

Physics of Our Days**NUCLEAR SHAPE ISOMERS**

S. M. POLIKANOV

Joint Institute for Nuclear Research, Dubna

Usp. Fiz. Nauk 107, 685-704 (August, 1974)

Experimental and theoretical investigations of recent years point to the existence of a new type of isomerism of atomic nuclei, due to the strong difference between the shapes of the nuclei in the initial and final stages.

THE concept that the atomic nucleus is a complicated system having a definite shape is used quite extensively in nuclear physics, and most frequently in the study of nuclear fission. In the latter case it is customary to regard the sequence of nuclear shapes from the initial elliptic ones to the dumbbell shapes prior to the fission of the nucleus into two fragments.

One of the problems in the theory of the atomic nucleus is the determination of the equilibrium nucleus deformation, corresponding to the minimum of the potential energy. To solve this problem one must calculate the dependence of the potential energy on the parameters that determine the shape of the nucleus. Generally speaking, the potential surface can be very complicated and one cannot exclude beforehand the possible appearance of additional minima of the potential energy, separated by a barrier from the principal deep well, the bottom of which is the ground state of the nucleus. If the barrier separating these minima is high enough then electromagnetic transitions between nuclear energy levels characterized by different equilibrium deformations will be strongly hindered. This means that the lower energy state of the additional potential-energy minimum turns out to be isomeric. The degree of hindrance of the transitions is determined mainly by the height of the barrier between the minima of the potential energy.

The question of "shape isomerism" of this kind of atomic nuclei has attracted attention recently, in connection with the discovery of spontaneously fissioning isomers^[1,2] and subbarrier fission resonances^[3], which are observed upon capture of slow neutrons. It was proposed in many experimental papers on the properties of certain spontaneously fissioning isomers that spontaneously fissioning isomers are "shape isomers" with a greatly decreased fission barrier^[4-6].

The publication of theoretical papers in which account was taken of the influence of shell effects on the fission barrier^[7] has made the "shape isomerism" hypothesis more concrete.

An analysis of the presently available experimental facts makes it possible to draw definite conclusions concerning the nature of the fissioning isomers and to discuss the possible future experimental research in this field.

SHAPE ISOMERISM OF MOLECULES

It was pointed out quite long ago that the collective motion in nuclear fission has much in common with the

oscillations of a diatomic molecule^[8]. The change in the shape of the ellipsoid in the case of the nucleus corresponds to the change in the distance between nuclei in the case of the molecule. The adiabatic realignment of the electrons when the nuclei of the diatomic molecules move closer together or move apart recalls the realignment of the nucleon orbit in an atomic nucleus whose shape is varied slowly.

This analogy is quite far reaching, and it is not surprising that attempts have been made recently to calculate the potential energy of the nucleus within the framework of the model of an oscillator with two centers^[9].

It is interesting to note that the shape isomerism concept is commonplace for molecules. It implies in this case the existence of two configurations of the atoms making up the molecule, and the transition from one to the other is hindered by the appearance of the potential barrier. An example of shape isomerism of this kind is provided by the rotation isomers of the dichlorethane molecule (Fig. 1). Indeed, for this molecule, which consists of two ClH_2 groups turned relative to the axis joining the two carbon atoms, the potential energy depends on the angle of rotation of one ClH_2 group relative to the other. As seen from Fig. 2, the dichlorethane molecule has equilibrium states separated by a barrier. There are also other known manifestations of shape isomerism in molecules. For example, mirror isomerism, in which one atom configuration is the mirror image of the other, is possible.

Drawing the analogy between the fission process and motion of the nuclei in a molecule, Hill and Wheeler pointed out back in 1953 the possible occurrence of shape isomerism in atomic nuclei^[8]. Hill and Wheeler proposed the existence of shape isomers of fissioning elements in the form of oblate ellipsoids of revolution¹⁾.

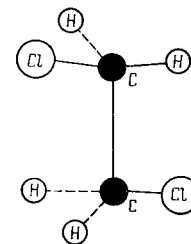


FIG. 1. Model of dichlorethane molecule.

¹⁾According to calculations by Pashkevich [10a], a non-axial deformation transforms an oblate ellipsoid into a prolate one.

EXPERIMENTAL INVESTIGATIONS OF THE PROPERTIES OF SPONTANEOUSLY FISSIONING ISOMERS

After the discovery in 1961 of the first spontaneously fissioning isomer ^{242}Am with a half-life 14 msec^[1], which incidentally turned out to be the longest-lived of all the isomers known to date, many laboratories have undertaken searches for new spontaneously fissioning isomers. Most of the investigations were made with beams of protons, deuterons, and alpha particles accelerated with electrostatic tandem generators and cyclotrons. These investigations have shown that spontaneously fissioning isomers exist for most investigated isotopes in the region from U to Cm. The half-lives of the observed isomers were found to range from 2 nsec to 14 msec. It should be noted that whereas we can state definitely that there are no isomers with lifetimes longer than 14 msec, the lower bound of the lifetime is imposed by the experimental conditions. The fact that the half lives of the observed fissioning isomers turned out to be very short has left its imprint on the experimental research. The experimenters were first forced to investigate the properties of the isomers in the immediate vicinity of the charged-particle accelerators. The observed effects are very small, and, as a rule, it is necessary to bombard highly active targets of heavy elements. All this has made the experiments quite difficult.

In practically all experiments with charged particles, thin targets were used, and therefore the nuclei of the isomers produced in any particular reaction acquired a momentum large enough to escape from the target. The fission fragments of the isomers decaying in flight were observed by placing fission-fragment detectors along the trajectory of the isomer nucleus. From the spatial distribution of the fission fragment tracks on the detectors it was possible to estimate the half-life of the corresponding isomer.

Pulsation of the accelerator particle beam was used in certain experiments. In this case, the isomer nuclei emitted from the target were decelerated in a collector

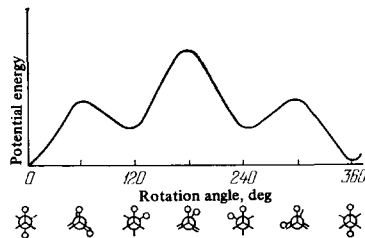


FIG. 2. Potential energy of dichlorethane molecule.

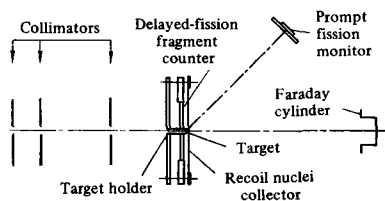


FIG. 3. Experimental setup used in the search of spontaneously fissioning isomers.

placed in their path, and the isomer fission fragments were observed in the time intervals between the bursts of the accelerated-particle beam. If the time distribution of the particles is analyzed, it is possible to calculate the half-life of the isomer. By way of example, Fig. 3 shows an experimental setup^[11] used in the search for isomers in the electrostatic tandem generator of the Niels Bohr institute in Copenhagen. The pulsating proton or deuteron beam was used to bombard a thin target placed, as indicated in Fig. 3, in the opening of a semiconducting surface-barrier detector. The nuclei produced in the reaction were emitted from the target and decelerated in a thin collector of formvar (organic film).

