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INTERACTION OF INTENSE OPTICAL RADIATION WITH FREE ELECTRONS (NONRELATIVISTIC CASE)

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A review is presented of the status of problems involving the interaction of matter and optical-band electromagnetic radiation of intensity such that any medium becomes a fully ionized plasma. The following are considered within the framework of the single-electron approximation in the case of nonrelativistic energies: stimulated bremsstrahlung and absorption of an electron in the field of a strong electromagnetic wave, stimulated two-photon Compton scattering, and scattering of electrons in the field of an intense standing electromagnetic wave (the Kapitza-Dirac effect). The role played by these processes in the heating of plasma by laser radiation is analyzed, as is the question of the possibility of obtaining amplification (negative absorption) of light in transitions in a continuous spectrum. The bibliography is brought up to date to the middle of 1971.

CONTENTS

1. INTRODUCTION

In connection with the development of quantum electronics in the optical band (lasers), it became realistically possible to observe a number of new physical phenomena due to the interaction of sufficiently intense electromagnetic radiation with matter. A large class of such new effects as generation of harmonics of optical radiation, parametric interactions, various types of stimulated scattering of radiation, self-focusing of wave beams in a medium, etc. are presently being intensely studied both theoretically and experimentally, and constitute the scope of nonlinear optics. A characteristic feature of this class of effects is that in final analysis all are due to nonlinear (in the electromagnetic field) polarization of the medium. It is customarily assumed that the medium itself does not change its aggregate state during the course of interaction with the radiation, and serves only as a nonlinear converter of the radiation. From this point of view, such processes as optical breakdown in gases or damage in solids are usually considered to be secondary phenomena that lead to the loss of the nonlinear-optical properties of the medium. The assumption that the state of the medium remains unchanged determines also the features of the theoretical approach to nonlinear-optics problems and is manifest in the fact that all can be described within the framework of macroscopic electrodynamics, i.e., with the aid of Maxwell's equations supplemented by nonlinear material equations.

Closely related to nonlinear optics is a more recent and much less developed field, also resulting from the appearance of coherent sources of optical radiation, namely nonlinear spectroscopy[1)]. Research in this field is aimed at studying the absorption spectra of intense optical radiation in gases, liquids, and solids, when an important role is played by multiphoton and multistage single-photon absorption mechanisms, and also the spectroscopic saturation effect and the high-frequency Stark effect.

As already noted, the phenomena investigated in nonlinear optics and in nonlinear spectroscopy are usually considered under conditions when the state of the medium does not change significantly during the time of interaction with the radiation. This leads to definite limitations on the intensity I of the external electromagnetic wave.

The condition that the aggregate state of the medium be constant certainly ceases to hold if the amplitude E_0 of the field intensity of the wave becomes comparable with the intensity of the intraatomic field E_a:

$$E_0 \sim E_a \approx m^{1/2}\Delta^{3/2}(e\hbar)^{-1},$$

where Δ is the binding energy of the external electron in the atom, e and m are the charge and mass of the electron, and $\hbar$ is Planck's constant. For example, for the first Bohr orbit of the hydrogen atoms we have $E_a \approx 5\times 10^9$ V/cm. The radiation intensity corresponding to the intensity E_a is equal to

$$I_a \approx c\,(4\pi)^{-1} m\Delta^3/e^2\hbar^2 \qquad (1.1)$$

(c is the speed of light). At such large radiation fluxes, any material medium loses its individuality and is rapidly converted into a fully ionized plasma[1-3]. For a typical atom first-ionization energy $\Delta \approx 10$ Ev, the intensity is $I_a \approx 5\times 10^{16}$ W/cm^2.

Actually, fast ionization of the atoms can occur also in fields much weaker than intraatomic, $E_0 \ll E_a$. The reason is that in the optical band we have $\omega \ll \Delta/\hbar \equiv \omega_a$. In this case, according to[1a,2], if the condition[2)]

[1)]This term is apparently not yet universally accepted, but it seems to us to reflect correctly the physical gist of the investigated phenomena.

[2)]The physical meaning of condition (1.2) is that the period ω^{-1} of the wave is large in comparison with the tunneling time of an electron of velocity $\sim(\Delta/m)^{1/2}$ through a potential barrier of width $\sim(\Delta/eE_0$. In this

$$E_0 \gg (\omega/\omega_a)E_a \approx \omega\, (m\Delta)^{1/2}e^{-1} \qquad (1.2)$$

is satisfied, the ionization of the atoms is described by Oppenheimer's tunnel formula[4]

$$w \sim \omega_a\, (E_a/E_0)^{1/2} \exp\,(-E_a/E_0), \qquad (1.3)$$

where w is the probability of ionization of the atom per unit time.

Under the typical conditions $\omega/a_a \sim 0.1$ and at an intensity

$$E_0 \approx (\omega/\omega_a)\, E_a \qquad (1.4)$$

formula (1.3) leads to a degree of photoionization $\sim 1\%$ within a period ω^{-1}. With increasing radiation intensity, however, the probability w increases exceedingly rapidly (the degree of ionization over the period increases to 100% when the intensity is increased by only a factor of 3). Thus, it can be assumed that rapid ionization of the atoms begins with an intensity $I \sim I_c$, where I_c corresponds to the field (1.4):

$$I_c \approx mc\omega^2\Delta/4\pi e^2 \approx (\omega/\omega_a)^2\, I_a.$$

For radiation from a neodymium glass laser (λ = 106 μ) and a ruby laser (λ = 0.69 μ) at Δ = 15.6 eV (N_2) we have respectively $I_c = 8 \times 10^{13}$ W/cm^2 and $I_c = 2 \times 10^{14}$ W/cm^2.

At intensities higher than critical, $I > I_c$, the atoms of the medium becomes ionized within times on the order of the period of the wave, and any substance is converted into a plasma regardless of its initial state. Consequently, at such high intensities the radiation interacts with a plasma practically during the entire pulse duration. Of course, processes analogous to those considered in nonlinear optics can occur in this case, e.g. one can have stimulated scattering by different types of natural oscillations of the plasma, self-focusing, etc. In this intensity region, however, great interest attaches apparently at the present time to the investigation of the mechanism of absorption of powerful radiation. In such strong fields (when $I > I_c$), the absorption coefficient can become (nonlinearly) dependent on the field intensity. To solve problems of this type, the phenomenological theory is no longer sufficient, and it is necessary to use a microscopic approach. Related to this region are also a number of problems involving the scattering of an electron beam in the presence of an intense electromagnetic field. It is precisely to this group of problems that the present review is devoted. In the main, we shall not deal here with collective effects that arise in the plasma under the the influence of a strong radiation field, and consider only the interaction between the field and free electrons.

At the present state of development of quantum electronics, it is quite feasible to obtain in experiments intensities on the order of I_c, or even on the order of I_a. Such intensities are easiest to realize with picosecond pulses from mode-locked neodymium-glass or ruby lasers. Moreover, the use of this laser regime together with specially developed focusing systems makes it possible to obtain even larger optical-radiation intensities, and one can hope to realize in experiments in the nearest future fluxes (intensities) $I \sim 10^{18}$–10^{20} W/cm^2. Such intensities will uncover new experimental possibilities in the study of the interaction of radiation with matter. The energy of the electron oscillations in the field of the wave then becomes comparable with the electron rest energy or, equivalently, the following condition is satisfied

$$eE_0/m\omega c \gtrsim 1.$$

The corresponding threshold (relativistic) intensity is

$$I_{\rm rel} = m^2\omega^2c^3/4\pi e^2 = (mc^2/\Delta)\, I_c.$$

The values of I_{rel} for neodymium and ruby laser emission are 2×10^{18} and 6×10^{18} W/cm^2, respectively.

At intensities $I > I_{rel}$, a number of new problems arise, e.g., the Compton scattering in a strong radiation field, and others. These questions are discussed in sufficient detail in the review[5a] and in the book[6], and will not be considered here.

Confining ourselves to intensites $I < I_{rel}$, we consider below the following questions: a) stimulated bremsstrahlung and absorption (linear and non-linear); b) stimulated two-photon Compton scattering; c) electrons scattered in the field of an intense standing wave (the Kapitza-Dirac effect).

Everywhere with the exception of Sec. a of Chap. 3, the electron translational motion is assumed to be nonrelativistic, and the quantum energy is assumed to be $\hbar\omega \ll mc^2$.

2. STIMULATED BREMSSTRAHLUNG AND ABSORPTION

When an electron is scattered by another particle (atom, ion, nucleus) in the presence of an electromagnetic wave of frequency ω, induced emission or absorption of one or several quanta $\hbar\omega$ is possible besides the spontaneous emission of a quantum $\hbar\omega_{sp} \le \epsilon$ (ϵ is the electron energy prior to scattering). Such electron transitions are usually referred to as stimulated bremsstrahlung and absorption (one- or multiquantum). To shorten the notation, we shall henceforth use also the term "stimulated bremsstrahlung effect" (SBE) to denote the entire aggregate of such processes.

The SBE was considered, of course, even before the appearance of quantum electronics, e.g., in the solution of the problem of radiation absorption in an electron plasma as a result of collisions. Quantum electronics has introduced two new aspects in the study of the SBE. First, the interest in the SBE is connected with searches for the possibility of obtaining negative absorption in transitions between states of a continuous electron energy spectrum ("free-free" transitions). An advantage of such systems is the absence of any limitations whatever on the frequencies of the emitted photons. In addition, in the case of transitions in the continuous spectrum it is relatively easy to solve the problem of inverting the population of the energy states. For example, a monoenergetic electron beam is in itself a system with inverted population relative to the entire region of the spectrum with lower energy. The conditions for negative absorption are determined in this case by the character of the interaction between

case the amplitude of the electron oscillations $\sim eE_0/m\omega^2$ is also much larger than the width of the barrier, and the amplitude of the oscillation velocity exceeds the intraatomic velocity of the electron.

the electrons and the external field (the electromagnetic and static fields of the scattering center).

Second, the development of high-power lasers has made it possible to observe multiphoton SBE, and at sufficiently high optical-radiation intensities I, the multiphoton SBE becomes the principal (nonlinear) mechanism of its absorption in the plasma.

In this chapter we examine the SBE from these two physical points of view.

a) **Negative absorption in free-free transitions.** We shall analyze the conditions of negative absorption of radiation in SBE as applied to a system of electrons scattered by heavy particles (atoms, molecules, or ions). In the general case, such an analysis calls for knowledge of the cross sections for the single-photon SBE. If $\sigma_{e,a}$ are the cross sections of stimulated emission (absorption) of a quantum $\hbar\omega$ in an electromagnetic field of intensity I, occurring when an electron is scattered by a force center, then the absorption coefficient α in the system under consideration is given by

$$\alpha = (N_e N \hbar\omega / I)\, \langle v(\sigma_a - \sigma_e) \rangle, \qquad (2.1)$$

where N_e and N are the average densities of the electrons and of the scattering centers in the system, v is the absolute value of the electron velocity, and the angle brackets denote averaging over the electron velocity distribution. The cross sections $\sigma_{e,a}$ depend both on the absolute value of the electron velocity v (prior to scattering) and on its direction relative to the polarization of the electromagnetic field. It will henceforth be assumed throughout that the radiation is linearly polarized and that the field of the scattering center has central symmetry. Then the cross sections depend not only on v, but also on the angle θ between the radiation polarization vector **e** and the direction $\mathbf{n} = \mathbf{v}/v$ of the electron velocity prior to scattering.

In the simplest case of isotropic distribution of electron velocity, an analysis of the conditions for negative absorption becomes much simpler. The first analysis applicable to this case was carried out in[7,8] (see also the pertinent papers in[9]). In this case (2.1) takes the form

$$\alpha = 4\pi N\hbar\omega I^{-1}\Big[\int\limits_0^\infty v^3\tilde{\sigma}_a(v)\, f(v)\, dv - \int\limits_{(2\hbar\omega/m)^{1/2}}^\infty v^3\tilde{\sigma}_e(v)\, f(v)\, dv\Big];$$

here

$$\tilde{\sigma}_{e,a}(v) = (4\pi)^{-1}\int \sigma_{e,a}(v,\theta)\, d\Omega = 0{,}5\int\limits_0^\pi \sigma_{e,a}(v,\theta)\sin\theta\, d\theta$$

are the SBE cross sections averaged over the angle variables and f(v) is the isotropic electron velocity distribution function, normalized by the condition

$$4\pi\int\limits_0^\infty v^2 f(v)\, dv = N_e. \qquad (2.2)$$

The average cross sections $\tilde{\sigma}_{e,a}(v)$ are convenient because, on the one hand, they satisfy the "integral" detailed balancing principle[10a]:

$$v^2\tilde{\sigma}_a(v) = w^2\tilde{\sigma}_e(w), \qquad (2.3)$$

$$w^2 = v^2 + (2\hbar\omega/m), \qquad (2.4)$$

and on the other hand they satisfy the Einstein relation between the coefficients A and B

$$\tilde{\sigma}_e(v) = n_c\sigma_{sp}(v,\omega)\,(\Delta\Omega/4\pi)\,\Delta\omega = (\pi^2c^2/\hbar\omega^3)\, I\sigma_{sp}(v,\omega), \qquad (2.5)$$

where $n_q = (4\pi^3c^2I/\hbar\omega^3\Delta\omega\Delta\Omega)$ is the average number of quanta for field oscillator ($\Delta\omega$ and $\Delta\Omega$ are respectively the width of the frequency and angular spectra of the linearly-polarized radiation with total intensity I), and $\sigma_{sp}(v,\omega)d\omega$ is the cross section for spontaneous emission, into a solid angle 4π, of a photon with a frequency in the interval $(\omega, \omega + d\omega)$, upon scattering of an electron of velocity v.

On the basis of (2.3) and (2.5), formula (2.1) takes the form

$$\alpha = -4\pi^3c^2N\omega^{-2}\int\limits_0^\infty vw^2\sigma_{sp}(w,\omega)\,[f(w) - f(v)]\, dv.$$

From this expression, when account is taken of (2.4), we see that to satisfy the condition $\alpha < 0$ it is necessary to satisfy the inequality $df/dv > 0$ in a certain finite interval of the velocities v. Obviously, this velocity interval has an inverted electron population (a "negative temperature")[3], and the physical meaning of the condition $\alpha < 0$ reduces to the fact that a velocity interval with "negative temperature" ($df/dv > 0$) makes a larger contribution to the absorption at the frequency ω than an interval with "positive temperature" ($df/dv < 0$).

The indicated condition for negative absorption is, generally speaking, only necessary but not sufficient, and it is possible to find one more independent necessary condition for $\alpha < 0$. The latter is easiest to find in the classical limit of low frequencies, when $\hbar\omega \ll \epsilon$, a condition that can usually be regarded as satisfied in the optical band, and all the more in the radio band. In this case a universal connection exists between the spontaneous emission cross section $\sigma_{sp}(v,\omega)$ and the transport cross section $\sigma_{tr}(v)$ of elastic scattering of an electron[10b] (see also[11], Chap. V, Sec. 2a)

$$\sigma_{sp}(v,\omega) = (4e^2v^2/3\pi\hbar\omega c^3)\,\omega^2\sigma_{tr}(v)\,[\omega^2 + \nu^2(v)]^{-1}, \qquad (2.6)$$

where $\nu(v) = Nv\sigma_{tr}(v)$ is the frequency of the elastic collisions of the electrons with scattering centers. After substituting (2.6) in (2.7) we obtain, accurate to first order in $\hbar\omega/\epsilon$, the known kinetic-theory expression for the absorption coefficient of a plasma with an isotropic electron distribution (see e.g.,[12])

$$\alpha = -(16\pi^2e^2/3mc)\int\limits_0^\infty [\omega^2 + \nu^2(v)]^{-1}\nu(v)\, v^3\,(df/dv)\, dv. \qquad (2.7)$$

The already mentioned second necessary condition $\alpha < 0$ is obtained when (2.7) is integrated by parts. It reduces obviously to the requirement that the inequality[4]

$$\frac{\partial}{\partial v}\Big[\frac{\nu(v)\, v^3}{\omega^2 + \nu^2(v)}\Big] < 0 \qquad (2.8)$$

be satisfied in a certain finite velocity interval. Satisfaction of this condition is determined completely by the character of the elastic scattering of the electron.

