging the work of the external field on the plasma over the period $2\pi/\omega_0$:

$$\langle\mathbf{E}(t)\,\mathbf{j}(t)\rangle = e\int d\mathbf{p}_e(\mathbf{E}_0\mathbf{v}_e)(2i)^{-1}[F_e^{(1)}(\mathbf{p}_e,t)-F_e^{(-1)}(\mathbf{p}_e,t)], \quad (3.5)$$

where $F_e^{(\pm 1)}$ are defined by Eq. (3.2).

Before turning to discussion of specific results obtained by means of this theory, we will point out here that in comparatively weak fields, when parametric instability of a plasma occurs in a narrow resonance region near the electron Langmuir frequency, the dispersion equation for plasma waves and the increment are given by Eqs. (2.9) and (2.10).

Here $\Delta\omega_0$ is defined by Eq. (2.4), $\omega_S = \omega_{Li}kr_{De}$ is the ion-acoustic wave frequency; $\tilde{\gamma}$ and γ_S, in contrast to Eqs. (2.5) and (2.11), are determined by the equations

$$\tilde{\gamma} = (1/2)\,\omega_0\delta\varepsilon_e''(\omega_0,k),$$
$$\gamma_s = (\omega^3/2\omega_L^2)\,[\delta\varepsilon_e''(\omega,k)+\delta\varepsilon_i''(\omega,k)],$$

where, as in all of the equations in this chapter, the dielectric permittivities are defined by the distribution functions $F_\alpha^{(0)}$.

Correspondingly the electron diffusion coefficient in momentum space in the weak-field approximation for the resonance region has the form

$$D_{ij}^{(e)}(\mathbf{p},t) = e^2\int\frac{d\mathbf{k}}{(2\pi)^3}k_ik_j\sum_u|\varphi_0(\omega_u,\mathbf{k},t)|^2\frac{\omega_s^4(\mathbf{k})}{\omega_u^4(\mathbf{k})}\left\{\pi\delta(L(0)\right.$$
$$-\gamma\frac{\partial}{\partial\omega_u}\frac{P}{L(0)}+\frac{(\mathbf{kv}_E)^2}{16k^4r_{De}^4}\left(\frac{1}{[\Delta\omega_0+\omega_u(\mathbf{k})]^2+\tilde{\gamma}^2}\left[\pi\delta(L(1))-\gamma\frac{\partial}{\partial\omega_u}\frac{P}{L(1)}\right]\right.$$
$$\left.\left.+\frac{1}{[\Delta\omega_0-\omega_u(\mathbf{k})]^2+\tilde{\gamma}^2}\left[\pi\delta(L(-1))-\gamma\frac{\partial}{\partial\omega_u}\frac{P}{L(-1)}\right]\right)\right\}.$$

Finally, Eq. (3.5) can be written in the form

$$\langle\mathbf{E}(t)\,\mathbf{j}(t)\rangle = (1/2)\,\sigma_T E_0^2,$$

where the turbulent conductivity σ_T is defined by the following simple formula:

$$\sigma_T = \frac{1}{4}\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{(\mathbf{kv}_E)^2}{E_0^2}\frac{k^2}{4\pi}\sum_u|\varphi_0(\omega_u(\mathbf{k}),\mathbf{k},t)|^2 \quad (3.6)$$
$$\times\frac{\omega_{Li}^4}{\omega_u^4}\tilde{\gamma}\left\{\frac{1}{(\Delta\omega_0+\omega_u)^2+\tilde{\gamma}^2}+\frac{1}{(\Delta\omega_0-\omega_u)^2+\tilde{\gamma}^2}\right\}.$$

b) In that stage of development of the instability in which the buildup of the field follows an exponential law with an increment characterized by the linear theory, we can already discern a number of important properties of the turbulent state of a parametrically unstable plasma.

We will evaluate first of all the energy associated with the harmonics of the electron distribution. This permits us to understand the scale of time in which the particle distribution will change substantially in parametric resonance.

Using Eqs. (3.2) and (3.3), we have for the energy of the n-th harmonic of the electron distribution

$$\mathcal{E}^{(n)} = \int d\mathbf{p}\,(p^2/2m_e)\,F_e^{(n)}(\mathbf{p},t) \quad (3.7)$$
$$= (n\omega_0)^{-1}\int d\mathbf{k}\,(2\pi)^{-3}\sum_u|\varphi_0(\omega_u,\mathbf{k},t)|^2(k^2/4\pi)\,|\delta\varepsilon_i(0)|^2\sum_{l=-\infty}^{+\infty}J_l(a)\,J_{l-n}(a)$$
$$\{[1+i\gamma(\omega_u,\mathbf{k})(\partial/\partial\omega_u)]\,[(l\omega_0+\omega_u)\,\delta\varepsilon_e(l)]\}\{[1+\delta\varepsilon_e(l)]\,[1+\delta\varepsilon_i^*(l-n)]\}^{-1}.$$

For parametric resonance at a harmonic of the external frequency $s\omega_0 \approx \omega_{Le}$ in a sufficiently strong field where $J_s(a) \sim 1$ and the thermal motion is a comparatively weak effect, Eq. (3.7) takes a simple form[4]. Here for the harmonic $n = 2s$ we obtain

$$\mathcal{E}^{(2s)} = \int d\mathbf{p}\,(p^2/2m_e)\,F_e^{(2s)}(\mathbf{p},t) \approx \int d\mathbf{k}/(2\pi)^{-3}\,|\varphi_0|^2\,(k^2/4\pi); \quad (3.8)$$

here the integration is carried out over the region of wave vectors $kr_E \sim 1$, in which the increment is determined by Eq. (2.1).

It follows from Eq. (3.8) that in the hydrodynamic stage of parametric plasma resonance, when the velocity spread of the electrons is unimportant for the field buildup law, the energy associated with a higher harmonic is comparable with the energy of the longitudinal perturbation field which is building up in the plasma. Taking as an initial condition the thermal value of the field energy density, determined by the electrons,

$$(k^2/4\pi)\,|\varphi_0(\mathbf{k},t=0)|^2 = \varkappa T_e, \quad (3.9)$$

we can write Eq. (3.8) in the form

$$\mathcal{E}^{(2s)} \approx (\varkappa T_e/r_E^3)\exp\left(2\int_0^t\gamma\,dt\right). \quad (3.10)$$

Since $r_E > r_{De}$ in the strong-field case being discussed, the pre-exponential factor in this formula amounts to a small fraction of the thermal energy of the plasma oscillations. The exponentially rapid rise of the right-hand side of (3.10) leads to the result that in times of order $(2\gamma)^{-1}\ln(n_e r_E^3)$ the energy $E^{(2S)}$ is equalized with the thermal energy, and for times of the order $(2\gamma)^{-1}\ln(r_E^3E_0^2/4\pi\kappa T_e)$ it is equalized with the energy of electron oscillations in the external field. It is apparent that in such times the anisotropic electron distribution, which is rapidly oscillating in velocity, can lead to qualitatively new behaviors of the subsequent plasma heating.

The increase in electron energy described by Eq. (3.10) simultaneously corresponds, evidently, to absorption of energy of the plasma pumping-wave field. This fact of the anomalously rapid dissipation of the field energy, which was pointed out by us in 1965[4], was subsequently discussed by Gurevich and Silin with application to the problem of radiative acceleration of a plasma, where it was shown that it is the most important of the possible limitations of this method.

Another result of the theory also refers to the situation in which the field buildup occurs according to an exponential law; it was obtained in ref. 29 for a nonisothermal plasma in the frequency region (2.2). Parametric resonance under these conditions has been studied in ref. 6, where the parametric instability was discussed as a kinetic instability due to the Cerenkov interaction of the waves with electrons. The corresponding discussion of ref. 26 showed that the kinetic parametric instability of a nonisothermal plasma relative to the buildup in it of potential oscillations leads to an anomalously strong interaction of electromagnetic waves with the plasma, which is expressed in a rapid absorption of the energy of the waves, due to an anomalous increase in the dissipative high-frequency conductivity of the plasma. We will dwell here on this result, having in mind its fundamental importance for the theory of the anomalous high-frequency conductivity of a plasma.

We will assume the external-field frequency to be appreciably greater than the electron Langmuir frequency and to lie in the region of (2.2). Then, according to ref. 5, instability turns out to be possible if the electric-field strength of the pumping wave becomes greater than the threshold value for which the electron oscillation velocity is roughly a factor of two greater than the thermal velocity. Let v_E be greater than the threshold value but not differ from it in order of magnitude; then oscillations turn out to build up over a wide region of wavelengths

$$r_E > \lambda\!\!\!^{-} = 1/k > 1/k_0 \equiv r_{De} I^{-1/2},$$

where

$$I = (|e|T_e/e_i T_i)\ln^{-1}(T_e^3 e_i^2 m_i/T_i^2 e^2 m).$$

Here the frequency of oscillations building up practically coincides with the ion Langmuir frequency, and Eq. (2.3) applies for the increment.

