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PROPAGATION OF DISCHARGES AND MAINTENANCE OF A DENSE PLASMA BY ELECTROMAGNETIC FIELDS

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I. INTRODUCTION

IN any processes referred to as discharges, a gas located in an external field remains in an ionized state. The plasma state of matter is not only the result of dissipation in it of electromagnetic energy, but is the cause of dissipation of the field, for un-ionized gases do not conduct electric current and, as a rule, do not absorb electromagnetic radiation over wide regions of the spectrum up to the far ultraviolet. (By designation of the frequency of the field we distinguish discharges in constant or slowly varying electric fields, high-frequency fields, microwave fields, and optical fields.)

Discharges have a characteristic tendency to propagate. In fact, processes exist which result in ionization of the gas layers adjacent to a discharge plasma: heating by a shock wave, thermal conduction, or thermal radiation, accompanied by thermal ionization; indirect ionization of atoms by plasma radiation or excitation of atoms with subsequent ionization; and so forth. If the ionized layers are still located in a sufficiently strong field, then a great deal of energy is also dissipated in them, ionization occurs in the next layers, and so forth. In other words, the discharge is propagated in the material. Here it is not at all necessary that the discharge also shift in space; the gas can flow through a stationary discharge. This principle is the basis of operation of plasmatrons—devices intended for continuous production of plasma by means of discharges.

Of course, the tendency for propagation of discharges occurs only under appropriate conditions. For example, the field can be localized in a limited region through which there is no gas flow, or the field intensity may turn out to be sufficient only for compensation of the energy loss from a given mass of plasma but not sufficient for conversion of new gas layers into plasma. In these cases there occurs not a production but only a maintenance of a definite plasma mass by the field.

It is important that for maintenance of a plasma and propagation of discharges it is sufficient to have relatively small fields, much smaller than those necessary for breakdown of the gas. This suggests use of auxiliary means for the initial formation of the discharge but, on the other hand, opens up even greater possibilities for control of discharges than with a breakdown, which ignites spontaneously.

The propagation of discharges is subject to laws which in some respects are general and independent of the nature of the field or the mechanism of propagation. The effect can often be considered as the propagation of some wave. The main theoretical problem—calculation of the rate of propagation and the parameters of the discharge produced—belongs in this case to the class of problems of the theory of "modes", which includes waves of many types: combustion, detonation, radiant cooling, and so forth. A static discharge maintained in a definite mass of gas is in some sense a limiting case of a propagated discharge, that is to say, propagation with zero velocity.

Effects of discharge propagation and plasma maintenance by electromagnetic energy are encountered in many physical processes and devices, sometimes very far removed from each other, such as, say, a laser spark and an induction plasma torch. These processes are the objects of physical investigations and are used for practical purposes; rather extensive experimental and theoretical information has been accumulated on them. It is therefore desirable to systematize, generalize, and analyze this information from a unified point of view, and that forms the subject of the present article. This analysis will aid in better understanding of well known phenomena and will create theoretical bases for the investigation and prediction of new phenomena.

1. Analogy with combustion, and modes of propagation. A deep analogy exists between discharge propagation processes in dissipation of the energy of a field in a plasma and the combustion associated with expenditure of chemical energy in matter. Resort to the ideas and methods of combustion and detonation theory (see the books by Zel'dovich[1] and Landau and Lifshitz[2]) has enabled us to discuss and understand certain important features of discharge propagation and has stimulated the theoretical study of the corresponding discharge phenomena.

Chemical reactions in combustible mixtures do not occur at ordinary temperatures and are sharply accelerated on heating. As a rule the reaction rates increase with increasing temperature according to a law of the Boltzmann type—the Arrhenius law $\exp(-U/kT)$, where U is the activation energy. At room temperature $kT \ll U$ and therefore the temperature dependence of the rate turns out to be extraordinarily rapid. When the mixture is ignited at some point, transfer of heat from the hot combustion products to layers have not yet reacted leads to their ignition, and a combustion wave is propagated over the material. Here two main mechanisms are possible for heating of the initial mixture and correspondingly two mechanisms of combustion propagation: detonation and slow combustion. In the first case the mixture is heated to ignition by a shock wave which is directly adjacent to the zone where the chemical reaction is occurring. The detonation wave is propagated in the material with ultrasonic velocity, and the reaction occurs at a high pressure and a density greater than the density of the initial mixture. In the second case the heat transfer is accomplished by the slow mechanism of thermal conduction, the flame is propagated with subsonic velocity, and the process occurs at almost constant pressure, i.e., in the combustion zone, where the temperature is high, the density is appreciably lower than the density of the cold material.

Like the chemical reaction rates, the degree of ionization rises very rapidly with increasing temperature, also following a law of the Boltzmann type, $\exp(-I/2kT)$, where I is the ionization potential of the atoms (molecules). The field energy is expended in the gas in the form of the Joule heating of currents or as the result of absorption of radiation only for sufficiently high ionization, so that in this case also it is appropriate to speak of the ignition temperature, more accurately the ionization temperature. The main (but not the only) mechanisms for discharge propagation—thermal conduction and shock wave—are the same as those which account for the propagation of combustion.

Of course, the analogy has limits. In chemical combustion there can be expended in a given mass only a limited amount of energy which is determined by the heat-producing ability of the material, and the temperature of the combustion products is more or less fixed by this value. The rate of propagation of detonation or slow combustion has a corresponding definite value. In discharges the energy expenditure and consequently also the plasma parameters (temperature) and rate of propagation depend on the intensity of the external field, which can be arbitrary. A static discharge which is maintained in a given mass of plasma in general has no analogy in combustion. A given mass of combustible material can react only once, and then the combustion either transfers to neighboring layers or ceases completely. However, a given mass of plasma with an appropriate heat outflow can take on the energy of the field in any quantity and for any length of time.

In order to make the discussion which follows systematic and for purposes of convenience in the physical discussion of the various phenomena, we will break them down into three groups, depending on the mode of propagation involved. The first group is the ultrasonic mode of propagation of a shock wave (Chap. II), the second is equilibrium thermal-conduction modes in which the gas can be considered as approximately in thermodynamic equilibrium and the discharge is propagated with subsonic velocity as in slow combustion (Chap. III), and the third group is the nonequilibrium modes in which a gas of heavy particles, as the result of the slowness of energy transfer from the electrons, remains cold and stationary and the discharge propagation has the nature of an ionization wave (Chap. IV). We will also include in the latter group certain other modes for which the velocity of sound also is not a characteristic quantity.

II. SHOCK-WAVE MODES

2. Optical detonation. Soon after discovery of the breakdown of a gas by the focused beam of a high-power pulsed laser in 1962, Ramsden and Davies[3] observed that the plasma front formed initially in the focal region where the light intensity is maximal moved rapidly during the laser pulse along the light path in the direction opposite to the beam (Fig. 1a). The motion of the front was indicated by the Doppler shift of the frequency of the laser light scattered from the front. The motion could be seen also in photographic scanning of the process. The greatest velocity (near the focus) reached 100 km/sec.

In order to explain this effect and to estimate the propagation velocity of the plasma front, Ramsden and Savič[4] proposed an optical detonation wave. The velocity of ordinary detonation D is determined by the heat-producing ability of the fuel q: $D \approx q^{1/2}$[1]. In this case we understand heat-producing ability to mean the amount of energy which is expended per unit mass of gas as the result of absorption of the light beam. By setting $q \approx S_0/\rho D$, where S_0 is the laser beam intensity and ρ is the gas density, Ramsden and Savič found $D \approx (S_0/\rho)^{1/3}$, in agreement with experiment. For $S_0 \sim 10^5$ MW/cm^2 = 10^{18} erg/cm^2-sec and $\rho \sim 10^{-3}$ g/cm^3 (air at atmospheric pressure), D turned out to be $\sim 10^7$ cm/sec. In the work of Mandel'shtam, Pashinin, Prokhorov, Raĭzer and Sukhodrev[5], motion of the front of a laser plasma was also detected and, most important of all, the plasma temperature was measured from the intensity of thermal x-rays. The experiments were performed in air with a ruby laser with an energy per pulse of 2.5 joules, a pulse length of 40 nsec, and a radius of the focused spot of 10^{-2} cm, i.e., the light intensity at the focus was $S_0 \approx 1 \times 10^{18}$ erg/cm^2-sec. Here the electron temperature turned out to be about 700 000°, and the velocity of the front 110 km/sec.[2]

A general analysis of ultrasonic modes of propagation of an optical discharge—of the wave of light absorption and gas heating, as it was then called—has been given in an article by the author[6]. In this article the shock adiabat was derived, possible propagation mechanisms were discussed, and the plasma temperature was calculated and found to be in good agreement with experiment[5].

Suppose that a plane ionization front is moving against the beam with a high ultrasonic velocity. The gas being heated is not able to expand during the absorption of the light, and the light flux is absorbed in a thin layer of plasma. For example, in air at atmospheric pressure for $T \sim 10^5-10^{6\circ}$ the absorption length for ruby laser light, which determines the width of the wave,

[1] In what follows the symbol ≈ denotes approximate equality, and the symbol ~ denotes equality in order of magnitude or proportionality; in the latter case the meaning will be clear from the context.

[2] Alcock, Pashinin, and Ramsden [33], in experiments with a more powerful laser, recorded by the same method an even higher temperature, above one million degrees.

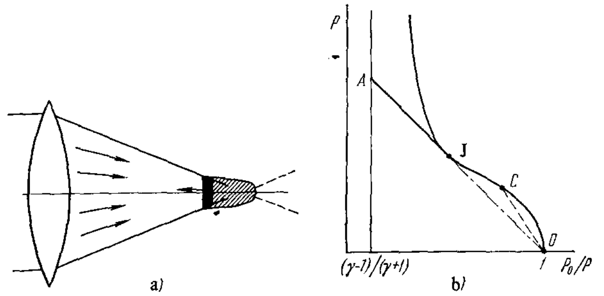

FIG. 1. a) Experimental arrangement for observation of optical detonation (the plasma is shaded, and the dark band is the wave front); b) shock adiabat of an ultrasonic light-absorption wave (the vertical straight line is the shock adiabat of a strong shock wave).

is $\sim 10^{-2}$ cm. In a certain sense the wave can be considered a hydrodynamic explosion. If we assume approximately that the light energy absorbed is expended only in heating of the gas to a final temperature T_p, then on writing down the energy balance we find

$$\rho_0 D \varepsilon_f = S_0, \qquad (1)$$

where ρ_0 is the density of the cold gas, and $\epsilon_f = \epsilon(T_f)$ is the specific internal energy acquired by it. This relation is valid for any mechanism of wave propagation. If ionization occurs as the result of heating of the gas by the shock wave, then the propagation velocity is obviously of the order $D \approx \sqrt{\epsilon_f}$ and we obtain from (1) the formula $D \approx (S_0/\rho_0)^{1/3}$ given above, and we also find that $\epsilon_f \approx (S_0/\rho_0)^{2/3}$.

In a more detailed discussion it is necessary to take into account the compression and the change in kinetic energy of the gas during the explosion, i.e., to depart from the general conditions of conservation of the fluxes of mass, momentum, and energy. As a result we obtain the equation for the shock adiabat of the light absorption wave, which relates the pressure p and the specific volume $1/\rho$ of the gas beyond the wave front with the initial density ρ_0 and the light flux S_0. The shock adiabat is shown in Fig. 1b. It differs from the shock adiabat of an explosive material in that it passes through the initial-state point O. The energy balance equation (1) now is satisfied with an accuracy to the coefficient in the right-hand side. However, this coefficient is close to unity, since the change in the kinetic energy of the gas turns out to be small in comparison with its heating.

As in the case of other hydrodynamic explosions, the rate of propagation of the explosion is characterized by the slope of the straight line drawn from the initial-state point O of the gas to the final-state point on the shock adiabat. It can be seen from Fig. 1b that for a given light intensity S_0 there exists a minimum possible propagation velocity of a wave in which compression of the gas occurs. It corresponds to the final-state point J. This is the so-called Jouguet point, which is well known from detonation theory. The wave velocity relative to the heated gas at this point coincides exactly with the local velocity of sound.

If other possible ionization mechanisms, say, heating of the gas by thermal plasma radiation or electronic thermal conduction (see Sec. 12) cannot provide a more rapid propagation of the light-absorption wave than does a shock wave, then just this detonation mode exists. The cold gas in this case is compressed and heated by a strong shock wave to state A, and then, absorbing luminous energy, expands along the straight line AJ and reaches the final state J at the moment of termination of the energy release. The optical detonation velocity D and the internal energy which the gas obtains in this case are

$$D = \left[\frac{2(\gamma^2-1)S_0}{\rho_0}\right]^{1/3}, \quad \varepsilon_\text{к} = \frac{2^{2/3}\gamma}{(\gamma^2-1)^{1/3}(\gamma+1)}\left(\frac{S_0}{\rho_0}\right)^{2/3}, \qquad (2)$$

where γ is the gas adiabat exponent.

If any propagation mechanism for a given intensity S_0 acts more rapidly than the shock wave, the wave velocity exceeds the "normal detonation" velocity. In this case the shock wave is not formed and the gas is continuously compressed to the final state C along the straight line OC. Here the wave is propagated in the heated gas with ultrasonic velocity. (These modes will be discussed in Section 12.) States lying on the shock adiabat to the left of the point J are unstable under ordinary conditions.