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[3]When $df/dv > 0$, the electron energy distribution function increases more rapidly than $\epsilon^{1/2}$.

[4]It is assumed here, naturally, that the function $f(v)\nu v^3/\omega^2 + \nu^2)$ vanishes at $v = 0$ and $v = \infty$.

For the case of a strongly ionized plasma, when electron-ion collisions are decisive (see, e.g.,[12]), we have

$$\nu(v) = N v \sigma_{tr}(v) = [4\pi Z^2 e^4 N_i/(m^2 v^3)]\, L(v), \qquad (2.9)$$

where $L(v)$ is the Coulomb logarithm, which is a monotonically increasing function of v.[5] We see therefore that condition (2.8) cannot be satisfied[6]. Accordingly, negative absorption of the radiation is impossible in a strongly ionized plasma with an arbitrary isotropic electron velocity distribution[7].

In the case of a weakly ionized plasma, when the collisions between the electrons and the neutral particles are decisive, it is impossible to obtain a similar universal answer. It can only be stated that at high radiation frequencies $\omega \gg \nu(v)$ the negative absorption is apparently likewise impossible. Indeed, in this case we have $\nu v^3/(\omega^2 + \nu^2) \sim \sigma_{tr}(v) v^4$, and this function has no negative slope for all the gases investigated to date. Violation of this property of the cross sections is physically unlikely.

To the contrary, at sufficiently low frequencies, when $\omega \ll \nu(v)$, we have $\nu v^3/(\omega^2 + \nu^2) \sim v^2/\sigma_{tr}(v)$, and for gases that exhibit a strong Ramsauer effect the condition (2.8) may turn out to be satisfied in a certain finite velocity interval. Consequently, in such gases, at definite electron distributions $f(v)$, negative absorption of sufficiently low-frequency radiation is possible as a result of the bremsstrahlung effect[8]. This phenomenon was apparently observed experimentally in[13], where a "decreasing section" was registered in the current-voltage characteristics of gas-discharge tubes filled with heavy noble gases exposed to ultraviolet radiation. Observation of an appreciable gain due to this effect at a frequency 60 MHz in a xenon gas-discharge plasma was reported in[14].

The rather limited possibilities of obtaining negative absorption in a plasma with an isotropic electron velocity distribution have the physical explanation, of course, that the state of such a plasma is already sufficiently close to equilibrium, since in such a plasma there is no directed electron motion relative to the radiation polarization. In the presence of such a motion, the situation can be appreciably changed. Let us consider first the case of the simplest non-isotropic distribution, when $f(v) = N_e \delta(v - v_0)$, i.e., the case of scattering of a monoenergetic beam of electrons of velocity v and density N_e. Then (2.1) takes the form

$$\alpha = (N_e N \hbar\omega/I)\, v_0 \sigma_t(v_0, \theta), \qquad (2.10)$$

where

$$\sigma_t(v, \theta) = \sigma_a(v, \theta) - \sigma_e(v, \theta)$$

is the total cross section of the bremsstrahlung photon absorption. An analysis of the conditions of negative absorption is determined in this case by the sign of the cross section $\sigma_t(v, \theta)$ as a function of the angle θ and the velocity v_0. This question in such a formulation was first investigated in[15], where the cross sections of single photon SBE were calculated for scattering of a nonrelativistic electron by fewer Coulomb and screened Coulomb centers.

We confine ourselves to the scattering of electrons by a pure Coulomb center in the Born approximation, i.e., under the condition[10a]

$$Ze^2/\hbar v_0 \ll 1. \qquad (2.11)$$

In the bremsstrahlung-effect problem, this condition should be satisfied for both values of the electron velocity, before and after the scattering. To calculate the emission cross sections σ_e, this condition must therefore be supplemented by the condition

$$2\xi \equiv \hbar\omega/0{,}5 m v_0^2 \ll 1. \qquad (2.12)$$

The sought cross section $\sigma_{e,a}$ are calculated in second order of perturbation theory (in first order in the radiation field and in first order in the Coulomb potential), and the initial and final wave functions of the electron must be chosen to be the wave functions of the free electron, i.e., plane waves. The summation over the intermediate states can be easily realized because the momentum conservation law holds. As a result we obtain the following expression for the differential cross sections of the SBE[15]

$$d\sigma_{a,e}/d\Omega = 2\pi Z^2 e^6 I\,(1 \pm 2\xi)^{1/2}\,[\mathbf{ne} - (1 \pm 2\xi)^{1/2}\,\mathbf{n'e}]^2 \times \{cm^3 (\hbar\omega)^2 v_0^2 \omega^2 [1 \pm \xi - (1 \pm 2\xi)^{1/2}\,\mathbf{nn'}]^2\}^{-1}; \qquad (2.13)$$

here $\mathbf{n}$ and $\mathbf{n}'$ are unit vectors in the directions of the velocities of the incident and scattered electrons, $\mathbf{e} = \mathbf{E}_0/E_0$ is the unit vector of the polarization of radiation with intensity $I = cE_0^2/8\pi$, and $d\Omega$ is a solid-angle element in the direction of the vector $\mathbf{n}'$.

For the emission cross section $d\sigma_e$, formula (2.13), strictly speaking, is valid only if condition (2.12) is satisfied. This, however, is precisely the region (of sufficiently soft quanta and high electron energies $\epsilon_0 = mv_0^2/2$) where the emission and absorption cross sections can be comparable in magnitude, and consequently, one can raise the question of negative absorption. We shall therefore assume $\xi \ll 1$ from now on.

On the basis of (2.13), in the scattering of an electron beam by Coulomb centers with density N_i, expression (2.10) for the absorption coefficient α takes the form

$$\alpha = (2\pi Z^2 e^6 N_i N_e/cm^3 v_0^3 \omega^2 \xi) \int d\Omega\, [(1+2\xi)^{1/2} \times \{[\mathbf{ne} - (1+2\xi)^{1/2}\,\mathbf{n'e}]\,[1+\xi-(1+2\xi)^{1/2}\,\mathbf{n'n}]^{-1}\}^2 - (1-2\xi)^{1/2}\,\{[\mathbf{ne} - (1-2\xi)^{1/2}\,\mathbf{n'e}]\,[1-\xi-(1-2\xi)^{1/2}\,\mathbf{nn'}]^{-1}\}^2].$$

Integrating over the solid angles $d\Omega$ and using then the smallness of ξ, we obtain ultimately

$$\alpha = (N_i N_e \hbar\omega v_0/I)\,\sigma_t(v_0, \theta) = (16\pi^2 Z^2 e^6 N_i N_e/cm^3 \omega^2 v_0^3)\,[2\cos^2\theta - (3\cos^2\theta - 1)\ln(2/\xi)]. \qquad (2.14)$$

We see therefore that negative absorption is possible for the considered simplest non-isotropic electron velocity distribution (a monoenergetic beam) if

$$\cos\theta > \{\ln(2/\xi)\,[3\ln(2/\xi) - 2]^{-1}\}^{1/2} \approx 1/\sqrt{3}, \qquad (2.15)$$

[5] The Coulomb logarithm $L(v)$ is defined by the formula [12] $L(v) = \ln[1 + (2p_{max}/p_{min})^2]^{1/2} = \ln[1 + \cot^2(\theta_{min}/2)]^{1/2}$, where p_{max} and p_{min} are respectively the maximum and minimum impact parameters, and θ_{min} is the minimum electron scattering angle. Only the parameter $p_{min} = \max\{2Ze^2/mv^2, h/mv\}$ depends on the velocity. The parameter p_{max} can be regarded equal to $\{v_T/\omega_p, v_T/\omega\}$, where $v_T = (3T_e/m)^{1/2}$ is the average thermal velocity of the plasma electrons and $\omega_p = (4\pi e^2 N_e/m)^{1/2}$ is the Langmuir (plasma) frequency.

[6] When $\omega \gg \nu$ and $\omega \ll \nu$, this is immediately evident from the fact that in this case we have $\nu v^3(\omega^2 + \nu^2)^{-1} \sim L(v)$ or $\sim v^6/L(v)$ respectively.

i.e., if the electron velocity vector $\mathbf{v}$ lies inside a cone whose axis coincides with the polarization direction $\mathbf{e}$ of the electric field, and whose generatrices are inclined to the axis at an angle

$$\theta_0 = \arccos\{\ln(2/\xi)[3\ln(2/\xi) - 2]^{-1}\}^{1/2} \approx \arccos(1/\sqrt{3}) \approx 55^\circ.$$

At a specified velocity v_0, the quantity $-\alpha(v_0, \theta)$ is maximal at $\theta = 0$, i.e., when the velocity of the scattered electrons is parallel to the polarization of the electric field of the radiation. The maximum gain, according to (2.14)[7)], is

$$-\alpha_0 = (32\pi^2 Z^2 e^6 N_i N_e / cm^3 v_0^3 \omega^2)[\ln(2/\xi) - 1] = (8\pi^2 Z^2 e^5 N_i j / cm\varepsilon_0^2 \omega^2)[\ln(2/\xi) - 1], \quad (2.16)$$

where $j = N_e v_0 e$ is the density of the current in the beam, and $\epsilon_0 = mv_0^2/2$.

Thus, in the presence of directed motion of the electrons relative to the radiation polarization, negative absorption in the SBE is possible also for Coulomb collisions. Numerical estimates show, however, that the gain attained in this case is quite small in the optical band. Only for the far infrared and the microwave bands can one hope to obtain an appreciable gain. Thus, at $\omega \approx 10^{13}\ \text{sec}^{-1}$ (wavelength $\lambda \approx 2 \times 10^{-2}$ cm), an electron velocity $v_0 \approx 10^8$ cm/sec, a solid-state ion density $N_i = 10^{22}\ \text{cm}^{-3}$, $N_e = 10^8\ \text{cm}^{-3}$, and $Z = 1$ an estimate in accordance with (2.21) yields $|\alpha_0| \approx 1\ \text{cm}^{-1}$. In such an estimate it is borne in mind that the electron beam interacts with ions of the solid or the liquid, and the absorption of the electromagnetic field at the frequency ω is assumed to be sufficiently weak.

The latter condition is usually not satisfied in interactions between an electron beam and a plasma. If the electron concentration of the plasma greatly exceeds the concentration of the particles in the beam, then bremsstrahlung absorption by the plasma electrons prevails over the amplification by the beam electrons. One can therefore speak of a possible negative absorption due to the SBE in a plasma only if a large number of electrons take part in the directional motion. In this connection, certain interest attaches to[16], where the possibility of negative absorption due to the SBE in a plasma with electron drift is predicted. The electron velocity distribution function is in this case

$$f(v) = N_e (\beta/\pi)^{3/2} \exp[-\beta(\mathbf{v} - \mathbf{v}_d)^2], \quad (2.17)$$

where $\mathbf{v}_d$ is the drift velocity. It is assumed that its direction coincides with the direction of the polarization vector $\mathbf{e}$. The electron drift can be produced by placing the plasma in an external constant electric field parallel to the vector $\mathbf{e}$.[8)]

On the basis of the foregoing analyses one should expect negative absorption in a plasma with distribution (2.17) at $v_d \gtrsim (T_e/m)^{1/2}$ and at frequencies $\omega \ll mv_d^2/\hbar$, for in this case, obviously, condition (2.15) with the parameter $\xi \ll 1$ is satisfied for most plasma electrons. It is further evident that when v_d increases the gain should tend to zero like v_d^{-3} in accordance with (2.16) ($v_0 = v_d$), and consequently, the negative absorption should have a maximum at a definite ratio v_d/v_T. A rigorous kinetic analysis of this question, carried out in[16], confirms these qualitative deductions. In the cited reference, an expression is derived for the absorption coefficient of a plasma with electron distribution (2.17), which can be represented in the form

$$\alpha = \alpha_{0T}\, Y(v_d, v_c, T_e), \quad (2.18)$$

where α_{0T} is the absorption coefficient of an equilibrium isotropic plasma[12],

$$Y(v_d, v_c, T_e) = (x\eta)^{-1}(1 + x^{-2}) \exp[-(x^2 + \eta^2)]\,\mathrm{sh}(2x\eta) - x^{-3} \int_{-x+\eta}^{x+\eta} \exp(-t^2)\,dt, \quad (2.19)$$

and $x = \beta^{1/2} v_d$ and $\eta = \beta^{1/2} v_c$ are parameters. The physical meaning of the velocity v_c is that at electron velocities $v \geq v_c$ the frequency of the electron-ion collisions is $\nu(v) \ll \omega$ ($\nu(v)$ is given by (2.9)). The results (2.18) and (2.19) pertain to the case when $v_c < v_T$, i.e., when the condition $\nu \ll \omega$ is satisfied for most plasma electrons. Figure 1 shows a family of plots of the function $Y(v_d, v_c, T_e)$ against the ratio v_d/v_T at different values of the parameter v_c/v_T. We see that at $v_d > 0.8\, v_T$ the function Y, and consequently also the absorption coefficient α, becomes negative, and the behavior of these two quantities as functions of the ratio v_d/v_T depends little on v_c/v_T. The maximum of the negative absorption sets in at $v_d \approx 1.3 v_T$ ($x \approx 1.8$), with a gain $(-\alpha_{max}) \approx 0.1\alpha_T$. With further increase of v_d, the coefficient α tends to zero (like v_d^{-3} at $x \gg 1$)[9)].

For radiation with wavelength $\lambda = 100\ \mu$ and for a plasma with temperature $T_e = 1$ eV and density $N_i = N_e = 10^{17}\ \text{cm}^{-3}$ ($Z = 1$), the maximum gain is approximately $1\ \text{cm}^{-1}$. This estimate shows that the considered system makes it possible, in principle, to obtain appreciable gains in the far IR, but the problem of

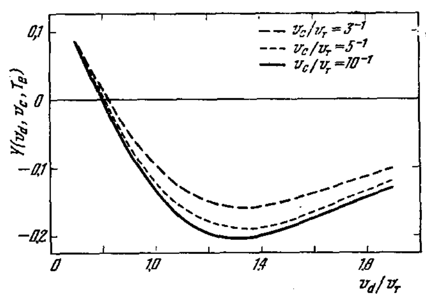

FIG. 1. Absorption coefficient referred to the absorption coefficient of the equilibrium plasma, $\alpha_T/\alpha_{0T} = Y(v_d, v_c, T_e)$ vs the ratio of the drift and thermal velocities v_d/v_T.

7) If the Born-approximation condition (2.11) is not satisfied, then according to [15] the expression for $-\alpha_0$ differs from (2.16) in that unity in the brackets is replaced by the term $xe^{-x}/(1-e^{-x})$, where $x = 2\pi Ze^2/\hbar v_0$.

8) It is known [17a] that the electron drift cannot be stationary in this case, since generally speaking the velocity v_d and the electron temperature $T_e = m/2\beta$ increase with time. In a weak electric field, however, this growth may not be noticeable in practice; in a strong field, the growth of the drift velocity ("electron whistler") can be compensated for by adding to the plasma a neutral gas with sufficiently high atom ionization energy. This difficulty is eliminated to a considerable degree under pulsed operation.

9) If the electron drift is due to a constant electric field, then according to [17a] the conditon $x \gtrsim 1$ can be reached only when the field intensity exceeds the critical value $E_c = m\beta^{-1/2}\nu_{ei}/e = 2\pi Z^2 e^3 N_i L/T_e$. In a fully ionized plasma, the bulk of the electrons is then subject to the "electron whistler" process (see the preceding footnote).

practical realization of such systems encounters certain difficulties connected with the production of rapid electron drift (see the last two footnotes). In addition, the foregoing analysis ignores completely the possible onset of plasma instability, which in practice can greatly hinder the practical realization of the model in question.