In accordance with Eq. (3.5) we can write the following expression for estimation of the average work of the external field on the plasma[26]:

$$\langle \mathbf{Ej}\rangle \sim \int \frac{d\mathbf{k}}{(2\pi)^3}\frac{k^2}{4\pi}|\varphi_0(\mathbf{k},t)|^2\frac{\omega_{Le}}{kr_{De}}. \tag{3.11}$$

The same expression determines the rate of buildup in time of the average electron kinetic energy

$$(d/dt)\int d\mathbf{p}\,(p^2/2m_e)\,F_e \approx \langle \mathbf{Ej}\rangle, \tag{3.12}$$

since the plasma oscillation energy amounts to a small fraction in the total energy balance, and the rate of change of the ion energy is small in comparison with (3.12).

It is extremely important that the frequency and increment of the developing oscillations under the conditions discussed do not depend on the detailed form of the particle distribution. This has permitted us to state[26] that the exponential rise of the field will continue until the electron energy increases to a value comparable with the initial electron energies, after which the instability can, generally speaking, be stabilized. For the change with time of the electron temperature, assuming that the initial field is determined by thermal noise (3.9), we have, according to (3.11) and (3.12), the following equation:

$$\frac{dn_e\varkappa T_e(t)}{dt} \sim \frac{\varkappa T_e(0)\,\omega_{Le}}{r_{De}(t)}\int_{r_E^{-1}}^{k_0(t)} k\,dk\,\exp\left(\frac{2\omega_{Li}^2\omega_{Le}^2}{k^2 v_E^3}\,t\right). \tag{3.13}$$

It is evident that the main contribution to the integral in the right-hand side is from the region of the maximum increment.

According to Eq. (3.13) the electron temperature increases by an amount comparable with the initial temperature in a time

$$t_T \sim (\omega_0^3/2\omega_{Li}^2\omega_{Le}^2)\ln(n_e r_{De}^3\omega_{Li}^2\omega_{Le}^3/\omega_0^5).$$

Since after this time the right-hand side of (3.13) will be in order of magnitude

$$n_e\varkappa T_e\omega_{Le}^2\omega_{Li}^2/\omega_0^5\ln(n_e r_{De}^3\omega_{Li}^2\omega_L$$

and, in addition, under the conditions discussed $v_E \sim v_{Te}$, we can state that in this case the plasma conductivity reaches a value

$$\sigma_T \sim \frac{e^2 n_e}{m_e\omega_0^2}\,\frac{\omega_{Le}^2\omega_{Li}^2}{\omega_0^3\ln(n_e r_{De}^3\omega_{Li}^2\omega_{Le}^3\omega_0^{-5})}. \tag{3.14}$$

We note that at a time t_T the plasma-oscillation energy density is

$$n_e\varkappa T_e\omega_{Li}^2/\omega_{Le}\omega_0\ln(n_e r_{De}^3\omega_{Li}^2\omega_{Le}^3\omega_0^{-5}).$$

This is a small quantity in comparison with the energy of the plasma electrons.

The turbulent high-frequency conductivity of the plasma (3.14) becomes substantially greater than the ordinary conductivity due to Coulomb collisions of charged particles if

$$\omega_{Le}\omega_{Li}^2 n_e r_{De}^3 \gg \omega_0^3\Lambda,$$

where $\Lambda \sim \ln(n_e r_{De}^3)$ is the Coulomb logarithm. This inequality is easily satisfied for plasmas in which $\Lambda \sim 15-20$.

In ref. 26 we made a qualitative generalization of the discussion given here to the case of hydrodynamic instability of a cold plasma for parametric resonance $\omega_0 \approx \omega_{Le}$, where the maximum increment (2.1) also does not depend on the form of the particle distribution. The latter permits us to expect an exponential rise in the field oscillations until the accumulated random energy of the electrons approaches the energy of their oscillations in the pumping-wave field. We will show that for such a rise the turbulent high-frequency conductivity of the plasma reaches a value equal in order of magnitude to the oscillation buildup increment (2.1). Specifically, in accordance with Eq. (3.4) for the parametric instability of a plasma in a strong field ($v_E \gg v_{Te}$) we have

$$\langle \mathbf{Ej}\rangle \sim \int \frac{d\mathbf{k}}{(2\pi)^3}\frac{k^2}{4\pi}|\varphi_0|^2\gamma\mathbf{kr}_E J_0(\mathbf{kr}_E)\,J_1(\mathbf{kr}_E). \tag{3.15}$$

From this, assuming that an exponential rise of the longitudinal plasma-perturbation field occurs, we obtain

$$\int dt\,\langle \mathbf{Ej}\rangle \sim \int \frac{d\mathbf{k}}{(2\pi)^3}\frac{k^2}{4\pi}|\varphi_0|^2\,\mathbf{kr}_E J_0(\mathbf{kr}_E)\,J_1(\mathbf{kr}_E). \tag{3.16}$$

Since the left-hand side of this equation determines the increase in the plasma-electron energy density, if we then assume such a rise equal in order of magnitude to the electron-oscillation energy density in the pumping-wave field

$$n_e e^2 E_0^2 / m_e \omega_0^2 \approx E_0^2 / 4\pi,$$

we thereby obtain an evaluation for the integral in the right-hand side of Eq. (3.16), at the moment of time when this rise in electron energy occurs. Since the main contribution to the right-hand sides of Eqs. (3.15) and (3.16) is from the region of maximum increment values (see Eq. (2.1)), it is clear that $\langle \mathbf{E}\mathbf{j} \rangle \sim \gamma \mathbf{E}_0^2$. Therefore we actually have for the turbulent conductivity $\sigma_T \sim \omega_{Le}(m_e/m_i)^{1/3}$, and outside the resonance region we correspondingly obtain $\sigma_T \sim \omega_{Li}$.

We note that, since in the region of the maximum increment value $|\mathbf{k} \cdot \mathbf{r}_E| \sim 1$, it is evident that on reaching such a high turbulent conductivity, the energy density of the longitudinal field of the perturbations is equalized in order of magnitude with the energy of the pumping-wave field. It is natural that under these conditions the validity of the discussion of the parametric properties of the plasma is destroyed, since it takes into account only the effect of the pumping field.

A special simplification of the quasilinear theory of parametric plasma instability occurs near the instability threshold, i.e., at comparatively weak fields. This region has been discussed by Gradov and Markeev[31], who have given a picture of parametric instability stabilization as the result of the increase in temperature of the plasma electrons. The conceptual side of ref. 31 is close to the postulates of our theory[26]. However, while the idea of the average electron energy (the temperature) is sufficient for estimation well above threshold, on the other hand this is no longer obvious near threshold. The very fact established in ref. 26 of the possibility of stabilization cannot produce any special questions for the near-threshold region of instability, although the pattern of the phenomenon does not turn out to be simple even in this respect[28,32].

c) According to the theory of the quasistationary turbulent state of a parametrically unstable plasma, one of the causes of establishment of such a state is the nonlinear interaction of the waves. The corresponding theory under conditions of a weak pumping field was constructed in ref. 27 for a nonisothermal plasma ($T_e \ll T_i$), where ion-acoustic waves turn out to build up for parametric resonance. We will relate the nonlinear interaction of these waves with their induced scattering by ions. We will assume that the detuning of the resonance lies in the region of (2.12), where the increment of the parametrically increasing perturbations is described by Eq. (2.15) and the threshold by Eq. (2.16). Then, with inclusion of the spontaneous radiation of waves and their induced scattering by ions, we can write down the following equation which determines the change with time of the intensity of ion-acoustic waves[27]:

$$\left(\frac{d}{dt} - 2\gamma\right) |\varphi_0(\mathbf{k})|^2 = \sqrt{\frac{\pi}{2}} \frac{\omega_{Li}}{\omega_{Le}} 8\pi r_{De}^2 k v_s \varkappa T_e + |\varphi_0(\mathbf{k})|^2 \int d\mathbf{k}' Q(\mathbf{k}, \mathbf{k}') |\varphi_0(\mathbf{k}')|^2; \tag{3.17}$$

here γ is defined by Eq. (2.15), v_s is the ion-acoustic wave velocity, and the integral kernel determined by induced scattering has the form*

$$Q(\mathbf{k}, \mathbf{k}') = -\sqrt{\frac{\pi}{2}} \frac{k v_s}{4\pi r_{De}^2} \frac{\omega_{Li} v_{Te}}{\omega_{Le} v_{Ti}} \frac{1}{n_e \varkappa T_e} \frac{k - k'}{k k'} \times \left(\frac{\mathbf{k}\mathbf{k}'}{k k'}\right)^2 \frac{[\mathbf{k}\mathbf{k}']^2}{|\mathbf{k} - \mathbf{k}'|^2} \exp\left[-\frac{r_{De}^2 (k - k')^2}{2 r_{De}^2 |\mathbf{k} - \mathbf{k}'|^2}\right] \approx -\frac{k v_s}{4 r_{De}^2} \frac{r_{Di}^2}{r_{De}^2} \frac{1}{n_e \varkappa T_e} \frac{(\mathbf{k}\mathbf{k}')^2 [\mathbf{k}\mathbf{k}']^2}{(k k')^3} \frac{\partial \delta(\mathbf{k} - \mathbf{k}')}{\partial k}.$$