Calculations based on Eq. (2) give good agreement with experiment[5]. Thus, for $S_0 = 2 \times 10^{18}$ erg/cm^2-sec, $\rho_0 = 1.3 \times 10^{-3}$ g/cm^3, and an effective value $\gamma = 1.33$, we obtain D = 133 km/sec and $\epsilon_f = 1.35 \times 10^{14}$ erg/g, which in the equilibrium case corresponds to a temperature $T_f = 900\,000°$. Inclusion of the loss to lateral expansion of the gas permits still better agreement with experiment to be obtained. Satisfactory agreement with experiment is obtained not only for the absolute values of the calculated rate but also for the theoretical dependence $D \sim (S_0)^{1/3}$. It should be noted that the shock wave and the detonation mechanism of propagation are possible only under the condition that the ionic gas has a high temperature. Estimates show that under the conditions of most experiments with a laser spark the exchange of energy between electrons and ions occurs rather rapidly.

The plane detonation mode discussed above, without inclusion of losses, can exist in principle even for very low light fluxes S_0, down to those values for which the velocity D calculated from Eq. (2) is no longer comparable with the velocity of sound in the cold gas. In actuality the threshold for existence of the mode in intensity is much higher, for it is determined by the energy loss which is always present under real conditions. The most important loss is due to the limitation of the transverse dimensions of the light beam and to the lateral expansion of the heated gas, which leads to outflow of energy beyond the luminous channel in the wave zone.

The limit of optical detonation has been evaluated by the author[7]. Consider a cylindrical light beam of radius R along which an optical-detonation wave is propagated. The high-temperature gas beyond the wave expands and the surface of the shock-wave front has the form shown in Fig. 2. The gas begins to expand immediately beyond the leading front of the shock wave, even

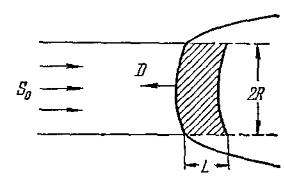

FIG. 2. The detonation mode with lateral expansion. The absorption wave is shaded.

inside the zone of energy dissipation, whose axial width L is of the order of the pathlength of the laser radiation at the plasma temperature $l_\nu(T_f)$. The role of energy loss to lateral expansion of the gas in the wave region is characterized by the ratio of the lateral and forward surfaces of the cylindrical volume of the region of energy dissipation (the shaded area in Fig. 2), i.e., by the quantity $2\pi RL/\pi R^2 \approx l_\nu/R$. If $l_\nu \ll R$, the loss is small and Eq. (2) is valid. If $l_\nu \gg R$, the loss is very great and only a small fraction of the light flux $\sim R/l_\nu$ is expended in advancement of the wave, this being absorbed in an axial distance of the order of the channel radius. However, at not too high temperatures of $\sim 20\ 000°$, in the vicinity of the first ionization of the atoms, the pathlength of the light increases extraordinarily rapidly as the temperature is reduced. Therefore, for maintenance of optical detonation at temperatures $T_f < T_t$ such that $l_\nu(T_t) \gg R$, the efficiency of utilization of the light energy is very low, and extremely high light power is required. Thus, the function $S_0(T_f)$ has a minimum S_t which lies immediately at $l_\nu(T_t) \approx R$. This condition, like the physical cause of the effect, completely corresponds to the well known phenomenon of the limits of detonation of cylindrical charges of small diameter.

For example, for air at atmospheric pressure a neodymium-glass laser with a beam radius R = 0.1 cm according to the calculation of ref. 7 produces a temperature $T_t \approx 19\ 000°$, a threshold light intensity $S_t \approx 80$ MW/cm^2 (a power of 2.6 MW), and a minimum optical-detonation velocity of 8.5 km/sec. As we can see, the threshold value S_t turns out to be three orders of magnitude lower than the intensities $\sim 10^5$ MW/cm^2 which ordinarily enter into experiments with a laser spark. These tremendous intensities are actually necessary not to maintain optical detonation but only to initiate it by breakdown of the gas by the laser pulse itself[3)].

We concluded from this[7] that forced ignition of a traveling laser spark is possible by means of an auxiliary plasma source for light intensities much lower than the threshold for breakdown.

III. EQUILIBRIUM THERMAL-CONDUCTION MODES

3. "Slow combustion" of a light beam. Forced ignition was achieved in the experiments of Bunkin, Konov, Prokhorov, and Fedorov[8]. A neodymium-glass laser beam with a millisecond pulse was focused in air with a lens of long focal length (f = 50 cm). The diameter of the focal spot was approximately 3 mm. The energy in the pulse was ≈ 1000 J, and the intensity of light at the focus was of the order of 10 MW/cm^2, which is far from sufficient for breakdown. The initial plasma in the focal region was produced by means of a spark discharge between two electrodes. After initiation the laser spark propagated along the slightly divergent light channel in both directions from the focus. However, in contrast to experiments with high-power laser pulses, the plasma front moved slowly, with an average velocity of ≈ 40 m/sec. The movement gradually slowed down and stopped even before the complete termination of the pulse (Fig. 3).

The experimentally observed slow propagation of the plasma front was interpreted by Bunkin et al.[8] as a slow combustion process[4)]. The light intensity in the experiments was actually insufficient for excitation of optical detonation, which requires as a minimum ~ 100 MW/cm^2. The velocity of the front was evaluated by means of the well known Zel'dovich formula for flame velocity. The rate of energy dissipation was expressed in terms of the absorption of light. The plasma temperature, which is necessary for the calculation, was estimated on the basis of experimental determination of the transparency of the plasma for the laser beam. The plasma actually absorbed only a small fraction of the light, which is responsible for the symmetry of the motion of the plasma front in both directions from the focus. In order to reconcile the theoretical front velocity with the experimental value, Bunkin et al.[8] invoked the idea of combustion in a tube from a closed end. The point is that the heated gas expands in all directions, including the direction of motion of the front, leading to motion of the cold gas in front of the front. Therefore the velocity of the front in the laboratory system turns out to be greater than its velocity of propagation in matter by roughly the ratio of the densities of the cold and heated gases (the pressures in them are the same). Bunkin et al.[8] also measured the threshold for occurrence of this mode. For pulse energies less than 730 J (intensity less than ≈ 10 MW/cm^2), combustion did not occur.

Thermal-conduction propagation of an optical discharge has been considered in detail by the author[11,12], and values have been calculated for the plasma temperature, wave-propagation velocity, and mode threshold. In contrast to the detonation mode (without loss) in which, on the basis of only the idea of a hydrodynamic explosion, the main parameters of the wave can be determined and we are not interested in its internal structure, in the case of slow combustion we cannot avoid investigation of the structure. The problems of the various equilibrium thermal-conduction modes have much in common. Therefore, in the first discussion of one of these modes, we will dwell in detail on the mathematical formulation, in order to avoid repetition as far as possible.

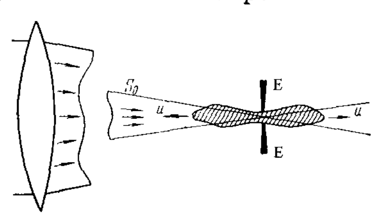

FIG. 3. Experimental arrangement and plasma configuration (shaded) from ref. 8. E denotes the ignition electrodes.

[3)]It is interesting that at other frequencies used in practice, for example, in the microwave region, the relation is most frequently the reverse. Thus, to maintain detonation at 1 atm, a radiation intensity greater than the breakdown threshold would be required, so that the detonation mode cannot be achieved in this case: breakdown sets in first.

[4)]The idea of similarity of the thermal-conduction propagation of a discharge to slow combustion was expressed for the first time by Velikhov and Dykhne [9], who discussed an ionization wave in a constant electric field, propagating as a result of electronic thermal conduction (see Sec. 10). The analogy was subsequently drawn in detail by Raĭzer [10] in study of a high-frequency discharge in a gas flow (see Sec. 5).

Let us consider an undamped process in which a thermal wave, propagated along a light channel of radius R in a direction opposite to a beam of constant power P, is maintained as the result of light absorption. We will assume the motion to be slow—subsonic, and the gas to be in equilibrium. Here the pressure p is equalized, so that the density ρ and temperature T are related (approximately $\rho T \approx$ const). We will consider the process as one-dimensional, neglecting the radial expansion of the gas (Fig. 4a), but taking into account in effect the energy loss due to thermal-conduction flow of heat in the radial direction beyond the limits of the light channel. The wave formed moves as a whole; $T(x, t) = T(x + ut)$, where u is the absolute propagation velocity of the wave in the cold gas, which is equal to the velocity of flow of the cold gas into the wave. The process is stationary in the coordinate system in which the wave is at rest, and in this system, to which we will convert, $dT/dt = v dT/dx$, where v is the velocity of the gas. Since the mass flow is conserved, $\rho v = \rho_0 u$ (ρ_0 is the density of the cold gas).

The temperature distribution T(x) in the wave is described by the energy-balance equation

$$\rho_0 u c_p \frac{dT}{dx} = -\frac{dJ}{dx} + F, \quad J = -\lambda \frac{dT}{dx} = -\frac{d\Theta}{dx}, \tag{3}$$

$$F = S\mu(T) - (A\Theta/R^2) - \Phi, \quad \Theta = \int_0^T \lambda\, dT; \tag{4}$$

here c_p is the specific heat, J is the heat flux, λ is the thermal conductivity, Θ is the heat-flow potential, F is a function of the heat source, S is the light intensity, μ is the light-absorption coefficient, and Φ is the loss by radiation, which, neglecting radiative heat-exchange effects, will be assumed to depend only on the temperature. The quantity $A\Theta/R^2$ describes the thermal-conduction loss. Actually, the heat flow through the side surface of the light channel is $\sim \Theta/R$, so that the loss per unit volume is $\sim (\Theta/R)(2\pi R/\pi R^2) \sim \Theta/R^2$. The coefficient A depends only on the radial profile of the temperature (for numerical calculations we can set[12] $A \approx 3$). The light intensity is given by the equation

$$\frac{dS}{dx} = -\mu S. \tag{5}$$

The order of the system of equations (3)—(5) can be reduced if we exclude x from them; we obtain

$$\frac{dJ}{dT} = -\frac{\lambda F(T, S)}{J} - \rho_0 u c_p, \quad \frac{dS}{dT} = \frac{\mu S}{J}. \tag{6}$$

Let us formulate the boundary conditions. In front of the wave (for $x = -\infty$) $T = 0$, $J = 0$, and the light intensity S_0 is given by the beam power $S_0 = P/\pi R^2$. As the wave moves into the medium the gas at first is heated and then, as the result of attenuation of the light flux and the existence of losses, is cooled (Fig. 4b). Behind the wave for $x = +\infty$ $T = 0$, $J = 0$. We define the plasma temperature as the maximum temperature to which the gas is heated. It is easy to see that one of the boundary conditions for the systems (3)—(5) and (6) is redundant. Consequently, the system can have a solution only for a chosen value of the parameter u. This also permits the unknown propagation velocity of the wave to be determined in the course of integration of the equations. The situation is completely analogous to that which occurs in combustion theory.

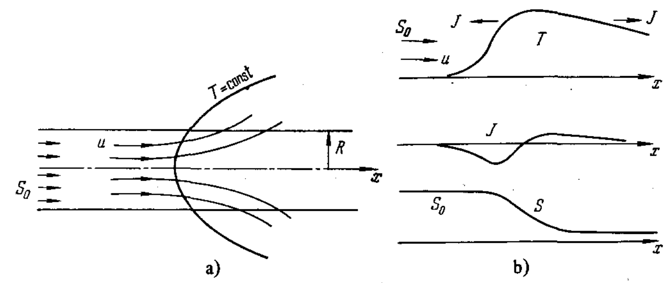

FIG. 4. a) Schematic diagram of heat flow and gas expansion (current lines and isotherms are shown); b) schematic profiles of temperature, heat flow, and electromagnetic-energy flow in the mode with loss.

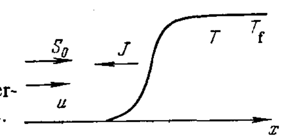

FIG. 5. Schematic profile of temperature for exclusion of the cooling zone.

In the two limiting cases the gas, after reaching T_{max}, cools much more slowly than it it is heated, and the zone of cooling can be excluded from consideration if we assume that behind the wave for $x = +\infty$ the temperature approaches some final value T_f (in actuality it coincides with the maximum temperature (Fig. 5)). These limits are as follows. If the plasma absorbs the radiation very strongly, the role of loss in the zone of intense heat dissipation is small and the heating is cut off when the light flux is exhausted. This case is realized when the penetration length of the radiation into the plasma $l(T_f) = 1/\mu \ll R$, i.e., when the width of the wave is much smaller than the channel radius. In this case we can omit the loss terms in F and impose behind the wave the condition: for $x = +\infty$ $J = 0$, $S = 0$. The temperature T_f to which the plasma is heated must be determined in the course of the solution. If the light is weakly absorbed, so that $l(T_f) \gg R$ (the plasma is transparent), we can approximately assume $S =$ const $= S_0$. The gas is heated in the stationary wave up to the time when the heat dissipation is not compensated by loss. After this the heating must be cut off, or else the process will be unstable. Thus, the final plasma temperature T_f is determined by the equation $F(S_0, T_f) = 0$; according to the stability condition, the final state corresponds to that root of the equation for which $(dF/dT)_{T = T_f} < 0$. Here behind the wave (when $x = +\infty$) $J = 0$ and $T = T_f$.

