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$K_L \to 2\mu$ DECAY

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A brief survey of the possible explanations of $K_L \to 2\mu$ decay is given. Some of these explanations cannot be rejected on the basis of the existing experiments, and special experiments must be set up to test them. If the problem of $K_L \to 2\mu$ decay is not brought about by experimental errors, the most plausible explanation appears to be the existence of a new interaction between kaons and muons.

CONTENTS

1. INTRODUCTION

A series of experiments[1] carried out with the aim of searching for the decay $K_L \to 2\mu$ led to a sensational negative result: the decay was not detected. The most accurate of the experiments[2] gave the following upper bound on the $K_L \to 2\mu$ decay width:

$$\Gamma(K_L \to 2\mu) \leqslant \bar{\Gamma}_L^\mu = 1.8 \cdot 10^{-9} \Gamma_L \quad (90\% \text{ confidence level}) \quad (1)$$

where Γ_L is the full width of the K_L meson. If the theoretical calculations are trusted, this result contradicts another experimental result concerning the decay $K_L \to 2\gamma$[1,3]:

$$\Gamma(K_L \to 2\gamma)/\Gamma_L = (5 \pm 1) \cdot 10^{-4}\,{}^{*}). \quad (2)$$

According to the theory[4],

$$\Gamma(K_L \to 2\mu)/\Gamma(K_L \to 2\gamma) \geqslant 1.2 \cdot 10^{-5}, \quad (3)$$

and consequently there exists a theoretical lower bound

$$\Gamma(K_L \to 2\mu)/\Gamma_L \geqslant (6 \pm 1.2) \cdot 10^{-9}.$$

a) Are the experiments reliable? The experts claim that the experiment[2] is reliable. In many respects it is a record-breaking one. For instance, the rare decay $K_L \to 2\pi$ served as the background for the decay $K_L \to 2\mu$ in this experiment, about a million events of this decay having been recorded. Nevertheless, we cannot consider the experimental result final on the basis of a single experiment, even a very good one. This history of elementary particle physics during the past 15 years forewarns us against this. It suffices to recall the claims concerning the tensor variant in the β decay of He^6, the absence of the decay $\pi \to e\nu$ to an accuracy 10^{-5}, the violation of the $\Delta Q = \Delta S$ rule in K^0 meson decays, the electron spectrum with $\rho = 0$ in the decay of the muon, the charge asymmetry in the decay $\eta \to \pi^+\pi^-\pi^0$, the violation of the equality $\eta_{00} = \eta_{+-}$, and other instances of experiments which seemed reliable but turned out to be incorrect. Therefore new experiments on the search for the decay $K_L \to 2\mu$ are essential.

As to the result (2), it is an average over several experiments[3] which differ in their methods and are in mutual agreement. Nevertheless, it would be highly desirable to determine the $K_L \to 2\gamma$ decay width with much greater accuracy.

In view of the fact that the experiments which we are discussing (especially searches for the decay $K_L \to 2\mu$) require great skill, much effort and a large amount of time, it is absolutely essential to analyze the theoretical side of the problem. (We note that, if both theory[4] and experiment on $K_L \to 2\gamma$ decay are correct, then the probability of obtaining experimentally the result (1) for $K_L \to 2\mu$ decay amounts to (2–3) $\times 10^{-3}$.)

b) Is the theory reliable? In the past year this question has been considered in a series of original works, which we shall discuss in detail below, as well as in reviews[5].

The derivation of the relation (3)—the theoretical lower bound on the $K_L \to 2\mu$ decay probability—seems sufficiently convincing. This is precisely why the result of the experiment[2] is so surprising.

We shall discuss the derivation of the relation (3) in detail in the following section. We point out now only that the basic idea consists in calculating the imaginary part of the amplitude. Allowance for the real part can only increase the value of the decay probability.

The imaginary part arises from $K_L \to n \to 2\mu$ transitions on the mass shell, where n is some state which actually occurs in the decay of the K_L meson. The two-photon intermediate state (Fig. 1) gives the main contribution to the imaginary part. This contri-

*The tables of [1] give the value $(5.6 \pm 0.5) \times 10^{-4}$ for this ratio. According to [3b], the world average values is equal to $(5 \pm 0.5) \times 10^{-4}$. The result $\Gamma(K_L \to 2\gamma)/\Gamma_L = (4.6 \pm 0.9) \times 10^{-4}$ of the ITEP group [3c] was not taken into account in the averaging in the tables of [1].

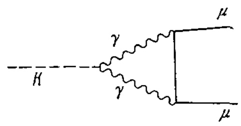

FIG. 1

bution can be calculated in a well-defined way if the $K_L \to 2\gamma$ decay amplitude is known and the transition $2\gamma \to 2\mu$ is described by quantum electrodynamics. The phase space of the other states (e.g., 3π and $2\pi\gamma$) is much smaller. This is why we can suppose that the contribution of the two-photon state is dominant in the imaginary part of the amplitude. The inequality (3) is then valid.

The theoretical foundation of the relation (3) has, as it were, three layers. The lowest layer is the hypothesis that such general principles of physics as the unitarity of the S-matrix and the CPT theorem are valid. On the basis of these principles alone, the imaginary part of the $K_L \to 2\mu$ decay amplitude can be expressed in terms of the transition amplitudes on the mass shell.

The next layer is the hypothesis that there is no "loss" of kaons into unknown decay channels, i.e., that there are no particles of mass less than (or of the order of) the mass of the kaon other than those which are listed in the tables of[1]: γ, ν, e, μ, π and K. Allowance for new decay channels in the unitarity condition, if they existed, could alter the theoretical value of the $K_L \to 2\mu$ decay probability.

Finally, the uppermost layer is the hypothesis that the known particles which are produced in K_L decay have only the usual interactions and that the estimates of the order of magnitude of the imaginary parts associated with the various intermediate states are valid. If, for example, pions interacted strongly with muons, it would not be justified to neglect the contribution of the 3π state in the imaginary part.

c) Plan of the review. Sec. 2 is devoted to a detailed discussion of the derivation of the relation (3). It is clear from the discussion in the preceding subsection that our ideas about elementary particles and their interactions must be changed in some way if the experimental and theoretical values of the $K_L \to 2\mu$ decay probability are really incompatible with each other. In accordance with the three layers of the theoretical foundation of the inequality (3), we consider in Secs. 3–5 these possible changes: 1) new interactions of the known particles (Sec. 3); 2) new particles and, in particular, new light particles (Sec. 4); and 3) violation of the fundamental principles of physics (Sec. 5).

The existence of new particles or interactions would obviously show up not only in the decay $K_L \to 2\mu$ but also in other experiments. One of the main tasks of this brief survey is to consider whether the possibilities 1) to 3) enumerated above are incompatible with the known experimental data.

2. CONSERVATIVE THEORY OF THE DECAY $K_L \to \mu^+\mu^-$

a) The unitarity condition. The bound (3) on the $K_L \to \mu^+\mu^-$ decay probability is derived with the help of the unitarity condition, which enables us to express the imaginary part of the decay amplitude in terms of a product of the matrix elements for the transitions $K_L \to n$ and $n \to 2\mu$ (n is some state into which the K_L meson can actually decay). The specific character of the present case consists in the fact that, to calculate the imaginary part, it is sufficient to use CPT invariance, without requring T invariance as usual. Let us explain this in greater detail.

Introducing the usual relation between the S- and T-matrices

$$S = 1 + iT,$$

the unitarity condition

$$SS^+ = 1$$

can be written in the form

$$T_i^f - (T_f^i)^* = i \sum_n \int d\tau_n \, T_i^n (T_f^n)^*,$$

where T_i^f is the amplitude for the transition from the state i to the state f, the summation is taken over all states n for which a real transition is possible, and τ_n is the phase space of the intermediate state. The asterisk denotes complex conjugation (see Appendix 1 for the normalizations of the amplitudes).

In the case of the decay $K_L \to \mu^+\mu^-$, the initial state is

$$K_L = K_2 + \varepsilon K_1, \tag{4}$$

where K_2 and K_1 have definite CP parity, −1 and +1, respectively. As ϵ is small ($|\epsilon| = 2 \times 10^{-3}$), we shall neglect the term ϵK_1 for the time being.

A muon pair with total orbital angular momentum equal to zero is formed as a result of the decay. The states of this pair ${}^1S_0$ and ${}^3P_0$ have CP parity −1 and +1 and are henceforth denoted by the indices μ_- and μ_+, respectively. If the polarization of the muons is not fixed, these states do not interfere.