For a stationary state Eq. (3.17) gives

$$\varkappa T_e = W_s(\mathbf{k}) \left\{ [1 - \cos^2\theta F(k)] + 2^{3/2} \pi^{1/2} \frac{r_{Di}^2 \omega_{Le}}{r_{De}^2 \omega_{Li}} \int d\mathbf{k}' \frac{W_s(\mathbf{k}')}{n_e \varkappa T_e} k k' \frac{\partial \delta(k' - k)}{\partial k} \frac{(\mathbf{k}\mathbf{k}')^2 [\mathbf{k}\mathbf{k}']^2}{(k k')^4} \right\}; \tag{3.18}$$

here θ is the angle between $\mathbf{E}_0$ and $\mathbf{k}$, and

$$F(k) = \frac{1}{\sqrt{2\pi}} \frac{v_E^2}{v_{Te}^2} \frac{\omega_0 \Delta\omega_0 \tilde{\gamma}(k) k v_{Te}}{[(\Delta\omega_0)^2 - k^2 v_s^2]^2 + 4 k^2 v_s^2 \tilde{\gamma}^2(k)},$$

$$W_s(\mathbf{k}) = \frac{k^2}{4\pi} |\varphi_0(k)|^2 \frac{\partial(\omega\varepsilon)}{\partial\omega} = \frac{|\varphi_0(k)|^2}{4\pi r_{De}^2}.$$

The last expression is the energy density of ion-acoustic oscillations per interval of the wave vectors, the total energy density of these oscillations being

$$W = \int \frac{d\mathbf{k}}{(2\pi)^3} W_s(\mathbf{k}).$$

We will assume that the intensity of ion-acoustic oscillations is so great in comparison with thermal oscillations that it satisfies the inequality

$$W_s(\mathbf{k}_0) [F(\mathbf{k}_0) - 1] \gg \varkappa T_e, \tag{3.19}$$

where k_0 is defined by the decay condition (2.16). Then in Eq. (3.18) we can neglect the left-hand side and consider a linear equation. At the same time we will assume that the electric field of the pumping wave is only slightly greater than the threshold value (2.15). Then, according to the linear theory of parametric instability, those waves will build up which are propagated almost parallel or antiparallel to the electric field $\mathbf{E}_0$ with wave vectors whose magnitude is close to k_0. Therefore an expansion in the corresponding small deviations is possible. As a result (cf. ref. 27) we obtain

$$W_s(\mathbf{k}) = \frac{2}{(2\pi)^{3/2}} \frac{n_e \varkappa T_e}{k_0^3} \frac{\omega_{Li} r_{De}^2}{\omega_{Le} r_{Di}^2} \left(\frac{E_0^2}{E_{\text{thr}}^2} - 1\right)^{-1/2} \frac{\tilde{\gamma}(k_0)}{k_0 v_{Te}} \times \left[\frac{\omega_{Li}^2}{\omega_{Le}^2} + 6 \frac{\omega_0 - \omega_p}{\omega_p}\right]^{-1/2} \frac{x + 2}{(1 - x)^4} [(1 - x)^2 - \psi^2], \tag{3.20}$$

where

$$\psi = \frac{\theta}{[F(k_0) - 1]^{1/2}},$$

$$x = \frac{k - k_0}{k_0} \frac{1}{[F(k_0) - 1]^{1/2}} \frac{k_0 v_{Te}}{\tilde{\gamma}(k_0)} \left(\frac{\omega_{Li}^2}{\omega_{Le}^2} + 6 \frac{\omega_0 - \omega_p}{\omega_p}\right)^{1/2}.$$

Here $W_s(\mathbf{k})$ is negligible outside the region $-2 \leq x \leq 1$ and $\psi^2 \leq \approx 1 - x$.

The distribution of ion-acoustic oscillations in wave vectors (3.20) satisfies inequality (3.19) if

$$\frac{E_0^2}{E_{\text{thr}}^2} - 1 \gg 32\pi^3 \frac{r_{Di}^4 k_0^2 v_{Te}^2}{r_{De}^4 \tilde{\gamma}^2(k_0)} \left[1 + 6 \frac{\omega_{Le}(\omega_0 - \omega_p)}{\omega_{Li}^2}\right] \frac{k_0^6}{n_e^2}.$$

Correspondingly, by means of (3.20) we obtain for the total energy density of ion-acoustic waves

$$W = n_e \varkappa T_e \left[\frac{E_0^2}{E_{\text{thr}}^2} - 1\right] \cdot \frac{9}{64 (2\pi)^{5/2}} \frac{\omega_{Li} r_{De}^2}{\omega_{Le} r_{Di}^2} \left[\frac{\omega_{Li}^2}{\omega_{Le}^2} + 6 \frac{\omega_0 - \omega_p}{\omega_p}\right]^{-1} \times \frac{1}{k_0^8 r_{De}^8} \exp\left(-\frac{\omega_0}{k_0^2 v_{Te}^2}\right). \tag{3.21}$$

These results for the quasistationary distribution of waves in a plasma permit determination of the turbulent conductivity of the plasma. For this purpose we will use Eq. (3.6), having in mind that the main contri-

*$[\mathbf{k}\mathbf{k}'] \equiv \mathbf{k} \times \mathbf{k}'$.

bution in the case considered here is from the region $k \sim k_0$ and small values of the angle between the vectors $\mathbf{E}_0$ and $\mathbf{k}$. Then

$$\sigma_T = (e^2 n_e / m_e \omega_0^2)\, \nu_T,$$

where

$$\nu_T = [\omega_{Le}^2 / \tilde{\gamma}(k_0)]\, W / n_e \varkappa T_e,$$

and W represents the total energy density of the oscillations, defined by Eq. (3.21). For example, for a hydrogen plasma for $\omega_0 - \omega_{Le} \approx \omega_{Le}[\ln(m_i/m_e)]^{-1}$, which corresponds to the right-hand edge of the detuning region (2.13), in the case of the turbulent effective collision frequency we obtain

$$\nu_T \sim \omega_{Li}(T_e/T_i)\,[(E_0^2/E_{thr}^2) - 1].$$

Here we must make a remark related to the limitation on the value of ν_T. Specifically, in accordance with the assumptions of the linear theory of parametric instability, this quantity must be small in comparison with $\tilde{\gamma}(k_0)$. This corresponds to the assumption that the dissipation of high-frequency oscillations of the plasma is determined by the Cerenkov absorption mechanism. For sufficiently high intensity of the ion-acoustic waves the turbulent dissipation becomes important also for the high-frequency oscillations. Then $\tilde{\gamma}(k_0)$ must be replaced by $\tilde{\gamma}(k_0) + (\nu_T/2)$. As a result we have for the turbulent collision frequency

$$\nu_T = 2\tilde{\gamma}(k_0)\left(\frac{E_0^2}{E_{thr}^2} - 1\right)\left\{1 + \frac{32}{81}(2\pi)^{7/2}\frac{\omega_{Le} r_{Di}^2}{\omega_{Li} r_{De}^2} \times \right. \tag{3.22}$$
$$\left. \times \left(\frac{\omega_{Li}^2}{\omega_{Le}^2} + 6\frac{\omega_0 - \omega_p}{\omega_p}\right)\left[\left(\frac{\omega_{Li}^2}{\omega_{Le}^2} + 6\frac{\omega_0 - \omega_p}{\omega_p}\right)^{1/2} - \frac{\omega_{Li}}{\omega_{Le}}\right]^2\right\}^{-1}.$$

From this it is clear that the turbulent effective frequency of collisions (3.22) can exceed the ordinary frequency of Coulomb collisions of electrons with ions when $\tilde{\gamma}(k_0) \gg \nu_{ei}$. Then in the vicinity of the right-hand edge of the detuning range (2.13) and for $\omega_{Li} r_{De}^2 > \omega_{Le} r_{Di}^2$ $\nu_T = 2\tilde{\gamma}(k_0)[(E_0^2/E_{thr}^2) - 1]$. This formula allows us to discuss the possibility of an anomalous increase in the high-frequency conductivity of a parametrically unstable plasma even in the near-threshold region where the pumping-wave electric-field strength is only slightly greater than the value determined by Eq. (2.15).

4. NUMERICAL EXPERIMENTS MODELING A PARAMETRICALLY UNSTABLE PLASMA, AND ANOMALOUS HIGH-FREQUENCY CONDUCTIVITY

One of the directions of theoretical investigation of the action of an electromagnetic field on a plasma is the numerical calculation of simple models simulating plasma properties. These calculations are called numerical experiments. The simplest model suitable for machine calculation is a one-dimensional charged-particle model. Here the interaction of the particles is determined by the Coulomb field described by the Poisson equation, and the particle motion obeys Newton's laws, which take into account the effect of the self-consistent field. The corresponding method of calculation has been described by Birdsall and Fust[33].

This one-dimensional model forms the basis of the calculations which have been published by Kruer et al.[34], who used a system of ten thousand electrons and ten thousand ions. The ions were taken as one hundred times heavier than the electrons, and the electron temperature at the initial moment of time was taken as thirty times greater than the ion temperature. The linear dimension of the system of charged particles was 256 Debye radii. The charged particles were subjected to the action of a uniform monochromatic electric field $E_0 \cos \omega_0 t$. Figure 1 shows illustrations of the results of a numerical experiment, obtained for an external-field frequency identical to the electron Langmuir frequency and for $v_E = 0.6 v_{Te}$.