Figure 6a shows the absorption coefficients for light of a neodymium laser ($\lambda_0 = 1.06\ \mu$) and a CO_2 laser ($\lambda_0 = 10.6\ \mu$) in air at atmospheric pressure. The smallest absorption lengths are 170 cm and 1.2 cm, respectively, whereas the radii of the light beams are ordinarily of the order of a millimeter. Consequently the limit of a transparent plasma is realized in this case. In this section we will discuss this limit.[5)]

[5)] This case has much in common with the situation arising in a constant electric field (sec. 10). The limit of low loss is characteristic of a

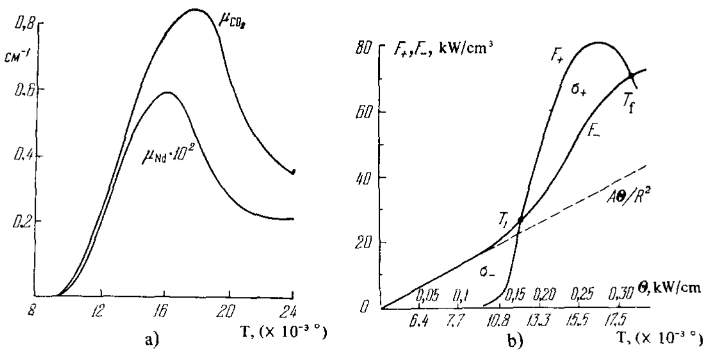

FIG. 6. a) Absorption coefficients for light of CO_2 and neodymium-glass lasers in air at atmospheric pressure; b) curves of heat dissipation and loss. Air, 1 atm; CO_2 laser light; $S_0 = 10$ kW/cm²; $A = 2.9$, $R = 0.15$ cm. At the bottom is given a temperature scale, i.e., the relation between T and θ.

We multiply the first of the equations (6) by J and integrate over the entire wave temperature interval from 0 to T_f. We obtain

$$\rho_0 u = \int_0^{\Theta_f} F(\Theta)\,d\Theta \Big/ \int_0^{T_f} |J|\,c_p\,dT. \qquad (7)$$

We will represent the source function F in the form of the difference between the heat dissipation $F_+ = S\mu$ and the loss $F_- = (A\Theta/R^2) + \Phi$. Figure 6b, in which the construction has been carried out for one specific case, permits us to judge the nature of the behavior of $F(\Theta)$ and of the integral occurring in the numerator of Eq. (7). The final temperature T_f corresponds to the upper point of intersection of the curves F_+ and F_- and at the lower point T_1 the state is unstable. The greater the light intensity S_0, the greater the upper area σ_+ between the curves F_+ and F_- in comparison with the lower area σ_-, and the greater the velocity of the wave. At some value S_t the areas are equal:

$$\int_0^{\Theta_f} F(S_t, \Theta)\,d\Theta = \sigma_+ - \sigma_- = 0, \qquad (8)$$

and the propagation velocity goes to zero. For $S < S_t$ $\sigma_+ < \sigma_-$ and $u < 0$, i.e., there is no discharge-wave mode (a cooling wave appears). Thus, the threshold for discharge propagation is determined by Eq. (8). It can be shown[12] that for $S > S_t$ we have approximately

$$\rho_0 u = \frac{\sqrt{2}}{w_f} \int_0^{\Theta_f} F\,d\Theta \Big/ \left(\int_{\Theta_1}^{\Theta_f} F\,d\Theta \right)^{1/2} = \frac{\sqrt{2}}{w_f} \frac{\sigma_+ - \sigma_-}{\sqrt{\sigma_+}}, \qquad (9)$$

where $w = \int_0^{\bar{T}} c_0 dT$ is the specific enthalpy. In the limit $S \gg S_t$, where $\sigma_+ \gg \sigma_-$, Eq. (9) goes over to the Zel'dovich formula for flame velocity[1]. For a small excess of the intensity over the threshold, the dependence of the velocity on the light power is different[6)].

Calculation of the threshold power for the conditions of the experiments of Bunkin et al.[8] according to Eq. (8) gave excellent agreement with the measured value. The calculated temperature of the air plasma was about 17 000°. In the case of a transparent plasma, as for optical frequencies, the final temperature necessarily corresponds to the falling part of the $\mu(T)$ curve, i.e., to almost complete single ionization of the gas. For infrared radiation from a CO_2 laser with radius $R = 0.15$ cm the theoretical threshold values are $S_t \approx 100$ kW/cm², $P_t = 7$ kW. For a diameter not exceeding 1 mm the loss by radiation Φ turns out to be small in comparison with the thermal-conduction loss $A\Theta/R^2$ and the threshold intensity is $S_t \sim 1/R^2$, i.e., the threshold power $P = \pi R^2 S_t$ has its smallest value and does not depend on the radius. For a CO_2 laser this value is theoretically 4 kW ($T_f \approx 18\ 000°$). For an excess of power above the threshold by 1.5–2 times the calculated velocities u are of the order of several meters per second. The laboratory propagation velocity can be an order of magnitude higher as a consequence of expansion of the heated gas in the direction of motion of the wave.

In the case of very thin channels where the loss by radiation is unimportant, a simple approximate formula for the threshold power can be obtained by replacement of the rapidly rising function $\mu(T)$ by a step function: $\mu = 0$ for $T < T_0$ ($\Theta < \Theta_0$), $\mu = \text{const}$ for $T > T_0$. The quantity T_0 evidently has the meaning of an ignition temperature, or more accurately the ionization temperature. It can be seen from Fig. 6a that for air and optical frequencies this is about 12 000°. In the step approximation and for the assumption $c_p(T)/\lambda(T) = \text{const}$, Eq. (3) becomes linear in $\Theta(x)$, and the problem can be solved completely in analytical form. The threshold power, as follows from Eq. (8), is equal in this case to

$$P_t = \pi A \Theta_0/\mu, \qquad (10)$$

where the final value for the plasma is $\Theta_f = 2\Theta_0$. Evaluations with Eq. (10) give quite satisfactory results.

Mul'chenko, Raĭzer, and Épshteĭn[13] have studied the forced ignition of a laser spark in argon at pressures of 16–80 atm. Combustion of the focused beam of a ruby laser operating in an ignition-free millisecond mode was initiated by breakdown of the gas at the focus by a pulse from another laser. The propagation of the plasma front was recorded by a high-speed device and the plasma temperature was determined photometrically. The velocity of the motion was of the order of 100 m/sec, and the temperature increased with increasing pressure from 18 000 to 33 000°. The threshold values for occurrence of combustion were 50 MW/cm² and 70 kW at 16 atm. With increasing pressure they decreased, at first rapidly and then very slowly, to 15 MW/cm² and 20 kW at 60 atm. The reduction of the threshold with increasing pressure is due to the increase in the absorbing ability of the gas for constancy of loss, since the loss by thermal conduction does not depend greatly on pressure. The slowing of the rate of reduction of the threshold is due to the fact that at higher pressures the most important losses are those by radiation, which increase with increasing pressure in roughly the same way as the absorbing ability of the gas. Therefore the threshold light intensity no longer depends on pressure. In all likelihood an important role in propagation of the wave at high pressures is played by radiant heat exchange, which with increasing density and opacity of the plasma takes on the nature of radiant thermal conduction.

high-frequency discharge (Sec. 6), and also for the "hyperdetonation" thermal-conduction mode at optical frequencies (Sec. 12); the general case must be considered at microwave frequencies (Sec. 7).

6) In the case of ordinary combustion the flame velocity near the limit is different from zero.

4. A stabilized optical discharge maintained by a focused beam. The continuous maintenance or production of a plasma by means of radiation in the optical region has one feature which is extremely attractive. No structural elements are needed to supply the energy to the discharge. There is no need for electrodes, coils, or waveguides as in the use of constant, high-frequency, or microwave fields—the energy is transported simply by the light beam. In principle an optical discharge can be ignited at any point, and can be moved in space as desired (but not too rapidly) by moving the beam; the discharge can be forced to run along the beam, and it can be localized, say, by focusing of the beam. The possibility of producing such an optical plasmatron have been discussed by the author[11] in an article devoted to estimation of the light power necessary for this purpose. If we have in mind the extended maintenance of a plasma, we must obviously think in terms of CO_2 lasers, since these are the most powerful continuous lasers at the present time.

The static discharge maintained by a parallel light beam of the threshold power, which was discussed at the end of the preceding section, is unstable. In fact, if the power is somewhat increased, the discharge begins to propagate, and if it is reduced, the discharge goes out. Stabilization of the discharge is easy by focusing of the beam. Here the discharge cannot go far from the focus, since in this case the light intensity becomes steadily weaker. Focusing of the light at a large angle generally facilitates the maintenance of a plasma, since the energy is concentrated; in addition the "useless" flow of heat from the region of energy dissipation beyond the limits of the light channel becomes relatively smaller.

A spherically symmetric model[12] can be used for a simplified description of the discharge at the focus, particularly if the plasma is transparent and heat is dissipated in the two cones of light touching at their vertices. Let us consider a stationary process which is maintained by a beam converging with spherical symmetry and of initial power P. All of the heat dissipated is removed from the discharge by thermal conduction, and the loss by radiation will be neglected. The distribution of the temperature, or more accurately of the heat-flow potential, is described by the equation

$$\frac{1}{r^2}\frac{d}{dr}r^2\frac{d\Theta}{dr}+\frac{P\mu(\Theta)}{4\pi\rho^2}\exp\left(-\int_r^\infty\mu\,dr\right)=0,\quad \rho=\begin{cases} r & \text{for } r>R,\\ R & \text{for } r<R;\end{cases} \tag{11}$$

here R is the equivalent focusing radius, which it is natural to define by equating the spherical focal volume $4\pi R^3/3$ to the true focal volume in focusing of the beam by a real optical system. The integrated curve $\Theta(r)$ must satisfy the boundary conditions that $\Theta(0)$ is finite and $\Theta(\infty) = 0$.

A complete representation of the laws governing the process, as well as numerical evaluations, can be obtained by specifying the function $\mu(\Theta)$ in the form of a step (see the end of Section 4), as a result of which Eq. (11) becomes linear. In the solution obtained, the discharge radius r_0, i.e., the radius of the sphere inside which the temperature exceeds the ionization temperature and where the light is absorbed: $r_0 = r(\Theta_0)$, depends on the beam power P. The function $r_0(P)$ turns out to be double valued, and the inverse function $P(r_0)$ has a minimum. On the assumption $\mu R \ll 1$, which is very well satisfied in the case of atmospheric pressure ($R \approx 10^{-2}$ cm, and $\mu \approx 1\ \text{cm}^{-1}$ for a CO_2 laser), $P(r_0)$ is described by the simple relations

$$P=(4\pi\Theta_0/\mu)\times\begin{cases} 3(R/r_0), & \text{if } r_0<R,\\ \mu r_0\{1-e^{-\mu r_0}[1+(2\mu R/3)]\}^{-1}, & \text{if } r_0>R.\end{cases} \tag{12}$$

The curve $P(r_0)$ passes through a minimum at $r_{0t} \approx \sqrt{4R/3\mu} > R$, and this minimum threshold power, below which a stationary solution does not exist, is given by[7)]

$$P_t \approx 4\pi\Theta_0/\mu. \tag{13}$$

For the two branches of $r_0(P)$, only the increasing branch in which the discharge radius increases with increasing power ($r_0 > r_{0t}$) corresponds to stable states. States on the falling branch ($r_0 < r_{0t}$) are unstable. If the radius increases slightly, a power less than the actual power P will correspond to a stable state and the discharge will begin to propagate although it has not yet reached the radius $r_0(P)$ on the rising branch. For air at 1 atm and a CO_2 laser, $T_0 \approx 12\,000°$, $\Theta_0 \approx 0.17$ kW/cm, $\mu \approx 0.8\ \text{cm}^{-1}$,[12] and the threshold power is found from Eq. (13) to be $P_t \approx 2.7$ kW. It is clear from Eq. (7) and also from physical considerations that a smaller power is required to maintain the plasma at increased pressures where the light is more strongly absorbed and a larger fraction of the beam energy is utilized. In addition, reduction of the power is made possible by use of gases with low heat conduction (such as the heavy inert gases). Thus, for example, numerical solution of Eq. (11) for argon at a pressure of 15 atm and for the light from a CO_2 laser gives $P_t \approx 43$ W (the calculation assumed R = 0.01 cm). Furthermore, $r_{0t} \approx 0.1$ cm; we note that $\mu_{max} \approx 80\ \text{cm}^{-1}$ and the corresponding temperature and potential are $T \approx 20\,000°$ and $\Theta \approx 0.18$ kW/cm.

A continuously burning optical discharge was first achieved experimentally by Generaloz, Zimakov, Kozlov, Masyukov, and Raĭzer[14]. A CO_2 laser beam with a power of 150 W was focused in a chamber filled with xenon at a pressure of several atmospheres. The beam was focused in the middle of the free volume far from all surfaces, in a circle of radius 0.005 cm. The discharge was ignited by another CO_2 laser which provided periodic pulses with a repetition frequency of 50—250 hertz, a power of 10 kW, and a duration of 0.3—1.5 μsec. On focusing of these pulses in the gas a breakdown occurred, as a result of which the initial plasma appeared. The focuses of the two lasers were carefully superposed. After the initiation the igniting laser was turned off, and the discharge, fed by the beam of the first laser, continued to burn under definite conditions—very stably and as long as desired.

The properties of a continuous optical discharge have been investigated in a study by the same authors[15]. Figure 7a shows a series of photographs of a discharge for different light powers and gas pressures. The plasma has dimensions of the order of millimeters. The discharge begins always at the focus and is then shifted somewhat along the beam opposite to the direction of the light and comes to rest. The measured veloc-

[7)] This formula differs from Eq. (10) for a cylindrical beam only by a numerical factor.

