LENS FOR SOUND WAVES IN AIR

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THE practical construction of an ordinary lens for effectively focusing acoustic waves propagating in air is a very complicated task. The point is that the acoustic resistance ρv of air differs so much from that of any other solid material of which the lens or its wall is to be prepared, that practically all the energy of the incident wave is reflected from the surface, and there is no gain at the point where the beam is focused. In fact, the published^[1] description of acoustic lenses and prisms show that their authors (Tyndall, Pol, Gesehus) used carbon dioxide as the lens filler material, walls of collodion film or thin silk, or net material filled with down, etc. The situation is radically changed if the waveguide principle is used in the lens construction, and the condition of tautochronism is attained not by shortening the wavelength in the lens material, but by artifically increasing the path length traversed by the lens in the waveguide channel in comparsion with its motion in free space.

This method was first used, insofar as we know, by W. Kock (see^[2]). He used inclined segments to construct a lens as seen from Fig. 1, having an effective refractive index $n = kL/ML = 1/\cos \theta$, taking into account the relative elongation of the path traversed by the wave in the waveguide compared with free space. A certain shortcoming of a lens of this type is that placement of the source at the lens focus produces a plane wavefront, but with an uneven intensity distribution over the front. The intensity is higher in that part of the front half-plane to which the segments are inclined.

Another possibility, which we have employed, is to assemble a lens of corrugated plates of roofing iron, aluminum, or cardboard in the shape of spherical halfsegments; the lens profile is shown in Fig. 2. We again obtain an effective refractive index equal to the relative increase of the path length in the waveguide. It is easiest to make the corrugations semicylindrical. In this case the refractive index is $n = \pi r/2r = 1.57$. If a sinusoidal profile is used it is necessary to calculate the length of a sinusoidal arc; this involves an elliptic integral that cannot be expressed in terms of elementary



functions, but can of course be calculated. For a profile in the form of an isosceles triangle with height equal to half the base, we obtain $n = a\sqrt{2}/a = 1.41$, etc.

In our construction, the acoustic lens is an assembly of corrugated plates having small flat initial sections; the envelope of the edges of the plates is a hemisphere. The initial flat plates have complicated configurations, and were therefore cut out by using a grapho-analytic method for developing the intersections of cylindrical surfaces with a sphere. The cut-out flat plates were then corrugated by using a very simple tool consisting of a number of pipes whose dimensions corresponded to the diameter of the cylindrical surfaces of the corrugated plates. These pipes were welded on alternate sides to a rigid base to form a special shaping tool. The flat iron sheet was secured to the first pipe of the tool, and the removable pipes were alternately pressed in to obtain a bent plate which was alternately manually shaped to the required form using alternately pipes of smaller diameter and the shaping tool. The plates were then stacked through holes drilled in their edges on assembly rods and were spaced apart by short tubes. The structure obtained in this manner acts as a lens.

The radius of curvature of the cylindrical corrugated surface was made equal to 20 mm, and the distance between the neighboring plates was also 20 mm.

The demonstration is made as follows: A signal from a GZ-1 sound generator is fed to a 0.5-GD loudspeaker. Reception is with a microphone. The voltage from the microphone goes to a U2-1A (28-IM) amplifier, the output of which is fed to an S1-1 (EO-7) oscilloscope. The acoustic lens is placed in the path of the wave in a position such that the sound waves are focused on the microphone. An appreciable amplification of the received signal is then observed, the amplitude increasing approximately fourfold at a frequency 6-8 kHz.

The lens prepared by us has an aperture of 40 cm and a curvature radius R = 20 cm, a hemispherical cross section (Fig. 3). The focal length of the lens is given by the usual formula

$$1/F = (n-1)(R_1^{-1} + R_2^{-1}),$$

which yields F = 40 cm at n = 1.5.

A corrugated lens bounded by a spherical surface does not ensure exact tautochronism. We therefore constructed, in accordance with the same principle. A sec-



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ond lens satisfying the requirement that the "optical" paths passing through different sections of the lens and gathered in the focus be equal. The equation of the surface of such a lens, in spherical coordinates, is obtained from the relations (Fig. 3):

 $n \cdot AB + r = \text{const} = nd + F, \quad AB = d + F - r \cos \theta,$ $r + n (d + F - r \cos \theta) = nd + F, \quad r = F (n - 1)/(n \cos \theta - 1).$

This is the equation of a hyperbola, i.e., the lens surface is a hyperboloid of revolution. The shapes of the initial flat laminations was obtained by the same graphoanalytic method, by determining the profile of the intersection of a hyperboloid of revolution with a cylindrical surface. The segments were then corrugated and assembled to form the lens shown in Fig. 4.

The amplification produced by the lens amounts to 5-6 times in amplitude at an approximate frequency 6 kHz. The lens is irradiated with a 0.2-GD loudspeaker, the small size of which ensures a sufficiently broad directivity pattern, so that it is possible to "sound" the entire surface of a lens with 50 cm aperture and approximate focal length 60 cm. The distance from the loudspeaker to the lens was 3.5-4 m. In the demonstration it was necessary to see to it that the lens not be situated in a node of a standing wave, since the reflection from its rear flat surface and subsequent beds was appreciable.

Waves passing through the bent segments with alternately increasing and decreasing cross sections are attenuated quite appreciably. This apparently explains why the gain produced by a tautochronic lens does not exceed the gain produced by a phase zone plate^[3], and is even somewhat smaller. In the absence of energy losses and in the case of ideal tautochronism such a lens should not only cause the oscillations from the even and odd Fresnel zones to be in phase as a whole, but the oscillations coming from different sections of each zone should also be in phase. The vector diagram due to a phase zone plate is thus so transformed by the lens that each half-turn is straightened out by the tautochronic lens.

The net increase in the reception amplitude should be $\pi R/2R = 1.57$. It must also be recognized that the discrete structure of the lens cannot ensure ideal tauto-chronism.



A corrugated 60° prism constructed in accordance with this principle is shown in Fig. 5. The prism refracts very effectively sound waves and deflects them approximately 30° . To demonstrate the action of the prism, a paper conical horn about 30 cm long is placed over the 0.5-GD loudspeaker. To maximize the received amplitude, the angle between the axis of the cone on the speaker and the microphone axis should be 30° . When the prism is removed, the received signal is appreciably weakened.

¹O. D. Khvol'son, Kurs fiziki (Course of Physics), Vol. 2, GIZ, 1923. J. Tyndall, Sound (Russ. Transl.), GIZ, 1927; R. W. Pohl, Mechanics, Acoustics, and Heat, (Russ. transl.), Gostekhizdat, 1927.

² V. Beketov, Antenny sverkhvysokikh chastot (Microwave Antennas), Oborongiz, 1957; W. E. Kock, Proc. IRE **37**, 852 (1949).

³ B. Sh. Perkal'skis and V. L. Larin, Usp. Fiz. Nauk 96, 374 (1968) [Sov. Phys.- Uspekhi 11, 771 (1969)].

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