In Fig. 1a the ordinate shows on a logarithmic scale the ratio of the energy of plasma waves (without the contribution of the pumping-field energy) to the initial thermal energy of the plasma, and the abscissa is $\omega_{Le} t$. It can be seen from the figure that the exponential rise with time of the plasma-field wave energy sets in quite rapidly, and is cut off after the wave energy increases by about one hundred times. Figure 1b shows how plasma heating occurs simultaneously with the increase in the wave energy. Here the ordinate is the ratio of the total plasma energy to the initial value of its thermal energy, and the abscissa is $\omega_{Le} t$. The effective collision frequency corresponding to the rate of heating at the right-hand edge of Fig. 1b is about $0.18\omega_{Le}$, which corresponds to an anomalous increase in the dissipative high-frequency conductivity of the plasma.

The results of Kruer et al.[34] obtained for $\omega = \omega_{Le}$ correspond to a periodic parametric instability of the plasma, for which the increment γ can be characterized by the equation[19]

$$(\gamma + \tilde{\gamma})^2 = (1/2)\{[((\Delta\omega_0)^2 + \tilde{\gamma}^2 - \omega_s^2]^2 + (\mathbf{k r}_E / k r_{De})^2 \omega_s^2 \omega_0 \Delta\omega_0]^{1/2} - (\Delta\omega_0)^2 + \tilde{\gamma}^2 + \omega_s^2]\}.$$

This formula, when used with the external-field frequency and velocity $v_E = 0.6 v_{Te}$ assumed by Kruer et al.[34], leads to a maximum increment value $\gamma_{max} = 0.016\omega_0$, which occurs for $k r_{De} = 0.14$. The corresponding numbers obtained by Kruer et al.[34] for these values are respectively $0.018\omega_0$ and 0.15, which shows good agreement of the numerical experiment with theory.

The later results reported by Kruer et al.[35] extend the earlier work[34] to the case of a pumping-wave frequency somewhat above the electron plasma frequency, where it turns out to be possible to build up almost periodic oscillations at the parametric instability threshold which coincide with ion-acoustic waves. Here also an exponential rise of the perturbations in the plasma was observed, which leads to appearance of an anomalous conductivity accompanied by anomalously rapid heating of the plasma electrons. However, the anomalous effective collision frequency turns out to be less than that found earlier[34].

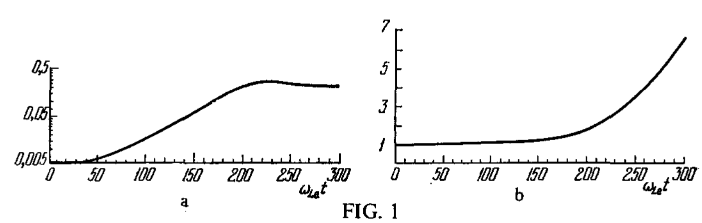

FIG. 1

Katz and Degroot[36] report a numerical experiment in which the ratio of the energy of the electromagnetic pumping wave to the thermal plasma energy varied from 0.72 to 5000. The pumping-field frequency was taken somewhat below or equal to, or somewhat above the electron Langmuir frequency. It was found that the parametric action of the radiation leads to plasma instability. The perturbations increase with a growth rate in agreement with the analytical theory, and at large times a saturation occurs. Katz and Degroot[36] note that for strong pumping fields, after saturation of the growth of the waves arising in the plasma, decay of the waves occurs which leads to heating of the plasma.

Kaw[37] reports a numerical experiment on the anomalous dissipation of an electromagnetic wave in a plasma for the case of a pumping-field frequency well above the electron Langmuir frequency. This work is reported as confirming the theory developed in ref. 26, whose results have been described in chapter 3 above.

On the whole it can be said that numerical experiments confirm the general theoretical ideas developed at the present time of parametric interaction of electromagnetic waves with plasma. Numerical experiments permit definite details to be obtained of the physical picture of development of parametric instabilities, energy dissipation, anomalous conductivity, and the entire dynamics of a parametrically unstable plasma. Among the studies which supplement our ideas on the parametric action of a wave on a plasma, we note that of Kruer and Dawson[38], devoted to numerical calculation of the interaction with a plasma of a strong traveling electromagnetic wave with a frequency close to twice the plasma frequency. Increase of the energy of electron plasma oscillations to saturation has also been observed, and a significant number of particles with high velocities, an order of magnitude greater than the thermal velocity, arise in the electron distribution.

5. EXPERIMENTAL STUDIES OF ANOMALOUS NONLINEAR DISSIPATION OF HIGH-FREQUENCY RADIO WAVES IN A PARAMETRICALLY UNSTABLE PLASMA

The theory of the parametric action of high-power radiation on a plasma arose to a certain degree in connection with the experimental studies carried out at the P. N. Lebedev Physics Institute, Academy of Sciences of the USSR, (FIAN) on radiative acceleration of plasma[39], which were undertaken, as was clear from the very beginning, under little studied physical conditions. Even the first results of the theory, reported in 1965[4], on the instability and the growth in energy of motion of the plasma particles under the action of an external high-frequency electromagnetic field, permitted the possibilities of radiative acceleration to be evaluated from positions which were completely new at that time. This was the subject of the work by Gurevich and Silin[30], in which, among other things, estimates were given of the growth time of electron thermal energy in the field of a high-frequency wave. In that work, on the basis of the results of ref. 4, we discussed the heating of electrons as the result of development of parametric instability against buildup of plasma potential oscillations, and a value $\sim 10/\omega_{Li}$ was given for the corresponding time of increase of the thermal energy up to a value of the order of the external-magnetic-field energy. It becomes clear that the action of high-power radiation on a plasma must be studied in more detail[1]). All of this served as a definite reorientation of research on the action of radiation on a plasma, which led subsequently at FIAN to the experimental observation of the very important phenomena of parametric action of high-power radiation on a plasma. However, the first confirmation of the theoretical ideas arose in another way.

Immediately after publication of the first theoretical studies on parametric resonance in plasma, Stern and Tzoar[41] experimentally observed, as the result of action of an external electromagnetic wave with a frequency close to the Langmuir frequency, the parametric excitation of ion-acoustic and high-frequency electron-plasma oscillations. In this experiment microwave radiation with a frequency $\omega_0 = 4.4 \times 10^9$ sec^{-1} entered a cylindrical plasma column whose average electron Langmuir frequency was approximately equal to the pumping-field frequency ($\omega_{Le} \approx \omega_0$). Qualitatively different patterns appeared at a low pumping-field power level and at a high level exceeding the experimentally observed threshold value, which corresponded to an electric field strength of ~15 V/cm. Thus, for a weak field the plasma reflected only the wave with frequency ω_0. On the other hand, above threshold an additional reflection was observed with waves with frequencies $\omega_0 - \omega_s$ and $\omega_0 + \omega_s$, where the frequency ω_s near threshold was identical with the frequency of ion-acoustic oscillations with a wavelength equal to the internal diameter of the vacuum pipe containing the plasma. The excitation of high-frequency plasma waves in a plasma was confirmed by the technique used by Stern and Tzoar[41] of scattering microwave radiation with frequency $\omega_{in} = 11.4 \times 10^9$ sec^{-1}. For a pumping-field power exceeding the threshold, three lines were observed in the scattered radiation spectrum: $\omega_{in} - \omega_0$, $\omega_{in} - \omega_0 - \omega_s$, $\omega_{in} - \omega_0 + \omega_s$. The excitation in a plasma of low-frequency ion-acoustic oscillations was subsequently established on the basis of the scattering spectrum, in which components $\omega_{in} - \omega_s$ and $\omega_{in} + \omega_s$ were observed, and also from the fluctuation current to a probe with the frequency of ion density fluctuations ω_s. Finally, Stern and Tzoar[41] reported that at a high pumping-field intensity (~200 V/cm) radiation from the plasma was observed with a broad spectrum near ω_0, which had sharp peaks in the vicinity of harmonics of ω_s and of the ion Langmuir frequency. Stern and Tzoar[41] associated their results with a plasma instability developed as the result of action of the external pumping field. Attempts at a theoretical analysis of the results of Stern and Tzoar[41] were undertaken later by Nichikawa[15] and Du Bois[16]. Here the qualitative aspects of the experimental results correspond

[1])The calculations of Gurevich and Silin [30] on the parametric action of radiation on plasma have made clear important limitations of the methods which have been discussed for radiative acceleration of plasma. We note that a critique of this method is contained in the review by Motz and Watson [10]. However, this critique did not involve analysis of a real model. We can cite, for example, the work of Hall and Gerwin [40], who carried out the necessary analysis and confirmed the physical ideas of ref. 30.

to the theory of plasma parametric instability, and quantitative agreement of the threshold value can be obtained with a reasonable value of the collision frequency[15].