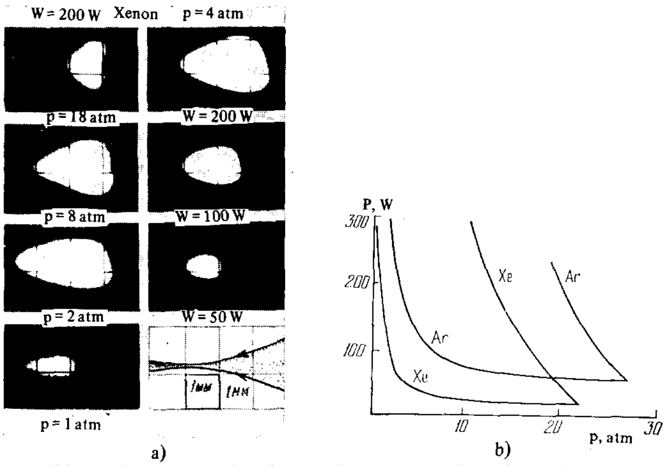

FIG. 7. a) Photographs of a continuous optical discharge in xenon at various pressures and light powers (at bottom right is a diagram of the behavior of the light rays; the rays are going from right to left, and one division is 1 mm); b) threshold powers for existence of a stable optical discharge in xenon and argon (the region of existence lies between the upper and lower curves, for a horizontal beam).

ities of propagation were of the order of meters per second. For a horizontal beam the discharge lost its stability at too high pressures and went out. This was due to the floating of the discharge under the action of Archimedes' principle and perhaps also to the action of convection currents in the gas. Supply of the feeding radiation vertically from below to above stabilizes the discharge; in floating the plasma enters a region of more intense light flux and again expands, displacing itself downward against the beam. In Fig. 7b we have shown the measured threshold powers of light for xenon and argon for various pressures for a horizontal beam configuration. For a vertical configuration there is no upper limit to the pressure. As in the experiments[13] with a pulsed discharge, on increasing the pressure the threshold decreases at first rapidly and then very slowly. The causes of this behavior have been discussed above.

Generalov et al.[15] also determined the plasma temperature. The electron density was determined from the Stark broadening of the H_β line of hydrogen. In argon at pressures of 4–16 atm the half-width of the line was 100–130 Å, corresponding to an electron density $N_e \approx 5 \times 10^{17}$ cm^{-3}. For 2 atm $N_e \approx 3.5 \times 10^{17}$ cm^{-3}, which corresponds under equilibrium conditions to a temperature of 23 000°. It is easy to see that the measured electron density corresponds to almost complete single ionization of the gas, and the range of possible variations of the temperature, with allowance for the temperature difference of electrons and ions, is extremely limited. Direct evaluation also shows that the temperature difference is small. Thus, the temperature of an argon plasma at 2 atm is about 23 000°. The plasma emits a blinding white light. It should be noted that continuous light sources of such high brightness did not previously exist.

5. Plasma temperature in a high-frequency discharge. An induction discharge is easily obtained by placing an evacuated vessel inside a solenoid carrying a sufficiently strong high-frequency current (of the order of a megahertz). Under the action of the rotational electric field which is induced by the variable magnetic flux, breakdown occurs in the gas and a discharge is ignited. A high pressure gas (atmospheric pressure) cannot be broken down in this manner, but if a discharge is ignited by some means, it will continue to burn, maintained by dissipation of Joule heat of the rotational currents. The foundations of the contemporary technology of induction discharges at high pressure was laid by the work of Babat[16] about 1940. These discharges have found serious practical application. They are used to produce a dense low-temperature plasma which, in contrast to an arc-discharge plasma which is contaminated by the electrode disintegration products, is absolutely pure.

One of the main questions which arises here is—what is the plasma temperature in the discharge? An answer can be obtained by considering the one-dimensional static combustion mode of a discharge in a stationary gas, in which the dissipated heat is removed by thermal conduction to the cooled walls of the tube; in high-pressure discharges the plasma can be considered approximately in thermodynamic equilibrium. This problem was formulated in the work of Soshnikov and Trekhov[17]. A cylindrical discharge of infinite length is described by the energy-balance equations and Maxwell's equations; in the latter we can neglect displacement currents:

$$-\operatorname{div}\mathbf{J}+\sigma\langle E^2\rangle-\Phi=0,\quad J=J_r=-\lambda\frac{dT}{dr},\tag{14}$$

$$\operatorname{rot}\mathbf{H}=4\pi\sigma\mathbf{E}/c,\quad \operatorname{rot}\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t},\quad H\equiv H_z,\ E\equiv E_\varphi;\tag{15}$$

here $\sigma = \sigma(T)$ is the conductivity, $\Phi = \Phi(T)$ is the loss by radiation, $E, H \sim e^{i\omega t}$, where ω is the frequency of the field, and the symbol $\langle E^2\rangle$ denotes averaging over time. On the axis (for $r = 0$), $J = 0$ and $E = 0$. At the internal surface of the cooled tube (for $r = R$) we can, for example, set $T = 0$. The magnetic-field strength is determined by the ampere-turns in the solenoid: $H_0 = 4\pi I_0 n/c$, where I_0 is the amplitude of the current and n is the number of turns per unit length. Equations (14) and (15) have been integrated for many situations and in this way all of the discharge parameters have been determined: temperature, input power, inductance, and so forth. The temperature distribution along the radius has the nature of a plateau (here $T \approx$ 7000–12 000°) with a small dip in the middle ($\approx$ 500–1000°) and drops sharply at the edges. The dip is due to the heat loss by radiation: heat is dissipated only in the peripheral layer as a result of the skin effect (numerous measurements also indicate the existence of a small temperature dip on the axis).

Gruzdev, Rovinskiĭ and Sobolev[18] discussed the same problem, but without including the loss by radiation. They obtained an integral of Eqs. (14) and (15) which permits determination of the plasma temperature T_f (the temperature on the axis). (It should be noted that for temperatures below about 10 000° the loss by radiation is actually small.) In Eq. (14) we will express the Joule heating in terms of the divergence of the electromagnetic energy flux S. Setting $\Phi \equiv 0$, we obtain

$$\operatorname{div}(\mathbf{J}+\mathbf{S})=0,\quad \mathbf{S}=\frac{c}{4\pi}\langle[\mathbf{EH}]\rangle.\tag{16*}$$

From this result obviously follows the integral repre-

*$[\mathbf{EH}] \equiv \mathbf{E}\times\mathbf{H}$.

senting conservation of total energy flow:

$$J + S = 0, \quad S \equiv S_r < 0, \; J \equiv J_r > 0. \tag{17}$$

However, Eqs. (14) and (15) with $\Phi \equiv 0$ also have a second integral. Actually, Maxwell's equations (15) in which the displacement current has been omitted permit the flux S to be represented in differential form,

$$S = -\frac{c^2}{32\pi^2\sigma}\langle dH^2/dr\rangle = -\frac{c^2}{64\pi^2\sigma}\frac{d}{dr}H_a^2, \tag{18}$$

where H_a is the real amplitude of the field. If we substitute (18) and the differential expression (14) for J in (17) and multiply the equation obtained by σ, it is easily integrable. In the case of practical importance in which the skin depth at the surface of the plasma column is small compared to its radius and the field is rapidly attenuated in the plasma, we obtain the simple relation

$$\int_0^{T_f} \sigma(T)\,\lambda(T)\,dT = \frac{c^2H_0^2}{64\pi^2} = \left(\frac{I_0 n}{2}\right)^2, \tag{19}$$

which determines the plasma temperature in terms of the number of ampere-turns.

Gruzdev et al.[18] have developed a method of successive approximations for finding the temperature distribution T(r), the power, and other quantities. We will give as an example the calculated values for argon at atmospheric pressure[18]. For $I_0n = 13.3$ A-V/cm, $T_f = 8000°$; at a frequency 12 MHz and for a tube radius R = 3.75 cm the input power per unit length of the discharge is W = 0.21 kW/cm; the radius of the surface with T = 4500° is $r_0 = 0.91$ R; the skin depth corresponding to a conductivity $\sigma_f = \sigma(T_f)$ is $\delta_f = c/\sqrt{2\pi\sigma_f\omega} \approx 0.45$ cm. For $I_0n = 33$ A-V/cm, $T_f = 10\,000°$, W = 1.1 kW/cm, $r_0 = 0.98$ R, and $\delta_f = 0.3$ cm.

The temperatures which are obtained in induction discharges usually correspond to rather small degrees of ionization of the gas, in which the conductivity is proportional to the electron density and $\sigma \sim N_e \sim \exp(-I/2kT)$, where $kT \ll I$. In order to achieve a noticeable increase in plasma temperature under the conditions of a rapidly rising function $\sigma(T)$, it is necessary according to Eq. (19) to increase substantially the current (and power) in the inductor, especially since at high temperatures loss by radiation appears. In induction discharges at atmospheric pressure, temperatures above about 10 000° are not obtained in practice.

The plasma temperature can be directly related also to the electromagnetic energy flow into the discharge (into the skin layer) S_0 ($W \approx 2\pi r_0 S_0$, where r_0 is the discharge radius). If we compare the plasma approximately to a wire with constant conductivity σ_f, then we can use for S_0 the well known formula[35]

$$S_0 = \frac{cH_0^2}{16\pi}\left(\frac{\omega}{2\pi\sigma_f}\right)^{1/2} = \frac{c^2H_0^2}{32\pi^2}\frac{1}{\sigma_f\delta_f}, \tag{20}$$

where $\delta_f = c/\sqrt{2\pi\sigma_f\omega}$ is the skin depth. From Eqs. (19) and (20) we obtain

$$\int_0^{T_f} \sigma\lambda\, dT \approx \frac{1}{2}\sigma_f\delta_f S_0. \tag{21}$$

The physical content of this formula becomes especially apparent if the temperature is low and $\sigma \sim \exp(-I/2kT)$ with $kT_f \ll I$. In this case the integral (21) can be approximately calculated by expanding the $1/T$ in the exponent about the value $1/T_f$ by the Frank-Kamenetskiĭ method.

The integral is approximately equal to $\sigma_f\lambda_f 2kT_f^2/I$, and according to (21)

$$S_0 = 4\lambda_f kT_f^2/I\delta_f. \tag{22}$$

However, except for the numerical coefficient this relation can easily be obtained from very simple qualitative reasoning. In fact, the Joule heat is dissipated mainly in the layer where the conductivity is sufficiently high, say, no more than e times smaller than the final value σ_f. The temperature in this layer varies from $T_f - \Delta T$, where $\Delta T \approx 2kT_f^2/I$, to T_f, and the thickness of the layer is of the order δ_f. Consequently the heat flow which carries to the walls the energy S_0 fed from the solenoid is of the order $J_0 \sim \lambda_f\Delta T/\delta_f$. Equating J_0 and S_0, we obtain Eq. (22).

As has been shown by Meĭerovich and Pitaevskiĭ[19], in the case of a thin skin layer and for the condition $kT_f \ll I$ the distributions of temperature, heat flow, heat dissipation, and so forth at the discharge boundary have a universal nature, i.e., the dimensionless quantities such as T/T_f depend only on r/δ_f. These authors[19] derived and numerically integrated the equation for the dimensionless temperature, constructed all the profiles, and found the exact relation between the plasma temperature and the electromagnetic energy flow into the skin layer. This relation is of course given by a formula similar to (22), but the correct value of the coefficient is 3.14, rather than 4.

6. High-frequency discharge in a gas flow. A high-frequency discharge-propagation wave arises in an electrodeless plasmatron. An induction plasma torch, as it is sometimes called, was constructed by Reed[20] in 1960. In this device a gas is blown through a solenoid in which a discharge is burning, and the gas flows out in the form of continuous plasma stream with a temperature of the order 10 000°. In a typical contemporary installation built by Kononov and Yakushin[21] (see Fig. 8) a coil consisting of several turns is supplied from a high-frequency vacuum-tube oscillator with a frequency range 6–18 mHz. A power up to 40 kW can be fed into the discharge. A quartz tube 6 cm in diameter and 35 35 cm long is placed in the coil. Air or argon is blown through the tube; the gas feed is tangential, with a helical flow. For this reason the discharge is pushed away from the walls of the tube. The axial components of the velocity of the cold gas at the periphery are of the order of a meter per second; on the tube axis, as the result of centrifugal forces, the pressure is reduced and a vortex is formed. There is practically no axial motion. The author[10] has proposed a model intended to explain how

FIG. 8. Photograph of a discharge and plasma jet in a plasmatron. Two turns of the coil and the end of the quartz tube are visible.

the cold gas is converted into a plasma in an induction plasma torch. The principal element of this model is the solution of the problem of the normal discharge propagation, just as the explanation of the configuration of the flame front in chemical combustion is based on the solution of the fundamental problem of the normal velocity of propagation of combustion.

As a beginning let us imagine the nonstationary process of expansion of the plasma column in a solenoid without gas flow at the stage in which the discharge being ignited on the axis has not reached the walls. Then each portion of the surface of the discharge wave can be considered plane. If the solenoid is long, the magnetic field is directed along the axis and the discharge layer will be a long circular cylinder. Here the longitudinal temperature gradients are very small. In addition, let the temperature be not so high that the loss by radiation plays an appreciable role. We arrive at the problem of propagation of a plane wave of a high-frequency discharge whose front is oriented parallel to the external magnetic field, in the absence of any energy loss.