Figure 4 shows typical decay curves of some of the isomers:

$^{238}\text{Pu}(d, 2n)^{238m}\text{Am}$, $^{238}\text{Pu}(d, p)^{239m}\text{Pu}$ (Fig. a),
 $^{239}\text{Pu}(d, pn)^{239m}\text{Pu}$ (b) and $^{241}\text{Pu}(d, pn)^{241m}\text{Pu}$ (c).

The large number of experiments performed by different procedures led to the discovery of more than 20 new spontaneously fissioning isomers^[11-14]. The half-lives of the known isomers are listed in Table I. The presently existing situation can be seen by examining Fig. 5, which shows the half-lives T_{sf} of the spontaneous fission of isotopes in the region from U to Cm. The upper group of points pertains to the ground state of the nuclei, and the lower to the isomer state. We see that the region of isomers drops downward on the average by 26 orders of magnitude. Thus, the experimental investigations show an increased probability of spontaneous fission of the isomers in comparison with the ground state, by an approximate factor 10^{26} . In all cases, only spontaneous fission was observed and one cannot exclude the possibility that the half-life is governed in some cases by another type of decay.

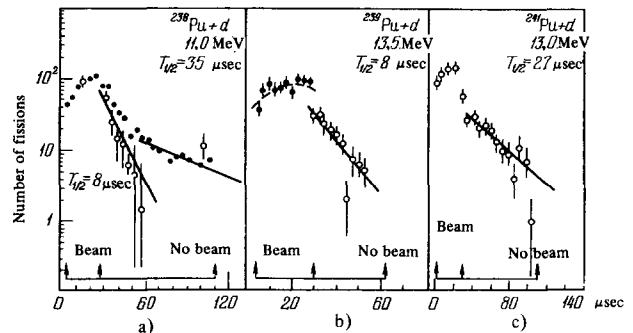


FIG. 4. Decay curves of plutonium and americium isomers [11].

Table I

Isomer	Half-life $T_{1/2}$, nsec.	Isomer	Half-life $T_{1/2}$, nsec.	Isomer	Half-life $T_{1/2}$, nsec.
^{234}U	< 110	^{240}Pu	4, 4	^{241}Am	1.5 *
^{236}U	200	^{241}Pu	27 *	^{242}Am	14 **
^{238}U	20	^{242}Pu	28	^{243}Am	6.5 *
^{235}Pu	20	^{243}Pu	33	^{244}Am	1.1 **
^{236}Pu	34	^{237}Am	5	^{240}Cm	< 2
^{237}Pu	80	^{238}Am	35 *	^{241}Cm	19
	900	^{239}Am	160	^{242}Cm	< 2
^{238}Pu	6.5	^{240}Am	0.9 **	^{243}Cm	38
^{239}Pu	8 *)				

*In μsec . **In msec.

In recent years, serious attempts were made in a number of laboratories to observe the γ -decay branch for certain fissioning isomers, but so far they did not lead to any positive results. The same can be said concerning attempts to observe the α decay of the isomer ^{242}Am . At the same time, it became possible to accumulate sufficient data concerning the character of the nuclear reactions in which isomer production is observed, particularly concerning the thresholds of some of these reactions. The most intensively investigated are reactions of the type $(p, 2n)$, $(n, 2n)$, $(\alpha, 2n)$, $(\alpha, 3n)$ and (γ, n) ^[5,6,12,15,16]. By way of example, Fig. 6 shows the excitation curve of the isomer ^{237}Am in the reaction $^{238}\text{Pu}(p, 2n)^{237}\text{Am}$. The ordinates in this figure represent the ratio of the events of spontaneous fission of the isomer to the number of prompt fission acts.

The data on the dependence of the cross sections of the indicated reactions on the energy of the incident particles were treated under the assumption that the isomer states are populated when the neutrons are evaporated from the produced excited compound nuclei. Calculations of the probability of evaporation of different numbers of neutrons, and accordingly of the cross section for the production of different isotopes, are given in^[7]. Analyzing the excitation curves for the investigated reactions in the same manner as in^[7], the authors of^[5,6,12,15,16] estimated the thresholds for the production of various isomers. In all cases, they turned out to be much higher than the thresholds for the production of the isotopes in the ground state. If it is assumed that the isomer states are populated in the

same manner as the ground state, then the difference between the reaction thresholds can be regarded as approximately equal to the energies of the isomer states.

Figure 7 and Table II show data on the energies of the isomer states, obtained by analyzing the excitation curves for certain spontaneously fissioning isomers. We see that the energies estimated in this manner lie mainly in the range from 2.5 to 4.5 MeV. Of course, it must be recognized that the employed method used to estimate the isomer-state energy is crude, but unfortunately it has not yet been possible to determine the isomer energy by a more accurate method.

The current hypothesis concerning the nature of the spontaneously fissioning isomer, based on deductions from the theoretical papers, will be presented in one of the following sections. We shall therefore touch upon here only those conclusions and hypotheses which can be drawn and were drawn only from experimental investigations of isomers, primarily from the study of the irregularities in the production of spontaneously fissioning isomers.

The simplest observed reaction in which spontaneously fissioning isomers are produced is the radiative neutron capture. The most complete data on the production of spontaneously fissioning isomers in the radiative capture of neutrons pertain to the isomers ^{242}Am and ^{244}Am ^[18-21]. First, we know the dependence of the cross section of the reaction (n, γ) for isomers ^{242}Am and ^{244}Am at a neutron energy from 0.5 to 4.0 MeV (Fig. 8). These results were obtained in experiments with the cyclotron of the Atomic Physics Institute in Bucharest^[18].

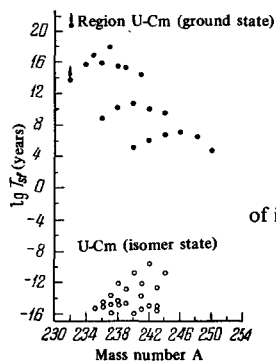


FIG. 5. Half-lives of spontaneous fission of isotopes in the region U-Cm.

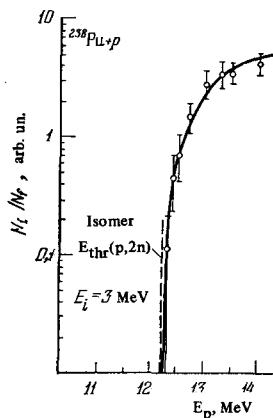


FIG. 6. Excitation curve of the isomer ^{237}Am in the reaction $^{238}\text{Pu}(p, 2n)^{237}\text{Am}$.

Isomer	Energy level, MeV	Reaction	Isomer	Energy level, MeV	Reaction
^{235}Pu	3.0 ± 0.2	$(\alpha, 2n)$	^{238}Am	3-4	$(p, 2n)$
^{236}Pu	4.1 ± 0.2	$(p, 2n)$	^{239}Am	2.9 ± 0.1	$(p, 2n)$
^{237}Pu	3.4 ± 0.2	$(\alpha, 2n)$	^{240}Am	3.15 ± 0.25	$(p, 2n)$
^{238}Pu	4.4 ± 0.4	$(\alpha, 2n)$	^{241}Am	3.15 ± 0.25	(γ, n)
^{239}Pu	3.0 ± 0.2	$(\alpha, 3n)$	^{242}Am	2.5 ± 0.1	$(p, 2n)$
^{240}Pu	3.0 ± 0.2	$(\alpha, 2n)$		2.9 ± 0.4	$(n, 2n)$
^{241}Pu	2.9 ± 0.1	(γ, n)		3.2 ± 0.25	(γ, n)
^{237}Am	3.0 ± 0.3	$(p, 2n)$			

FIG. 7. Energy of isomer states.