A program of experimental investigations intended to check the theory of parametric action of high-power radiation on a plasma was set up and carried out at FIAN. One of the first phenomena observed in this work was the anomalous decay of a plasma[42-46]. In the first work in this direction in study of the interaction of a plasma flow with the potential barrier of a microwave field, it was shown that, over a wide range of plasma density values where the plasma is transparent for the microwave field, the energy spectra of plasma ions which have passed through the microwave barrier turn out to be identical. However, this situation was destroyed as the plasma density approached the value at which the electron Langmuir frequency is comparable with the frequency of the microwave field. Thus, in the work of Sergeĭchev[43] it was reported that "at a density determined with an accuracy of 30% and corresponding to $\omega_{Le}^2 \approx 0.4\omega_0^2$, a sharp change was already observed in the nature of the passage of the stream through the microwave barrier". This situation occurred both for energies of the oscillating electrons an order of magnitude greater than their thermal energy and for energies close to the thermal energy. Sergeĭchev associated this change in the nature of the plasma stream passage with parametric instabilities of the plasma. Subsequent experimental studies have confirmed this point of view. A definitive summary of research on anomalous decay of a plasma in a microwave field has been given by Sergeĭchev[45]. We will therefore dwell on this work in somewhat greater detail.

A diagram of the apparatus used in the decay experiments is shown in Fig. 2. Microwave radiation in the 10-cm range in the TE_{01} mode was varied from a power of tens of watts to 10^6 W. It was turned on before injection of the plasma and was maintained during the entire time of its flow. The duration of the field pulse was 10—20 μsec. The injected plasma contained mainly hydrogen ions H^+ and H_2^+ and carbon ion C^+ with an average energy of directed motion 200 eV, while the electron temperature was 4—5 eV. The transverse components of the ion kinetic energy after passing through the diaphragm did not exceed 0.2 eV. These data permit us to say that for a plasma with a critical density

$$n_0 = \omega_0^2 m_e / 4\pi e^2 \approx 10^{11}\ \text{cm}^{-3}$$

the electron-ion collision frequency turns out to be $\nu_{ei} = 3 \times 10^5\ \text{sec}^{-1}$. It should be noted that, since $v_{Te} = 7 \times 10^7$ cm/sec, the frequency $\nu_e = 3.5 \times 10^7\ \text{sec}^{-1}$ of collisions of electrons with the lateral boundaries of the plasma beam, whose dimensions are characterized by the diaphragm diameter of 2 cm, turns out to be substantially higher than the electron-ion collision frequency. This fact hinders the direct application of the theory of parametric action of high-power radiation on a plasma to an experimental situation of this type in the case where the results of this theory are determined substantially by electron-ion collisions. The main result of the work of Sergeĭchev[45] is the measurement

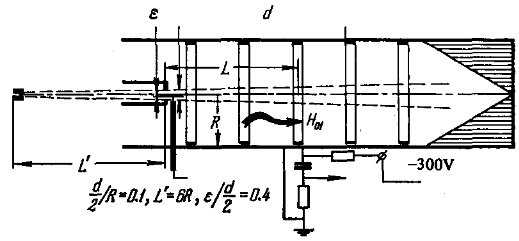

FIG. 2

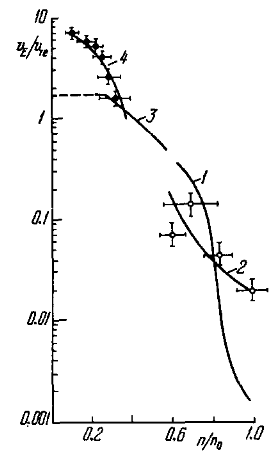

FIG. 3

of the threshold value of microwave electric-field strength at which, as a function of the plasma density, decay occurs which leads to appearance of current at the collector. This current is absent without the microwave field and if it is present with low intensity. The experimental results are shown in Fig. 3, where the ordinate is the ratio of the electron oscillation velocity amplitude to its thermal velocity corresponding to the vacuum electric-field strength for which decay occurs; the abscissa is the ratio of the maximum plasma density measured near the entrance diaphragm to the critical density value n_0. The experimental points correspond to measurements of refs. 42—46. Curves 4 and 2 were drawn on the basis of these points. The horizontal dashed line corresponds to the formula $v_E = 1.8\ v_{Te}$, theoretical curve 3 was obtained by calculation of the threshold from the formulas of ref. 5, and, finally, threoretical curve 1 was obtained from the formula for the threshold due to the Cerenkov effect: $v_E/v_{Te} = 2(kr_{De})^{-7/2} \exp(-\omega_0^2/2k^2v_{Te}^2)$, where the wave vector is determined by the relation

$$\Delta\omega_0 = \omega_0 - \omega_p[1 + (3/2)k^2 r_{De}^2] = \tilde{\gamma} \gg \omega_s.$$

We note that for low field intensities the parametric instability threshold is determined by collisions. In this case for the decay threshold $v_E/v_{Te} \sim 10^{-3}$, and for the aperiodic instability threshold of an opaque plasma $v_E/v_{Te} \sim 10^{-2}$. If we use in these evaluations instead of the electron-ion collision frequency the frequency of electron collisions with the plasma beam boundaries, these thresholds increase by an order of magnitude. Figure 3 permits us to conclude that the experimental data are similar to the theoretical predictions. However, for complete agreement of the theory with experiment, more data are necessary on

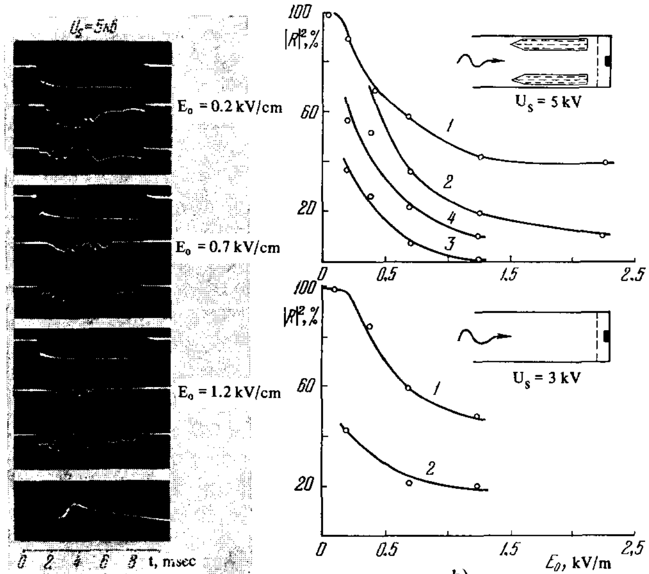

FIG. 4

the plasma state, so that the plasma behavior can be compared with various theoretical predictions.

Among the studies carried out at FIAN to check the theory of parametric action of high-power radiation on a plasma, the experimental studies of electromagnetic-wave absorption are important. The work of Gekker and Sizukhin[47 48] (see also refs. 46, 49) led to experimental observation of anomalously strong absorption in plasma, arising for sufficiently high power of the radiation. Gekker and Sizukhin[47,48] studied the absorption by a plasma of a high-power TE_{11} wave in the 10-cm region in a circular wave guide. Here the plasma column injected against the traveling wave filled the entire cross section of the waveguide and had a particle distribution which was practically uniform over the radius. The injected plasma temperature was ~4 eV. The velocity of the plasma front injected into the equipment was $\sim 10^7$ cm/sec. The density gradient of the number of plasma particles in the leading front was 10^9—10^{10} cm^{-4}, and the maximum plasma density on leaving the region of microwave loading (in the traveling-wave case) reached 10^{12} cm^{-3}. As the plasma propagated along the waveguide (along the z axis) the maximum density of the number of particles fell off in proportion to z^{-3}. Here the electron Langmuir frequency of the plasma ω_{Le} for some value of z became equal to the frequency of the electromagnetic wave, which was terminated in the waveguide. In ref. 47 a traveling wave was used (in the absence of a plasma); here the apparatus was supplied with a matched microwave load, and in ref. 48 results are presented also for another arrangement in which there is no matched termination, but the plasma is injected into a TE_{11} standing wave.

For a weak field (E_0 = 0.1 V/cm), where the electron oscillation velocity in the field of the wave is small in comparison with the thermal velocity, Gekker and Sizukhin[47] obtained practically 100% reflection of the wave. This corresponds to the usual ideas of interaction of a high-frequency field with a plasma under conditions where the inequalities $\omega_{Le} > \omega_0 \gg \nu_{ei}$ which occur in this experiment are satisfied. The electromagnetic wave reflected from the plasma was detected by means of a directional coupler, and the wave passing through the waveguide was detected by means of antennas introduced through side tubes on the circular waveguide. These measurements were carried out over a range of electric-field strengths E_0 from 0.2 kV/cm to 2 kV/cm. Typical oscillograms of microwave signals, shown in Fig. 4a, correspond to the traveling wave scheme. For each of the three cases of pumping-field intensity, the lower oscillogram gives the reflected-wave signal, the middle oscillogram gives the transmitted-wave signal measured by an antenna at a distance z = 45 cm from the plasma source, and the upper oscillogram is a complete-reflection signal for calibration. At the bottom in the same figure we have shown separately an oscillogram of the ion current determined by the signal from a plasma probe introduced at the center of the circular waveguide at z = 45 cm.