Let us consider a transition to one of them. It follows from CPT invariance that

$$T_f^i = T_{\tilde{i}}^{\tilde{f}},$$

where the state $\tilde{i}$ is obtained (to within phase factors) from the state i by CP conjugation. Since the initial and final states are eigenstates of the operator CP in the case in question, the CPT transformation reduces to the T transformation (to within a phase factor) and we have

$$T_f^i = \pm T_i^f. \tag{5}$$

The amplitudes T_i^f can always be defined in such a way (by extracting a factor i) that the unitarity condition has the same form independently of the sign on the right-hand side of (5):

$$\mathrm{Im}\, T_2^{\mu\pm} = \frac{1}{2} \int d\tau_n \, T_2^n (T_{\mu\pm}^n)^*, \tag{6}$$

where the index 2 denotes the state K_2^0.

b) The two-photon imaginary part. As we have already mentioned, the two-photon intermediate state (see Fig. 1) dominates in the unitarity condition (6). The corresponding contribution to the imaginary part is calculated in Appendix 1. Restricting ourselves to this contribution alone and assuming that the CP parity is conserved in the decay $K_2 \to 2\gamma$, we have

$$\Gamma(K_2 \to 2\mu)/\Gamma(K_2 \to 2\gamma) = (\alpha^2/2v)(m_\mu/m_K)^2 \ln^2[(1+v)/(1-v)] = 1{,}2\cdot 10^{-5}, \quad (7)$$

where v is the velocity of the μ meson in the rest system of the kaon ($v \approx 0.9$).

If we assume that CP invariance is violated maximally in the decay $K_2 \to 2\gamma$, then the lower bound on the ratio of the $K_2 \to 2\mu$ and $K_2 \to 2\gamma$ decay probabilities will be the same as for the decay of the K_1 meson with CP conservation[6]:

$$\Gamma(K_1 \to 2\mu)/\Gamma(K_1 \to 2\gamma) = (\alpha^2 v/2)(m_\mu/m_K)^2 \ln^2[(1+v)/(1-v)] = 1\cdot 10^{-5} \quad (8)$$

c) **The other imaginary parts.** In addition to the two-photon contribution, the states $2\pi\gamma$, 3π and $3\pi\gamma$ give a contribution of the same order in the weak and electromagnetic coupling constants. However, their phase space is considerably smaller and we can expect that allowance for these states does not alter the result in an essential way. Some quantitative estimates of the contributions of the various states to the absorptive part of the $K_L \to 2\mu$ decay amplitude are presented in the table.

The following remarks are appropriate in connection with these estimates:

1) **The $2\pi\gamma$ state** (see Fig. 2). The calculation[6] is somewhat less definite here than in the case of the two-photon imaginary part. If the form factor of the pion and the form factors in the decay $K_2 \to 2\pi\gamma$ are neglected, the answer is expressed in terms of the $K_2 \to 2\pi\gamma$ decay probability. Allowance for the form factors can alter the result by about a factor of two[7,8]. Only an upper bound on the $K_L \to 2\pi\gamma$ decay probability is known experimentally at the present time. In the table we give an estimate of the upper bound on the $2\pi\gamma$ contribution from[7]. We note that this estimate is smaller than that in[8] by about an order of magnitude.

2) **The 3π state** (see Fig. 3). The calculation uses model-dependent ideas to a much greater extent. The $3\pi - 2\gamma$ transition amplitude can be found within the framework of current algebra[9,10,18]. The value 10^{-5} presented in the table is obtained[10] on the basis of these calculations and dispersion relations. Simple order-of-magnitude estimates (allowing for the ratio of the 3π and 2γ phase spaces) give the value $\sim 10^{-4}$.

d) **Allowance for the terms $\sim\epsilon$.** So far, we have neglected the term ϵK_1 in formula (4), assuming that

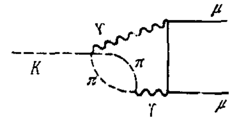

FIG. 2

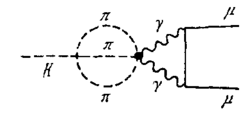

FIG. 3

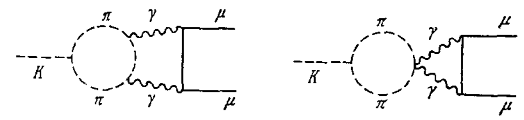

FIG. 4

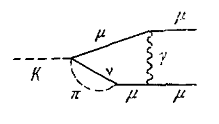

FIG. 5

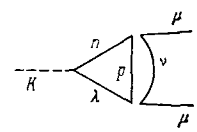

FIG. 6

$K_L = K_2$. We shall now estimate the contribution of this term. Unfortunately, this estimate will be extremely unreliable, owing to the fact that the decays $K_S \to 2\mu$ and $K_S \to 2\gamma$ have not been observed experimentally, so that the $K_1 \to 2\gamma$ and $K_1 \to 2\mu$ transition amplitudes are unknown. We recall that, if CPT is conserved, then

$$K_S = K_1 + \varepsilon K_2.$$

The experimental upper bounds are

$$\Gamma(K_S \to 2\mu) \leqslant \bar{\Gamma}_S^\mu = 7\cdot 10^{-6}\Gamma_S \text{ [1]}$$

(see also formula (14) below) and

$$\Gamma(K_S \to 2\gamma) \leqslant \bar{\Gamma}_S^\gamma = 1{,}2\cdot 10^{-3}\Gamma_S \text{ [35]}. \quad (9)$$

Adopting the conservative hypotheses

$$\Gamma(K_S \to 2\mu) \sim \Gamma_{\text{теор}}(K_L \to 2\mu) \sim 10^{-8}\Gamma_L \sim 10^{-11}\Gamma_S,$$
$$\Gamma(K_S \to 2\gamma) \sim \Gamma(K_L \to 2\gamma) \sim 10^{-6}\Gamma_S,$$

the terms proportional to ϵ give a contribution $\sim 10^{-3}$ to the $K_L \to 2\mu$ amplitude. We note that the phase of this contribution depends not only on the phase of ϵ (experimentally $\epsilon \approx 2\times 10^{-3} e^{i\pi/4}$ but also on the presence of the absorptive part, which depends on the contribution of the real two-pion intermediate state.

e) **Terms of order G^2.** The initial interest in the decay $K_L \to 2\mu$ was connected with the search for neutral currents in the weak interaction. Even if neutral currents do not enter the initial weak interaction Lagrangian, they can, and in general must, arise in second order in the weak interaction constant (Fig.

State n	2γ	$\pi^+\pi^-\gamma$	3π	2π	$\pi\mu\nu$
Im $T(K_L \to n \to 2\mu)$	1	10^{-2} *	$\sim 10^{-5}$ **	$\sim 10^{-3}$ ***	10^{-7} ****

*The result is based on a calculation of the diagram of Fig. 2 in [7]. In [8] the result 5×10^{-2} was obtained, which in our opinion is an overestimate.

**The 3π–2γ block in the (3π–2γ–2μ) amplitude (Fig. 3) is calculated within the framework of current algebra [9,10]. Using the simple dimensional estimate $(3\pi|2\mu) = (\alpha^2/m_K^2)\bar{\mu}\gamma^5\mu\varphi_\pi^3$ for the (3π–2μ) amplitude, the result is about an order of magnitude greater than that given in the table.

***The result is based on a dimensional estimate for the 2π–2μ transition amplitude: $(2\pi|2\mu) = (\alpha^2/m_K)\bar{\mu}\mu\varphi_\pi^2$. Calculations according to perturbation theory are given in [6] (Fig. 4).

****The result is based on a rough estimate of the diagram of Fig. 5.

6). The amplitude of Fig. 6 is quadratically divergent, and according to[11] the result can be written in the form

$$\Gamma (K_L \to 2\mu)/\Gamma (K^+ \to \mu\nu) = G^2 (\Lambda/4\pi)^4 \times \begin{cases} 2 \text{ in the theory with an intermediate W boson,} \\ 32 \text{ in the four-fermion theory;} \end{cases}$$

here $G = 10^{-5}/m_p^2$ and Λ is a cut-off parameter. It should be stressed that, if such general principles as the unitarity of the S-matrix and CPT invariance are valid, then the amplitude corresponding to Fig. 6 is purely real and consequently cannot compensate the two-photon absorptive part (see Fig. 1). An absorptive part in the order G^2 could appear as a result of a real intermediate $\pi\mu\nu$ state (see the diagram in Fig. 5). However, the contribution of this state on the mass shell is several orders of magnitude smaller than the contributions of the $2\pi\gamma$, 3π and 2π states considered above.

f) **Violation of the conservative theory and compensation.** We see that, in comparison with the contribution of the 2γ state, the contribution of all the other intermediate states is small. However, the estimates presented above make use of theoretical values of the coupling constants of kaons, pions, photons and muons. At the present ime, experiment allows significantly larger values of these coupling constants in a number of cases. We can therefore not exclude the possibility that the contribution of some of the channels is much greater than the conservative estimates. We have also already mentioned other possible violations of the theory: "loss" of kaons into hitherto unobserved decay channels of the K_L, and violation of fundamental principles.