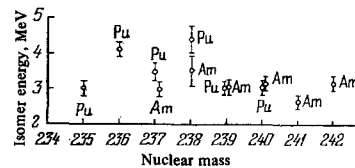
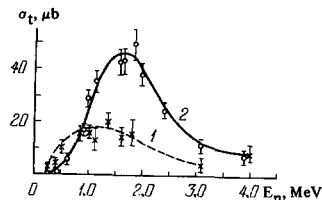


FIG. 8. Cross section of the reactions $^{241}\text{Am}(n, \gamma)^{242}\text{Am}$ (curve 1; $T_{1/2} = 14$ msec) and $^{243}\text{Am}(n, \gamma)^{244}\text{Am}$ (2; $T_{1/2} = 1$ msec).



Attention is called to two circumstances. First, the probability of production of ^{242}Am by radiative capture of neutrons with energy 0.5–4.0 MeV is larger than, for example, in reactions with heavy ions. Second, the dependence of the cross section of the reaction (n, γ) on the neutron energy in the range 0.5–1.0 MeV is quite unusual. In the radiative capture of neutrons with such energy one always observed a decrease in the reaction cross section with increasing neutron energy. In this case, however, the situation is reversed. It is also remarkable that at neutron energies 0.5–1.0 MeV a similar character is possessed by the dependence of the prompt fission cross section σ_f on the neutron energy. In the capture of neutrons with energies 0.5–1.0 MeV, the spin of the produced compound nucleus is relatively small, and the probability of isomer production is not less but, as already mentioned, larger than in reactions with heavy ions. This indicates that the spin of the fissioning isomers is not large^[19]. Previously advanced arguments favoring a small spin for the isomers were based on approximately the same argument.

The growth of the cross section above 0.5 MeV, which is unusual for the (n, γ) reaction, can be interpreted in simple fashion by assuming the existence of a certain potential barrier that separates the isomer state from the ground state, and that the height of this barrier is close to that of the fission barrier.

Thus, the experimental results lead to the conclusion that in the case of spontaneously fissioning isomers we are dealing with a situation in which the energy of the isomer states is sufficiently high (~ 3 MeV), and the spin is small. In addition, it follows from the experiment that for isomer production it is necessary to excite the nucleus initially to 6.5–7.0 MeV.

Having this information, we should explain two main properties of spontaneously fissioning isomers: 1) the hindrance of the electro-magnetic transitions from the isomer state, 2) the sharp increase of the probability of spontaneous fission of isomers.

Whereas the acceleration of the spontaneous fission process can still be explained relatively simply by assuming that the energy of the isomer states is high enough, the situation with the reason for the hindrance of the electromagnetic transitions is more complicated. A natural explanation that comes to mind and has been advanced in a number of papers (see, for example, [4]) is that shape isomerism comes into play in the case of fissioning isomers. Indeed, if we assume that the potential surface has apart from the deepest minimum also an additional minimum that is less deep but for which the fission barrier is lower than for the ground state, then all the facts can be explained very simply. The probability of the spontaneous fission from the lower state of the additional potential well may turn out to be large enough, and the electromagnetic transitions from the second well will be hindered because of the difference between the nuclear shapes in the two states. The assumption that spontaneously fissioning isomers are shape isomers can be advanced on the basis of experimental facts only, without using any theoretical calculations.

A more detailed investigation of radiative capture

of neutrons with formation of spontaneously fissioning isomers of americium yielded some additional data favoring the "shape isomerism" hypothesis.

Experiments were performed at the JINR (Joint Institute for Nuclear Research, Dubna) on the radiative capture of neutrons by americium isotopes. Thermal neutrons^[19], neutrons with energy up to 20 eV^[20], and 14–16 MeV^[21] neutrons were used. In the first case, the neutron source was a lead target bombarded by 660 MeV protons from the JINR synchrocyclotron. The neutrons produced by interaction of fast protons with lead nuclei had, in the main, an energy of several MeV, and to moderate them to thermal energy the lead block was surrounded by water as a neutron moderator.

In the second case, a beryllium target was bombarded with an intense beam of deuterons accelerated by the JINR two-meter cyclotron. The spectra of the neutrons emitted in the $\text{Be} + d$ reaction were shaped with the aid of a paraffin moderator and boron filters. Different energy spectra of the neutrons were obtained by varying the ratio of the number of nuclei of the neutron moderator (paraffin) and the slow-neutron absorber (boron). Naturally, in such a method of shaping the neutron spectrum, the data obtained on the cross sections for the production of fissioning isomers were averaged over a large number of resonances.

Figure 9 shows the results of experiments with thermal neutrons (σ_g is the cross section for the production of a nucleus in the ground state), while Table III gives experimental results obtained with neutrons of somewhat higher energies—the cross sections σ_f for prompt fission and σ_i for the production of spontaneously fissioning isomers ^{242}Am and ^{244}Am by radiative capture of neutrons.

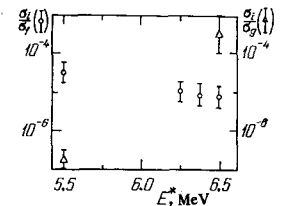


FIG. 9. Dependence of σ_i/σ_f and σ_i/σ_g on the excitation energy of the nucleus ^{242}Am . ($E^* = 5.5$ MeV—thermal neutrons).

Table III

E_n , eV	$^{241}\text{Am} + n$			$^{243}\text{Am} + n$		
	σ_i/σ_f , 10^{-4}	σ_f , b	σ_i , μb	σ_i/σ_f , 10^{-4}	σ_f , b	σ_i , μb
0.2	1.0 ± 0.8	3.13	300 ± 100		0.5	< 10
0.2–20	0.8 ± 0.3	0.5	40 ± 15			
> 20	0.2 ± 0.06	1.2	24 ± 6	0.3 ± 0.1	1.4	42 ± 15

Table IV

Neutron energy E_n , MeV	Target	$T_{1/2}$, 10^{-9} sec	σ_i/σ_f , 10^{-4}	σ_f , b	σ_i , 10^{-4} b
2.2	^{239}Pu	29.0 ± 3.8	4.1	2.0	8.2
	^{233}U	4.1–5.2	43	2.0	86.0
	^{234}U	30.4 ± 4.9	4.7	2.1	9.6
	^{234}U	19.7 ± 4.9	1.2	1.5	1.8
	^{236}U	66.6 ± 8.7	3.1	1.3	4.0
0.55	^{238}U	34.9 ± 4.5	(No isomer observed)		
	^{239}U		7.4	2.0	15.0

Attention is called to the very strong correlation between prompt fission and the formation of fissioning isomers. It can be stated that fissioning isomers are produced quite effectively only when the prompt-fission probability is high enough. If the fission probability is low, then the probability of isomer production is correspondingly low, regardless of the probability of radiative capture of the neutrons. The existence of this correlation allows us to conclude that the production of fissioning isomers is preceded by the onset of oscillations corresponding to the fission degrees of freedom.