In Fig. 4b we have plotted the reflection coefficient of the TE_{11} wave as a function of the electric-field strength for both the traveling-wave and standing-wave resonances. Curve 1 corresponds to the maximum averaged over different plasma-injector bursts, curve 2 corresponds to the maximum values for the best bursts from the plasma source, curve 3 was drawn from the values obtained from the dips in the reflected-signal oscillograms, and, finally, curve 4 was plotted by Gekker and Sizukhin[48] in determination of the standing-wave coefficient from the oscillograms. All of these data indicate a significant decrease in the reflection coefficient for a high-power wave in comparison with the reflection coefficient in a weak field, which is practically indistinguishable from unity. Since for a certain time during the experiment the electromagnetic wave does not travel beyond the plasma layer, the decrease in reflection coefficient corresponds to a substantial increase in the absorption of the plasma-wave energy. Here, as follows from Fig. 4b, the energy absorbed by the plasma increases with increasing electric-field strength of the wave. Gekker and Sizukhin[48] showed that with increasing field strength the time for which the plasma is preserved in the burst injected into the waveguide decreases, i.e., the dissipation of the plasma under the action of the external microwave field occurs more rapidly. The entire set of facts demonstrated by Gekker and Sizukhin[47,48] permits us to state that anomalous absorption of electromagnetic waves by a plasma occurs for an electric-field strength greater than 100 V/cm ($v_E/v_{Te} \approx 0.1$) and rises up to E_0 = 1 kV/cm ($v_E/v_{Te} \sim 1$). We note here that propagation of a wave in the waveguide becomes impossible for a plasma density greater than the critical value n_0. In the experiments of Gekker and Sizukhin[47,48] this blocking of the waveguide was shifted substantially in plasma density as the result of the low waveguide modes used. On the other hand, if we have in mind this displacement of the resonance, the absorption threshold obtained by Gekker and Sizukhin[47,48] is consistent with the estimates of the theory of parametric action of radiation on a plasma (see the review by Gekker[49]).

An additional phenomenon occurring under the ac-

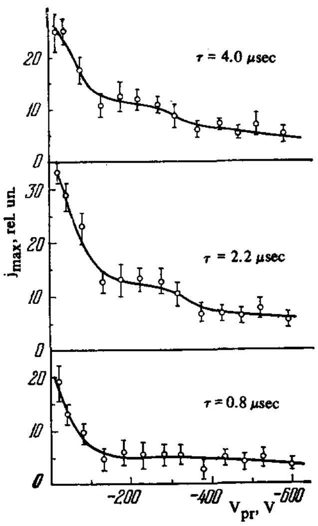

FIG. 5

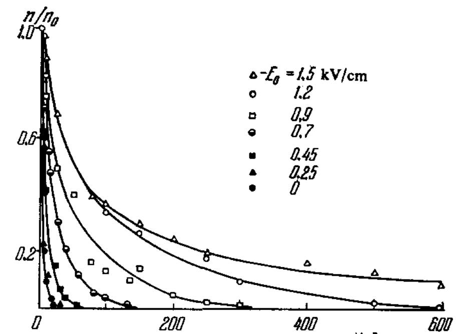

FIG. 6

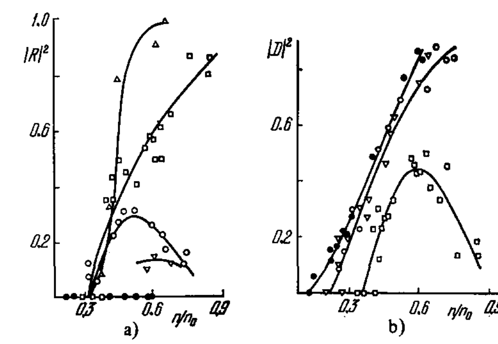

FIG. 7. The curves drawn through the symbols □, ○, ∇, and ○, correspond to E_0 = 0.7, 2, and 3 kV/cm, respectively.

tion of a strong microwave field on a plasma, observed by Batanov, Sarksian, and Silin[50] (see also ref. 46) and associated by them with the parametric development of instability in the plasma, is the anomalously intense heating of electrons by a strong microwave field. The authors of ref. 50 used a circular waveguide in which a traveling TE_{11} wave was propagated. A strong magnetic field was placed transverse to the guide, and perpendicular to the microwave electric field, along which was propagated the plasma stream subjected to the action of the microwave field. Here the conditions $\omega_{Le} \leq \omega_0 < |\Omega_e| = |e|B/m_ec$ were satisfied. For an increase of power with a rise

$$e\Phi_0 = e^2E_0^2/4m_e(\omega_0^2 - \Omega_e^2)$$

an electron current occurs to the plasma probe which for negative or zero values of the probe electron potential corresponds to an increase in the average energy of the electrons, to heating of the plasma. By changing the negative potential of the probe it is possible to obtain a representation of the energy distribution of the plasma electrons. Figure 5 shows the corresponding experimental results[50]. We emphasize that electron heating occurs for field strengths greater than the threshold value, which corresponds to $e\Phi_0 \approx 0.01\kappa T_e$.

Batanov et al.[50] observed electrons reaching energies of motion along the magnetic field of ~600 eV in the cyclotron resonance. Since the magnetic field was uniform in this case, the achievement of high energies by the electrons indicates an effective mechanism of transfer of the energy of oscillation of the electrons to their energy of motion along the magnetic field. The effective collision frequency corresponding to accumulation of an electron energy of ~20–30 eV near the threshold of anomalous heating corresponds to a value an order of magnitude higher than the frequency of Coulomb electron-ion collisions. With an increase of the radiation power, this effective collision frequency rises.

The appearance of fast electrons in the action of a microwave field on a plasma was observed by Barinov et al.[51] even in the absence of a magnetic field; here the experiments were carried out in the 10-cm range and a TE_{11} wave interacted with a plasma moving against it, which corresponds to the conditions of the experiments of Gekker and Sizukhin[47,48] on anomalous absorption. Here electron acceleration occurs preferentially along the direction of the electric field of the wave. From the energy distributions of the electrons (Fig. 6) Barinov et al.[51] concluded that accelerated electrons appeared for a threshold field strength of ≈0.25 kV/cm, which is close to the thresholds of Gekker and Sizukhin[47,48] for anomalous absorption. With increasing E_0, as can be seen from Fig. 6, the maximum electron energy rises rapidly and turns out to be substantially greater than the energy of oscillations of an electron in the field of the wave. The data of Barinov et al.[51] on the dependence of the accelerated electron energy for E_0 = 0.7 kV/cm indicate that the accelerated electron energies exceeded 50 eV only for $n = (0.4 \pm 0.2)n_0$, became more than 100 eV for $n - (0.5 \pm 0.2)n_0$, and reached a maximum energy of 300 eV for $n \approx n_0$. Further increase of the plasma density led to a decrease in the electron energy.

The problem of simultaneously studying the absorption of an electromagnetic wave and the heating of the electrons was formulated and solved in the work of Sergeĭchev and Trofimov[52], who employed a plasma flux uniform in cross section and intersecting a rectangular waveguide. Figures 7a and b show the reflection coefficient $|R^2|$ and absorption coefficient $|D|^2$ for the energy of the wave in the plasma as a function of the number-of-particle density of the plasma n for various electric-field strengths. Simultaneously with the absorption of energy by the microwave field, they observed an increase in the electron energy (a heating). Figure 8 shows the current density of accelerated

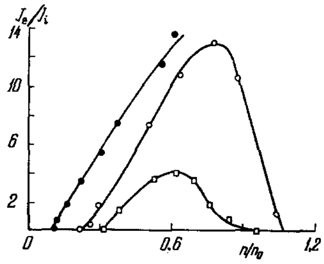

FIG. 8. The curves drawn through the symbols □, ○, and corresponds to E_0 = 0.7, 2, and 3 kV/cm, respectively.

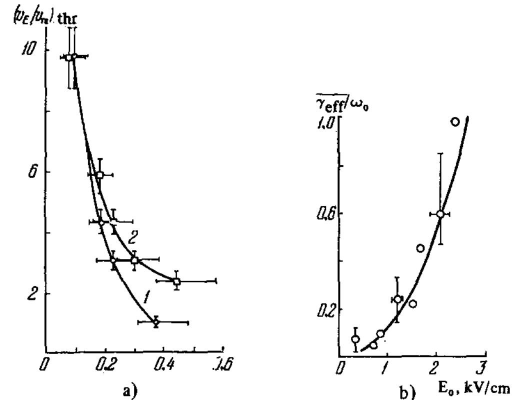

FIG. 9

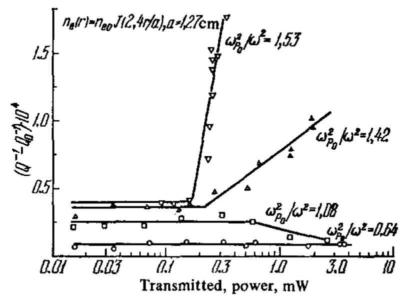

FIG. 10

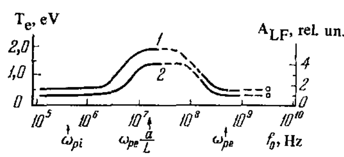

FIG. 11

plasma electrons in the direction of the lines of force of the electric field, normalized to the ion current density. In regard to the electron energies, for example, for E_0 = 7 kV/cm and $n = 0.6n_0$, the average accelerated-electron energies reached 4 keV, and the maximum energies reached 11 keV. In Fig. 9a we have shown the threshold field strength as a function of plasma concentration, plotted from the beginning of anomalous absorption (curve 1) and from the beginning of accelerated-electron current (curve 2). The difference between the curves is due to inaccuracy in measurement of the accelerated-electron current threshold. The two curves have qualitatively identical behavior, although the threshold values appreciably exceed those determined by Sergeĭchev[45], which, however, refer to a different field geometry and its interaction with the plasma.