At the same time, the analysis presented in this section shows that it is possible in principle to use the SBE to develop media with negative absorption of radiation. It is obvious that the results obtained above pertain, with certain stipulations, also to the scattering of electrons by impurities in a solid, and by the same token point to the possibility of obtaining a negative absorption by free carriers in semiconductors[15,16]. Insofar as we know, these possibilities have not yet been realized in the laboratory, although it is not excluded that they play a definite role under cosmic conditions.

We note in conclusion that SBE in scattering of relativistic electrons was investigated in[18a,c], where the conditions for negative absorption were analyzed for the case of scattering by isolated ions (in a plasma), and also by the crystal lattice points in an ionic crystal.

b) Bremsstrahlung and absorption in a strong electromagnetic field. The results described in the preceding section pertain to the case when the radiation field intensity I is low enough. In the language of quantum theory, this condition means that in a single act of electron scattering by a scattering center an important role is played only by single-photon SBE: the cross sections $\sigma_{e,a}^{(n)}$ of multiphoton processes, when two, three, etc. photons are absorbed or emitted in a single act, are small in comparison with the cross section $\sigma_{e,a}^{(n)}$ given by (2.13). In classical theory, weakness of the radiation field in SBE processes means that the amplitude of the electron vibrational velocity v_E in the wave field, is small in comparison with its translational velocity v, i.e., that

$$v_E \equiv eE_0/m\omega \ll v. \tag{2.20}$$

As applied to processes in a plasma, the velocity v should be taken to mean the average thermal velocity of the electron $v_T = (3T_e/m)^{1/2}$; the quantity $eE_0/m\omega$ has then the meaning of the vibrational velocity of the electron, naturally only if the radiation frequency ω is large in comparison with the effective frequency of the collisions between the electrons and the plasma particles. Since we are dealing with the optical band, we shall assume the last condition to be satisfied in all cases.

If (2.20) is not satisfied, then the relative velocity of the scattered electron and of the scattering center, and consequently also the effective frequency of their collisions in the plasma, begin to depend on the wave amplitude E_0. This in turn causes the coefficient α of plasma absorption due to the SBE to become dependent on the radiation intensity I, i.e., the absorption becomes nonlinear. This nonlinearity should become particularly pronounced in the case of a strongly ionized plasma, owing to the strong dependence of the Coulomb-collision cross section on the relative particle velocity. The foregoing considerations enable us to predict directly in this case that the absorption coefficient α will depend on the intensity I in the limit of large I, when $v_E \gg v_T$. Indeed, according to (2.9), one should expect ν_{eff}, and consequently also α, to be proportional in this case to $v_E^{-3} \sim E_0^{-3} \sim I^{-3/2}$. Thus, the absorption of the radiation in a strongly ionized plasma, due to the SBE, should decrease with increasing intensity. This conclusion is important, in particular, for the problem of high-temperature heating of plasma by laser radiation.

The condition (2.20) is certainly not satisfied in a plasma produced when the medium is irradiated with light of intensity $I > I_c$, for in this case (in accordance with the definition of I_c, see Chap. 1) we have $eE_0/m\omega > (\Delta/m)^{1/2} \sim v_T^{(0)}$, where $v_T^{(0)}$ is the average velocity of the plasma electrons during the initial stage of irradiation (at a time on the order of ω^{-1} after the start of the irradiation). It must be emphasized that the condition "at the initial stage of the irradiation" is a particular manifestation of the general condition that the plasma irradiation regime be nonstationary, which is the only case when the condition (2.20) can be violated. In the stationary irradiation regime, owing to the plasma heating, we have (see[12] and the review[19])

$$v_T(E_0) = (3T_e(E_0)/m)^{1/2} \gtrsim eE_0/m\omega\delta^{1/2} \gg v_E$$

(δ is the average relative fraction of the energy transferred in the collisions between the electron and the heavy scattering particle; $\delta = 2m/M \ll 1$ in Coulomb collisions). If $(eE_0/m\omega) \gtrsim \delta^{1/2} v_T^{(0)}$, where $v_T^{(0)}$ is the thermal velocity of the electrons in the absence of external radiation, then the plasma can also exhibit nonlinear properties, but they are due only to its heating (to the dependence of the electron temperature T_e on E_0), and not to the nonlinear bremsstrahlung absorption considered by us.

The nonlinear SBE can be observed and can play an important role only during times much shorter than the time necessary to establish the steady state. Indeed, if the field is strong during the initial instant of the irradiation, $v_E \lesssim v_T^{(0)}$, then this condition can be satisfied only during time intervals on the order of the thermalization time of the electron vibrational motion. The electronic components of the plasma become heated within a time $t \sim \nu_{eff}^{-1}$, after which the electromagnetic field becomes weak (the condition (2.20) is satisfied) and the absorption follows the usual linear laws. If $v_T^{(0)} \ll v_E$, then the effective collision frequency is $\nu_{eff} \sim (v_T^{(0)}/v_E)^3 \tau_{ei}^{-1}$, where τ_{ei} is the time of the electron-ion collisions in the plasma in the absence of a field. Thus, to observe the nonlinear SBE by measuring the absorbed energy, the pulse duration τ should be bounded by the inequality $\tau < (v_E/v_T^{(0)})^3 \tau_{ei}$.

This condition together with the requirement $v_E \gg v_T^{(0)}$, as applied to laser experiments, is satisfactorily fulfilled in the case of picosecond pulses ($\tau = 10^{-12}$ sec). For example, at an ion concentration $N_i = 10^{19}$ cm^{-3}, a frequency $\omega = 3 \times 10^{15}$ sec^{-1}, and a radiation field intensity $I = 10^{16}$ W/cm^2 we have $\nu_{eff}^{-1} \approx 10^{-11}$ sec $\gg \tau$. In order for a field of such intensity to be considered as strong, it is necessary that the initial electron temperature not exceed $\sim 10^{8}$°K.

We present below the classical and quantum-mechanical treatment of the influence of intense radiation on the SBE processes.

1) Classical treatment. The classical solution of the problem of absorption of strong radiation in a fully ionized plasma was first obtained in[20a] on the basis of the kinetic theory.

A strong radiation field can influence the SBE in a plasma, generally speaking, in two ways. First, the cross sections for emission and absorption in the elementary scattering act can change (in comparison with the case of a weak field). Second, the electron distribution function can also change. These two nonlinearity mechanisms can not be separated in the general case. It is precisely such a common approach which is developed in[20a]. An elementary attempt was made in[21] to take into account the nonlinearity connected only with the change of the electron distribution function, but the results are apparently in error. We present below a simplified classical analysis that takes into account only the influence of a strong radiation field on the elementary scattering act, and compare the results of such an approach with the rigorous theory.

Assume that at a certain instand of time t_0 an electron having a translational velocity v is scattered by a force center with a centrally-symmetrical potential. We assume that the scattering time is $\Delta\tau \sim (a/v) \ll 1/\omega$ (a is the effective radius of the potential)[10], i.e., the scattering act can be regarded as instantaneous, and therefore the scattering is elastic. We orient the axes of a rectangular coordinate system in such a way that the z axis coincides with the direction $\mathbf{e} = \mathbf{E}_0/E_0$ of the radiation polarization, and the velocity vector $\mathbf{v}$ lies in the plane (x, z) at an angle θ to the vector $\mathbf{e}$. Then the vector $\mathbf{u} = \mathbf{u}(t_0)$ of the total electron velocity in the alternating electric field $\mathbf{E} = \mathbf{E}_0 \cos\omega t$ prior to scattering has the coordinates

$$u_x = v\sin\theta, \quad u_y = 0, \quad u_z = v\cos\theta + v_E \sin\psi \tag{2.21}$$

and its absolute value is

$$u = v\,[1 + 2\zeta\cos\theta\sin\psi + \zeta^2\sin^2\psi]^{1/2}, \tag{2.22}$$

where $\psi = \omega t_0$, $\zeta = v_E/v$, and v_E is given by (20).

Immediately after the elastic scattering of the electron through an angle ϑ, the coordinates of its total velocity $\mathbf{u}'(t_0)$

$$u'_x = u_x\cos\vartheta + u_z\sin\vartheta\cos\varphi, \qquad u'_y = u\sin\vartheta\sin\varphi, \qquad u'_z = u_z\cos\vartheta - u_x\sin\vartheta\cos\varphi, \tag{2.23}$$

where φ is the azimuthal scattering angle with polar axis along the vector $\mathbf{u}(t_0)$. When $t > t_0$, the electron is acted upon only by the wave field, and therefore

$$u'_x(t) = u'_x(t_0), \qquad u'_y(t) = u'_y(t_0), \tag{2.24}$$
$$u'_z(t) = u'_z(t_0) + \int_{t_0}^{t} v_E\cos\omega t\,dt = u'_z(t_0) + v_E(\sin\omega t - \sin\psi).$$

Prior to scattering, the electron has an average total energy

$$\varepsilon_0 = 0.5m\,(v^2 + 0.5v_E^2) = 0.5mv^2(1 + 0.5\zeta^2).$$

The scattering in the field of the wave changes the average energy of the electron by an amount

$$\Delta\varepsilon = (\vartheta,\ \varphi;\ \psi) = \frac{m}{2}\langle u'^2(t)\rangle - \varepsilon_0,$$

where the brackets $\langle \ldots \rangle$ denote averaging over the time t. On the basis of (2.21)–(2.24) we obtain for $\Delta\epsilon(\vartheta, \varphi; \psi)$:

$$\Delta\varepsilon = mv^2\zeta\sin\psi\,[(1-\cos\vartheta)(\cos\theta + \zeta\sin\psi) + \sin\theta\sin\vartheta\cos\varphi].$$

If $d\sigma(u, \vartheta)$ is the differential cross section for elastic scattering of the electron and N_i is the density of the scattering centers, then the rate of change of the electron energy is

$$d\varepsilon/dt = N_i\int u\Delta\varepsilon(\vartheta,\ \varphi)\,d\sigma(u,\ \vartheta) = N_i\sigma_{tr}(u)muv^2\zeta\sin\psi(\cos\theta + \zeta\sin\psi), \tag{2.25}$$

where $\sigma_{tr}(u) = \int(1-\cos\vartheta)d\sigma$ is the transport cross section.

The expression for the plasma absorption coefficient α is obtained from the expression for the rate $d\epsilon/dt$ by averaging the latter over the phase and over the electron velocities $\mathbf{v}$:

$$\alpha = (2\pi I)^{-1}\int f(\mathbf{v})\,d\mathbf{v}\int_0^{2\pi}(d\varepsilon/dt)\,d\psi. \tag{2.26}$$

We confine ourselves henceforth to Coulomb collisions, for which the cross section σ_{tr} is given by (2.9). We then obtain for $d\epsilon/dt$ on the basis of (2.25)

$$d\varepsilon/dt = (4\pi Z^2e^4N_i/mv) \times \zeta\sin\psi(\cos\theta + \zeta\sin\psi)\,L[(1 + 2\zeta\cos\theta\sin\psi + \zeta^2\sin^2\psi)^{3/2}]^{-1}$$
$$L = \ln\,[1 + (2p_{max}/p_{min})]^{1/2}, \tag{2.27}$$
$$p_{max} = \min(u/\omega_p,\ u/\omega), \quad p_{min} = \max(2Ze^2/mu^2,\ \hbar/mu).$$

We consider first the case of scattering of an electron beam, when $f(\mathbf{v}) = N_e\delta(\mathbf{v} - \mathbf{v}_0)$. In a weak field, when $v_E \ll v_0$, we have[11]

$$u = v\,(1 + \zeta\cos\theta\sin\psi + \ldots), \quad L = \ln(2/\xi) + 2\zeta\cos\theta\sin\psi, \tag{2.28}$$

and on the basis of (2.26)–(2.27) we obtain for the absorption coefficient α, accurate to term $\sim\zeta^2$, a formula that coincides exactly with the quantum-mechanical formula (2.14).

The case of arbitrary values of ζ will be considered for an electron-beam orientation parallel to the polarization of the electric field of the wave ($\theta = 0$). Then $u = v_0|1 + \zeta\sin\psi|$, and

$$d\varepsilon/dt = (4\pi Z^2e^4N_i/mv_0)\,\zeta\sin\psi L(\psi)/(1 + \zeta\sin\psi)^2, \tag{2.29}$$
$$L(\psi) = \begin{cases} \ln\,[1 + (\hbar v_0/\xi Ze^2)^2(1 + \zeta\sin\psi)^6]^{1/2}, & |1 + \zeta\sin\psi| < 2Ze^2/\hbar v_0, \\ \ln\,[1 + (2/\xi)^2(1 + \zeta\sin\psi)^4]^{1/2}, & |1 + \zeta\sin\psi| > 2Ze^2/\hbar v_0. \end{cases}$$

It is seen from (2.16) that when the velocity v_0 of the scattered electrons decreases, the negative-absorption coefficient in a weak field increases sharply[12] and tends to zero like v_0^{-3} with increasing v_0. We can therefore expect in the now-considered strong-field

[10] For Coulomb collisions, this condition is not satisfied precisely in the frequency region at which the field penetrates in the plasma, i.e., at $\omega \gtrsim \omega_p$ (ω_p is the plasma frequency). It is known, however, [12], that in the case of a weak external field the transition from the case $\omega \ll \omega_p$ to the case $\omega \ll \omega_p$ changes only the logarithmic factor in the expression for the effective collision frequency. One can therefore hope that in a strong field the relation between the frequencies ω and ω_p will not greatly influence the main relations of the SBE. This assumption is partly justified by numerical calculations for the case $\omega > \omega_p$ in [20a] and by the results of the following section.

[11] We assume that $p_{min} = \hbar/mu$, which in this case ($\zeta \ll 1$) corresponds to the condition (2.11).

[12] See footnote 7.

case the negative-absorption coefficient $(-\alpha)$ to increase sharply a $\zeta \to 1$ and tend to zero like $E^{-3} \sim I^{-3/2}$ as $\zeta \to \infty$. Calculation of α on the basis of (2.26) and (2.29) confirms this prediction fully; it is impossible, however, to obtain a single elementary formula for arbitrary values of ζ, and we present below three formulas for α at $\zeta \ll 1$, $\zeta = 1$, and $\zeta \gg 1$:

$$\zeta \ll 1: \ \alpha = \alpha_0 (1 + 1{,}5\zeta^2 + \ldots); \qquad (2.30)$$

$$\zeta = 1: \alpha \approx \alpha_0 (\sqrt{2/9\pi}) (\hbar v_0/\xi Z e^2)^{1/2} [\ln (16\Delta_{эфф}/\hbar\varphi)/\ln (2/\xi)];$$

$$\zeta \gg 1: \alpha \approx (\alpha_0/3\pi\zeta^2) (2\hbar v_0/\xi 2Ze^2)^{1/3} [\ln (16\Delta_{эфф}/\hbar\omega)/\ln (2/\xi)] [1+(2\zeta^2)^{-1}+\ldots]$$
$$= -(64\pi^2/3) (Z^2e^3 N_i N_e/\lambda E_0^3) (2/\xi) (\hbar v_0/2Ze^2)^{1/3} \ln (16\Delta_{эфф}/\hbar\omega) [1+(2\zeta^2)^{-1}+\ldots]; \qquad (2.31)$$

here α_0 is the negative-absorption coefficient at $\theta = 0$ in a weak radiation field, as given by (2.16); $\Delta_{eff} \equiv (Z^2e^4m/2\hbar^2)$ is the effective ionization potential of the scattering ion, and $\lambda = 2\pi c/\omega$. The approximate equality signs in (2.30) and (2.31) denote that the calculations are performed with logarithmic accuracy.

From (2.30) we see that $|\alpha|$ increases rapidly as $\zeta \to 1$ in comparison with the case of a weak field, inasmuch as the parameter $(2/\xi)\hbar v_0/2Ze^2 = mv_0^3/Ze^2\omega$ is numerically large. Thus, at $v_0 = 2 \times 10^8$ cm/sec and $Z = 1$, for radiation of wavelength $\lambda = 0.1$ cm, this parameter is approximately equal to 2×10^4; with this, $\alpha(\zeta = 1) \approx 8\alpha_0$, and the radiation intensity corresponding to the condition $\zeta = 1$ is approximately 6×10^7 W/cm^2.