A compensation of the two-photon imaginary part is introduced in all the models for violation of the theory which are discussed below. Let us note one trivial numerical fact. To eliminate the discrepancy between theory and experiment, it suffices to compensate not the full imaginary part of the amplitude but 0.45 of its value. If the width of the decay $K_L \to 2\gamma$ is not $5 \times 10^{-4}\Gamma_L$ but $4 \times 10^{-4}\Gamma_L$, which does not contradict a single individual experimental measurement of $\Gamma(K_L \to 2\gamma)$[3], then the fraction of the compensating term is decreased from 0.45 to 0.35.

3. NEW INTERACTIONS?

a) **Anomalous $K_1\bar{\mu}\mu$ interaction.** Christ and Lee[12] (see also[13]) advanced the hypothesis that the $K_L \to 2\mu$ decay probability is suppressed as a result of a small admixture of the K_1 state in the wave function of the K_L meson:

$$K_L = K_2 + \varepsilon K_1.$$

It is clear that two conditions must be satisfied in this case. Firstly, the $K_1 \to 2\mu$ decay amplitude must be much larger than the $K_2 \to 2\mu$ decay amplitude, in order to compensate the small value of ϵ. Secondly, CP invariance must be strongly violated in the decays $K^0 \to 2\mu$. Otherwise, the final states in the decays $K_1 \to 2\mu$ and $K_2 \to 2\mu$ are different, they do not interfere in the total $K_L \to 2\mu$ decay probability, and compensation of the two-photon imaginary part cannot occur.

1) Violation of CP invariance in the decay $K_S \to 2\mu$. Let us first assume that there exists a CP-odd interaction[14]

$$iK_1\bar{\mu}\gamma_5\mu, \tag{10}$$

corresponding to an amplitude $T_1^\mu = i\tilde{T}_1^\mu K_1\bar{\mu}\gamma_5\mu$ having a magnitude such that

$$(m_K v/8\pi)\,|\tilde{T}_2^{\mu-} + \varepsilon\tilde{T}_1^{\mu-}|^2 < \bar{\Gamma}_L^\mu \tag{11}$$

(see Appendix 1, formulas (1.3) and (1.5)).

It is easily seen from formulas (11) and (7) that the inequality

$$(1.2\cdot 10^{-5}\Gamma_L^\gamma)^{1/2} + (\bar{\Gamma}_L^\mu)^{1/2}]^2 \geqslant |\operatorname{Im}\varepsilon|^2\,|\tilde{T}_1^{\mu-}|^2 (m_K v/8\pi) \geqslant [(1.2\cdot 10^{-5}\Gamma_L)^{1/2} - (\bar{\Gamma}_L^\mu)^{1/2}]^2.$$

must be satisfied. This in turn means that the $K_S \to 2\mu$ decay probability must be bounded by

$$1.2\cdot 10^{-5} \geqslant \Gamma (K_S \to 2\mu)/\Gamma_S \geqslant 1\cdot 10^{-6}. \tag{12}$$

We note that the probability of the hypothetical CP-odd decay $K_S \to 2\mu$ turned out to be of the same order of magnitude as the probability of the known CP-odd decay $K_L \to 2\pi$, so that the interaction (10) has a natural order of magnitude. The possible existence of a CP-odd $K_1\bar{\mu}\mu$ interaction of such a magnitude was considered several years ago by Lipman[15a] and by Marshak and his coworkers[15b].

We shall now consider a somewhat more exotic possibility.

2) Violation of CP invariance in the decay $K_L \to 2\gamma$. Let CP be conserved in the decay $K_S \to 2\mu$ (whose amplitude is assumed to be much larger than the amplitude T_L^μ, as in the preceding subsection) but maximally violated in the decay $K_2 \to 2\gamma$. Then the two-photon imaginary part is smaller (see Sec. 2b) and the amplitude $\tilde{T}_1^{\mu+}$ satisfies the inequalities

$$[(1\cdot 10^{-5}\Gamma_L^\gamma)^{1/2} + (\bar{\Gamma}_L^\mu)^{1/2}]^2 \geqslant |\operatorname{Im}\varepsilon|^2\,|T_1^{\mu+}|^2 (m_K v/8\pi) \geqslant [(1\cdot 10^{-5}\Gamma_L^\gamma)^{1/2} - (\bar{\Gamma}_L^\mu)^{1/2}]^2,$$

while the $K_S \to 2\mu$ decay probability is bounded by

$$1.0\cdot 10^{-5} \geqslant \Gamma (K_S \to 2\mu)/\Gamma_S \geqslant 6\cdot 10^{-7}. \tag{13}$$

Professor Kleinknecht has informed one of the authors of a preliminary result obtained at CERN:

$$\Gamma(K_S \to 2\mu) \leqslant \bar{\Gamma}_S^\mu = 1.5\cdot 10^{-6} \text{ (90\% confidence level).} \tag{14}$$

The possibilities (12) and (13) could be closed down by improving this result by a factor of three. However, the uncertainty associated with the error in measuring the $K_L \to 2\gamma$ decay probability should be borne in mind when comparing the theory with experiment. If this probability amounts to 4×10^{-4} of the full decay probability, then the lower bounds in the relations (12) and (13) decrease to 6×10^{-7} and 3×10^{-7}, respectively.

b) **Anomalous $(2\pi)(2\mu)$ interaction**[16]. So far we have assumed that the decay $K_1 \to 2\mu$ is a result of a direct interaction of K_1 mesons and muons and that, by virtue of the hermitian character of the effective Hamiltonian (more precisely, by virtue of unitarity and CPT invariance of the S-matrix), the phase of the $K_1 \to 2\mu$ decay amplitude is fixed. However, it is clear that the $K_2 \to 2\mu$ transition amplitude can be maximally compensated when the matrix element T_2^μ and the product ϵT_1^μ have opposite phases. If the amplitude T_2^μ is purely imaginary, then the lowest limit on $\Gamma(K_S \to 2\mu)$ is attained when the phase of T_1^μ is equal to $\pi/4$. (Recall that $\varphi_\epsilon = \pi/4$.) In this case the lower bounds (12) or (13) can obviously be reduced by a factor of two:

$$\Gamma(K_S \to 2\mu)/\Gamma_S \geqslant 5\cdot 10^{-7}, \qquad (12')$$

if CP parity is violated in the decay $K_S \to 2\mu$, and

$$\Gamma(K_S \to 2\mu)/\Gamma_S \geqslant 3\cdot 10^{-7}, \qquad (13')$$

if CP parity is violated in the decay $K_2 \to 2\gamma$.

A large phase of the $K_S \to 2\mu$ amplitude could be caused by an anomalously strong $(2\pi)(2\mu)$ interaction. If the $K_2 \to 2\gamma$ decay amplitude is CP-even, then the anomalous $(2\pi)(2\mu)$ interaction must violate CP invariance and its effective Lagrangian is of the form

$$c_- m_K^{-1}(\varphi_\pi\varphi_\pi^*)(\bar{\mu}\gamma^5\mu). \qquad (15)$$

If CP parity is not conserved in the decay $K_2 \to 2\gamma$, the anomalous $(2\pi)(2\mu)$ interaction can be CP invariant. In this case its effective Lagrangian is of the form

$$c_+ m_K^{-1}(\varphi_\pi\varphi_\pi^*)(\bar{\mu}\mu). \qquad (16)$$

The latter possibility was considered in[16]. From the requirement that the phase of the amplitude $T_S^{\mu\pm}$, which we denote by $\varphi_S^{\mu\pm}$, be close to $\pi/4$, it is easy to obtain[16a], by using the unitarity condition,

$$c_-^2 = (256\pi^2 \sin^2\varphi_S^{\mu-}/3v_\pi v_\mu)(\Gamma_S^\mu/\Gamma_S) \approx 3\cdot 10^{-4},$$
$$c_+^2 = (256\pi^2 \sin^2\varphi_S^{\mu+}/3v_\pi v_\mu^3)(\Gamma_S^\mu/\Gamma_S) \approx 2\cdot 10^{-4},$$

where we have adopted the corresponding lower bounds (12′) and (13′) for the ratio Γ_S^μ/Γ_S.