It would be extremely interesting to trace such a correlation also for other isotopes, for example ^{236}U , ^{240}P , etc. Unfortunately, however, the presently available experimental data on the formation of fissioning isomers of uranium and plutonium in reactions with neutrons are incomplete and contradictory. The formation of uranium and plutonium isomers by radiative capture of neutrons with energies 2.2 and 0.5 MeV was reported from Harwell^[22]. Table IV lists the results of this study, the cross sections for the production of fissioning isomers and radiative capture of neutrons^[22], but it must be noted that the deduced existence of the isomers ^{233}U , ^{235}U , ^{237}U and the isomer ^{240}Pu with 20 nsec half-life is not confirmed by investigations with charged particles.

At the same time, Romanian physicists^[23] have reported the production of the isomer ^{236}U in capture of neutrons of energy ~ 2 MeV, with a cross section approximately 10 times larger than obtained by the British physicists.

It would be extremely important to obtain more reliable experimental data on the formation of spontaneously fissioning uranium and plutonium isomers in radiative capture of slow neutrons.

SUBBARRIER FISSION RESONANCES

As already mentioned in the preceding section, the experimental studies suggest a manifestation of shape isomerism in the case of spontaneously fissioning isomers.

We shall return later on to the feasibility of a theoretical description of this phenomenon, and consider for the time being other experimental data on nuclear

fission, which were reported quite recently and, in our opinion, have a direct bearing on the question at hand.

Even quite recently, when speaking of fissioning isotopes, we usually had in mind only those experiencing fissioning by capture of slow neutrons. At the same time, there are many isotopes that experience fission only when the neutron energy exceeds a certain threshold, which is different for different isotopes but amounts on the average to several hundred keV. At neutron energies below this threshold, one usually observes many resonances corresponding to the radiative capture of the neutrons.

The advances in experimental accuracy have made it possible to investigate individual resonances more thoroughly. Surprisingly, groups of resonances with relatively large fission widths were observed for certain isotopes heretofore regarded as non-fissioning^[3].

As a typical example, we can cite the data for the isotope ^{241}Pu , which were obtained by bombarding the ^{240}Pu target with slow neutrons (Fig. 10). Figure 10 shows clearly the individual groups of resonances with large fission widths. Attention is called to the fact that the distance between the groups of fissioning resonances that are customarily called subbarrier fission resonances greatly exceeds the distance between the ordinary resonances of radiative capture of neutrons. Similar results were obtained also for targets of ^{237}Np and ^{242}Pu .

The existence of rather broad (~ 100 keV) resonances in the fission of certain nuclei by neutrons with energy equal to several hundred keV, and also in the reaction (d, pf), was pointed out a relatively long time ago.

The last reaction is of particular interest, since in this case one can observe nuclear fission in a wide range of excitation energies, including the region below the neutron binding energy. If the energy of the proton observed in coincidence with the fission act is determined, then it is possible to trace the variation of the fission probability with increasing excitation energy.

Interest in broad fission resonances has increased recently. The observation of narrow subbarrier fission resonances and of spontaneously fissioning isomers, together with new ideas concerning the structure of the fission barrier, has stimulated precision measurements of the cross sections of the reactions (d, pf), (p, p'f), and (n, f).

As to the reactions (d, pf) and (p, p'f), they were investigated most successfully with electrostatic tandem generators^[24,25]. The proton energies were measured either with surface-barrier detectors^[24] or with magnetic spectrometers^[25].

By way of example, Fig. 11 shows the results of experiments (^[27] (a) and ^[28] (b)) on the fission of the isotope ^{240}Pu obtained in the reaction $^{239}\text{Pu}(d, pf)$. The peak on the Pf(E^*) curve for ^{240}Pu at the excitation energy ~ 4.9 MeV is clearly seen. Experiments with a magnetic spectrometer have shown that this peak actually splits into several peaks with distances from 10 to 40 keV between them.

An example of a resonance observed in the fission of nuclei by neutrons is the peak on the curve^[26] of Fig. 12, which shows the cross section of the reaction $^{230}\text{Th}(n, f)$.

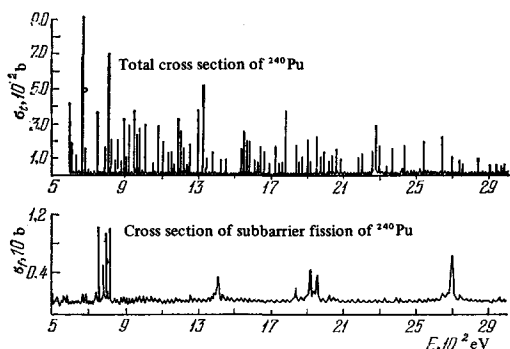


FIG. 10. Subbarrier fission resonances in the reaction $^{240}\text{Pu} + n$.

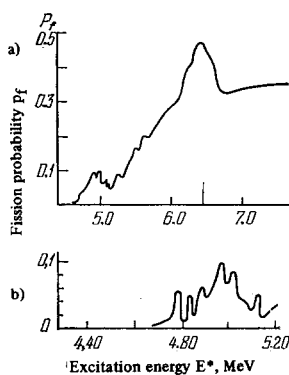


FIG. 11. Subbarrier fission resonances in the reaction $^{239}\text{Pu}(d, pf)$.

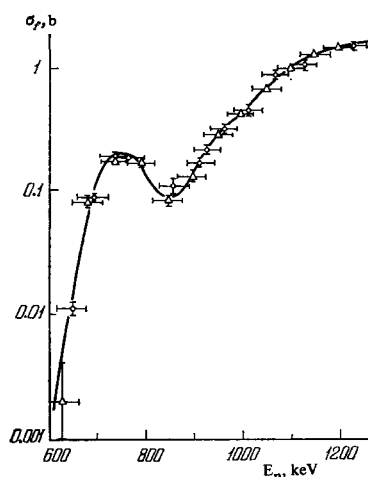


FIG. 12. Cross section of the reaction $^{230}\text{Th}(n, f)$.

MODEL OF NUCLEAR SHAPE ISOMERISM

Following the discovery of spontaneously fissioning isomers of americium, several hypotheses were advanced concerning the nature of the observed phenomenon. These hypotheses were discussed in sufficient detail in the review^[27]. A common premise in most of these hypotheses was the existence of states with different natures and with high spins for nuclei of heavy elements^[28-31].

The calculations of Zel'dovich^[30], Sliv and Kharitonov^[29], Malov, Polikanov, and Solov'ev^[31] and Peker^[28] have shown that different effects can lead to the appearance of high-spin isomer states in heavy nuclei. One cannot exclude the possibility that some of these states, as already mentioned in the cited papers, are characterized by an increased spontaneous-fission probability. For example, in the case of vortex isomers^[30] there should occur states with very large spin ($\sim 40\hbar$). It is possible that the nuclei, which in these states have the form of an oblate ellipsoid of revolution, can experience spontaneous fission via an entirely different approach to the scission point than in the heretofore investigated cases.

The question of the existence of high-spin isomer states with different characteristics and of spontaneous fission from these states is of great interest, and re-

search in this direction is extremely desirable. Undoubtedly, the most effective method of obtaining such high-spin states should be reactions with heavy ions.

As already mentioned, experiments on spontaneously fissioning isomers have led to the conclusion that the isomers have low spin and accordingly to the conclusion of shape isomerism^[4-6,32]. This conclusion forces us to give preference to a shape-isomerism model that does not require a high spin, namely the model developed by Strutinskiĭ^[7].