Thus, in the case of non-isotropic distribution of the electron velocities, negative absorption in the SBE for Coulomb collisions is possible also for a strong radiation field, and the negative-absorption coefficient depends in this case on the radiation intensity and has a maximum point. In the limiting particular case of a distribution such that the electron beam is directed along the radiation polarization, this maximum is reached at the point $\zeta = eE_0/m\omega v_0 = 1$, where the value of $|\alpha|$ greatly exceeds $|\alpha_0|$; at $\zeta \gg 1$, the absorption, while remaining negative, becomes small $(|\alpha| \sim \zeta^{-3} \sim I^{-3/2})$. A plot of $|\alpha/\alpha_0|$ against ζ for this particular type of non-isotropic distribution is shown in Fig. 2a.

An entirely different dependence of the absorption coefficient α on the radiation intensity is obtained in the case of isotropic electron velocity distribution. We have seen in Sec. a of Chap. 2 that in a weak radiation field an isotropic distribution always leads to a positive absorption $(\alpha > 0)$. On going to a strong field, the absorption coefficient, remaining positive, tends monotonically to zero. This follows directly from the general formulas (2.26) and (2.27). In this case (2.26) takes the form

$$\alpha = I^{-1} \int_0^\infty v^2 f(v)\, dv \int_0^{2\pi} d\psi \int_0^\pi \sin\theta\, (d\varepsilon/dt)\, d\theta, \qquad (2.32)$$

where the distribution function $f(v)$ is normalized by the condition (2.2). The calculation of α thus reduces primarily (according to (2.27)) to the calculation of the function $B(\zeta)$, defined by the integral

$$B(\zeta) = \zeta \int_0^{2\pi} d\psi \sin\psi \int_0^\pi d\theta \sin\theta\, (\cos\theta + \zeta \sin\psi) \times L\, (1 + 2\zeta \cos\theta \sin\psi + \zeta^2 \sin^2\psi)^{-3/2}. \qquad (2.33)$$

When $\zeta \ll 1$ we can use the expansions (2.28), and we obtain for $B(\zeta)$

$$B(\zeta) = (4\pi/3)\zeta^2 + \ldots \qquad (2.34)$$

When $\zeta \gtrsim 1$, we can discard the term $2\zeta \cos\theta \sin\psi$ in the denominator under the integral sign in (2.33)[13], and we can write accordingly for the Coulomb logarithm

$$L \approx \ln (2\zeta^2/\xi) = \ln (2/\xi_E),$$

where $\xi_E = \hbar\omega/mv_E^2$.

The integration can be readily performed in this case, and we get for $B(\zeta)$

$$B(\zeta) \approx [8\zeta^2/(1+\zeta^2)^{3/2}] D(\zeta/(1+\zeta^2)^{1/2}) \ln (2/\xi_E), \qquad (2.35)$$

where $D(k) = k^{-2}(K(k) - E(k))$ is a complete elliptic integral.

Since $B(\zeta) > 0$ for all values of ζ, the absorption coefficient α defined by (2.32) is also positive at all values of the radiation field intensity. We perform the final calculation of α for a Maxwellian distribution $f(v)$. In this case we obtain on the basis of (2.32), (2.27), and (2.33)

$$\alpha_T = (4/\pi^{1/2}) (Z^2e^4 N_i N_e/mI) \beta^{1/2} \int_0^\infty x B(\zeta_T/x) e^{-x^2} dx, \qquad (2.36)$$

where $\zeta_T = eE_0\beta^{1/2}/m\omega$ and $\beta = m/2T_e$. A weak radiation field corresponds in this case to the condition $\zeta_T \ll 1$. On the basis of (2.36) we obtain the well known expression for the absorption coefficient α_{0T} of a Maxwellian plasma (see, e.g.,[12]). To carry out the integration of (2.36) at $\zeta_T \ll 1$, the integration interval is broken up into two: from 0 to ζ_T and from ζ_T to ∞. In a second interval, we can use for the function $B(\zeta)$ the expansion (2.34), and in the first interval we can use the representation (2.35).

We turn now to the case of a strong field, when $\zeta_T \gtrsim 1$. We can use in this case in the entire integration interval the representation (2.35) for the function $B(\zeta)$, and this yields

$$\alpha_T = 24\pi\sqrt{3}\, (Z^2e^6 N_i N_e/cm^{3/2}\omega^2\varepsilon_T^{3/2}) \ln (2/\xi_E) (1+\zeta_T^2)^{-3/2} B_1(\zeta_T), \qquad (2.37)$$

where $\epsilon_T = 1.5T_e$, and

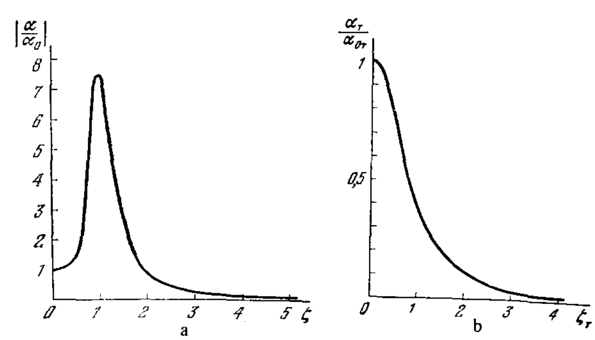

FIG. 2. Coefficient of negative absorption of an electron beam (parallel to the polarization of the electric field of the wave), referred to its value in a weak field, as a function of the field quantity $\zeta = v_E/v$(a), and absorption coefficient of isotropic Maxwellian plasma, referred to its value in a weak field, as a function of the field quantity $\zeta_T = v_E \cdot \beta^{1/2}$, $\beta = m/2T_e$ (b).

[13] When $\zeta \gg 1$, the validity of such an operation is obvious. When $\zeta \sim 1$, it follows from the fact that the main contribution to the integral with resepct to θ is made by the vicinity of the point $\pi/2$, where $|2\zeta \cos\theta \sin\varphi| \ll 1$.

$$B_1(\zeta_T) = (4/\sqrt{\pi}) \int_0^\infty [(1+\zeta_T^2)/(x^2+\zeta_T^2)]^{3/2} D(\zeta_T/(x^2+\zeta_T^2)^{1/2}) x^2 e^{-x^2} dx$$
$$\approx D(\zeta_T/(1+\zeta_T^2)^{1/2}) \quad (1 \leqslant \zeta_T < \infty). \qquad (2.38)$$

At $\zeta_T \gg 1$ we have

$$D(\zeta_T/(1+\zeta_T^2)^{1/2}) \approx \ln[4\zeta_T] - 1 = 0{,}5 \ln(4{,}3\varepsilon_E/T_e),$$

and we therefore obtain on the basis of (2.37)

$$(Z^2e^6N_iN_e/cm^{3/2}\omega^2\varepsilon_T^{3/2}\zeta_T^3)\ln(4\varepsilon_E/\hbar\omega)\ln(4{,}3\varepsilon_E/T_e)$$
$$= 32\pi(Z^2e^2N_iN_e\omega/cE_0^3)\ln(4\varepsilon_E/\hbar\omega)\ln(4{,}3\varepsilon_E/T_e), \qquad (2.39)$$

where $\epsilon_E = 0.5mv_E^2$.

The approximation of the function $B_1(\zeta_T)$ by the elliptic integral $D(\zeta_T/(1+\zeta_T^2)^{1/2})$ makes it possible to write down an expression for the absorption coefficient α_T of a Maxwellian plasma at arbitrary values of ζ_T:

$$\alpha_T \approx 32\pi(Z^2e^6N_iN_e/cm^{3/2}\omega^2\varepsilon_T^{3/2}) D(\zeta_T/(1+\zeta_T^2)^{1/2})$$
$$\times (1+\zeta_T^2)^{-3/2} \ln[(T_e/\hbar\omega)(1+\zeta_T^2)].$$

When $\zeta_T \ll 1$, this formula coincides exactly with the known expression for the linear absorption coeffficient α_{0T} [16]. When $\zeta_T \lesssim 1$, this formula leads to (2.37)), accurate to the approximation (2.38) and a numerical factor $4/3\sqrt{3}$. A plot of α_T/α_{0T} against ζ_T is shown in Fig. 2b.

Let us compare our results with the conclusions of the rigorous theory of nonlinear bremsstrahlung absorption in a plasma[20a]. In that reference, an expression was obtained for the effective frequency of the electron-ion collisions in a strong field of a monochromatic electromagnetic wave. The general formulas are quite complicated, but they become much simpler if the initial electron distribution (prior to the interaction with the field) is Maxwellian, and the field frequency ω is much lower than the plasma frequency, $\omega \ll \omega_p$. The latter condition is equivalent to the instantaneous-scattering assumption used above (see footnote 10). In this case the absorption coefficient α can be written, in accordance with[20a], in the form

$$\alpha = (32(2\pi)^{3/2} N_iN_eZ^2e^6/m^3cv_T^3\omega^2)(v_T/v_E)^3 Q(v_E/2v_T)\ln(k_{\max}/k_{\min}),$$
$$Q(r) = \int_0^r dz\, z^2 e^{-z^2}[I_0(z^2) - I_1(z^2)]; \qquad (2.40)$$

here $I_n(z^2)$ is a Bessel function of imaginary argument, and k_{max} and k_{min} are cutoff factors. As usual, $k_{min} = r_D^{-1} \approx (\omega_p/v_T)$. The quantity k_{max} is determined from the condition of the applicability of perturbation theory or the classical approach, but according to[20a] it is necessary to use for the electron energy in this case the sum of the energy of the translational motion $mv_T^2/2$ and of the oscillation energy in the external field $mv_E^2/2$. Therefore, if $v_E \ll v_T$, it is necessary to use for k_{max} the smaller of the quantities mv_T^2/Ze^2 and $mv_T/\hbar$, and if $v_E \gg v_T$, then $k_{max} = \min\{mv_E^2/Ze^2, mv_E/\hbar\}$.

In this case of a weak field, $v_E \ll v_T$, we obtain from (20) the usual expression for the weak-field absorption coefficient[12]. On the other hand, if $v_E \gg v_T$, then calculation of the asymptotic form of the function $Q(r)$ at $r \gg 1$ yields

$$\alpha \approx (32N_iN_eZ^2e^2\omega/cE_0^3)\ln(eE_0/2m\omega v_T)\ln(E_0^2v_T/Zm\omega^2\omega_p).$$

This result is valid if $Ze^2/\hbar v_E > 1$, but if the inverse inequality holds, then the expression under the sign of the second logarithm is replaced by $mv_Ev_T/\hbar\omega_p$.

Thus, in the asymptotic case of a very strong field, $v_E \gg v_T$, the simplified approach (see (2.39) and the rigorous kinetic theory lead to identical results, accurate to a slow logarithmic dependence. We note in conclusion that the influence of a strong monochromatic field of frequency ω_0 on the SBE at an arbitrary frequency $\omega \neq \omega_0$ was considered in[18d] for an isotropic Maxwellian plasma. It was shown that this case has certain distinguishing features, and under certain conditions the influence of a strong field on the corresponding absorption coefficient can be observed in principle in experiments on the interaction of laser radiation with a plasma at the presently available source powers.

We proceed now to the quantum mechanical treatment of the problem of the SBE in a strong radiation field.

2) Quantum-mechanical treatment. The quantum theory of the SBE has the advantage that it enables us to calculate the differential cross sections $d\sigma_{e,a}^{(n)}$ of electron scattering in the presence of an external monochromatic field (with frequency ω), accompanied by emission or absorption of a definite (n) number of quanta $\hbar\omega$ ($n = 1, 2, 3, \ldots$). In the case of a sufficiently weak radiation field, the cross sections for the multiquantum SBE can be obtained in principle with the aid of perturbation theory in terms of the interaction of the electron with this field. To describe the processes of emission or absorption of n photons, the perturbation-theory calculations must be carried out to order n. This fact determines the dependence of the cross sections $d\sigma_{e,a}^{(n)}$ on the radiation-field intensity I, $\sigma_{e,a}^{(n)} \sim I^n$.

The first attempts at calculations of this type were made in[22], but the calculations were not rigorously performed even within the framework of perturbation theory. This caused the small perturbation-theory parameter $\gamma^2 = I/I_0$ to be incorrectly determined, namely, the characteristic intensity I_0 was underestimated by a factor $(c/v)^2$ (see below).

Stimulated bremsstrahlung absorption in a strong radiation field was considered in[23] by a quantum-mechanical approach. The method used there, however, was quite cumbersome. In addition, only the asymptotic total absorption cross section was obtained in finished form for a very strong field in the case of scattering by ions, which is in essence equivalent to the results of the classical results. The differential cross sections $d\sigma_{e,a}^{(n)}$ for the SBE in a strong radiation field were not obtained in[23].

A quantum theory of the SBE in a strong radiation field, not restricted to perturbation theory, was developed in[24] (nonrelativistic electrons) and[25] (relativistic electrons). The main features of the case of the strong field are, first, the multiphoton processes become generally speaking just as probable as the single-photon processes ($\sigma_{e,a}^{(n)} \sim \sigma_{e,a}^{(1)}$), and, second, the role of processes of emission and absorption virtual photons increases. The latter circumstance causes the general result $\sigma_{e,a}^{(n)} \sim I^n$ of perturbation theory to become incorrect. We confine ourselves below to the nonrelativistic case.

To calculate the cross sections $\sigma_{e,a}^{(n)}$ of interest to us, for a strong radiation field, we can use a semi-classical approach in which the electron motion is described quantum-mechanically and the electromagnetic field is considered classically. The Hamiltonian of a nonrelativistic electron situated in the field of a plane monochromatic wave is

$$\mathscr{H} = [-i\hbar\nabla - (e/c)\,\mathbf{A}]^2/2m,$$

where $\mathbf{A} = \mathbf{A}_0 \cos\omega t = -(c\mathbf{E}_0/\omega)\cos\omega t$ is the vector potential, which can be regarded as independent of the spatial coordinates. The exact wave functions of the electron, corresponding to such a Hamiltonian, are well known:

$$\psi_p = (2\pi\hbar)^{-3/2} \exp\left\{(i/\hbar)\left[\mathbf{pr} - \int_0^t [\mathbf{p} - (e/c)\,\mathbf{A}(\tau)]^2 (2m)^{-1}\,d\tau\right]\right\}, \qquad (2.41)$$

where $\mathbf{p}$ is the electron momentum. The electron energy in the state (2.41) is an oscillating function of the time

$$\varepsilon(t) = \langle\psi_p^* \,|\, \mathscr{H} \,|\, \psi_p\rangle = (2m)^{-1}(\mathbf{p} - (e/c)\,\mathbf{A}(\tau))^2$$

with a mean value

$$\varepsilon = \overline{\varepsilon(t)} = (p^2/2m) + (e^2E_0^2/4m\omega^2).$$

When the electron is scattered by a static potential, transitions between the states (2.41) take place: $\psi_{p0} \to \psi_p$; in this case, as will be shown below, the average electron energy can change only by an amount $\Delta\epsilon = (p^2 - p_0^2)/2m$, which is a multiple of the photon energy $\hbar\omega$, that is, $\Delta\epsilon = \pm n\hbar\omega$, where $n = 1, 2, 3, \ldots$ We shall calculate the probability of such a transition, taking the interaction of the electron with the scattering center into account in the first Born approximation. Thus, in scattering by a Coulomb center, there should be satisfied the condition (2.1) supplemented, when applied to the emission cross section $\sigma_e^{(n)}$, by a condition analogous to (2.12):

$$2n\xi \ll 1, \quad \xi = \hbar\omega/mv_0^2.$$

The probability amplitude for the transition of an electron situated in a state ψ_{p0} at $t = 0$ to the state ψ_p is determined by the expression

$$C_{pp_0}(t) = (-i/\hbar)\int_0^t dt' \exp\left\{(i/\hbar)\left[(p^2 - p_0^2)(2m)^{-1}t' - e\,(\mathbf{p} - \mathbf{p}_0)\,(mc)^{-1}\int_0^{t'} \mathbf{A}(\tau)\,d\tau\right]\right\} \times \int V(\mathbf{r}) \exp[-(i/\hbar)(\mathbf{p} - \mathbf{p}_0)\,\mathbf{r}]\,d\mathbf{r}, \qquad (2.42)$$

where $V(\mathbf{r})$ is the energy of the interaction of the electron with the scattering center. The quantity under the integral with respect to t' contains a periodic function of the time, which can be expanded in a Fourier series. As a result, the amplitude C_{pp_0} is represented in the form

$$C_{pp_0}(t) = -\sum_n \{\exp[(i/\hbar)(\Delta\varepsilon + n\hbar\omega)\,t] - 1\}\,(\Delta\varepsilon + n\hbar\omega)^{-1} J_n[(e\mathbf{E}_0/m\hbar\omega^2)(\mathbf{p}_0 - \mathbf{p})] \times \int V(\mathbf{r}) \exp[(-i/\hbar)(\mathbf{p} - \mathbf{p}_0)\,\mathbf{r}]\,d\mathbf{r}, \qquad (2.43)$$

where $\Delta\epsilon = (p^2 - p_0^2)/2m$, J_n is a Bessel function, and n takes on all positive and negative integer values. In the calculation of the transition probability per unit time $w_{pp_0} = \lim_{t\to\infty} (|C_{pp_0}(t)|^2/t)$ Eq. (2.43) leads to a double sum over n and n′. It is easily seen, however, that in the limit as $t \to \infty$ the crossing term of this sum $(n \neq n')$ make no contribution, and the diagonal terms $(n = n')$ lead to energy δ-functions

$$w_{pp_0} = \sum_{n=-\infty}^{+\infty} w_{pp_0}^{(n)}\,\delta(\Delta\varepsilon + n\hbar\omega).$$