Sergeĭchev and Trofimov[52] for $n = 0.6n_0$ calculated from the measured data for the reflection coefficient the effective collision frequency, which we have shown as a function of field strength in Fig. 9b; as can be seen, it corresponds to a nonlinear anomalously high dissipation of the field in the plasma.

The program of experimental studies formulated at FIAN as a whole has shown that high-power electromagnetic radiation has an anomalously strong action on plasma, leading to an increased dispersal of the plasma, anomalously strong absorption of the energy of the electromagnetic wave, and the appearance of electrons of very high energy. The phenomena observed here correspond to the concepts and predictions of the theory of parametric action of radiation on plasma.

A somewhat different arrangement was used in the work of Dreicer, Henderson, and Ingraham[53], who measured the high-frequency resistance of a magnetized thermally ionized potassium plasma column as a function of intensity of the electric field parallel to a constant magnetic field. Here a TM_{010} resonator was used at a frequency near 2 GHz. For a plasma particle density $n < n_0$ the measured Q of the resonator with the plasma is consistent with the theory of field absorption due to electron-ion Coulomb collisions. If $n > n_0$, this agreement occurs only for a sufficiently weak field. If v_E reaches $\sim 0.15 v_{Te}$ and increases still further, the high-frequency resistance of the plasma increases with increasing field, which is similar to the results obtained at FIAN (Fig. 10).

Parametric resonance in a plasma located in a strong magnetic field has been observed by Demirkhanov, Khorasanov, and Sidorova[54]. These authors investigated the behavior of a thermally isolated cesium plasma with a temperature $\kappa T_e \sim \kappa T_i \sim 0.2$ eV and with a density $n \sim 10^8$–10^9 cm^{-3}. The degree of ionization was 20–30%, and the longitudinal magnetic field of the single-ended Q machine was $(2-5)\times 10^3$ Oe. The radius of the plasma filament was a = 2 cm, and the length of the filament L = 25 cm. A high-frequency voltage was applied to electrodes bounding the plasma on the ends. The frequency of the variable electric field was changed over the range 10^5 Hz $< (\omega_0/2\pi) < 3\times 10^7$ Hz, 1.5×10^8 Hz $< (\omega_0/2\pi) < 10^9$ Hz, and $(\omega_0/2\pi) \approx 3\times 10^9$ Hz. As a result of the action of the high-frequency electric field, the plasma was heated. In addition to this, intense low-frequency oscillations of the density appeared in the plasma, with the fundamental harmonic in the range 1–5 kHz. In Fig. 11 we have shown the electron temperature (curve 1) and the amplitude of low-frequency plasma oscillations (curve 2) as a function of the frequency of the electric field. The solid curves represent averaged experimental data, and the dashed portions are extrapolations; the points correspond to data for 3×10^9 Hz.

Demirkhanov et al.[54] reached the conclusion that the explanation of the results shown in Fig. 11 can be understood in terms of the concept of parametric resonance of a magnetized plasma. For natural frequencies

$$\omega_{re}^{\pm} = [(1/2)\{\omega_{Le}^2+\Omega_e^2 \pm [(\omega_{Le}^2+\Omega_e^2) - 4\omega_{Le}^2\Omega_e^2\cos^2\theta]^{1/2}\}]^{1/2}$$

under the experimental conditions of ref. 54 where $\Omega_e^2 \gg \omega_{Le}^2$, we have $\omega_{re}^+ = |\Omega_e|$, $\omega_{re}^- = \omega_{Le} \cos\theta$. On the assumption that the principal mode of natural oscillations is determined by the geometry of the system,

$$k_z \approx 2\pi/2L, \quad k_\perp \approx 1/a, \quad k_\perp \gg k_z;$$

therefore

$$\omega_{re}^- = \omega_{Le} k_z/k \approx (\pi a/L)\, \omega_{Le}.$$

The range of frequencies 5–30 MHz turns out to be of just this order; this is the range in which Demirkhanov et al.[54] most clearly observed development of plasma instability. The width of the resonance region can be related to the development of waves with different values of cos θ. The drop in the curves of Fig. 11 in the region of frequencies of the order of the electron Langmuir frequency corresponds to the region of parametric instability discussed by Aliev and Silin[55].

Anomalous dissipation of electromagnetic waves with frequency close to the electron Langmuir frequency has been observed by Eubank[56] for a plasma in a magnetic field B = 2000–4000 G. A decrease was observed in the reflection coefficient with increasing field strength of the wave ($\omega_0/2\pi = 10.5 \times 10^9$ Hz). Eubank also observed intense oscillations and an increase in the plasma temperature transverse to the magnetic field, for which, as can be seen from Fig. 12, there is a threshold of ~300 W ($E_0 \approx 500$ V/cm), and for strong fields a saturation occurs. For the transverse temperature the saturation amounts to ~2 eV, while the longitudinal temperature reaches 10–15 eV. From this Eubank[56] concludes that the Cerenkov interaction of electrons with plasma waves predominates over the interaction due to Coulomb collisions.

In conclusion we note the interest in experiments on heating of the ionosphere by radio waves[57-61], in which the energy flux of the waves is large. In fact, estimates made by Perkins and Kaw[62] have shown that the energy flux used in these experiments leads to a field strength in the ionospheric plasma substantially above the threshold field defined by Eq. (1.12). Therefore we can speak of the departure of research on the parametric action of radiation on plasma from the laboratory to the ionosphere.

The direct experimental study of parametric excitation of waves in ionospheric plasma was undertaken by Wong and Taylor[63], who reported the parametric action of pumping radiofrequency waves of frequency 5.62 MHz, which led to excitation of ion-acoustic waves at a height of about 200 km. The excited radiation was observed in the effect of scattering of radio waves with frequency 430 MHz. Here the scattered-radiation line shape is determined by the intensity of ion-acoustic waves and by their spectrum. A substantial increase in the intensity of ion-acoustic waves occurs under conditions where the pumping field is of an order greater than the threshold value given by Eq. (2.7). The intensity of ion-acoustic plasma perturbations, according to Wong and Taylor[63], turns out to be nonlinearly dependent on the intensity of the pumping wave. These results have shown the practical possibility of artificial change of the turbulent state of ionospheric plasma by means of the parametric action of high-power radio waves.

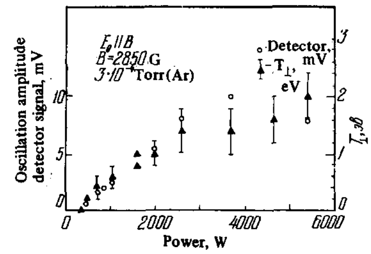

FIG. 12

6. CONCLUSION

In summarizing the material set forth above, we can state that the theoretical results obtained during the last few years relating to parametric action on a plasma have allowed the prediction of a qualitatively new group of phenomena in the interaction of electromagnetic waves with plasma. This group of phenomena involves development in the plasma of parametric instabilities due to the nonlinear action on the plasma of a sufficiently strong high-frequency electric field. The development of parametric instabilities leads to appearance of a turbulent state of the plasma, one of the manifestations of which is an anomalous high-frequency conductivity of the plasma. Theoretical predictions of anomalous nonlinear dissipation of high-frequency electromagnetic waves in plasma have found confirmation in a number of experimental studies, which have led, as we have described above, to the experimental discovery of anomalously strong absorption of the electromagnetic wave by the plasma. These important first steps in the experimental study of the action of high-power electromagnetic radiation on a plasma show us the need for extensive and detailed studies of the phenomena occurring in a plasma under these conditions, for the physical nature of the anomalous dissipation, like the more general phenomenon of parametric action of radiation on a plasma, is incomparably richer and more complicated than the nature of the ordinary dissipation due to particle collisions. A significant working out of details must be further provided in the theoretical investigations.

It must be emphasized that in selection of the material comprising the present view we have limited ourselves in many respects. Thus, we have not begun to discuss the possibilities associated with parametric excitation in a plasma of nonpotential perturbations, which, although they are very interesting and important, are nevertheless as yet not sufficiently well studied. At the same time we have left to one side almost completely the discussion of the role of constant magnetic fields, which, as is well known, in many respects qualitatively change the picture of parametric action of radiation on plasma. The only justification for this limitation is our desire to set forth completely and at the same time compactly the material in the region which has been comparatively best studied at the present time. At the same time it must be clear that the less studied regions of the physics of parametric action of radiation on plasma conceal extraordinary interesting possibilities which undoubtedly will attract our attention in the near future.