The value $c_\pm \sim 10^{-2}$ obtained for the $(2\pi)(2\mu)$ interaction constant is much larger than in perturbation theory[6], according to which this constant is a quantity of second order in α. Nevertheless, the CP-invariant interaction (16) could have failed to show up in the experiments with muons which have been performed so far (measurement of the anomalous magnetic moment of the muon, the cross sections for scattering and production of muons, and the levels of μ-mesonic atoms). As to the CP-noninvariant interaction (15), for a coupling constant $\sim 10^{-2}$ it would give a neutron dipole moment exceeding the present upper bound on d_n by 2 to 4 orders of magnitude. This is a serious argument against the hypothesis that such an interaction exists.

We note that Gaillard[17] obtained a lower limit than (13′): $(K_S \to 2\mu) \geq 1.6\times 10^{-7}\,\Gamma_S$. This limit can be attained if the phase of $T(K_S \to 2\mu)$ is equal to 45°. The anomalous $(2\pi)(2\mu)$ interaction was introduced in[17] in a manner which is not clear. Moreover, in analyzing the decay $K_L \to 2\gamma$ in[17] it was assumed that the modulus of the term ϵT_1^γ in the expression $T_L^\gamma = T_2^\gamma + \epsilon T_1^\gamma$ is equal to its maximal value (9) allowed by experiment and that the phase of this term coincides with the phase of T_L^γ; this requires, in addition, an anomalously strong $(2\pi)(2\gamma)$ interaction. Such a conjunction of four anomalies in the decays $K_L \to 2\gamma$ and $K_S \to 2\gamma$ and in the $(2\pi - 2\mu)$ and $(2\pi - 2\gamma)$ interactions seems to us extremely improbable.

Thus, if one succeeded in demonstrating that $\Gamma_L^\gamma = 5\times 10^{-4}\,\Gamma_L$ and $\Gamma_S^\mu < 3\times 10^{-7}\,\Gamma_S$, then the hypothesis[12] that the amplitude T_1^μ and T_2^μ mutually cancel could be considered to be refuted. We have already discussed above how the limiting value of Γ_S^μ given by (12′) and (13′) decreases as a function of the magnitude of Γ_L^γ. If $L_L^\gamma = 4\times 10^{-4}$, we would have $\Gamma_S^\mu \geq 2.9\times 10^{-7}\,\Gamma_S$ and $\Gamma_S^\mu \geq 1.6\times 10^{-7}\,\Gamma_S$, respectively, instead of (12′) and (13′).

We turn now to the discussion of those mechanisms of compensation of the two-photon absorptive part of the amplitude T_L^μ for which the term ϵT_1^μ is not significant.

c) Anomalous $(3\pi)(2\mu)$ interaction[16c,18]. Can the contribution of the 3π channel to the imaginary part of the $K_L \to 2\mu$ decay amplitude be 3 to 4 orders of magnitude larger than the natural estimates presented in the table? It appears that it cannot be, if this enhancement must be dependent on an anomalously large value of the $3\pi \to 2\gamma$ transition amplitude.

In fact, such an interaction would lead, generally speaking, to a large 3π production cross section in colliding lepton beams[18,19] (Fig. 7), which is incompatible with experiment. The discrepancy can be avoided by assuming[19] that the $(3\pi)(2\gamma)$ interaction is strong for an invariant 3π mass $\leq m_K$ but that its amplitude drops sharply for large mass. However, such a possibility is extremely artificial.

If the enhancement of the contribution of the 3π channel is dependent on a direct $3\pi \to 2\mu$ transition caused by some unknown interaction of muons with hadrons, then no direct inconsistencies with experiment appear. In particular, a rather strong interaction of hadrons with muons is compatible with the data on the energy levels of μ-mesonic atoms, on $g - 2$ of the muon, on elastic scattering of muons by protons, and on inelastic scattering of muons by protons with pion production (see Figs. 8–10, where the small black circle represents the anomalous $(3\pi)(2\mu)$ interaction under consideration). The experiments which are most sensitive to such an anomalous interaction are apparently those on elastic scattering of muons by protons at not very high energies (~1 GeV) and at large angles (~180°) and those which measure $g - 2$. (The diagram of Fig. 8 gives a value close to the experimental upper limit for a cut-off value $\Lambda = 1$ GeV.)

Nevertheless, the explanation of the result (1) by an anomalously strong $(3\pi)(2\mu)$ interaction seems highly implausible for the following reason, noted by M. Zh. Shmatikov. The small value of the imaginary part of the $K_L \to 3\pi \to 2\mu$ transition amplitude is explained by the small phase space of three pions, which is proportional to Q^2, where $Q \approx 70$ MeV is the energy release in the decay $K \to 3\pi$. This suppression is generally absent in the real part of the amplitude.

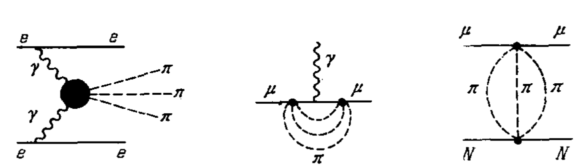

FIG. 7 FIG. 8 FIG. 9

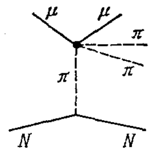

FIG. 10

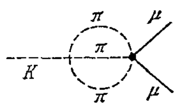

FIG. 11

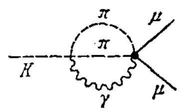

FIG. 12

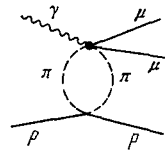

FIG. 13

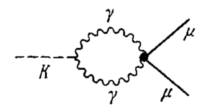

FIG. 14

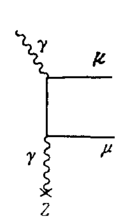

FIG. 15

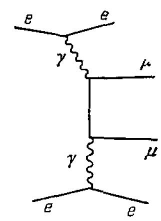

FIG. 16

Hence, if the imaginary part of the amplitude of Fig. 11 is compared with the imaginary part of the amplitude of Fig. 1, the real part corresponding to Fig. 11 will be greater by a factor $(m_X/Q)^2$, where m_X is some hadron mass, while the contribution of the real part to the probability will be larger by a corresponding factor $(m_X/Q)^4$. For example, if $m_X = m_K$, then $(m_X/Q)^4 \approx 2.5 \times 10^3$. The real part of the amplitude for the transition $K_L \to 3\pi \to 2\mu$ must therefore cancel the contributions of the other diagrams to high accuracy. The necessity of such a cancellation seems to us to be a very serious argument against the possibility in question.

d) **Anomalous $(2\pi\gamma)(2\mu)$ interaction**[20]. An anomalous $(2\pi\gamma)(2\mu)$ interaction (Fig. 12) with the coupling constant required for the compensation of the two-photon absorptive part of the amplitude T_L^μ by the contribution of the $2\pi\gamma$ state would lead to an anomalously large cross section for the photoproduction of muon pairs (Fig. 13). The possibility of such an interaction can be excluded on the basis of experimental data.

e) **Anomalous $(2\gamma)(2\mu)$ interaction**[21]. The contribution of the diagram in Fig. 1 can be compensated if one assumes that there is a violation of quantum electrodynamics for the muon as a result of a $(2\gamma)(2\mu)$ interaction (Fig. 14). However, to avoid a contradiction with the data on $g - 2$ and on muon photoproduction, it must be assumed that the anomalous interaction is maximal when the mass of the two muons is of the order of the kaon mass and that it drops sharply if the mass of the two muons either increases or decreases (more precisely, if the photon energy decreases). Such a behavior requires in turn that the effective dimensions of the muon be large and is qualitatively incompatible with the beautiful agreement of the data on $g - 2$ with calculations according to quantum electrodynamics. This possibility can be definitely "closed" by measuring the cross section for muon pair production in the processes represented in Figs. 15 and 16 to within a few percent.

f) **Strong interaction between muons**. A strong interaction between muons, if it existed, could, of course, alter the value of the diagram of Fig. 1. However, it would alter the value of $g - 2$ to just as great an extent, which is inadmissible in view of the great precision with which theory and experiment agree on $g - 2$. It seems to us that the only possibility which is not excluded is the existence of a dimuon resonance with a mass equal to m_K and which is so narrow that its contribution to $g - 2$ becomes tolerably small. In essence, such a resonance would be a new particle, and we shall return to it in the following section.