We will consider a stationary mode in the rest system of the wave. The energy-balance equation is similar to Eq. (3):

$$\rho_0 u c_p \frac{dT}{dx} = -\frac{dJ}{dx} + \sigma\langle E^2\rangle, \quad J = -\lambda\frac{dT}{dx}. \tag{23}$$

The field is described by Maxwell's equations (15), except now $H \equiv H_z$, $E \equiv E_y$, $S \equiv S_x$. In front of the discharge at $x = -\infty$ $T = 0$ and $H = H_0 = (4\pi/c)I_0 n$. In the plasma (for $x = +\infty$) $J = 0$, $H = 0$. The temperature in the wave behaves as shown in Fig. 5. The plasma temperature T_f (for $x = +\infty$), like the discharge-propagation velocity u, must be determined by solution of the equations. The system of equations (23) and (15) has an obvious integral in which the total energy flow is conserved,

$$\rho_0 u w\ (T) + J + S = \text{const} = S_0, \tag{24}$$

from which follows the trivial energy-balance equation for the discharge wave as a whole

$$\rho_0 u w_f = S_0, \quad w_f = \int_0^{T_f} c_p\, dT. \tag{25}$$

An approximate solution of the system (23) and (15) or (24) and (15) can be found if we utilize the sharpness of the function $\sigma(T)$. We introduce an ionization temperature T_0 such that for $T < T_0$ we can neglect the dissipation of the field. Obviously T_0 is only slightly less than T_f. By placing the origin of coordinates at the point where $T = T_0$, we replace the sharp transition function $\sigma[T(x)]$ by a step function: $\sigma = 0$ for $x < 0$, $\sigma = \text{const} = \sigma_f$ for $x > 0$, $T_0 < T \leq T_f$. In this approximation Maxwell's equations give the well known solution (see, for example, ref. 35): $H = H_0$, $S = S_0$ for $x < 0$ and $H = H_0 e^{-x/\delta_f}$, $S = S_0 e^{-2x/\delta_f}$ for $x > 0$, where the energy flow S_0 into the skin layer is determined by Eq. (20). We can now integrate Eq. (24) and find the temperature distribution T(x) in the following approximation. In the heating zone $x < 0$ $T = T_0 e^{-|x|/\Delta}$, where $\Delta = \lambda_f/\rho_0 u c_{pf}$ (if we assume for simplicity that $c_p(T)/\lambda(T) = \text{const}$). For $x > 0$ T asymptotically approaches T_f, where T_0 is related to T_f. For determination of the final temperature we can write an integral equation which is a generalization of (19).

The calculations lead to a quite natural result. For a rapid dependence $\sigma(T)$ the final temperature T_f is nearly the same as the temperature of the static discharge at the same field H_0 (it is less than the static temperature). For the specific dependence $\sigma \sim e^{-I/2kT}$ and $kT_f \gg I$ the correction to the static temperature is of the order $2k(T_f)^2/I$, as is the difference $T_f - T_0$. The physical reason for these results is that in the dissipation zone the flow-balance equation (24) is not very different from Eq. (17) which is valid in the static case. In practice all of the field energy dissipated here is carried away by thermal conduction to the forward zone of the wave and is expended in heating gas to the ionization temperature. Since the plasma temperature is determined mainly by the flow balance in the heat-dissipation zone itself, it is not very sensitive to how the energy carried away from this zone is expended; either it goes away to the walls or it goes into heating of new portions of the gas. The situation is very similar to that which occurs in ordinary combustion.

The wave-propagation velocity for a known temperature T_f is given by Eqs. (25) and (20). The characteristic widths of the heating and dissipation zones Δ and $\delta_f/2$ are related as T_f and $T_f - T_0$, as follows directly from the condition of continuity of the flow for $x = 0$, $T = T_0$. From this follows the formula for the velocity

$$u \approx \frac{2\chi_f}{\delta_f}\frac{T_f - T_0}{T_f}\frac{\rho_f}{\rho_0}, \quad \chi_f = \frac{\lambda_f}{\rho_f c_{pf}}, \tag{26}$$

which is characteristic of the thermal-conduction propagation mechanism (χ_f is the thermal conductivity of the heated gas). We note that the velocity can be represented also in a form which is practically identical to the Zel'dovich formula for combustion velocity. For real parameter values the velocities u are of the order of centimenters per second. Thus, for the examples given in Section 5 (argon, 1 atm) $u \approx 2$ cm/sec for $I_0 n = 13.3$ A-V/cm, $T_f \approx 8000°$, and $u \approx 7$ cm/sec for $I_0 n = 33$ A-V/cm, $T_f \approx 10\,000°$.

Let us return to the process of radial expansion of the discharge. Let us assume that at some moment ther there is turned on an axial gas flow which is concentrated preferentially at the periphery of the tube, as in a plasmatron. The thermal conduction wave maintained by dissipation of Joule heat is propagated in the radial direction, and simultaneously heat is carried away by the gas flow. Obviously heat will propagate further along the radius in the rear part of the solenoid where gas particles arrive already heated in passing through the foward part. Therefore the isotherm $T = T_0$ which bounds the discharge begins to be deflected relative to the flow until the axial removal of heat exactly compensates the radial supply. When the velocity of gas flow into the discharge along the normal, which increases with deviation of the front, becomes equal to the discharge propagation velocity u, further rotation of the front is cut off and the state becomes stable. The pattern established in the induction torch has the form shown in Fig. 9. In this figure the discharge wave, i.e., the skin layer together with the preceding heating zone, is shaded (it is assumed that the skin layer is thin). Also shown are lines of gas flow or more accurately the projections of the actually helical lines onto the plane of

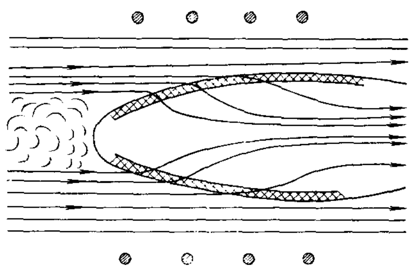

FIG. 9. Qualitative diagram of the combustion process in a high-frequency plasmatron. The "flame" is crosshatched. Lines of gas flow are shown. Vortices can be seen on the axis in front of the discharge.

a section of the tube through a diameter. The flow lines are bent in the wave since, on being heated, the gas expands and is accelerated mainly in the direction perpendicular to the discharge front, the tangential velocity component here changing only slightly. The internal cavity of the discharge ring is filled with plasma heated to a final temperature T_f and the flow here is now straight. The angle of inclination of the discharge front to the axis is approximately equal to the ratio of the normal discharge-propagation velocity u to the axial flow velocity v_0. Since $v_0 \sim 1$ m/sec, while $u \sim 1-10$ m/sec, the angle of inclination is small, and this also permits us in calculation of u to assume approximately that the magnetic field is parallel to the surface of the front. In the paraxial region of the tube in front of the discharge the flow velocity is small (a vortex is formed in that region), so that the cold gas flows into the discharge mainly through the side surface.

More detailed information on the physics of high-frequency discharges and the plasmatron can be found in the author's review[22]. A detailed bibliography on experiments and applications is contained in the review by Yakushin[23]. Some considerations relating to the stability of high-frequency discharges, which are close in spirit to the principles of combustion theory, have been published by Frank-Kamenetskiĭ[24].

7. "Flame" propagation in a waveguide containing atmospheric air. In high-power (kilowatt) continuous-wave microwave devices it is common to observe this "flame" propagation. Suddenly at some point in the waveguide a discharge is ignited, and the plasma production runs opposite to the high-frequency wave; this occurs at a microwave power much less than the power necessary for breakdown of air. The discharge is always initiated by some nonuniformity—an impurity or foreign material, for example, a metal chip accidentally left in the guide, which is strongly heated in the microwave field and results in a cloud of ionized gas. This effect often creates serious difficulties, and to avoid it a careful cleaning of the waveguide is recommended.

This phenomenon was described by Beust and Ford[25] in 1961. They intentionally initiated a discharge by introduction of a small steel screw into the waveguide. The experiments were performed with a rectangular guide 2.29×1.02 cm intended for the X band (5.2–11.9 GHz, vacuum wavelength $\lambda_0 = 3.8-2.5$ cm).

The effect had a threshold at a power of about 0.25 kW, whereas breakdown of air requires a thousand times more power. The velocity with which the discharge moved increased with increasing power from ≈ 25 cm/sec near threshold to 6 m/sec at 2.5 kW. The plasma formation, judging from the photographs presented, has the shape of a column located in the center of the waveguide perpendicular to the axis and parallel to the narrow wall, i.e., along the electric field (the TE_{01} mode was used). The diameter of the column, as far as can be seen from the photographs, is several millimeters. In typical situations 75% of the incident microwave power was absorbed in the plasma, and the remainder was reflected.

An acquaintance with the above facts leaves no doubt that we are dealing here with an explicit case of discharge propagation, this time a microwave discharge, in the slow-combustion mode. On this basis the author[26] has given a physical interpretation of the effect and has calculated the principal quantities.

Let us consider a plane stationary mode of discharge wave. The loss by radiation is small in microwave discharges, since the temperatures obtained here are not high (in air at 1 atm about 5000°). We will assume at the start that there are no losses by thermal conduction; this is permissible for powers which are appreciably greater than the threshold for existence of the mode. The energy balance in the wave is described in this case by Eq. (23). It is now impossible to neglect the displacement currents in Maxwell's equations, as was done for the high-frequency discharge, and the field is a wave field. A monochromatic plane wave satisfies the equation (see the books of Ginzburg[27] and of Landau and Lifshitz[35])

$$\frac{d^2E}{dx^2} + \frac{\omega^2}{c^2}\left(\varepsilon + i\,\frac{4\pi\sigma}{\omega}\right)E = 0. \tag{27}$$

The dielectric constant ϵ and the high-frequency conductivity σ are[27]

$$\varepsilon = 1 - \frac{4\pi e^2 N_e}{m(\omega^2 + \nu_m^2)}, \quad \sigma = \frac{e^2 N_e \nu_m}{m(\omega^2 + \nu_m^2)}, \tag{28}$$

where N_e is the number of electrons per cm^3, which we take as the equilibrium value, and ν_m is the effective frequency of electron collisions. For $x = -\infty$, $T = 0$ and $J = 0$, and the field strength or energy flux S_i in the incident electromagnetic wave are given. For $x = +\infty$ in the case of no loss, $J = 0$ and $E = 0$. Equations (24) and (25) remain valid, and the energy flow S_0 into the plasma, which appears in (24) and (25), is equal to $S_0 = S_i(1 - R_r)$, where R_r is the reflection coefficient for the electromagnetic wave from the plasma front (which is not known beforehand).

The solution of Eqs. (23) and (27) presents difficulties of two kinds. One of them, as before, is due to the nonlinearity of the equations and the presence of the unknown parameter u. However, in the present case we have an additional difficulty due to the need of solving the wave equation in a nonuniform medium. The first difficulty can be avoided approximately in roughly the same way as before if we take advantage of the extreme rapidity of the dependence of $\epsilon - 1$ and σ on T. In the zone where the field is dissipated and where the reflected wave is produced, the temperature is extremely close to the final temperature and in this region Eq. (17), which is characteristic of a static discharge, is valid with an accuracy to order $2kT_f/I \ll 1$.

In the high-frequency case it was possible to integrate this equation by reason of the fulfillment of the condition $4\pi\sigma/\omega \gg |\epsilon|$ ($\omega \to 0$), which permits displacement currents to be neglected and the flow S to be represented in the form of Eq. (18). The same procedure turns out to be possible also in the opposite limiting case $4\pi\sigma/\omega \ll \epsilon \approx 1$ ($\omega \to \infty$), in which the flux absorption equation (5) is valid (the geometrical optics approximation). Generally speaking, in a microwave plasma at atmospheric pressure $4\pi\sigma/\omega \sim |\epsilon|$. However, Eq. (5) can be preserved to a certain approximation if we understand by S the flux in the traveling electromagnetic wave and calculate the absorption coefficient μ in terms of local values $\epsilon(x)$ and $\sigma(x)$ by the formulas applying to a uniform medium. The reflection coefficient can be evaluated to a first approximation if we consider the plasma boundary as sharp.

Under these approximations we quickly obtain from (17) and (5) the integral relation

$$\int_0^{T_f} \mu(T)\lambda(T)\,dT = S_0 = S_i[1 - R_r(T_f)], \tag{29}$$

which determines the discharge temperature.[8] After calculation of T_f we can also find the velocity u from Eq. (25). Baltin, Batenin, Gol'dberg, Devyatkin, and Tsemko[28], who studied a stabilized microwave discharge in nitrogen at atmospheric pressure (see Section 8), numerically solved the exact integral equation for the function S(T), which follows from Eqs. (24), (25), and (5), which are the parent system for Eq. (29). The results of the calculations—the dependence of T_f on S_0, are in satisfactory agreement with measurements.

Equation (29), which determines the final state of the discharge plasma, permits a remarkable interpretation. It can be shown[26] that it is practically equivalent to the condition of equality to unity of the optical thickness of the heating zone in the discharge wave, $\tau_0 \approx \int_{-\infty}^{x(T_0)} \mu\,dx$. This condition is completely admissible. In fact, we will assume that $\tau_0 < 1$. This would mean that at the end of the heating zone the absorption coefficient is still so small that in the extent of some adjacent part of the dissipation zone the absorption also will be small. If we assume that $\tau_0 > 1$, this will mean that even in the heating zone the electromagnetic wave is strongly absorbed. Both assumptions are inconsistent with the very definitions of the concepts of dissipation zone and heating zone.

When energy loss is taken into account, the problem of the propagation mode is formulated in the same way as was done in Section 3. Since Eq. (5) is used to describe the field, the entire system (3) and (5) is preserved. For $x = +\infty$ we now have $T = 0$ and $J = 0$. In this formulation the problem is substantially complicated. An approximate solution[9] has been obtained by the author[26]. It permits determination of a threshold flux S_t such that for $S_i < S_{it}$ the wave is not propagated.