The presence of δ-functions, which determine the energy conservation law, makes it possible to interpret individual terms of this sum as the probabilities of emission (at $n > 0$) or absorption (at $n < 0$) of $|n|$ photons. The transition probabilities w_{pp_0} should be summed over the final state of the electron. The integration with respect to the modulus $|\mathbf{p}|$ of the momentum can be easily carried out because of the δ-functions. Changing over from the probabilities $w_{pp_0}^{(n)}$ to the differential cross sections $\sigma_{e,a}^{(n)}$ for the emission or absorption of n photons following the scattering of an electron having a velocity $v_0 = p_0/m$ into a solid angle $d\Omega$, we obtain on the basis of (2.43)[24]

$$d\sigma_{a,e}^{(n)}/d\Omega = \beta_{a,e} J_n^2\,[\gamma\mathbf{e}\,(\mathbf{n}_0 - \beta_{a,e}\mathbf{n})]\,d\sigma_s\,(\mathbf{n}_0 - \beta_{a,e}\mathbf{n})/d\Omega, \qquad (2.44)$$

where $\gamma = eE_0v_0/\hbar\omega^2$, $\beta_{a,e} = (1 \pm 2n\xi)^{1/2}$, $\mathbf{e} = \mathbf{E}_0/E_0$, $\mathbf{n}_0 = \mathbf{v}_0/v_0$, $\mathbf{n}$ is a unit vector in the direction of the electron scattering, and

$$d\sigma_s\,(\mathbf{n}_0 - \mathbf{n})/d\Omega = (m/2\pi\hbar^2)^2 \left|\int V(\mathbf{r}) \exp[(i/\hbar)\,p_0\,(\mathbf{n}_0 - \mathbf{n})\,\mathbf{r}]\,d\mathbf{r}\right|^2$$

is the differential cross section for elastic scattering of the electron in the Born approximation.[10a]. For a centrally symmetrical potential $V(r)$ we have

$$d\sigma_s\,(\mathbf{n}_0 - \beta\mathbf{n})/d\Omega = (2m/\hbar^2)^2 \left|\int V(r)\,q^{-1}\sin(qr)\,r\,dr\right|^2,$$

where $q^2 = (p_0/\hbar)^2(1 + \beta^2 - 2\beta\mathbf{n}_0\cdot\mathbf{n})$.

In the case of the emission cross section there is a limitation on n (following from the condition that β_e be real, and expression the energy conservation law), namely $n < 1/2\xi$. For Coulomb scattering, (2.44) determines $d\sigma_e^{(n)}$, strictly speaking, only if this inequality is strongly fulfilled, i.e., under the condition (2.42). We emphasize that the dependence of the cross sections $d\sigma_{a,e}^{(n)}$ on the intensity I and the polarization $\mathbf{e}$ of the radiation field enters only via the argument of the Bessel function. Obviously, when $n = 0$, Eq. (2.44) determines the cross section for elastic scattering of the electron in a strong radiation field:

$$d\sigma^{(0)}/d\Omega = J_0^2\,[\gamma\,(\mathbf{e}\Delta\mathbf{n})]\,d\sigma_s/d\Omega, \qquad (2.45)$$

where $d\sigma_s/d\Omega$ is the cross section for elastic scattering without allowance for the influence of the radiation field, and $\Delta\mathbf{n} = \mathbf{n}_0 - \mathbf{n}$; thus, (2.45) can be regarded as a generalization of the classical Born formula for elastic scattering.

It is seen from (2.44) that the applicability of perturbation theory to the analysis of SBE (and consequently also the validity of the power-law relations $\sigma_{a,e}^{(n)} \sim I^n$ and of the inequalities $\sigma_{a,e}^{(1)} \gg \sigma_{a,e}^{(2)} \gg \sigma_{a,e}^{(3)} \gg \ldots$) is determined by the condition[14)]

$$\gamma^2 = (eE_0v_0/\hbar\omega^2)^2 = I/I_0 \ll 1, \quad I_0 = c\hbar^2\omega^4/8\pi e^2v_0^2. \qquad (2.46)$$

[14)]The condition (2.46) is sufficient for the applicability of perturbation theory to the calculation of the cross sections $\sigma_{a,e}(n)$ at arbitrary $n = 0, 1, 2, \ldots$ The applicability of perturbation theory to the calculation of $\sigma_{a,e}(n)$ for a specified value of n is determined, generally speaking, by a less stringent condition that follows from the power-law expansion of the Bessel functions $J_n(x)$: $\gamma^2 \ll 2(n + 1)$.

The general behavior of the cross section $\sigma_{a,e}^{(n)}$ with increasing parameter γ can be established on the basis of (2.44) from the following asymptotic representations of the cylindrical functions

$$J_n(x) \approx \pi^{-1}[2(n-x)/3x]^{1/2} K_{1/3}([2(n-x)]^{3/2}/3x^{1/2}), \quad n \gg 1, \quad n > x,$$

where $K_p(z)$ is the Macdonald function, for which we have in turn, at $|z| \gg 1$, the representation

$$K_p(z) \approx (\pi/2z)^{1/2} e^{-z}.$$

These representations show that it is possible to introduce the concept of the maximum degree of the multiquantum character of the SBE, n_{max}, which depends on the parameters γ and ξ and on the orientation of the unit vectors $\mathbf{n}_0$, $\mathbf{n}$, and $\mathbf{e}$. At given values of these parameters, the value of n_{max} is such that for all $n \lesssim n_{max}$ the cross sections $\sigma_{a,e}^{(n)}$, generally speaking, are of the same order of magnitude, whereas at $n \gg n_{max}$ the cross sections $\sigma_{a,e}^{(n)}$ are exponentially small. It is clear that for the emission processes the value of n_{max} cannot exceed the integer part of $1/2\xi$.

In the case of a weak field $\gamma \ll 1$ we have $n_{max} = 1$, i.e., the multiquantum processes have little probability in comparison with the emission and absorption of a single photon. If $\gamma\xi \ll 1$ but the parameter γ is not small (this is possible, since $\xi \ll 1$ for optical frequencies), then $n_{max} \sim \gamma$ at $|\mathbf{e}\cdot\mathbf{n}_0 - \beta\mathbf{e}\cdot\mathbf{n}| \sim 1$. The latter condition is satisfied in a wide range of angles determining the orientation of the vector $\mathbf{n}$. On the other hand, if $\mathbf{n} \parallel \mathbf{n}_0$, then we have again $n_{max} \sim 1$.

Thus, the condition $\gamma \sim 1$ determines the critical value of the intensity I of the radiation field, starting with which the results of perturbation theory no longer hold for the differential cross sections $d\sigma_{e,a}^{(n)}$. A change takes place in this case both in the dependences of the cross sections $d\sigma_{e,a}^{(n)}$ on the intensity and the number of photons emitted are absorbed when the electrons are scattered by the potential $V(r)$. The latter circumstance, i.e., the change of n_{max}, can be observed in principle by investigating the energy distributions of the scattered electrons.

However, when calculating the integral quantities, e.g., the absorption coefficient, the nonlinear corrections may cancel each other to a considerable degree and the effective nonlinearity parameter may decrease. According to the classical results (see the discussion in Sec. b of Chap. 2), the nonlinearity parameter determining the absorption coefficient is the quantity $\zeta = v_E/v = \gamma\xi \ll 1$. The quantum-mechanical approach leads apparently to an analogous result: accurate to small quantum corrections ($\sim\xi$), the absorption coefficient is determined by the same classical parameter $\gamma\xi$. There is no rigorous proof of this statement at present. However, the calculation of the first nonlinear correction to the absorption coefficient shows that its relative value is determined by the parameter $(\gamma\xi)^2$. In addition, the calculation of the absorption coefficient in the asymptotic strong-field limit leads to a result similar to (2.39) precisely under the condition $\gamma\xi \gg 1$.

In the general case the absorption coefficient α can be expressed as follows in terms of the cross section $\sigma_{e,a}^{(n)}$

$$\alpha = -(N_i N_e \hbar\omega/I) v_0 \sigma_t, \quad \sigma_t = \sum_n n(\sigma_e^{(n)} - \sigma_a^{(n)}). \tag{2.47}$$

Let us examine this expression for scattering by a Coulomb potential $V(r) = -Ze^2/r$, assuming the field of wave to be sufficiently weak so that it is meaningful to expand the absorption coefficient in powers of the intensity. Let us find the first correction to the absorption coefficient, $\delta\alpha \sim E_0^2$. Using (2.44), (2.40), and the series expansion of the Bessel functions, we obtain in the lowest order in the field intensity E_0 the already known results (2.14) of perturbation theory. The next (first nonvanishing) order of the expansion yields the sought corrections:

$$\begin{aligned}\sigma_{e,a}^{(2)} &\approx (\pi Z^2 e^4/2m^2v^4 (eE_0 v/2\hbar\omega^2)^4 \{1 + 2(\mathbf{n}_0\mathbf{e})^2 - [(\mathbf{n}_0\mathbf{e})^4/3] \\ &\qquad \mp 6\xi(1 - \mathbf{n}_0\mathbf{e})^2 \pm 8\xi^3[3 - 42(\mathbf{n}_0\mathbf{e})^2 + 47(\mathbf{n}_0\mathbf{e})^4]\ln\xi^{-1}\}, \\ \sigma_t^{(2)} &= \sigma_e^{(2)} - \sigma_a^{(2)} \approx (2\pi Z^2 e^4/m^2v^4)(evE_0/2\hbar\omega^2)^4(\hbar\omega/mv^2) \\ &\qquad \times \{-3(1 - \mathbf{n}_0\mathbf{e}) + 4\xi^2[3 - 42(\mathbf{n}_0\mathbf{e})^2 + 47(\mathbf{n}_0\mathbf{e})^4]\ln\xi^{-1}\}.\end{aligned} \tag{2.48}$$

Similar formulas can be obtained also for the nonlinear corrections to the first-order cross sections $\sigma_{e,a,t}^{(1)}$. It follows from these formulas that the cross section $\sigma_{e,a,t}^{(2)}$ and the nonlinear corrections to $\sigma_{e,a,t}^{(1)}$ are determined by the parameter γ. However, in the calculation of the total cross section σ_t, as already mentioned, the principal terms in formula (2.48) cancel each other

$$\delta\sigma_t = (3\pi Z^2 e^8 E_0^4/4m^5\hbar\omega^5 v^6)[3 - 42(\mathbf{n}_0\mathbf{e})^2 + 47(\mathbf{n}_0\mathbf{e})^4]\ln(mv^2/\hbar\omega);$$

here $\delta\sigma_t$ is the correction of second order in γ^2 to the total cross section σ_t. From this we obtain with the aid of (2.47) the first correction to the absorption coefficient:

$$\delta\alpha = -(3\pi^2 Z^2 e^8 E_0^2 N_i N_e/m^5\omega^4 v^5 c)[3 - 42(\mathbf{n}_0\mathbf{e})^2 + 47(\mathbf{n}_0\mathbf{e})^4]\ln(mv^2/\hbar\omega). \tag{2.49}$$

Comparison with (2.14) shows that $\delta\alpha/\alpha \sim (v_E/v)^2 = (\gamma\xi)^2$.

In the case of a plasma with an isotropic distribution, Eq. (2.49) should be averaged over the directions of the vector $\mathbf{n}_0$; this yields

$$\overline{\delta\alpha} = (24\pi^2 Z^2 e^8 E_0^2 N_i N_e/5m^5\omega^4 v^5 c)\ln(mv^2/\hbar\omega).$$

Formulas (2.44) and (2.47) enable us also to estimate the asymptotic behavior of the absorption coefficient in the case of a very strong field $\gamma\xi \gg 1$[24]. Using the asymptotic representation of the Bessel functions at large values of the arguments, averaging the rapidly oscillating factors, and integrating, we obtain

$$\alpha \approx (32\pi Z^2 e^3 \omega N_i N_e/cE_0^3)\ln(eE_0/m\omega v)\ln(evE_0/\hbar\omega^2).$$

This result differs from the classical formula (2.39) only in the form of the second logarithm, a difference that can apparently be attributed to the approximate character of the calculations.

In[26], by a method similar to that described in the present section, they considered the multiquantum SBE in scattering by a screened Coulomb potential $V(r) = \exp(-r/R)/r$. We note, however, that allowance for the screening must be made only in the case of a sufficiently small screening radius $R < v/n\omega$. For a plasma at $R = R_D$ this yields $\omega < \omega_p/n$, i.e., in the case $\omega > \omega_p$ of interest it is not necessary to take the screening into account.

As already mentioned earlier, the nonlinear SBE can be appreciable only for sufficiently short pulses, since in the case of long durations the plasma has time to become heated and the subsequent dissipation of the field energy is determined by the usual linear bremsstrahlung absorption. Actually, there is one more limitation of the same type on the pulse duration.

To study the SBE by measuring the light energy absorbed in the plasma, it is necessary that the pair collisions of the electrons and ions be the principal mechanism responsible for the absorption of the light. This requirement may not be satisfied if the action of the strong radiation field on the plasma leads to development of instabilities. It is known[20b,27-29] that in the case when $v_E \gg v_T$ the thresholds of many instabilities are strongly exceeded. One can, however, use pulses so short that the buildup of the growing oscillations in the plasma is small during the pulse time τ. This leads to the condition $\gamma\tau \lesssim 1$, where γ is the maximum instability increment. The strongest instabilities arise in the case when the frequency ω of the external field is close to the plasma frequency ω_p[27]. The limitation on the pulse duration then turns out to be too stringent, so that even for picosecond pulses it is impossible to guarantee that the absorption will be determined exclusively by pair collisions.

In the case $\omega \gg \omega_p$, the situation is much more favorable. A plasma frequency lower than that of the light leads to an appreciable increase of the thresholds of the possible instabilities and to a decrease of the increments[28,29]. The best conditions for the observation of the SBE can apparently be produced by using an isothermal plasma, in which the initial electron and ion temperatures are equal[29]. Estimates show that in this case the condition $\gamma\tau \ll 1$ can be satisfied for picosecond pulses up to very appreciable laser-radiation powers ($I \lesssim 10^{17}$ W/cm^2 at $N_i \lesssim 10^{19}$ cm^{-3}).