It should be noted that the available experimental data do not contradict an analysis of shape isomerism within the framework of the model of Urin and Zaretskiĭ^[33], according to which the breaking of the nucleon pairs can cause a phase transition accompanied by a sharp decrease of the mass coefficient that determines the vibrations of the nucleus. The available experimental data were not analyzed on the basis of the Urin-Zaretskiĭ model.

Strutinskiĭ's work has been described in great detail in a number of papers^[34,35], and we confine ourselves here to a mention of those of its main conclusions which are important for the explanation of the experimental results. In the shell model, which has been around for many years, it was always customary to speak to closed shells only for spherical nuclei. The question of the appearance of shell effects when the nucleus is deformed was first raised by Geilikman^[36]. The procedure developed by Strutinskiĭ for taking shell effects into account has led to the very important conclusion that the shell corrections to the fission barrier calculated in accordance with the liquid-drop model can be comparable with the height of the barrier. One can say that the shell effects produced upon deformation of the nucleus are the results of fluctuations of the level density of the nucleus near the Fermi surface.

According to Strutinskiĭ, the potential energy of the nucleus can be represented in the form of the sum

$$V = V_{LDM} + \delta,$$

where V is the potential energy of the nucleus, V_{LDM} is the part of the potential energy of the nucleus described by the liquid-drop model, and δ describes the shell corrections that depend strongly on the level density near the Fermi surface. While the liquid-drop term varies smoothly from nucleus to nucleus, the shell corrections can be affected quite strongly by small changes in the number of nucleons in the nucleus.

An interesting result of Strutinskiĭ's work is the conclusion that an additional minimum of the potential energy of the nucleus occurs at the saddle point (Fig. 13). The concluded existence of an additional minimum on the fission barrier is quite important, since it uncovers the possibility of explaining, at least qualitatively, the nature of the spontaneously fissioning isomers and of the subbarrier fission resonances^[34,35]. Indeed, within the framework of the described model, the discussed isomer states can be regarded as low states of the second potential well. The probability of spontaneous fission can then indeed be greatly increased, since the fission barrier is much lower for the isomers than for the ground state, as seen from Fig. 13. The barrier separating the potential wells is

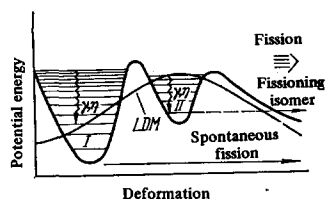


FIG. 13. Fission barrier of nuclei with allowance for shell effect.

high enough and can greatly hinder the electromagnetic transitions from the lower state of the second potential well. In essence, we encounter here practically the same situation as for shape isomers in molecules.

The subbarrier fission resonances also find a perfectly natural explanation. Indeed, the existence of two equilibrium deformations of the nucleus makes it possible to speak of two classes of nuclear states corresponding to different nuclear shapes^[37]. If the excitation energy of the nucleus is small compared with the height of the internal barrier (see Fig. 13), then a sufficiently strong mixing of the states of both classes takes place. It should be borne in mind here that the fission width is much larger for nuclear states with larger deformation than for states of the principal potential well. At the same time, the level density of the excited nucleus is smaller for equilibrium deformation at the saddle point, since a fraction E_1 of the energy of the nucleus goes to deformation of the nucleus, and the nucleus is accordingly cooled.

Within the framework of such an analysis, narrow subbarrier fission resonances occur as a result of mixing of states of classes I and II (the principal and additional potential wells), and the distance between the groups of the fission resonances is a reflection of the level density at large deformation. Mixing of the states of classes I and II takes place also for nuclei that have a large probability of fission by capture of thermal neutrons. The observed modulation of the fission cross section is a result of such a mixing.

Lynn^[37] analyzed in detail the data on the fission resonances and calculated both the level density for two equilibrium deformations and the position of the bottom of the second well.

Some results of these calculations can be seen in Table V, which gives the average distance between the levels \bar{D}_I and \bar{D}_{II} of the first and second potential wells and the energy E_1 of the isomer level.

The broad fission resonances observed in reactions (d, pf), (n, f), etc. are presently interpreted as vibrational states in the second potential well^[34]. These states should be characterized by a very sharp increase of the penetrability of the first potential barrier^[34]. The existence of a connection between the oscillations and the different states at large nuclear deformations can cause the splitting of the resonance into several levels, and this was indeed observed^[25] in precision measurements of the spectrum of the protons in the reaction $^{239}\text{Pu}(d, pf)$. The authors have concluded that in this case the distance between resonances corresponds apparently to the distance between the levels of the nucleus for the second equilibrium deformation. A detailed analysis of these results is not very simple, since it is necessary to know

Table V

Isotope	\bar{D}_I, eV	\bar{D}_{II}, eV	E_1, MeV
^{235}U	$12 \left(J^\pi = \frac{1}{2}^+ \right)$	$7 \cdot 10^3$	2,7
^{236}U	$0,5 \left(J^\pi = 3^-4^- \right)$	≈ 260	$> 2,6$
^{238}Np	$0,67 \left(J^\pi = 2^+3^+ \right)$	54	2,2
^{239}Pu	$13 \left(J^\pi = \frac{1}{2}^+ \right)$	10^3	2,1
^{240}Pu	$3 \left(J^\pi = 1^+ \right)$	460	2,4
		$\approx 50 \cdot 10^3$	2,3
	$160 \left(J^\pi = 0^+ \right)$	700	1,9

the spins of the individual resonances, and we shall not stop to discuss this.

We have already mentioned estimates the isomer-state energy on the basis of the thresholds of the reactions that lead to the isomers (p. 000). It is interesting to see how these data agree with isomer-level energy estimates based on data on the subbarrier resonances. Unfortunately, such comparisons can be made at present only for individual isomers.

Determination of the energy of the isomer ^{240}Pu from the threshold of the (γ, n) reaction yielded for the level energy a value $\sim 3.0 \text{ MeV}$ ^[15], whereas an analysis of the data on the resonances gives more readily the value 2.1 MeV. The earlier estimates of the energy from the reaction threshold were based on the use of the statistical theory of neutron evaporation, without allowance for the existence of a second well^[5].

Yagere^[38] analyzed recently the data on the threshold reactions within the framework of the double-hump fission barrier model, with allowance for the change in the level density and the fission width in the second potential well. The results of this analysis show that the energies of the isomer levels of the americium isotopes turn out to be on the average 1 MeV lower than previously assumed, i.e., more readily closer to 2 MeV than to 3 MeV. This should apparently be valid also for isomers of other elements.

It can thus be assumed that the isomer-level data calculated from the results of measurements of the reactions thresholds are close to those obtained by analyzing the data on the subbarrier fission resonances.

SYSTEMATICS OF THE HALF-LIVES OF SPONTANEOUS ISOMER FISSION

By now the number of synthesized fissioning isomers is large enough to discern certain regularities in the variation of the half-lives of the spontaneous fission of isomers^[11]. Figure 14 shows the dependence of the half-lives of the spontaneous fission of isomers on the number of neutrons. Examination of this figure leads to several conclusions.

