

We note that the dispute "filament or multifocus structure" does not affect the question of waveguide limitation of the beam divergence, which was observed in many experiments both in the optical band and in the radio band in a plasma (such experiments were performed recently by Litvak and co-workers^[12]).

The multifocus structure is highly inefficient in practice, since it distributes the energy over a large volume and does not carry it over large distances, since the power is absorbed and scattered in the foci, and only a power close to the threshold value, i.e., the same power that is incident on one focus, reaches the receiver. Therefore calculations for the multifocus structure can be useful only insofar as they determine the range of conditions in which one should not operate.

The energy fed to a single focus or to the waveguide can be increased by setting an initial beam divergence $\theta \gg \theta_{\text{diff}}$, so that the trapping conditions are $\theta^2 \sim n_2 E^2 \gg n_2 E_{\text{thr}}^2$. The problem of gathering energy into one focus is quite interesting in practice.

Self-focusing of powerful beams can be used to obtain pre-thermonuclear temperatures. The fast motion of the focus^[13] makes it possible to use concentrated field regions for particle acceleration^[14].

It is possible that meandering of the focus in the target leads to the appearance of an overheated group of particles and to hard x-ray or neutron emission.

Intensive studies have been made recently of self-focusing of acoustic waves with allowance for strong nonlinearity of acoustic waves in dense media as a result of heating^[1b], cavitation^[1c], changes in the compressibility, change in the carrier density^[15], etc.

¹G. L. Askar'yan, a) Zh. Eksp. Teor. Fiz. **42**, 1567 (1962) [Sov. Phys.-JETP **15**, 1088 (1962)]; ZhETF Pis. Red.: b) **4**, 144 (1966); c) **13**, 395 (1971); d) **4**, 400 (1966) [JETP Letters, b) **4**, 99 (1966) c) **13**, 283 (1971) d) **4**, 270 (1966)].

²Physics Dictionary (in Russian), vol. 1, entry "Waveguide," G. V. Kisun'ko et al., Soviet encyclopedia, 1960, p. 302.

³V. I. Talanov, Izv. vuzov (Radiofizika) **7**, 564 (1964).

⁴R. Chiao et al., Phys. Rev. Lett. **14**, 479 (1964).

⁵E. Carmire et al., ibid. **16**, 347 (1966).

⁶H. F. Pilipetskiĭ and A. R. Rustamov, ZhETF Pis. Red. **2**, 88 (1965) [JETP Lett. **2**, 55 (1965)].

⁷G. A. Askar'yan et al., ibid. **14**, 452 (1971) [JETP Lett. **14**, 308 (1971)].

⁸A. L. Dyshko et al., ibid. **6**, 655 (1967); **7**, 153 (1968); [JETP Lett. **6**, 146 (1967); **7**, 117 (1968)]; Zh. Eksp. Teor. Fiz. **61**, 2305 (1971) [Sov. Phys.-JETP **34**, 1235 (1972)].

⁹V. M. Bespalov and V. I. Talanov, ZhETF Pis. Red.

*The articles by V. N. Lugovoi and by V. M. Eleonskii, L. G. Ogan'es'yants, and V. P. Silin abstracted below contain the erroneous statement that self focusing is described in the paper by T. F. Volkov (in the collection "Fizika plazmy i problema upravlyaemykh termoyadernykh reaktsii" [Plasma Physics and the Problem of Controlled Thermonuclear Reactions], Vol. 3, AN SSSR, 1958, p. 336). That paper deals with longitudinal redistribution of the field and plasma in plane waves, and the question of the change of divergence as a result of the appearance of transverse gradients was neither posed nor considered. The article by T. F. Volkov therefore does not deal with self-focusing.

3, 471 (1966) [JETP Lett. **3**, 307 (1966)].

¹⁰V. E. Zakharov et al., ZhETF Pis. Red. **14**, 564 (1971) [JETP Lett. **14**, 390 (1971)]; Zh. Eksp. Teor. Fiz. **60**, 136 (1971) [Sov. Phys.-JETP **33**, 77 (1971)].

¹¹V. V. Korobkin et al., ZhETF Pis. Red. **11**, 153 (1970) [JETP Lett. **11**, 94 (1970)]; N. I. Lipatov et al., ibid. **11**, 444 (1970) [11, 300 (1970)].

¹²Yu. Ya. Brodskii et al., ibid. **13**, 136 (1971) [13, 95 (1971)]; B. G. Eremin and A. G. Litvak, ibid. **13**, 603 (1971) [13, 430 (1971)].