3. STIMULATED COMPTON SCATTERING

Prior to the appearance of powerful optical-radiation sources, the Compton effect was not considered as a possible mechanism for absorption of optical radiation. The reason was that in the usual (spontaneous) Compton scattering the effective wave absorption coefficient is given by

$$\alpha \sim N_e \sigma_0 \Delta\varepsilon/\hbar\omega, \tag{3.1}$$

where $\sigma_0 = (8\pi/3)r_0^2 \approx 6.6 \times 10^{-25}$ cm^{-2} is the Thompson cross section, N_e is the electron density, $\hbar\omega$ is the quantum energy, and $\Delta\epsilon$ is the average energy transferred to the electron in a single scattering act. This energy is positive and amounts to $\Delta\epsilon \sim (\hbar\omega/mc^2)\hbar\omega$, if $\hbar\omega \gg kT_e$, and is negative with absolute value $|\Delta\epsilon| \sim (kT_e/mc^2)\hbar\omega$, if $\hbar\omega \ll kT_e$ (see Sec. b of the present chapter below; T_e is the electron temperature). In the second case, the electrons become cooled—this is the so-called inverse Compton effect. In both cases, however, even at electron densities $N_e \sim 10^{21}$ cm^{-3}, the corresponding absorption (incoherent amplification) coefficients α are quite small according to (3.1) (of the order of 10^{-9}–10^{-6} cm^{-1}).

The effectiveness of the Compton absorption mechanism can be greatly increased if the scattering becomes stimulated. From now on, except for Sec. c of the present chapter, we shall consider only the two-quantum stimulated Compton effect, wherein only one photon is absorbed and the emission of another photon is stimulated simultaneously in a single scattering act.[15)] The effectiveness of the mechanism of stimulated Compton scattering increases with increasing radiation intensity. As a result, in particular, the main mechanism of its absorption becomes the stimulated Compton effect if the optical radiation interacting with the medium has sufficiently high intensity.

The study of this effect is of interest not only from the point of view of the mechanism of electron heating by intense optical radiation, but also from the point of view of obtaining a negative absorption, i.e., developing a "Compton laser." The development of such a laser, using photons scattered backward from a relativistic electron beam, was suggested by Pantell and co-workers[32].

For two-quantum Compton scattering, the number of photons is conserved, and the energy-momentum conservation law leads to the well known relation between the frequencies ω_1 and ω_2 of the incident and scattered photon[33]

$$\omega_2 = \omega_1 [1 - (v/c)\cos\theta_1] [1 - (v/c)\cos\theta_2 + (\hbar\omega_1/\varepsilon)(1-\cos\theta)]^{-1} \tag{3.2}$$

where v and ϵ are the velocity and energy of the initial electron, θ_1 and θ_2 are the angles between **v** and the wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ of the initial and scattered photons, and θ is the angle between $\mathbf{k}_1$ and $\mathbf{k}_2$.

The cross section for spontaneous Compton scattering is determined by the Klein-Nishina-Tamm formula, which takes the following form in the laboratory frame[33]:

$$d\sigma_{sp} = F\, d\Omega_2, \tag{3.3}$$

where

$$F = [2r_0^2(\hbar\omega_2)^2/(mc^2)^2\varkappa_1^2]\, u_0,$$

$$u_0 = 4(\varkappa_1^{-1} + \varkappa_2^{-1})^2 - 4(\varkappa_1^{-1} + \varkappa_2^{-1}) - (\varkappa_1\varkappa_2^{-1} + \varkappa_2\varkappa_1^{-1}),$$

$$\varkappa_1 = 2(p_1k_1)/(mc^2)^2, \qquad \varkappa_2 = -2(p_1k_2)/(mc^2)^2,$$

$$p_\alpha k_\beta = c^2\hbar\mathbf{p}_\alpha\mathbf{k}_\beta - \varepsilon_\alpha\hbar\omega_\beta,$$

$r_0 = e^2/mc^2 = 2.8 \times 10^{-13}$ cm is the classical radius of the electron.

In the case of stimulated scattering, the frequency ω_2 and the wave vector $\mathbf{k}_2$ of the emitted photon should be regarded as fixed, and the scattering cross section itself increases to a value

$$d\sigma = (1 + n_{\mathbf{k}_2})\, d\sigma_{cn}, \tag{3.4}$$

where $n_{\mathbf{k}_2}$ is the number of photons per field oscillator with wave vector $\mathbf{k}_2$:

$$n_{\mathbf{k}} = 4\pi^3c^2 J(\omega, \mathbf{q})/\hbar\omega^3 = 4(\pi c)^3 N(\omega, \mathbf{q})/\omega^2. \tag{3.5}$$

In this formula, $J(\omega, \mathbf{q})$ is the spectral density of the energy flux of the unpolarized radiation in a unit solid angle whose axis is directed along $\mathbf{q} = \mathbf{k}/k$, and $N(\omega, \mathbf{q}) = J(\omega, \mathbf{q})/c\hbar\omega$ is the spectral (and volume) density of the photons propagating in the same unit solid angle. If I is the total (integral) intensity of a light beam with uniform energy distribution over the frequency and angle spectra, having respectively a width $\Delta\omega$ and a solid angle $\Delta\Omega \ll 1$, then $J(\omega, \mathbf{q}) = I/\Delta\omega\Delta\Omega$.[16)]

15)The multiquantum Compton effect is treated in [6,30a] (see also the review [5a], where a detailed bibliography is given); its role in the heating of the electronic component of a plasma was considered in [31].

16)In the general case, the connection between I and $J(\omega, \mathbf{q})$ is of the form $I = \int I_\omega d\omega$, $I_\omega = \int J(\omega, \mathbf{q}) \cos\vartheta\, d\Omega$, where $\cos\vartheta = \mathbf{q}\cdot\mathbf{n}$; **n** is the

We note that in the presence of one radiation beam, the finite character of the width of its frequency ($\Delta\omega$) and angular ($\Delta\Omega$) spectra is a necessary condition for stimulated Compton scattering with energy transferred to the electrons ($\Delta\epsilon \neq 0$). Indeed, for one monochromatic wave we always have ($\Delta\epsilon = \hbar(\omega_1 - \omega_2) = 0$, for one plane wave we always have $\theta = 0$ and $\theta_1 = \theta_2$, and consequently, according to (3.2), we always have $\omega_1 = \omega_2$ and $\Delta\epsilon = 0$. In the presence of several wave beams, the condition that $\Delta\omega$ and $\Delta\Omega$ be finite is in general not necessary.

In this chapter of the article we consider the following question: One of the possibilities of obtaining negative absorption via the stimulated Compton effect, absorption of intense optical radiation by the electronic component of a plasma, and elastic scattering of electrons by plane standing electromagnetic waves—the Kapitza-Dirac effect.

a) Negative absorption in stimulated Compton scattering. The most interesting possibility of obtaining negative absorption in the stimulated Compton effect is connected with the use of a relativistic electron beam and backward scattering[32]. This is precisely the case which we consider below.

We assume that the electron energy in the beam is $\epsilon \gg mc^2$, and then we have for backward scattering ($\theta = \pi$, $\theta_1 = \pi$, $\theta_2 = 0$), according to (3.2)

$$\omega_2 = 4\omega_1 (\varepsilon/mc^2)^2. \tag{3.6}$$

Thus, in this case it becomes possible to produce a "Compton laser" with a very large frequency conversion coefficient, $4(\epsilon/mc^2)^2$. Inasmuch as the energy ϵ of the electron beam can change in this case in a wide range, the generation frequency of this laser can be tuned in a considerable range.

Let's calculate the gain α_2 at the frequency ω_2 at a specified radiation intensity I_1 at the pump frequency ω_1. Let σ_e be the total scattering cross section at which the quantum $\hbar\omega_1$ is absorbed and stimulated emission of the quantum $\hbar\omega_2$ takes place, σ_a is the total cross section of the inverse process, in which the quantum $\hbar\omega_2$ is absorbed and stimulated emission of the quantum $\hbar\omega_1$ takes place. Then, according to the definition of the coefficient α_2, we have

$$\alpha_2 = \sigma_t N_e (\omega_2/\omega_1) I_1/I_2, \tag{3.7}$$

where $\sigma_t = \sigma_e - \sigma_a$. In the calculation of the cross sections $\sigma_{e,a}$ we assume (following[32]) that the electron beam has a certain energy distribution $f(\epsilon)$, and the width $\Delta\epsilon$ of this distribution is large enough so that

$$\Delta\varepsilon/\varepsilon_0 \gg \Delta\omega_1/\omega_1, \quad \Delta\omega_2/\omega_2 \tag{3.8}$$

(ϵ_0 is the average energy of the electrons in the beam). When this condition is satisfied, the cross sections $\sigma_{e,a}$ are obtained by averaging the cross section (3.4) over the electron energies, and the conservation laws determine the energy of the electrons participating in this process as a function of the frequencies and wave vectors of the emitted and absorbed photons[17)]. As a result we get

$$\sigma_e \approx (\pi^3/4)\, r_0^2 (m^4c^{10}I_2/\hbar\omega_2\omega_1^2\varepsilon^3)\, f(\varepsilon),$$
$$\sigma_a \approx (\pi^3/4)\, r_0^2 (m^4c^{10}I_2/\hbar\omega_2\omega_1^2\varepsilon^3)\, f(\varepsilon - \hbar\omega_2); \tag{3.9}$$

here ϵ is that value of the electron energy which satisfies relation (3.6) for given ω_1 and ω_2 (the function $f(\epsilon)$ is assumed normalized by the condition $\int f(\epsilon)d\epsilon = 1$). If $\Delta\epsilon \gg \hbar\omega_2$, then $f(\epsilon - \hbar\omega_2)$ can be expanded in powers of $\hbar\omega_2$, and we obtain for the gain α_2 on the basis of (3.7) and (3.9)

$$\alpha_2 = \pi^3 r_0^2 (m^2 N_e c^6 I_1/\omega_1^2\varepsilon)\, df/d\varepsilon.$$

Negative absorption (amplification) at the frequency ω_2 corresponds to the condition $df/d\epsilon > 0$, i.e., in the distribution $f(\epsilon)$ the energy ϵ should fall in an interval corresponding to the inverted population (with condition (3.8) satisfied). This condition can always be satisfied by choosing the frequencies ω_1 and ω_2. For a Gaussian distribution

$$f(\varepsilon) = [(2\pi^{1/2}/\Delta\varepsilon]^{-1} \exp[-(\varepsilon - \varepsilon_0)^2/2(\Delta\varepsilon)^2]$$

the maximum of $df/d\epsilon$ is reached at the point $\epsilon_1 = \epsilon_0 - \Delta\epsilon$ with a value $(df/d\epsilon)_{max} \approx (2\Delta\epsilon)^{-2}$. In this case

$$\alpha_{2\,max} \approx (\pi^3/4)\, r_0^2 N_e (m^2c^6 I_1/\omega_1^2\varepsilon(\Delta\varepsilon)^2).$$

If $\epsilon_0 = 10$ MeV, $(\Delta\epsilon/\epsilon_0) \approx 10^{-4}$, $N_e = 10^9$ cm^{-3}, $\omega_1 = 10^{10}$ sec^{-1}, and $I_1 = 3 \times 10^6$ W/cm^2, then amplification takes place at the frequency $\omega_2 \approx 1.6 \times 10^{13}$ sec^{-1} ($\lambda_2 \approx 120\mu$) with a coefficient $\alpha_{2\,max} \approx 0.3$ cm^{-1}.

We note that when condition (3.8) is satisfied, negative absorption can be obtained also for a nonrelativistic electron beam. In this case, however, the frequency conversion coefficient turns out to be, according to (3.2), of the order of unity.

b) Absorption of optical radiation in a plasma as a result of stimulated Compton scattering. The Compton mechanism of interaction of radiation with plasma electrons can be investigated consistently on the basis of the kinetic theory. The collision integral for this case was first obtained in[17b,34], but no investigations were made there of the distinguishing features of the stimulated Compton effect. This was done in a series of papers by Peyraud[35], who analyzed in detail the kinetic equations obtained in[17b,34] for the case when the intensity of the radiation is high enough and it is necessary to take stimulated scattering into account. In particular, he obtained results concerning the rate of heating of the electrons by the radiation, and also the ensuing shift into the "red" side of the frequency spectrum of the radiation transmitted through the plasma (see also[36], which deals with plasma heating by the stimulated Compton effect in two opposing light beams, and[37], devoted to a study of the Compton mechanism of heating of relativistic electrons). These questions are of greatest interest, and we analyze them below on the basis of a more elementary approach (than in[35]).

Just as elsewhere, we consider the case of nonrelativistic electrons ($v \ll c$) and soft photons ($\hbar\omega \ll mc^2$), and the small quantities $(v/c)^2$ and $\hbar\omega/mc^2$ are assumed to be quantities of the same order.

From (3.2) we obtain in this case for the frequency of the scattered photon

$$(\omega_1 - \omega_2)/\omega_1 = (v/c)(\cos\theta_1 - \cos\theta_2) + (v/c)^2 \cos\theta_2 (\cos\theta_1 - \cos\theta_2) + (\hbar\omega_1/mc^2)(1 - \cos\theta). \tag{3.10}$$

normal to the surface with respect to which the radiation intensity is considered (I_ω is the spectral intensity).

17)In this case ($\epsilon \gg mc^2$) we have in (3.3) $\kappa_1 = -\kappa_2$ and $U_0 = 2$.

It was already noted above that in the presence of only one light beam the process of stimulated Compton scattering with energy transfer to the electrons is possible only if the frequency and angular spectral widths $\Delta\omega$ and $\Delta\Omega$ are finite. Formula (3.10) makes it possible to formulate this condition more rigorously. It is seen from it that in a light beam with nonmonochromaticity $(\Delta\omega/\omega)$, stimulated scattering through an angle θ is possible only if the following condition is satisfied

$$\Delta\omega/\omega > |(v/c)(\cos\theta_1 - \cos\theta_2)| = 2(v_{\parallel}/c)\sin(\theta/2),$$

where $v_{\parallel}$ is the component of the velocity of the scattering electron in the direction of the vector $(\mathbf{k}_1 - \mathbf{k}_2)$. The maximum stimulated scattering angle in a light beam with divergence angle $2\theta_0$ (and accordingly with a solid angle $\Delta\Omega = 4\pi\sin^2(\theta_0/2)$) is equal to $2\theta_0$. Consequently, in order for all the photons of the beam in question to be able to take part in the process of stimulated scattering by electrons with maximum velocity v, it suffices to satisfy the condition

$$\Delta\omega/\omega > 2(v/c)\sin(\theta_0/2) = (\Delta\Omega/\pi)^{1/2} v/c. \tag{3.11}$$

If the electrons have a Maxwellian velocity distribution with temperature T_e, then the condition under which almost all the electrons and almost all the beam photons can take part in the stimulated scattering takes the form[18)]

$$\Delta\omega/\omega > [(kT_e/mc^2)\Delta\Omega]^{1/2}. \tag{3.12}$$

From the conditions (3.11) and (3.12) we see, in particular, that the smaller the angular divergence of the beam, the smaller the required nonmonochromaticity. However, as will be shown later, when the beam divergence is decreased, its absorption coefficient due to the effect under consideration also decreases (since the energy transferred to the electron and the stimulated scattering decreases).