[1] V. L. Ginzburg and A. V. Gurevich, Usp. Fiz. Nauk 70, 201, 393 (1960) [Sov. Phys.-Uspekhi 3, 115, 175 (1960)].
[2] V. P. Silin, Zh. Eksp. Teor. Fiz. 47, 2254 (1964) [Sov. Phys.-JETP 20, 1510 (1965)].
[3] V. P. Silin, A Survey of Phenomena in Ionized Gases, Vienna, IAEA, 1968.
[4] V. P. Silin, Zh. Eksp. Teor. Fiz. 48, 1679 (1965) [Sov. Phys.-JETP 21, 1127 (1965)].
[5] V. P. Silin, Zh. Eksp. Teor. Fiz. 51, 1842 (1966) [Sov. Phys.-JETP 24, 1242 (1967)].
[6] E. A. Jackson, Phys. Rev. 153, 235 (1967).
[7] Jun-ichi Okutani, Tokyo Univ. Preprint, 1969.
[8] J. R. Sanmartin, Phys. Fluids 13, 1533 (1970).
[9] Yu. M. Aliev, L. M. Gorbunov, V. P. Silin, and H. Watson, Plasma Physics and Controlled Nuclear Fusion Research, vol. 1, Vienna, IAEA, 1966.
[10] H. Motz and C. J. H. Watson, Adv. Electron. and Electron Phys. 23, 154 (1967).
[11] D. F. DuBois and M. V. Goldman, Phys. Rev. Letters 14, 544 (1965).
[12] M. V. Goldman, Ann. Phys. (N.Y.) 38, 95 (1966).
[13] Y. C. Lee and C. H. Su, Phys. Rev. 152, 129 (1966).
[14] D. F. DuBois and M. V. Goldman, Phys. Rev. 164, 207 (1967).
[15] K. Nishikawa, J. Phys. Soc. Japan 24, 1152 (1968).
[16] D. F. DuBois, Statistical Physics of Charged-Particle Systems (1968 Tokyo Summer Lectures), ed. by R. Kubo and T. Kihara, Tokyo-New York, 1969.
[17] V. P. Silin, ZhETF Pis. Red. 7, 242 (1968) [JETP Letters 7, 187 (1968)].
[18] N. E. Andreev, A. Yu. Kiriĭ, and V. P. Silin, Proc. of the 9th Intern. Conf. on Phenomena in Ionized Gases, Bucharest, IAEA, 1969.
[19] N. E. Andreev, A. Yu. Kiriĭ, and V. P. Silin, Zh. Eksp. Teor. Fiz. 57, 1024 (1969) [Sov. Phys.-JETP 30, 559 (1970)].
[20] N. E. Andreev, A. Yu. Kiriĭ, and V. P. Silin, Izv. vyzov (Radiofizika) 13, 1321 (1970).
[21] V. N. Oraevskiĭ and R. Z. Sagdeev, Zh. Tech. Fiz. 32, 1291 (1962) [Sov. Phys.-Tech Phys. 7, 955 (1963)].
[22] Yu. M. Aliev and E. Ferlengi, Zh. Eksp. Teor. Fiz. 57, 1623 (1969) [Sov. Phys.-JETP 30, 877 (1970)].
[23] J. H. Krenz and G. S. Kino, J. Appl. Phys. 36, 2387 (1965).
[24] R. R. Ramazashvili, Zh. Eksp. Teor. Fiz. 53, 2168 (1967) [Sov. Phys.-JETP 26, 1225 (1968)].
[25] F. W. Perkins and J. Flick, Princeton Univ. Preprint MATT-833, 1971.
[26] V. P. Silin, Zh. Eksp. Teor. Fiz. 57, 183 (1969) [Sov. Phys.-JETP 30, 105 (1970)].
[27] V. V. Pustovalov and V. P. Silin, Zh. Eksp. Teor. Fiz. 59, 2215 (1970) [Sov. Phys.-JETP 32, 1198 (1971)].
[28] A. Yu. Kiriĭ, Zh. Eksp. Teor. Fiz. 60, 955 (1971) [Sov. Phys.-JETP 33, 517 (1971)].
[29] E. J. Valeo and C. Oberman, Princeton Univ. Preprint MATT-835, 1971.
[30] A. V. Gurevich and V. P. Silin, Yad. Fiz. 2, 250 (1965) [Sov. J. Nucl. Phys. 2, 179 (1966)].
[31] O. M. Gradov and B. M. Markeev, Kr. soobshch. Fiz. (Brief Reports in Physics), P. N. Lebedev Physics Institute (FIAN), No. 5, 15 (1971).
[32] V. V. Pustovalov and V. P. Silin, ZhETF Pis. Red. 14, 439 (1971) [JETP Letters 14, 299 (1971)].
[33] C. K. Birdsall and D. Fuss, J. Comput. Phys. 3, 494 (1969).
[34] W. L. Kruer, P. K. Kaw, J. M. Dawson, and C. Oberman, Phys. Rev. Letters 24, 987 (1970).
[35] W. L. Kruer, P. K. Kaw, and J. M. Dawson, Bull. Am. Phys. Soc. 15, 1407 (1970).
[36] J. I. Katz and J. S. Degroot, Bull. Am. Phys. Soc. 15, 1472 (1970).
[37] P. K. Kaw, Princeton Univ. Preprint, 1970 (Report at the Conf. on Laser Plasma, Moscow, November, 1970).
[38] W. L. Kruer and J. M. Dawson, Princeton Univ. Preprint, October, 1970; J. M. Dawson, W. L. Kruer, and P. K. Kaw, Bull. Am. Phys. Soc. 15, 1408 (1970).
[39] V. I. Veksler, I. R. Gekker, É. Ya. Gol'ts, G. A. Delone, B. P. Kononov, O. V. Kudrevatova, G. S. Luk'yanchikov, M. S. Rabinovich, M. M. Savchenko, K. A. Sarksyan, K. F. Sergeĭchev, V. P. Silin, et al., Trudy Mezhdunarodnoĭ konferentsii po uskoritelyam (Proceedings, Intern. Conf. on Accelerators), August, 1963, Dubna, JINR, 1964.
[40] R. B. Hall and R. A. Gerwin, Phys. Laboratory (Boeing Scientific Research Laboratories, Seattle, Washington 98126) Preprint D1-82-0977, May, 1970.
[41] R. A. Stern and N. Tzoar, Phys. Rev. Letters 17, 903 (1966).
[42] K. F. Sergeĭchev and I. R. Gekker, Proc. of the 18th Intern. Conf. on Phenomena in Ionized Gases, Vienna, IAEA, 1967.
[43] K. F. Sergeĭchev, Zh. Eksp. Teor. Fiz. 52, 575 (1967) [Sov. Phys.-JETP 25, 377 (1967)].
[44] K. F. Sergeĭchev, see ref. 18.
[45] K. F. Sergeĭchev, Zh. Eksp. Teor. Fiz. 58, 1157 (1970) [Sov. Phys.-JETP 31, 620 (1970)].
[46] G. M. Batanov, I. R. Gekker, K. A. Sarksian, K. F. Sergeĭchev, and V. P. Silin, 3rd European Conf. on Controlled Fusion and Plasma Physics (Symposium on Beam-Plasma Interactions), Utrecht, 1969.
[47] I. R. Gekker and O. V. Sizukhin, ZhETF Pis. Red. 9, 408 (1969) [JETP Letters 9, 243 (1969)].
[48] I. R. Gekker and O. V. Sizukhin, see ref. 18.
[49] I. R. Gekker, Physik and Technik des Plasmas II (Zusammenfassende Vortrage der Arbeiterstagung, Rostock, 18—23 Oktober 1970), Phys. Ges. der DDR, 1970.
[50] G. M. Batanov, K. A. Sarksian, and V. P. Silin, see ref. 18.
[51] V. I. Barinov, I. R. Gekker, O. V. Sizukhin, and É. G. Khachaturyan, Kr. soobshch. fiz. (Brief Reports in Physics), P. N. Lebedev Physics Institute (FIAN), No. 3, 41 (1971).
[52] K. F. Sergeĭchev and V. E. Trofimov, ZhETF Pis. Red. 13, 236 (1971) [JETP Letters 13, 166 (1971)].
[53] H. Dreicer, D. B. Henderson, and J. C. Ingraham, Phys. Rev. Letters 26, 1616 (1971).
[54] R. A. Demirkhanov et al., Zh. Eksp. Teor. Fiz. 59, 1873 (1970) [Sov. Phys.-JETP 32, 1013 (1971)].
[55] Yu. M. Aliev, V. P. Silin, and C. J. H. Watson, Zh. Eksp. Teor. Fiz. 50, 943 (1966) [Sov. Phys.-JETP 23, 626 (1966)].
[56] H. P. Eubank, Princeton Univ. Preprint MATT-825, Princeton, January, 1971.
[57] W. F. Utlaut, J. Geophys. Res. 75, 6402 (1970).
[58] A. A. Biondi, et al., J. Geophys. Res. 75, 6421

(1970).
[59]W. F. J. Evans, et al., J. Geophys. Res. 75, 6425 (1970).
[60]W. F. Utlaut, et al., J. Geophys. Res. 75, 6429 (1970).
[61]R. Cohen and J. D. Whitehead, J. Geophys. Res. 75, 6439 (1970).
[62]F. W. Perkins and P. K. Kaw, Princeton Univ. Preprint MATT-812, 1970.
[63]A. Y. Wong and R. J. Taylor, Phys. Rev. Letters 27, 644 (1971).

Translated by C. S. Robinson