4. NEW PARTICLES?

a) **What purpose do they serve?** The simplest way in which some unknown particles could reduce the lower theoretical bound on the $K_L \to 2\mu$ decay probability is to contribute to the imaginary (absorptive) part of the $K_2 \to 2\mu$ amplitude. In this case, they must first of all appear in the decay of the K_L meson and consequently must be lighter than the K_L meson and, secondly, they must interact with muons. We note that, the smaller the width of the decay of the K_L meson into these particles, the stronger they must interact with muons in order to give the required value of the absorptive part of the $K_L \to 2\mu$ amplitude.

b) **Fundamental restrictions**. What restrictions must be imposed on these new particles? They must be neutral, since otherwise their photoproduction would be observed. They cannot interact strongly with muons, since this would lead to the wrong value of $g - 2$. The relative probability of decay of the K_L meson into these particles cannot be large. This assertion is undoubtedly correct if these particles decay rapidly and if their decay products are charged. The situation is more uncertain if these particles are "unobserved" (stable or decaying through unobserved channels, e.g., neutral ones).

If there existed a large loss into an unobserved channel, the sum of the partial decay widths of the K_L meson would not be equal to the full width. However, as far as we know, *no experiments have been performed* in which the number of produced and decaying K_L mesons are directly compared. That the loss into unobserved channels cannot exceed ten percent is clear from the following considerations.

The absence of unobserved channels is easily confirmed experimentally for the K_S^0 meson, in which case not only $K_S^0 \to \pi^+\pi^-$ decays but also $K_S^0 \to \pi^0\pi^0$ decays are measured. The K_S^0 lifetime is small, and it is easy to see that the numbers of produced and decaying K_S^0 mesons are identical. (One can judge the number of K_S^0 mesons produced, for example, by selecting those events in which the decay of a Λ hyperon is detected in the reaction $\pi^- p \to K^0\Lambda^0$ or by observing the charge-exchange reaction $K^- p \to K^0 n$.)

Moreover, the so-called "vacuum regeneration" phenomenon, in which the interference of the $\pi^+\pi^-$ decays of the K_L^0 and K_S^0 mesons is observed, permits a direct determination of the parameters $|\eta_{+-}| = |T_{K_L}^{\pi^+\pi^-}/T_{K_S}^{\pi^+\pi^-}|$ and, consequently, the absolute and not the relative magnitude of $\Gamma(K_L \to \pi^+\pi^-)$. On the other hand, the ratio $\Gamma(K_L \to \pi^+\pi^-)/\Gamma(K_L \to \text{all observed channels})$ is known. Thus, it is possible to find the width of all the observed channels and, by comparing it with the inverse lifetime of the K_L^0 meson determined by the exponential decay curve, we can see that these quantities agree to good accuracy.

The small loss of K_L^0 mesons into unobserved

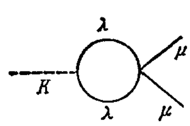

FIG. 17

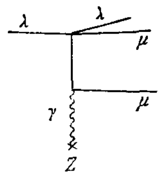

FIG. 18

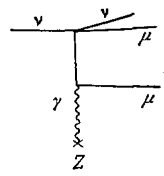

FIG. 19

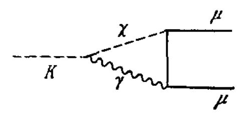

FIG. 20

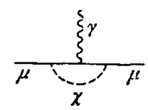

FIG. 21

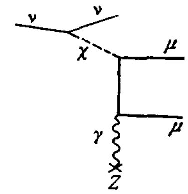

FIG. 22

channels can also be judged indirectly on the basis of the fact that the $\Delta T = 1/2$ rules for leptonic and for nonleptonic decays, which relate the partial widths of the K_L^0 and K^+ mesons, are satisfied to within a few percent and the fact that K^+ mesons have no unobserved decays.

What are the other experiments which could throw light upon the existence of new neutral light particles? To answer this question, let us consider certain concrete models which have been studied in the literature.

c) The λ particle model[22]. In this model one assumes the existence of neutral λ particles of spin $1/2$ and mass less than $0.5m_K$, which contribute to the absorptive part of the $K_L \to 2\mu$ amplitude (Fig. 17). If the λ particles are stable and do not have a strong interaction, they must be penetrating and are similar to the neutrino in this respect. By passing through the shielding, they should lead to observable effects in neutrino experiments, e.g., the production of $\mu^+\mu^-$ pairs (Fig. 18) in spark chambers.

Estimates show[23] that the number of such pairs produced by λ particles under the conditions of the neutrino experiment carried out at CERN would have to be of the order 0.1—1, which is two orders of magnitude greater than the expected number of muon pairs produced by the neutrino (Fig. 19).

We note that the expected number of muon pairs produced by λ particles is uniquely determined by the product of the $K_L \to \lambda\bar{\lambda}$ decay width and the $\lambda Z \to \lambda\mu^+\mu^-Z$ reaction cross section. But just this product is fixed by the value of the imaginary part of the diagram in Fig. 17. From this point of view, an improvement in the accuracy of the neutrino experiment by an order of magnitude would be of interest. Experiments in which the source of neutrinos (and possibly other penetrating particles) is not the charged mesons $\pi^\pm$ and $K^\pm$, as usual, but the neutral ones K^0 and $\bar{K}^0$, would be of special interest.

d) **The χ^0 meson model**[24,25]* In this model one assumes[24] the existence of a neutral vector meson with mass of the order of 350 MeV, which contributes to the absorptive part of the $K_L \to 2\mu$ amplitude owing to the diagram of Fig. 20. An interaction of the χ meson with the muon would give a contribution to the magnetic moment of the muon (Fig. 21). A bound on the constant of this interaction follows from the data on g − 2. This in turn implies the fulfillment of the inequality†.

$$\Gamma (K_L \to \chi^0\gamma)/\Gamma_L > 0.6 \cdot 10^{-2}.$$

*We are grateful to A. G. Dolgolenko, A. G. Meshkovskiĭ and W. A. Shebanov for discussions of the experimental bounds presented in this subsection. The χ meson was also proposed independently by A. N. Moskalev.

†This number differs from that presented in [24] by a factor of two; see [25b].

The existing experiments leave almost no "lifetime space" for the χ^0 meson. The $\chi^0 \to e^+e^-$ decay width is bounded by[25a]

$$\Gamma (K_L \to \chi^0\gamma \to e^+e^-\gamma)/\Gamma_L < 2.7 \cdot 10^{-5}.$$

The $\chi^0 \to \pi^0\gamma$ decay width is bounded by[25a]

$$\Gamma (K_L \to \chi^0\gamma \to \pi^0\gamma\gamma)/\Gamma_L < 1.5 \cdot 10^{-4}.$$

The $\chi^0 \to \mu^+\mu^-$ decay width is apparently bounded by

$$\Gamma (K_L \to \chi^0\gamma \to \mu^+\mu^-\gamma)/\Gamma_L < 4 \cdot 10^{-4}$$

(This last inequality is obtained on the basis of the experimental work[26] on the search for the decay $K_L \to \pi^+\pi^-\gamma$ and may prove to be incorrect if the detection efficiency for muons in this work is less than that for pions).

If the χ^0 meson is stable (for $m_\chi < 2m_\mu$) or decays by unobserved channels[25b], then for $m_\chi \leq 300$ MeV the width $\Gamma(K_L \to \chi^0\gamma)$ is apparently bounded experimentally[27] by

$$\Gamma (K_L \to \chi^0\gamma)/\Gamma_L < 4 \cdot 10^{-4}.$$

The decay $\chi^0 \to 2\mu$ must certainly take place if $m_{\chi^0} > 2m_\mu$. It is possible that this decay is not observed because it constitutes only a small fraction of the χ^0 decays and the main decay channel is unobserved (e.g., $\chi^0 \to \nu\bar{\nu}$). For $m_{\chi^0} \gtrsim 300$ MeV, the experimental search[27] for the decays $K_L \to$ (γ + neutral particles) does not exclude this possibility. However, the $\chi \to \nu\bar{\nu}$ decay width can be bounded by the data of the neutrino experiment, since events of the type in Fig. 22 were not observed.

e) **Pseudokaons and a "narrow" dimuon.** A special class of models is comprised by those in which one assumes that the physical long-lived K^0 mesons (which we shall denote by $\hat{K}_L$) represent a coherent superposition of "ordinary" K_L mesons and some other unknown particle K_L'—a pseudokaon with mass equal to m_K such that

$$\hat{K}_L = K_L + \hat{\epsilon} K_L'.$$

But such a model immediately raises a number of questions. Do K_S' mesons also exist? Do the K_S' and K_L' mesons have strong interactions? What symmetry will ensure the degeneracy of the K′ and K mesons?? And so on. Leaving all these questions unanswered, let us explain how the existence of K′ mesons could alter the theoretical bound on the $K_L \to 2\mu$ decay probability. The idea is the same as in the mechanism of compensation of the K_2 and K_1 decays which was discussed above: The $K_L \to 2\mu$ decay amplitude and the $K_L' \to 2\mu$ decay amplitude, multiplied by $\hat{\epsilon}$, must interfere destructively.