First, it is seen quite clearly that the probability of spontaneous fission of the isomers differs strongly for nuclei with even and odd numbers of nucleons. The isomers of odd-neutron isotopes have a large spontaneous fission half-life. This is clearly seen for the isomers of plutonium and americium. Second, one can note a decrease, in the mean, of the half-lives of the spontaneous fission of isomers on going to nuclei with

larger atomic numbers. Thus, for example, the half lives of the odd isomers of curium are approximately one-thousandth the half-lives of the odd isomers of plutonium, which has the same number of neutrons. As seen from Fig. 14, the isomer state of interest to us could not be observed in the case of the even-even curium isotopes. This is due most readily to the fact that the half-lives of the even-even isomers of curium are too small to be observed with the aid of the procedure employed so far.

Attention is called also to the fact that it was impossible to observe spontaneously fissioning isomers of neptunium and odd isomers of uranium. The only reasonable explanation that can be suggested at present is that in the case of the neptunium isotopes and the odd uranium isotopes, the shape isomers exist but decay mainly via an electromagnetic transitions to the principal potential well. This agrees qualitatively with the fission barriers calculated by Tsang and Nilsson^[39], according to whom the internal barrier becomes narrower for nuclei with $A < 242$.

As seen from Fig. 14, the half-lives of spontaneous fission of the americium isomers and of the odd isomers of plutonium vary smoothly from isotope to isotope, decreasing for the lighter isotopes. This, however, is not observed for the even-even isotopes of plutonium, where the spontaneous-fission half-lives of the isomers turn out to be practically equal.

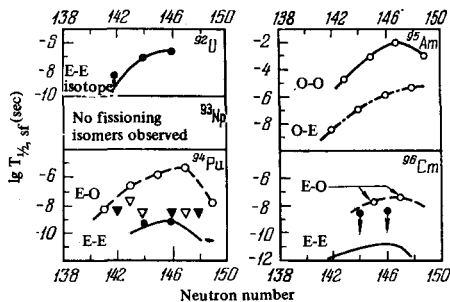


FIG. 14. Dependence of the half-lives of spontaneous fission of isomers on the number of neutrons.

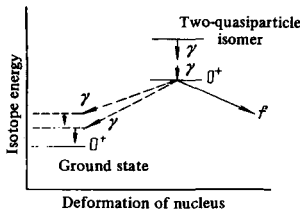


FIG. 15. Decay of two-quasiparticle isomer in the second potential well.

Table VI

State	Element	Number of neutrons								
		141	143	145	147	149	151	153	155	157
Ground	Uranium Plutonium Curium Californium Fermium	$4 \cdot 10^3$	10	$1.1 \cdot 10^5$						
Isomer	Plutonium Americium Curium	$8 \cdot 10^3$	$6 \cdot 10^3$	$6.5 \cdot 10^3$ $9 \cdot 10^3$ > 10	$1.5 \cdot 10^2$		$< 10^5$	$> 5 \cdot 10^2$	$1.5 \cdot 10^4$	$3.6 \cdot 10^5$

It was suggested that in the case of even-even (E-E) plutonium isomers one observes not only the ground states in the second potential well but also excited states ($\nabla, \blacktriangledown$) that are isomeric with respect to the lower state of the second well^[11,14]. For example, one cannot exclude the appearance in the second potential well of two-quasiparticle isomers whose spontaneous-fission half-lives are larger than in the lower state, owing to the addition of a certain energy to the barrier as a result of the breaking of the nucleon pair. In a certain sense, the structure of the fission barrier for two-quasiparticle states should be similar to the structure of the barrier of odd-odd (O-O) nuclei. In this case, a situation can arise (Fig. 15), in which the half-life of the isomer is determined by the time of the

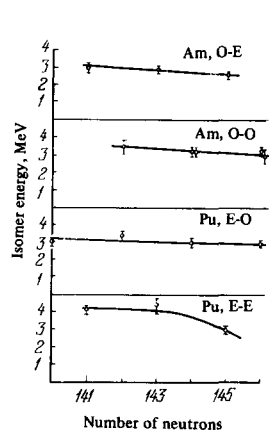


FIG. 16

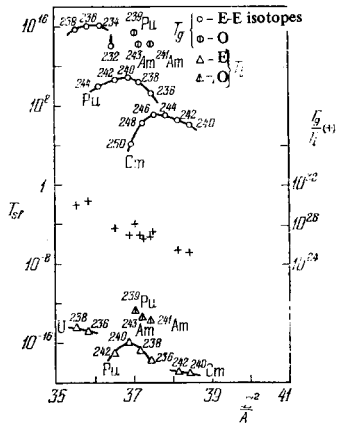


FIG. 17

FIG. 16. Dependence of the isomer-level energy on the number of neutrons.

FIG. 17. Spontaneous-fission half-lives of nuclei in the region U-Cm.

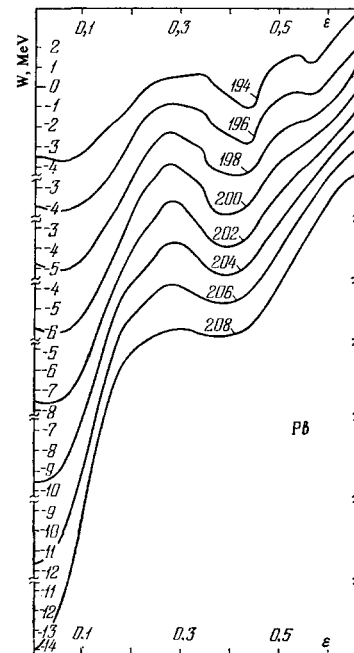


FIG. 18. Potential energy of lead isotopes.

electromagnetic transition to the lower ground state of the second well, with a very short half-life of spontaneous fission.

The spontaneous-fission half-lives $T_{1/2}$ of even-even isomers of plutonium and curium were estimated in^[11] under the assumption that the hindrance factor $h = T_{1/2}(Z, N)/T_{1/2}(Z, N - 1)$ for spontaneous fission, estimated from the data for the americium isomers, has approximately the same value for plutonium and curium isotopes with the same neutron number N . Table VI lists the values of h for certain isomers.

The only even-even plutonium isomer corresponding to the lower state of the second potential well is apparently, ^{240}Pu , with a half-life 4 nsec^[14]. This assumption seems quite reasonable if it is recognized that the energy of the isomer state of ^{240}Pu is approximately 800 keV lower than for the lighter even-even neighbors (Fig. 16).

Two isomer states of one and the same isotope exist also in isotopes with an odd number of nucleons. For example, it has been reliably established that there exist two fissioning isomers of ^{237}Pu , with half-lives 900 and 82 nsec^[11,40]. In this case we are apparently dealing with two states of close energy but differing in spin. The entire difference is due to the fact that the odd nucleon is on different orbitals.

When speaking of the spontaneous-fission half-lives of isomers, one can attempt to establish their correlation with the spontaneous-fission half-lives of the same isotopes in the ground state. Figure 17 shows a plot of the spontaneous-fission half-lives in the ground state T_g and in the isomer state T_i against Z^2/A for certain isotopes. This plot was constructed both from experimental data on the half lives and from calculated data (for the isomers) taken from^[11]. We see that the ratio of the half-lives fluctuates much less than the half lives themselves. One can also note a certain decrease of T_g/T_i with increasing Z^2/A .