We now calculate the absorption coefficient α_C of a light beam in an electron gas, assuming that condition (3.11) is satisfied for an overwhelming majority of the electrons. The rate of change of the energy of such electrons, due to Compton scattering, is determined by the general formula

$$d\varepsilon/dt = \int d\omega_1 \int c\,[1-(v/c)\cos\theta_1]\,N(\omega_1, \mathbf{q}_1)\,d\Omega_1 \int \hbar(\omega_1-\omega_2)\,F\,\{1 + 4(\pi c)^3 N(\omega_2, \mathbf{q}_2)\,\omega_2^{-2}\}\,d\Omega_2, \tag{3.13}$$

where F and $N(\omega, \mathbf{q})$ are determined by formula (3.3)–(3.5). To explain the structure of (3.13), we note that the integral with respect to $d\Omega_1$ contains an expression for the photon flux density relative to the moving electron. We consider first the contribution made to $d\epsilon/dt$ by the induced processes, corresponding to the second term in the curly brackets of the integral with respect to $d\Omega_2$. The calculations are greatly facilitated by the nonrelativistic character of the motion of electrons and by the satisfaction of condition (3.11). The first makes it possible to write, in analogy with (3.10)

$$F/\omega_2^2 = (r_0^2/2\omega_1^2)\{1+\cos^2\theta - 2(v/c)[\cos\theta(1-\cos\theta)(\cos\theta_1+\cos\theta_2) - (1+\cos^2\theta)\cos\theta_1] + \ldots\}. \tag{3.14}$$

On the other hand, on the basis of (3.11), assuming $\Delta\Omega \ll 1$, we have

$$N(\omega_1, \mathbf{q}_1)\,N(\omega_2, \mathbf{q}_2) \approx (\Delta\Omega)^{-2}[N_{\omega_1}^2 - 0{,}5(\omega_1-\omega_2)\,dN_{\omega_1}^2/d\omega_1], \tag{3.15}$$

where

$$N_\omega = I_\omega/c\hbar\omega = \int d\Omega N(\omega, \mathbf{q})\cos\vartheta \approx \Delta\Omega N(\omega, \mathbf{q}_0)$$

is the spectral (and volume) density of the photons in the beam (see footnote 16). Substitution of (3.14) and (3.15) in (3.13) yields

$$d\varepsilon/dt = 2\pi^3 r_0^2\hbar c^4(\Delta\Omega)^{-2}\int d\omega\,(N_\omega^2/\omega)\int d\Omega_1\,d\Omega_2\{(\hbar\omega/mc^2)(1-\cos\theta) \times (1+\cos^2\theta) + (v/c)(\cos\theta_1-\cos\theta_2)(1+\cos^2\theta) + (v^2/c^2)(\cos^2\theta_1-\cos^2\theta_2)[1+\cos^2\theta-\cos\theta(1-\cos\theta)]\},$$

where the integration with respect to $d\Omega_1$ and $d\Omega_2$ extends over the width of the solid angle of the light beam. Obviously, upon integration with respect to these variables the terms $\sim v/c$ and $(v/c)^2$ vanish, and the first term $(\sim\hbar\omega/mc^2)$ yields

$$\int d\Omega_1\,d\Omega_2\,(1-\cos\theta)(1+\cos^2\theta) \approx (\Delta\Omega)^3/\pi.$$

Thus, we get finally (first derived in[35])

$$d\varepsilon/dt = 2\pi^2 r_0^2\Delta\Omega I^2/m\omega^2\Delta\omega, \tag{3.16}$$

where $I \approx c\hbar\omega N_\omega\Delta\omega$ is the total intensity of the light beam and ω is the central frequency of its spectrum.

The contribution made to $d\epsilon/dt$ by spontaneous scattering, corresponding to the first term in the curly brackets of (3.13), is calculated analogously, and we obtain

$$(d\varepsilon/dt)_{sp} \approx (8/3)\,r_0^2 I\,[(\hbar\omega/mc^2) - 2(v^2/c^2)]. \tag{3.17}$$

We note that the condition (3.11) is not used in the derivation of this formula, so that the latter is valid for all nonrelativistic electrons. For the change in the average energy $\langle\epsilon\rangle$ of the plasma electrons we get on the basis of (3.17)

$$(d\langle\varepsilon\rangle/dt)_{sp} = (8/3)\,r_0^2 I\,(\hbar\omega/mc^2)\,[1 - 4(\langle\varepsilon\rangle/\hbar\omega)].$$

We see therefore that heating of the electrons by the spontaneous Compton effect occurs only if $\hbar\omega > 4\langle\epsilon\rangle$. When the inverse condition is satisfied, the electrons are cooled, and this is sometimes called the inverse Compton effect. The reason for such cooling is obviously that in spontaneous scattering by fast electrons the scattered quanta turned out to be harder than the incident ones.

We now return to formula (3.16), which determines the rate of change of the electron energy in stimulated scattering. At sufficiently large values of the intensity I, the contribution made to $d\epsilon/dt$ by this process will prevail over the contribution from spontaneous scattering[19)]. It is very important that when only the condi-

[18)] For laser radiation with $\Delta\omega/\omega \approx 10^{-3}$ and angular divergence $\Delta\Omega \approx 10^{-4}$ sr, the analysis presented here is suitable, according to (3.12), up to temperatures $T_e \sim 10^6 - 10^7$ deg. After this paper was sent to press, there appeared other papers [38] also dealing with a case that is significant for $\Delta\Omega \gtrsim 1$ (when condition (3.12) is not satisfied). In the same reference, an attempt was made to take into account the influence of the collective interactions in the plasma (the so called dynamic polarization), which are important in a dense plasma, i.e., at $\omega \sim \omega_p$.

[19)] The condition for this, obviously, takes the following form (it is assumed that $\hbar\omega \gg 4\langle\epsilon\rangle$): $I > (4/3\pi^2)\,\hbar\omega^3\,\Delta\omega/c^2\Delta\Omega$. For radiation with tion (3.11) is satisfied, the rate of change of the energy $d\epsilon/dt$, as seen from (3.16), does not depend on the energy itself and is always positive. This means, in particular, that the stimulated Compton effect leads to equal heating of all (nonrelativistic) electrons satisfying the condition (3.11)[20)]. The effective coefficient of (Compton) absorption α_C, due to the considered effect, in a plasma with arbitrary electron velocity distribution, subject only to the condition (3.11) (or (3.12)) (in particular, in an electron beam) is given by

$$\alpha_C = (N_e/I)\, d\varepsilon/dt = 2\pi^2 (N_e r_0^2 \Delta\Omega / m\omega^2 \Delta\omega)\, I. \qquad (3.18)$$

It is of interest to compare the contributions made to the heating of the plasma electrons by the bremsstrahlung effect and by the stimulated Compton effect. As follows from the analysis in the preceding chapter, the bremsstrahlung absorption coefficient is $\alpha_T \sim N_e^2$, and is independent of the intensity I when $I \ll (kT_e/mc^2) I_{rel}$, decreasing like $I^{-3/2}$ when $I > (kT_e/mc^2) I_{rel}$. It is seen in turn from (3.18) that the coefficient $\alpha_C \sim N_e$ and it increases linearly with I at $I \ll I_{rel}$. This means that in a sufficiently rarefied plasma a light beam with sufficiently high intensity is absorbed principally as a result of stimulated Compton-effect processes. This question was investigated in greater detail in[31a]. It was shown that the Compton absorption mechanism prevails over the bremsstrahlung mechanism at $I > I_C$, where

$$I_c \sim [(\omega_p/\omega)^2 (\Delta\omega\ \omega^{-1}/\Delta\Omega)\, r_0/\lambda]^{2/5} I_{rel}\,;$$

λ is the wavelength of the radiation, and $\omega_p = (4\pi e^2 N_e/m)^{1/2}$ is the plasma frequency. At $\omega_p/\omega \sim 10^{-1}$, $(\Delta\omega/\omega\Delta\Omega) \sim 1$ and $\lambda \approx 1$, the threshold intensity is $I_C \sim 10^{-4} I_{rel}$.

In two-photon Compton scattering, the only one which we are now considering, the number of photons is conserved, and consequently the absorption of the light beam by electrons, due to stimulated Compton effect, should always be accompanied by a "red" shift of the spectral distribution of the beam. The presence of such a shift can serve as the basis for an experimental procedure of observing stimulated Compton scattering. The magnitude of the shift can be easily estimated by assuming, as before, that the angular width of the beam is $\Delta\Omega \ll 1$ and that condition (3.11) is fulfilled for almost all the electrons. When such a beam passes through a plasma layer of thickness L, each of its spectral components shifts (without changing intensity) by an amount (see[35])

$$\delta\omega = 2\pi^2 r_0^2 N_e c\hbar \Delta\Omega L m^{-1} N_\omega (L),$$

where the spectral (and volume) density of the photons $N_\omega(L)$ pertains to the radiation emerging from the plasma layer. From the expression for $\delta\omega$ we see that the largest shift is experienced by the spectral components having maximum intensity. This should cause the shape of the spectrum to become sharper on the "red" side and flatter on the "blue" side. In the case of small absorption of the radiation in the layer, the total width $\Delta\omega$ of the spectrum can be assumed unchanged, and we obtain for the shift of its central frequency

$$\delta\omega_0/\omega = (2\pi^2 r_0^2 N_e L \Delta\Omega / m\omega^2 \Delta\omega)\, I = \alpha_C L.$$

This formula is convenient for the interpretation of the experimental data. The first observations of such a spectral shift were reported in[39].

We have considered above the heating of the electronic component of a plasma by absorption of one light beam that has a sufficiently narrow but finite width of the angular spectrum. A special interest attaches also to the case of heating of electrons by two opposing light beams, in which mutual transfer of the photons from one beam to another takes place (i.e., absorption of a photon from one beam and stimulated emission in the other beam). In this case the angular widths of the beams do not play a fundamental role, and we can assume for simplicity that the heating is produced by two opposing flat quasimonochromatic waves. The rate of heating can be calculated in this case from the general formula (3.13), in which we must put $N(\omega, q) = N_\omega^{(1,2)} \delta(q \pm q_0)$, where the superscripts (1, 2) and the $\pm$ sign pertain respectively to the two beams.[21)] Upon satisfaction of condition (3.11), which in this case takes the form $\Delta\omega/\omega > 2v/c \sim (kT_e/mc^2)^{1/2}$ and is quite stringent for laser radiation, calculation yields (first obtained in[36]

$$d\varepsilon/dt \approx 16\pi^3 r_0^2 I_1 I_2 / m\omega^2 \Delta\omega,$$

where I_1 and I_2 are the total intensities of the beams, whose spectral widths are assumed to be the same.

It is of interest to consider the change in the spectral composition of the opposing beams, due to stimulated scattering. Inasmuch as photons are transferred in this case from one beam to the other, but their total number is conserved, there should be intensification (in terms of the photon density) of one beam at the expense of the other. If we use the stationary kinetic equations obtained in[35], we can in turn obtain the following system of equations for the spectral densities $N_{1\omega}(z)$ and $N_{2\omega}(z)$ of beams propagating opposite to each other along the z axis (the direction of which coincides with the direction of the first beam):

$$\frac{\partial N_{1\omega}}{\partial z} = \frac{16\pi^3 r_0^2 N_e c\hbar}{m} N_{1\omega} \frac{\partial N_{2\omega}}{\partial \omega},$$
$$\frac{\partial N_{2\omega}}{\partial z} = -\frac{16\pi^3 r_0^2 N_e c\hbar}{m} N_{2\omega} \frac{\partial N_{1\omega}}{\partial \omega}.$$

As above, it is assumed here that the condition (3.11) is satisfied, i.e., $\Delta\omega/\omega > 2v/c$.

It is easy to find for this system a solution that describes the passage of the beams through a plasma layer of thickness L, in the case of a sufficiently small absorption of the beams, when $\mu_{1,2} \ll 1$, where

$$\mu_{1,2} = \alpha_{12,21} L\omega/\Delta\omega, \text{ and } \alpha_{12,21} = 16\pi^3 r_0^2 I_{2,1} N_e / m\omega^2 \Delta\omega$$

is the absorption coefficient of the first (second) beam in the presence of the opposing second (first) beam. In this case we can use perturbation theory and represent

wavelength $\lambda = 1\mu$ and divergence $\Delta\Omega = 2 \times 10^{-4}$ sr and $(\Delta\omega/\omega) = 10^{-4}$ we obtain the condition $I > 2.5 \times 10^5$ W/cm².

[20)]In [38] it is shown for the case of isotropic radiation ($\Delta\Omega = 4\pi$) that for a hot plasma, i.e., when a condition inverse to (3.12) is satisfied, $d\epsilon/dt$ decreases like $T^{-3/2}$.

[21)]Formula (3.13) must of course, be symmetrized in this case with respect to the superscripts (1, 2).

the solution in the form

$$N_{1\omega}(z)=N_{1\omega}^{(0)}(0)+N_{1\omega}^{(1)}(z)+N_{1\omega}^{(2)}(z)+\ldots,$$
$$N_{2\omega}(z)=N_{2\omega}^{(0)}(L)+N_{2\omega}^{(1)}(z)+N_{2\omega}^{(2)}(z)+\ldots$$

It is assumed that the plasma occupies a layer $0 \leq z \leq L$ ($N_{1\omega}^{(0)}$ and $N_{2\omega}^{(0)}$ are the spectral densities of the first and second beams as they enter the layer, respectively). The first-order corrections take the form

$$N_{1\omega}^{(1)}(z)=\mu_2 (c\hbar\omega(\Delta\omega)^2 I_1^{-1}N_{1\omega}^{(0)}(dN_{2\omega}^{(0)}/d\omega)\, z/L,$$
$$N_{2\omega}^{(1)}(z) = \mu_1 c\hbar\omega(\Delta\omega)^2 I_2^{-1}N_{2\omega}^{(0)}(dN_{1\omega}^{(0)}/d\omega)(L-z)/L. \qquad (3.19)$$

The functions $N_{1\omega}^{(1)}(z)$ and $N_{2\omega}^{(1)}(z)$ satisfy the obvious relation

$$\int N_{1\omega}^{(1)}(L)\,d\omega=-\int N_{2\omega}^{(1)}(0)\,d\omega, \qquad (3.20)$$

which means that the change (in first order) of the total photon density of the first beam on leaving the layer is equal to the analogous change in the second beam, but with opposite sign. This in turn is a reflection of the circumstance that the total number of photons in the system should be conserved (in all orders in $\mu_{1,2}$). Consequently, generally speaking, amplification (in terms of the photon density) of one of the light beams at the expense of the other takes place already in first order. If, however, the spectral compositions of both beams are the same when they enter the layer, i.e., if

$$N_{1\omega}^{(0)}=AN_{2\omega}^{(0)} \equiv A\varphi(\omega), \quad A=\text{const}, \qquad (3.21)$$

then, as seen from (3.19), the photon density in each beam is conserved in first order (the integrals (3.20) vanish). Changes of the photon density in the individual beams occur only in the second order in $\mu_{1,2}$. It is easy to show that these changes take the form

$$\int N_{1\omega}^{(2)}(L)\,d\omega=-\int N_{2\omega}^{(2)}(0)\,d\omega$$
$$=0{,}25A(1-A)\mu_1\mu_2[c\hbar\omega(\Delta\omega)^2]^2(I_1I_2)^{-1}\int\varphi(\omega)(d\varphi/d\omega)^2\,d\omega.$$

We see therefore that the sign of the change of the photon density in the individual beam depends on the ratio of the total intensities of the beams I_1 and $I_2 = I_1/A$: the beam that is amplified (in terms of the photon density) is always the weaker one (in terms of the intensity) (see the experimental data of[40]).