Near the threshold of the mode the longitudinal and transverse heat flows are comparable, i.e., the width of the dissipation zone $1/\mu_f$ is roughly equal to the transverse dimension of the discharge r_0 (the radius of the discharge "column"). The condition $1/\mu(T_{ft}) \approx r_0$ gives us the minimum possible temperature of the discharge plasma.

For the experimental conditions of Beust and Ford[25]—air at 1 atm, $\lambda_0 = 3$ cm, $\omega = 6.3 \times 10^{10}$ sec^{-1}, and $r_0 \approx 0.3$ cm—the calculations give $T_{ft} \approx 4200°$ and threshold fluxes $S_{ot} \approx 0.2$ kW/cm^2, $S_{it} \approx 0.28$ kW/cm^2 ($R_r = 0.28$), which is in good agreement with the experimental value of the threshold power if we calculate it for the surface of the discharge column. As the energy flow fed to the discharge increases, the temperature increases slowly and the propagation velocity rises. For example, for $S_0 \approx 1$ kW/cm^2, $T_f \approx 6000°$ and $1/\mu_f \approx 0.02$ cm, the velocity is $u \approx 30$ cm/sec. The discharge-propagation velocity relative to the heated gas is $v = (\rho_0/\rho_f)u \approx 8.7$ m/sec. The measured velocities are in agreement with these values and not with the values of u. This indicates that the situation in a waveguide is to some extent close to combustion in a tube from the closed end (in contrast to a discharge inside a solenoid, which is stabilized, being "attached" to the coil). In a waveguide, where the discharge region is quite concentrated, the heated gas expands in all directions, including the direction of propagation, and therefore the laboratory velocity of the discharge wave also is found to be much greater than u. In general the hydrodynamic process in the waveguide is greatly complicated by the fact that the discharge does not cover the entire tube. The transverse dimensions of the discharge column are limited as the result of the sharp drop in the field in the transverse direction in the presence of a plasma in the waveguide; apparently the radius is affected also by the hydrodynamics of the flow.

8. Microwave discharge in a gas flow. Discharge in a resonator. A reverse pattern of microwave discharge propagation, quite similar to that observed in electrodeless plasmatrons, arises in microwave plasmatrons. One of the first designs, described by Aksënov et al.[29] and Blinov et al.[30], is typical; a waveguide carrying a wave from a magnetron intersects a quartz tube (Fig. 10). Gas is blown along the tube, usually in a twisted flow. A stabilized discharge burns in the region of intersection. It is pushed away from the walls of the tube by about half the radius as a result of the twisting of the flow; a plasma stream flows from the tube. Microwave radiation with $\lambda_0 \sim 5$–12 cm is ordinarily used, with a power of the order of a kilowatt; the tube radii are of the order of a centimeter; the axial gas-flow velocity is of the order of tens of cm/sec. In microwave plasmatrons an extremely high efficiency is reached—more

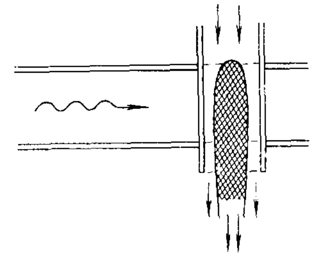

FIG. 10. Diagram of microwave plasmatron (the plasma is crosshatched).

[8]It should be noted that Eq. (29), which is strictly valid in the limit $\omega \to \infty$, gives reasonable results even in the opposite limit $\omega \to 0$, in which Eq. (19) is accurately satisfied [26]. This also provides partial justification of use of the limit $\omega \to \infty$ at microwave frequencies.

[9]We note that the method of solution can be somewaht simplified in comparison with ref. 26 if we use as one of the conditions $\tau_0 = 1$.

than 50% of the power generated can be fed to the plasma, and in some cases 80–90%. The temperatures obtained in a microwave discharge are lower than those in a high-frequency discharge.[10)] In nitrogen at atmospheric pressure only 5000–6500° is obtained. Measurements of the vibrational and rotational temperatures, on the one hand, and of the electron concentration on the other hand (see ref. 31), show that the plasma is in a state close to thermodynamic equilibrium. On the other hand, in argon the electron temperature of 6500–7000° appreciably exceeds the atomic temperature of 4500° (see ref. 32). This also is natural: in atomic gases there is no effective mechanism for equalization of the temperatures, such as excitation of molecular vibrations by electron impact.

We must assume that the shape of the surface of a microwave discharge in a gas flow (the flame surface), as in the case of an induction discharge, is determined by the relation between the incident-flow velocity and the normal discharge-propagation velocity u (see Section 6). For powers appreciably above threshold, the depth of penetration of the field into the plasma is small (the skin depth is small) and the situation is close to that which occurs in a high-frequency discharge, although with the difference that in a waveguide the field distribution is not symmetric about the axis of the discharge column, as it is inside the coil. As was clear from the preceding discussion, the temperature in the discharge is almost independent of the gas-flow velocity and should be close to the static temperature.

One of the difficulties which arises in calculation of the dependence of the temperature on the power generated is due to the need of taking into account what fraction of this power is dissipated in the discharge[11)]. This fraction has been calculated[30,31] on the basis of the known solution of the problem of scattering of a TE_{01} wave in a waveguide from a very thin conducting rod located in the center of the waveguide parallel to the electric vector[34]. In this approximation a maximum of half of the incident power can be absorbed; here a fourth is reflected and a fourth is transmitted. It cannot be said that this approximation is satisfactory; the discharge "rod" is in no way thin and the absorption often amounts to more than 50% experimentally. In order to relate the plasma temperature to the energy flow into the discharge, it is necessary to use a relation similar to (29). Use of a minimum principle in analogy with the channel model of an arc (see Section 9), as done by Baltin et al.[31], is not justified and can result in erroneous results.

Blinov et al.[30] have described a microwave discharge of still another geometry. The discharge burns on the axis of a circular waveguide along which a TM_{01} wave is propagated. The cylindrical surface of the waveguide and the conducting plasma cylinder on the axis form a coaxial line for the electromagnetic wave. Gas is forced along the waveguide tube with a helical motion, and a plasma stream flows out from the end through an orifice. In this system the generated power is absorbed almost completely and a very high plasma temperature is reached. Note that the temperature can be related to the input power by the same relations (29) or (21) in which the flow into the plasma must be expressed in terms of the imaginary part of the propagation constant of the electromagnetic wave along the coaxial line. The latter itself depends on the conductivity, i.e., on the plasma temperature. It should be emphasized that theoretical determination of the radius of microwave discharges in a gas flow, as in the case of a high-frequency discharge, requires consideration of the hydrodynamic process with inclusion of the radial distribution of the incident-gas velocity and the vortex motion at the axis. Up to the present time this problem has not been solved. The optimal mode of gas flow is usually chosen empirically.

In the experiments of P. L. Kapitza, which were begun in 1950, long before the creation of microwave plasmatrons, and which became well known after publication of a detailed article in 1969,[36] a microwave discharge was produced not in a waveguide but in a resonator. The resonator was supplied with a specially developed oscillator which could provide a continuous power up to 175 kW and which produced oscillations with $\lambda_0 \approx 20$ cm. Discharges were studied in hydrogen, deuterium, helium, and other gases at pressures of one to several atmospheres. The discharge was ignited in the middle of the resonator, in the region of maximum field. The discharge had the shape of a filament drawn out horizontally along the electric vector. On increasing the power the length of the filament reached 10 cm (a half wavelength), and the diameter 1 cm. A power of up to 15–20 kW could be fed into the discharge. In order to stabilize the discharge the gas was given a helical motion in the resonator; in the absence of forced stabilization the discharge filament twisted and floated under the action of Archimedes buoyant forces. In the outer part of the discharge the temperature (in hydrogen) amounted to 6000–8000°, which is typical for microwave discharges (see above). The article described the results of detailed theoretical and experimental studies of the electrodynamic characteristics of the process, the plasma parameters and the effect of an external magnetic field.

In Kapitza's work special attention was concentrated on discussion of the proposed effect of formation inside the discharge of a highly heated cavity with an electron temperature of the order of a million degrees. It is assumed that the high-temperature region is thermally isolated from the surrounding low-temperature plasma by a double electrical layer and is maintained at the expense of the energy which is dissipated as a result of occurrence of the anomalous skin effect.

9. Plasma temperature in an arc discharge. Many studies have been made of arcs and arc plasmatrons, and we do not intend to discuss here the various aspects of this subject. We will be concerned only with the question of determining the temperature of the discharge plasma in order to demonstrate the complete unity in

[10)] At high temperatures the reflection increases rapidly, but the reflected wave must be diverted from the magnetron and the reflected power is lost. If we discuss the high-frequency discharge in wave language, although the wavelength here is much greater than the dimensions of the system, the reflection from the plasma is of course almost complete in this case, but the reflected power is returned. The generator supplies only the small difference between the incident and reflected powers.

[11)] Calculation of the reflection coefficients on the basis used for plane waves does not give correct results [[26]].

this respect of equilibrium discharges in all frequency ranges. We will consider a cylindrical arc column. The column is the part of the discharge sufficiently removed from the electrodes that the effect of the near-electrode processes is not felt. A radial temperature distribution and longitudinal electric-field strength E are automatically established in the column such that a current of definite magnitude I_0 flows through the arc. The theoretical problem is to obtain the temperature and the field as a function of the current (see the book by Finkelnburg and Maecker[37].

In the absence of gas flow a stationary state in the column is provided by radial outflow of the heat dissipated in the plasma by Joule heating, and the energy balance is described by Eq. (14), where $\langle E\rangle^2 \equiv E^2$. The loss by radiation we will not take into account in what follows; this is permissible for low-current arcs. On the axis at r = 0 dT/dr = 0, and at a sufficiently large distance for r = R we can set $T = T_R \approx 0$ (the "cooled screen"). The current is

$$I_0 = E\int_0^R 2\pi r\sigma\, dr. \tag{30}$$

This problem is ordinarily solved[12)] by use of the so-called channel model[37], in which the arc column is approximately divided into a conducting channel of radius r_0 with a constant temperature T_f and conductivity $\sigma_f = \sigma(T_f)$ and a nonconducting zone of heat outflow $r_0 < r < R$, where $\sigma = 0$. Here

$$I_0 = E\sigma_f \pi r_0^2. \tag{31}$$

Integrating Eq. (14) in the zone of heat outflow and noting that here the heat flow through the entire cylindrical surface is equal to the power dissipated in the channel, $I_0E = I_0^2/\pi r_0^2\sigma_f$ per unit length, we obtain the equation

$$\sigma_f(\Theta_f - \Theta_R) = \frac{I_0^2}{2\pi^2 r_0^2}\ln\frac{R}{r_0}, \quad \Theta = \int_0^T \lambda\, dT, \tag{32}$$

which relates the two unknown parameters r_0 and T_f.

The missing equation in the channel model can be obtained by use of Steenbeck's minimum principle, according to which for given I_0, R, and T_R that temperature distribution should be established for which the power dissipated, and consequently also E, are minimal. Differentiating Eq. (31), say, with respect to r_0, substituting the derivative dT_f/dr_0 found from Eq. (32), and setting $dE/dr_0 = 0$, the missing second relation between T_f and r_0 is found to be:

$$\lambda_f\sigma_f^2 \Big/ \left(\frac{d\sigma}{dT}\right)_{T=T_f} = \frac{I_0^2}{4\pi^2 r_0^2}. \tag{33}$$

Calculations for arcs on the basis of Eqs. (31)—(33) give excellent agreement with experiment, but the question of use of the minimum principle has been the subject of many discussions[37]. As a result the use of the principle for arcs has been accepted as permissible. It has been shown that it is an expression of the general condition of the minimum production of entropy, which follows from the thermodynamics of nonequilibrium processes, and Eq. (14) can be considered as the Euler-Lagrange equation for the corresponding variational problem.

Nevertheless, there must remain a feeling of dissatisfaction with the fact that solution of the problem on the basis of such a simple and natural model as the channel model requires use of this additional and physically not completely clear condition. The fact is that there is no necessity of invoking the minimum principle, and the missing relation follows from the equations of energy balance and electrodynamics in the same way as in the case of other discharges, as has been shown by the author[39].

We will use the first Maxwell equation (15) to express E in terms of H and substitute the expression for the flux S ($E \equiv E_z$, $H \equiv H_\varphi$, $S \equiv S_r$) into Eq. (16) which is rigorously valid in this case. Multiplying the equation obtained by σ and integrating it over r from 0 to R, we find

$$\int_{T_R\approx 0}^{T_f} \sigma(T)\lambda(T)\, dT = \left(\frac{c^2}{16\pi^2}\right)\int_0^R \frac{H}{r}\frac{d}{dr}(rH)\, dr. \tag{34}$$

We will convert this exact relation into an approximate relation by using the channel-model approximation $\sigma = 0$, const in order to express the right-hand part in terms of a given quantity—the current. Outside the channel $H = H_0(r_0/r)$, where $H_0 = 2I_0/cr_0$, and in the channel $H = H_0(r/r_0)$. As the result of integration we obtain the relation

$$\int_{T_R\approx 0}^{T_f} \sigma\lambda\, dT = \frac{I_0^2}{4\pi^2 r_0^2}. \tag{35}$$

which provides the missing relation between T_f and r_0, determines the plasma temperature, and corresponds completely to Eqs. (19) and (29) for the discharges discussed earlier. For a rapid dependence $\sigma(T)$ the new equation (35) gives practically the same result as the old equation—(33), i.e., it provides agreement with experiment. For example, when $\sigma \sim \exp(-I/2kT)$, Eqs. (35) and (34) differ by a quantity of order $2kT_f/I \ll 1$.