¹³P. D. McWane, Nature **211**, 1081 (1966).

¹⁴G. A. Askar'yan and S. D. Manukyan, Zh. Eksp. Teor. Fiz. **62**, 2156 (1972) [Sov. Phys.-JETP **35**, 1127 (1972)].

¹⁵G. A. Askar'yan and V. I. Pustovoit, Zh. Eksp. Teor. Fiz. **58**, 647 (1970) [Sov. Phys.-JETP **31**, 346 (1970)].

V. E. Zakharov. Theory of Self-Focusing. Stationary self-focusing^[1] of waves of nonlinear media, including self-focusing of electromagnetic waves in a nonlinear dielectric, is described by the equation for the complex envelope of the wave^[2,3]

$$2i \frac{\partial \Psi}{\partial z} + \Delta_{\perp} \Psi + |\Psi|^2 \Psi = 0. \quad (1)$$

This equation describes, in particular, beams (waveguides) that are homogeneous along the propagation axis: plane^[2] $\Psi = \sqrt{2} \eta e^{i1\eta^2 x^2 / ch 2\eta x}$, and cylindrical^[3] $\Psi = e^{iz/\lambda^2} \lambda^{-1} R(r/\lambda)$. The functions $R(\xi)$ were calculated in this case with a computer.

The problem of the theory is to determine what happens to a wave with transverse distribution $\Psi_0(r)$ at $z = 0$ incident on a nonlinear half-space $z > 0$, and in particular determine the feasibility of trapping the wave energy in a waveguide propagation mode, and also the possibility of the formation of singularities (foci) at finite values of z .

The theory is entirely different for two-dimensional (x, z) and three-dimensional (x, y, z) beams. In the two-dimensional case the beam energy is trapped in the waveguide propagation mode^[4]. As $z \rightarrow \infty$, a finite number of planar waveguides is produced, which generally speaking are inclined to the axis. The waveguide parameters can be calculated from the initial distribution of the field $\Psi_0(x)$. To this end it is necessary to solve the eigenvalue problem

$$\begin{bmatrix} id/dx & -\Psi_0/\sqrt{2} \\ \Psi_0/\sqrt{2} & -id/dx \end{bmatrix} \begin{bmatrix} \chi_1 \\ \chi_2 \end{bmatrix} = \zeta \begin{bmatrix} \chi_1 \\ \chi_2 \end{bmatrix}.$$

As $z \rightarrow \infty$, each complex eigenvalue $\zeta_i = \xi_i + i\eta_i$ corresponds to a waveguide with amplitude η_i and slope (relative to the axis) $\tan \varphi_i = 4\xi_i$.

Actually, however, such two-dimensional waveguide propagation can be realized only in a strongly anisotropic medium (e.g., in a plasma with propagation transverse to the magnetic field), since a planar waveguide in an isotropic medium is unstable to transverse modulation.

In the three-dimensional case, the description of the nonlinear medium with the aid of Eq. (1) is not accurate enough, for in the case of sufficiently intense beams it leads to the formation of a singularity of a pointlike focus^[5]. Near the focus, the field behaves like^[5a]

$$\Psi \approx \lambda^{-1}(z) R(r/\lambda(z)) \exp \left\{ i \int \lambda^{-2}(z) dz + [i^2 \lambda_z^2 / 2\lambda(z)] \right\} + A_0;$$

$\lambda(z) \sim (z_0 - z)^{2/3}$, where z_0 is the point of singularity. The power concentrated in the singularity is equal to the critical value:

$$\int |\Psi|^2 dr = 2\pi \int_0^\infty R^2 r dr \sim 1.86.$$

Near the singularity it is necessary to take into account the influence of factors that were not taken into account in the derivation of (1). In a conservative medium with purely cubic nonlinearity, the first to come into play is violation of the quasioptical approximation. This leads to the appearance of a backward wave reflected from the focus. A similar effect (but with change of frequency) is obtained when account is taken of stimulated Raman scattering. Some influence on the behavior near the focus can be exerted by the vector structures of the electromagnetic field, namely the appearance of its longitudinal component. The cases most investigated, however, are those in which the field at the focus is limited either by nonlinear damping or by saturation of the nonlinearity. Nonlinear damping (two-photon or higher) of the energy leads to absorption of the energy that enters in the focus. For intense beams this produces the sequence of foci first described in^[7]. Quasi-stationary variation of the beam amplitude at the entrance to the medium gives rise to motion of the foci^[8].