SEARCHES FOR NEW REGIONS OF FISSIONING ISOMERS

The calculations of the shell corrections to the fission barrier^[10b,39] have led to the conclusion that fissioning isomers can occur in the region of lead (Fig. 18). Searches for short-lived spontaneously fissioning isomers of Pu, Rn, and other elements were therefore attempted in reactions with heavy ions.

After the first experiments by Ruddy et al.^[41], who bombarded gold and other neighboring elements with heavy ions, it was concluded that such isomers exist in the region of polonium, and moreover even in the region of the rare-earth elements. Recognizing that the fission barrier of the isotopes in the region of polonium, and especially in the rare-earth region, is high, it was necessary to assume very high energies for such states. The existence of fissioning isomers of polonium and of rare earths was so surprising, that very careful experiments were immediately performed to verify the observed phenomenon. The experiments were performed in Dubna^[42] and in Berkeley^[43]. Figure 19 shows a diagram of the Dubna experiments.

As seen from the experimental setup, the nuclei of the isotopes produced in interactions of heavy ions with the target, having acquired a large momentum, escape from the target, and if short-lived spontaneously-fissioning isomers were produced thereby, then tracks of fission fragments would be noted on the mica detector placed in the immediate vicinity of the target (the notation in Fig. 19 is as follows: 1-collimator; 2-cassette for target, 3-target, 4-annular mica detector for the delayed-fission fragments, 5-mica detector for prompt-fission fragments, 6-ion-current collector).

The geometry in the Berkeley experiment was practically the same, but surface-barrier silicon detectors were used in place of mica detectors.

Several series of experiments, in which different targets were bombarded with heavy ions, led both groups to the same conclusion, namely, that there are no spontaneously-fissioning isomers. In any case, it can be stated that the cross sections for the production of spontaneously-fissioning isomers of radon and of neighboring elements is less than 10^{-33} sec^2 according to the Dubna data and less than 10^{-31} cm^2 according to the Berkeley data. In both cases, these values are smaller by factors of hundreds and thousands than indicated by Ruddy et al.^[41]

Thus, the assumed existence of fissioning isomers of polonium and lighter elements was not confirmed, and the question whether shape isomers exist in this region of nuclei remains open.

FURTHER WAYS OF EXPERIMENTALLY INVESTIGATING SHAPE ISOMERS OF ATOMIC NUCLEI

When speaking of the present-day situation, one can state apparently that the available experimental data on spontaneously fissioning isomers and subbarrier fission resonances agree with the concept of the existence of a second potential well in the case of large nuclear deformation.

At the same time it must be admitted that we have so far obtained no direct experimental proof that the nuclei in the discussed isomer states are anomalously strongly deformed. If we admit of the existence of a second potential well on the fission barrier, then we can suggest a number of experiments in which one can obtain apparently certain information on the nuclear shape in the isomeric state.

It is quite attractive, above all, to obtain information on electromagnetic transitions preceding the population of the isomer state. For example, it would be very valuable to obtain data on the rotational band constructed on the isomer state. This would yield the moment of inertia of the nucleus in the isomer state²⁾.

By now, certain preliminary data were obtained^[44] on the emission of internal conversion electrons in the formation of the isomer- ^{236}U in the reaction $^{235}\text{U}(n, \gamma)^{236}\text{U}$. It would be of interest also to study in greater detail those isomers of the even-even pluton-

²⁾ Recently in Munich H. G. Specht et al., measuring the spectra of the conversion electrons in the reaction, obtained data pointing to the existence of a rotational $^{238}\text{U}(\alpha, 2n)^{238\text{m}}\text{Pu}$ band constructed on the isomer state. The moment of inertia for this band is approximately double the ordinary one.

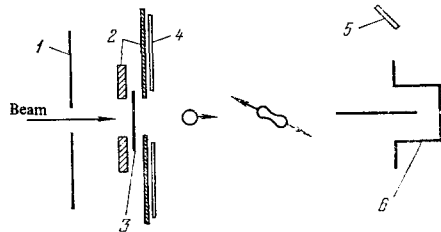


FIG. 19. Setup for the search for short-lived fissioning isomers [42].

ium isotopes which apparently are two-quasiparticle isomers (^{236}Pu , ^{238}Pu , ^{242}Pu).

If these are indeed two-quasiparticle isomers, then, as is usually the case for such isomers, electromagnetic transitions to lower terms of the rotational band will take place. Consequently, if we measure the spectrum of the γ quanta emitted in coincidence with fission fragments of the isomer, then we can hope to observe rotational levels of the isomer. As before, it is necessary to continue the search of the γ -decay mode of certain isomers. This pertains primarily to the isomers of uranium and neptunium.

Completely new approaches to the problem are uncovered when beams of negative muons are used. When speaking of the study of fissioning isomers with the aid muons, mention should be made of the nonradiative muon capture by heavy nuclei^[45]. It was established that for the heaviest elements the transition of the negative muon in a mesic atom from the 2P orbit to 1S is not accompanied in 25% of the cases by emission of an x-ray quantum, and the entire energy of the 2P \rightarrow 1S transition is transferred directly to the nucleus, which becomes excited approximately to 6.5 MeV. This energy suffices to cause fission, and this does indeed take place, for example, in ^{239}Pu (Fig. 20). The figure shows the exponential plot corresponding to the lifetime of the muon on the 1S orbit, and a certain excess of the number of fissions at the instant of the muon capture (the start of the time reckoning is evidence of nonradiative fission).

Nonradiative capture of negative muons should undoubtedly be accompanied by the formation of fissioning isomers, just as in the capture of slow neutrons. The probability of this process is undoubtedly also low, but nonetheless we see here ways of obtaining direct information on the nuclear shape, so that a study of the decay of fissioning isomers in the presence of a negative muon on the 1S orbit is worth while.

Figure 21 shows schematically the variation of the fission barrier of the nucleus in the presence of a muon. Owing to the strong Coulomb interaction between the muon and the nucleus, one can expect rather appreciable changes in the properties of the isomer. First, the energy of the isomer level increases somewhat. At the same time, a change takes place in the ratio of the fission probabilities and the radiative transition to the first potential well. A change takes place in the half-life of the shape isomer. Observation of the change in the lifetime of the isomer in the presence of a muon would be one of the proofs that shape isomerism comes into play in the case of fissioning isomers. Finally, speaking of further investigations

FIG. 20. Temporal distribution [46] of the fission act occurring when negative muons are stopped by a target of ^{239}Pu .

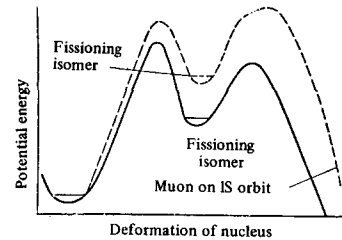
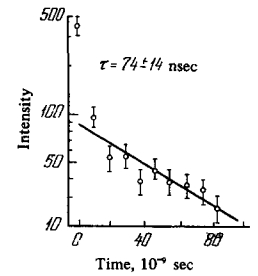


FIG. 21. Change of fission barrier in the presence of a muon on the 1S orbit.

of shape isomerism, we wish to note that a search for new region of shape isomerism remains as pressing as before.