It must be emphasized that use of the minimum power principle requires extraordinary caution. Thus, in the case of an induction discharge it gives incorrect results (see ref. 22 in this regard). The same is true also of microwave discharges.

We will also call attention to the fundamental similarity of the spherical-model problem for an optical discharge with the cylindrical problem for the arc column.

IV. NONEQUILIBRIUM AND OTHER MODES

10. The ionization wave and discharge contraction in a constant field. Volkov's experiments[40] on pulsed discharges in inert gases with small additions of cesium vapor revealed a rapid expansion of the current-carrying channel. For example, in argon with 1% cesium at a pressure of 100 mm Hg and a discharge current of 80 A, the velocity was initially $\sim 10^5$ cm/sec; after a time of ~ 100 μsec it fell to $\sim 10^3$ cm/sec. Here the field decreased from ~ 50 to 5 V/cm. In the early stage of the process the gas clearly could not be heated and brought into motion, especially since a small energy was fed to the discharge and the electron concentration

[12)] Equation (14) linearized in Θ, which is obtained by linear approximation of the function $\sigma(\Theta)$: $\sigma = 0$ for $\Theta \leqslant \Theta_1$, $\sigma = B(\Theta - \Theta_1)$ for $\sigma \geqslant \Theta_1$, permits exact analytic solution (see ref. 38).

was very small, even for complete ionization of the cesium. It was suggested that the ionization is propagated in the stationary gas as the result of thermal conduction.

In this connection Velikhov and Dykhne[9] have considered a plane stationary mode of ionization wave, propagating in a constant electric field E (in a direction x perpendicular to the field) as the result of electronic thermal conduction, and have noted the similarity of the process to slow combustion. This was the first formulation of a problem of this type. It was assumed that the electron density N_e is related to the electron temperature T by the condition of thermodynamic equilibrium, where $kT \ll I$ and the degree of ionization is very small. The electrons collide with neutral atoms. Although the electrons transfer energy to the atoms in these collisions, the gas of heavy particles remains cold and stationary, as a result of its large heat capacity.

Under these assumptions (and without inclusion of other losses) the electron balance is described by the equation

$$uI\frac{dN_e}{dx} = -\frac{d}{dx}N_e j_1 + N_e\sigma_1 E^2 - N_e\frac{2m}{M}\frac{3k}{2}\frac{T-T_a}{\tau}, \quad j_1 = -\lambda_1\frac{dT}{dx}, \quad N_e \sim e^{-I/2kT}; \tag{36}$$

here $\sigma_1 = e^2\tau/m$ and $\lambda_1 \approx v_e^2\tau k \approx k^2 T\tau/m$ are the electrical and thermal conductivities calculated for one electron, τ is the time between collisions of the electrons (it was assumed constant), v_e is the thermal velocity of the electrons, M is the mass of the atom, and T_a is the temperature of the atoms (it is assumed that $T_a \ll T$). For $x \to +\infty$ the temperature approaches the final value T_f, which is determined by the condition of equality of the heat dissipation and the elastic loss:

$$T_f = \frac{e^2E^2\tau^2 M}{3km^2}, \tag{37}$$

and $j_1 = 0$. As can be seen from Eq. (36), the leading edge of the wave is sharp. The boundary condition at the edge is that here, for $T = 0$, the heat flow for one electron is finite, or more accurately, $j_1 = -uI$.

The order of magnitude of the propagation velocity and the width of the wave can be determined by comparison of the various terms in Eq. (36). On comparing the leading term in the divergence of the heat flow, which is proportional to the derivative with respect to N_e, with the term due to elastic loss, we find that the length scale in the wave is the quantity $L = (M/m)^{1/2}(I/m)^{1/2}\tau$. By comparing the divergence of the flow with the convection term, we find the velocity scale to be

$$U = \left(\frac{kT_f}{M}\right)^{1/2}\left(\frac{kT_f}{I}\right)^{3/2} \approx v_e\left(\frac{m}{M}\right)^{1/2}\left(\frac{kT_f}{I}\right)^{3/2}. \tag{38}$$

In the work of Velikhov and Dykhne[9], Eq. (36) is transformed to the dimensionless variables $\vartheta = T/T_f$, $\xi = x/L$, and then, by elimination of the coordinate, to the variables $y = \vartheta d\vartheta/d\xi$ and ϑ. A qualitative study of the field of the integrated curves as a function of the parameter $\nu = u/U$ showed that the boundary condition can be satisfied for one value $\nu \sim 1$. Thus, the wave velocity is $u \approx U$. The velocity estimates given by Volkov[40] on the basis of Eq. (38) gave agreement with experiment in order of magnitude.

Munt, Ong, and Turcotte[41,42a] have discussed the same problem but without the important assumption of equilibrium of the electron density and with inclusion of electron diffusion. The rigorous equation for balance of the number of electrons has the form

$$u\frac{dN_e}{dx} = \frac{d}{dx}\left(D\frac{dN_e}{dx} + \frac{8}{13}\frac{DN_e}{T}\frac{dT}{dx} + \frac{DeN_e}{kT}E_x\right) + q; \tag{39}$$

here $D \approx v_e^2\tau$ is the electron-diffusion coefficient, E_x is the longitudinal polarization field which arises as the result of separation of charges in diffusion (it satisfies the Poisson equation), $q = K_i N N_e - K_r N_e^2 N_+$, where K_i and K_r are the ionization and recombination rate constants for the atoms, which are related by the principle of detailed balance and which depend on the electron temperature T. If ion diffusion is neglected, $u dN_+/dx = q$. The energy-balance equation, with the expression for heat flow corresponding to Eq. (39), has the form

$$u\frac{d}{dx}\left(\frac{3}{2}N_e kT\right) = \frac{d}{dx}\left(\frac{80}{26}DN_e k\frac{dT}{dx} + \frac{55}{26}DkT\frac{dN_e}{dx} + \frac{55}{26}N_e DeE_x\right) + \frac{N_e De^2E^2}{kT} - \frac{3N_e kT}{DM}(T-T_a) - Iq.$$

Beyond the wave, at $x = \infty$, the electron temperature T_f is determined by the same formula (37) ($T_f \gg T_a$), $N_e = N_+ = N_{ef}$, where N_{ef} is the equilibrium density at temperature T_f, $E_x = 0$. In front of the wave under these conditions, with inclusion of electron diffusion and the final ionization velocity, the front is diffuse and at $x = -\infty$ $N_e = N_+ = 0$, while the temperature $T_{-\infty}$ and the polarization field $E_{x,-\infty}$ are bounded but unknown beforehand.

In practice the Debye radius always turns out to be so small that the charge separation is negligible and the diffusion is ambipolar. These authors note that the characteristic time of the ionization reaction, i.e., the time for establishment of thermodynamic equilibrium, $\tau_{reac} = (K_{if}N_{ef})^{-1}$, is most frequently greater than the time of energy transfer from the electrons to the atoms $\tau_{exch} = \tau(M/m)$, and the equations are solved for just this limiting case, which is directly opposite to that considered in ref. 9. Here the temperature is almost constant over the entire space. The width of the wave is $L' \sim u\tau_{reac}$. On the other hand, $u \sim D/L$, and hence the velocity scale is $U' = v_e(\tau/\tau_{reac})^{1/2}$. The parameter $\nu' = u/U'$ is the eigenvalue of the dimensionless system of equations. It turns out that the system has two eigenvalues, one of which is of the order of unity, and the second corresponds to the slow propagation velocity and is apparently superfluous[13)]. For hydrogen and for the conditions $T_f = 10^{4\circ}$, $N = 10^{15}$ cm^{-3}, $N_{ef} = 10^{12}$ cm^{-3}, and $\psi \approx \tau_{reac}/\tau_{exch} = 10^5$, we obtain a numerical value $u \approx 36$ m/sec; for argon for the same parameters $\psi \approx 54$ and $u \approx 180$ m/sec.

In the formulations of the problem set forth above, a plane ionization front is propagated in an arbitrarily weak field. However, the phenomenon of discharge contraction is well known; here the static discharge does not fill the entire region where there is electric field, and the region where the current flows coexists in a stationary mode with the un-ionized current-free region. The cause of the contraction may be the dependence of the electron-collision frequency on temperature, which is due to the thermal expansion of the heated gas and to the increasing role of Coulomb collisions with ions[43].

13) Note that in combustion theory, on inclusion of losses, two flame velocities appear [42b].

Equation (36) in this case permits the existence of a static solution with $u = 0$.[44] A radiative mechanism of contraction is also possible[44] 14). Vitshas, Golubev and Malikov[46] have shown that in the case of very small electron concentrations (for a discharge in argon with admixture of cesium vapor), when the electronic thermal conduction plays a small role in comparison with the atomic conduction, the cause of the contraction is heat release to the walls.

A corresponding plane model of the contraction of a nonequilibrium discharge in a constant field for low ionization (Fig. 11) has been constructed by Vitshas, Dykhne, Naumov, and Panchenko[47]. In order to show more graphically the physical meaning of the relationships, we will follow the course of the primary discussions[47] and consider here the much simpler case of an equilibrium discharge of the same geometry. In complete analogy with the derivation of Section 3 we have

$$\rho_0 u c_p \frac{dT}{dx} = -\frac{dJ}{dx} + \sigma E^2 - \frac{A'\Theta}{\Lambda^2}, \qquad J = -\lambda \frac{dT}{dx}, \tag{40}$$

where Λ is the layer thickness (R in Eq. (3)). The plasma temperature T_f corresponds to the upper, stable, point of intersection of the heat dissipation and loss curves: $\sigma_f E^2 = A'\Theta_f/\Lambda^2$. For $x = \infty$ $T = T_f$, $J = 0$; for $x = -\infty$ $T = 0$, $J = 0$. The static mode of the contracted discharge ($u = 0$) corresponds to the threshold for propagation of the discharge wave, which is determined by the condition of equality of areas, Eq. (8):

$$\int_0^{\Theta_f(E)} \left(\sigma E^2 - \frac{A'\Theta}{\Lambda^2}\right) d\Theta = 0. \tag{41}$$

From this we find the field E_t for which the discharge boundary is stationary. For $E > E_t$ $u > 0$ and the discharge will be propagated; for $E < E_t$ ($u < 0$), the discharge decays (a wave of cooling and deionization occurs). In contrast to the cylindrical arc (Section 9), in a static plane discharge neither the field nor the current density $j = \sigma E$ depend on the total current I_0. Only the extent d of the current region along the x axis depends on the current: $I_0 = jd\Lambda$.

The calculation made by Vitshas et al.[47] for discharges in argon with a small concentration of cesium is complicated because of the absence of equilibrium: in that case the difference in the electron temperature T and the atom temperature T_a is important. An equation similar to (40) is written down for the atomic temperature T_a, and the relation between T and T_a is given by the condition $\sigma E^2 = W_{el}$, where W_{el} is the last term

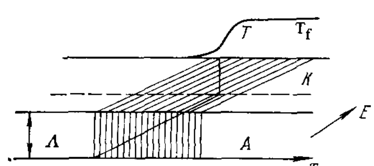

FIG. 11. Drawing of a plane model of discharge contraction. The plasma is shaded. A–anode, K–cathode. A temperature profile is shown at the top.

14)The closely related question of the stability of a power discharge from which energy is removed by thermal radiation has been discussed by Pis'mennyĭ and Rakhimov [45].

in Eq. (36). The analysis of the curves of heat dissipation and loss and of an equation similar to (41) is correspondingly complicated.

11. Ionization waves in waveguides. This phenomenon has been observed and studied experimentally in a series of papers by Bethke, Frohman, and Ruess[48-50]. If a localized plasma is created by a shock wave or a spark discharge in a waveguide field with an inert gas at an end remote from the microwave source, the plasma front is separated from its initial location and moves rapidly in the direction to the source. The experiments were carried out in a cylindrical waveguide of radius 2.5 cm and length more than a meter at a frequency of 8.35 GHz ($\lambda_0 = 3.6$ cm) in Xe, Kr, and Ar at pressures of 0.3–3 mm Hg. The effect occurred even at low radiation fluxes, the threshold being only 0.2–1 W/cm^2. A power of 40–200 W/cm^2 is necessary for breakdown of the gas under these conditions. When the microwave power was increased the velocity of the front increased from tens of meters per second near the threshold to tens of kilometers per second at ~ 50 W/cm^2. The maximum electron concentrations were $(0.7-9) \times 10^{12}$ cm^{-3} (the critical concentration in the waveguide was 0.72×10^{12} cm^{-3}). A special check showed that the gas remains stationary, i.e., the plasma-front propagation has the nature of an ionization wave. Dielectric windows transparent to the microwave radiation were placed along the path of the ionization wave. The ionization wave stopped in front of a window of plastic with a short-wavelength transmission limit $\lambda_0 \approx 2000$ Å in the ultraviolet region of the spectrum, but the wave penetrated a window of LiF, which transmits the ultraviolet to about 1100 Å, and continued to propagate with the same velocity. This indicated that a dominant role in the propagation mechanism is played by the transport of atomic resonance radiation, whose wavelength in the inert gases lies just in the range 1000–1500 A. As we can see, this process differs quite substantially from that which occurs in a waveguide filled with atmospheric air (Section 7), where the discharge is propagated quite slowly.

V. I. Myshenkov and the author[51] developed an approximate theory for an ionization wave propagated in a waveguide as the result of transfer of resonance radiation. The simplest kinetic scheme was adopted: electrons acquire energy in the microwave field and excite atoms to a single resonance level, and the excited atoms are ionized by electron impact. (Since the field is substantially below the threshold for breakdown, the electrons are not in a state capable of ionizing unexcited atoms.) The excitation from the plasma is transferred to the unperturbed layer by resonance photons.