How can such a model be tested? The compensation caused by the destructive interference of K_L and $\hat{\epsilon}K_L'$

will be violated when a $\hat{K}'_L$ beam passes through a plate, if the strong interactions of K_L and K'_L are different. The number of decays into 2μ must therefore rise behind the regenerator. The size of this effect depends on the value of $\hat{\epsilon}$ and consequently on the $K'_L \to 2\mu$ decay probability, whose value we cannot predict.

Certain bounds on the $K'_L\bar{\mu}\gamma^5\mu$ interaction constant and consequently on the $K'_L \to 2\mu$ decay probability can be obtained from the existing experimental data. Thus, the decay probability cannot be very small, since otherwise the K′ mesons, if they do not have strong interactions, would penetrate through the shielding in the neutrino experiment and would produce 2μ decays in the detectors, which have not been observed experimentally. An upper bound on the $K'_L\bar{\mu}\gamma^5\mu$ interaction constant follows from the data on measurements of $g-2$ of the muon and the $\mu Z \to \mu\mu\mu Z$ reaction cross section.

The influence of the regenerator on the $K_L \to 2\mu$ decay probability which was discussed above may be substantial if the $K'_L \to 2\mu$ decay probability is close to its lower limit and is of the order of magnitude of the $K_S \to 2\pi$ decay probability. The effect is extremely small if the $K'_L \to 2\mu$ decay probability is close to its upper limit.

An attempt to explain both the decay $K_L \to 2\mu$ and the decay $K_L \to 2\pi$ (without violation of CP invariance) by introducing pseudokaons is contained in[28].

We conclude the discussion of particles with mass close to the kaon mass by mentioning one further probability. As noted by A. L. Lyubimov, there would be no inconsistency with the decay $K_L \to 2\mu$ if the decay $K_L \to 2\gamma$ had a partial probability much smaller than 5×10^{-4} and if one observed in the experiments[3] the 2γ decays not of the K_L meson but of some other particle with a mass, say, 20 MeV smaller than the K_L mass. The 2μ decays of this particle could then not be detected in the experiment[2] owing to the $K_{\mu 3}$ background in this mass region of the 2μ system. The fact that the same number 5×10^{-4} is obtained in various experiments with different energies of the K^0 mesons constitutes an argument against such a possibility.

5. NEW VIOLATIONS OF PRINCIPLES?

To exhaust all the remaining possible solutions of the problem of the decay $K_L \to 2\mu$, it becomes necessary to reconsider those principles of physics which are the basis of the theoretical inequality (3). First of all, the unitarity of the S-matrix and the CPT theorem will come under suspicion.

a) **Violation of CPT invariance**[29]. It is well known that the imaginary parts of the matrix elements of the processes have two origins. Firstly, there is the presence of real intermediate states which give a non-zero absorptive part. Secondly, there is violation of T invariance. In discussing the imaginary part of the $K_2\bar{\mu}\gamma_5\mu$ matrix element, we have so far considered the first possibility—the various real intermediate states 2γ, 3π, $2\pi\gamma$, $\lambda\bar{\lambda}$, $\chi\gamma$, etc.

Let us now consider the second possibility—violation of T invariance. Let us assume that the $K_2\bar{\mu}\gamma_5\mu$ amplitude has an imaginary part which is dependent on a T-noninvariant interaction with a dimensionless constant $\sim 10^{-12}$, partially compensating the imaginary part which is dependent on the real intermediate state 2γ. Since the CP parities of the K_2 meson and the muon pair in the $^1S_0$ state are identical, CP parity is conserved in the $K_2\bar{\mu}\gamma_5\mu$ amplitude. It follows from this that a violation of T invariance implies a violation of CPT invariance in this case:

$$CP = +1, \quad T = -1, \quad CPT = -1. \tag{17}$$

If the violation of CPT invariance satisfies the conditions (17), it is very complicated to detect it in other effects. (This may not be so if, at the same time, the interaction violates some other selection rules such as $\Delta S < 2$, conservation of muonic charge, etc.) However, if there is also violation of CPT invariance with the selection rules

$$CP = -1, \quad T = +1, \quad CPT = -1, \tag{18}$$

then there must appear non-zero differences between the lifetimes of particles and antiparticles and between the masses of particles and antiparticles. As to the masses, it follows from the accuracy with which the so-called Wu-Yang triangle is verified experimentally that

$$(m_{K^0} - m_{\bar{K}^0})/m_K \leqslant 10^{-18}.$$

This excludes a universal violation of CPT invariance with the selection rules (18) and a dimensionless constant $\sim 10^{-12}$, since a P-even interaction with $\Delta S = 0$ and such a constant would yield

$$(m_{K^0} - m_{\bar{K}^0})/m_K \sim 10^{-12}. \tag{19}$$

As to differences in the lifetimes, they are expected in this case at the level 10^{-4} to 10^{-5}, which is about one or two orders of magnitude smaller than the present upper limit. Since we can regard as established the existence of the CP-noninvariant interaction responsible for the decay $K_L \to 2\pi$, with the selection rules

$$CP = -1, \quad T = -1, \quad CPT = +1, \tag{20}$$

an interaction with the selection rules (17) in conjunction with the interaction with the selection rules (20) must lead to processes having the selection rules (18). To be sure, the dimensionless constant characterizing these processes will then be significantly smaller than 10^{-12}. The value of this constant will be different in different models for the violation of CP and CPT with the selection rules (20) and (17). Thus, for example, if a superweak interaction with a dimensionless constant $\sim 10^{-16}$ is responsible for the violation of CP, then the dimensionless constant characterizing the amplitudes with the selection rules (18) will be of the order 10^{-28}. It will be of the order 10^{-21} if CP is violated in a milliweak interaction with a constant of the order 10^{-9}. The experimentally most accessible CPT-odd effect in this case would apparently be in the neutral kaon system, owing to the well-known enhancement mechanism associated with the small difference between the masses of the K_L and K_S mesons. The point is required in the theoretical analysis below.

Returning to the amplitudes of first order in the CPT- and T-odd interaction with the selection rules

(17) (without the corrections due to the CP- and T-odd interaction), it should be stressed that such an interaction leads to the appearance of an additional phase factor in the amplitudes (an identical one for particle and antiparticle decay) which is not associated with real intermediate states and is not determined by the unitarity condition. Such an interaction does not lead to a difference between the particle and antiparticle decay amplitudes. The search for it is therefore very difficult. In order to demonstrate that it is the violation of CPT invariance which is responsible for the observed anomaly in the case of the decay $K_L \to 2\mu$, it is necessary to experimentally "close" all the other possible explanations. If a CPT-noninvariant interaction contains pseudoscalar currents, it could show up in the decay $K_L \to 2e$ at the level of accuracy which is already attained. However, if this interaction contains not pseudoscalar but axial currents, the sensitivity of experimental searches for the decay $K_L \to 2e$ will have to be increased by 4 to 5 orders of magnitude.

We considered above the violation of CPT invariance within the framework of the S-matrix formalism in a purely phenomenological way. A violation of CPT invariance is possible in quantum field theory only if such fundamental principles as causality, Lorentz invariance and positivity of the energy are violated. The question as to whether these principles can be reconciled with a violation of CPT invariance within the framework of the S-matrix formalism remains open at the present time.

b) **Unitarity of the S-matrix?** As is well known, the condition of unitarity of the S-matrix

$$S^{+}_{ik}S_{km}=\delta_{im} \qquad (21)$$

constitutes a compact mathematical expression of two physical principles: conservation of probability and the superposition principle. Conservation of probability means that the sum of the probabilities of all the transitions from a given initial state to all possible final states is equal to unity. Conservation of probability is described by the diagonal terms of the equality (21), for which i = m:

$$\sum_k S_{ik}S^{*}_{ik}=1. \qquad (22)$$

The nondiagonal terms of (21) are of the form

$$\sum_k S_{ik}S^{*}_{mk}=0, \quad i\neq m, \qquad (23)$$

and express the superposition principle*.

It should be noted that the direct application of the unitarity relation to kaon decay (the transition from the relation (22) to the relation (6)) requires special reservations. The point is that the S-matrix relates stable states to one another, while the kaon is unstable. Strictly speaking, the relation (6) is therefore not exact but approximate. However, the corrections to it are negligibly small, since they are certainly smaller than[30] Γ_{K_L}/m_K.