If the nonlinearity is saturated (deviates from cubic), the diffraction effects turn out to be stronger than the nonlinear-focusing effects, and the dimension of the focus is finite. However, the rays emerging from the focus are again gathered into a focus some distance away, so that a pulsating waveguide is produced^[6b]. Several randomly oscillating waveguides can be produced if the initial-beam power is high enough.

From the point of view of experimental observation, the picture of oscillating waveguides and the picture of absorbing foci are difficult to distinguish, and their identification calls for organization of special experiments. It is probable, however, that absorption foci are produced when light propagates in Kerr-type dielectrics; the waveguides should be observed in the case of self-focusing in a plasma.

Notice should also be taken of the role of nonstationary processes of nonlinearity relaxation and of parametric four-photon instability. Whereas these processes are negligible for long pulses ($\tau \sim 10^{-9}$ sec), they may become decisive for short pulses ($\tau \sim 10^{-11} - 10^{-12}$ sec).

¹G. A. Askar'yan, Zh. Eksp. Teor. Fiz. **42**, 1567 (1962) [Sov. Phys.-JETP **15**, 1088 (1962)].

²V. I. Talanov, Izv. vuzov (Radiofizika) **7**, 564 (1964).

³P. Y. Chiao et al., Phys. Rev. Lett. **13**, 479 (1964).

⁴V. E. Zakharov and A. B. Shabat, Zh. Eksp. Teor. Fiz. **61**, 118 (1971) [Sov. Phys.-JETP **34**, 62 (1972)].

⁵V. N. Vlasov et al., Paper at Fifth All-Union Conference on Nonlinear Optics (Kishinev, 1970); see also "Abstracts of Papers" [of this conference], Moscow, MGU, 1970, p. 66.

⁶V. E. Zakharov et al., a) ZhETF Pis. Red. **14**, 564 (1971) [JETP Lett. **14**, 390 (1971)]; b) Zh. Eksp. Teor. Fiz. **60**, 136 (1971) [Sov. Phys.-JETP **33**, 77 (1971)].

⁷A. L. Dyshko et al., ZhETF Pis. Red. **6**, 655 (1967) [JETP Lett. **6**, 146 (1967)].

⁸V. N. Lugovoĭ and A. M. Prokhorov, ZhETF Pis. Red. **7**, 153 (1968) [JETP Lett. **7**, 117 (1968)].

V. N. Lugovoĭ. Theory of Propagation of Giant Laser Pulses in a Nonlinear Medium. The propagation of intense light beams in nonlinear media has recently attracted much attention. The greatest interest attaches to light beams obtained in pulsed lasers, in which the main contribution to the nonlinearity of the medium is made by the Kerr effect. Consequently, most papers dealing with the propagation of light in nonlinear media, following the first (1964) paper of Chiao, Garmire, and Townes^[1], deal with a Kerr-type nonlinearity, where the refractive index of the medium is a function of the light intensity.

The concept of critical beam power was introduced in^[1], and it was subsequently shown by Kelley in 1965^[2] that a light beam with above-critical power begins to propagate in a medium with Kerr nonlinearity in the following manner. The intensity on the axis of this beam increases without limit (within the framework of the employed parabolic equation) when it approaches a certain point on the axis (the "collapse" point). The propagation of the beam beyond the collapse point, however, was not considered. The point of view commonly accepted at that time was that self-trapping of the beam in the waveguide-propagation regime takes place beyond the collapse point^[1] (the beam-intensity profile in the waveguide regime in a Kerr medium was calculated in^[1]). The experimentally observed thin luminous filaments in liquids, glasses, and subsequently also in gases were regarded as realization of such a regime. We note that the possibility of self-trapping of an electromagnetic beam in the waveguide regime was noted back in 1958 by Volkov^[3], who was the first to calculate the beam intensity profile in the case of self-trapping in a plasma. Subsequently, such a possibility was mentioned also in^[4] (the intensity profile considered in^[4b] coincides with that obtained in^[3]).

Many experimental results, however, could not be explained by the hypothesis of self-trapping of the beam in the waveguide regime beyond the collapse point. In 1967 Dyshko, Lugovoĭ, and Prokhorov^[5a] have proposed, on the basis of a numerical solution of the problem, a new (multifocus) picture of the propagation of light beam beyond this point in media with Kerr nonlinearity, and in 1968 Lugovoĭ and Prokhorov^[6] explained the thin luminous filaments previously observed in experiment not as being due to waveguide propagation, but as trajectories of moving foci.

The multifocus structure of a light beam is a finite series of individual foci produced on the beam axis as a result of successive focusing of different annular zones of the beam. The multifocus structure is shown schematically in the figure. The collapse point itself