If we do not take into account recombination and electron diffusion to the walls of the tube, which under the experimental conditions[50] occur slowly and are unimportant in the wave region, the electron density satisfies the kinetic equation

$$u \frac{dN_e}{dx} = \alpha N_e N^*, \tag{42}$$

where N^* is the density of excited atoms, and α is the rate constant for their ionization by electron impact (with inclusion of the electron energy spectrum). The density of excited atoms is described a certain integral-differential equation. For simplification the latter was converted to a differential equation of the diffusion type. This equation, which corresponds to the energy-balance equation in the case of equilibrium thermal-conduction modes, has the form

$$u\frac{dN^*}{dx}=D^*\frac{d^2N^*}{dx^2}+\frac{\sigma\langle E^2\rangle}{I^*}-\frac{N^*}{T^*}; \quad (43)$$

here D* is the diffusion-excitation coefficient. It is given by $D^* = l^2/3\tau^*$, where τ^* is the lifetime of the excited atom with respect to photon emission, l is the mean free path of the photons with allowance for the dispersion shape of the resonance line in the wings, $l \approx 0.7\, l_0^{1/4}R^{3/4}$, l_0 is the mean free path in the center of the line, and R is the radius of the tube. The last term on the right-hand side describes the loss due to removal of excitation to the walls of the tube, where $T^* = R^2/3D^*$. The excitation sources were chosen on the assumption that the field energy dissipated $\sigma\langle E^2\rangle$ is mainly expended in excitation of atoms; I* is the excitation potential; the field E satisfies the wave equation (27); σ and ϵ are given by Eqs. (28).

The ionization rate can be considered approximately independent of the field at sufficiently strong fields and equal to zero in weak fields where the elastic loss prevents the electrons from reaching the energy I − I* necessary for ionization of the excited atoms. Thus, α = const for $\langle E^2\rangle > E_f^2$ and $\alpha = 0$ for $\langle E^2\rangle < E_f^2$, where we have, in Eq. (37),

$$E_f^2 = 2m^2(I-I^*)\nu_m(\omega^2+\nu_m)^2/e^2M.$$

A similar formula with I* instead of I − I* determines the field below which the electrons do not reach the energy sufficient for excitation of the atoms, i.e., below which the mode under discussion cannot exist. An evaluation gives for the corresponding energy fluxes ≈ 0.4–1 W/cm², in good agreement with the experimental thresholds for the effect. Thus, the mode threshold is determined by the elastic loss. The boundary conditions for the system of equations (42), (43), and (27) are as follows: for $x = -\infty$ $N^* = 0$, and the given quantities are the energy flow in the incident electromagnetic wave S_i and some low density of "bare" electrons N_{e_0}. For $x = +\infty$ $E = 0$ (since N_e = const = N_{ef}) and $N^* = 0$ (as a consequence of the removal of excitation to the walls). As usual, the system is overdetermined.

The approximate solution of the system is based on three principal factors. In the first approximation the source of excited atoms is assumed concentrated: $\sigma\langle E^2\rangle = S_0\delta(x)$. This permits integration of (43) and then of (42), i.e., finding the form of the function $N_e(x)$. Then, instead of the wave equation (27), Eq. (5) is taken for the flux S; finally, to establish the equation which determines the propagation velocity, use is made of the condition of equality to unity of the optical thickness of the preionization zone (the heating zone in equilibrium modes), where the ionization rises to such a value that intense dissipation of the field begins. For purposes of illustration we will present the results of a numerical example for one version of the experiments of Bethke and Ruess[50]: xenon, 3 mm Hg, $\omega = 5.3\times10^{10}$ sec⁻¹, R = 2.5 cm. We have $l_0 = 2.6\times10^{-6}$ cm, $\tau^* = 3.74\times10^{-9}$ sec, $D^* = 3.2\times10^5$ cm²/sec, $T^* = 6.5\times10^{-6}$ sec, I* = 9 eV, $\alpha \approx 4\times10^{-8}$ cm³/sec, $\nu_m \approx 2.4\times10^{10}$ sec⁻¹. For a change of S_i from 0.6 to 40 W/cm², N_{ef} increases from 1.8×10^{12} to 9×10^{12} cm⁻³, which is in good agreement with experiment, N^*_{max} from 0.8×10^{12} to 23×10^{12} cm⁻³, and u from 70 m/sec to 2 km/sec. The function $u(S_i)$ is found to be correct, but the theoretical velocities turn out to be too low by several times in comparison with the measured values. This is due first of all to the fact that too simple a scheme was assumed for the ionization. The gradual ionization must occur more rapidly, and u in the solution is proportional to the ionization rate constant α. It is interesting that the experiments[50] have revealed jumps in the rate (seeming transitions to other modes) whose nature remains unexplained.

The propagation of ionization fronts in a waveguide has been studied also by Batenin et al.[52], who worked with argon at pressures of 0.1–1 atm—much higher than those used by Bethke et al.[48-50], and also with nitrogen at pressures of 16–40 mm Hg. The microwave discharge was initiated by a spark gap and propagated inside a quartz tube of radius 1 cm placed along the waveguide axis. A frequency of 2.4 GHz (λ_0 = 12.6 cm) was used, and the power was varied from 200 to 1300 W. The plasma absorbed approximately 70% of the power. The electron concentrations in the argon were $\sim10^{13}$ cm⁻³ and in the nitrogen $\sim10^{12}$ cm⁻³. The electron temperature was of the order of 10 eV. The discharge in nitrogen had the shape of a column oriented along the electric vector, and in argon at higher pressures the discharge had a complex shape of individual filaments. Figure 12 shows the ionization-front velocities measured by Batenin et al.[52]

It can be seen that for the same power level and approximately the same pressures (the lower curve in nitrogen at 40 mm Hg and the upper curve in argon at 76 mm Hg) the velocity in argon, which is measured in kilometers per second, is three orders of magnitude greater than that in nitrogen. The velocities in nitrogen at the highest pressure studied are of the same order as the discharge velocity in air at 1 atm—meters per second (see Section 7). It is clear that the discharge-propagation mechanisms in molecular gases (nitrogen, air) and the inert gases are completely different. In molecular gases, where diffusion of resonance radiation does not occur, the mechanism is thermal conduction, even at comparatively low pressures, while in the inert gases a much more efficient mechanism is acting—the transfer of resonance radiation. This question requires further theoretical study.

Another paper by the same authors[53] showed that a microwave discharge wave can be slowed down and stopped completely if a sufficiently strong longitudinal magnetic field is produced in it. In nitrogen at a pressure of 40 mm Hg and a power of 1.3 kW, where the velocity was 4 m/sec, a field of 1.7 kOe was required to stop the discharge. The reason for the slowing down and stopping of the discharge wave is the decrease in the effective value of the electric field or the decrease in the absorption coefficient of the plasma on application of the longitudinal magnetic field.

12. Hyperdetonation thermal-conduction and radiation modes. Breakdown wave. We have discussed above the subsonic thermal-conduction mechanism of optical-discharge propagation, which is similar to slow combustion (Sec. 3), and the ultrasonic-detonation mechan-

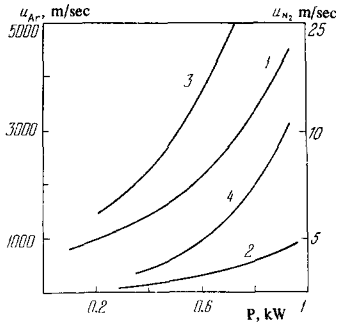

FIG. 12. Velocity of a microwave discharge wave in a waveguide [52]. In argon (left-hand scale): curve 1–for a pressure of 76 mm Hg, 2–760 mm Hg; in nitrogen (right-hand scale): 3–16 mm Hg, 4–40 mm Hg.

ism (Sec. 2). The first mechanism acts at moderate light intensities, and the second at high intensities. It is interesting that even at higher intensities, where the plasma temperature reaches millions of degrees, the thermal-conduction mechanism is most prominent, but in this case it is not only ultrasonic, but even ultra-detonation, for it provides propagation of the discharge wave with a velocity exceeding the velocity of a shock wave. The situation is similar to that which occurs in the earliest stage of very strong explosions, where the explosion energy is first propagated in air by a heat wave and only later does the shock wave come forth (see the book by Zel'dovich and Raĭzer[54]). It is true that in explosions the heat wave is due to radiant thermal conduction, while here we are speaking of electronic thermal conduction (the radiative mechanism will be discussed below).

In ultrasonic heat propagation the gas in the discharge wave does not expand, but is compressed (or does not change density) according to the shock adiabat of the discharge wave (see Fig. 1b). Therefore, in contrast to the case of the subsonic mode, the plasma is opaque and the light flux is absorbed in a comparatively thin layer, and the role of the losses is small. Consequently, Eq. (29) can be applied to this case and by its means we can determine the plasma temperature T_f (at optical frequencies the reflection is negligible, $R_r \approx 0$). The propagation velocity of the wave after this is found from Eq. (25) or Eq. (1), which are practically identical. In a heavy gas in which at temperatures of $\sim 10^{6\circ}$ the atoms are multiply but still not completely ionized, the average charge of the ions and the number of electrons per atom Z, roughly speaking, are proportional to $T^{1/2}$ (the ionization potential is $I(Z) \sim Z^2$, $I(Z)/kT \approx$ const).[54] The absorption (stopping) coefficient for the light quanta is $\mu \sim Z^3T^{-3/2} \sim$ const and the thermal conductivity, which is completely electronic, is $\lambda \sim T^{5/2}Z^{-1} \sim T^2$. Hence it is found from Eq. (29) that the plasma temperature is $T_f \sim S_0^{1/3}$. The internal energy in the region of multiple ionization, roughly speaking, is $\epsilon \sim T^{3/2}$,[15] and consequently the propagation velocity is $u \sim S_0/\epsilon \sim T_f^{3/2} \sim S_0^{1/2}$.

The detonation velocity is $\sim S_0^{1/3}$ and depends slowly on S_0. This means that for small light fluxes the wave should be a detonation wave with a small tongue of thermal-conduction heating in front of the shock-wave front[54], while beginning with some value S_0 the thermal-conduction wave should travel more rapidly than a detonation wave and in general there will be no shock wave (see Chapter II, Sec. 2). Estimates show that this transition should occur at $T_f \sim 3 \times 10^{6\circ}$ and $S_0 \sim 10^{19}$–10^{20} erg/cm^2-sec, which corresponds to gigawatt powers in experiments with high-power laser pulses.

An additional mechanism of discharge wave propagation is possible—radiative propagation, in which the energy transport and ionization of the cold gas in front of the discharge are due to radiant heat exchange. Light quanta produced in the discharge plasma are absorbed in the cold gas and ionize atoms, as a result of which a propagation of the discharge occurs. It must be noted that energy transport by equilibrium radiation is accomplished more rapidly than by electronic thermal conduction, and in general it does not play a role in such large-scale phenomena as strong explosions.[54]

The heat wave in explosions arises as the result of radiant thermal conduction. However, if the size of the heated region is small, as in experiments with discharges, the plasma usually turns out to be optically thin, its radiation has a volume nature, and the thermal-radiation density is much less than the equilibrium value. The efficiency of radiant heat exchange is also correspondingly small.

The radiative propagation mechanism has been discussed by the author[6] with application to experiments on the propagation of a laser spark in the case of giant laser pulses. For plasma temperatures of $\sim 10^5$–10^6 deg, photons are emitted with energies of tens and hundreds of electron volts, whose mean free paths in plasma are $\sim 10^{-1}$–10 cm, which is much larger than the size of the heated region, $\sim 10^{-2}$–10^{-1} cm. These photons are easily absorbed in the cold gas, since their energy is greater than the ionization potential of the atoms and molecules. In ref. 6 the propagation velocity was calculated for the radiative mode at optical frequencies. It turned out to be of the same order as the detonation velocity, and its dependence on the light flux is approximately the same, so that it is difficult even to say which of the mechanisms, detonation or radiative, is more effective. Discussion of the radiative mode involves considerable difficulties, and all of the calculations have too approximate a nature to enable us to draw reliable conclusions.

In the case of discharges with comparatively lower temperatures of $\sim 10\ 000°$, the radiative mechanism is less important than the thermal conduction, since the fraction of photons with energies above the ionization potential of the atoms and molecules is small in the thermal radiation of a plasma, and the cold gas is simply transparent for plasma radiation; the thermal radiation is an energy loss in pure form. It should be noted that this question has not been studied in detail, and conditions may be possible in ordinary discharges at higher temperatures in which radiant heat exchange also plays a role.

We will note further the phenomenon of the "apparent" propagation of a discharge—the breakdown wave.

[15] The translational energy per atom is theoretically $\sim ZT \sim T^{3/2}$; the energy expended in removal of electrons is $\int_0^Z I(Z)dZ \sim Z^3$, i.e., it is also $\sim T^{3/2}$.

The breakdown wave differs fundamentally from all of the discharge-propagation mechanisms discussed above in that its propagation velocity is a phase velocity, and here there is no energy transport. The breakdown is achieved *if the field intensity is sufficient for breakdown* of the gas but for one reason or another the breakdown occurs at different times in different places. Thus, in experiments with a focused laser beam the discharge begins first of all in the focal region where the light intensity is maximal, while at points located further and further from the focus and closer to the lens an electron shower develops with greater and greater delay. In this way a breakdown wave arises. This mechanism has been considered by us[6] and independently by Ambartsumyan et al.[55], who explained on this basis the experiments described in their paper. The breakdown wave travels more rapidly, the smaller the focusing angle of the light beam. Breakdown waves are realizable also in other field-frequency regions.

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