At the beginning of this review we discussed how the unitarity of the S-matrix in conjunction with CPT invariance leads to a theoretical bound on the $K_L \to 2\mu$ decay probability. We have just seen how this bound may change if we abandon CPT invariance. Unfortunately, we cannot consider violation of the unitarity of the S-matrix at even such a phenomenological level as violation of CPT, since in this case the S-matrix apparatus itself must be altered.

The question as to the need to test the unitarity of the S-matrix and the superposition principle experimentally in connection with the discovery of violation of CP invariance has been discussed in the literature[31].

The best method of testing the unitarity relation is apparently to compare with experiment the well-known Bell-Steinberger relation[32], which constitutes an expanded transcription of the relation (23) in the case of $K_{L,S}$ mesons:

$$[2i(m_L-m_S)+\Gamma_S+\Gamma_L](2\Gamma_S)^{-1}\langle K_S|K_L\rangle=\sum_i B_i\eta_i, \qquad (24)$$

where $m_{L,S}$ are the masses of the $K_{L,S}$ mesons, $B_i = \Gamma(K_S \to i)/\Gamma_S$, and $\eta_i = T^i_{K_L}/T^i_{K_S}$; here $i = \pi^+\pi^-$, $\pi^0\pi^0$, $\pi e\nu$, $\pi\mu\nu$, $\pi^+\pi^-\pi^0$, $3\pi^0$, $2\pi\gamma$, etc. As is well known, all the quantities which enter the relation (24) can be measured experimentally. We note in particular that it would be desirable to measure $\Gamma(K_S \to 3\pi^0)$ or to obtain a good bound on this quantity.

It is also of interest to test the unitarity condition in other decays: β decay of the neutron, $K_{\mu 3}$ decay, nonleptonic decays of hyperons, etc. The phases in these processes are measured to an accuracy in the range 1–10°. This can apparently be regarded as an experimental demonstration that unitarity of the S-matrix for these processes is valid, at least to such an accuracy.

c) **Other principles.** At the present time we see no serious basis for casting doubt upon the other principles of modern physics in connection with the decay $K_L \to 2\mu$, in particular, such principles as Lorentz invariance, conservation of angular momentum, and conservation of energy and momentum, although the accuracy with which these principles are verified in the high-energy region is not great. In our opinion, it would be better to test these fundamental principles more accurately without any reference to the $K_L \to 2\mu$ problem.

However, such a point of view is not universally accepted. Thus, for example, according to a hypothesis of B. A. Arbuzov, nonconservation of momentum is a specific property of the decay $K_L \to 2\mu$ itself. In this case the muons from the decay $K_L \to 2\mu$ have momenta different from those required by the ordinary kinematics and are therefore not detected in the experiment[2].

6. CONCLUSIONS

During the past 20 years, kaons have undoubtedly contributed more to our understanding of the laws of the microworld than any one of the other known elementary particles: the discovery of kaons played a major rule in the introduction of the strangeness quantum number, and the study of kaon decays led to the discovery that C, P and CP invariance are violated.

For all these years, muons have remained one of

*We are grateful to I. Yu. Kobzarev for calling our attention to the fact that the relation (6), interpreted as a relation among experimental quantities, can be violated in theories in which the superposition principle does not hold.

the most profound puzzles. It can even be said that they have become a more and more profound puzzle. We have learnt very much about them during these years, but the question "why is the muon heavier than the electron?" has no answer, as before. If the measurements[2,3] are correct, it is possible that they are a kaonic key to the mystery of the muon. In this case it is highly probable that the 70's will lead us to the unravelling of this mystery.

The brief survey presented above shows that we lack the experimental facts required to exclude a whole series of more or less plausible anomalies in that part of elementary particle physics which is usually considered to be well studied.

We shall give here a short list of the experiments which have been discussed in connection with the $K_L \to 2\mu$ problem.

1) A continuation of searches for the decay $K_L \to 2\mu$ and the measurement of the $K_S \to 2\gamma$ decay probability in several independent experiments.

2) Searches for the decays $K_S \to 2\mu$ with an accuracy up to 10^{-7} and the measurement of the $K_S \to 2\gamma$ decay probability.

3) Searches for anomalous interactions of muons (elastic and inelastic scattering of muons by nucleons, production of muon pairs in hadron and in photon collisions, g − 2, and μ-mesonic atoms).

4) Searches for new light particles in the decays of K_L mesons (by photon decays, by the equilibrium of the produced and decaying K_L mesons, searches for anomalous events in neutrino experiments, and searches for the decay $K_L \to 2\mu$ under various conditions, in particular after a regenerator).

5) A test of CPT invariance and, in particular, a comparison of the lifetimes and partial widths of particles and antiparticles at the level $\sim 10^{-5}$.

6) A careful quantitative test of the Bell-Steinberger relation, which is based on the unitarity of the S-matrix, and a measurement of the so-called T-odd correlations in $K_{\mu 3}$ decay, β decay of the neutron, and the decays of hyperons.

7) A test of the fundamental conservation laws (of energy, momentum, angular momentum, and Lorentz invariance).

We have attempted to analyze above how serious the problem of $K_L \to 2\mu$ decays is and to what extent the proposed solutions of this problem are inconsistent with the existing experimental data. In judging the various hypotheses, however, no only the absence of discrepancies with experiment but also the elegance of these hypotheses is a criterion. Nearly all the possible explanations of the $K_L \to 2\mu$ problem which are discussed in this review seem artificial and unattractive at the present time. From this point of view, it would be natural to expect that the experimental data on either $K_L \to 2\mu$ or on $K_L \to 2\gamma$, or on both decays, will change and come into agreement with the theory. However, it would be of much greater interest if the experimental data did not change and the basis of the theory had to be altered. In this case, the criteria for elegance would also change, as has already happened more than once in the past.

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APPENDICES

1. CONTRIBUTION OF THE TWO-PHOTON STATE TO THE ABSORPTIVE PART OF THE $K_2 \to 2\mu$ DECAY AMPLITUDE

The contribution of the two-photon state to Abs T_2^μ is determined by the graph of Fig. 1 and can be written symbolically as

$$\text{Abs}\, T_2^\mu = \frac{1}{2}\int d\tau \sum_{e_1 e_2} T_2^\gamma (T_\mu^\gamma)^*, \qquad (1.1)$$

where

$$d\tau = (2!)^{-1}\,[d^3k_1/(2\pi)^3\, 2\omega_1)]\,[d^3k_2/(2\pi)^3\, 2\omega_2]\,(2\pi)^4\,\delta^{(4)}\,(\mathscr{P} - k_1 - k_2)$$

is the phase space of the two γ quanta, the sum is taken over the polarizations (e_i), and $\mathscr{P}$ is the kaon momentum.

The $2\gamma - 2\mu$ transition amplitude is described by quantum electrodynamics and is equal to

$$T_\mu^\gamma = e^2\bar{\mu}_1\,\{\hat{e}_1\,[(\hat{p}_1 - \hat{k}_1 + m_\mu)/2p_1k_1]\,\hat{e}_2 + \hat{e}_2\,[(\hat{p}_1 - \hat{k}_2 + m_\mu)/2p_1k_2]\,\hat{e}_1\}\,\mu_2, \qquad (1.2)$$

where p_1 and p_2 are the μ^- and μ^+ momenta, respectively.

The amplitudes T_2^μ and T_2^γ are theoretically unknown and we parametrize them in the most general way, without assuming the conservation of CP invariance.

A $\mu^+\mu^-$ pair produced in the decay of a K_2^0 meson can be in the $^1S_0$ or the $^3P_0$ state by virtue of conservation of angular momentum. Since the CP parity of a fermion-antifermion system is equal to $(-1)^{S+1}$, where S is the total spin of the pair, CP is conserved ($CP(\bar{\mu}\gamma_5\mu) = -1$) in the decay into the state $^1S_0$, while it is violated ($CP(\bar{\mu}\mu) = +1$) in the decay into the state $^3P_0$. In view of this, the amplitude T_2^μ can be represented in the form of a sum of two terms:

$$T_2^\mu = \varphi_{K_2}\bar{\mu}_1\,(\widetilde{T}_2^{\mu -}\gamma_5 + iv^{-1}\widetilde{T}_2^{\mu +})\,\mu_2, \qquad (1.3)$$

where v = 0.9 is the velocity of the muon in the kaon rest system, and the indices "±" indicate the CP parity of the $(\mu^+\mu^-)$ pair. The factor i is extracted in order to express the absorptive part in terms of Im $T_2^{\mu\pm}$ or, in other words, to make the quantities $\widetilde{T}_2^{\mu\pm}$ real in the absence of real intermediate states (i.e., if the decay $K_L \to 2\gamma$ were forbidden). This statement is easily understood by considering that in the absence of real intermediate states the decay amplitude can be regarded as a matrix element of some effective Lagrangian, which must be hermitian. In this case the assertion that $\widetilde{T}_2^{\mu\pm}$ are real follows from the fact that the quantities φ_{K_2}, $\bar{\mu}\gamma^5\mu$ and $i\bar{\mu}\mu$ are antihermitian.