We have emphasized above that the amplification of one beam at the expense of the other is only in terms of the photon density, and not in terms of the total intensity. It is clear already from the first-order formulas (3.19) that the intensities of both beams should be less when they leave the layer. Indeed, at identical spectral distributions (satisfaction of (3.31)) we have in the first order in $\mu_{1,2}$

$$I_1(L)-I(0)=I_2(0)-I_2(L)$$
$$=A(\mu_1\mu_2)^{1/2}c\hbar\omega(\Delta\omega)^2(I\,I_2)^{-1/2}\int c\hbar\omega\varphi(\omega)(d\varphi/d\omega)\,d\omega<0.$$

An increase of the photon density of a light beam with simultaneous decrease of its total intensity is possible obviously only in the presence of a "red" shift in its spectrum. Such a shift does indeed take place. For identical spectral distributions, in first order in $\mu_{1,2}$, the "red" shift is $(\delta\omega)_{1,2} = \mu_{1,2}\Delta\omega \sim I_{2,1}$, i.e., the weaker beam (in terms of intensity) experiences a larger "red" shift.

c) **Scattering of electrons in a field of an intense standing wave.** The scattering of electrons in a field of standing electromagnetic wave, known as the Kapitza-Dirac effect, is a particular case of stimulated Compton scattering. In the first nonvanishing order in the interaction of the electrons with the field, the scattering by the standing wave is accompanied by simultaneous absorption of a photon with momentum $\hbar\mathbf{k}$ $|\mathbf{k}| = \omega/c$) and stimulated emission of a photon with momentum $-\hbar\mathbf{k}$. In higher orders, scattering with participation of a larger number of photons becomes possible. The energy and momentum conservation laws make the scattering possible only at certain definite angles between the directions of the electron and photon momenta $\mathbf{p}$ and $\hbar\mathbf{k}$. In the nonrelativistic approximation, the only kinematically allowed processes are those conserving the electron energy, i.e., process in which an equal number of photons of the type $\mathbf{k}$ and $-\mathbf{k}$ take part.

Kapitza and Dirac[41] considered two-photon scattering and showed that such a process can be interpreted as diffraction of electrons by a periodic "grating" formed by the standing wave. The probability of the electron reflection increases sharply when the Bragg condition is satisfied. From this point of view, the diffraction maximum of order n corresponds to scattering with absorption of n photons propagating in one direction and emission of n photons in the opposite direction of the momentum. Multiple reflection correspond to processes with absorption and emission of virtual photons with momenta $\pm\hbar\mathbf{k}$.

The experimental observation of the Kapitza-Dirac effect became possible only relatively recently[42-45], thanks to the use of powerful laser sources. In this connection, attention was called again to this phenomenon in a number of theoretical papers[5b,46], and different assumptions were made with respect to the possibilities of observing the Kapitza-Dirac effect and its utilization. A brief review of several aspects of this phenomenon, based on the presently available literature, is given in[47]. To avoid repetition, the emphasis in the present review is on other features of the Kapitza-Dirac effect; in particular, we consider in greater detail the specific features of strong and very strong radiation fields.

The probability of electron reflection can be easily obtained in the lowest order of perturbation theory by using the usual formulas for the differential cross section of Compton scattering[33], if account is taken of the presence of an external field and a transition is made to the induced processes. The result takes the form

$$w = (8\pi^3e^4t/\hbar^2m^2\omega^4c^2)\int I_\omega I_{\omega'(\omega)}\,d\omega,$$
$$\omega' = \varepsilon(\varepsilon-cp_z)/(\varepsilon+cp_z+2\hbar\omega), \qquad (3.22)$$

where w is the total probability of scattering of one electron, t is the interaction time, ϵ is the electron energy, p_z is the projection of its momentum on the propagation direction of the absorbed photons (the z axis), and I_ω is the spectral intensity of each of the plane waves making up the standing wave (see footnote 16). It is assumed that these waves differ little from pure monochromatic ones, so that the function I_ω has a sharp maximum in the vicinity of a certain frequency. The connection between ω' and ω follows from the energy and momentum conservation laws in Compton scattering (formula (3.2)). At low frequencies $\hbar\omega \ll \epsilon$ and at small momenta $|cp| \ll \epsilon$, the $\omega'(\omega)$ relation takes the form

$$\omega' = \omega \{1 - (\hbar\omega/\varepsilon) [1 + (\sin\theta/\sin\theta_0)]\}, \quad (3.23)$$

where $\sin\theta_0 = \lambda_e/\lambda = \hbar\omega/c|p|$, p is the total momentum, $\lambda_e = \hbar/|p|$ is the de Broglie wavelength of the electron, $\sin\theta = p_z/|p|$, θ is the glancing angle, and λ is the wavelength of the light.

Thus, the probability of electron reflection, as expected, increases sharply when the Bragg condition is satisfied ($\theta = -\theta_0$)[22]. The degree of stringency of this condition is determined by the spectral width of the electromagnetic wave. If $\Delta\theta$ is the deviation of the glancing angle from $-\theta_0$, at which the probability w decreases substantially, we get in our case

$$\Delta\theta/\theta_0 \sim (mc^2/\hbar\omega)\,\Delta\omega/\omega \ll 1.$$

At very high monochromaticity of the field and not too long action times, formula (3.22) cannot be used, and the scattering probability is determined in the lowest order of perturbation theory by the expression

$$w = (4\pi^2e^4c^2/\hbar^4\omega^3)I^2[1 + (\sin\theta/\sin\theta_0)]^{-2}\sin^2\{(\hbar\omega^2t/mc^2)[1+(\sin\theta/\sin\theta_0)]\}, \quad (3.24)$$

where $I = \int I_\omega d\omega$ is the total intensity of each of the traveling waves. In this case the reflection probability w also increases sharply when the Bragg condition is approached. The degree of stringency of this condition is determined by the interaction time $\Delta\theta/\theta_0 \sim mc^2/\hbar\omega \cdot \omega t$. The probability at the maximum is

$$w_0 = 4\pi^2e^4I^2t^2/\hbar^2\omega^4m^2c^2. \quad (3.25)$$

The condition for the applicability of formula (3.22) is determined by the inequality $\Delta\omega/\omega \gg 1/\omega t$. Formulas (3.24) and (3.25) are valid for the inverse relation between the spectral width and the interaction time, $\Delta\omega/\omega \ll 1/\omega t$. We shall henceforth confine ourselves just to the last case and describe the electromagnetic field of a standing wave, neglecting both the angular and the frequency spreads.

In both cases (formulas (3.22)–(3.25)), the region of applicability of the lowest order of perturbation theory is limited by the condition that the intensity I and that the duration t of the interaction be small. This follows from the fact that at sufficiently large I and t the probability w becomes larger than unity, indicating that the employed approach is incorrect. An attempt was made in[18b] to get rid of these limitations for the case of nonrelativistic electron energies.

The nonrelativistic character of the electron motion allows us to make certain assumptions that simplify the problem greatly. The time required for a nonrelativistic electron to traverse a distance on the order of the wavelength of light, $\sim\lambda/v$, is much larger than the period of the field oscillations ($\sim\lambda/c$). This allows us to assume that a satisfactory description of the slow motion of the electron (averaged over the fast oscillations) can be obtained by using a time-averaged Hamiltonian of the electron in the external field. Such a procedure is analogous to the method of gauge potentials, used in the classical description of the motion of an electron in an inhomogeneous rapidly alternating field[30b,48]. This approximation was used in[49] to make a rather full investigation of the dependence of the scattering probability w in the lowest order of perturbation theory on different physical characteristics of the electromagnetic field and of the electron beam.

The averaged Hamiltonian of the electron, after changing to dimensionless variables, takes the form

$$\mathscr{H} = \frac{\hbar^2\omega^2}{2mc^2}\left(-\frac{\partial^2}{\partial x^2} + 2q\cos 2x\right), \quad (3.26)$$

where $q = (4\pi e^2c/\hbar^2\omega^4)I$, $x = (\omega/c)z$, and z is the spatial coordinate along the wave vector of the absorbed photons.

Owing to the spatial periodicity of the Hamiltonian (3.26), the time-dependent wave function of an electron situated at the initial instant in a state with a definite momentum value $|p\rangle$ can be written in the form

$$\psi(\tau) = \sum_{n=-\infty}^{+\infty}(-i)^n e^{2inx}F_n(\tau, p)\,e^{-ip^2\tau}|p\rangle; \quad (3.27)$$

Here $\tau = (\hbar\omega^2/2mc^2)t$ is the dimensionless time, p is the projection of the initial electron momentum on the z axis in dimensionless coordinates (referred to the value of the photon momentum $\hbar\omega/c$), and $F_n(\tau, p)$ are the scattering probability amplitudes and satisfy the equations

$$\frac{\partial F_n}{\partial\tau} = i\gamma_nF_n + q(F_{n+1} - F_{n-1}), \quad (3.28)$$

where $\gamma_n = -4n(n + p)$, with initial conditions $F_n(0, p) = \delta_{n,0}$.

From the representation of the wave function $\psi(\tau)$ in the form (3.27) it follows that the electron scattering takes place with a momentum change $\Delta p = -2n$, $n = 0, \pm1, \pm2, \ldots$ The probability amplitudes of scattering and the direction of the n-th diffraction maximum are determined by the functions $F_n(\tau, p)$. The condition $\Delta p = -2n$ at small glancing angle of the incident (θ) and scattered (θ') electrons coincides with the Laue condition

$$\lambda(\theta - \theta')/2 = n\lambda_e. \quad (3.29)$$

At low field intensity, the parameter q is small, and the system (3.28) can be solved by the iteration method. This is equivalent to the employed perturbation theory, and leads in first order to formula (3.24). The condition for the applicability of this result is of the form $q\tau \ll 1$.

In real experimental situations, this condition may not be satisfied. For optical frequencies, the parameter value q = 1 corresponds to an electromagnetic radiation intensity $I \approx 10$ MW/cm^2. At a laser-beam width $d \approx 1$ cm and at an electron energy $mv^2/2 \approx 9$ keV, the interaction time is such that $\tau = \tau = (\hbar\omega/2mc^2)d/v \approx 1$. One can attempt to lift the limitation on the interaction time τ, retaining the weak-field condition $q \ll 1$.

[22] In the paper of Kapitza and Dirac [41], formula (3.22) was derived only if the Bragg condition is strictly satisfied, $\theta = -\theta_0$, $\omega' = \omega$, $\int I^1_\omega\, d\omega \approx I^2/\Delta\omega$, so that $w_0 = 8\pi^3e^4tI^2/\hbar^2m^2\omega^4c^2\Delta\omega$.

In this approximation, the amplitude F_1 is not small if the direction of the initial momentum is close to the direction determined by the first-order Bragg condition. The system (3.28) reduces in this case to a system of two equations for the two quantities F_0 and F_1. This system can be easily solved and leads to the following equation for the scattering probability in the direction of the first diffraction maximum:

$$w = q^2 \sin^2 \tau \cdot (q^2 + \xi^2)^{1/2} (q^2 + \xi^2)^{-1}, \xi = 2 [1 + (\sin \theta / \sin \theta_0)]. \quad (3.30)$$

Formula (3.30) describes the saturation of the electron scattering probability at a large interaction duration $q\tau \gtrsim 1$. Under real conditions, the electron reflection probability averages out because, say, of the spread of the electron velocities. Under strong saturation conditions $q\tau \gg 1$ we have $\overline{W} = 1/2$, i.e., near the Bragg angle approximately half of the electrons are scattered.

Similar results can be obtained also for the probabilities of scattering in the direction of the diffraction maxima of higher orders as the direction of the momentum p approaches the direction determined by the Bragg condition of the corresponding order.

Formula (3.30) can be obtained also by expanding the exact solution in terms of the eigenfunctions of the Hamiltonian (3.26), i.e., in Mathieu functions[18b].

Under the same assumptions, the stationary problem with an adiabatically slow decrease of the potential at infinity was considered in[50]. A solution by perturbation theory again led to formulas (3.22) and (3.24), while the use of the quasiclassical approximation led to formulas of the type (3.30).

Expression (3.30) for the scattering probability w shows that at $q\tau > 1$ the degree of stringency of the Bragg condition is determined by the value of the field $\Delta\theta/\theta_0 \sim q$, and $\Delta\theta$ increases with increasing field, i.e., the dependence of the scattering probability on the direction of the initial momentum becomes smoother.

The system (3.28) enables us to analyze in part the asymptotic behavior in a very strong field $q \gg 1$. In this case the system (3.28) reduces to a system of recurrence relations for the Bessel functions, so that we have ultimately for the functions $F_n(\tau, p)$

$$F_n(\tau, p) \approx J_n(-2q\tau). \quad (3.31)$$

A more rigorous analysis[18b] shows that the necessary condition for the applicability of this result is of the form $q\tau^2 \ll 1$, i.e., formula (3.31) is valid at not too large interaction times.

Thus, with increasing field the degree of stringency of the Bragg condition decreases, and the intensity of the diffraction maxima of higher orders increases. In the limit of a very strong field, the distribution of the scattering probabilities does not depend on the direction of the initial momentum and is determined by (3.31). As a result of the interaction with a field, the initial beam breaks up into a "fan" that is symmetrical with respect to the initial direction, and the directions of the scattered beams are determined by the Laue conditions (3.29). The first reports of experimental observation of the Kapitza-Dirac effect were published by two groups of workers in 1965 [42a,49]. These first investigations, however, turned out to be contradictory, and it was doubted whether the scattering of electrons by a standing wave was observed[49]. The two groups later improved their initial experiments[42b,43]. Finally, a new report of experimental observation of the Kapitza-Dirac effect appeared in 1968[45].

It can apparently be stated on the basis of these investigations that scattering of electrons by a standing wave has indeed been observed. The reflection angle was close to the Bragg angle, but the fraction of the scattered electrons was quite small.

The results of[42b,43,45] do not contradict the predictions of the theory. In all these investigations, the parameters characterizing the electromagnetic field and the electron beam were such that the case $q\tau \ll 1$ was realized, i.e., the conditions for applicability of perturbation theory were satisfied. It would be of great interest to perform an experiment in which the theoretical results could be verified for a strong field. At the present time, however, we do not have any data of this kind.

4. CONCLUSION

In this review we touched upon only two effects of the interaction of optical radiation with electrons—the bremsstrahlung effect and two-photon Compton effect. These, of course, do not exhaust all the features of the interaction between intense radiation and electrons. As already mentioned in Chap. 1, when $I > I_{rel}$ the Compton scattering acquires essentially a multiphoton character, i.e., several photons are absorbed in one single scattering act, and several photons are emitted simultaneously. So far, the only theory developed for the spontaneous multiphoton Compton effect (see the review[5a] and the book[6]) is for the case when s photons are absorbed simultaneously with spontaneous emission of one photon. The theory of stimulated Compton scattering at radiation intensities $I > I_{rel}$ has not been developed at all to date. Yet at such high radiation intensities it is precisely the stimulated scattering processes which should determine the main features of the phenomenon.

In our analysis we made no mention at all of one very interesting aspect of the interaction between intense optical radiation and electrons. We have in mind the possible production of electron-positron pairs in a laser experiment. This question is discussed in part (but quite inadequately) in the literature (see[31c,51-53]). The most realistic possibility of observing such an effect is connected with scattering of electrons having in the wave field a total energy $\epsilon > 3\,mc^2$ by nuclei[1c,31c]. This thereshold condition imposed on the energy can be obtained in a laser experiment, in which there is realized a radiation intensity I several times larger than the intensity I_{rel} (the question of "how many time" is determined not only by the threshold condition of the reaction $\epsilon > 3mc^2$, but also by the polarization of the irradiation itself, since the electron energy ϵ depends on the polarization; see[10b], Sec. 47).

The possibility of "drawing out" electron positron pairs from vacuum by optical radiation was discussed in[1c,51] (see also[52]). Estimates show that this problem still remains far from practical realization, since observation of the effect in this case calls for radiation intensities on the order of 10^{26}–10^{27} W/cm^2.

Without mentioning other possible effects of interaction of intense laser radiation with electrons (and other charged particles), we can state with assurance that the development of laser physics has proceeded to a degree that it has already become an experimental basis for the observation of many new effects of quantum electrodynamics. The first effects in this region will undoubtedly be observed when radiation intensities $I > I_{rel}$ are realized.

Note added in proof. A number of new papers were published recently. A classical analysis of bremsstrahlung in a strong field, analogous to Chap. 2b, is given in [54]. An attempt to calculate the bremsstrahlung absorption coefficient averaged over a Maxwellian distribution, for arbitrary intensity is made in [55] (quantum approach). The experimental investigation of induced Compton scattering, started in [39], is continued in [56]. Induced Compton interaction of Maxwellian electrons with spectrally narrow radiation is considered in [57].

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