If we turn to Fig. 18, which shows the results of the calculations by Pashkevich^[10D] for lead isotopes, then it becomes clear that even in the case of this element one can expect the appearance of shape isomers with sufficiently large excitation energies. It is difficult to state at present whether the isomer states with excitation energy exceeding the binding energy of the neutron will be sufficiently stable. If they turn out to be sufficiently stable, then the main type of their decay will most readily be emission of delayed neutrons, α particles, and γ quanta, and not fission. Of course, one can likewise exclude not a weak spontaneous-fission mode.

¹C. M. Polikanov et al., Zh. Eksp. Teor. Fiz. 42, 1464 (1962) [Sov. Phys.-JETP 15, 1016 (1962)].

²G. N. Flerov and S. M. Polikanov, Comptes Rendus de Congres International de Physique Nucleaire (Paris, 1964), vol. 1, P., Centre National de la Recherche Scientifique, 1964, p. 407.

³A. Fubini et al., Phys. Rev. Lett. 20, 1373 (1968); E. Migneco and J. P. Theobald, Nucl. Phys. A112, 603 (1968).

⁴G. N. Flerov, Yu. P. Gangrskii, B. N. Markov, A. A. Pleve, S. M. Polikanov, and H. Jungclaussen, Yad. Fiz. 6, 17 (1967) [Sov. J. Nucl. Phys. 6, 12 (1968)].

⁵S. Bjørnholm et al., Nucl. Phys. A95, 513 (1967).

⁶G. N. Flerov, A. A. Pleve, and S. M. Polikanov et al., ibid. A97, 444.

⁷V. M. Strutinsky, ibid. A95, 420 (1967); A122, 1 (1968).

⁸D. Hill and A. Wheeler, Phys. Rev. 89, 1102 (1953).

⁹P. Holzer et al., Nucl. Phys. A138, 241 (1969).

- ¹⁰V. V. Pashkevich, a) Symposium "Nuclear Structure," Dubna, 1968; Dubna JINR, Contribution 94; b) "Shells" in Nuclei with $Z > 80$, JINR Preprint R4-4383, Dubna, 1969.
- ¹¹S. M. Polikanov, and G. Sletten, Nucl. Phys. **A151**, 656 (1970).
- ¹²N. L. Lark et al., *ibid.* **A139**, 481 (1969).
- ¹³R. Repnov et al., *ibid.* **A147**, 183 (1970).
- ¹⁴R. Vandenbosh and K. L. Wolf, 2nd IAEA Symposium on Physics and Chemistry of Fission, Vienna, IAEA, 1969, p. 449.
- ¹⁵Yu. P. Gangrskii et al., Yad. Fiz. **11**, 54 (1970) [Sov. J. Nucl. Phys. **11**, 30 (1960)].
- ¹⁶S. C. Burnett et al., Phys. Lett. **B31**, 52[?] (1970).
- ¹⁷J. D. Jackson, Canad. J. Phys. **34**, 341 (1958).
- ¹⁸G. N. Flerov, A. A. Pleve, and S. M. Polikanov et al., Nucl. Phys. **A102**, 443 (1967); I. Boca, N. Martologu, M. Sezon, I. Vilkov, N. Vilkov, G. N. Flerov, A. A. Pleve, S. M. Polikanov, S. P. Tretiakova, *ibid.* **A134**, 541 (1969).
- ¹⁹Yu. P. Gangrskii, K. A. Gavrilov, et al., Yad. Fiz. **10**, 65 (1969) [Sov. J. Nucl. Phys. **10**, 38 (1970)].
- ²⁰P. Dalhsuren et al., Nucl. Phys. **A148**, 492 (1970).
- ²¹T. Nagy et al., Investigations of Fast-neutron Capture Reactions Leading to Spontaneously Fissioning Isomers ²⁴²Am and ²⁴⁴Am, JINR Preprint R7-5162, Dubna, 1970.
- ²²A. Y. Elwin and A. T. G. Ferguson, *cm.*¹⁴, p. 457.
- ²³I. Boca et al., A Study of the ^{238m1}U-Isomeric Fission through the ²³⁵U(n, γ)-Reaction in the Energy Range 0.25-4 Mev. Preprint comitetul pentru energia nucleara institutul de fizica atomica CRD-42-1970, Bucharest, 1970.
- ²⁴B. B. Back et al., *cm.*¹⁴, SM-122/74; H. C. Britt et al., *ibid.*, p. 375.
- ²⁵H. Y. Specht et al., *ibid.*, p. 363.
- ²⁶P. E. Vorotnikov, Yad. Fiz. **7**, 83 (1968) [Sov. J. Nucl. Phys. **7**, 60 (1968)].
- ²⁷S. M. Polikanov, Usp. Fiz. Nauk **94**, 43 (1968) [Sov. Phys.-Usp. **11**, 22 (1968)].
- ²⁸L. K. Peker, Izv. Akad. Nauk SSSR, ser. fiz. **28**, 298 (1964).
- ²⁹L. A. Sliv, Yu. I. Kharitonov, Zh. Eksp. Teor. Fiz. **46**, 811 (1964) [Sov. Phys.-JETP **19**, 553 (1964)].
- ³⁰Ya. B. Zel'dovich, ZhETF Pis. Red. **4**, 78 (1966) [JETP Lett. **4**, 53 (1966)].
- ³¹A. L. Malov, S. M. Polikanov, and V. G. Solov'ev, Yad. Fiz. **4**, 528 (1966) [Sov. J. Nucl. Phys. **4**, 376 (1967)].
- ³²G. N. Flerov and V. A. Druin, in: Struktura slozhnykh yader (Structure of Complex Nuclei) Gosatomizdat, 1966, p. 249.
- ³³M. Urin and D. Zaretski, Nucl. Phys. **75**, 101 (1966).
- ³⁴V. M. Strutinsky and S. Bjørnholm, *cm.*^{16a}, p. 431.
- ³⁵V. M. Strutinsky and H. C. Pauli, *cm.*¹⁴, p. 155.
- ³⁶B. T. Geilikman, Proc. of the Intern. Conference on Nuclear Structure (Kingston, Canada, 1960), Toronto, Univ. of Toronto Press, -Amsterdam, North Holland, 1960, p. 874.
- ³⁷E. Lynn, *see*¹⁴, p. 249.
- ³⁸S. Yâgere, Phys. Lett. **B32**, 571 (1970).
- ³⁹C. F. Tsang and S. G. Nilsson, Nucl. Phys. **A140**, 275 (1970).
- ⁴⁰P. A. Russo et al., Phys. Rev. **C3**, 1555 (1971).
- ⁴¹F. H. Ruddy and Y. M. Alexander, *ibid.* **187**, 1672 (1969).
- ⁴²G. N. Flerov et al., Searches for Spontaneously-fissioning Isomers with Nanosecond Lifetimes in Reactions with Heavy Ions, JINR Preprint R-5018, Dubna, 1970.
- ⁴³S. Bjørnholm et al., Nucl. Phys. **A156**, 561 (1970).
- ⁴⁴A. G. Belov et al., Yad. Fiz. **14**, 685 (1971) [Sov. J. Nucl. Phys. **14**, 385 (1972)].
- ⁴⁵D. F. Zaretsky and V. V. Novikov, Nucl. Phys. **28**, 177 (1961).
- ⁴⁶V. Cojocararu et al., Phys. Lett. **20**, 53 (1966).

Translated by J. G. Adashko