The $K_2 \to 2\gamma$ amplitude can be written in an analogous way as

$$T_2^\gamma = i\varphi_{K_2}\,[\widetilde{T}_2^{\gamma -}\,((k_1k_2)\,(e_1e_2) - (k_1e_2)\,(k_2e_1)) + \widetilde{T}_2^{\gamma +}\,\varepsilon_{\alpha\beta\gamma\delta}e_{1\alpha}e_{2\beta}k_{1\gamma}k_{2\delta}]. \qquad (1.4)$$

The first term in the expression (1.4) corresponds to CP-conserving decay and the second to CP-violating decay.

The amplitudes $\widetilde{T}$ are related to the decay probability as follows:

$$\Gamma(K_2 \to 2\mu) = (m_K v/8\pi)\,[|\widetilde{T}_2^{\mu-}|^2 + |\widetilde{T}_2^{\mu+}|^2], \tag{1.5}$$

$$\Gamma(K_2 \to 2\gamma) = (m_K/64\pi)\,[|\widetilde{T}_2^{\gamma-}|^2 + |\widetilde{T}_2^{\gamma+}|^2]. \tag{1.6}$$

Substituting (1.2), (1.3) and (1.4) in (1.1), we obtain the following relation, which determines the imaginary parts of the quantities $\widetilde{T}_2^{\mu\pm}$:

$$\bar{\mu}_1 [\mathrm{Im}\,\widetilde{T}_2^{\mu-}\gamma_5 + iv^{-1}\,\mathrm{Im}\,\widetilde{T}_2^{\mu+}]\,\mu_2 =$$
$$= ie^2 \int d\tau\, [\widetilde{T}_2^{\gamma-}(g_{\alpha\beta}k_1k_2 - k_{2\alpha}k_{1\beta}) + \widetilde{T}_2^{\gamma+}\varepsilon_{\alpha\beta\gamma\delta}k_{1\gamma}k_{2\delta}]\,\bar{\mu}_1\gamma_\alpha\,[(\hat{p}_1 - \hat{k}_1 + m_\mu)/2p_1k_1]\,\gamma_\beta\mu_2. \tag{1.7}$$

We note that both terms in (1.2) give an identical contribution to $\mathrm{Im}\,\widetilde{T}_2^{\mu\pm}$.

Using now the relations

$$\bar{\mu}_1\gamma_\alpha(\hat{p}_1 - \hat{k}_1 + m_\mu)\gamma_\beta\mu_2 = \bar{\mu}_1(2p_{1\alpha} - \gamma_\alpha\hat{k}_1)\gamma_\beta\mu_2,$$
$$\gamma_\alpha\hat{k}_1\gamma_\beta\varepsilon_{\alpha\beta\gamma\delta}k_{1\gamma}k_{2\delta} = 2i\,(k_1k_2)\,\gamma_5\hat{k}_1 \tag{1.8}$$

and taking into account that the term $\sim p_{1\alpha}$ in (1.8) reduces to zero in the integration over $d\tau$, (1.7) leads to the following two equalities:

$$\mathrm{Im}\,\widetilde{T}_2^{\mu-}\bar{\mu}_1\gamma_5\mu_2 = (e^2/2)\,m_K^2\widetilde{T}_2^{\gamma-}\int d\tau\,(p_1k_1)^{-1}\,\bar{\mu}_1\gamma_5\hat{k}_1\mu_2, \tag{1.9}$$

$$\mathrm{Im}\,\widetilde{T}_2^{\mu+}\bar{\mu}_1\mu_2 = (v/2)\,e^2\widetilde{T}_2^{\gamma+}\int d\tau\,(p_1k_1)^{-1}\,\bar{\mu}_1\,(m_\mu m_K^2 + 2p_1k_1\hat{k}_1)\,\mu_2. \tag{1.10}$$

The integration in (1.9) and (1.10) is performed trivially and we finally obtain

$$\mathrm{Im}\,\widetilde{T}_2^{\mu-}/\widetilde{T}_2^{\gamma-} = v^{-1}\,(\mathrm{Im}\,\widetilde{T}_2^{\mu+}/\widetilde{T}_2^{\gamma+}) = (\alpha m_\mu/4m_K)\,v^{-1}\ln[(1+v)/(1-v)]. \tag{1.11}$$

We implicitly assumed that $\mathrm{Im}\,\widetilde{T}_2^{\gamma} = 0$ in the derivation of this result. Consideration of the unitarity condition for the decay $K \to 2\gamma$ shows that this is actually so, if anomalously strong $(3\pi\,|\,2\gamma)$ and $(2\pi\,|\,2\gamma)$ interactions do not exist.

Substituting (1.11) in (1.5) and using the relation (1.6), we obtain the lower bound on the $K_2 \to \mu^+\mu^-$ decay probability (formulas (7) and (8)).

2. THE UNITARITY CONDITION FOR DECAYS OF THE K_L MESON

According to the arguments given in Sec. 2a, the unitarity condition permits a direct calculation of the imaginary part only for transitions between states with definite CP (or C) parity. Let us therefore first consider the $K_{1,2} \to 2\mu$ transition amplitudes:

$$T(K_1 \to 2\mu) = T_1^{\mu+} + iT_1^{\mu-}, \quad T(K_2 \to 2\mu) = T_2^{\mu-} + iT_2^{\mu+};$$

here the indices "±" indicate the CP parity of the $\bar{\mu}\mu$ pair. The unitarity condition for these amplitudes takes the form

$$\mathrm{Im}\,T_1^{\mu\pm} = (1/2)\sum_n \int d\tau_n\,[T_1^{n\pm}(T_{\mu\pm}^{n\pm})^* \mp T_1^{n\mp}(T_{\mu\pm}^{n\mp})^*], \tag{2.1}$$

$$\mathrm{Im}\,T_2^{\mu\pm} = (1/2)\sum_n \int d\tau_n\,[T_2^{n\pm}(T_{\mu\pm}^{n\pm})^* \pm T_2^{n\mp}(T_{\mu\pm}^{n\mp})^*]; \tag{2.2}$$

here $T_{\mu\pm}^{n\pm}$ is the CP-conserving $(\bar{\mu}\mu)\pm \to n\pm$ transition amplitude, and $T_{\mu\pm}^{n\mp}$ is the CP-violating $(\bar{\mu}\mu)$ $\pm \to n\mp$ transition amplitude.

Considering now that $K_2 = K_L - \epsilon K_1$, we have

$$T_2^{n-} = T_L^{n-} - i\varepsilon T_1^{n-}, \tag{2.3}$$

$$T_2^{n+} = T_L^{n+} + i\varepsilon T_1^{n+}. \tag{2.4}$$

Substituting (2.3) and (2.4) in the relation (2.2), we obtain

$$\mathrm{Im}\,T_L^{\mu\pm} \pm \mathrm{Re}\,(\varepsilon T_1^{\mu\pm}) = \frac{1}{2}\sum_n \int d\tau_n\,\{T_L^{n\pm}(T_{\mu\pm}^{n\pm})^* \pm T_L^{n\mp}(T_{\mu\pm}^{n\mp})^* \pm$$
$$\pm i\varepsilon\,[T_1^{n\pm}(T_{\mu\pm}^{n\pm})^* \mp T_1^{n\mp}(T_{\mu\pm}^{n\mp})^*]\}. \tag{2.5}$$

By virtue of the relation (2.1), the last two terms in (2.5) are equal to $\pm i\epsilon\,\mathrm{Im}\,T_1^{\mu\pm}$. Using this, we finally obtain

$$\mathrm{Im}\,T_L^{\mu\pm} = (1/2)\sum_n \int d\tau_n\,[T_L^{n\pm}(T_{\mu\pm}^{n\pm})^* \pm T_L^{n\mp}(T_{\mu\pm}^{n\mp})^*] \mp (\mathrm{Re}\,\varepsilon)\,(T_S^{\mu\pm})^*$$

(here we have replaced ϵT_1 by ϵT_S). This relation expresses the imaginary part of the $K_L \to 2\mu$ decay amplitude in terms of amplitudes for physical processes and is the basis of the theoretical analysis.

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